

# Two-body charmless decays in QCD Factorisation: status and prospects

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Based on

G. Bell, M. Beneke, X.-Q. Li, TH '09, '15, and w.i.p.

CKM Conference Heidelberg, September 17-21, 2018

- Introduction
- Theory approaches to charmless non-leptonic  $B$ -decays
- Status of two-body charmless decays in QCD Factorisation
- Prospects
- Conclusion

# Introduction to non-leptonic $B$ decays

- Non-leptonic  $B$  decays offer a rich and interesting phenomenology
  - Large data sets from  $B$ -factories, Tevatron, LHCb, in future Belle II
  - $\mathcal{O}(100)$  final states
  - Numerous observables:
    - branching ratios
    - CP asymmetries
    - polarisations
    - Dalitz plot analyses
    - Combinations thereof
- Test of CKM mechanism (CP violation)
- Indirect search for New Physics
  - Not as sensitive as rare or radiative  $B$  decays, but large data sets
- Theoretical description complicated by purely hadronic initial and final state
  - QCD effects from many different scales

## Theory approaches based on factorisation

- Disentangle long and short distances

- QCD Factorisation

[Beneke,Buchalla,Neubert,Sachrajda'99-'01]

- Systematic framework to all orders in  $\alpha_s$  and leading power in  $\Lambda/m_b$
- Problems with factorisation of power suppressed and annihilation contributions. Endpoint divergences.
- Countless pheno applications

[Beneke,Neubert'03; Cheng,Yang'08; Cheng,Chua'09; Bell,Pilipp'09; Beneke,Li,TH'09; Bobeth,Gorbahn,Vickers'14; Bell,Beneke,Li,TH'15; ...]

- PQCD

[Keum,Li,Sanda'00; Lü,Ukai,Yang'00]

- Based on  $k_T$ -factorisation. Organises amplitude differently
- Generates larger strong phases. Avoids endpoint divergences.
- Discussion of theoretical uncertainties difficult since no complete NLO ( $\mathcal{O}(\alpha_s^2)$ ) analysis available
- Also countless pheno applications

[e.g. Ali,Kramer,Li, Lü,Shen,Wang,Wang'07]

## More theory approaches

- Flavour symmetries:

[Zeppenfeld'81]

Isospin, U-Spin ( $d \leftrightarrow s$ ), V-Spin ( $u \leftrightarrow s$ ), Flavour SU(3)

[see e.g. Chiang,Gronau,Rosner'08; Chiang,Zhou'06'08; Cheng,Chiang,Kuo'14'16]

- Only few a priori assumptions about scales needed
- Implementation of symmetry breaking difficult

[Jung,Mannel'09; Cheng,Chiang'12]

- Combination:

- Recently: factorization-assisted topological-amplitude approach (FAT)

[Li,Lü,Yu'12; Li,Lü,Qin,Yu'13; Wang,Zhang,Li,Lü'17]

- Many more analysis

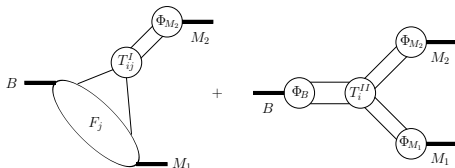
[Descotes-Genon,Matias,Virto'06; Ciuchini,Silvestrini et al.; Datta,London,Imbeault'03,'12]

[Fleischer'99+; Fleischer,Jaarsma,Vos'16-'18; Nandi,Soni'10; ...]

→ See also Jaarsma's talk later this session

## More theory approaches

- Dalitz plot analysis. Applied to 3-body decays
  - Mostly a fit to data, but also QCD-based predictions possible  
[Kränkl,Mannel,Virto'15; Klein,Mannel,Virto,Vos'17]
  - Also Flavour-symmetry analyses  
[e.g. Bhattacharya,Gronau,Imbeault,London,Rosner'14; Bhattacharya,London'15]
  - Important for phenomenology  
→ see talks by Bhattacharya and Magalhães in WG 5 on Tue



- Amplitude in the limit  $m_b \gg \Lambda_{\text{QCD}}$

[Beneke, Buchalla, Neubert, Sachrajda'99-'04]

$$\begin{aligned} \langle M_1 M_2 | Q_i | \bar{B} \rangle &\simeq m_B^2 F_+^{B \rightarrow M_1}(0) f_{M_2} \int_0^1 du T_i^I(u) \phi_{M_2}(u) \\ &+ f_B f_{M_1} f_{M_2} \int_0^1 d\omega dv du T_i^{II}(\omega, v, u) \phi_B(\omega) \phi_{M_1}(v) \phi_{M_2}(u) \end{aligned}$$

