Overview of Kaon physics

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Why Kaon?

- Kaon observables are sensitive to NP at a very high scale, which is not accessible at the LHC.
  - FCNC and CP violation in Kaon system are suppressed in the SM.

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}^{SM} + \frac{1}{\Lambda_{NP}^2} \sum_i C_i \mathcal{O}_i^{\text{dim6}} \]

\[ \Lambda_{NP} \sim \begin{cases} \mathcal{O}(10^5 \text{TeV}) & : K^0 \\ \mathcal{O}(10^4 \text{TeV}) & : D^0 \\ \mathcal{O}(10^3 \text{TeV}) & : B_{d,s} \end{cases} \]

- Several on-going experiments for Kaon observables
  (KOTO/NA62/LHCb + KLOE-2/TREK...)

- Using recent result of lattice calculation, there is discrepancy in $\varepsilon'/\varepsilon$ between SM value and data (discuss in detail later)
Outline

- $\epsilon$
- $\epsilon'/\epsilon$
- $K_L \rightarrow \pi^0 \nu \bar{\nu}$
- $K^+ \rightarrow \pi^+ \nu \bar{\nu}$
- $K_S \rightarrow \mu \mu$
\( \varepsilon \) and \( \varepsilon' \)

1964 \( K_L \rightarrow 2\pi \) was observed \( \text{Discovery of CP violation} \)

\[
|K_L\rangle = |K_2\rangle + \varepsilon'|K_1\rangle
\]

\( \eta_{00} = \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)} \equiv \varepsilon - 2\varepsilon' \)

\( \eta_{+-} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} \equiv \varepsilon + \varepsilon' \)

\( |\varepsilon| \approx \frac{1}{3} \left( |\eta_{00}| + 2|\eta_{+-}| \right) \)

\[
\epsilon = \mathcal{O}(10^{-3}) \quad \text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \mathcal{O}(10^{-3}) \quad \varepsilon' = \mathcal{O}(10^{-6})
\]

\( \text{Highly suppressed and sensitive to NP} \)
Indirect CP violation $\epsilon$ gives severe constraint on NP

SM prediction of $\epsilon$ is sensitive to $|V_{cb}|$

$$\epsilon(SD) \propto \text{Im}\lambda_t \left[ \text{Re}\lambda_c \eta_{cc} S_0(x_c) - \text{Re}\lambda_t \eta_{tt} S_0(x_t) - (\text{Re}\lambda_c - \text{Re}\lambda_t) \eta_{ct} S_0(x_c, x_t) \right]$$

$$\simeq |V_{cb}|^2 \lambda^2 \bar{\eta} \left[ |V_{cb}|^2 (1 - \bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} S_0(x_c) \right]$$

$\epsilon$ evaluated from inclusive $|V_{cb}|$ is consistent with the measured value. On the other hand there is $4\sigma$ tension with exclusive $|V_{cb}|$ (still tension in averaged).

See e.g. Bailey, Lee, Lee, Leem 1808.09657

Vcb exclusive vs. inclusive problem

$$|\epsilon|_{\text{exp}} = 2.228(11) \times 10^{-3}$$

$O(10\%)$ NP room on $\epsilon$ is still allowed
**Vcb exclusive vs. inclusive**

![Diagram showing exclusive vs. inclusive Vcb]

**2 different methods for functional form of form factors:**

1. **Model-dependent method**: *CLN*
   - Caprini, Lellouch, and Neubert (CLN) hep-ph/9712417

2. **Model-independent method**: *BGL*
   - Boyd, Grinstein, and Lebed (BGL) hep-ph/9705252

**Recent discussions on exclusive Vcb:**

The gap might be explained in part with *BGL* method

Large deviation from heavy quark symmetry?

The situation of exclusive Vcb is still unclear

*See talk WG2*
\[ \frac{\epsilon'}{\epsilon} = -\frac{\omega}{\sqrt{2} |\epsilon|_{\text{exp}} \Re A_0} \]

\[ A(K^0 \to (\pi\pi)_{I=0,2}) = A_{0,2}e^{i\delta_{0,2}} \]

\[ \left( \begin{array}{c} \text{Im}A_0 \\ \frac{1}{\omega} \end{array} \right) \]

QCD penguin operator

EW penguin operator

\[ \Delta I = 1/2 \text{ rule} \]

\[ \frac{\Re A_0}{\Re A_2} \equiv \frac{1}{\omega} = 22.46 \text{ (exp.)} \]

In the SM, there is accidental cancellation between Im\( A_0 \) and Im\( A_2 \) due to the enhancement factor \( 1/\omega \)

