Status of Vus determination from Kaon decays

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Outline :

1. Introduction and Motivation
2. V_{us} from $K_{\ell 3}$ decays
3. V_{us}/V_{ud} from $K_{\ell 2}/\pi_{\ell 2}$ decays
4. V_{us} and Unitarity of the CKM matrix
5. Conclusion and outlook

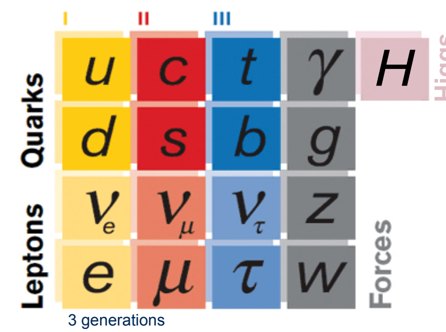
1. Introduction and Motivation

1.1 Test of the Standard Model: V_{us} and CKM unitarity

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model

    Description of the weak interactions:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_{\alpha}^{+} \left(\bar{D}_L V_{CKM} \gamma^{\alpha} U_L + \bar{e}_L \gamma^{\alpha} \nu_{e_L} + \bar{\mu}_L \gamma^{\alpha} \nu_{\mu_L} + \bar{\tau}_L \gamma^{\alpha} \nu_{\tau_L} \right) + \mathrm{h.c.}$$



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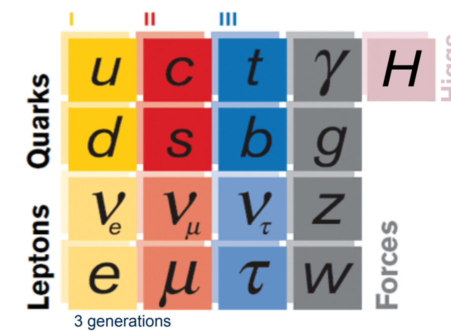
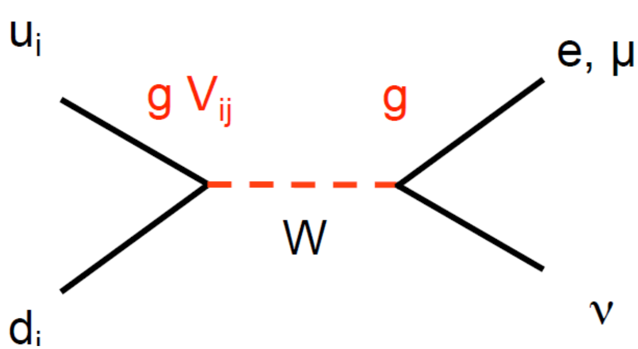
Unitary matrix



  - ➢ Check unitarity of the first row of the CKM matrix:

Cabibbo Universality: $\boxed{|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} \stackrel{?}{=} 1}$

Negligible ~2×10^{-5}
(B decays)



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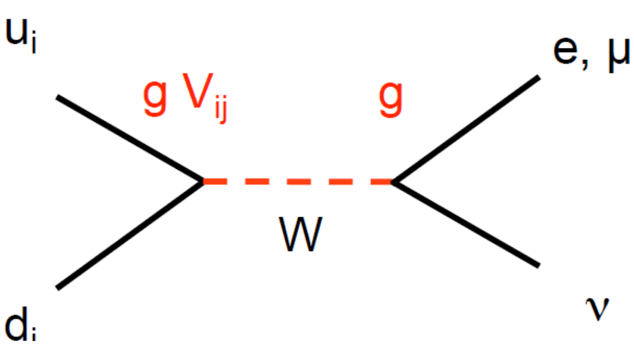
Gauge coupling

  - Universality: Is G_F from μ decay equals to G_F from π, K, nuclear β decay?

$$\boxed{G_\mu^2 = (g_\mu g_e)^2 / M_W^4 \quad \stackrel{?}{=} \quad G_{\rm CKM}^2 = (g_q g_\ell)^2 \left(|V_{ud}|^2 + |V_{us}|^2 \right) / M_W^4}$$