- $T^{I,II}$ : Hard scattering kernels, perturbatively calculable
- |  |   |                                       |
|--|---|---------------------------------------|
| <ul style="list-style-type: none"> <li><math>F_+</math>: <math>B \rightarrow M</math> form factor</li> <li><math>f_i</math>: decay constants</li> <li><math>\phi_i</math>: light-cone distribution amplitudes</li> </ul> | } | Universal.<br>From Sum Rules, Lattice |
|--|---|---------------------------------------|
- Strong phases are  $\mathcal{O}(\alpha_s)$  and/or  $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$

# Anatomy of QCD factorisation

$T^I$   
vertex

$T^{II}$   
spectator

tree

penguin

tree

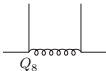
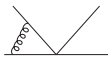
penguin

LO:  $\mathcal{O}(1)$

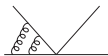


NLO:  $\mathcal{O}(\alpha_s)$

[Beneke, Buchalla, Neubert, Sachrajda '99-'04]



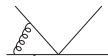
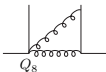
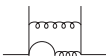
NNLO:  $\mathcal{O}(\alpha_s^2)$



[Bell '07, '09]

[Beneke, Li, TH '09]

[Kränkl, TH in progress]



[Beneke, Jäger '05]

[Kivel '06; Pilipp '07]



[Beneke, Jäger '06]

[Jain, Rothstein,

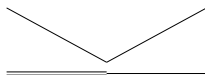
Stewart '07]

[Bell, Beneke, Li, TH in progress]

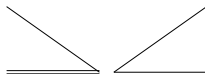


# Classification of amplitudes

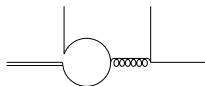
- $\alpha_1$  : colour-allowed tree amplitude



- $\alpha_2$  : colour-suppressed tree amplitude



- $\alpha_4^{u,c}$  : QCD penguin amplitudes



$$\begin{aligned}\sqrt{2} \langle \pi^- \pi^0 | \mathcal{H}_{\text{eff}} | B^- \rangle &= A_{\pi\pi} \lambda_u [\alpha_1(\pi\pi) + \alpha_2(\pi\pi)] \\ \langle \pi^+ \pi^- | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle &= A_{\pi\pi} \{ \lambda_u [\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi)] + \lambda_c \alpha_4^c(\pi\pi) \} \\ - \langle \pi^0 \pi^0 | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle &= A_{\pi\pi} \{ \lambda_u [\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi)] - \lambda_c \alpha_4^c(\pi\pi) \}\end{aligned}$$

$$\langle \pi^- \bar{K}^0 | \mathcal{H}_{\text{eff}} | B^- \rangle = A_{\pi\bar{K}} \left[ \lambda_u^{(s)} \alpha_4^u + \lambda_c^{(s)} \alpha_4^c \right]$$

$$\langle \pi^+ K^- | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle = A_{\pi\bar{K}} \left[ \lambda_u^{(s)} (\alpha_1 + \alpha_4^u) + \lambda_c^{(s)} \alpha_4^c \right]$$

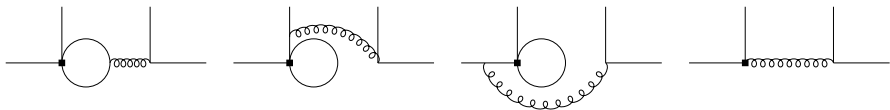
[Beneke, Neubert'03]

- Tree amplitudes  $\alpha_1$  and  $\alpha_2$  known analytically to NNLO

[Bell'07'09; Beneke, Li, TH'09]

# Penguin amplitudes $a_4^U$ and $a_4^C$ to NLO

- NLO:



$$\begin{aligned}\alpha_4^U(\pi\pi) &= -0.029 - [0.002 + 0.001i]_V + [0.003 - 0.013i]_P + [?? + ??i]_{\mathcal{O}(\alpha_s^2)} \\ &\quad + \left[ \frac{r_{sp}}{0.485} \right] \{ [0.001]_{LO} + [0.001 + 0.000i]_{HV+HP} + [0.001]_{tw3} \} \\ &= (-0.024^{+0.004}_{-0.002}) + (-0.012^{+0.003}_{-0.002})i\end{aligned}$$

$$\begin{aligned}\alpha_4^C(\pi\pi) &= -0.029 - [0.002 + 0.001i]_V - [0.001 + 0.007i]_P + [?? + ??i]_{\mathcal{O}(\alpha_s^2)} \\ &\quad + \left[ \frac{r_{sp}}{0.485} \right] \{ [0.001]_{LO} + [0.001 + 0.001i]_{HV+HP} + [0.001]_{tw3} \} \\ &= (-0.028^{+0.005}_{-0.003}) + (-0.006^{+0.003}_{-0.002})i\end{aligned}$$

