New approaches to exclusive predictions for charm mixing

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• Charmed mixing: short and long distance
  ➡ exclusive approach: no experiment
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Introduction: experimental facts

★ Experimental fact: charm mixing parameters are non-zero

★ ... and rather large

- if CP-violation is neglected...

\[
x = 0.50^{+0.13}_{-0.14} \%
\]

\[
y = 0.63 \pm 0.08 \%
\]

- if CP-violation is allowed

\[
x = 0.36^{+0.21}_{-0.16} \%
\]

\[
y = 0.67^{+0.06}_{-0.13} \%
\]

... maybe still on the fence about x

★ Theoretical fact: exclusive and inclusive predictions differ by an order of magnitude
Intro: quark-hadron duality, lifetimes

★ New Physics couples to quark degrees of freedom, we observe hadrons!
- need to know how to compute non-perturbative matrix elements
- need to understand how quark-hadron duality works

★ Observables computed in terms of hadronic degrees of freedom...

\[
\Gamma_{\text{hadron}}(H_b) = \sum_{\text{all final state hadrons}} \Gamma(H_b \rightarrow h_i)
\]

★ ... must match observables computed in terms of quark degrees of freedom

\[
\Gamma(H_b) = \frac{1}{2M_b} \langle H_b \mid T \mid H_b \rangle = \frac{1}{2M_b} \langle H_b \mid \text{Im} \int d^4x \left\{ H_{\text{eff}}^{A^B=1}(x) H_{\text{eff}}^{A^B=1}(0) \right\} \mid H_b \rangle
\]

\[
\Gamma(H_b) = \frac{G_F^2 m_Q^5}{192\pi^3} \left[ A_0 + \frac{A_2}{m_Q^2} + \frac{A_2}{m_Q^3} + \ldots \right]
\]

HQ expansion converges reasonably well...

Bloom, Gilman; Poggio, Quinn, Weinberg
How to define quark-hadron duality and quantify its violations?

- Compute quark correlator in Euclidean space and analytically continue to Minkowski space [exact calculation in ES = exact result in MS]
- Expand it in $a_\Sigma$ and "1/Q ~ 1/m_Q": series truncation
- Any deviation beyond “natural uncertainty” is treated as violation of quark-hadron duality [resonances, instantons,…]

This definition is due to M. Shifman
Quark-hadron duality: lifetimes

★ In case of b-flavored hadrons can compare directly to experiment

<table>
<thead>
<tr>
<th>Lifetime ratio</th>
<th>Experimental average</th>
<th>HQE prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau(B^+)/\tau(B^0)$</td>
<td>$1.076 \pm 0.004$</td>
<td>$1.04^{+0.05}_{-0.01} \pm 0.02 \pm 0.01$</td>
</tr>
<tr>
<td>$\tau(B_s^+)/\tau(B^0)$</td>
<td>$0.990 \pm 0.004$</td>
<td>$1.001 \pm 0.002$</td>
</tr>
<tr>
<td>$\tau(\Lambda_b^+)/\tau(B^0)$</td>
<td>$0.967 \pm 0.007$</td>
<td>$0.935 \pm 0.054$</td>
</tr>
<tr>
<td>$\tau(\Xi_b^+)/\tau(\Xi_b^-)$</td>
<td>$0.929 \pm 0.028$</td>
<td>$0.95 \pm 0.06$</td>
</tr>
</tbody>
</table>

... works surprisingly well...

<table>
<thead>
<tr>
<th>Channel</th>
<th>Expansion parameter $x$</th>
<th>Numerical value</th>
<th>exp$[-1/x]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \to c\bar{c}s$</td>
<td>$\Delta \sqrt{m_b^2 - 4m_c^2}$</td>
<td>$0.054 - 0.58$</td>
<td>$9.4 \cdot 10^{-9} - 0.18$</td>
</tr>
<tr>
<td>$b \to c\bar{c}u$s</td>
<td>$\Delta \sqrt{m_b^2 - m_c^2}$</td>
<td>$0.045 - 0.49$</td>
<td>$1.9 \cdot 10^{-10} - 0.13$</td>
</tr>
<tr>
<td>$b \to u\bar{u}s$</td>
<td>$\Delta \sqrt{m_b^2 - 4m_u^2}$</td>
<td>$0.042 - 0.48$</td>
<td>$4.2 \cdot 10^{-11} - 0.12$</td>
</tr>
</tbody>
</table>

★ How does it work for charmed hadrons?
### Quark-hadron duality: mixing

★ How can one tell that a process is dominated by long-distance or short-distance?

