Progress in lattice in the kaon system

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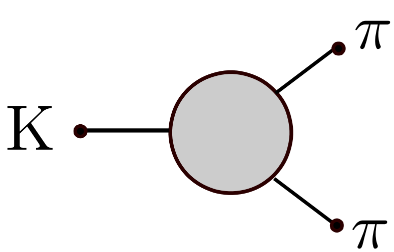
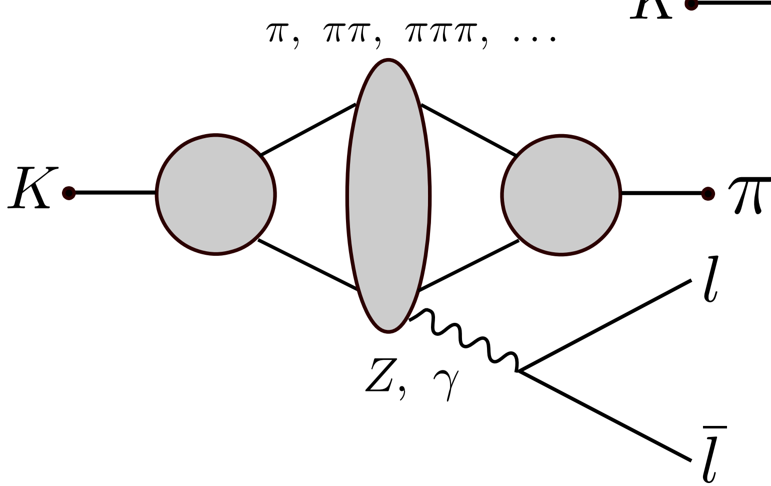
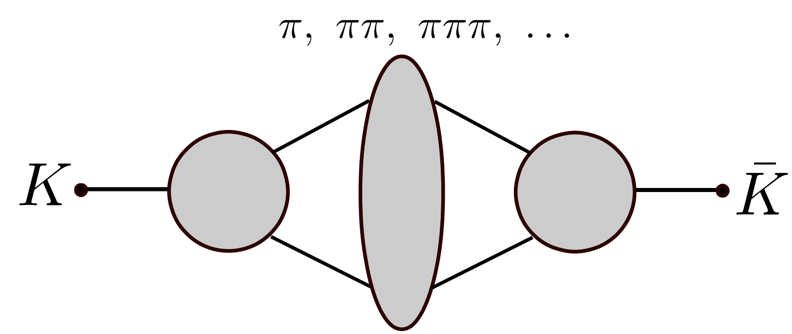
COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Introduction

- Lattice QCD directly simulates low-energy hadronic interactions on supercomputers.
- Only known *ab initio*, ***systematically improvable*** technique for studying non-perturbative QCD.
- Simulations have now reached sufficient precision to make significant impact on the search for BSM physics.
- In particular, with recent advances in theoretical and computational techniques, now able to directly compute matrix elements involving multi-particle states.
- In this talk we focus upon lattice calculations of kaonic matrix elements that are highly sensitive to BSM physics:







Time is our greatest enemy...

With many apologies to my colleagues and the audience, I will not have time to cover many exciting areas of lattice kaon physics including:

- Precision studies of B_K , the short-distance contribution to ε_K and it's impact on CKM matrix unitarity tests.
- Recent advances in non-perturbatively computing electromagnetic corrections to kaon decay amplitudes.
- Lepton flavor universality constraints from leptonic kaon decays (“K_{l2}”)
- Precision determinations of $|V_{us}|$ from $K\rightarrow\pi$ semileptonic decays (“K_{l3}”)

However on this topic I would like to highlight a brand new result from FNAL/MILC

[arXiv:1809.02827]

$$|V_{us}| = 0.22333(43)_{f_+(0)}(42)_{exp}$$

For the first time the theoretical error on this quantity is at the same level as the experimental error.

Schematic lattice kaon physics calculation

- We are typically interested in hadronic matrix elements of some Weak process.
- Lattice energy scales are $O(\Lambda_{QCD}) << M_W$ hence to high precision we can use weak effective theory.
- E.g. 1st order matrix elements (such as $K \to \pi\pi$) computed as

$$\langle 0|\mathcal{O}_{\rm snk}(t_{\rm snk})H_W(t)\mathcal{O}_{\rm src}(t_{\rm src})|0\rangle$$

$$= \sum_{n,m}\langle 0|\mathcal{O}_{\rm snk}(t_{\rm snk})|n\rangle\langle n|H_W(t)|m\rangle\langle m|\mathcal{O}_{\rm src}(t_{\rm src})|0\rangle \times e^{-E_n(t_{\rm snk}-t)}e^{-E_m(t-t_{\rm src})}$$

- Here O_{src} and O_{snk} are operators with the quantum numbers of the desired states.
- Extract matrix elements by fitting time dependence in limit of large $(t_{snk}-t)$, $(t_{src}- t)$

Weak Hamiltonian and associated systematics

$$H_W = \sum_{ij} c_i(\mu) Q_i(\mu)$$

where c_i are (perturbative) Wilson coefficients encapsulating high energy physics and Q_i are four-quark operators.

- c_i and Q_i are renormalization scheme dependent.
- To avoid using perturbation theory (PT) in strongly-coupled regime we renormalize Q_i in a non-perturbative scheme and run to high energies where we can reliably match using PT.
- Matching energy is limited by need to avoid finite-lattice spacing effects, and PT is usually only performed to 1-loop
- As a result the perturbative truncation error is typically one of the larger systematic errors.
- If we want a dynamical charm we require a fine lattice spacing. However this is often computationally intractible

→ Either suffer large charm discretization errors **or**
→ Compute matrix element in 3 flavor theory.

Tradeoff is sys error from using PT to cross charm threshold (~1.3 GeV) in Wilson coeffs

<u>$K \to \pi\pi$ and ε'/ε</u>

Motivation

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0 \rightarrow \pi\pi$:

$$\eta_{00} = \frac{A(K_{\mathrm{L}} \to \pi^0\pi^0)}{A(K_{\mathrm{S}} \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_{\mathrm{L}} \to \pi^+\pi^-)}{A(K_{\mathrm{S}} \to \pi^+\pi^-)}.$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \quad \text{(experiment)}$$

measure of direct CPV

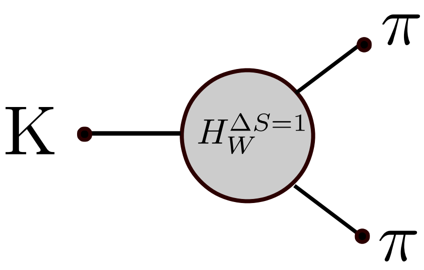
measure of indirect CPV

- Small size of ε' makes it particularly sensitive to new direct-CPV introduced by many BSM models.

Lattice calculation

- K → ππ decays require single insertion of ΔS=1 Hamiltonian:

10 effective four-quark operators



$$H_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{j=1}^{10} [z_j(\mu) + \tau y_j(\mu)] Q_j$$

$$\left(\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \right)$$

perturbative Wilson coeffs.

RI-SMOM NPR
renormalization matrix
(mixing)

Lellouch-Luscher finite-volume correction

$$A^I = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[(z_i(\mu) + \tau y_i(\mu)) \, Z_{ij}^{\mathrm{lat} \to \overline{\mathrm{MS}}} M_j^{I,\,\mathrm{lat}} \right]$$

isospin-definite amplitude

$M_j^{I,lat} = \langle (\pi\pi)_I | Q_j | K \rangle$ (lattice)

$$\epsilon' = \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)$$

$\boxed{\omega = \mathrm{Re}A_2 / \mathrm{Re}A_0}$

(δ_I are strong scattering phase shifts.)

Key challenges of lattice calculation

- Primary challenge is to assure physical kinematics: For periodic BCs, amplitude with 2 stationary pions in final state dominates. However

$$2m_\pi \approx 260\ \mathrm{MeV} \ll m_K \approx 500\ \mathrm{MeV}$$

- Desired state with moving pions is next-to-leading term: require 2exp fits?