1.2 Constraining New Physics

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
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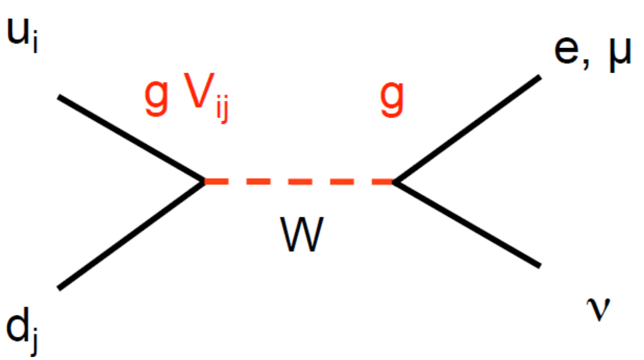
$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{eL} + \bar{\mu}_L \gamma^\alpha \nu_{\mu L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau L} \right) + \mathrm{h.c.}$$

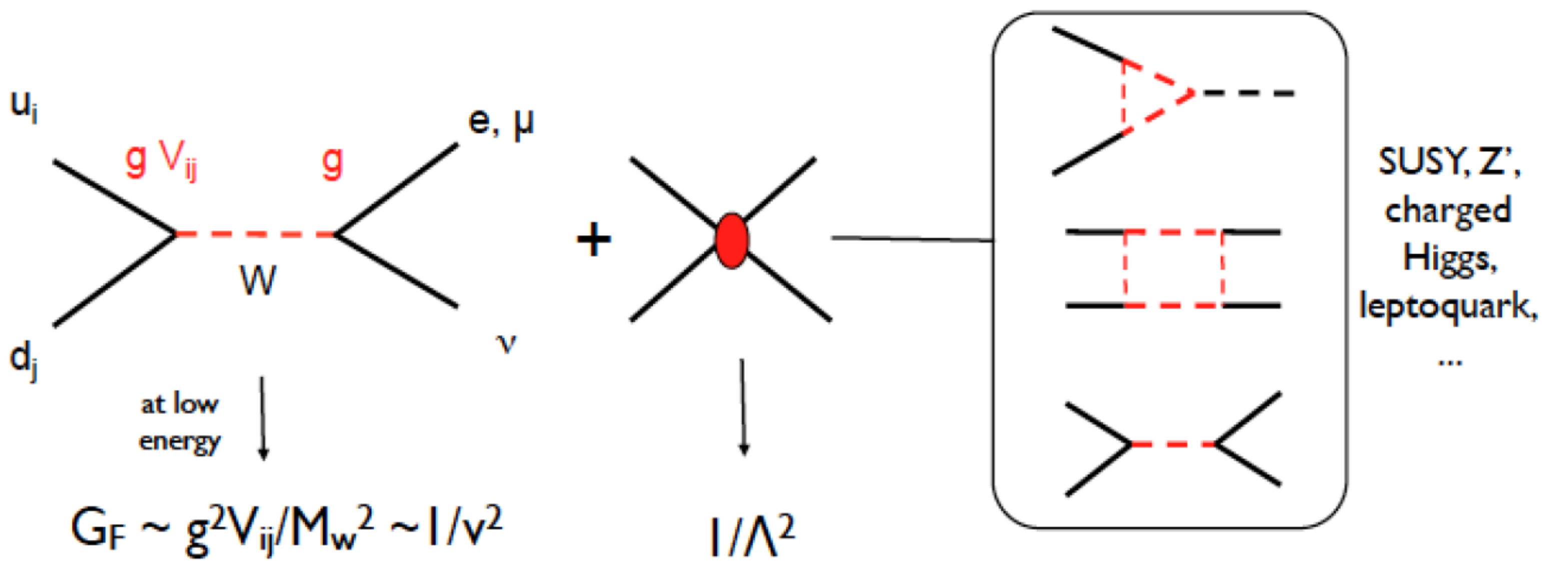
- Look for *new physics*
  - ➢ In the Standard Model : W exchange ⇨ only V-A structure