★ To start thing off, mass and lifetime differences of mass eigenstates...

\[ x_D = \frac{M_2 - M_1}{\Gamma_D}, \quad y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D} \]

★ ...can be calculated as real and imaginary parts of a correlation function

\[ y_D = \frac{1}{2M_D\Gamma_D} \text{Im} \langle D^0 | i \int d^4 x \, T \left\{ \mathcal{H}_w^{\Delta C = 1} (x) \, \mathcal{H}_w^{\Delta C = 1} (0) \right\} | D^0 \rangle \]

\[ x_D = \frac{1}{2M_D\Gamma_D} \text{Re} \left[ 2\langle D^0 | H^{\Delta C = 2} | D^0 \rangle + \langle D^0 | i \int d^4 x \, T \left\{ \mathcal{H}_w^{\Delta C = 1} (x) \, \mathcal{H}_w^{\Delta C = 1} (0) \right\} | D^0 \rangle \right] \]

★ ... or can be written in terms of hadronic degrees of freedom...

\[ y = \frac{1}{2\Gamma} \sum_n \rho_n \left[ \langle D^0 | H_W^{\Delta C = 1} | n \rangle \langle n | H_W^{\Delta C = 1} | D^0 \rangle + \langle D^0 | H_W^{\Delta C = 1} | n \rangle \langle n | H_W^{\Delta C = 1} | D^0 \rangle \right] \]
Mixing: short vs long distance

★ How can one tell that a process is dominated by long-distance or short-distance?

\[
y = \frac{1}{2\Gamma} \sum_n \rho_n \left[ \langle D^0 | \mathcal{H}_W^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_W^{\Delta C=1} | D^0 \rangle + \langle D^0 | \mathcal{H}_W^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_W^{\Delta C=1} | D^0 \rangle \right]
\]

★ It is important to remember that the expansion parameter is \(1/E_{\text{released}}\)

\[
y_D = \frac{1}{2M_D \Gamma_D} \text{Im} \left\langle D^0 \right| i \int d^4 x \mathcal{T} \left\{ \mathcal{H}_w^{\Delta C=1}(x) \mathcal{H}_w^{\Delta C=1}(0) \right\} \left| D^0 \rightangle
\]

OPE-leading contribution:

★ In the heavy-quark limit \(m_c \to \infty\) we have \(m_c \gg \sum m_{\text{intermediate quarks}}\), so \(E_{\text{released}} \sim m_c\)
  - the situation is similar to B-physics, where it is "short-distance" dominated
  - one can consistently compute pQCD and 1/m corrections

★ But wait, \(m_c\) is NOT infinitely large! What happens for finite \(m_c\)???
  - how is large momentum routed in the diagrams?
  - are there important hadronization (threshold) effects?
Threshold (and related) effects in OPE

★ How can one tell that a process is dominated by long-distance or short-distance?

★ Let's look at how the momentum is routed in a leading-order diagram
- injected momentum is $p_c \sim m_c$
- thus, $p_1 \sim p_2 \sim m_c / 2 \sim O(\Lambda_{QCD})$?

Still OK with OPE, signals large nonperturbative contributions

★ For a particular example of the lifetime difference, have hadronic intermediate states

- let's use an example of KKK intermediate state
- in this example, $E_{\text{released}} \sim m_D - 3 m_K \sim O(\Lambda_{QCD})$

★ Similar threshold effects exist in B-mixing calculations
- but $m_b \gg \Sigma m_{\text{intermediate quarks}}$, so $E_{\text{released}} \sim m_b$ (almost) always
- quark-hadron duality takes care of the rest!

