New approaches to exclusive predictions for charm mixing

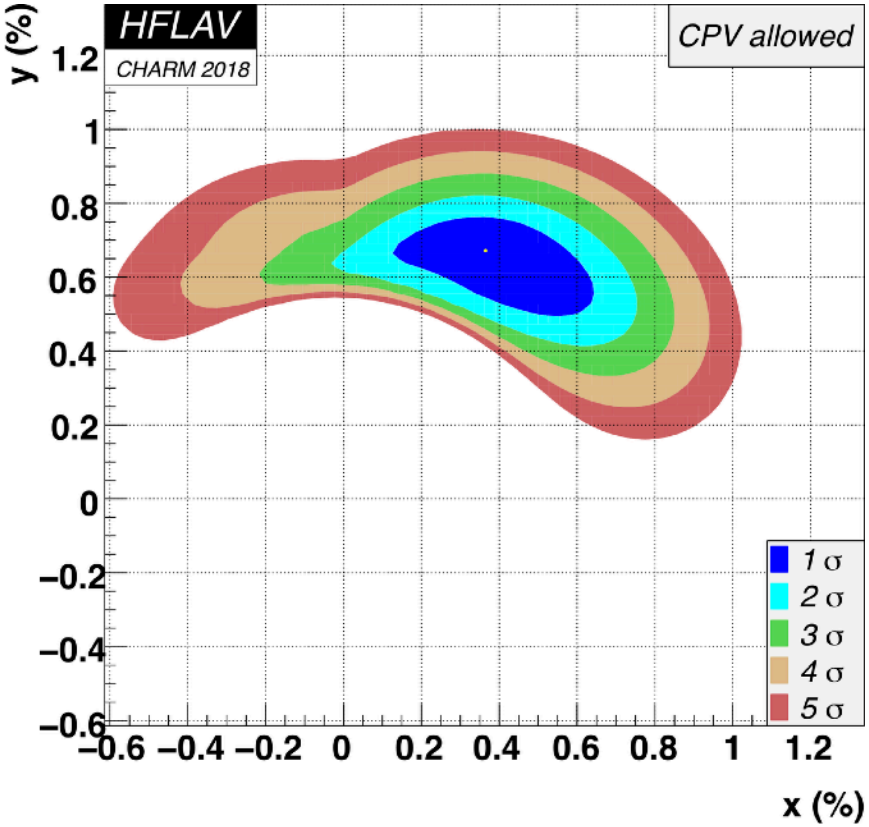
Alexey A. Petrov
Wayne State University
Leinweber Center for Theoretical Physics

Table of Contents:

- Introduction: quark-hadron duality
- Charmed mixing: short and long distance
  - ➡ exclusive approach: no experiment
  - ➡ exclusive approach: how to use experimental data
- Conclusions: things to take home

Introduction: experimental facts

★ Experimental fact: charm mixing parameters are non-zero



★ … and rather large

- if CP-violation is neglected…

$$x = 0.50^{+0.13}_{-0.14}\%$$
$$y = 0.63 \pm 0.08\%$$

- if CP-violation is allowed

$$x = 0.36^{+0.21}_{-0.16}\%$$
$$y = 0.67^{+0.06}_{-0.13}\%$$

… maybe still on the fence about x

★ Theoretical fact: exclusive and inclusive predictions differ by an order of magnitude

Intro: quark-hadron duality, lifetimes

★ New Physics couples to quark degrees of freedom, we observe hadrons!

- ➡ need to know how to compute non-perturbative matrix elements
- ➡ need to understand how quark-hadron duality works

★ Observables computed in terms of hadronic degrees of freedom…

$$\Gamma_{hadron}(H_b) = \sum_{\substack{all\ final\ state \\ hadrons}} \Gamma(H_b \to h_i)$$

Bloom, Gilman;
Poggio, Quinn, Weinberg

★ … must match observables computed in terms of quark degrees of freedom

$$\Gamma(H_b) = \frac{1}{2M_b}\langle H_b | T | H_b \rangle = \frac{1}{2M_b}\langle H_b | \mathrm{Im}\, i\int d^4x\, T\left\{H_{eff}^{\Delta B=1}(x) H_{eff}^{\Delta B=1}(0)\right\} | H_b \rangle$$

$$\Gamma(H_b) = \frac{G_F^2 m_Q^5}{192\pi^3}\left[A_0 + \frac{A_2}{m_Q^2} + \frac{A_2}{m_Q^3} + \ldots\right]$$

HQ expansion converges reasonably well…

Intro: quark-hadron duality, lifetimes

★ How to define quark-hadron duality and quantify its violations?