# Motivation for NNLO

- Direct CP asymmetries start at  $\mathcal{O}(\alpha_s)$ 
  - Large (scale) uncertainties
  - NNLO is only first perturbative correction
  - NNLO is NLO for direct CP asymmetries!
- NLO results for tree amplitudes

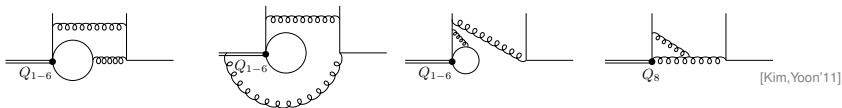
$$\alpha_1(\pi\pi) = 1.009 + [0.023 + 0.010 i]_{\text{NLO}} - \left[ \frac{r_{\text{sp}}}{0.445} \right] \{ [0.014]_{\text{LOsp}} + [0.008]_{\text{tw3}} \} = 1.010 + 0.010i$$

$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077 i]_{\text{NLO}} + \left[ \frac{r_{\text{sp}}}{0.445} \right] \{ [0.114]_{\text{LOsp}} + [0.067]_{\text{tw3}} \} = 0.222 - 0.077i$$

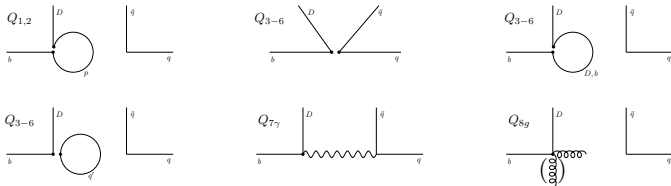
- Large cancellation in LO + NLO in  $\alpha_2$ . Particularly sensitive to NNLO
- Problems with colour-suppressed, tree-dominated decays (e.g.  $\bar{B}^0 \rightarrow \pi^0 \pi^0$ )
  - However: Recent prelim. result by Belle:  $\mathcal{B}(\bar{B}^0 \rightarrow \pi^0 \pi^0) = (0.90 \pm 0.16) \cdot 10^{-6}$
- Does factorisation hold at NNLO?

# Penguin amplitudes at two loops

- $\mathcal{O}(70)$  diagrams at NNLO.

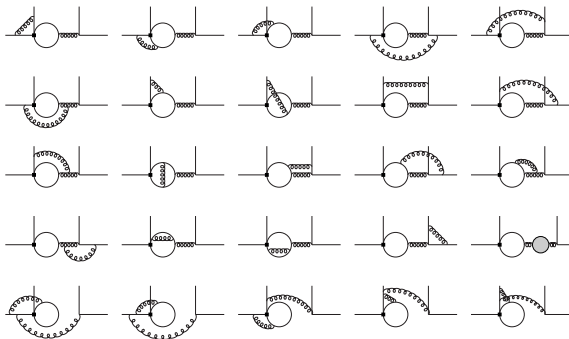


- Quite some book-keeping due to various insertions



# Penguin amplitudes at two loops

- For  $Q_{1,2}^{u,c}$  only a subset of  $\sim 25$  diagrams contributes



- Technical difficulties:
  - Genuine two-loop two-scale problem with threshold
    - Apply state-of-the-art multi-loop techniques
  - Perform matching from QCD onto SCET

# Results: Penguin Amplitudes

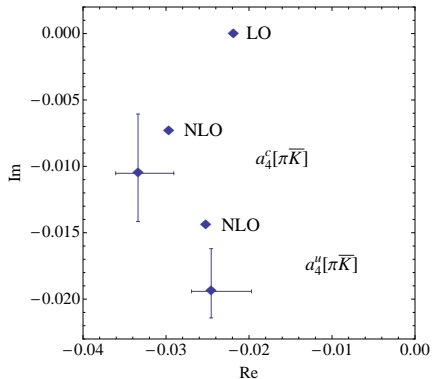
- Only  $Q_{1,2}$  contribution. Inputs from [Beneke,Li,TH'09]

$$\begin{aligned} a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{V_1} + [0.49 - 1.32i]_{P_1} - [0.32 + 0.71i]_{P_2} \\ &\quad + \left[ \frac{r_{sp}}{0.434} \right] \left\{ [0.13]_{LO} + [0.14 + 0.12i]_{HV} - [0.01 - 0.05i]_{HP} + [0.07]_{tw3} \right\} \\ &= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i, \end{aligned}$$