Thus, two approaches:
1. insist on $1/m_c$ expansion, hope for quark-hadron duality
2. saturate correlators by hadronic states
Aside: classification of charm decays

★ Can be classified by SM CKM suppression

★ Cabibbo-favored (CF) decay
- originates from $c \rightarrow s \, u\bar{d}$
- examples: $D^0 \rightarrow K\pi^+$

★ Singly Cabibbo-suppressed (SCS) decay
- originates from $c \rightarrow q \, uq\bar{q}$
- examples: $D^0 \rightarrow \pi\pi$ and $D^0 \rightarrow KK$

★ Doubly Cabibbo-suppressed (DCS) decay
- originates from $c \rightarrow d \, u\bar{s}$
- examples: $D^0 \rightarrow K^+\pi^-$

★ “Common final states” for $D$ and $\bar{D}$ generate mixing in exclusive approach
Exclusive approach to mixing: use data?

★ LD calculation: saturate the correlator by hadronic states, e.g.

\[
y = \frac{1}{2\Gamma} \sum_n \rho_n \left[ \langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle + \langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]
\]

... with \( n \) being all states to which \( D^0 \) and \( \bar{D}^0 \) can decay. Consider \( \pi \pi, \pi K, KK \) intermediate states as an example...

\[
y_2 = Br(D^0 \to K^+ K^-) + Br(D^0 \to \pi^+ \pi^-) - 2 \cos \delta \sqrt{Br(D^0 \to K^+ \pi^-) Br(D^0 \to \pi^+ K^-)}
\]

If every \( Br \) is known up to \( O(1%) \), the result is expected to be \( O(1%) \)!

The result here is a series of large numbers with alternating signs, \( SU(3) \) forces 0.

If experimental data on \( Br \) is used, are we only sensitive to exit. uncertainties?

★ The experimental uncertainties of hadronic \( Br \) have significantly improved

- can we now use experimental data to predict \( y \)?
- is it a prudent method to ``predict'' \( y \)?

J. Donoghue et. al.
L. Wolfenstein
P. Colangelo et. al.
H.Y. Cheng and C. Chiang

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★ The experimental uncertainties of hadronic \( Br \) have significantly improved

- can we now use experimental data to predict \( y \)?
- is it a prudent method to ``predict'' \( y \)?
Exclusive approach to mixing: no decay data!

★ Need to “repackage” the analysis: look at complete multiplet contribution

\[
y = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \rightarrow n)
\]

Numerator:

\[
A_{N,8} = |A_0|^2 s_1^2 \left[ \frac{1}{2} \Phi(\eta, \eta) + \frac{1}{2} \Phi(\pi^0, \pi^0) + \frac{1}{3} \Phi(\eta, \pi^0) + \Phi(\pi^+, \pi^-) - \Phi(K^0, \pi^0) \right.
\]
\[
+ \Phi(K^+, K^-) - \frac{1}{6} \Phi(\eta, K^0) - \frac{1}{6} \Phi(\eta, \bar{K}^0) - \Phi(K^+, \pi^-) - \Phi(K^-, \pi^+) \right]
\]

Denominator:

\[
A_{D,8} = |A_0|^2 \left[ \frac{1}{6} \Phi(\eta, K^0) + \Phi(K^+, \pi^-) + \frac{1}{2} \Phi(K^0, \pi^0) + O(s_1^2) \right]
\]

★ This contribution is calculable....

\[
y_{2,8} = \frac{A_{N,8}}{A_{D,8}} = -0.038 s_1^2 = -1.8 \times 10^{-4}
\]

.... but completely negligible!

1. Repeat for other states
2. Multiply by Br_{Fr} to get y
Old results

★ Repeat for other intermediate states:

- Product is naturally $O(1\%)$
- No (symmetry-enforced) cancellations
- Disp relation: compute $x$ (model-dependence)