EW penguin is comparable to QCD penguin due to the enhancement factor
**ε′/ε discrepancy**

Decay amplitude

\[ \langle (\pi\pi)_I | \mathcal{H} | K^0 \rangle = \sum_n C_n \langle (\pi\pi)_I | O_n | K^0 \rangle \]

- **Short distance**
  - NLO result has been available since early 90’s
  - NNLO QCD calculation is in progress

- **Long distance (Matrix elements)**
  - First lattice result by RBC-UKQCD in 2015

From the lattice result, ε′/ε has been calculated in SM using data for ReA_{0,2}

\[
\begin{align*}
\text{SM with Lattice} & \quad \left( \frac{\varepsilon'}{\varepsilon} \right)_{\text{SM}} = (1.06 \pm 5.07) \times 10^{-4} \\
\text{Exp} & \quad \left( \frac{\varepsilon'}{\varepsilon} \right)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}
\end{align*}
\]

Kitahara, Nierste and Tremper, 1607.06727

c.f. RBC-UKQCD / Buras, Gorbahn, Jager and Jamin 1507.06345

**2.8σ difference** \( \mathcal{O}(1) \) NP in ε′/ε?
\( \varepsilon'/\varepsilon \) discrepancy

- \( O_6 \) & \( O_8 \) have dominant effects on \( \varepsilon'/\varepsilon \) due to chiral enhancement
  
  \[
  \langle (\pi\pi)_0 | O_6 | K \rangle \propto B_6^{(1/2)} \\
  \langle (\pi\pi)_2 | O_8 | K \rangle \propto B_8^{(3/2)}
  \]

- Values extracted from the lattice result
  
  \[
  B_6^{(1/2)} = 0.57 \pm 0.19 \\
  B_8^{(3/2)} = 0.76 \pm 0.05
  \]

- Error for \( \varepsilon'/\varepsilon \) is dominated by \( B_6^{(1/2)} \)

- Two ways of analytic approaches

  \begin{align*}
  \text{Large } N_C & \quad \text{Dual QCD approach} \\
  B_6^{(1/2)} & \leq B_8^{(3/2)} < 1 \\
  \left( \frac{\varepsilon'}{\varepsilon} \right)_\text{SM} & < (6.0 \pm 2.4) \times 10^{-4}
  \end{align*}

  \begin{align*}
  \text{ChPT (FSI)} \\
  B_6^{(1/2)} & \sim 1.5 \\
  B_8^{(3/2)} & \sim 0.9 \\
  \left( \frac{\varepsilon'}{\varepsilon} \right)_\text{SM} & = (15 \pm 7) \times 10^{-4}
  \end{align*}

Result in DQCD approach gives a strong support to lattice result. On the other hand, result in ChPT is consistent with data

- Wait for improved lattice results

See J. Aebischer talk (WG3, Wed)
Interpretation of $\varepsilon'/\varepsilon$ discrepancy

Motivated by $\varepsilon'/\varepsilon$ discrepancy, several new physics models have been studied:

- **Little Higgs Model with T-parity**
  - Blanke, Buras and Recksiegel 1507.06316
  - Buras, Buttazzo and Knegjens 1507.08672/Buras, 1601.00005
  - Endo, Kitahara, Mishima and KY 1612.08839/Bobeth, Buras, Celis and Jung 1703.04753

- **Modified Z scenario**
  - Buras, Buttazzo and Knegjens 1507.08672 /Buras 1601.00005

- **Z’ models**
  - Buras, Buttazzo, Knegjens 1507.08672 /Buras 1601.00005

- **331 model**
  - Buras and De Fazio 1512.02869/1604.02344

- **MSSM Chargino Z penguin**
  - Endo, Mishima, Ueda and KY 1608.01444
  - Tanimoto and KY 1603.07960
  - Endo, Goto, Kitahara, Mishima, Ueda and KY 1712.04959

- **Gluino Z penguin**
  - Kitahara, Nierste and Tremper 1604.07400,1703.05786
  - Crivellin, D’Ambrosio, Kitahara, Nierste 1712.04959
  - Chobanova, D’Ambrosio, Kitahara, Martínez, Santos, Fernández and KY 1711.11030

- **Gluino Box**
  - Kitahara, Nierste and Tremper 1604.07400,1703.05786

- **Vector-like quarks**
  - Bobeth, Buras, Celis and Jung 1609.04783

- **Right handed current**
  - Cirigliano, Dekens, Vries and Mereghetti 1612.03914
  - Alioli, Cirigliano, Dekens, de Vries and Mereghetti 1703.04751

