Extracting γ from three-body B-meson decays

Bhubanjyoti Bhattacharya
(bbhattach@ltu.edu)

Lawrence Tech
THEORY AND PRACTICE 1932

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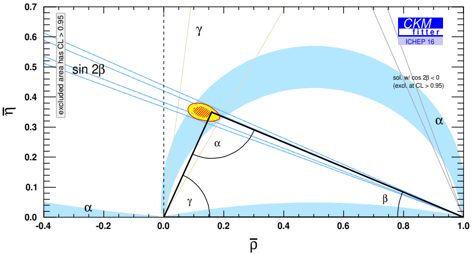
Motivation



CKM fitter, ICHEP 2016

- α, β quite precise
- Still room in γ
- Precise γ from LHCb: soon?
- B-factory program
  $\rightarrow$ highly successful!

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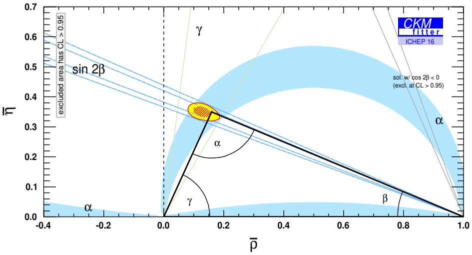

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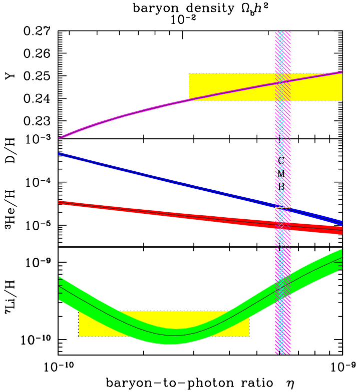

η from CMB & BBN, PDG

- CKM paradigm not enough
- Need new sources of CPV
- Deviations from CKM?

Two-body vs three-body γ

Two-body (tree-level)

- $B \to DK, B \to D\pi$
  $D \to 2P, 3P, \ldots$
- Interfere: $D^0, \overline{D}^0, D_{\rm CP}$
- Methods: (J. Brod talk WG6 17/09)
  GLW (Gronau-London-Wyler)
  ADS (Atwood-Dunietz-Soni)
  GGSZ(Giri-Grossman-Sofer-Zupan)
- n parameters, m observables
  $m \geq n \quad \Rightarrow$ solution!
- Theoretically very clean
  $\delta\gamma \sim \mathcal{O}(10^{-4}) - \mathcal{O}(10^{-7})$
  Brod + Zupan, 1308.5663

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Three-body (tree + loop)

- Charmless B decays
  $B \to PPP \quad (P = \pi, K)$
- Observables:
  $\mathcal{B}, A_{\rm CP}$ (direct, indirect)
- Find "scenarios" with more observables than parameters
- Need CP eigenstate
  $\to$ measure indirect CPV
- Use SU(3) symmetry
  $\to$ diagrammatic approach
  $\to$ dynamical assumptions
  $\to$ How clean is it?

Observables and Parameters

Example 3-body process: $B^0 \to K^0\pi^+\pi^-$ $(\bar{b} \to \overline{s}$ transition)

Amplitude $\to$ $$\begin{aligned}\mathcal{A}(B^0 \to K^0\pi^+\pi^-) &= V_{tb}V_{ts}^* a_t + V_{ub}V_{us}^* a_u \\ &= \tilde{a}_t + e^{-i\gamma}\,\tilde{a}_u\end{aligned}$$

$|\tilde{a}_t|,\ |\tilde{a}_u|,\ \delta$ (relative strong phase) $\to$ **momentum dependent**

Momentum dependent observables: need Dalitz amplitude analysis

$$\begin{aligned}X_{\rm DP} &= |\mathcal{A}_{\rm sym}|^2 + |\overline{\mathcal{A}}_{\rm sym}|^2 & \textit{CP averaged branching fraction} \\ Y_{\rm DP} &= |\mathcal{A}_{\rm sym}|^2 - |\overline{\mathcal{A}}_{\rm sym}|^2 & \textit{Direct CP asymmetry} \\ Z_{\rm DP} &= {\rm Im}\left(\mathcal{A}_{\rm sym}^*\overline{\mathcal{A}}_{\rm sym}\right) & \textit{Indirect CP asymmetry}\end{aligned}$$

Momentum dependent hadronic parameters

Standard Model Weak Phase : momentum independent!