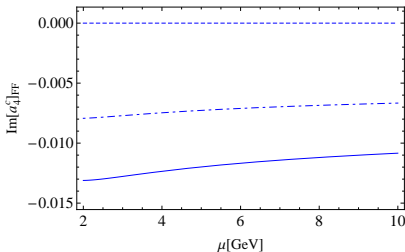
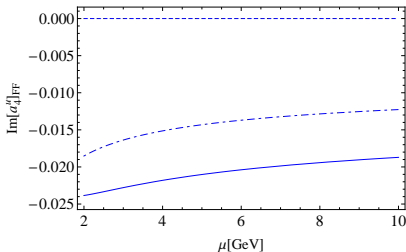
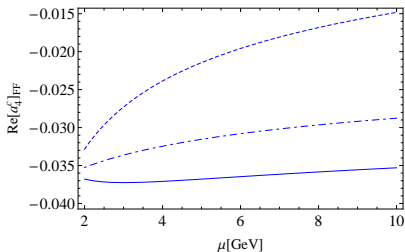
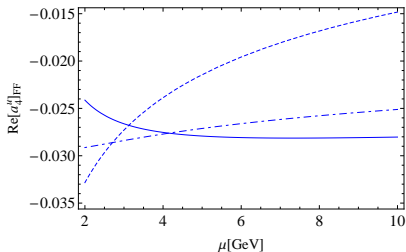
$$\begin{aligned} a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{V_1} + [0.05 - 0.62i]_{P_1} - [0.77 + 0.50i]_{P_2} \\ &\quad + \left[ \frac{r_{sp}}{0.434} \right] \left\{ [0.13]_{LO} + [0.14 + 0.12i]_{HV} + [0.01 + 0.03i]_{HP} + [0.07]_{tw3} \right\} \\ &= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i. \end{aligned}$$

- NNLO correction sizable, but no breakdown of perturbative expansion

# Results: Penguin Amplitudes



# Results: Scale dependence



- Only form factor term, no spectator scattering

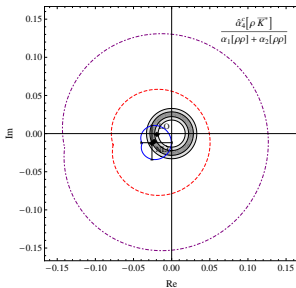
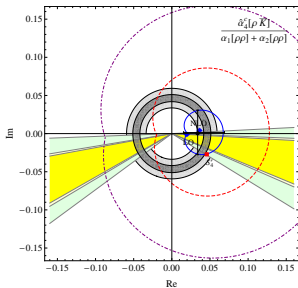
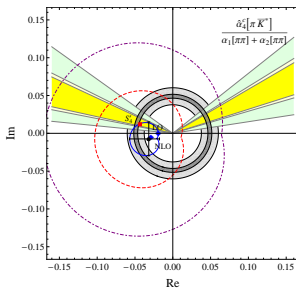
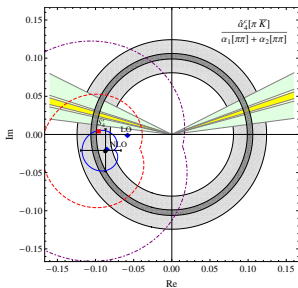