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Final state representation} & \frac{y_{F,R}}{s_1^2} & y_{F,R} (\%) \\
\hline
PP & 8 & -0.0038 & -0.018 \\
& 27 & -0.00071 & -0.0034 \\
PV & 8_S & 0.031 & 0.15 \\
& 8_A & 0.032 & 0.15 \\
& 10 & 0.020 & 0.10 \\
& 10' & 0.016 & 0.08 \\
& 27 & 0.040 & 0.19 \\
(VV)_{s\text{-wave}} & 8 & -0.081 & -0.39 \\
& 27 & -0.061 & -0.30 \\
(VV)_{p\text{-wave}} & 8 & -0.10 & -0.40 \\
& 27 & -0.14 & -0.70 \\
(VV)_{d\text{-wave}} & 8 & 0.51 & 2.5 \\
& 27 & 0.57 & 2.8 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Final state representation} & \frac{y_{F,R}}{s_1^2} & y_{F,R} (\%) \\
\hline
(3P)_{s\text{-wave}} & 8 & -0.48 & -2.3 \\
& 27 & -0.11 & -0.54 \\
(3P)_{p\text{-wave}} & 8 & -1.13 & -5.5 \\
& 27 & -0.07 & -0.36 \\
(3P)_{\text{form-factor}} & 8 & -0.44 & -2.1 \\
& 27 & -0.13 & -0.64 \\
4P & 8 & 3.3 & 16 \\
& 27 & 2.2 & 9.2 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Final state} & \text{fraction} \\
\hline
PP & 5\% \\
PV & 10\% \\
(VV)_{s\text{-wave}} & 5\% \\
(VV)_{d\text{-wave}} & 5\% \\
3P & 5\% \\
4P & 10\% \\
\hline
\end{array}
\]

- Note dominance of near-threshold states!


naturally implies that $x,y \sim 1\%$ is expected in the Standard Model
Exclusive approach to mixing: use data!

★ What if we insist on using experimental data anyway?