➡ Compute quark correlator in Eucledian space and analytically continue to Minkowski space [exact calculation in ES = exact result in MS]

➡ Expand it in a_S and "$1/Q \sim 1/m_Q$": series truncation

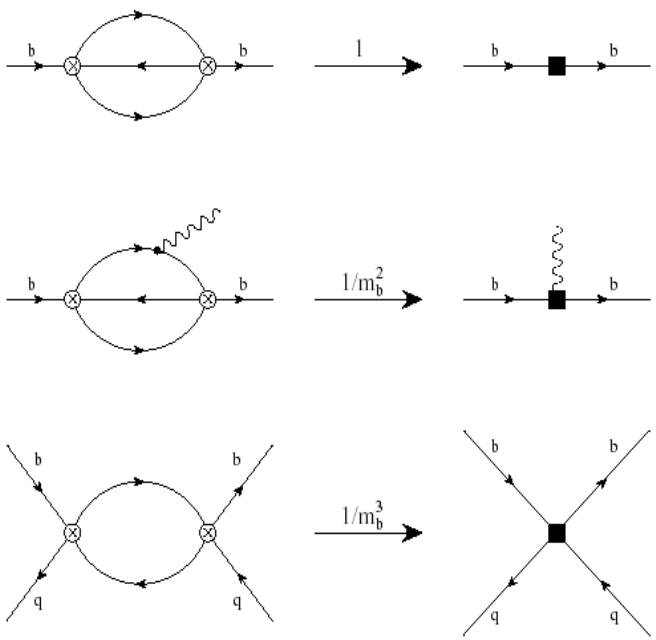
➡ Any deviation beyond "natural uncertainty" is treated as violation of quark-hadron duality [resonances, instantons,...]

This definition is due to M. Shifman

Rob Gonzalves

Quark-hadron duality: lifetimes

★ In case of b-flavored hadrons can compare directly to experiment



Lifetime ratio	Experimental average	HQE prediction
$\tau(B^+)/\tau(B^0)$	1.076 ± 0.004	$1.04^{+0.05}_{-0.01} \pm 0.02 \pm 0.01$
$\tau(B_s^0)/\tau(B^0)$	0.990 ± 0.004	1.001 ± 0.002
$\tau(\Lambda_b^0)/\tau(B^0)$	0.967 ± 0.007	0.935 ± 0.054
$\tau(\Xi_b^0)/\tau(\Xi_b^-)$	0.929 ± 0.028	0.95 ± 0.06

HFLAV, 2017

★ … works surprisingly well…

Channel	Expansion parameter x	Numerical value	$\exp[-1/x]$
$b \to c\bar{c}s$	$\frac{\Lambda}{\sqrt{m_b^2-4m_c^2}} \approx \frac{\Lambda}{m_b}\left(1+2\frac{m_c^2}{m_b^2}\right)$	$0.054 - 0.58$	$9.4 \cdot 10^{-9} - 0.18$
$b \to c\bar{u}s$	$\frac{\Lambda}{\sqrt{m_b^2-m_c^2}} \approx \frac{\Lambda}{m_b}\left(1+\frac{1}{2}\frac{m_c^2}{m_b^2}\right)$	$0.045 - 0.49$	$1.9 \cdot 10^{-10} - 0.13$
$b \to u\bar{u}s$	$\frac{\Lambda}{\sqrt{m_b^2-4m_u^2}} = \frac{\Lambda}{m_b}$	$0.042 - 0.48$	$4.2 \cdot 10^{-11} - 0.12$

Jubb, Kirk, Lenz, Tetlalmatzi-Xolocotzi, 2017

★ How does it work for charmed hadrons?

Quark-hadron duality: mixing

★ How can one tell that a process is dominated by long-distance or short-distance?

★ To start thing off, mass and lifetime differences of mass eigenstates...

$$x_D = \frac{M_2 - M_1}{\Gamma_D}, \; y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D}$$

★ ...can be calculated as real and imaginary parts of a correlation function

$$y_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} \, {\rm Im} \, \langle \overline{D^0} | \, i \int {\rm d}^4x \, T \Big\{ \mathcal{H}_w^{|\Delta C|=1}(x) \, \mathcal{H}_w^{|\Delta C|=1}(0) \Big\} | D^0 \rangle$$

bi-local time-ordered product

$$x_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} \, {\rm Re} \, \Big[2 \langle \overline{D^0} | H^{|\Delta C|=2} | D^0 \rangle + \langle \overline{D^0} | \, i \int {\rm d}^4x \, T \Big\{ \mathcal{H}_w^{|\Delta C|=1}(x) \, \mathcal{H}_w^{|\Delta C|=1}(0) \Big\} | D^0 \rangle \Big]$$

local operator
(b-quark, NP): small?

bi-local time-ordered product

★ ... or can be written in terms of hadronic degrees of freedom...

