$|V_{us}|$ from taus

Taku Izubuchi

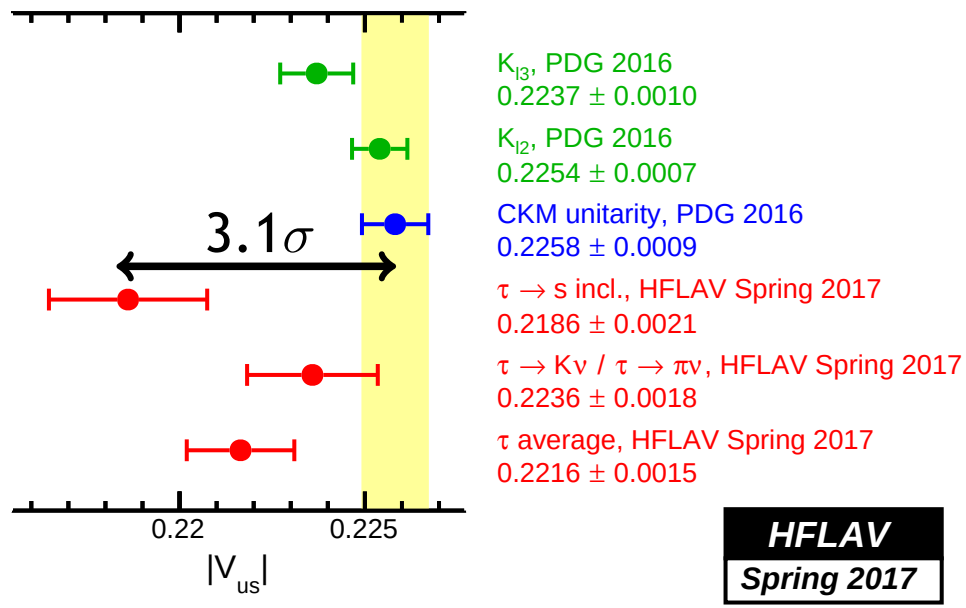
based on

P. Boyle, R. Hudspith, T. Izubuchi, A. Jüttner, C. Lehner, R. Lewis,
K. Maltman, H. Ohki, A. Portelli, M. Spraggs

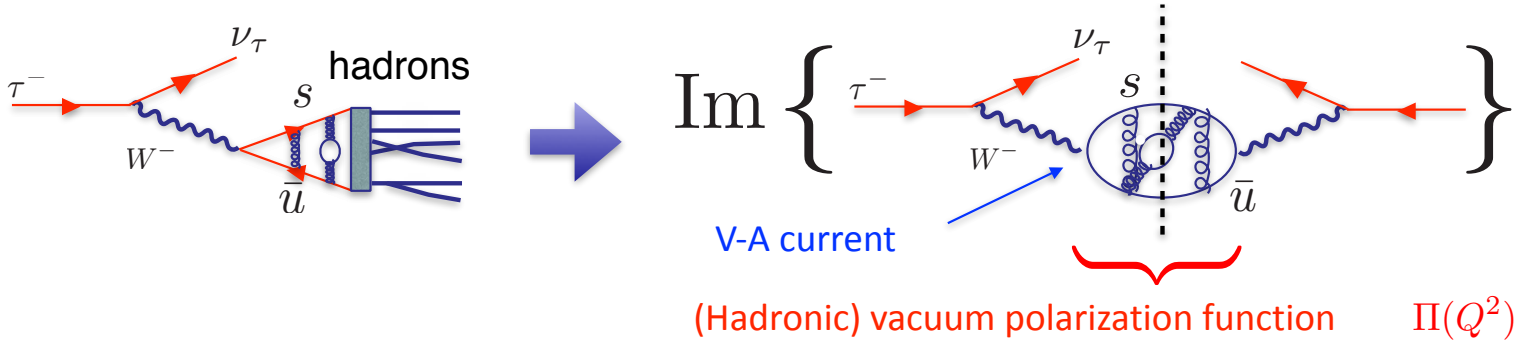
"Novel $|V_{us}|$ Determination Using Inclusive Strange τ Decay and Lattice HVPs"
`arXiv:1803.07228 [hep-lat]`

(RBC and UKQCD Collaborations)

- Tau inclusive decay, Finite Energy Sum Rule
- New method: Combining FESR + Lattice HVP
- Results & Updates
- Conclusion



Tau decay



- $\tau \to \nu + had$ through V-A vertex. V_{ij} : CKM, EW correction S_{EW} [Marciano, Sirlin]

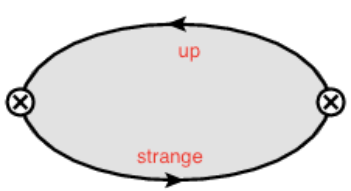
$$R_{ij,V/A} = \frac{\Gamma(\tau^- \to \text{hadrons}_{ij}\ \nu_\tau)}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}$$

$$= \frac{12\pi^2|V_{ij}|^2 S_{EW}}{m_\tau^2}\int_0^{m_\tau^2} ds\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\rho^{(0+1)}_{ij,V/A}(s)+\rho^{(0)}_{ij;V/A}(s)\right]$$

- The Spin=0 and 1, vacuum polarization, Vector(V) or Axial (A) current-current 2pt

$$\Pi^{\mu\nu}_{ij;V/A}(q^2) = i\int d^4x e^{iqx}\left\langle 0|T J^\mu_{ij;V/A}(x) J^{\dagger\mu}_{ij;V/A}(0)|0\right\rangle$$

$$= (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi^{(1)}_{ij;V/A}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij;V/A}$$



Finite Energy Sum Rule (FESR)

[Shifman, Vainstein, Zakharov 1979]

- FESR = Optical theorem (Unitarity) + Dispersion relation (Analyticity)
- Optical theorem relate S=-1 spectral function $\rho^{0/1}_{V/A,ij}(s)$ and HVP $\Pi^{0/1}_{V/A,ij}(s)$ for given quantum number: flavor (us or ud), spin (0 or 1), parity (V or A)

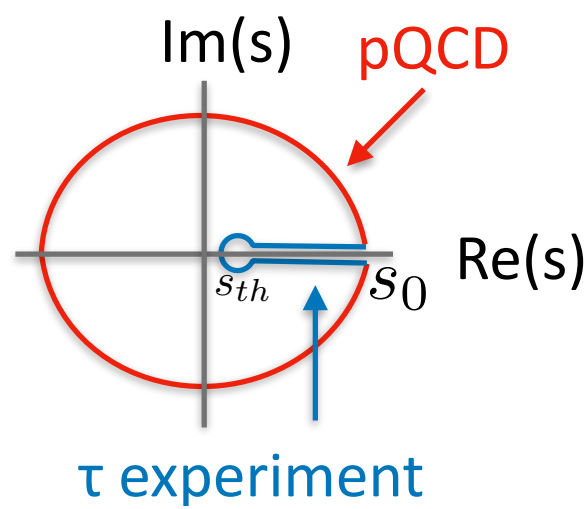
$$\frac{1}{\pi}\mathrm{Im}\Pi(s) = \rho(s)$$

- Do *finite* radius contour integral for arbitrary regular weight function $w(s)$

$$\int_{s_{th}}^{s_0} ds\rho(s)w(s) = +\frac{i}{2\pi}\oint_{|s|=s_0} ds\Pi(s)w(s)$$

- Real axis integral is extracted from experimental decay energy distribution dR_τ/ds

$$\frac{dR_{ij;V/A}}{ds} = \frac{12\pi^2|V_{ij}|^2 S_{EW}}{m_\tau^2}\omega_\tau(s)\rho(s)$$