1.2 Constraining New Physics

- BSM: sensitive to tree-level and loop effects of a large class of models

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$$

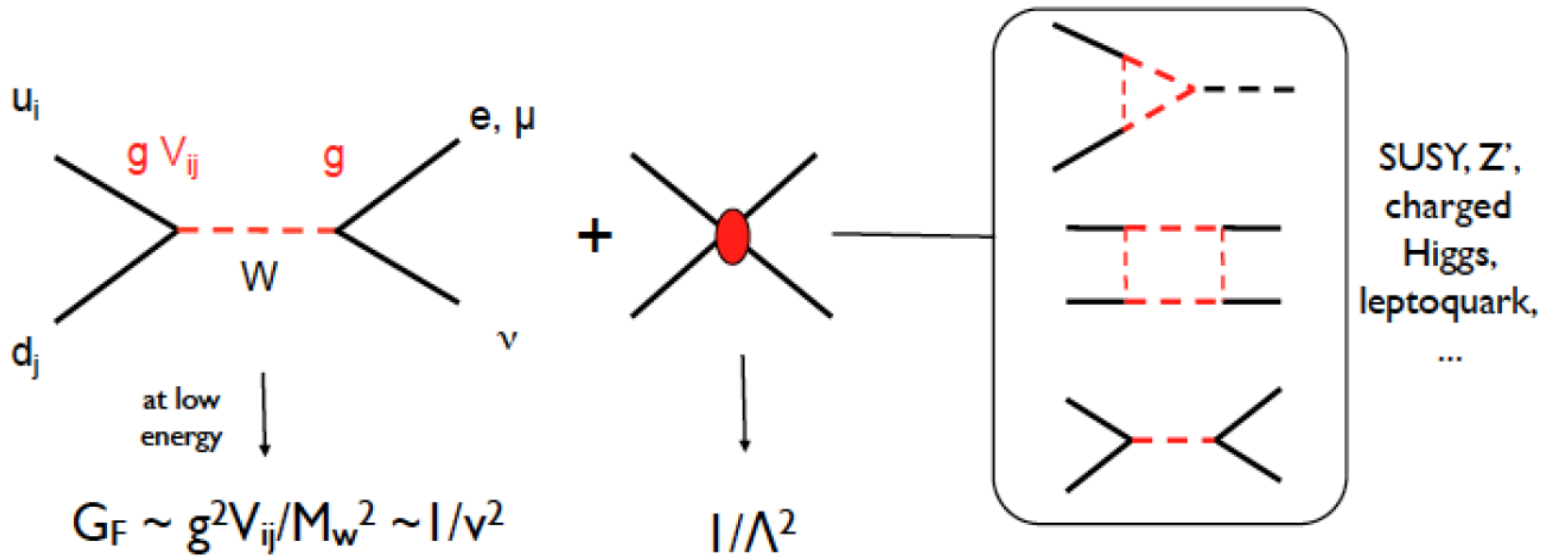


BSM effects : $\boxed{\Delta \sim \frac{c_n}{g^2}\frac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \longleftrightarrow \Lambda \sim 1\text{-}10\ \text{TeV}}$

1.2 Constraining New Physics

➤ BSM: sensitive to tree-level and loop effects of a large class of models

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$$



➤ Look for new physics by comparing the extraction of V_{us} from different processes: helicity suppressed $K_{\mu 2}$, helicity allowed K_{l3}, hadronic τ decays

1.3 Some history

- 2002: Old $K \rightarrow \pi l \nu_l$ data give $\boxed{\Delta_{CKM} = 1 - |V_{ud}|^2 - |V_{us}|^2 = 0.0035(15)}$

  ⇨ PDG 2004: a 2.3σ hint of *unitarity violation*?