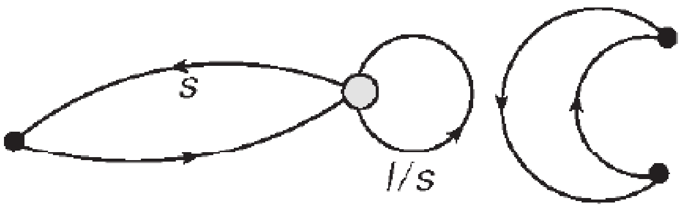
- Avoid 2-exp fits by removing stationary pion state from system through manipulating lattice spatial boundary conditions:

  - Antiperiodic BCs on down-quark for A_2
  - G-parity BCs on both quarks for A_0

$$p_\pi = 0 \rightarrow \pi/L$$

tune L to match E_K and $E_{\pi\pi}$

- For A_0 serious noise issue due to "disconnected diagrams"



type4

- Use "all-to-all" propagators to tune source to minimize overlap with vacuum "1s hydrogen wavefunction pion source" $e^{-r/a}$ with a=2 appears optimal

(r is inter-quark separation)

Summary of RBC/UKQCD calculations

[Phys.Rev. D91 (2015) no.7, 074502]

- A_2 computed on two large, ~ (5.5 fm)^3 volume with physical quark masses and two lattices (2.36 GeV and 1.73 GeV) → Continuum limit taken.
- <1% statistical error!
- 10% and 12% total errors on Re(A_2) and Im(A_2) resp.
- Dominant sys. errors due to truncation of PT series in computation of renormalization and Wilson coefficients.

[Phys.Rev.Lett. 115 (2015) 21, 212001]

- A_0 computed with physical quark masses on single, coarse lattice (a^{-1}= 1.38 GeV) but with large (4.6 fm)^3 physical volume to control FV errors.
- 21% and 65% stat errors on Re(A_0) and Im(A_0) due to disconn. diagrams and, for Im(A_0) a strong cancellation between Q_4 and Q_6.
- Dominant, 15% systematic error is due again to PT truncation errors exacerbated by low renormalization scale 1.53 GeV required by coarse lattice spacing.

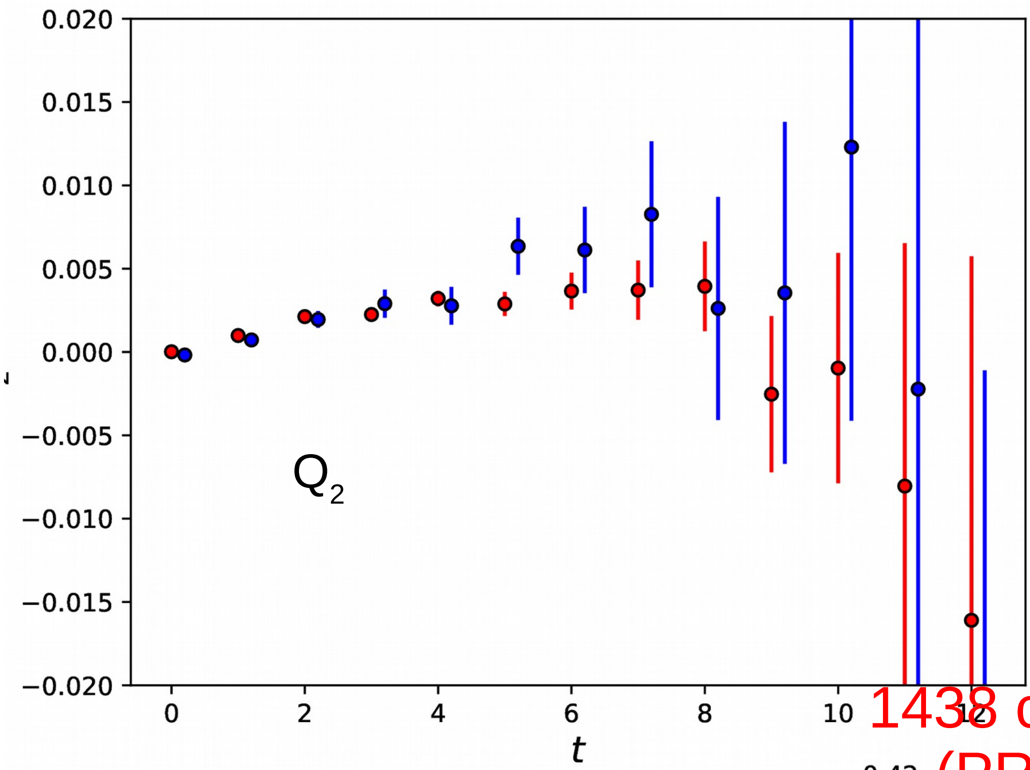
Results for ε' [Phys.Rev.Lett. 115 (2015) 21, 212001]

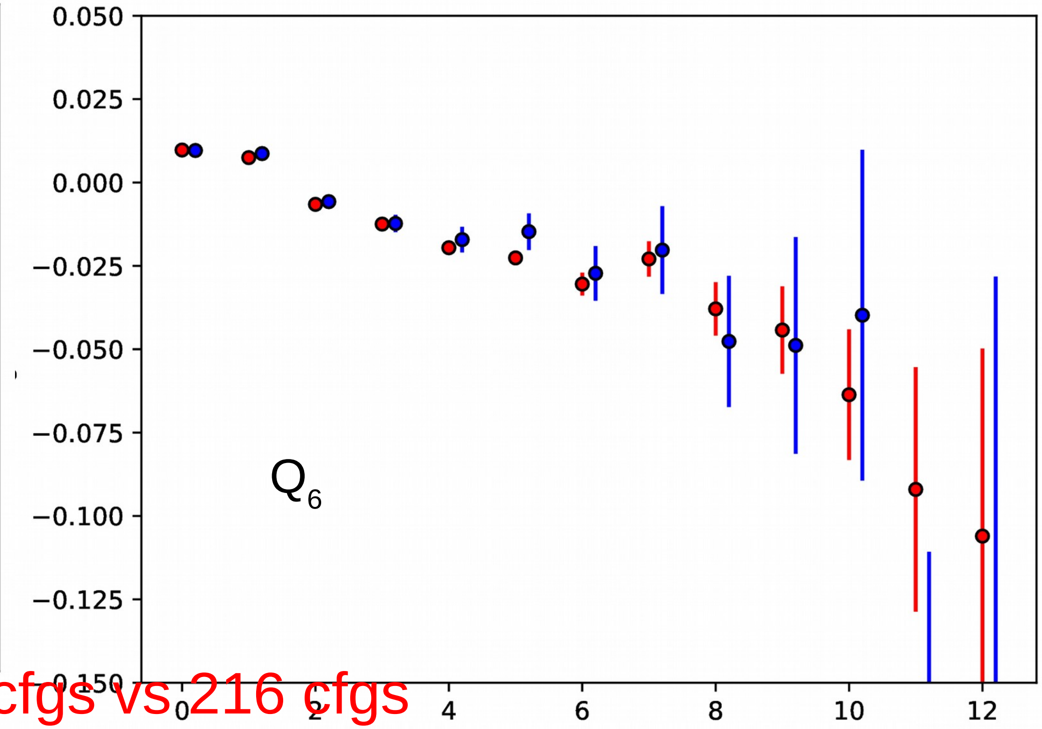
- Re(A_0) and Re(A_2) from expt.
- Lattice values for Im(A_0), Im(A_2) and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}\mathrm{A}_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{\begin{array}{ll} = \; 1.38(5.15)(4.43)\times 10^{-4}, & \text{(our result)} \\ \;\; 16.6(2.3)\times 10^{-4} & \text{(experiment)} \end{array}}$$

- Total error on Re(ε'/ε) is ~3x the experimental error
- Find reasonable (2.1σ) consistency with Standard Model
- “This is now a quantity accessible to lattice QCD”!
- Focus since has been to improve statistics and reduce / improve understanding of systematic errors.