$|V_{us}|$ determination from FESR

[E. Gamiz, *et al.*, 2003, 2005, Maltman et al 2006]

- Inclusive differencial τ decay rate with wieght $w(s)$

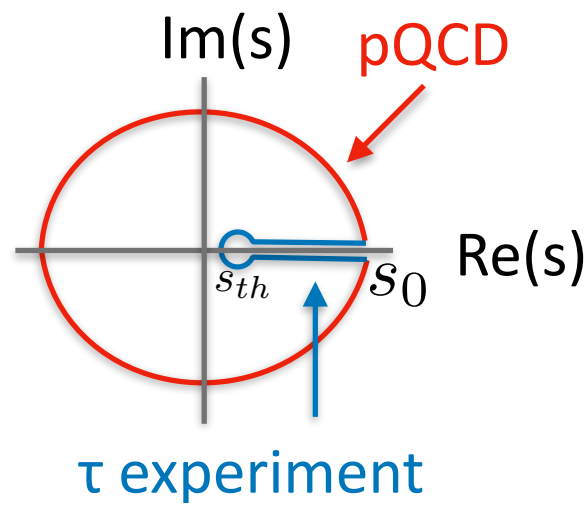
$$R^{\omega}{}_{ij}(s_0) \equiv \int_{s_{th}}^{s_0} ds \frac{dR_{ij}}{ds} \frac{\omega(s/s_0)}{\omega_\tau(s/m_\tau^2)}$$

- Take difference between up-down and up-strange channel

$$\Delta R^{\omega} = \frac{R^{\omega}_{ud}}{|V_{ud}|^2} - \frac{R^{\omega}_{us}}{|V_{us}|^2}$$

- $|V_{ud}|$ and m_s as input, selecting $s_0 = m_\tau^2, \omega = \omega_\tau(s/s_0)$

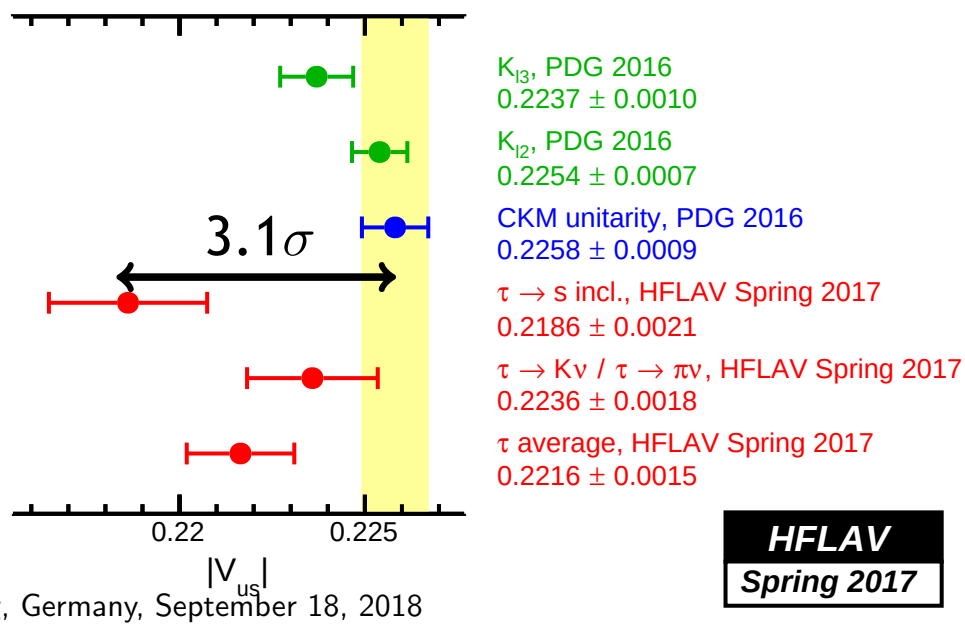
$$|V_{us}| = \sqrt{\frac{R^{\omega}_{us}(s_0)}{\frac{R^{\omega}_{ud}(s_0)}{|V_{ud}|^2} - [\Delta R^{\omega}(s_0)]^{\text{pQCD}}}}$$



- For $s > s_0$, fixed-order or contour-improved pQCD is used. OPE condensations at dim=4,6 ... are input/assumed. (a source of unaccounted uncertainties)

V_{us} extraction from FESR

- -3.1σ deviation from CKM unitarity
- Use Vaccum Saturation assumption estimate for OPE's D =6 condensations, and D=8 is dropped.
- Tests of these sumptions with different BF, or by varying s_0 and/or $\omega(s)$ suggest unaccounted systematic error. Further more, a simultaneous fit of the condensates along with $|V_{us}|$, yielded significantly large and stable $|V_{us}|$ [Antonelli *et al.* 2013, Hudspith *et al.* 2017, HFLAV 2017] .
- Uncertainties in us spectral data at high energy dominate error.
- How to improve ?



Our proposal : Combining FESR and Lattice QCD

- By combining both the first principle lattice QCD and experimental data, more reliable precise analysis could be carried out.

  Idea : generalized dispersion relations using weight function with poles at space-like (Euclidean) $s = -Q^2 < 0$ region.

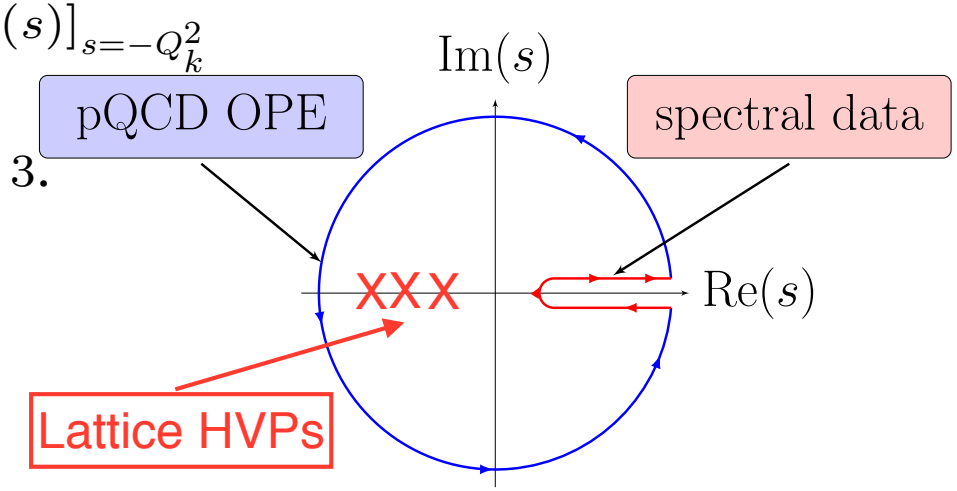
- One could control uncertainties from pQCD OPE, quark-hadron duality violation (contribution of hadron resonance above m_τ^2) in theoretical estimation, as well as the higher hadron multiplicity data, which have larger error, in experimental data.

  Fit for OPE term is no longer necessary.