★ Ex., one can employ Factorization-Assisted Topological Amplitudes

<table>
<thead>
<tr>
<th>Modes</th>
<th>$B_{\text{exp}}$</th>
<th>$B_{\text{FAT}}$</th>
<th>Modes</th>
<th>$B_{\text{exp}}$</th>
<th>$B_{\text{FAT}}$</th>
<th>Modes</th>
<th>$B_{\text{exp}}$</th>
<th>$B_{\text{FAT}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0 K^0$</td>
<td>24.0 ± 0.8</td>
<td>24.2 ± 0.8</td>
<td>$\pi^0 K^{*0}$</td>
<td>37.5 ± 2.9</td>
<td>35.9 ± 2.2</td>
<td>$K^0 \rho^0$</td>
<td>12.8$^{+1.4}_{-1.6}$</td>
<td>13.5 ± 1.4</td>
</tr>
<tr>
<td>$\pi^+ K^-$</td>
<td>39.3 ± 0.4</td>
<td>39.2 ± 0.4</td>
<td>$\pi^+ K^{*-}$</td>
<td>54.3 ± 4.4</td>
<td>62.5 ± 2.7</td>
<td>$K^- \rho^+$</td>
<td>111.0 ± 9.0</td>
<td>105.0 ± 5.2</td>
</tr>
<tr>
<td>$\eta K^0$</td>
<td>9.70 ± 0.6</td>
<td>9.6 ± 0.6</td>
<td>$\eta K^{*0}$</td>
<td>9.6 ± 3.0</td>
<td>6.1 ± 1.0</td>
<td>$K^0 \omega$</td>
<td>22.2 ± 1.2</td>
<td>22.3 ± 1.1</td>
</tr>
<tr>
<td>$\eta' K^0$</td>
<td>19.0 ± 1.0</td>
<td>19.5 ± 1.0</td>
<td>$\eta' K^{*0}$</td>
<td>$&lt; 1.10$</td>
<td>0.19 ± 0.01</td>
<td>$K^0 \phi$</td>
<td>8.47$^{+0.66}_{-0.34}$</td>
<td>8.2 ± 0.6</td>
</tr>
<tr>
<td>$\pi^+ \pi^-$</td>
<td>1.421 ± 0.025</td>
<td>1.44 ± 0.02</td>
<td>$\pi^+ \rho^-$</td>
<td>5.09 ± 0.34</td>
<td>4.5 ± 0.2</td>
<td>$\pi^- \rho^+$</td>
<td>10.0 ± 0.6</td>
<td>9.2 ± 0.3</td>
</tr>
<tr>
<td>$K^+ K^-$</td>
<td>4.01 ± 0.07</td>
<td>4.05 ± 0.07</td>
<td>$K^+ K^{*-}$</td>
<td>1.62 ± 0.15</td>
<td>1.8 ± 0.1</td>
<td>$K^- K^{*-}$</td>
<td>4.50 ± 0.30</td>
<td>4.3 ± 0.2</td>
</tr>
<tr>
<td>$K^0 K^{*-}$</td>
<td>0.36 ± 0.08</td>
<td>0.29 ± 0.07</td>
<td>$K^0 K^{*-}$</td>
<td>0.18 ± 0.04</td>
<td>0.19 ± 0.03</td>
<td>$K^0 K^{*0}$</td>
<td>0.21 ± 0.04</td>
<td>0.19 ± 0.03</td>
</tr>
<tr>
<td>$\pi^0 \eta$</td>
<td>0.69 ± 0.07</td>
<td>0.74 ± 0.03</td>
<td>$\eta \rho^0$</td>
<td>1.4 ± 0.2</td>
<td></td>
<td>$\pi^0 \omega$</td>
<td>0.117 ± 0.035</td>
<td>0.10 ± 0.03</td>
</tr>
<tr>
<td>$\pi^0 \eta'$</td>
<td>0.91 ± 0.14</td>
<td>1.08 ± 0.05</td>
<td>$\eta' \rho^0$</td>
<td>0.25 ± 0.01</td>
<td></td>
<td>$\pi^0 \phi$</td>
<td>1.35 ± 0.10</td>
<td>1.4 ± 0.1</td>
</tr>
<tr>
<td>$\eta \eta$</td>
<td>1.70 ± 0.20</td>
<td>1.86 ± 0.06</td>
<td>$\eta \omega$</td>
<td>2.21 ± 0.23</td>
<td>2.0 ± 0.1</td>
<td>$\eta \phi$</td>
<td>0.14 ± 0.05</td>
<td>0.18 ± 0.04</td>
</tr>
<tr>
<td>$\eta \eta'$</td>
<td>1.07 ± 0.26</td>
<td>1.05 ± 0.08</td>
<td>$\eta' \omega$</td>
<td>0.044 ± 0.004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^0 \pi^0$</td>
<td>0.826 ± 0.035</td>
<td>0.78 ± 0.03</td>
<td>$\pi^0 \rho^0$</td>
<td>3.82 ± 0.29</td>
<td>4.1 ± 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^0 K^0$</td>
<td>0.069 ± 0.002</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^- K^+$</td>
<td>0.133 ± 0.009</td>
<td>0.133 ± 0.001</td>
<td>$\pi^- K^{*-}$</td>
<td>0.345$^{+0.180}_{-0.102}$</td>
<td>0.40 ± 0.02</td>
<td>$K^+ \rho^-$</td>
<td>0.144 ± 0.009</td>
<td></td>
</tr>
<tr>
<td>$\eta K^0$</td>
<td>0.027 ± 0.002</td>
<td></td>
<td>$\eta K^{*0}$</td>
<td>0.017 ± 0.003</td>
<td></td>
<td>$K^0 \omega$</td>
<td>0.064 ± 0.003</td>
<td></td>
</tr>
<tr>
<td>$\eta' K^0$</td>
<td>0.056 ± 0.003</td>
<td></td>
<td>$\eta' K^{*0}$</td>
<td>0.00055 ± 0.00004</td>
<td></td>
<td>$K^0 \phi$</td>
<td>0.024 ± 0.002</td>
<td></td>
</tr>
</tbody>
</table>

Jiang, Yu, Qin, Li, and Lu, 2017

★ ... but it appears to yield a smaller result, $y_{PP+PV} = (0.21 ± 0.07)\%$. 

Alexey A Petrov (WSU & LCTP)
Exclusive approach to mixing: use data!

★ What if we insist on using experimental data anyway?

★ Possible additional contributions?
  - each intermediate state has a finite width, i.e. is not a proper asymptotic state
  - within each multiplet widths experience (incomplete) SU(3) cancelations
  - this effect already happens for the simplest intermediate states!