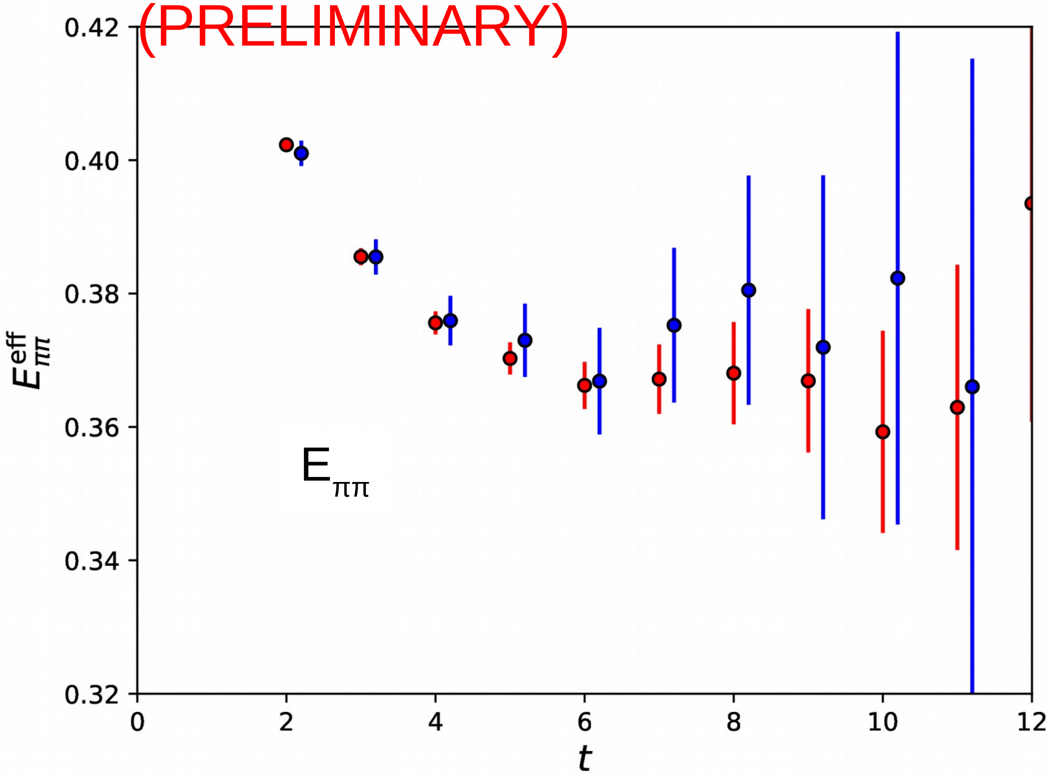




1438 cfgs vs 216 cfgs
(PRELIMINARY)

- Since 2015 have increased statistics from 216 to over 1400!

6.7x increase



Systematic error improvements

- Improved NPR error 15% → 8% (prelim) by increasing scale 1.53 → 2.29 GeV using step-scaling procedure. [PoS LATTICE2016 (2016) 308]
- Inclusion of dim.6 gauge-invariant operator G_1 which mixes with Q_i under renormalization, effects demonstrated to be %-scale as expected. [G. McGlynn arxiv:1605.08807]

  Do not expect significant improvement in Wilson coeffs from increasing scale as error dominated by use of PT to cross the charm threshold (1.29 GeV).
- Working on circumventing this by computing 3→4 flavor matching non-perturbatively
- Requires $\mu \ll m_c$. At these low energies, gauge-fixed MOM-scheme approach severely hampered by increased mixing with tower of gauge-noninvariant operators.
- Circumvent using position-space NPR which does not require gauge fixing.

- Laying the groundwork for non-perturbatively computing the effects of isospin breaking and electromagnetism. [EPJ Web Conf. 175 (2018) 13016]
- Study possibility of complete, non-perturbative calculation of Wilson coefficients [EPJ Web Conf. 175 (2018) 13014, arXiv:1711.05768]

Resolving the ππ puzzle

- 2015 result has 2σ+ discrepancy between our I=0 ππ phase shift ($\delta_0=23.8(4.9)(1.2)^o$) and dispersion theory prediction ($\sim 34^o$).

[RBC&UKQCD PRL 115 (2015) 21, 212001]
[Colangelo *et al*, Nucl.Phys. B603 (2001) 125-179]

- Observed discrepancy more significant (~5σ) with 6.5x stats.
- Most likely explanation is excited-state contamination.
- To address added scalar (σ=ūd) ππ operator to the 2-pt function calculation.
- Combined fits (or GEVP) to ππ→ππ, σ→ππ and σ→σ correlators result in considerably lower ground-state energy:

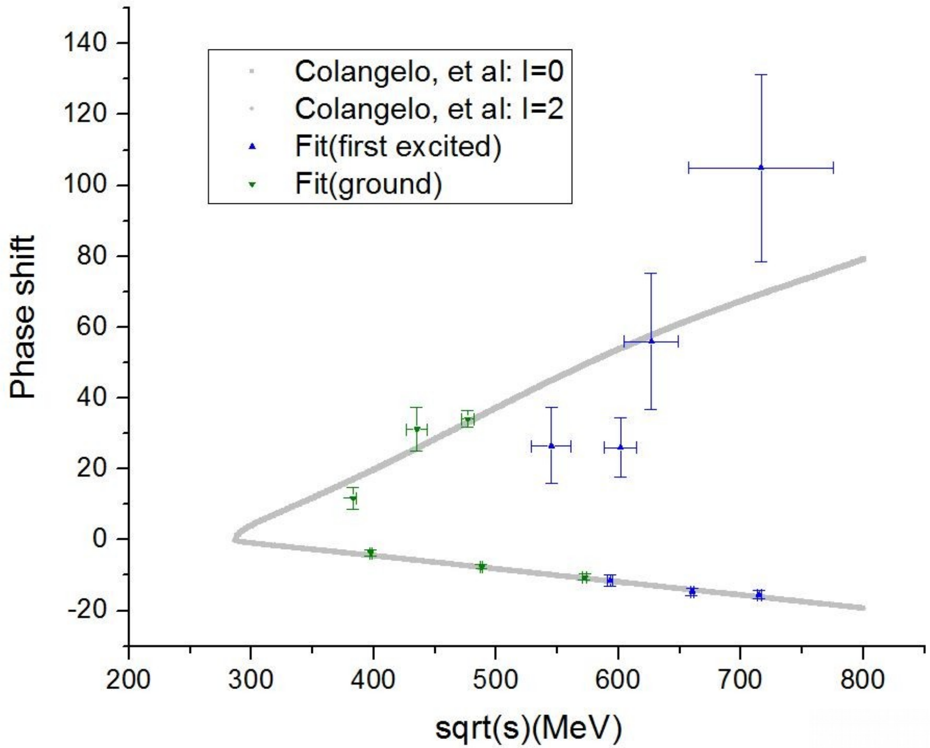
508(5) MeV [1386 cfgs] from ππ→ππ alone
vs
483(1) MeV [501 cfgs] from sim. fit of all 3 correlators.

- New phase shift $\delta_0=30.9(1.5)(3.0)^o$ [prelim] compatible with dispersive result.

- Strong evidence for nearby excited finite-volume ππ state.
Indeed such a state with E ~ 770 MeV is predicted by dispersion theory.