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left[\langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | \overline{D}^0 \rangle + \langle \overline{D}^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]$$

Mixing: short vs long distance

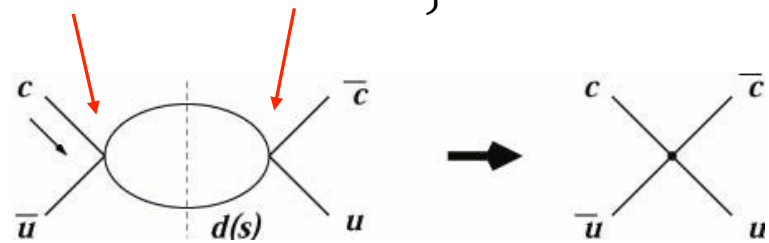
★ How can one tell that a process is dominated by long-distance or short-distance?

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left[\langle D^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|\overline{D}^0\rangle + \langle \overline{D}^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|D^0\rangle\right]$$

★ It is important to remember that the expansion parameter is $1/E_{released}$

$$y_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}}\,{\rm Im}\,\langle\overline{D^0}|\,i\int {\rm d}^4x\,T\Big\{\mathcal{H}_w^{|\Delta C|=1}(x)\,\mathcal{H}_w^{|\Delta C|=1}(0)\Big\}|D^0\rangle$$

OPE-leading contribution:



★ In the heavy-quark limit $m_c \to \infty$ we have $m_c \gg \sum m_{intermediate\ quarks}$, so $E_{released} \sim m_c$

- the situation is similar to B-physics, where it is "short-distance" dominated
- one can consistently compute pQCD and 1/m corrections

★ But wait, m_c is NOT infinitely large! What happens for finite m_c???

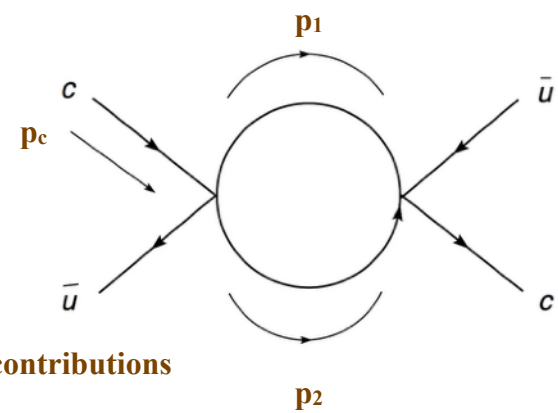
- how is large momentum routed in the diagrams?
- are there important hadronization (threshold) effects?

Threshold (and related) effects in OPE

★ How can one tell that a process is dominated by long-distance or short-distance?

★ Let's look at how the momentum is routed in a leading-order diagram

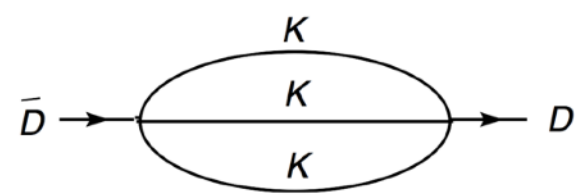
- injected momentum is $p_c \sim m_c$
- thus, $p_1 \sim p_2 \sim m_c/2 \sim O(\Lambda_{QCD})$?



Still OK with OPE, signals large nonperturbative contributions

★ For a particular example of the lifetime difference, have hadronic intermediate states

- let's use an example of KKK intermediate state
- in this example, $E_{released} \sim m_D - 3\, m_K \sim O(\Lambda_{QCD})$



★ Similar threshold effects exist in B-mixing calculations

- but $m_b \gg \sum m_{\text{intermediate quarks}}$, so $E_{released} \sim m_b$ (almost) always
- quark-hadron duality takes care of the rest!

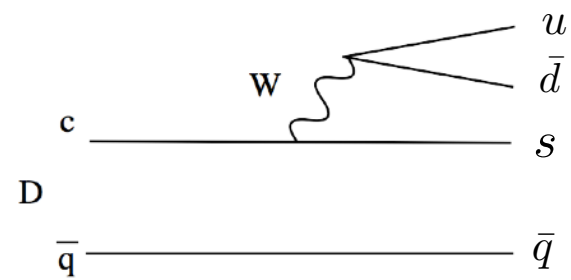
Thus, two approaches: **1. insist on $1/m_c$ expansion, hope for quark-hadron duality**
2. saturate correlators by hadronic states

Aside: classification of charm decays

★ Can be classified by SM CKM suppression

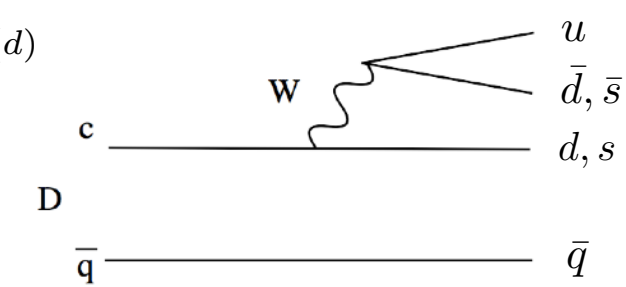
★ Cabibbo-favored (CF) decay
- originates from $c \to s\, u\bar{d}$
- examples: $D^0 \to K^- \pi^+$