- **Leptoquark**
  - Bobeth and Buras 1712.01295

- **LR symmetric model**
  - Haba, Umeeda and Yamada 1802.09903/1806.0342

- **Type-III 2HDM**
  - Chen and Nomura 1805.07522/1808.04097

- **Chiral-flavorful vectors**
  - Matsuzaki, Nishiwaki and KY 1806.02312

- **Diquark model**
  - Chen and Nomura 1808.04097

Different implications (correlations & predictions) for other observables appear depending on models $\Rightarrow$ Possibility of model discriminations

Clean signal: $K \to \pi\nu\nu$
**$K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$**

- Highly suppressed in the SM: $\text{BR}_{\text{SM}} \approx 10^{-11}$

  \[
  \text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} = (3.00 \pm 0.30) \times 10^{-11}
  \]

  \[
  \text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (9.11 \pm 0.72) \times 10^{-11}
  \]

- Theoretically clean (Hadronic matrix element can be estimated using isospin symmetry)

- Neutral decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is purely CP violating mode
K_L → π^0νν and K^+ → π^+νν

- KOTO at J-PARC reported new result from the 2015 data this summer

\[
BR(K_L \rightarrow \pi^0\nu\bar{\nu})_{SM} = (3.00 \pm 0.30) \times 10^{-11} \\
BR(K_L \rightarrow \pi^0\nu\bar{\nu})_{exp} < 2.6 \times 10^{-8} \ (90\% C.L.)
\]
\(< 3.0 \times 10^{-9} \ (90\% C.L.) \quad (90\% C.L.)

- KOTO-phase2 aims to measure at 10% accuracy

- NA62 at CERN observed one event in 2016 data

\[
BR(K^+ \rightarrow \pi^+\nu\bar{\nu})_{SM} = (9.11 \pm 0.72) \times 10^{-11} \\
BR(K^+ \rightarrow \pi^+\nu\bar{\nu})_{exp} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}
\]
\(< 14 \times 10^{-10} \ (95\% C.L.)

- Expected about 20 SM events from the 2017-2018 data sample

See M. Koval talk (WG3, Wed)

12/19
There are interesting correlations between Kaon observables depending on the chiral structure of coupling (LH and/or RH)

\[
\text{Re}\left(\frac{\epsilon'}{\epsilon}\right) \propto -\text{Im} \Delta^s_{L} - 3 \text{Im} \Delta^s_{R} + \cdots \quad \text{CPV} \Rightarrow \text{Strong correlation}
\]

\[
\text{BR}(K^+ \to \pi^+ \nu \bar{\nu}) \propto |X + \cdots|^2
\]

\[
\text{BR}(K_L \to \pi^0 \nu \bar{\nu}) \propto (\text{Im } X)^2
\]

\[
|\epsilon_K| \propto \text{Im}\left[(\Delta^s_{L})^2 + (\Delta^s_{R})^2 - 240 \Delta^s_{L} \Delta^s_{R}\right]
\]

**LH scenario**

**LH+RH scenario**
$\frac{\epsilon'}{\epsilon} \leftrightarrow K \rightarrow \pi \nu \bar{\nu}$ - Examples -

**Chargino Z penguin**

- Large trilinear couplings bring enhancement of $\frac{\epsilon'}{\epsilon}$
- LH Z scenario $\Rightarrow$ negative correlation between $\frac{\epsilon'}{\epsilon}$ and $K \rightarrow \pi 0 \nu \bar{\nu}$

**Gluino box**

- Large isospin breaking ($m_{\tilde{U}} \neq m_{\tilde{D}}$) gives effect on $\text{Im}A_2$

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**BR($K_L \rightarrow \pi^0 \nu \bar{\nu}$) $< 0.6$ SM**

**BR($K^+ \rightarrow \pi^+ \nu \bar{\nu}$) O(10~100%) effect**

**KOTO**

**NA62**

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Different correlations between $\frac{\epsilon'}{\epsilon}$ and $K \rightarrow \pi \nu \bar{\nu}$ may allow to distinguish among models
Recent other progress for $\epsilon'/\epsilon$

- **Chromomagnetic operator**
  
  - ETM collaboration has reported the first Lattice result for the $K\to\pi$ matrix element of the chrome magnetic operator
  
  - DQCD gives a similar result

  CMO contribution is not significant in the SM, but it could be important in some NP models

- **BSM operators**

  Matrix elements of BSM operator are calculated in DQCD
  
  Master formula including BSM operators is derived with DQCD

  Scalar & tensor operators, which have chiral enhancement, are important

- **SMEFT study** : *SM effective field theory* $[SU(2) \times U(1) \text{ inv.}] \ (\mu_{EW} < \mu < \mu_{NP})$

  \[
  \mathcal{L} = \mathcal{L}_{SM} + \sum_{i} C_{i} O_{d}^{i,5} \]