Implications for $K \to \pi\pi$ and resolution

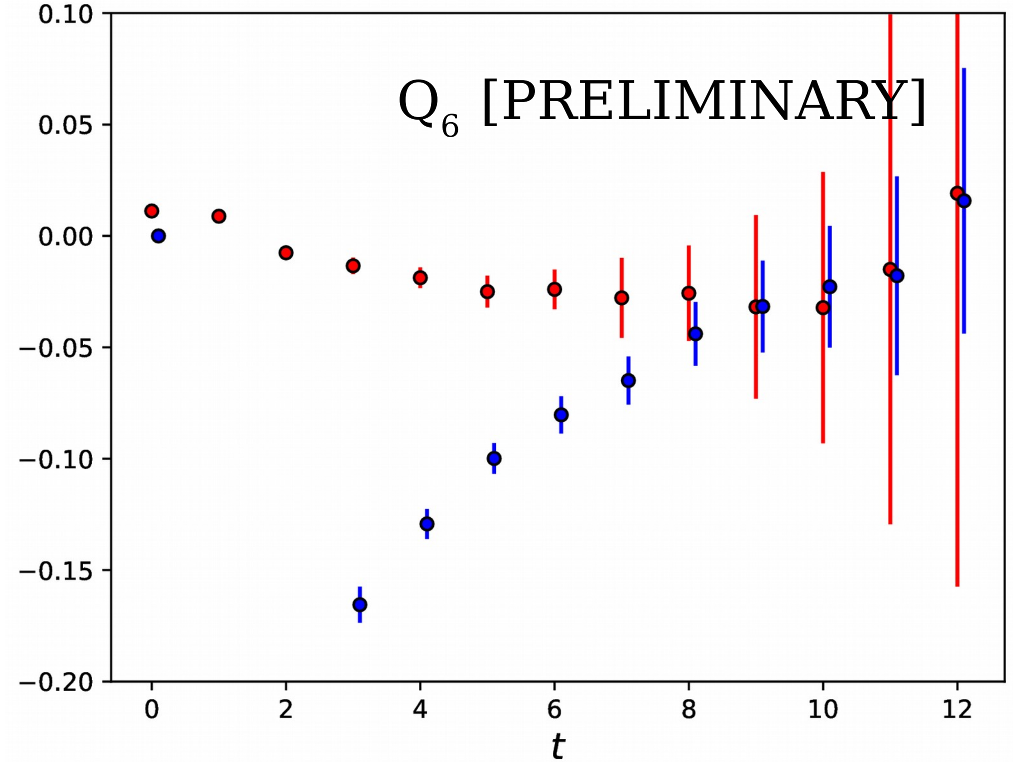
- Despite vast increase in statistics, *this second state cannot be resolved from the time dependence using only a single ππ operator.*
- Possibly a significant underestimate of excited state systematic error in $K \to \pi\pi$ calculation that can only be resolved by adding additional operators.
- In response we have **expanded the scope of the calculation**:
  - Added $K \to \sigma$ matrix elements
  - Added $K \to \pi\pi$ matrix element of new ππ operator with larger relative pion momenta (still $p_{CM}=0$)
- Result is **3x increase in the number of I=0 ππ operators in $K \to \pi\pi$ calc.**
- Also added ππ 2pt functions with non-zero total ππ momenta.
  Calculate phase shift at several (smaller) additional center-of-mass energies.
  - Additional points that can be compared to dispersive result / experiment
  - Improve ~11% systematic on Lellouch-Luscher factor associated with slope of phase shift.
- Currently have 152 measurements with new operators!



- Preliminary results for 132 configs of ππ phase shifts including $P_{CM}\neq 0$
- Excellent agreement for I=2 and for I=0 ground-state and to reasonable degree for I=0 excited state.

Q_6 matrix element with K→σ (blue) and K→ππ (red)

- Hope to have updated result by the end of the year!



- Also note recent calculation of ε' by Ishizuka *et al* with energy-conserving kinematics but unphysical pion/kaon masses
  m_π = 260 MeV and m_K= 570 MeV

[arXiv:1809.03893]

Obtain $$\mathrm{Re}(\epsilon'/\epsilon) = 18(53)_{\mathrm{stat}} \times 10^{-4}$$

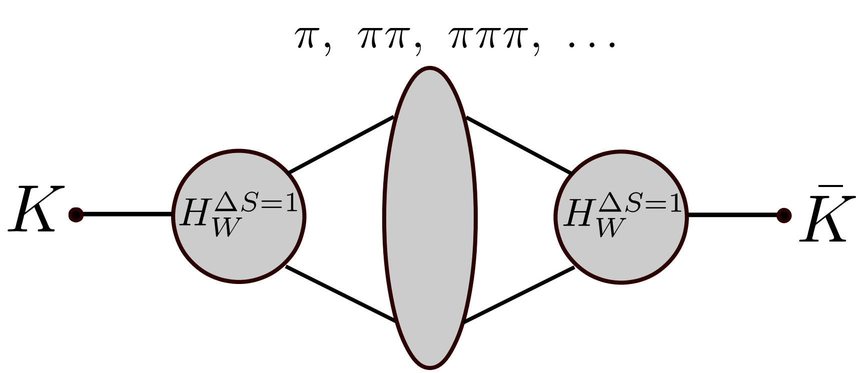
K_L-K_S mass difference

Introduction

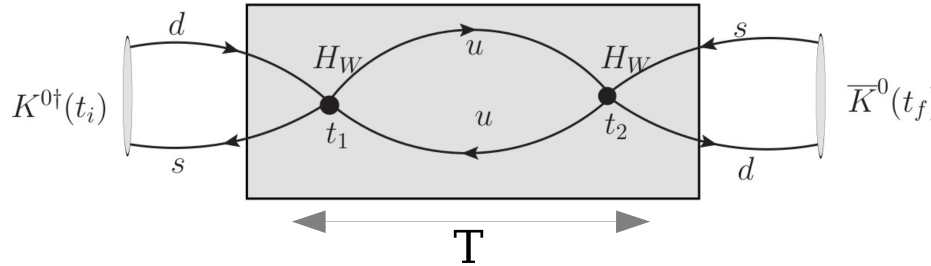
- Neutral kaon mixing induced by 2nd order weak processes gives rise to mass difference between K_L and K_S

$$\Delta M_K = 2\sum_n \frac{\langle \overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K - E_n}$$

- FCNC → highly suppressed in SM due to GIM mechanism: $\Delta m_K = 3.483(6)\times 10^{-12}$ MeV small and highly sensitive to new BSM FCNC.
- PT calc using weak EFT with $\Delta S=2$ eff. Hamiltonian (charm integrated out) dominated by $p\sim m_c$: poor PT convergence at charm scale → ~36% PT sys error.
- PT calc neglects long-distance effects arising when 2 weak operators separated by distance $\sim 1/\Lambda_{QCD}$.
- Use lattice to evaluate matrix element of product of $H_W^{\Delta S=1,\,\mathrm{eff}}$ directly:



Lattice method and challenges

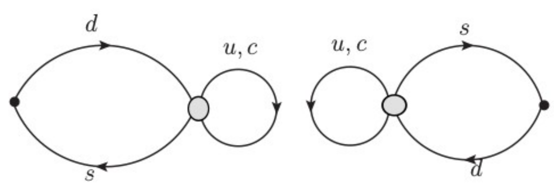


$$\mathscr{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \boxed{\frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n}} \left(-T - \frac{1}{M_K - E_n} + \frac{e^{(M_K - E_n)T}}{M_K - E_n} \right)$$

- Vary integration window T to extract desired matrix element as term linear in T
- Require subtraction of exponentially-growing terms when $E_n < m_K$: |π>, |ππ>, |0>
- Use ability to shift H_W by total divergence $\bar{s}\gamma^5 d$ to directly remove |0>
  Similarly use $\bar{s}d$ to remove |η>, which although $m_\eta > m_K$ gives noisy contribution

$$\langle \eta | H_W - c_s \bar{s} d | K \rangle = 0 \qquad \langle 0 | H_W - c_p \bar{s}\gamma^5 d | K \rangle = 0$$

- Pion and two-pion terms contributions explicitly subtracted
- Disconnected diagrams make the calculation noisy. Requires large statistics and maximal translation of sources.