Hadronic Parameters

- Hadronic parameters are momentum dependent
- $\tilde{a}_{u,t} \equiv \tilde{a}_{u,t}(p_1, p_2, p_3) \; = \; \tilde{a}_{u,t}(s_{12}, s_{13})$
- Recent work on momentum dependence
  in context of CP asymmetries in $B \to 3\pi$
  K. Keri Vos, et al. 1708.02047
- Disentangle weak and strong phases through resonances
  Charles, et al. 1704.01596
- Need some external hadronic input from QCDf
- This work: treat hadronic parameters as observables
  $\rightarrow$ No external hadronic input (such as QCDf) used
  $\rightarrow$ Hadronic parameters may be extracted and studied independently

γ from U-spin

- In two-body decays:
  - $\rightarrow$ U-spin pair: $B_s^0 \rightarrow K^+K^-$, $B_d^0 \rightarrow \pi^+\pi^-$ $(a_d = a_s)$
  - $\rightarrow$ 4 observables: $\mathcal{B}_d, \mathcal{B}_s, A_{\rm CP}^d, A_{\rm CP}^s$; 4 unknowns: $|a_1|, |a_2|, \delta, \gamma$
  - $\rightarrow$ U-spin relation: $A_{\rm CP}^d \mathcal{B}_d \tau_s = -A_{\rm CP}^s \mathcal{B}_s \tau_d$ (1 less observable)
  - $\rightarrow$ Measure indirect $A_{\rm CP}$ and use $B - \bar{B}$ mixing (β, β_s)

    Fleischer method, hep-ph/9903456
  - $\rightarrow$ Enough info (5 observables, 4 unknowns): γ can be extracted
- Test U-spin breaking: check U-spin relation
- 1503.00737 (with D.London) $\rightarrow$ γ from 3-body U-spin pairs
  - $\rightarrow$ $B_s^0 \rightarrow K_S K^+ K^-$, $B_d^0 \rightarrow K_S \pi^+ \pi^-$
  - $\rightarrow$ Need time-dependent Dalitz analysis for indirect CPV
  - $\rightarrow$ Test momentum dependence of U-spin breaking

Binning and averaging

- Local (momentum dependent) observables:

  $\rightarrow \; \Gamma(t) \; = \; \dfrac{\Gamma(B^0 \to f) + \Gamma(\overline{B}^0 \to f)}{2} \,, \quad A_{\rm CP}(t) \; = \; \dfrac{\Gamma(B^0 \to f) - \Gamma(\overline{B}^0 \to f)}{\Gamma(B^0 \to f) + \Gamma(\overline{B}^0 \to f)}$

  $\rightarrow \; |f\rangle \neq |\overline{f}\rangle$ $\qquad A_{\rm CP}$ is not true CP Asymmetry

- Time-dependent Dalitz analysis : extract coefficients

- Observables are coefficients of $\sin, \cos, \sinh, \cosh$ of $\Delta m, \Delta\Gamma$

  $\rightarrow$ For fit use redefinitions:

  $$\int_{\rm bin} |\mathcal{A}|^2 \; \equiv \; |\mathcal{A}'|^2 \,, \quad \int_{\rm bin} |\tilde{\mathcal{A}}|^2 \; \equiv \; |\tilde{\mathcal{A}}'|^2 \,, \quad \int_{\rm bin} {\rm Im} \left[(q/p) \mathcal{A}^* \tilde{\mathcal{A}} \right] \; \equiv \; ?$$

  $\rightarrow$ Approximate : $\displaystyle\int_{\rm bin} {\rm Im} \left[(q/p) \mathcal{A}^* \tilde{\mathcal{A}} \right] \; \approx \; {\rm Im} \left[(q/p) \mathcal{A}'^* \tilde{\mathcal{A}}' \right]$

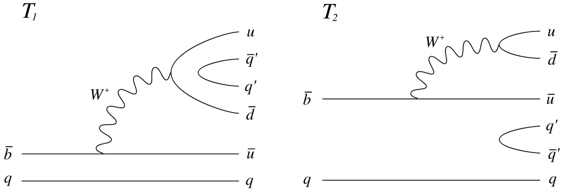
- U-spin relation in Dalitz analysis: $a^d_{\rm CP} b_d \tau_s = -a^s_{\rm CP} b_s \tau_d$

  $\rightarrow$ Momentum-dependent relationship

  $\rightarrow$ Unequal U-spin breaking in pockets, but satisfied on average?

Flavor SU(3) diagrammatics

- 2-body $\bar{b} \to \bar{s}$ transitions: $T', C', P'_{tc}, P'_{uc}, P'_{EW}, P'^{C}_{EW}$
- No weak-exchange diagrams (suppressed, dynamical assumption)
- 2 choices for 3-body diagrams (subscripts 1 or 2)



- Three-body topologies: $T'_1, T'_2, C'_1, C'_2, \ldots$
- In SU(3) limit: 3 identical particles in final state
- Equivalent to SU(3) matrix element for Fully Symmetric case