$V_{cs} V_{ud}^*$



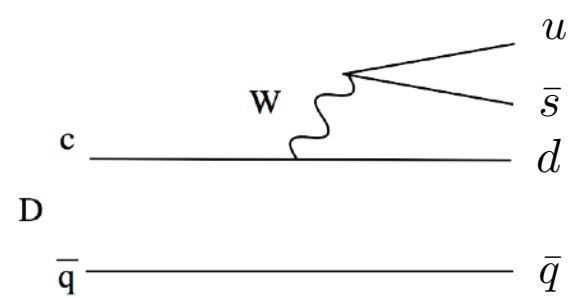
★ Singly Cabibbo-suppressed (SCS) decay
- originates from $c \to q\, u\bar{q}$
- examples: $D^0 \to \pi\pi$ and $D^0 \to KK$

$V_{cs(d)} V_{us(d)}^*$



★ Doubly Cabibbo-suppressed (DCS) decay
- originates from $c \to d\, u\bar{s}$
- examples: $D^0 \to K^+ \pi^-$

$V_{cd} V_{us}^*$



★ "Common final states" for D and $\bar{D}$ generate mixing in exclusive approach

Exclusive approach to mixing: use data?

★ LD calculation: saturate the correlator by hadronic states, e.g.

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left[\langle D^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|\overline{D}^0\rangle + \langle \overline{D}^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|D^0\rangle\right]$$

... with n being all states to which D^0 and $\overline{D}^0$ can decay. Consider $\pi\pi$, πK, KK intermediate states as an example...

J. Donoghue et. al.
L. Wolfenstein
P. Colangelo et. al.

H.Y. Cheng and C. Chiang

$$\begin{aligned} y_2 &= Br(D^0 \to K^+K^-) + Br(D^0 \to \pi^+\pi^-) \\ &- 2\cos\delta\sqrt{Br(D^0 \to K^+\pi^-)Br(D^0 \to \pi^+K^-)} \end{aligned}$$

cancellation expected

If every Br is known up to O(1%) ➡ the result is expected to be O(1%)!

The result here is a series of large numbers with alternating signs, SU(3) forces 0

If experimental data on Br is used, are we only sensitive to exit. uncertainties?

★ The experimental uncertainties of hadronic Br have significantly improved

- can we now use experimental data to predict y?
- is it a prudent method to ``predict" y?

Exclusive approach to mixing: no decay data!

★ Need to "repackage" the analysis: look at complete multiplet contribution

$$y = \sum_{F_R} y_{F,R}\, Br(D^0 \to F_R) = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \to n)$$

Falk, Grossman, Ligeti, Nir. A.A.P. Phys.Rev. D69, 114021, 2004
Falk, Grossman, Ligeti, and A.A.P. Phys.Rev. D65, 054034, 2002

Numerator:

$$A_{N,8} = |A_0|^2 s_1^2 \Big[\frac{1}{2}\Phi(\eta,\eta) + \frac{1}{2}\Phi(\pi^0,\pi^0) + \frac{1}{3}\Phi(\eta,\pi^0) + \Phi(\pi^+,\pi^-) - \Phi(\overline{K}^0,\pi^0)$$
$$+ \Phi(K^+,K^-) - \frac{1}{6}\Phi(\eta,K^0) - \frac{1}{6}\Phi(\eta,\overline{K}^0) - \Phi(K^+,\pi^-) - \Phi(K^-,\pi^+)\Big]$$

Denominator:

phase space function

$$A_{D,8} = |A_0|^2 \Big[\frac{1}{6}\Phi(\eta,K^0) + \Phi(K^+,\pi^-) + \frac{1}{2}\Phi(K^0,\pi^0) + O(s_1^2) \Big]$$

★ This contribution is calculable....

$$y_{2,8} = \frac{A_{N,8}}{A_{D,8}} = -0.038 s_1^2 = -1.8 \times 10^{-4}$$

1. Repeat for other states
2. Multiply by Br_{Fr} to get y

.... but completely negligible!

Old results

★ Repeat for other intermediate states:

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
PP	8	−0.0038	−0.018
	27	−0.00071	−0.0034
PV	8_S	0.031	0.15
	8_A	0.032	0.15
	10	0.020	0.10
	$\overline{10}$	0.016	0.08
	27	0.040	0.19
$(VV)_{s\text{-wave}}$	8	−0.081	−0.39
	27	−0.061	−0.30
$(VV)_{p\text{-wave}}$	8	−0.10	−0.48
	27	−0.14	−0.70
$(VV)_{d\text{-wave}}$	8	0.51	2.5
	27	0.57	2.8

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
$(3P)_{s\text{-wave}}$	8	−0.48	−2.3
	27	−0.11	−0.54
$(3P)_{p\text{-wave}}$	8	−1.13	−5.5
	27	−0.07	−0.36
$(3P)_{\text{form-factor}}$	8	−0.44	−2.1
	27	−0.13	−0.64
$4P$	8	3.3	16
	27	2.2	9.2
	$27'$	1.9	11

- Product is naturally O(1%)
- No (symmetry-enforced) cancellations
- Disp relation: compute x (model-dependence)

naturally implies that x,y ~ 1% is expected in the Standard Model

Final state	fraction
PP	5%
PV	10%
$(VV)_{s\text{-wave}}$	5%
$(VV)_{d\text{-wave}}$	5%
$3P$	5%
$4P$	10%

A.F., Y.G., Z.L., Y.N. and A.A.P. Phys.Rev. D69, 114021, 2004

E.Golowich and A.A.P. Phys.Lett. B427, 172, 1998

Note dominance of near-threshold states!