- Divergence when operators approach removed by GIM – requires (valence) charm on lattice. Need fine lattice to control discretization errors.

Calculation status

- First complete calculation in 2014 with large statistics (800 configs) on single, somewhat coarse (a^{-1}=1.73 GeV) lattice

[Phys.Rev.Lett. 113 (2014) 112003]

Unphysical masses: 330 MeV pions (no ππ intermediate state) , m_c = 950 MeV

est. of dominant charm disc err. only

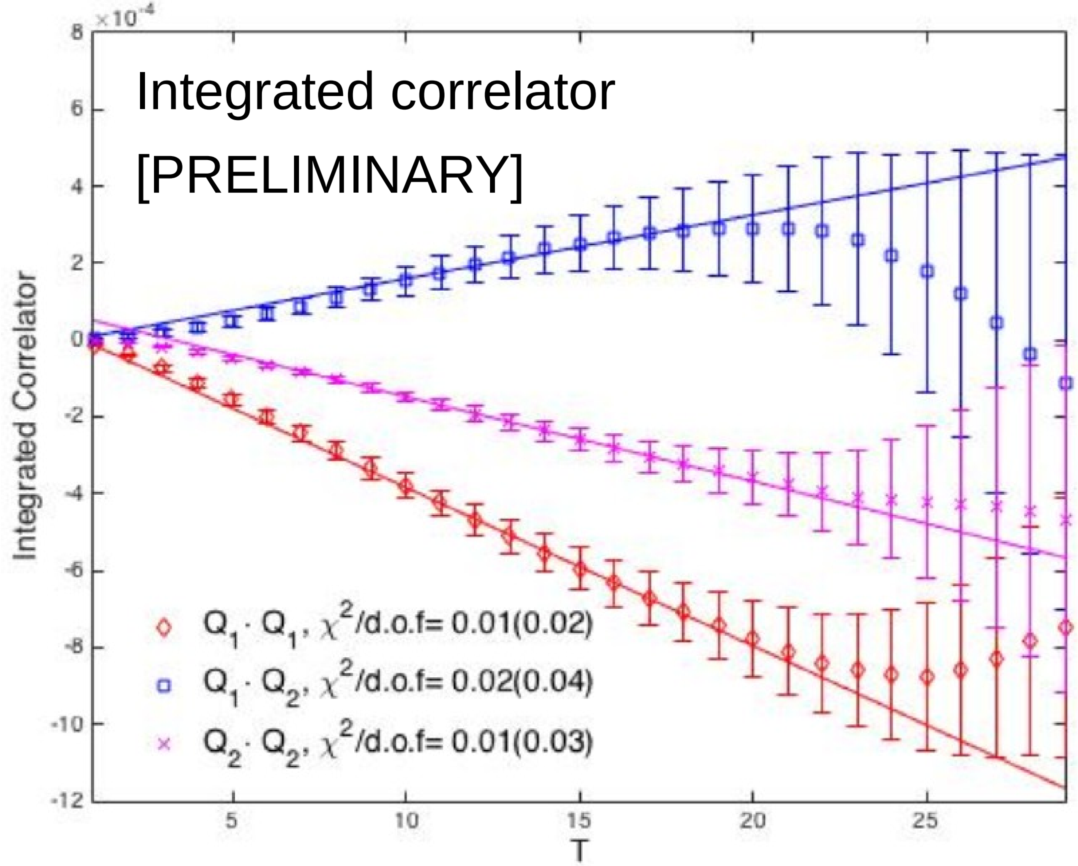
lattice

$$\Delta M_K = 3.19(41)(96) \times 10^{-12}\ \mathrm{MeV}$$

experiment

$$3.483(6) \times 10^{-12}\ \mathrm{MeV}$$

- Presently repeating calculation on large $(5.5\ \mathrm{fm})^3$, fine a^{-1}=2.36 GeV lattice
- Physical charm and pion mass
- Prelim. results for 129 configs presented by B.Wang at Lattice 2018



$$\Delta M_K = 7.0(1.7)_{\mathrm{stat}} \times 10^{-12}\ \mathrm{MeV}$$

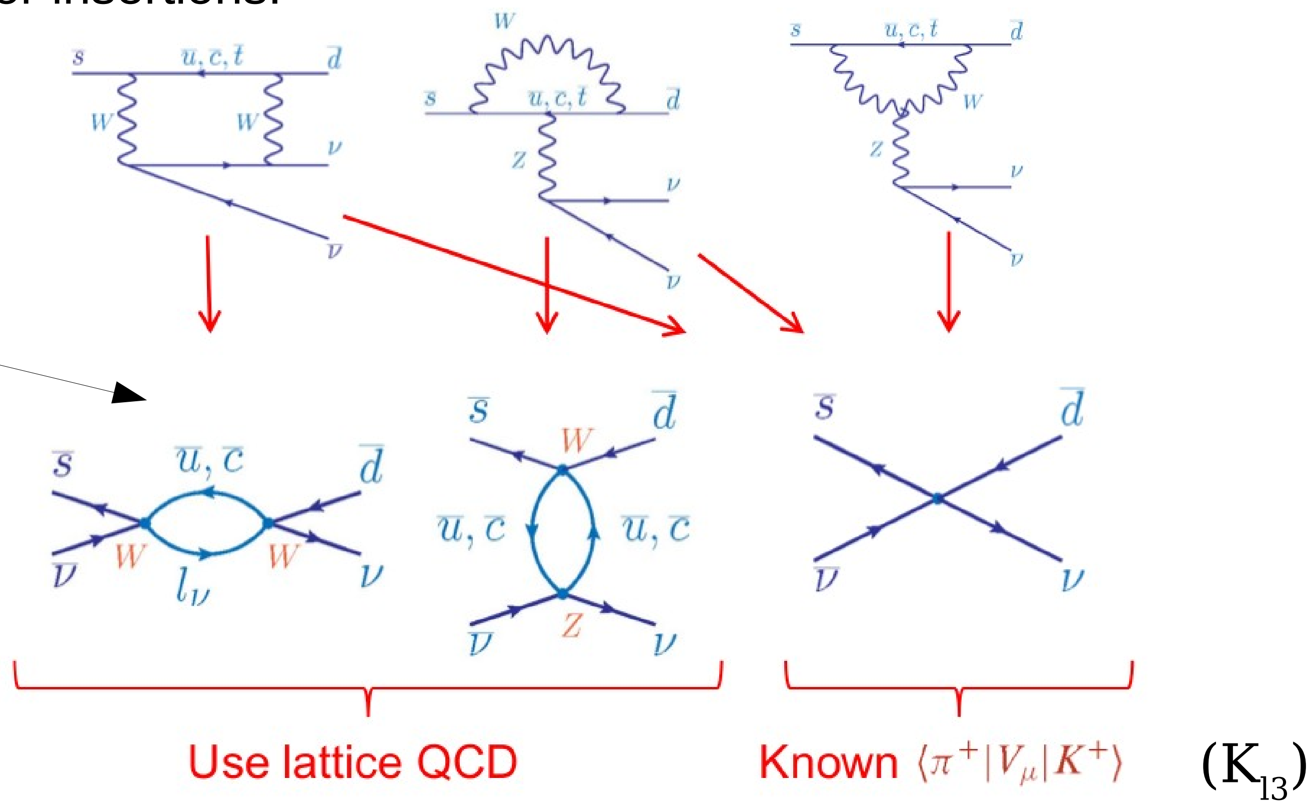
- η-state gives significant stat. err. contrib as divergent op. subtraction coeff noisy
- Charm discretization error estimate from naive $(m_c a)^2 \sim 25\%$
- However only 3-10% observed errors in f_D and dispersion relation of η_c
- Aim to continue measurements on ORNL Summit computer and ultimately a second lattice spacing to understand disc. effects.