- If we have a reliable estimate for $\Pi(s)$ in Euclidean (space-like) points, $s = -Q_k^2 < 0$, we could extend the FESR with weight function $\omega_N(s)$ to have N poles,

$$\int_{s_{th}}^{\infty} w_N(s)\rho(s) = \sum_{k=1}^{N} \text{Res}[w_N(s)\Pi(s)]_{s=-Q_k^2}$$

- $|s| \to \infty$ circle integral vanishes for $N_p \geq 3$.



weight function $w_N(s)$

- Weight function

$$w(s) = \prod_k^{N_p} \frac{1}{(s+Q_k^2)} = \sum_k a_k \frac{1}{s+Q_k^2}, \quad a_k = \sum_{j\neq k} \frac{1}{Q_k^2 - Q_j^2}$$
$$\Longrightarrow \sum_k (Q_k)^M a_k = 0 \ \ (M = 0, 1, \cdots, N_p - 2)$$

- The residue constraints automatically subtracts $\Pi^{(0,1)}(0)$ and $s\Pi^{(1)}(0)$ terms.
- For experimental data, $w(s) \sim 1/s^n, n \geq 3$ suppresses
  - ▷ *larger error from higher multiplicity final states at larger* $s < m_\tau^2$
  - ▷ *uncertanties due to pQCD+OPE at* $m_\tau^2 < s$
- For lattice, Q_k^2 should be not too small to avoid large stat. error, $Q^2 \to 0$ extrapolation, Finite Volume/Time error. Also not too larger compared to m_τ^2 otherwise the higher energy, higher multiplicity, OPE region will be enhanced.

Experimental τ data

- K pole contributions, $\omega_N(m_K^2)\gamma_K$: γ_K either from $\Gamma(\tau \to K\nu)$ or $\Gamma(K_{\mu 2})$.

  Exclusive determination, $\gamma_K = 2|V_{us}|^2 f_K^2$ with lattice f_K:

$$|V_{us}| = \begin{cases} 0.2233(15)_{exp}(12)_{th} & \text{for } \gamma_K[\tau_K] = 0.0012061(167)_{exp}(13)_{IB} \\ 0.2260(3)_{exp}(12)_{th} & \text{for } \gamma_K[K_{\mu 2}] = 0.0012347(29)_{exp}(22)_{IB} \end{cases}$$

- Belle, BarBar, ALEPH, strange dR/ds distribution data for exclusive, semi-inclusive modes, in the form of

$$\rho(s) = \left(1 + 2\frac{s}{m_\tau^2}\right)\rho^{(1)}(s) + \rho^{(0)}(s)$$

- Unit normalized invariant hadron mass distribtuion $1/N \times dN(s)/ds$ and Branching Fraction. HFLAV $K\pi$ normalization. Alternate results using the ACLP normalization [Antonelli *et al.* 2013] are given in the Supplementary Material of `arXiv:1803.07228`.

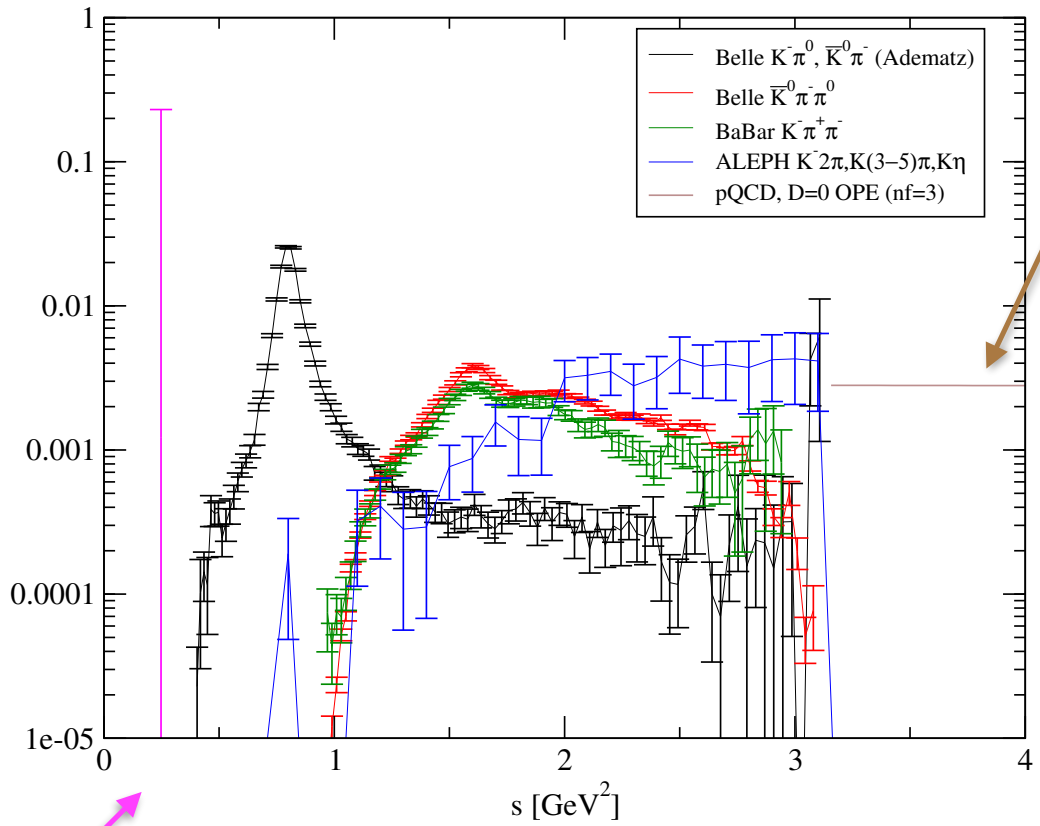
- higher multiplicity final states, at larger s, have larger errors

- In each channels, correlation matrix for unit normalized distribution are used for available channels ($K\pi$) and added by the Branching Fraction in quadrature. Error from all channel are added in quadrature.

$$\rho(s) \equiv |V_{us}|^2 \left[\left(1 + 2\frac{s}{m_\tau^2} \right) \mathrm{Im}\Pi^1(\mathrm{s}) + \mathrm{Im}\Pi^0(\mathrm{s}) \right]$$