Exclusive approach to mixing: use data!

★ What if we insist on using experimental data anyway?

★ Ex., one can employ Factorizaton-Assisted Topological Amplitudes

in units of 10^{-3}

Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)
$\pi^0\overline{K}^0$	24.0 ± 0.8	24.2 ± 0.8	$\pi^0\overline{K}^{*0}$	37.5 ± 2.9	35.9 ± 2.2	$\overline{K}^0\rho^0$	$12.8^{+1.4}_{-1.6}$	13.5 ± 1.4
π^+K^-	39.3 ± 0.4	39.2 ± 0.4	π^+K^{*-}	54.3 ± 4.4	62.5 ± 2.7	$K^-\rho^+$	111.0 ± 9.0	105.0 ± 5.2
$\eta\overline{K}^0$	9.70 ± 0.6	9.6 ± 0.6	$\eta\overline{K}^{*0}$	9.6 ± 3.0	6.1 ± 1.0	$\overline{K}^0\omega$	22.2 ± 1.2	22.3 ± 1.1
$\eta'\overline{K}^0$	19.0 ± 1.0	19.5 ± 1.0	$\eta'\overline{K}^{*0}$	< 1.10	0.19 ± 0.01	$\overline{K}^0\phi$	$8.47^{+0.66}_{-0.34}$	8.2 ± 0.6
$\pi^+\pi^-$	1.421 ± 0.025	1.44 ± 0.02	$\pi^+\rho^-$	5.09 ± 0.34	4.5 ± 0.2	$\pi^-\rho^+$	10.0 ± 0.6	9.2 ± 0.3
K^+K^-	4.01 ± 0.07	4.05 ± 0.07	K^+K^{*-}	1.62 ± 0.15	1.8 ± 0.1	K^-K^{*+}	4.50 ± 0.30	4.3 ± 0.2
$K^0\overline{K}^0$	0.36 ± 0.08	0.29 ± 0.07	$K^0\overline{K}^{*0}$	0.18 ± 0.04	0.19 ± 0.03	$\overline{K}^0K^{*0}$	0.21 ± 0.04	0.19 ± 0.03
$\pi^0\eta$	0.69 ± 0.07	0.74 ± 0.03	$\eta\rho^0$		1.4 ± 0.2	$\pi^0\omega$	0.117 ± 0.035	0.10 ± 0.03
$\pi^0\eta'$	0.91 ± 0.14	1.08 ± 0.05	$\eta'\rho^0$		0.25 ± 0.01	$\pi^0\phi$	1.35 ± 0.10	1.4 ± 0.1
$\eta\eta$	1.70 ± 0.20	1.86 ± 0.06	$\eta\omega$	2.21 ± 0.23	2.0 ± 0.1	$\eta\phi$	0.14 ± 0.05	0.18 ± 0.04
$\eta\eta'$	1.07 ± 0.26	1.05 ± 0.08	$\eta'\omega$		0.044 ± 0.004			
$\pi^0\pi^0$	0.826 ± 0.035	0.78 ± 0.03	$\pi^0\rho^0$	3.82 ± 0.29	4.1 ± 0.2			
π^0K^0		0.069 ± 0.002	π^0K^{*0}		0.103 ± 0.006	$K^0\rho^0$		0.039 ± 0.004
π^-K^+	0.133 ± 0.009	0.133 ± 0.001	π^-K^{*+}	$0.345^{+0.180}_{-0.102}$	0.40 ± 0.02	$K^+\rho^-$		0.144 ± 0.009
ηK^0		0.027 ± 0.002	ηK^{*0}		0.017 ± 0.003	$K^0\omega$		0.064 ± 0.003
$\eta' K^0$		0.056 ± 0.003	$\eta' K^{*0}$		0.00055 ± 0.00004	$K^0\phi$		0.024 ± 0.002

Jiang, Yu, Qin, Li, and Lu, 2017

★ … but it appears to yield a smaller result, $y_{PP+PV} = (0.21 \pm 0.07)\%$.

Exclusive approach to mixing: use data!

★ What if we insist on using experimental data anyway?