Rare kaon decays $K \to \pi\nu\bar{\nu}$

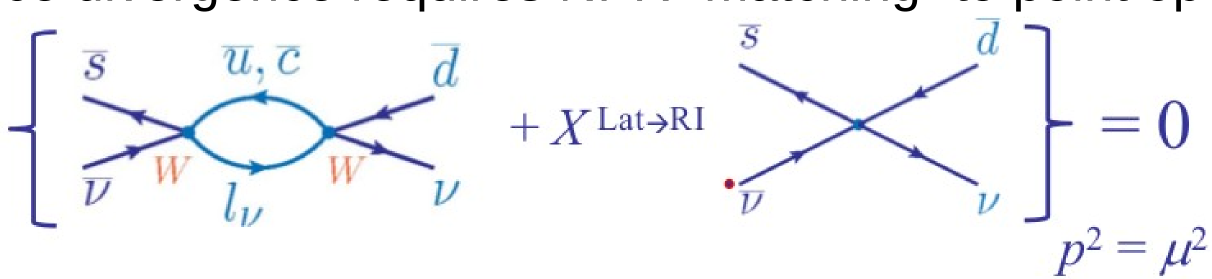
Overview

- Another FCNC thus far not particularly well experimentally measured.
- CERN NA62 expt expected to provide result with target of ~10% error
- Short-distance dominated but expect ~5% LD effect: Lattice!
- Again 2x operator insertions:

Effective operators representing both W and Z required

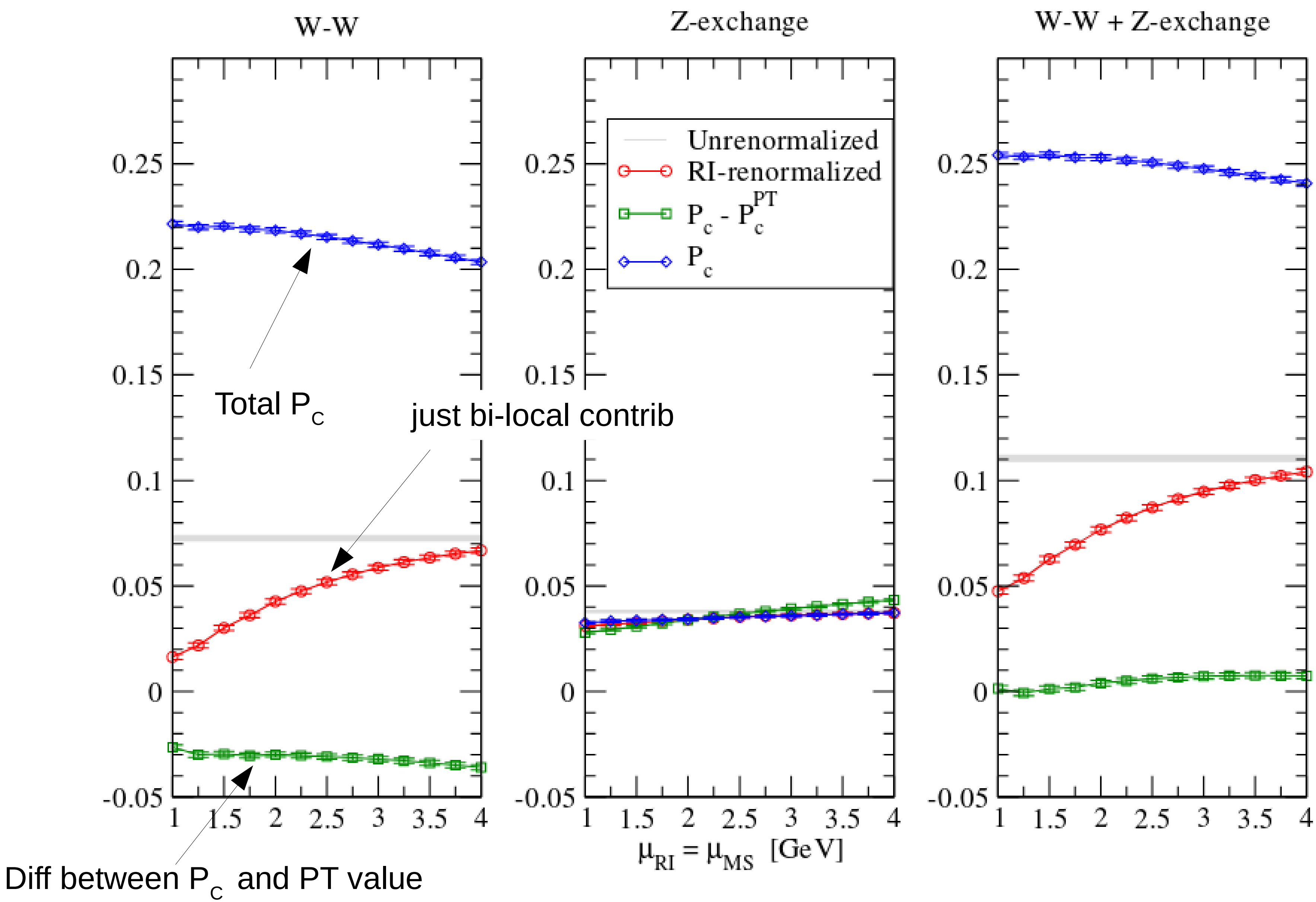


- Short-distance divergence requires NPR "matching" to point operator.



Lattice calculation and results

- Technique largely the same as for ΔM_K :
  - Integrate operator insertion times over window.
  - Remove exponentially-growing and noisy exponentially falling terms through total divergence / explicit subtraction
- Remove stationary pion state using APBC on down-quark. Tune momentum such that the neutrinos travel in opposite directions with $p = p_\pi/2$
- More complicated intermediate states including $l^+\nu$, $\pi^0 l^+\nu$, $(\pi^0\pi^+)_{I=2}$ require subtraction and finite-volume correction
- First complete lattice calculation on single, small volume a^{-1}=1.7 GeV with large statistics but unphysical masses: [Phys.Rev.Lett. 118 (2017) no.25, 252001]
  - m_π=420 MeV, m_K=560 MeV, m_C^{MSbar}(2 GeV) = 860 MeV
- Disconnected part computed but at different kinematics to avoid APBC. However s-dependence expected to be small and disconn size only ~3% of connected.



- $\Delta P_{c,u}$ (unphys) = $0.0040(13)_{stat}(55)_{sys}$ (only FV and ren. scale-dependence sys errs)
- Expect within 4 years a physical calc will be possible

Related calculations: $K \to \pi l\bar{l}$ and LD contribution to ε_K

$K \to \pi l\bar{l}$

- $K \to \pi l\bar{l}$ also a rare FCNC like $K \to \pi\nu\bar{\nu}$ although long-distance dominated.
- Similar technique can be used. Exploratory calculation performed with unphysical masses m_π=430 MeV, m_K=600 MeV, m_C^{MSbar}(2 GeV) = 530 MeV and no disconnected diagrams. [Phys.Rev. D94 (2016) no.11, 114516]
- GIM mechanism cancels bilocal operator divergence, no local/bilocal matching required.
- Full calculation would require more physical charm mass.
- Intriguing possibility: integrate out the charm and remove divergences through local/bilocal matching thus avoid charm discretization effects in exchange for increased PT error.
- Plan to shortly begin 3f calculation on (5.5 fm)^3 1/a=1.73 GeV ensemble with physical masses and disconnected diagrams.

LD contributions to ε_K

- ε_K short-distance dominated and SD contribution B_K known to ~1% from lattice
- Largest errors from V_{cb} but missing LD contribution will become significant as V_{cb} error improves.
- Amplitude is the imaginary part of that used to compute ΔM_K
- However more operators contribute to imaginary part including penguin operators.
- Requires bilocal/local operator matching to remove all divergences.
- Exploratory calculation performed with unphysical masses: m_π=330 MeV, m_K = 580 MeV, m_c^{MSbar}(2 GeV)=940 MeV

[PoS LATTICE2015 (2016) 342]

- Conclude LD effects ~3% with unphysical masses.
- Significant charm effects (as always) require finer lattices to control.
- Intend to begin physical mass calculation in near future.