To compare with experiments,

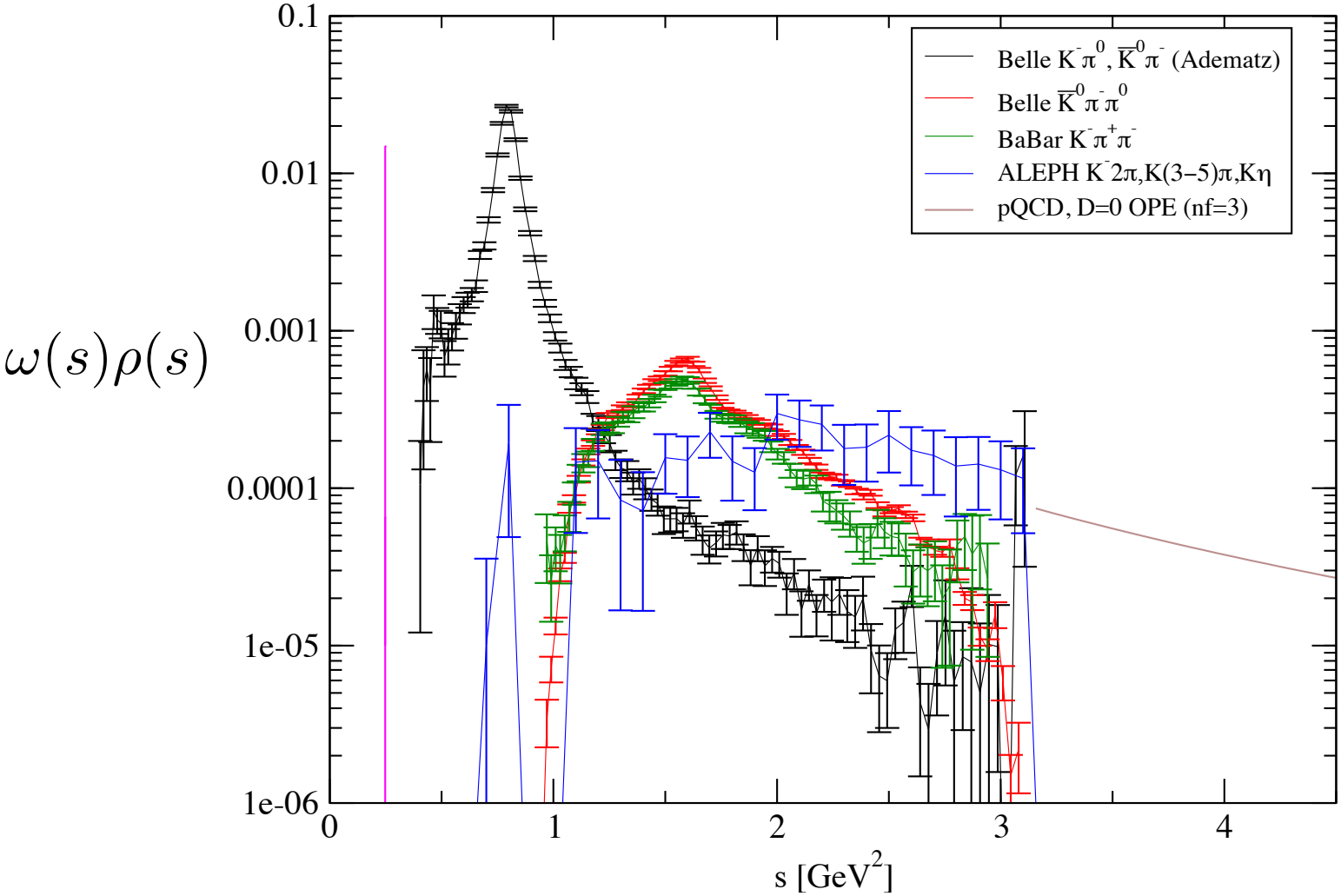
a conventional value of |Vus|=0.2253 is used



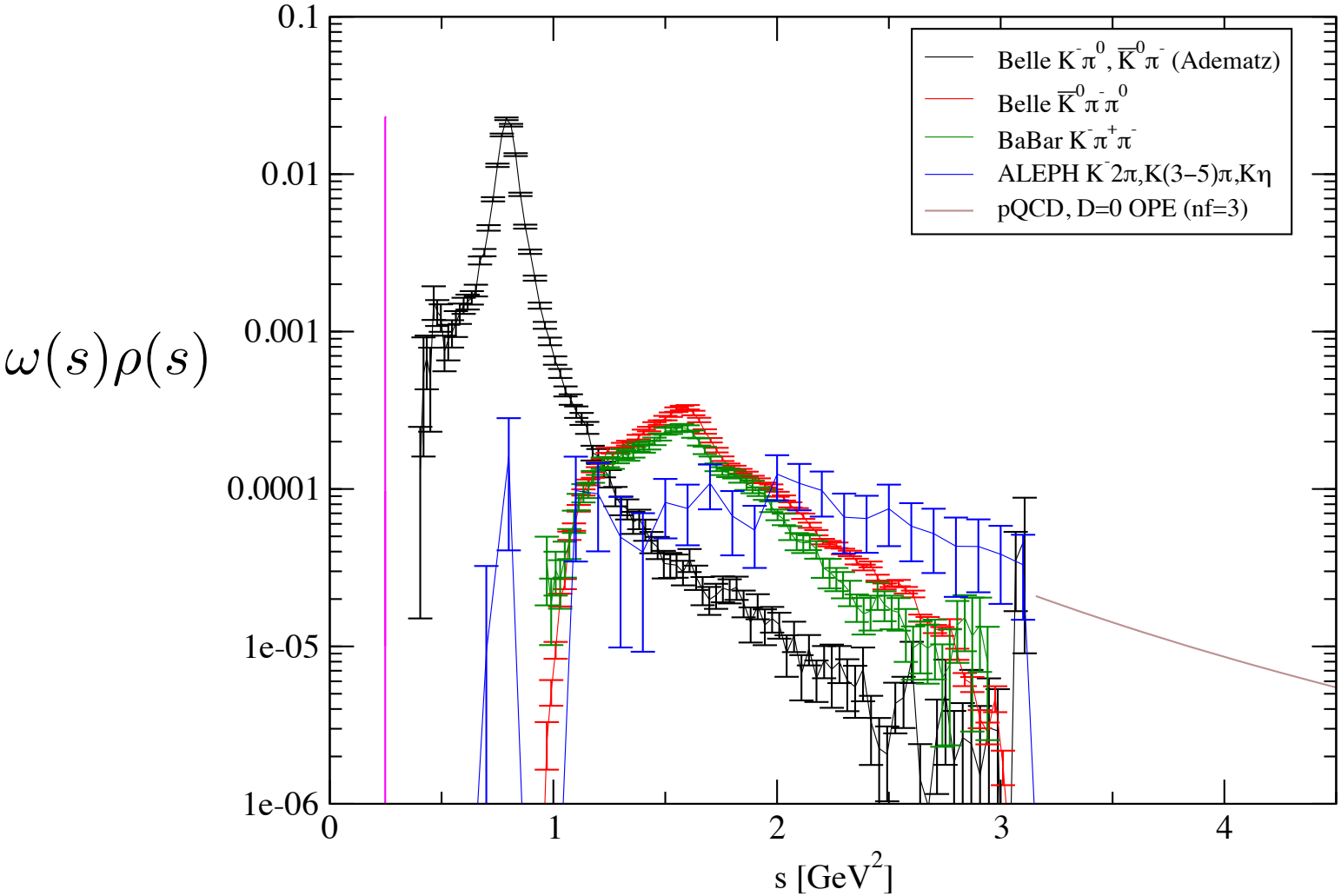
For K pole, we assume a delta function form, whose coefficient is obtained from the experimental value of τ-> K or K-> μ decay widths