# Results: Amplitude ratios

Ratio	NLO	NNLO
$\frac{P_{\pi\pi}}{T_{\pi\pi}}$	$-0.121 - 0.021i$	$-0.124^{+0.031}_{-0.060} + (-0.026^{+0.045}_{-0.046})i$
$\frac{P_{\rho\rho}}{T_{\rho\rho}}$	$-0.035 - 0.009i$	$-0.041^{+0.020}_{-0.016} + (-0.014^{+0.019}_{-0.018})i$
$\frac{P_{\pi\rho}}{T_{\pi\rho}}$	$-0.038 - 0.005i$	$-0.040^{+0.016}_{-0.030} + (-0.009^{+0.026}_{-0.026})i$
$\frac{P_{\rho\pi}}{T_{\rho\pi}}$	$0.040 + 0.002i$	$0.036^{+0.042}_{-0.023} + (-0.001^{+0.033}_{-0.033})i$
$\frac{C_{\pi\pi}}{T_{\pi\pi}}$	$0.317 - 0.040i$	$0.320^{+0.255}_{-0.142} + (-0.030^{+0.150}_{-0.091})i$
$\frac{C_{\rho\rho}}{T_{\rho\rho}}$	$0.165 - 0.064i$	$0.176^{+0.187}_{-0.133} + (-0.054^{+0.142}_{-0.104})i$
$\frac{C_{\pi\rho}}{T_{\pi\rho}}$	$0.219 - 0.064i$	$0.212^{+0.197}_{-0.112} + (-0.062^{+0.114}_{-0.079})i$
$\frac{C_{\rho\pi}}{T_{\rho\pi}}$	$0.092 - 0.080i$	$0.112^{+0.189}_{-0.144} + (-0.065^{+0.152}_{-0.115})i$
$\frac{T_{\rho\pi}}{T_{\pi\rho}}$	$0.821 + 0.016i$	$0.810^{+0.262}_{-0.200} + (0.010^{+0.062}_{-0.062})i$
$\frac{\alpha_4^C(\pi K)}{\alpha_1(\pi\pi) + \alpha_2(\pi\pi)}$	$-0.085 - 0.019i$	$-0.087^{+0.022}_{-0.036} + (-0.021^{+0.029}_{-0.029})i$
$\frac{\alpha_4^C(\pi K^*)}{\alpha_1(\pi\pi) + \alpha_2(\pi\pi)}$	$-0.029 - 0.005i$	$-0.030^{+0.015}_{-0.026} + (-0.007^{+0.023}_{-0.023})i$
$\frac{\alpha_4^C(\rho K)}{\alpha_1(\rho\rho) + \alpha_2(\rho\rho)}$	$0.037 + 0.004i$	$0.034^{+0.039}_{-0.021} + (0.001^{+0.030}_{-0.030})i$
$\frac{\alpha_4^C(\rho K^*)}{\alpha_1(\rho\rho) + \alpha_2(\rho\rho)}$	$-0.023 - 0.010i$	$-0.027^{+0.027}_{-0.016} + (-0.012^{+0.024}_{-0.023})i$

- Unpublished numbers. Only  $Q_{1,2}$  contribution. Inputs from [Beneke,Li,TH'09].

# Results: Amplitude ratios



# Results: Direct CP asymmetries I

- Direct CP asymmetries in percent.

Errors are CKM and hadronic, respectively.

$f$	NLO	NNLO	NNLO + LD	Exp
$\pi^- \bar{K}^0$	$0.71^{+0.13+0.21}_{-0.14-0.19}$	$0.77^{+0.14+0.23}_{-0.15-0.22}$	$0.10^{+0.02+1.24}_{-0.02-0.27}$	$-1.7 \pm 1.6$
$\pi^0 K^-$	$9.42^{+1.77+1.87}_{-1.76-1.88}$	$10.18^{+1.91+2.03}_{-1.90-2.62}$	$-1.17^{+0.22+20.00}_{-0.22-6.62}$	$4.0 \pm 2.1$
$\pi^+ K^-$	$7.25^{+1.36+2.13}_{-1.36-2.58}$	$8.08^{+1.52+2.52}_{-1.51-2.65}$	$-3.23^{+0.61+19.17}_{-0.61-3.36}$	$-8.2 \pm 0.6$
$\pi^0 \bar{K}^0$	$-4.27^{+0.83+1.48}_{-0.77-2.23}$	$-4.33^{+0.84+3.29}_{-0.78-2.32}$	$-1.41^{+0.27+5.54}_{-0.25-6.10}$	$1 \pm 10$
$\delta(\pi \bar{K})$	$2.17^{+0.40+1.39}_{-0.40-0.74}$	$2.10^{+0.39+1.40}_{-0.39-2.86}$	$2.07^{+0.39+2.76}_{-0.39-4.55}$	$12.2 \pm 2.2$
$\Delta(\pi \bar{K})$	$-1.15^{+0.21+0.55}_{-0.22-0.84}$	$-0.88^{+0.16+1.31}_{-0.17-0.91}$	$-0.48^{+0.09+1.09}_{-0.09-1.15}$	$-14 \pm 11$

$$\delta(\pi \bar{K}) = A_{\text{CP}}(\pi^0 K^-) - A_{\text{CP}}(\pi^+ K^-)$$

$$\Delta(\pi \bar{K}) = A_{\text{CP}}(\pi^+ K^-) + \frac{\Gamma_{\pi^- \bar{K}^0}}{\Gamma_{\pi^+ K^-}} A_{\text{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma_{\pi^0 K^-}}{\Gamma_{\pi^+ K^-}} A_{\text{CP}}(\pi^0 K^-) - \frac{2\Gamma_{\pi^0 \bar{K}^0}}{\Gamma_{\pi^+ K^-}} A_{\text{CP}}(\pi^0 \bar{K}^0)$$