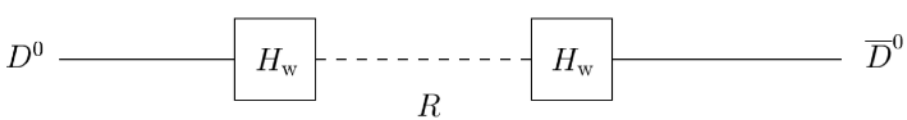
A.A.P. and R. Briere
arXiv:1810.xxxx

★ Possible additional contributions?

- each intermediate state has a finite width, i.e. is not a proper asymptotic state
- within each multiplet widths experience (incomplete) SU(3) cancelations
- this effect already happens for the simplest intermediate states!

★ Consider, for illustration, a set of single-particle intermediate states:

$$\Sigma_{p_D}(p_D)\Big|_{\rm tot}^{\rm res} = \frac{1}{2m_D}\sum_R Re\,\frac{\langle D_L|\mathcal{H}_W|R\rangle\langle R|\mathcal{H}_W^\dagger|D_L\rangle}{m_D^2 - m_R^2 + i\Gamma_R m_D} \quad - \quad (D_L \to D_S)$$



$$\Delta m_D\Big|_{\rm R}^{\rm res} \propto \frac{m_D^2 - m_R^2}{(m_D^2 - m_R^2)^2 + \Gamma_R^2 m_D^2}$$

$$\Delta\Gamma_D\Big|_{\rm R}^{\rm res} \propto -\frac{\Gamma_R m_D}{(m_D^2 - m_R^2)^2 + \Gamma_R^2 m_D^2}$$

★ Each resonance contributes to $\Delta\Gamma$ only because of its finite width!

Finite width effects: one-body contributions

★ Multiplet effects for (single-particle) intermediate states

- in this simple example: heavy pion, kaon and eta/eta'
- each single-particle intermediate state has a rather large width

$$\Delta\Gamma_D|^{\rm res}_{\rm octet} = \Delta\Gamma_D^{(K_H)} - \frac{1}{4}\Delta\Gamma_D^{(\pi_H)} - \frac{3\cos^2\theta_{\rm H}}{4}\Delta\Gamma_D^{(\eta_H)} - \frac{1\sin^2\theta_{\rm H}}{4}\Delta\Gamma_D^{(\eta'_H)}$$

- where for each state $\Delta\Gamma_D^{\rm res} = -Cf_R^2\,\frac{\mu_R\gamma_R}{(1-\mu_R)^2+\gamma_R^2}$ with $\mu_R = m_R^2/m_D^2$, $\gamma_R = \Gamma_R/m_D$
- … and a model calculation gives $C \equiv 2m_D(G_F a_2 f_D \xi_d/\sqrt{2})^2$
- SU(3) forces cancellations: a new SU(3) breaking effect due to widths!

Table: Magnitudes of Pseudoscalar Resonance Contributions.

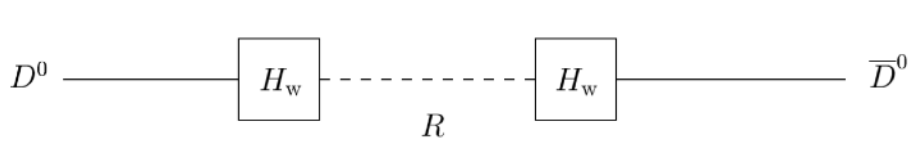
Resonance	$\lvert\Delta m_D\rvert \times 10^{-16}$ (GeV)	$\lvert\Delta\Gamma_D\rvert \times 10^{-16}$ (GeV)
$K(1460)$	$\sim 1.24\ (f_{K(1460)}/0.025)^2$	$\sim 0.88\ (f_{K(1460)}/0.025)^2$
$\eta(1760)$	$(0.77 \pm 0.27)\ (f_{\eta(1760)}/0.01)^2$	$(0.43 \pm 0.53)\ (f_{\eta(1760)}/0.01)^2$
$\pi(1800)$	$(0.13 \pm 0.06)\ (f_{\pi(1800)}/0.01)^2$	$(0.41 \pm 0.11)\ (f_{\pi(1800)}/0.01)^2$
$K(1830)$	$\sim 0.29\ (f_{K(1830)}/0.01)^2$	$\sim 1.86\ (f_{K(1830)}/0.01)^2$

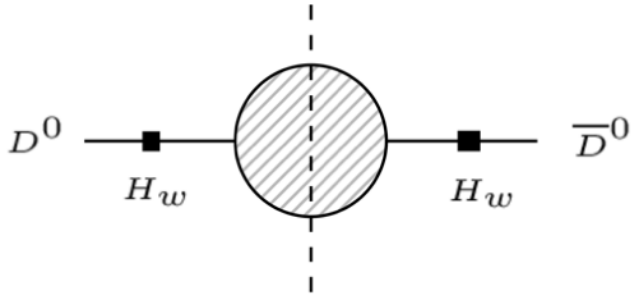
E. Golowich and A.A.P.
PLB427 (1998) 172-178

Finite width effects: one-body contributions

★ Let us take another look at those one-body contributions

- the width of each excited light quark state $\Gamma_R = \Gamma(R \to P_1 P_2) + \Gamma(R \to P_1 P_2 P_3) + \ldots$
- … which is equivalent to accounting for resonant FSI in 2-body intermediate state!