$$\delta(s - m_k^2) 0.0012299(46) \sim 2 f_k^2 |V_{us}|^2$$

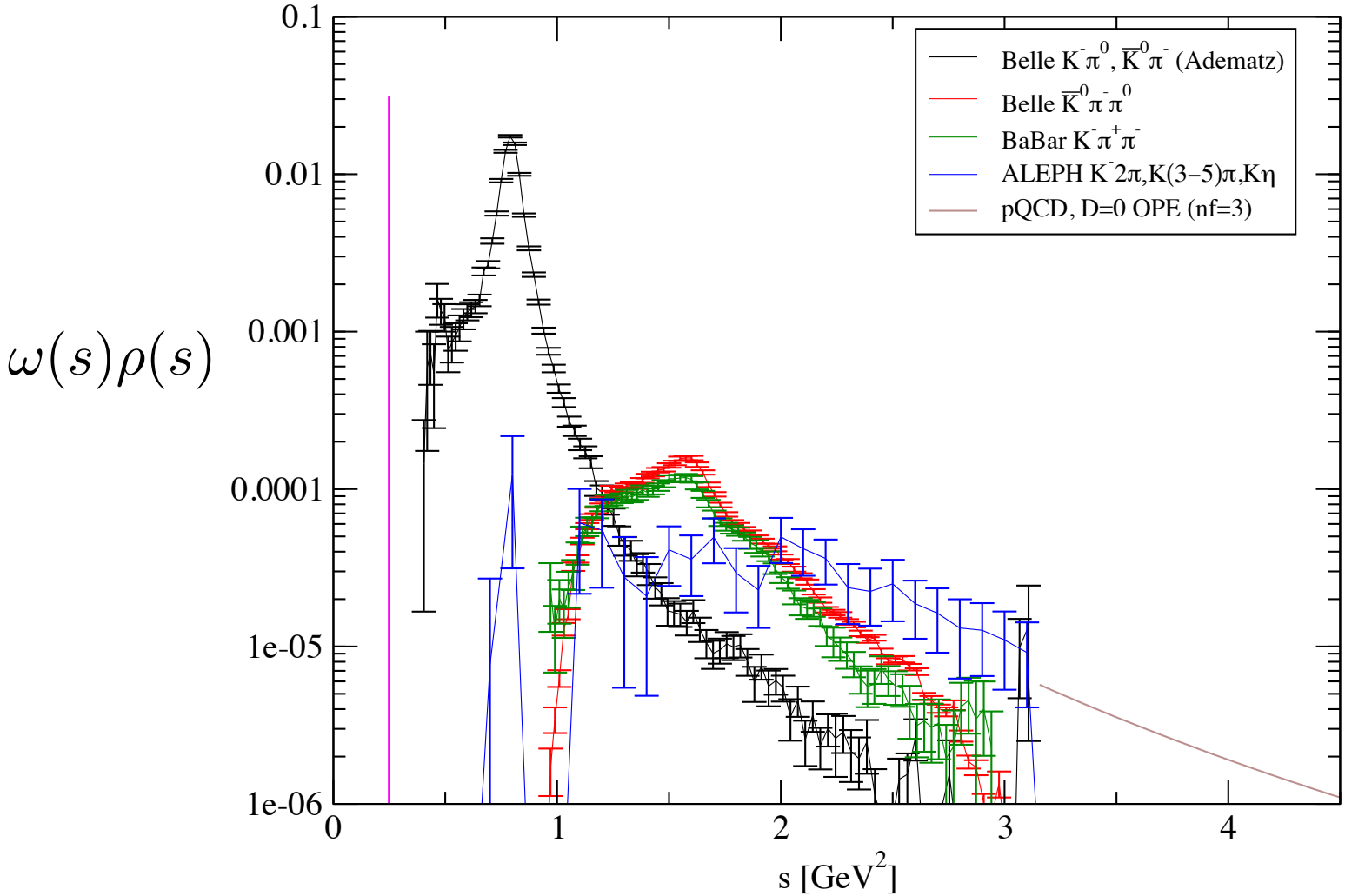
- example: N=3, $\{Q_1^2, Q_2^2, Q_3^2\} = \{0.1, 0.2, 0.3\}$ $[\mathrm{GeV}^2]$



- example: N=4, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2\} = \{0.1, 0.2, 0.3, 0.4\}$ $[\mathrm{GeV}^2]$



- example: N=5, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2, Q_5^2\} = \{0.1, 0.2, 0.3, 0.4, 0.5\}$ $[\mathrm{GeV}^2]$



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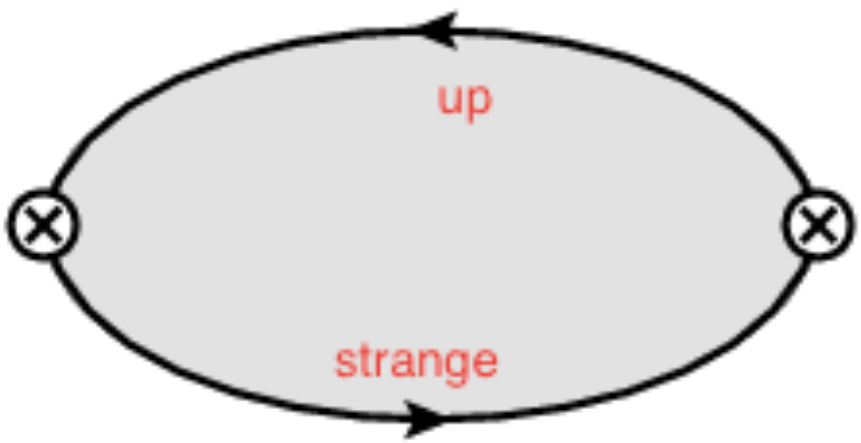
Lattice HVPs

- From Vector-Vector and Axial-Axial correlation function made of up and strange quarks $\Pi^{V/A}_{\nu\nu}(Q^2)$
- Local currents with appropriate renormalization factors, Z_V, Z_A computed non-perturbatively, or conserved currents.
- Tensor $\Pi_{\mu\nu}$ are decomposed into spin 1 from C^{zz} and spin 0 from C^{tt} components, with tensor zero-mode subtraction [Blum 2001, Bernecker & Myer 2011, ...] [$(g-2)_\mu$ HVP calculations].
- Smaller Q^2 data has a larger statistical error, while larger Q^2 suffers discretization errors.



QCD ensemble and statistics

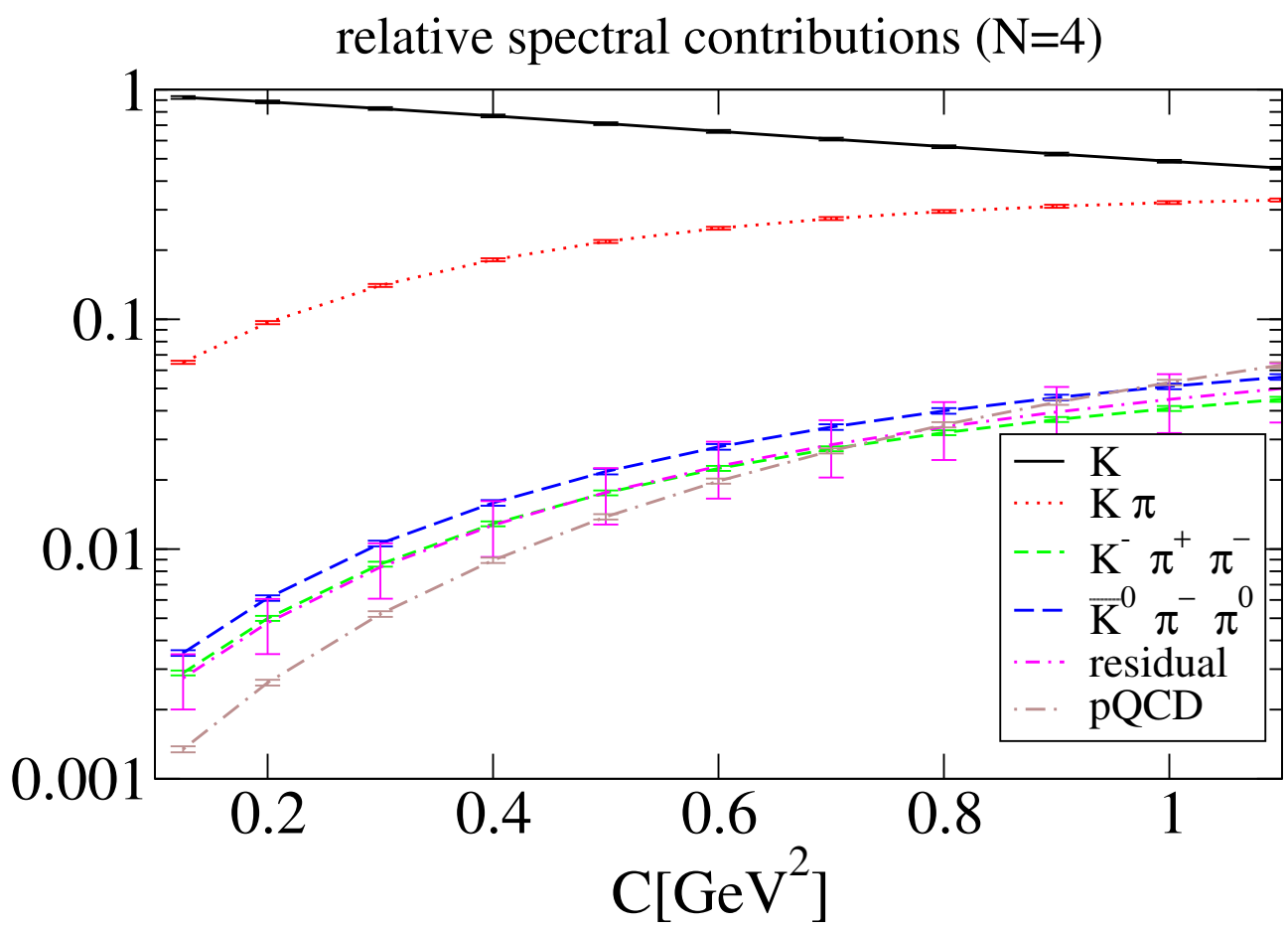
- Main analysis is on two ensemble, at almost physical quark masses ($M_\pi \approx 140$ MeV, $M_K \approx 499$ MeV), V=(5 fm)^3.
- Correct the residual up and strange quark mass error by partially quenched calculation.
- Consistent with results on other heavier / smaller ensemble, which are used to estimate size and direction of discretization errors.