# Results: Direct CP asymmetries II

- Direct CP asymmetries in percent

$f$	NLO	NNLO	NNLO + LD	Exp
$\pi^- \bar{K}^{*0}$	$1.36^{+0.25+0.60}_{-0.26-0.47}$	$1.49^{+0.27+0.69}_{-0.29-0.56}$	$0.27^{+0.05+3.18}_{-0.05-0.67}$	$-3.8 \pm 4.2$
$\pi^0 K^{*-}$	$13.85^{+2.40+5.84}_{-2.70-5.86}$	$18.16^{+3.11+7.79}_{-3.52-10.57}$	$-15.81^{+3.01+69.35}_{-2.83-15.39}$	$-6 \pm 24$
$\pi^+ K^{*-}$	$11.18^{+2.00+9.75}_{-2.15-10.62}$	$19.70^{+3.37+10.54}_{-3.80-11.42}$	$-23.07^{+4.35+86.20}_{-4.05-20.64}$	$-23 \pm 6$
$\pi^0 \bar{K}^{*0}$	$-17.23^{+3.33+7.59}_{-3.00-12.57}$	$-15.11^{+2.93+12.34}_{-2.65-10.64}$	$2.16^{+0.39+17.53}_{-0.42-36.80}$	$-15 \pm 13$
$\delta(\pi \bar{K}^*)$	$2.68^{+0.72+5.44}_{-0.67-4.30}$	$-1.54^{+0.45+4.60}_{-0.58-9.19}$	$7.26^{+1.21+12.78}_{-1.34-20.65}$	$17 \pm 25$
$\Delta(\pi \bar{K}^*)$	$-7.18^{+1.38+3.38}_{-1.28-5.35}$	$-3.45^{+0.67+9.48}_{-0.59-4.95}$	$-1.02^{+0.19+4.32}_{-0.18-7.86}$	$-5 \pm 45$

$$\hat{\alpha}_4^p(M_1 M_2) = a_4^p(M_1 M_2) \pm r_\chi^{M_2} a_6^p(M_1 M_2) + \beta_3^p(M_1 M_2)$$

# Results: Branching ratios

- Unpublished numbers. Only  $Q_{1,2}$  contribution. Inputs from [Beneke,Li,TH'09].
- Branching ratios in  $10^{-6}$ .

	NNLO	NLO	Experiment
$B^- \rightarrow \pi^- \pi^0$	$5.43^{+2.66+2.05+1.27+0.52}_{-2.14-1.73-0.57-0.50}$	5.33	$5.48^{+0.35}_{-0.34}$
$\bar{B}_d^0 \rightarrow \pi^+ \pi^-$	$7.47^{+3.15+3.36+0.30+1.18}_{-2.61-2.76-0.60-0.66}$	7.30	$5.10^{+0.19}_{-0.19}$
$\bar{B}_d^0 \rightarrow \pi^0 \pi^0$	$0.35^{+0.14+0.19+0.33+0.20}_{-0.11-0.11-0.09-0.10}$	0.33	$1.33^{+0.46}_{-0.46}$
$B^- \rightarrow \pi^- \bar{K}^0$	$16.03^{+0.79+9.66+0.87+13.51}_{-0.77-6.68-1.28-5.61}$	14.94	$23.79^{+0.75}_{-0.75}$
$B^- \rightarrow \pi^0 K^-$	$9.57^{+0.79+5.00+0.18+7.15}_{-0.74-3.50-0.39-3.01}$	8.97	$12.94^{+0.52}_{-0.51}$
$\bar{B}_d^0 \rightarrow \pi^+ K^-$	$14.01^{+1.09+8.43+0.12+11.92}_{-1.03-5.76-0.26-4.92}$	12.88	$19.57^{+0.53}_{-0.52}$
$\bar{B}_d^0 \rightarrow \pi^0 \bar{K}^0$	$5.82^{+0.31+4.05+0.07+5.58}_{-0.31-2.72-0.16-2.26}$	5.31	$9.93^{+0.49}_{-0.49}$

- Errors are CKM, scale and inputs (masses, decay constants, FFs), Gegenbauer moments, power corrections