Flavor SU(3) diagrammatics

- Flavor SU(3): triplet of B mesons $\rightarrow$ octet of π, K, η
- Final state with 3 identical particles: S_3 symmetry
  $\rightarrow$ Fully symmetric (FS), fully antisymmetric (FA)
  $\rightarrow$ Mixed symmetric (MS1, MS2, MS3, MS4)
- FS amplitude stays same when any 2 particles are interchanged
- 1402.2909 (with Gronau, Imbeault, London, Rosner) considered FS
  $\rightarrow$ 9 independent SU(3) matrix elements
  $\rightarrow$ *Already includes rescattering effects to all orders in* α_s
  $\rightarrow$ 16 $b \rightarrow s$ and 16 $b \rightarrow d$ 3-body channels
  $\rightarrow$ Several SU(3) relationships that can be tested
  Example: $\mathcal{A}(B^+ \rightarrow K^0\pi^+\pi^0)_{\rm FS} \;=\; -\;\mathcal{A}(B^0 \rightarrow K^+\pi^0\pi^-)_{\rm FS}$

γ from 3-body B decays

- SU(3) limit gives relations between tree and electroweak penguins
- 4 effective combinations (a, b, c, d) in the SU(3) limit
- SU(3) FS amplitudes expressed in terms of effective diagrams:

$$
\begin{aligned}
A(B^0 \to K^0 K^0 \overline{K}^0)_{\rm FS} &= a \\
\sqrt{2}A(B^0 \to K^+ K^0 K^-)_{\rm FS} &= -ce^{i\gamma} - a + \kappa b \\
2A(B^0 \to K^+ \pi^0 \pi^-)_{\rm FS} &= be^{i\gamma} - \kappa c \\
\sqrt{2}A(B^0 \to K^0 \pi^+ \pi^-)_{\rm FS} &= -de^{i\gamma} - a + \kappa d
\end{aligned}
$$

- Construct momentum dependent observables (X, Y, Z)
- Amplitude analysis (Example: isobar) to get amplitude at a point
- One Dalitz plot point sufficient for γ: average over multiple points (more details in the next talk by E. Bertholet)

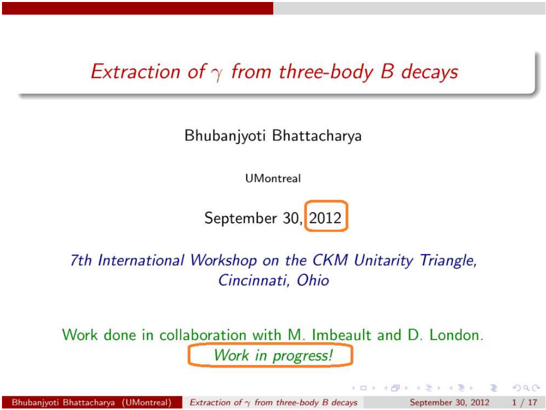
SU(3) breaking: How to test it?

- Single parameter SU(3) breaking $\rightarrow \quad |f_{SU(3)}|$

$$
\begin{aligned}
A(B^0 \rightarrow K^0 K^0 \overline{K}^0)_{\rm FS} &= f_{SU(3)}\ a \\
\sqrt{2}A(B^0 \rightarrow K^+ K^0 K^-)_{\rm FS} &= f_{SU(3)}(-ce^{i\gamma} - a + \kappa b) \\
2A(B^0 \rightarrow K^+ \pi^0 \pi^-)_{\rm FS} &= be^{i\gamma} - \kappa c \\
\sqrt{2}A(B^0 \rightarrow K^0 \pi^+ \pi^-)_{\rm FS} &= -de^{i\gamma} - a + \kappa d \\
\sqrt{2}A(B^+ \rightarrow K^+ \pi^+ \pi^-)_{\rm FS} &= -ce^{i\gamma} - a + \kappa b
\end{aligned}
$$

- Extract $|f_{SU(3)}|$ from a fit; Expect $|f_{SU(3)}| = 1$ for exact SU(3)
- $|f_{SU(3)}|$ should vary widely over allowed Dalitz area
- Average $|f_{SU(3)}| - 1$ may indicate the level of SU(3) breaking (Similar to U-spin breaking test by averaging over Dalitz area)

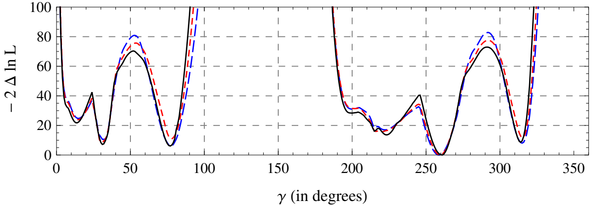
From CKM 2012



- Completed :
  1303.0846
  PLB 726 (2018) 337
- More work done since
  $\rightarrow$ E. Bertholet's talk
- More work to come
  fully-antisymmetric
  amplitudes

Results

- γ extracted by applying SU(3) to fully symmetric state
  $\rightarrow$ Full Analysis in 1303.0846 (with Imbeault, London)



$\rightarrow$ SM-like : 77°
$\rightarrow$ Other solutions : $32^\circ,\ 259^\circ,\ 315^\circ$
$\rightarrow$ Discrete Ambiguity

- Key : γ extraction by applying SU(3) to other symmetry states
  $\rightarrow$ Break discrete ambiguity : more information from other states
- Also key : estimate systematic uncertainties in γ extraction
- Interesting situation : γ widely different from SM value
  $\rightarrow$ Significant SU(3) breaking?
  $\rightarrow$ NP in three-body B decays? $K\pi\pi - KKK$ puzzle?