Vol	a^{-1} [GeV]	M_π [MeV]	M_K [MeV]	conf
$48^3 \times 96$	1.7295(38)	139	499	88
		135	496	5 (PQ-correction)
$64^3 \times 128$	2.359(7)	139	508	80
		135	496	5 (PQ-correction)

Tuning "inclusiveness"

- N : number of poles
- C [GeV^2] : center of poles
- Δ : spacings of poles (insensitive)

- $N = 4, \Delta = 0.05$ GeV^2 example
- $K, K\pi$ contribution are largest
- higher multiplicity modes and OPE strongly suppressed.



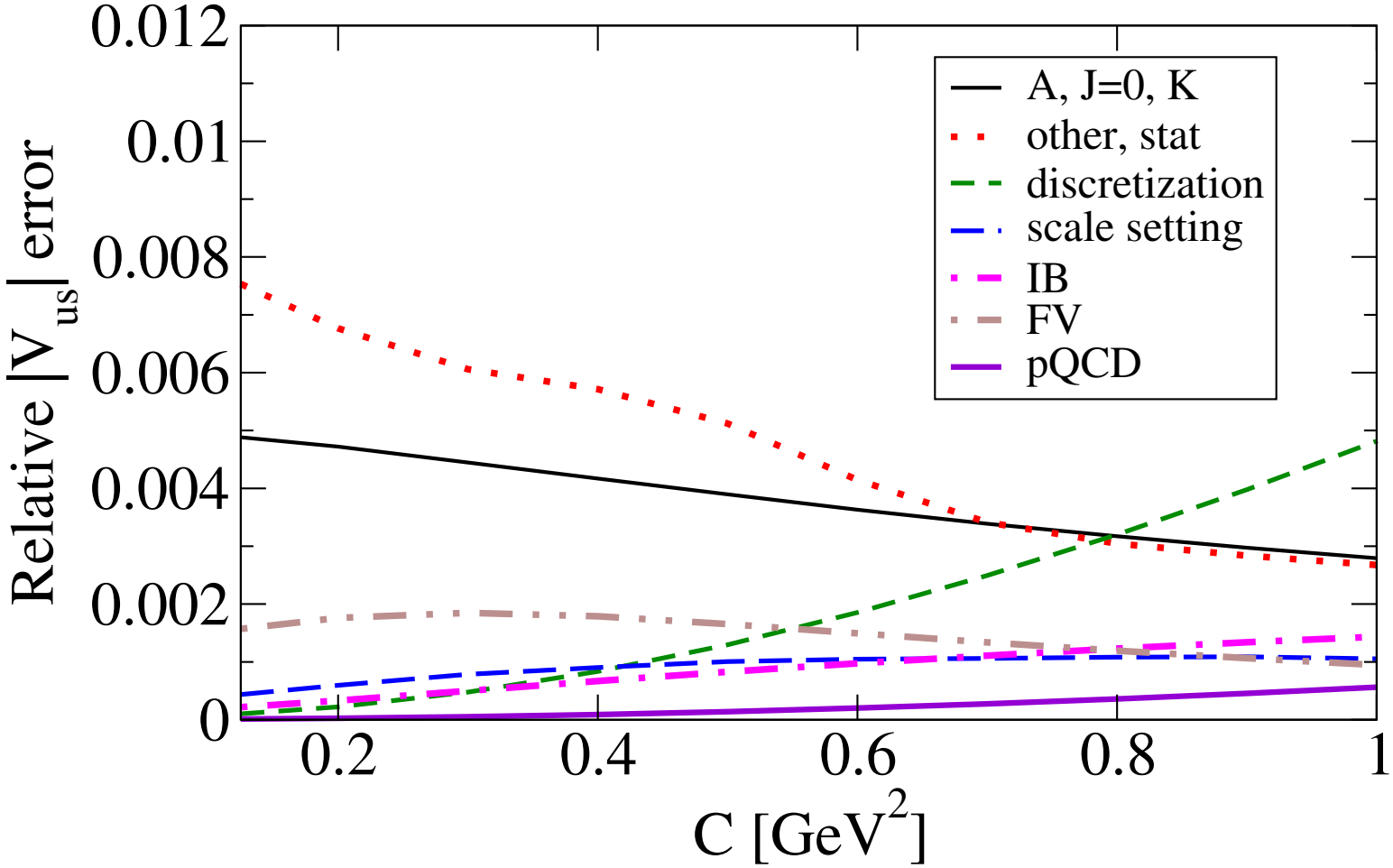
Error budget

- Lattice stat errors are still significant (could be improved)
- Discretization error from $O(a\Lambda_{QCD})^4$
- Finite Volume error, estimated from ChPT in FV for $K\pi$ channel
- Isospin breaking effects, corrected from IB in K_{l2} and $K \to \pi$ analysis [Antonelli, Cirigliano, Lusiani, Passemar, JHEP10(2013)070] ($\pi^0 - \eta$ mixing, EM, IB phase space) Residual theory error: 0.2 %
- pQCD/OPE uncertanty 2% of pQCD contribution
- Experimental specral funcation error is currently comparable.