- Perturbative QCD corrections
  - Compute scalar penguin amplitude  $a_6$  to NNLO
    - Power-suppressed, but chirally enhanced contribution
    - Factorises at NLO

$$\hat{\alpha}_4^p(M_1 M_2) = a_4^p(M_1 M_2) \pm r_\chi^{M_2} a_6^p(M_1 M_2) + \beta_3^p(M_1 M_2)$$

$$r_\chi^\pi = \frac{2m_\pi^2}{m_b(\mu) 2m_q(\mu)}, \quad r_\chi^K = \frac{2m_K^2}{m_b(\mu) (m_q + m_s)(\mu)}, \quad m_q = (m_u + m_d)/2$$

- QED corrections
  - Relevant for observables that are sensitive to isospin-violating effects
  - Formulation of factorisation theorem not straightforward
- Make connection to QCDF in three-body decays
  - New nonperturbative input:  $B \rightarrow \pi\pi$  FF,  $2\pi$ LCDA
  - Would benefit from  $\bar{B}^0 \rightarrow D^+(\pi\pi)^-$  data!

- Power corrections

- Were argued to be small based on NLO analysis of  $\bar{B}^0 \rightarrow D^+ \pi^-$  [BBNS'00]
  - NNLO analysis gives hints for natural size ( $\sim 10 - 15\%$ ) [Kränkl,Li,TH'16]
- To date: Parametrisation of the unknown [BBNS'01] or using data [Bobeth,Gorbahn,Vickers'14]
- Optimal solution: Understanding of power corrections on field-theoretic grounds
  - Complicated, but ideas in this direction exist based on the 'collinear anomaly' [Becher,Neubert'10; Chiu,Jain,Neill,Rothstein'12]
- Until then: Attempt to combine Flavour-symmetries with QCDF, e.g.

$$P^{s0} = f P^{d0} \left[ 1 + (A_{KK}^d / P^{d0}) \left\{ \delta\alpha_4^c - \delta\alpha_{4EW}^c / 2 + \delta\beta_3^c + 2\delta\beta_4^c - \delta\beta_{3EW}^c / 2 - \delta\beta_{4EW}^c \right\} \right]$$

[Descotes-Genon,Matias,Virto'06]

- Aim for comprehensive analysis for all channels and observables

# Conclusion and Outlook

- QCDF at leading power and NNLO QCD is established and almost complete
- Yet many improvements possible in the future
- Aim at comprehensive pheno analysis and precise predictions for plethora of observables
  - Hope to be able to disentangle perturbative, parametric and hadronic uncertainties from potential (small) NP effects



# Backup slides

- Topological tree amplitudes at NNLO

[Beneke, Li, TH '09]

$$\begin{aligned}\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010 i]_{\text{NLO}} + [0.026 + 0.028 i]_{\text{NNLO}} \\ &\quad - \left[ \frac{r_{\text{sp}}}{0.445} \right] \{ [0.014]_{\text{LOsp}} + [0.034 + 0.027 i]_{\text{NLOsp}} + [0.008]_{\text{tw3}} \} \\ &= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050}) i\end{aligned}$$

$$r_{\text{sp}} = \frac{9f_{M_1} \hat{f}_B}{m_b \lambda_B f_+^{\text{B}\pi}(0)}$$

$$\begin{aligned}\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077 i]_{\text{NLO}} - [0.031 + 0.050 i]_{\text{NNLO}} \\ &\quad + \left[ \frac{r_{\text{sp}}}{0.445} \right] \{ [0.114]_{\text{LOsp}} + [0.049 + 0.051 i]_{\text{NLOsp}} + [0.067]_{\text{tw3}} \} \\ &= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078}) i\end{aligned}$$

- In agreement with G. Bell

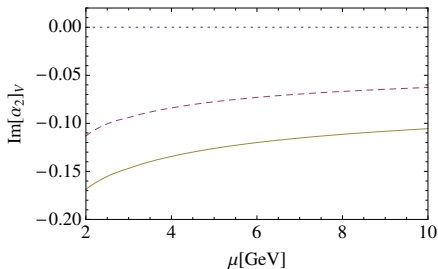
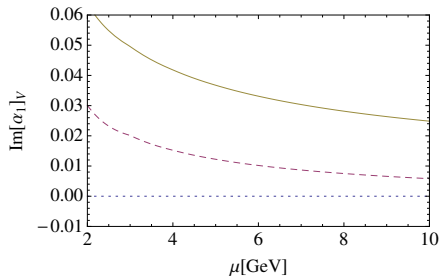
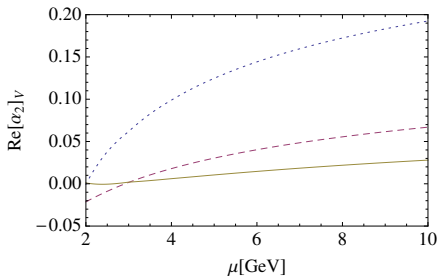
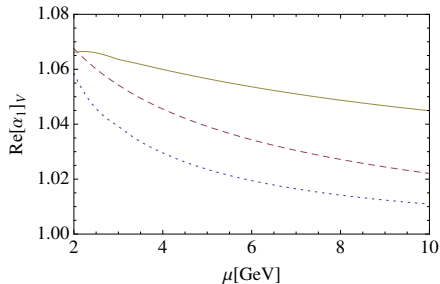
[Bell'09]