Conclusions

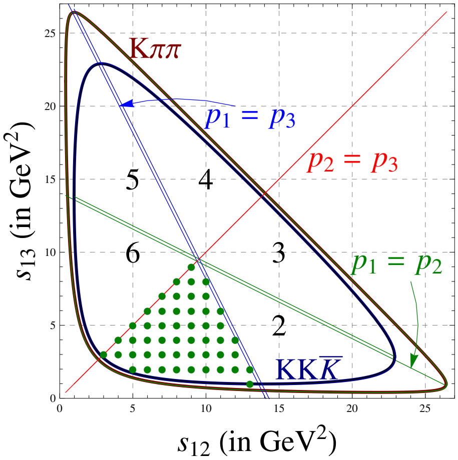
- γ from tree is theoretically very clean!
- Precise γ from LHCb will strongly test CKM unitarity
- γ from tree + loop can expose interesting physics
- Flavor symmetries provide effective path to study γ
- γ from U-spin + time-dependent Dalitz analysis
- γ from SU(3) + fully-symmetric amplitude analysis
- Discrete ambiguities may be broken using other symmetries
- γ tree + loop may present hints of new physics

Thank You!

Back-up Slides

Three-body decays : Dalitz plots

- Three-body final state : $|P_1(p_1)P_2(p_2)P_3(p_3)\rangle \quad s_{ij} = (p_i + p_j)^2$
  $\rightarrow$ Momentum dependent. One relation $s_{12} + s_{23} + s_{13} = \text{constant}$



Features of a Dalitz plot:

- Independent measurements at different points may be possible
- Same SM weak phase (γ); Hadronic parameters are local
- Consistency checks : Flavor symmetries (SU(3), U-spin) provide amplitude relationships

Three-body SU(3) relations

$b \to s$ relationships from isospin and SU(3) symmetry :

$$
\begin{aligned}
\mathcal{A}(B^+ \to K^0\pi^+\pi^0)_{\rm FS} &= -\,\mathcal{A}(B_d^0 \to K^+\pi^0\pi^-)_{\rm FS} \\
\sqrt{2}\mathcal{A}(B_s^0 \to 3\pi^0)_{\rm FS} &= -\,\sqrt{3}\mathcal{A}(B_s^0 \to \pi^0\pi^+\pi^-)_{\rm FS} \\
\sqrt{2}\mathcal{A}(B^+ \to K^0\pi^+\pi^0)_{\rm FS} &= \mathcal{A}(B_d^0 \to K^0\pi^+\pi^-)_{\rm FS} + \sqrt{2}\mathcal{A}(B_d^0 \to K^0\pi^0\pi^0)_{\rm FS} \\
\sqrt{2}\mathcal{A}(B_d^0 \to K^+\pi^0\pi^-)_{\rm FS} &= \mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm FS} + \sqrt{2}\mathcal{A}(B_d^0 \to K^0\pi^0\pi^0)_{\rm FS} \\
\mathcal{A}(B^+ \to K^+K^+K^-)_{\rm FS} &+ \sqrt{2}\mathcal{A}(B^+ \to K^+K^0\overline{K}^0)_{\rm FS} \\
&= \sqrt{2}\mathcal{A}(B_d^0 \to K^0K^+K^-)_{\rm FS} + \mathcal{A}(B_d^0 \to K^0K^0\overline{K}^0)_{\rm FS} \\
\mathcal{A}(B_s^0 \to \pi^0K^+K^-)_{\rm FS} &+ \sqrt{2}\mathcal{A}(B_s^0 \to \pi^0K^0\overline{K}^0)_{\rm FS} \\
&= -\,\sqrt{2}\mathcal{A}(B_s^0 \to \pi^-K^+\overline{K}^0)_{\rm FS} - \mathcal{A}(B_s^0 \to \pi^+K^-K^0)_{\rm FS} \\
\sqrt{2}\mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm FS} &= \mathcal{A}(B^+ \to K^+K^+K^-)_{\rm FS}
\end{aligned}
$$

7 relations involving 16 amplitudes that depend on 9 matrix elements

Similar situation also in $b \to d$ FS amplitudes