★ Consider, for illustration, a set of single-particle intermediate states:

$$-\sum_{p_D} (p_D^D) \bigg|_{\text{tot}}^{\text{res}} = \frac{1}{2m_D} \sum_R \text{Re} \frac{\langle D_L | H_W | R \rangle \langle R | H_W^\dagger | D_L \rangle}{m_D^2 - m_R^2 + i\Gamma_R m_D} - (D_L \to D_S)$$

★ Each resonance contributes to $\Delta \Gamma$ only because of its finite width!
Finite width effects: one-body contributions

★ Multiplet effects for (single-particle) intermediate states
- in this simple example: heavy pion, kaon and eta/eta'
- each single-particle intermediate state has a rather large width

\[
\Delta \Gamma_D^{\text{res}} = \Delta \Gamma^{(K_H)}_D - \frac{1}{4} \Delta \Gamma^{(\pi_H)}_D - \frac{3 \cos^2 \theta_H}{4} \Delta \Gamma^{(\eta_H)}_D - \frac{1}{4} \sin^2 \theta_H \Delta \Gamma^{(\eta'_H)}_D
\]

- where for each state \( \Delta \Gamma_D^{\text{res}} = -C f_R^2 \frac{\mu_R \gamma_R}{(1 - \mu_R)^2 + \gamma_R^2} \) with \( \mu_R = m_R^2/m_D^2 \) and \( \gamma_R = \Gamma_R/m_D \)
- ... and a model calculation gives \( C \equiv 2m_D (G_F a_2 f_D \xi_d / \sqrt{2})^2 \)
- SU(3) forces cancellations: a new SU(3) breaking effect due to widths!

| Resonance | \( |\Delta m_D| \times 10^{-16} \) (GeV) | \( |\Delta \Gamma_D| \times 10^{-16} \) (GeV) |
|-----------|---------------------------------|---------------------------------|
| \( K(1460) \) | \( \sim 1.24 (f_{K(1460)}/0.025)^2 \) | \( \sim 0.88 (f_{K(1460)}/0.025)^2 \) |
| \( \eta(1760) \) | \( (0.77 \pm 0.27) (f_{\eta(1760)}/0.01)^2 \) | \( (0.43 \pm 0.53) (f_{\eta(1760)}/0.01)^2 \) |
| \( \pi(1800) \) | \( (0.13 \pm 0.06) (f_{\pi(1800)}/0.01)^2 \) | \( (0.41 \pm 0.11) (f_{\pi(1800)}/0.01)^2 \) |
| \( K(1830) \) | \( \sim 0.29 (f_{K(1830)}/0.01)^2 \) | \( \sim 1.86 (f_{K(1830)}/0.01)^2 \) |

E. Golowich and A.A.P.
PLB427 (1998) 172-178
Let us take another look at those one-body contributions
- the width of each excited light quark state \( \Gamma_R = \Gamma(R \rightarrow P_1 P_2) + \Gamma(R \rightarrow P_1 P_2 P_3) + \ldots \)
- ... which is equivalent to accounting for resonant FSI in 2-body intermediate state!

Since we shall be using experimental data to compute 2-body contributions, this effect will be taken into account automatically!

It's consistent to omit 1-body IS if experimental data is used.
Let us apply similar logic to two-body contributions
- consider contributions from the stable (wrt strong interactions) octet of pions, kaons, etas

\[ y_{PP} = (0.1 - 20) \times 10^{-4} \]
Falls short of the experimentally observed value of \( y \)

What about other two body contributions (PV, SP, SS, etc.)?
- can use similar techniques to evaluate contribution to mixing as above 2BIS...
- ... but \( V, P', S \) states are not good asymptotic states!
- we get new SU(3)-breaking contribution from the widths of those states!

Since we are to use experimental data,
use Dalitz plot analyses to get at these contributions
1. To counteract the effects of finite widths and avoid double counting, one should work directly with Dalitz plot decays of D-mesons.
   - BESIII data on CP-fractions and DP would prove important
     
     T. Gershon, J. Libby, G. Wilkinson
     PLB 750 (2015) 338

2. There is no need to perform isobar parameterization of those Dalitz plots, use model-independent parameterizations are advantageous.