- NNLO corrections to vertex and spectator terms significant but tend to cancel! ☹

# Renormalization scale dependence

- Only vertex correction, no spectator scattering

[Beneke,Li,TH '09]



# Some definitions

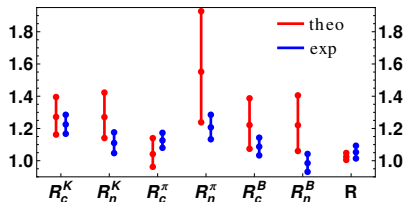
$$A_{\pi\pi} = i \frac{G_F}{\sqrt{2}} m_B^2 F_+^{B \rightarrow \pi}(0) f_\pi$$

$$r_{\text{sp}} = \frac{9 f_\pi \hat{f}_B}{m_b \lambda_B F_+^{B \rightarrow \pi}(0)}$$

$$\lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega, \mu)$$

$$\Delta A_{\text{CP}}^-(\pi K) = A_{\text{CP}}(B^- \rightarrow \pi^0 K^-) - A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^+ K^-) = \Delta A_{\text{CP}}(\pi K)$$

$$\Delta A_{\text{CP}}^0(\pi K) = A_{\text{CP}}(B^- \rightarrow \pi^- \bar{K}^0) - A_{\text{CP}}(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$$



# Results: Direct CP asymmetries III

- Direct CP asymmetries in percent

$f$	NLO	NNLO	NNLO + LD	Exp
$\rho^- \bar{K}^0$	$0.38^{+0.07+0.16}_{-0.07-0.27}$	$0.22^{+0.04+0.19}_{-0.04-0.17}$	$0.30^{+0.06+2.28}_{-0.06-2.39}$	$-12 \pm 17$
$\rho^0 K^-$	$-19.31^{+3.42+13.95}_{-3.61-8.96}$	$-4.17^{+0.75+19.26}_{-0.80-19.52}$	$43.73^{+7.07+44.00}_{-7.62-137.77}$	$37 \pm 11$
$\rho^+ K^-$	$-5.13^{+0.95+6.38}_{-0.97-4.02}$	$1.50^{+0.29+8.69}_{-0.27-10.36}$	$25.93^{+4.43+25.40}_{-4.90-75.63}$	$20 \pm 11$
$\rho^0 \bar{K}^0$	$8.63^{+1.59+2.31}_{-1.65-1.69}$	$8.99^{+1.66+3.60}_{-1.71-7.44}$	$-0.42^{+0.08+19.49}_{-0.08-8.78}$	$6 \pm 20$
$\delta(\rho \bar{K})$	$-14.17^{+2.80+7.98}_{-2.96-5.39}$	$-5.67^{+0.96+10.86}_{-1.01-9.79}$	$17.80^{+3.15+19.51}_{-3.01-62.44}$	$17 \pm 16$
$\Delta(\rho \bar{K})$	$-8.75^{+1.62+4.78}_{-1.66-6.48}$	$-10.84^{+1.98+11.67}_{-2.09-9.09}$	$-2.43^{+0.46+4.60}_{-0.42-19.43}$	$-37 \pm 37$

# More on theory approaches to nonleptonic $B$ -decays

- Perturbative QCD (PQCD) approach based on  $k_T$ -factorisation

[see e.g. Keum,Li,Sanda'01]

- Factorises amplitudes according to

$$A(B \rightarrow M_1 M_2) = \phi_B \otimes H \otimes J \otimes S \otimes \phi_{M_1} \otimes \phi_{M_2}$$

- Generates larger strong phases. Avoids endpoint divergences.
- However: Organises amplitude differently
- Introduces additional infrared prescriptions, e.g. exponentiation of Sudakov logs, phenomenological model for transverse momentum effects
- Discussion of theoretical uncertainties difficult, since no complete NLO ( $\mathcal{O}(\alpha_s^2)$ ) analysis available
- Independent information on hadronic input functions not available