Error budget

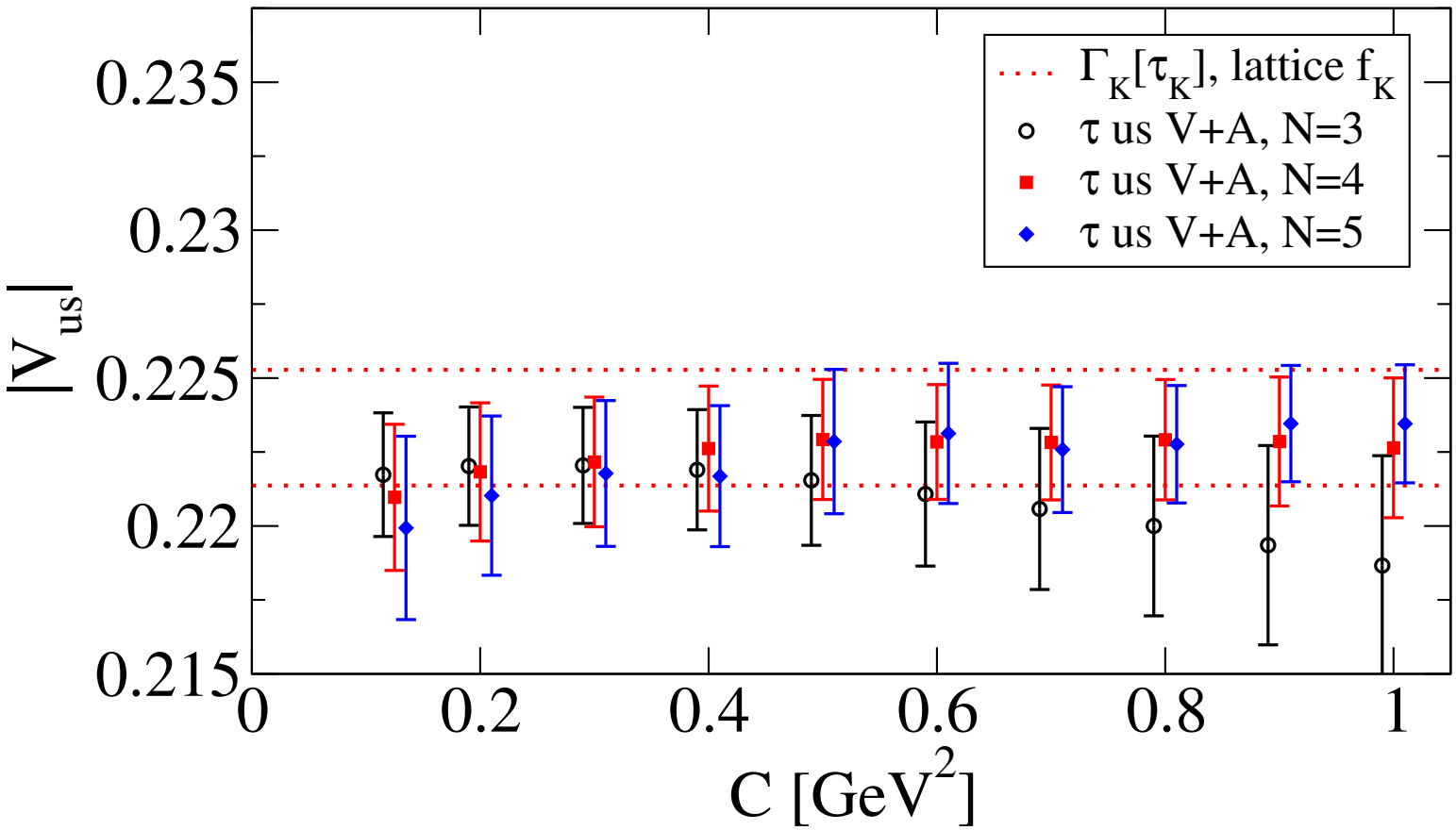
contribution		relative error (%)			
	[N, C[GeV^2]]	$[3, 0.3]$	$[3, 1]$	$[4, 0.7]$	[5, 0.9]
theory	f_K	0.37	0.20	0.34	0.36
	others, stat.	0.41	0.19	0.34	0.41
	discretization	0.10	0.80	0.25	0.27
	scale setting	0.09	0.08	0.11	0.11
	IB	0.10	0.21	0.11	0.10
	FV	0.10	0.04	0.13	0.18
	pQCD	0.05	0.26	0.03	0.01
	total	0.59	0.91	0.58	0.65
experiment	K	0.48	0.27	0.44	0.47
	$K\pi$	0.20	0.32	0.23	0.22
	$K^-\pi^+\pi^-$	0.06	0.16	0.06	0.05
	$\bar{K}^0\pi^-\pi^0$	0.03	0.09	0.03	0.03
	residual	0.41	1.35	0.41	0.28
	total	0.66	1.43	0.65	0.59
Combined	total	0.88	1.70	0.87	0.88