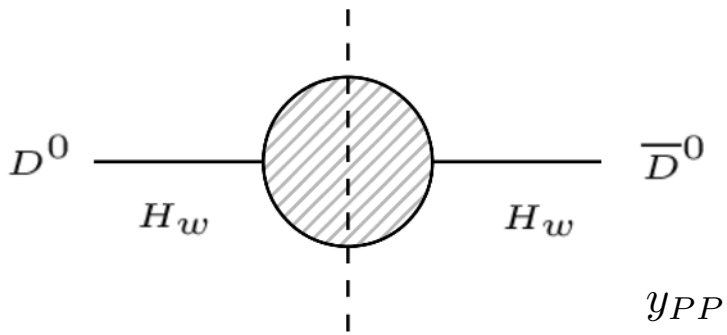
Since we shall be using experimental data to compute 2-body contributions, this effect will be taken into account automatically!

It's consistent to omit 1-body IS if experimental data is used

Finite width effects: two-body contributions

★ Let us apply similar logic to two-body contributions

- consider contributions from the stable (wrt strong interactions) octet of pions, kaons, etas



$$y_{PP} = (0.1 - 20) \times 10^{-4}$$

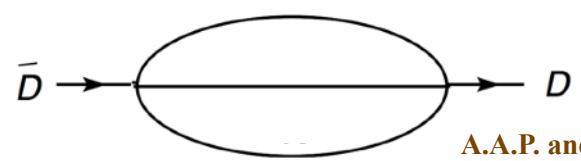
Falls short of the experimentally observed value of y

★ What about other two body contributions (PV, SP, SS, etc.)?

- can use similar techniques to evaluate contribution to mixing as above 2BIS…
- … but V, P′, S states are not good asymptotic states!
- we get new SU(3)-breaking contribution from the widths of those states!

Jiang, Yu, Qin, Li, and Lu, 2017

Since we are to use experimental data,
use Dalitz plot analyses to get at these contributions



A.A.P. and R. Briere
arXiv:1810.xxxx

Finite width effects: (near) future

1. To counteract the effects of finite widths and avoid double counting, one should work directly with Dalitz plot decays of D-mesons.
   - BESIII data on CP-fractions and DP would prove important

T. Gershon, J. Libby, G. Wilkinson
PLB 750 (2015) 338

2. There is no need to perform isobar parameterization of those Dalitz plots, use model-independent parameterizations are advantageous.

A.A.P. and R. Briere
arXiv:1804.xxxx

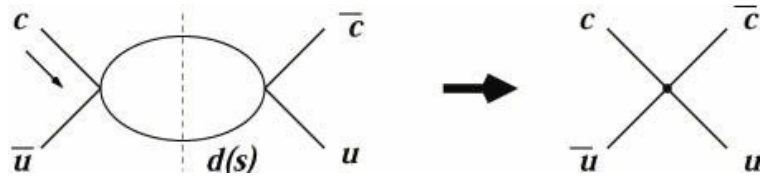
4. Things to take home

- Philosophy: does exclusive approach to mixing constitute a prediction?
- Computation of charm mixing amplitudes is a difficult task
  - no dominant heavy dof, as in beauty decays
  - light dofs give no contribution in the flavor SU(3) limit
- Charm quark is neither heavy nor light enough for a clean application of well-established techniques
  - "heavy-quark-expansion" techniques miss threshold effects
  - "heavy-quark" techniques give numerically leading contribution that is parametrically suppressed by $1/m^6$
  - "hadronic" techniques need to sum over large number of intermediate states, AND cannot use current experimental data on D-decays
  - "hadronic" techniques currently neglect some sources of SU(3) breaking
- Finite width effects complicate use of experimental data in exclusive analyses to obtain mass and lifetime differences
  - instead, direct use of Dalitz decays of D-mesons is desirable
- Quantum-coherent initial states allow for unique measurements


The Evening Herald
EXTRA!
WEATHER
FROM THE WVLRLD

Mixing: short-distance estimates

★ SD calculation: expand the operator product in $1/m_c$, e.g.



