Extracting $\gamma$ from three-body $B$-meson decays

Bhubanjyoti Bhattacharyya
(bbhattach@ltu.edu)

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Motivation

CKM fitter, ICHEP 2016

- $\alpha, \beta$ quite precise
- Still room in $\gamma$
- Precise $\gamma$ from LHCb: soon?
- B-factory program
  $\rightarrow$ highly successful!
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$\eta$ from CMB & BBN, PDG
- CKM paradigm not enough
- Need new sources of CPV
- Deviations from CKM?
Two-body vs three-body $\gamma$

Two-body (tree-level)

- $B \to DK, B \to D\pi$
  $D \to 2P, 3P, \ldots$
- Interfere: $D^0, \overline{D}^0, D_{\text{CP}}$
- Methods: (J. Brod talk WG6 17/09)
  - GLW (Gronau-London-Wyler)
  - ADS (Atwood-Dunietz-Soni)
  - GGSZ (Giri-Grossman-Sofer-Zupan)
- $n$ parameters, $m$ observables
  - $m \geq n \implies$ solution!
- Theoretically very clean
  - $\delta\gamma \sim \mathcal{O}(10^{-4}) - \mathcal{O}(10^{-7})$
  - Brod + Zupan, 1308.5663
Two-body vs three-body $\gamma$

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Three-body (tree + loop)
- Charmless $B$ decays
  - $B \to PPP \ (P = \pi, K)$
- Observables:
  - $\mathcal{B}, A_{\text{CP}}$ (direct, indirect)
- Find “scenarios” with more observables than parameters
- Need CP eigenstate
  - $\to$ measure indirect CPV
- Use SU(3) symmetry
  - $\to$ diagrammatic approach
  - $\to$ dynamical assumptions
  - $\to$ How clean is it?
Observables and Parameters

Example 3-body process: $B^0 \rightarrow K^0\pi^+\pi^-$  (\(\bar{b} \rightarrow \bar{s}\) transition)

Amplitude $\rightarrow \mathcal{A}(B^0 \rightarrow K^0\pi^+\pi^-) = V_{tb}V_{ts}^*a_t + V_{ub}V_{us}^*a_u$

$$= \tilde{a}_t + e^{-i\gamma}\tilde{a}_u$$

|\(\tilde{a}_t|, |\tilde{a}_u|, \delta\) (relative strong phase) $\rightarrow$ momentum dependent

Momentum dependent observables: need Dalitz amplitude analysis

$$X_{DP} = |A_{sym}|^2 + |\overline{A}_{sym}|^2 \quad CP\ averaged\ branching\ fraction$$

$$Y_{DP} = |A_{sym}|^2 - |\overline{A}_{sym}|^2 \quad Direct\ CP\ asymmetry$$

$$Z_{DP} = \text{Im}(\overline{A}_{sym}A_{sym}) \quad Indirect\ CP\ asymmetry$$

Momentum dependent hadronic parameters

Standard Model Weak Phase: momentum independent!
Hadronic Parameters

- Hadronic parameters are momentum dependent
  
  \[ \tilde{a}_{u,t} \equiv \tilde{a}_{u,t}(p_1, p_2, p_3) = \tilde{a}_{u,t}(s_{12}, s_{13}) \]

- Recent work on momentum dependence
  in context of CP asymmetries in \( B \to 3\pi \)
  K. Keri Vos, et al. 1708.02047

- Disentangle weak and strong phases through resonances
  Charles, et al. 1704.01596

- Need some external hadronic input from QCDf

- This work: treat hadronic parameters as observables
  \[ \rightarrow \] No external hadronic input (such as QCDf) used
  \[ \rightarrow \] Hadronic parameters may be extracted and studied independently
\[ \text{\textbf{\LARGE $\gamma$ from U-spin}} \]

- In two-body decays:
  - $B^0_s \to K^+ K^-$, $B^0_d \to \pi^+ \pi^-$ ($a_d = a_s$)
  - 4 observables: $B_d, B_s, A^d_{\text{CP}}, A^s_{\text{CP}}$; 4 unknowns: $|a_1|, |a_2|, \delta, \gamma$
  - U-spin relation: $A^d_{\text{CP}} B_d \tau_s = -A^s_{\text{CP}} B_s \tau_d$ (1 less observable)
  - Measure indirect $A_{\text{CP}}$ and use $B - \bar{B}$ mixing ($\beta, \beta_s$)
  - Fleisheimer method, hep-ph/9903456
  - Enough info (5 observables, 4 unknowns): $\gamma$ can be extracted
- Test U-spin breaking: check U-spin relation
- 1503.00737 (with D.London) \to $\gamma$ from 3-body U-spin pairs
  - $B^0_s \to K_S K^+ K^-$, $B^0_d \to K_S \pi^+ \pi^-$
  - Need time-dependent Dalitz analysis for indirect CPV
  - Test momentum dependence of U-spin breaking
Binning and averaging