Conclusions

Conclusions

- Lattice calculations now able to significantly impact search for BSM physics.
- Complete, physical calculation of ε'/ε possible. 2015 result likely has underestimated excited-state systematic - expect update by end of year
- Matrix elements with 2 operator insertions now possible with controlled systematics
- Calculation of FCNC suppressed Δm_K with physical parameters underway.
- Rare kaon decays $K \to \pi\nu\bar{\nu}$ and $K \to \pi l\bar{l}$ also accessible although work needed to control charm discretization (4f) / PT truncation errors (3f).
- LD contributions to ε_K also on the radar.

Thank you!

ΔI=1/2 rule

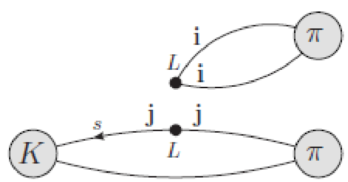
- In experiment kaons approx 450x (!) more likely to decay into I=0 pi-pi states than I=2.

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \simeq 22.5 \qquad \text{(the ΔI=1/2 rule)}$$

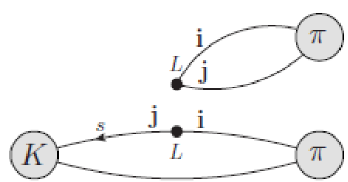
- Perturbative running to charm scale accounts for about a factor of 2. Is the remaining 10x non-perturbative or New Physics?
- The answer is **low-energy *QCD*!**

RBC/UKQCD [arXiv:1212.1474, arXiv:1502.00263]

Strong cancellation between the two dominant contractions



Contraction ①.

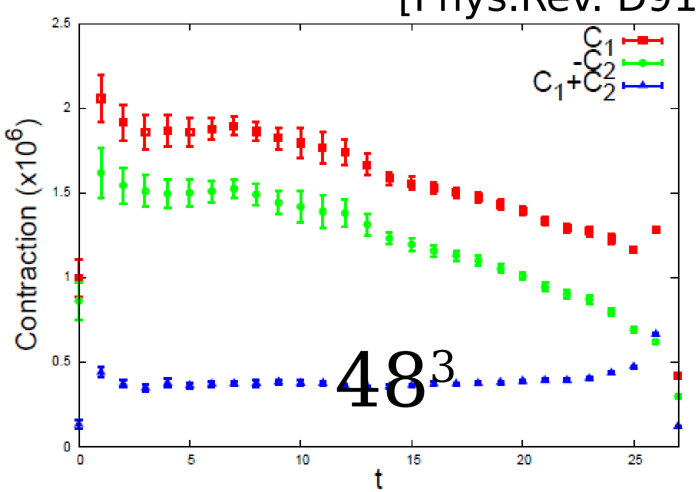


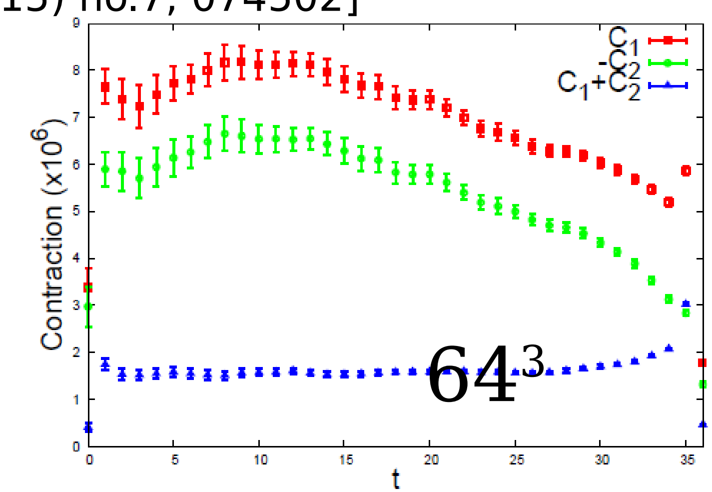
Contraction ②.

$$\mathrm{Re}(A_2) \sim ① + ②$$

find $② \approx -0.7①$ heavily suppressing Re(A_2).

[Phys.Rev. D91 (2015) no.7, 074502]





Pure-lattice calculation

$$\frac{\mathrm{Re}(A_0)}{\mathrm{Re}(A_2)} = 31.1(11.2)$$

[Re(A_0) agrees with expt.]

Statistics increase

- Original goal was a 4x increase in statistics over 216 configurations used in 2015 analysis.
- 4x reduction in configuration generation time obtained via algorithmic developments (exact one-flavor implementation)
- Large-scale programme performed involving many machines:

Source	Determinant computation	Independent configs.
Blue Waters	RHMC	34+18+4+3
KEKSC	RHMC	106
BNL	RHMC	208
DiRAC	RHMC	151
KEKSC	EOFA	275+215
BNL	EOFA	245
		1259 total

- Measurements performed using IBM BG/Q machines at BNL and the Cori computer (Intel KNL) at NERSC largely complete.
- Including original data, now have 6.7x increase in statistics!

Finite-volume effects and other systematic error sources

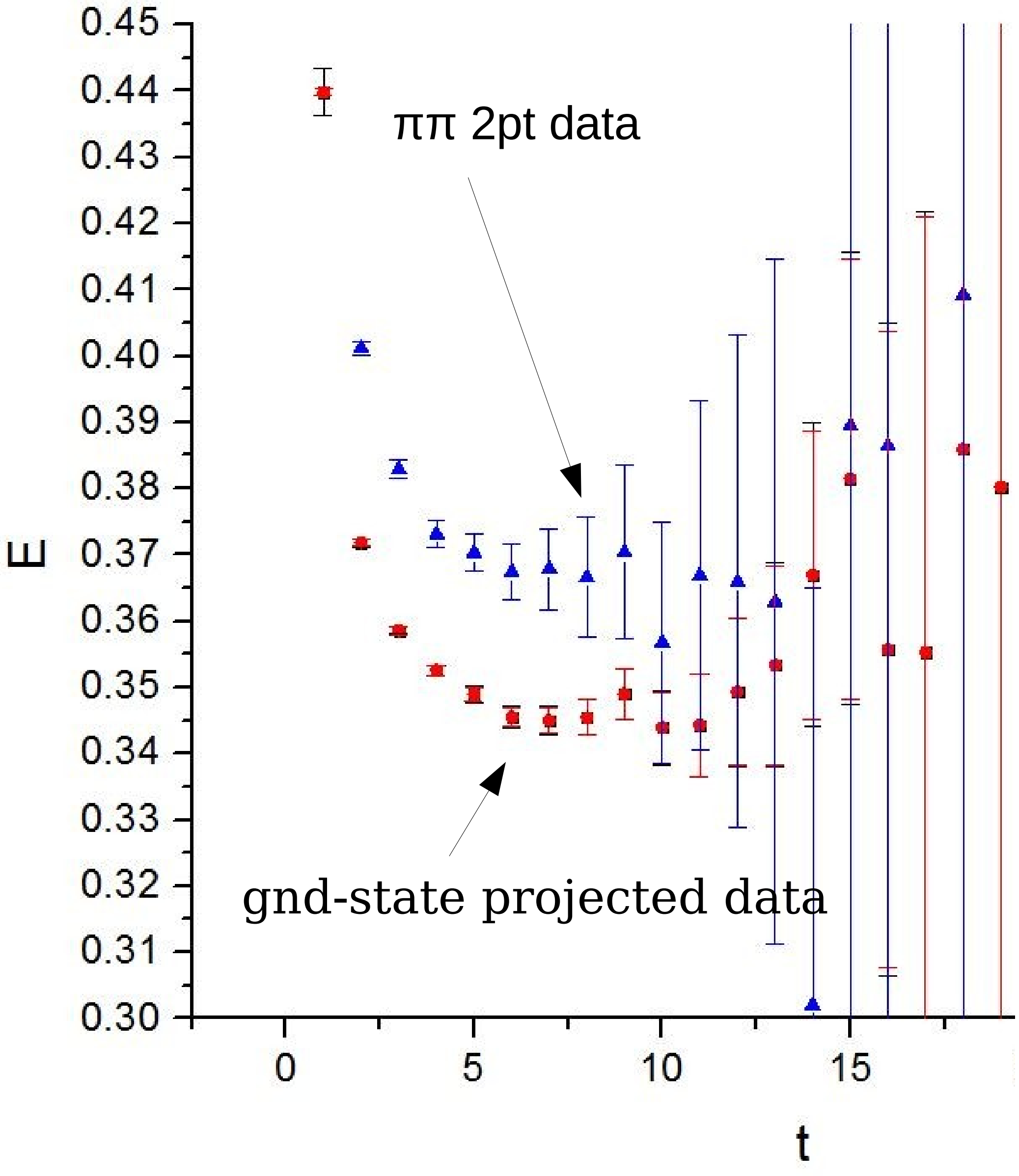
- Lattice calculations necessarily performed in finite-volume.
  Typically FV error are exponentially suppressed in lattice size L.
- However multi-particle states have only power law suppression ($1/L^N$) due to their interactions as they are squeezed by the box.
- Fortunately power-law effects can be removed using Luscher's formula (for 2-particle scattering) and the Lellouch-Luscher formula (2 particle decay)