E. Golowich and A.A.P.
Phys. Lett. B625 (2005) 53

★ Note that $1/m_c$ is not small, while factors of m_s make the result small

- keep $V_{ub} \neq 0$, so the leading SU(3)-breaking contribution is suppressed by $\lambda_b^2 \sim \lambda^{10}$
- ... but it is tiny, so look for SU(3)-breaking effects that come from mass insertions and quark condensates

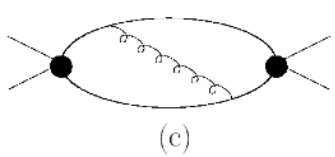
H. Georgi, …
I. Bigi, N. Uraltsev

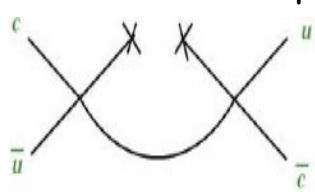
M. Bobrowski et al
JHEP 1003 (2010) 009

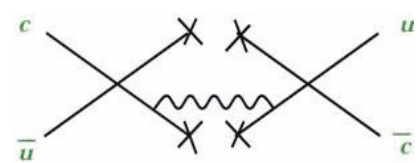
$$\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}\right) + 2\lambda_s\lambda_b \left(\Gamma_{12}^{sd} - \Gamma_{12}^{dd}\right) - \lambda_b^2\, \Gamma_{12}^{dd}$$

LO:	$O(m_s^4)$	$O(m_s^2)$	$O(1)$
NLO:	$O(m_s^3)$	$O(m_s^1)$	$O(1)$

- ... main contribution comes from dim-12 operators!!!







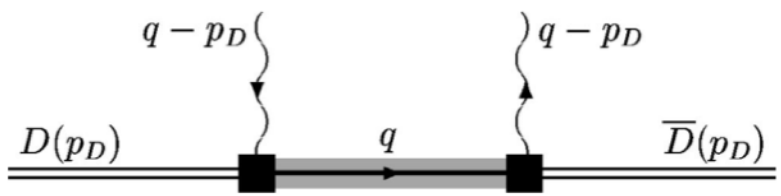
Guestimate: $x \sim y \sim 10^{-3}$?

Exclusive approach to mixing: no data

Falk, Grossman, Ligeti, Nir. A.A.P.
Phys.Rev. D69, 114021, 2004
Falk, Grossman, Ligeti, and A.A.P.
Phys.Rev. D65, 054034, 2002

★ LD calculation: consider the correlation

$$\Sigma_{p_D}(q) = i\int d^4z \left\langle \overline{D}(p_D)\right| T\left[H_w(z)H_w(0)\right]\left| D(p_D)\right\rangle e^{i(q-p_D)z}$$



$$-\frac{1}{2m_D}\Sigma_{p_D}(p_D) = \left(\Delta m - \frac{i}{2}\Delta\Gamma\right)$$

★ $\Sigma_{p_D}(q)$ is an analytic function of q. To write a disp. relation, go to to HQET:

$$H_w = \frac{4G_F}{\sqrt{2}} V_{cq_1} V^*_{uq_2} \sum_i C_i O_i = \hat{H}_w \left[e^{-im_c v\cdot z} h_v^{(c)} + e^{im_c v\cdot z}\tilde{h}_v^{(c)}\right] + \ldots$$

$$\left| D(p = mv)\right\rangle = \sqrt{m}\left| H(v)\right\rangle + \ldots$$

Now we can interpret $\Sigma_{p_D}(q)$ for all q

Dispersion relations for mixing

★ ...this implies for the correlator

Rapidly oscillates for large m_c

$$\Sigma_{p_D}(q) = i\int d^4z \left\langle \overline{H}(v)\middle| T e^{i(q-p_D-m_c v)z}\left[\hat{H}_w h_v^{(c)}(z), \hat{H}_w \tilde{h}_v^{(c)}(0)\right]\middle| H(v)\right\rangle +$$
$$i\int d^4z \left\langle \overline{H}(v)\middle| T e^{i(q-p_D+m_c v)z}\left[\hat{H}_w \tilde{h}_v^{(c)}(z), \hat{H}_w h_v^{(c)}(0)\right]\middle| H(v)\right\rangle + \ldots$$

★ HQ mass dependence drops out for the second term, so for $\overline{\Sigma}_v(q) = \Sigma_{p_D}(q)/m_D$

$$\boxed{\overline{\Sigma}_v(q) = -2\Delta m(E) + i\Delta\Gamma(E)}$$

mass and width difference of a heavy meson with mass E

★ Thus, a dispersion relation

$$\boxed{\Delta m = -\frac{1}{2\pi} P \int_{2m_\pi}^{\infty} dE \left[\frac{\Delta\Gamma(E)}{E - m_D} + O\left(\frac{\Lambda_{QCD}}{E}\right)\right]}$$

Compute $\Delta\Gamma$, then find Δm!

No data: $SU(3)_F$ and phase space

★ "Repackage" the analysis: look at the <u>complete</u> multiplet contribution

$$y = \sum_{F_R} y_{F,R}\, Br(D^0 \to F_R) = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \to n)$$

y for each SU(3) multiplet

Each is **0** in SU(3)

★ Does it help? If only phase space is taken into account: **<u>mild</u>** model dependence

$$y_{F,R} = \frac{\sum_{n \in F_R} \langle \overline{D}^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \Gamma(D^0 \to n)} = \frac{\sum_{n \in F_R} \langle \overline{D}^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \langle D^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}$$

Can consistently compute