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     arXiv:1804.xxxx
4. Things to take home

- Philosophy: does exclusive approach to mixing constitute a prediction?
- Computation of charm mixing amplitudes is a difficult task
  - no dominant heavy dof, as in beauty decays
  - light dofs give no contribution in the flavor SU(3) limit
- Charm quark is neither heavy nor light enough for a clean application of well-established techniques
  - “heavy-quark-expansion” techniques miss threshold effects
  - “heavy-quark” techniques give numerically leading contribution that is parametrically suppressed by $1/m^6$
  - “hadronic” techniques need to sum over large number of intermediate states, AND cannot use current experimental data on D-decays
  - “hadronic” techniques currently neglect some sources of SU(3) breaking
- Finite width effects complicate use of experimental data in exclusive analyses to obtain mass and lifetime differences
  - instead, direct use of Dalitz decays of D-mesons is desirable
- Quantum-coherent initial states allow for unique measurements
Mixing: short-distance estimates

★ SD calculation: expand the operator product in $1/m_c$, e.g.

$$\Gamma_{12} = -\lambda_s^2 \left( \Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd} \right) + 2\lambda_s \lambda_b \left( \Gamma_{12}^{sd} - \Gamma_{12}^{dd} \right) - \lambda_b^2 \Gamma_{12}^{dd}$$

- keep $V_{ub} \neq 0$, so the leading SU(3)-breaking contribution is suppressed by $\lambda_b^2 \sim \lambda^{10}$
- ... but it is tiny, so look for SU(3)-breaking effects that come from mass insertions and quark condensates

LO: \( O(m_s^4) \) \hspace{1cm} NLO: \( O(m_s^2) \) \hspace{1cm} \( O(1) \)

- ... main contribution comes from dim-12 operators!!!
Exclusive approach to mixing: no data

★ LD calculation: consider the correlation

\[ \Sigma_{PD}(q) = i \int d^4z \left< \overline{D}(p_D) \right| T \left[ H_w(z) H_w(0) \right] |D(p_D)\rangle e^{i(q-p_D)z} \]

\[ -\frac{1}{2m_D} \Sigma_{PD}(p_D) = \left( \Delta m - \frac{i}{2} \Delta \Gamma \right) \]

★ \( \Sigma_{PD}(q) \) is an analytic function of \( q \). To write a disp. relation, go to HQET:

\[ H_w = \frac{4G_F}{\sqrt{2}} V_{cq_1} V^*_{uq_2} \sum_i C_i O_i = \hat{H}_w \left[ e^{-im_{\nu \nu} z} \hat{h}_v^{(c)} + e^{im_{\nu \nu} z} \tilde{\hat{h}}_v^{(c)} \right] + \ldots \]

\[ |D(p = mv)\rangle = \sqrt{m} |H(v)\rangle + \ldots \]

Now we can interpret \( \Sigma_{PD}(q) \) for all \( q \)

Dispersion relations for mixing

★ ...this implies for the correlator

\[ \Sigma_{p_D}(q) = i \int d^4z \left\langle H(v) \right| T e^{i(q-p_D-m_v)z} \left[ \hat{H}_w h_v^{(c)}(z), \hat{H}_w \tilde{h}_v^{(c)}(0) \right| H(v) \rangle + \]

\[ i \int d^4z \left\langle H(v) \right| T e^{i(q-p_D+m_v)z} \left[ \hat{H}_w \tilde{h}_v^{(c)}(z), \hat{H}_w h_v^{(c)}(0) \right| H(v) \rangle + \ldots \]

★ HQ mass dependence drops out for the second term, so for \( \Sigma_v(q) = \Sigma_{p_D}(q)/m_D \)

\[ \Sigma_v(q) = -2\Delta m(E) + i\Delta \Gamma(E) \]

★ Rapidly oscillates for large \( m_c \)

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★ Thus, a dispersion relation

\[ \Delta m = -\frac{1}{2\pi} P \int_{2m_\pi}^{\infty} dE \left[ \frac{\Delta \Gamma(E)}{E - m_D} + O\left( \frac{\Lambda_{QCD}}{E} \right) \right] \]

Compute \( \Delta \Gamma \), then find \( \Delta m! \)
No data: SU(3)_F and phase space

★ “Repackage” the analysis: look at the complete multiplet contribution

\[ y = \sum_{F_R} y_{F,R} \, Br(D^0 \rightarrow F_R) = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \rightarrow n) \]

y for each SU(3) multiplet

Each is 0 in SU(3)

★ Does it help? If only phase space is taken into account: mild model dependence

\[ y_{F,R} = \frac{\sum_{n \in F_R} \langle D^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \Gamma(D^0 \rightarrow n)} = \frac{\sum_{n \in F_R} \langle D^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \langle D^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle} \]

Can consistently compute