  Due to the RG effect, observables have correlation

  - Model independent approach

  The constraints from $K^0$ and $D^0$ mixing as well as EDM are potentially important

  - Z penguin scenario

  $\Delta S=1$ operators generate $\Delta S=2$ contributions, through top-Yukawa enhanced RG evolution
**K→μμ**

It could be used to probe NP contributions. However, there are LD contributions (K→γγ) in SM:

\[ K_S \rightarrow \mu\mu \]

K→γγ is evaluated in ChPT at O(p^4)

\[ \mathcal{B}(K_S \rightarrow \mu^+\mu^-)_{SM} = (4.99 \text{ (LD)} + 0.19 \text{ (SD)}) \times 10^{-12} \]
\[ = (5.18 \pm 1.50 \pm 0.02) \times 10^{-12} \]

_Isidori and Unterdorfer  hep-ph/0311084_

\[ \mathcal{B}(K_S \rightarrow \mu^+\mu^-)_{exp} < 0.8 \times 10^{-9} \text{ LHCb Run1} \]

**SM sensitivity at LHCb Run3 (2021-)**

\[ \mathcal{B}(K_L \rightarrow \mu\mu)_{sm} \]

Leading O(p^4) contributions for K→γγ vanish

\[ K_L \rightarrow \mu\mu \]

Determined from \[ B(K_L \rightarrow \gamma\gamma)_{exp} \]

Sign ambiguity in \[ A(K_L \rightarrow \mu\mu)_{LD} \]

\[ \mathcal{B}(K_L \rightarrow \mu^+\mu^-)_{SM} = \begin{cases} 
(6.85 \pm 0.80 \pm 0.06) \times 10^{-9} & \text{destructive} \\
(8.11 \pm 1.49 \pm 0.13) \times 10^{-9} & \text{constructive}
\end{cases} \]

_Isidori and Unterdorfer  hep-ph/0311084_
_Gorbahn and Haisch  hep-ph/0605203_

\[ \mathcal{B}(K_L \rightarrow \mu^+\mu^-)_{exp} = (6.84 \pm 0.11) \times 10^{-9} \]
$K_S \rightarrow \mu\mu$ - Interference contribution -

A state of $K^0$ (or $\bar{K}^0$) at $t=0$ evolves into mixture of $K_S$ and $K_L$ states

If # of $K^0$ & $\bar{K}^0$ in beam are different from each other, interference contribution btwn $K_S$ and $K_L$ exists

The decay intensity of neutral Kaon beam into $f$:

$$I(t) = \frac{N(K^0)}{N(K^0) + N(\bar{K}^0)} \left| \langle f | \mathcal{H}_{eff} | K^0(t) \rangle \right|^2 + \frac{N(\bar{K}^0)}{N(K^0) + N(\bar{K}^0)} \left| \langle f | \mathcal{H}_{eff} | \bar{K}^0(t) \rangle \right|^2$$

$$= \frac{1}{2} |A(K_S)|^2 e^{-\Gamma_st} + \frac{1}{2} |A(K_L)|^2 e^{-\Gamma_Lt} + D \text{ Re}\left[ e^{-i\Delta M_{K}t} A(K_S)^* A(K_L) \right] e^{-\frac{\Gamma_s + \Gamma_L}{2}t} + O(\bar{\epsilon})$$

$\text{Dilution factor } D \equiv \frac{N(K^0) - N(\bar{K}^0)}{N(K^0) + N(\bar{K}^0)}$

※Nonzero $D$ can be achieved by an accompanying charged kaon tagging $pp \rightarrow K^0 K^- X$

Decay length of the interference contribution is around $2\tau_S$

$O(1\text{m})$: LHCb detector size

$K_L : \tau_L$

$K_S : \tau_S$

$\text{Interference: } \tau \sim 2\tau_S$

$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{eff}$

Decay inside of LHCb detector
**$K_S \rightarrow \mu\mu$ - Interference contribution -**

\[
\mathcal{B}(K_S \rightarrow \mu^+\mu^-)_{\text{eff}} = \mathcal{B}(K_S \rightarrow \mu^+\mu^-) + D \mathcal{B}(K \rightarrow \mu^+\mu^-)_{\text{int}}
\]

\[
\mathcal{B}(K \rightarrow \mu^+\mu^-)_{\text{int}} \propto A(K_S \rightarrow \mu^+\mu^-)^* A(K_L \rightarrow \mu^+\mu^-) \supset \text{Im}A_{SD}A(K_L \rightarrow \mu\mu)_{\text{LD}}
\]

SD effect becomes comparable in size to LD, due to large $A(K_L \rightarrow \mu\mu)_{\text{LD}}$ in interference effect.