  These breakthroughs made the study of scattering and decay amplitudes possible!

- Excited state contamination effects also can be significant in decay and scattering amplitudes.
- Operators project onto all states with corresponding quantum numbers and we must rely on fitting time dependence to extract desired term.
- If nearby energies then fits cannot easily distinguish the states leading to excited state contamination. This can be resolved by increasing statistics and performing simultaneous fits including more operators.

- It is typically desired to compute using multiple lattice spacings and to extrapolate to the continuum. However this can be impractical for expensive calculations hence there are discretization effects that must also be estimated.

Effect of projecting 2pt data onto ground-state using existing data:

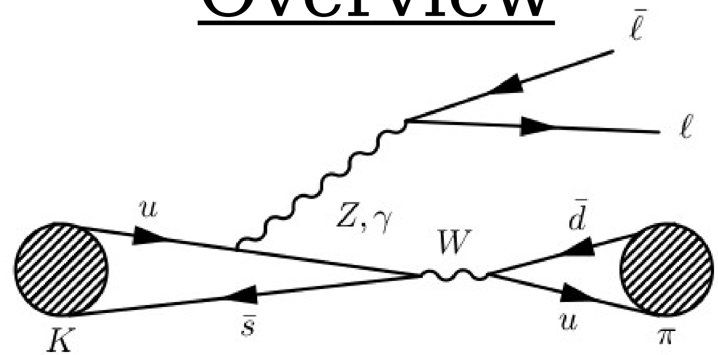


Expect even better ground-state projection with new higher-momentum operators in upcoming analysis

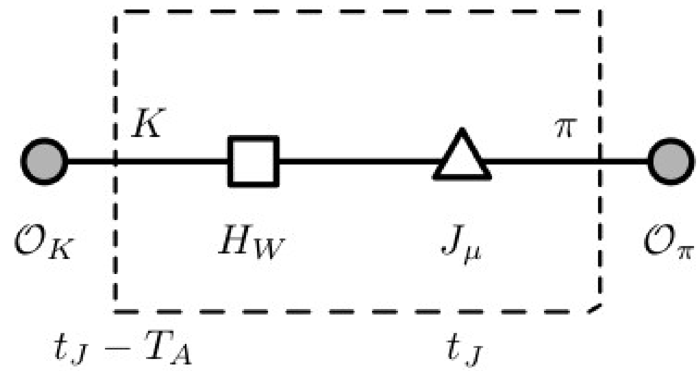
Rare kaon decays $K \to \pi l \bar{l}$

Overview

[arXiv:1507.03094]



- Additional FCNC processes “rare kaon decays”: $K^+ \to \pi^+ l^+ l^-$, $K_S \to \pi^0 l^+ l^-$
- Amplitude is long-distance dominated: Compute $K \to \pi\gamma^*$ on lattice.
- Lattice approach very similar to Δm_K but with EM-current insertion:

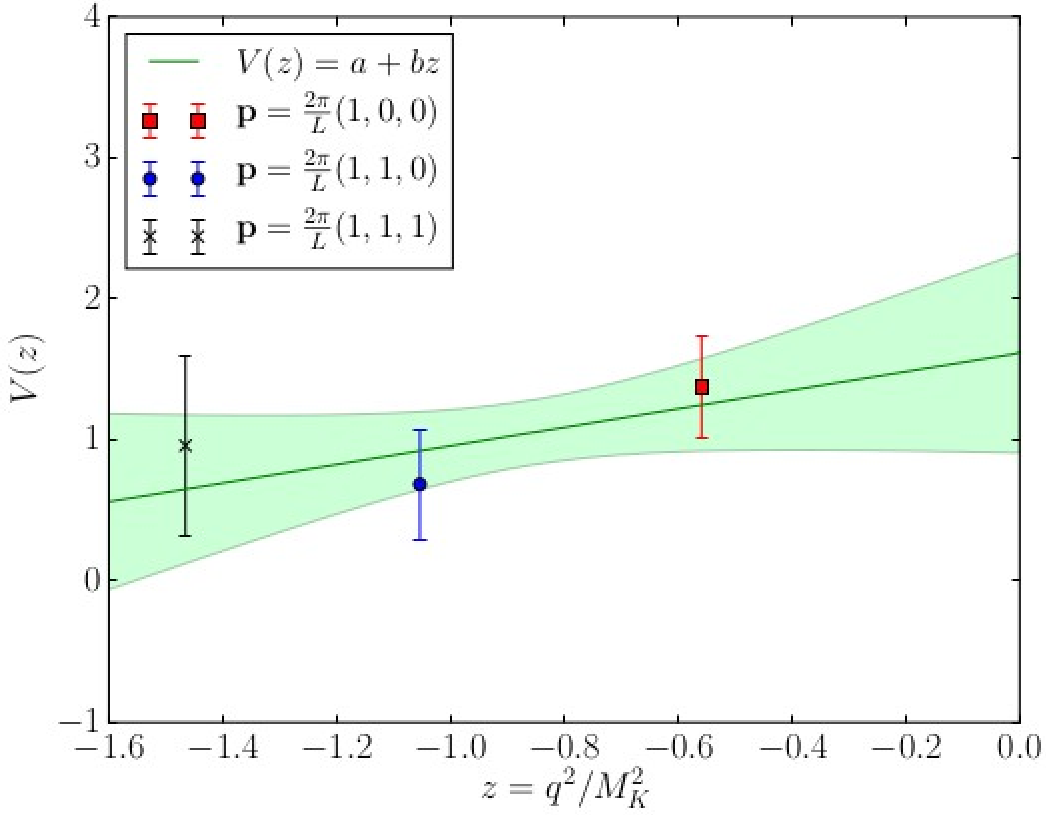


Momentum applied to pion to study dependence

- Unphysical initial calculation presently underway on 24^3x64 ensemble with 430 MeV pions, 620 MeV kaons and m_c~ 533 MeV.
- Multiple pion momenta allow extraction of form factor.

- Preliminary results from Lattice 2016 talk by A.Lawson (July 28th):

$$A_\mu\left(q^2\right) \equiv G_F \frac{V(z)}{(4\pi)^2}\left(q^2(k+p)_\mu - \left(M_K^2 - M_\pi^2\right) q_\mu\right), \qquad z = q^2/M_K$$



- Future steps are physical pion and kaon masses.
- No statistically significant charm mass dependence observed – is going to physical charm necessary?