- Local (momentum dependent) observables:
  \[ \Gamma(t) = \frac{\Gamma(B^0 \to f) + \Gamma(\bar{B}^0 \to f)}{2}, \quad A_{\text{CP}}(t) = \frac{\Gamma(B^0 \to f) - \Gamma(\bar{B}^0 \to f)}{\Gamma(B^0 \to f) + \Gamma(\bar{B}^0 \to f)} \]
  \[ |f\rangle \neq |\bar{f}\rangle \quad A_{\text{CP}} \text{ is not true CP Asymmetry} \]

- Time-dependent Dalitz analysis: extract coefficients

- Observables are coefficients of \( \sin, \cos, \sinh, \cosh \) of \( \Delta m, \Delta \Gamma \)
  \[ \rightarrow \text{For fit use redefinitions:} \]
  \[ \int |A|^2 \equiv |A'|^2, \quad \int |\tilde{A}|^2 \equiv |\tilde{A}'|^2, \quad \int \text{Im} \left[ (q/p)A^*\tilde{A} \right] \equiv ? \]
  \[ \rightarrow \text{Approximate:} \quad \int \text{Im} \left[ (q/p)A^*\tilde{A} \right] \approx \text{Im} \left[ (q/p)A'^*\tilde{A}' \right] \]

- U-spin relation in Dalitz analysis: \( a_{\text{CP}}^d b_d \tau_s = -a_{\text{CP}}^s b_s \tau_d \)
  \[ \rightarrow \text{Momentum-dependent relationship} \]
  \[ \rightarrow \text{Unequal U-spin breaking in pockets, but satisfied on average?} \]
Flavor SU(3) diagrammatics

- 2-body $\bar{b} \rightarrow \bar{s}$ transitions: $T', C', P_{tc}', P_{uc}', P_{EW}', P_{EW}''$
- No weak-exchange diagrams (suppressed, dynamical assumption)
- 2 choices for 3-body diagrams (subscripts 1 or 2)

- Three-body topologies: $T_1', T_2', C_1', C_2', \ldots$
- In SU(3) limit: 3 identical particles in final state
- Equivalent to SU(3) matrix element for Fully Symmetric case
Flavor SU(3) diagrammatics

- Flavor SU(3): triplet of $B$ mesons $\rightarrow$ octet of $\pi, K, \eta$
- Final state with 3 identical particles: $S_3$ symmetry
  $\rightarrow$ Fully symmetric (FS), fully antisymmetric (FA)
  $\rightarrow$ Mixed symmetric (MS1, MS2, MS3, MS4)
- FS amplitude stays same when any 2 particles are interchanged
- 1402.2909 (with Gronau, Imbeault, London, Rosner) considered FS
  $\rightarrow$ 9 independent SU(3) matrix elements
  $\rightarrow$ Already includes rescattering effects to all orders in $\alpha_s$
  $\rightarrow$ 16 $b \rightarrow s$ and 16 $b \rightarrow d$ 3-body channels
  $\rightarrow$ Several SU(3) relationships that can be tested

Example: $A(B^+ \rightarrow K^0 \pi^+ \pi^0)_{FS} = - A(B^0 \rightarrow K^+ \pi^0 \pi^-)_{FS}$
γ from 3-body B decays

- SU(3) limit gives relations between tree and electroweak penguins
- 4 effective combinations \((a, b, c, d)\) in the SU(3) limit
- SU(3) FS amplitudes expressed in terms of effective diagrams:

\[
A(B^0 \to K^0 K^0 \bar{K}^0)_{FS} = a \\
\sqrt{2}A(B^0 \to K^+ K^0 K^-)_{FS} = -ce^{i\gamma} - a + \kappa b \\
2A(B^0 \to K^+ \pi^0 \pi^-)_{FS} = be^{i\gamma} - \kappa c \\
\sqrt{2}A(B^0 \to K^0 \pi^+ \pi^-)_{FS} = -de^{i\gamma} - a + \kappa d
\]

- Construct momentum dependent observables \((X, Y, Z)\)
- Amplitude analysis (Example: isobar) to get amplitude at a point
- One Dalitz plot point sufficient for \(\gamma\): average over multiple points
  (more details in the next talk by E. Bertholet)
SU(3) breaking: How to test it?