The sign of LD $A(K_L \rightarrow \mu\mu)_{\text{LD}}$ can be determined by a measurement of the interference.

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**The interference changes Br. at O(60%) level in SM [more significant in MSSM] and the sign of the LD in $K_L \rightarrow \mu\mu$ can be determined if D=O(1) → LHCb**

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*Figure 1: D'Ambrosio and Kitahara 1707.06999*

*Figure 2: Chobanova, D'Ambrosio, Kitahara, Martínez, Santos, Fernández and KY 1711.11030*
Summary

Many recent progresses in Kaon physics

\[ O(10\%) \text{ NP} \]

\[ \epsilon \quad \epsilon' / \epsilon \quad \text{Vcb} \]

Discrepancy ?

\[ K_L \rightarrow \pi^0 \nu \bar{\nu} \]
\[ K^+ \rightarrow \pi^+ \nu \bar{\nu} \]
\[ K_S \rightarrow \mu \mu \]

Lattice

NP studies

NP studies

Many other interesting topics (not covered in this talk)

e.g. \[
\begin{align*}
\text{LFU test} & : \left( \frac{K_{e2}}{K_{\mu2}} \right) \\
\text{LFV} & : \left( K_L \rightarrow \mu e \right) \\
K & \rightarrow \pi \ell \ell & \text{see A. Juettner talk (WG3 Wed)} \\
\text{correlation with B physics} & : \left( R_D(\ast), R_K(\ast), \ldots \right) & \text{see M. Bordone talk (WG3 Wed)}
\end{align*}
\]

Kaon physics will continue to offer a powerful probe for NP!
Current situation of $\varepsilon'_K/\varepsilon_K$

\[ \propto \frac{\text{Im} A_0}{\frac{\text{Re} A_0}{\text{Re} A_2}} \frac{\text{Im} A_2}{B_6^{(1/2)}} \propto B_8^{(3/2)} \]

- $B_6^{(1/2)} \sim 1.6$, $B_8^{(3/2)} \sim 0.9$
- $B_6^{(1/2)} \sim 1.6$, $B_8^{(3/2)} \sim 0.9$
- $B_8^{(1/2)} \approx 3$, $B_8^{(3/2)} \approx 3.5$
- $B_6^{(1/2)} \leq B_8^{(3/2)} \leq 1$
- $B_6^{(1/2)} \leq B_8^{(3/2)} \leq 0.76$
- $B_6^{(1/2)} \sim 1.5$
- $B_6^{(1/2)} = 0.57$, $B_8^{(3/2)} = 0.76$

Current QCD predictions:
- $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$, $B_8^{(3/2)} = 0.8$

<table>
<thead>
<tr>
<th>Theory</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ChPT</td>
<td>$\sim 14$</td>
</tr>
<tr>
<td>dual QCD</td>
<td>$16.0 \pm 1.5$</td>
</tr>
<tr>
<td>Lattice</td>
<td>$31.0 \pm 11.1$</td>
</tr>
</tbody>
</table>

$\Delta I = 1/2$ rule: $\left( \frac{\text{Re} A_0}{\text{Re} A_2} \right)$

Exp. $22.45 \pm 0.05$
RBC-UKQCD lattice result

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Lattice QCD</th>
<th>Exp. data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re$A_0$</td>
<td>$10^{-7}$ GeV</td>
<td>4.66 ± 1.00 ± 1.26</td>
</tr>
<tr>
<td>Im$A_0$</td>
<td>$10^{-11}$ GeV</td>
<td>−1.90 ± 1.23 ± 1.08</td>
</tr>
<tr>
<td>Re$A_2$</td>
<td>$10^{-8}$ GeV</td>
<td>1.50 ± 0.04 ± 0.14</td>
</tr>
<tr>
<td>Im$A_2$</td>
<td>$10^{-13}$ GeV</td>
<td>−6.99 ± 0.20 ± 0.84</td>
</tr>
</tbody>
</table>

- The real parts are consistent with those extracted from the data.
- Scattering phase-shifts are determined from the two-pion energy levels in a finite Euclidean volume on the lattice.

*Caveat:* The calculated $I = 0$ $\pi \pi$ phase-shift is smaller than the data:

$$\delta_0 = 23.8(4.9)(1.2) \degree \quad \leftrightarrow \quad (\delta_0)_{\text{exp}} = 38.3(1.3) \degree$$

[3] RBC-UKQCD, 1505.07863
[15] RBC-UKQCD, 1502.00263