Errors Breakup $|V_{us}|$

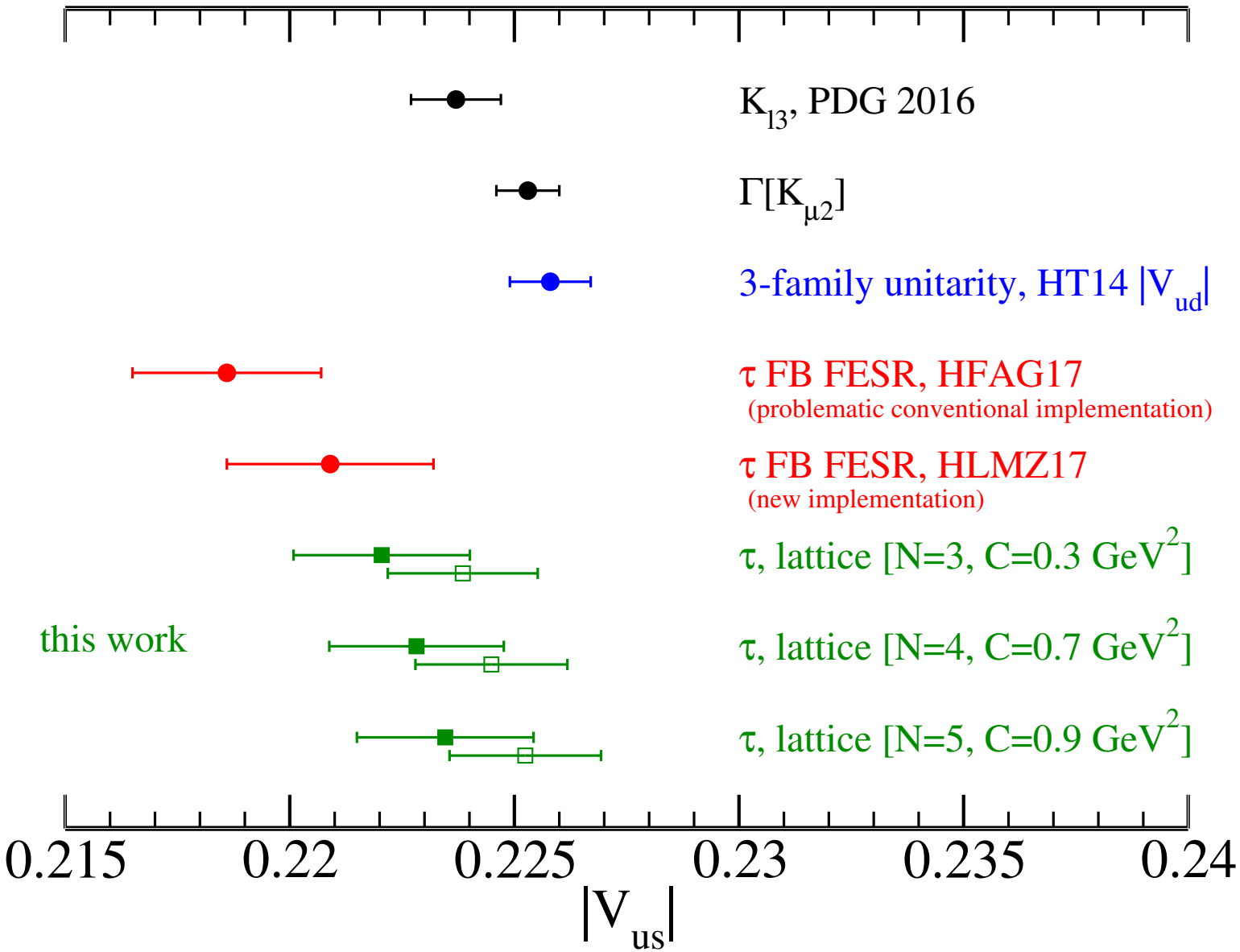


$N = 4$ case

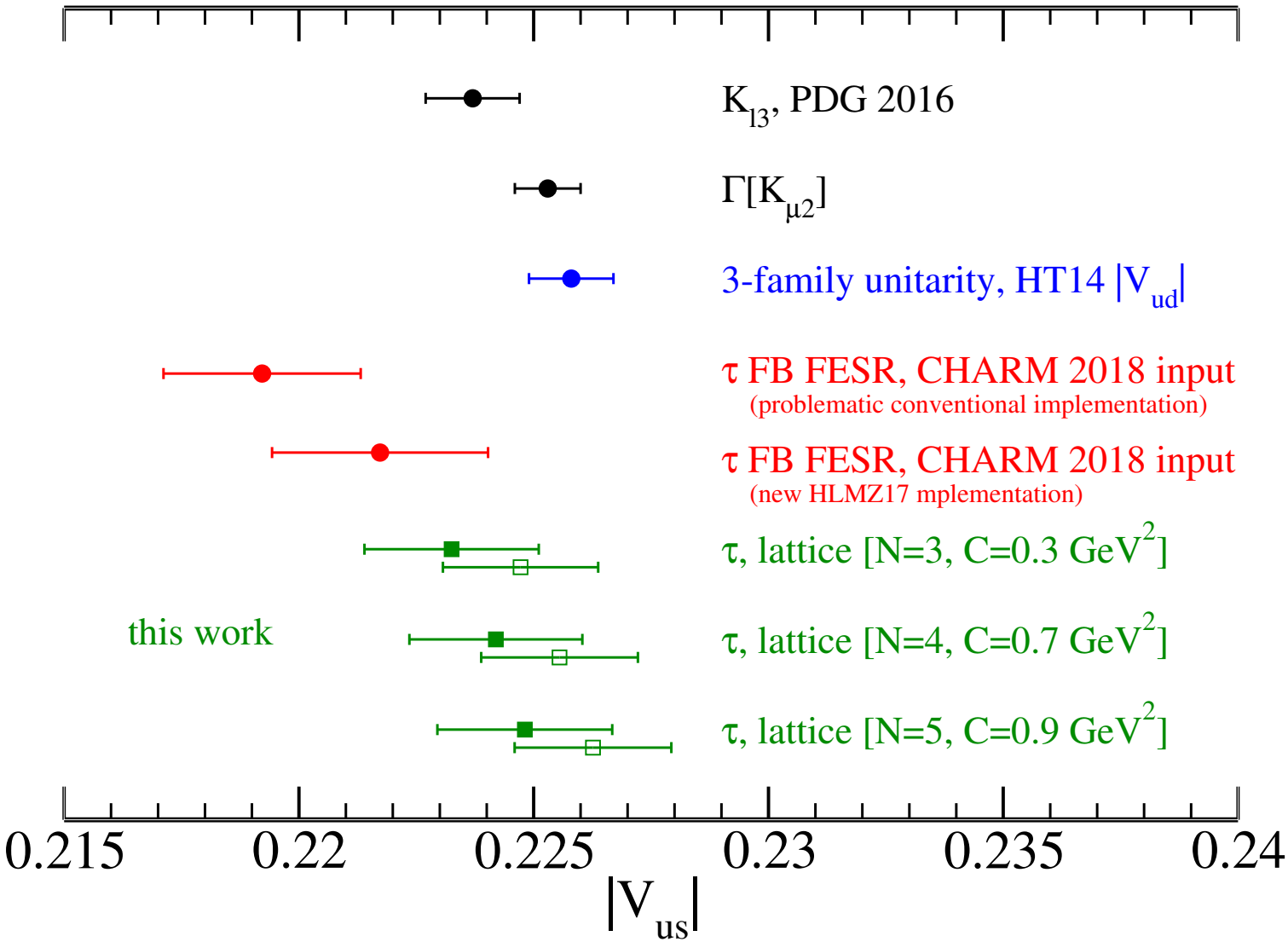
$|V_{us}|$ Results



- $N = 3, 4, 5$. Full error. Holizontal dots is exclusive $\tau \to K$ detemination using f_K.
- All estimated systematic erros included



- filled square: K pole from $\gamma_K[\tau_K]$ empty square: K pole from $\gamma_K[K_{\mu 2}]$
- HLMZ17 uses the fitted OPE condensations for D >4 varying s_0 and $\omega(s)$.



- BFs are updated using those reported at Charm 2018 by A. Lusiani
- HLMZ17 is also updated

Error budget (updated BF at Charm 2018 by A. Lusiani)

contribution		relative error (%)			
	[N, C[GeV^2]]	$[3, 0.3]$	$[3, 1]$	$[4, 0.7]$	[5, 0.9]
theory	f_K	0.37	0.20	0.34	0.36
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experiment	K	0.39	0.22	0.36	0.39
	$K\pi$	0.16	0.27	0.19	0.18
	$K^-\pi^+\pi^-$	0.06	0.16	0.06	0.05
	$\bar{K}^0\pi^-\pi^0$	0.03	0.09	0.03	0.03
	residual	0.40	1.34	0.41	0.27
	total	0.59	1.39	0.58	0.51
Combined	total	0.83	1.67	0.82	0.83

- Exp error reduces about 10%. Now theory and experimental errors are same for the preferred $N = 4$ value

Discussions and Conclusions

- By introducing a weight function with poles at space-like momentum region with Lattice QCD's first principle calculation replacing OPE, we propose a new type of |Vus| determination. Systematical robust cross check by changing weight function N, C (controlling ''inclusiveness''). Contribution from higher multiplicity could be suppressed

- Our results including all sys error from τ inclusive decay

$$|V_{us}| = \begin{cases} 0.2228(15)_{exp}(13)_{th}[0.9\%], & for\ \gamma_K[\tau_K] \\ 0.2245(11)_{exp}(13)_{th}[0.8\%], & for\ \gamma_K[K_{\mu 2}] \end{cases}$$

for $N = 4, C = 0.7$ [GeV^2], vs $V_{us} = 0.2258(9)[0.4\%]$ from PDG17
($c.f.$ $V_{us} = 22333(60)[0.3\%]$ FNAL/MILC 18 [A. Kronfeld's plenary talk])

- By using updated BF for $K, K\pi$ [A. Lusiani CHARM 2018] ,

$$|V_{us}| = \begin{cases} 0.2242(13)_{exp}(13)_{th}[0.8\%], & for\ \gamma_K[\tau_K] \\ 0.2256(10)_{exp}(13)_{th}[0.7\%] & for\ \gamma_K[K_{\mu 2}] \end{cases}$$

- Future improvements of this analysis will provide further information on new physics, which may be sensitive to τ inclusive decays. The combined experimental uncertainty can be further reduced through improvements to the experimental $\tau \to K, K\pi$ BFs, while the largest of the Lattice error is due to statistics, easy to improve.