- Single parameter SU(3) breaking $\rightarrow |f_{SU(3)}|$
  $$A(B^0 \rightarrow K^0 K^0 \overline{K}^0)_{FS} = f_{SU(3)} a$$
  $$\sqrt{2}A(B^0 \rightarrow K^+ K^0 K^-)_{FS} = f_{SU(3)} (-ce^{i\gamma} - a + \kappa b)$$
  $$2A(B^0 \rightarrow K^+ \pi^0 \pi^-)_{FS} = be^{i\gamma} - \kappa c$$
  $$\sqrt{2}A(B^0 \rightarrow K^0 \pi^+ \pi^-)_{FS} = -de^{i\gamma} - a + \kappa d$$
  $$\sqrt{2}A(B^+ \rightarrow K^+ \pi^+ \pi^-)_{FS} = -ce^{i\gamma} - a + \kappa b$$

- Extract $|f_{SU(3)}|$ from a fit; Expect $|f_{SU(3)}| = 1$ for exact SU(3)
- $|f_{SU(3)}|$ should vary widely over allowed Dalitz area
- Average $|f_{SU(3)}| - 1$ may indicate the level of SU(3) breaking
  (Similar to U-spin breaking test by averaging over Dalitz area)
From CKM 2012

Extraction of $\gamma$ from three-body $B$ decays

Bhubanjoyti Bhattacharya

UMontreal

September 30, 2012

7th International Workshop on the CKM Unitarity Triangle,
Cincinnati, Ohio

Work done in collaboration with M. Imbeault and D. London.

Work in progress!

- Completed:
  1303.0846
  PLB 726 (2018) 337

- More work done since
  $\rightarrow$ E. Bertholet’s talk

- More work to come
  fully-antisymmetric amplitudes
Results

- $\gamma$ extracted by applying SU(3) to fully symmetric state
  $\rightarrow$ Full Analysis in 1303.0846 (with Imbeault, London)

$\rightarrow$ SM-like: $77^\circ$
$\rightarrow$ Other solutions: $32^\circ, 259^\circ, 315^\circ$
$\rightarrow$ Discrete Ambiguity

- Key: $\gamma$ extraction by applying SU(3) to other symmetry states
  $\rightarrow$ Break discrete ambiguity: more information from other states

- Also key: estimate systematic uncertainties in $\gamma$ extraction

- Interesting situation: $\gamma$ widely different from SM value
  $\rightarrow$ Significant SU(3) breaking?
  $\rightarrow$ NP in three-body $B$ decays? $K\pi\pi - KK\pi$ puzzle?
Conclusions

- $\gamma$ from tree is theoretically very clean!
- Precise $\gamma$ from LHCb will strongly test CKM unitarity
- $\gamma$ from tree + loop can expose interesting physics
- Flavor symmetries provide effective path to study $\gamma$
- $\gamma$ from U-spin + time-dependent Dalitz analysis
- $\gamma$ from SU(3) + fully-symmetric amplitude analysis
- Discrete ambiguities may be broken using other symmetries
- $\gamma$ tree + loop may present hints of new physics
Thank You!
Back-up Slides
Three-body decays: Dalitz plots

- Three-body final state: \( |P_1(p_1)P_2(p_2)P_3(p_3)\rangle \quad s_{ij} = (p_i + p_j)^2 \rightarrow \) Momentum dependent. One relation \( s_{12} + s_{23} + s_{13} = \text{constant} \)

Features of a Dalitz plot:

- Independent measurements at different points may be possible
- Same SM weak phase (\( \gamma \)); Hadronic parameters are local
- Consistency checks: Flavor symmetries (SU(3), U-spin) provide amplitude relationships
Three-body SU(3) relations

*b* → *s* relationships from isospin and SU(3) symmetry:

\[
\begin{align*}
A(B^+ \to K^0\pi^+\pi^0)_{FS} &= - A(B_d^0 \to K^+\pi^0\pi^-)_{FS} \\
\sqrt{2}A(B_s^0 \to 3\pi^0)_{FS} &= -\sqrt{3}A(B_s^0 \to \pi^0\pi^+\pi^-)_{FS} \\
\sqrt{2}A(B^+ \to K^0\pi^+\pi^0)_{FS} &= A(B_d^0 \to K^0\pi^+\pi^-)_{FS} + \sqrt{2}A(B_d^0 \to K^0\pi^0\pi^0)_{FS} \\
\sqrt{2}A(B_d^0 \to K^+\pi^0\pi^-)_{FS} &= A(B^+ \to K^+\pi^+\pi^-)_{FS} + \sqrt{2}A(B_d^0 \to K^0\pi^0\pi^0)_{FS} \\
A(B^+ \to K^+K^+K^-)_{FS} &= \sqrt{2}A(B^+ \to K^+K^0\overline{K}^0)_{FS} \\
&= \sqrt{2}A(B_d^0 \to K^0K^+K^-)_{FS} + A(B_d^0 \to K^0K^0\overline{K}^0)_{FS} \\
A(B_s^0 \to \pi^0K^+K^-)_{FS} &= \sqrt{2}A(B_s^0 \to \pi^0K^0\overline{K}^0)_{FS} \\
&= -\sqrt{2}A(B_s^0 \to \pi^-K^+\overline{K}^0)_{FS} - A(B_s^0 \to \pi^+K^-K^0)_{FS} \\
\sqrt{2}A(B^+ \to K^+\pi^+\pi^-)_{FS} &= A(B^+ \to K^+K^+K^-)_{FS}
\end{align*}
\]

7 relations involving 16 amplitudes that depend on 9 matrix elements

Similar situation also in *b* → *d* FS amplitudes