- 2003 BNL 865 measures $BR\left(K^+ \rightarrow \pi^0 e^+ \nu\right) = 5.13(10)\%$
  ⇨ value of V_{us} consistent with unitarity

- 2004 – present: Many new measurements from KTeV, ISTRA+, KLOE, NA48
  - BRs, lifetimes, form-factors
  - Much higher statistics than older measurements
  - Proper account of correlations between measurements

  ⇨ Isospin breaking, radiative corrections start to matter: computed within ChPT

- 2008 – beyond: Progress in the computation of hadronic elements from lattice QCD

- Value of V_{us} used in precise test of the SM

1.4 Experiment, Theory & Evaluation

V_{us} from $K_{\ell 3}$ & $K_{\ell 2}$

- ~100 measurements of ~10 experimental parameters
- 50+ (and counting!) lattice results for 2 hadronic matrix elements
- Radiative and SU(2)-breaking corrections, ChPT results, etc.

FlaviA net Kaon WG

2006-2010 (EU 6FP)

Experimental averages, fits, etc

Selection of results (experiments, corrections)

Evaluation, discussion and intepretation

Final report: EPJC 69 (2010) 399

This talk is an attempt at an update to 2018

Corresponding effort to synthesize results from **lattice QCD:**

Flavor Lattice Averaging Group (FLAG):

http://itpwiki.unibe.ch/flag

Participation by all major lattice collaborations

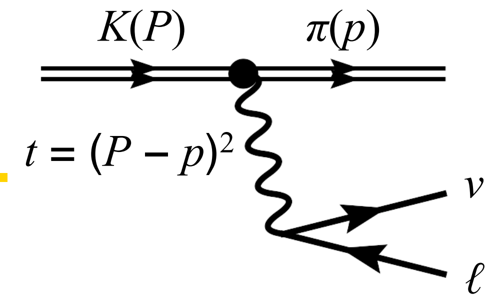
Biannual review of lattice results for π, K, B, D physics

2013 review: EPJC 74 (2014) 2890

2016 review: EPJC77 (2017) no.2, 112

2. V_{us} from $K_{\ell 3}$ decays

2.1 V_{us} from K_{l3} decays



- Master formula for $K \to \pi l \nu_l$: $K = \{K^+, K^0\}$, $l = \{e, \mu\}$

$$\boxed{\Gamma\left(K \to \pi l \nu[\gamma]\right) = Br(K_{l3}) * \tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl}\left(1 + 2\Delta_{\mathrm{EM}}^{Kl} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)}$$

Experimental inputs:

$\Gamma(K_{l3})$ Rates with well-determined treatment of radiative decays
- Branching ratios
- Kaon lifetimes

$I_{Kl}(\lambda_{Kl})$ Integral of form factor over phase space: λs parametrize evolution in $t=q^2$

Inputs from theory:

S_{EW}^K Universal short distance EW corrections

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer (t=0)

$\Delta_{\mathrm{EM}}^{Kl}$ Form-factor correction for long-distance EM effects

$\Delta_{\mathrm{SU(2)}}^{K\pi}$ Form-factor correction for SU(2) breaking

2.2 Modern experimental data for V_{us} from $K_{\ell 3}$

Experiment	Measurement	Year
BNL865	**BR($K^+ \to \pi^0{}_D e^+ \nu$)/BR($K^+ \to \pi^0{}_D X^+$)**	**2003**
KTeV	**$\tau(K_S)$**	**2003**
	BR(K_{Le3}), BR($K_{L\mu 3}$), $\lambda_+(K_{Le3})$, $\lambda_{+,0}(K_{L\mu 3})$	**2004**
ISTRA+	**$\lambda_+(K^-{}_{e3})$, $\lambda_{+,0}(K^-{}_{e3})$**	**2004**
KLOE	**$\tau(K_L)$**	**2005**
	BR(K_{Le3}), BR($K_{L\mu 3}$), BR(K_{Se3}), $\lambda_+(K_{Le3})$	**2006**
	$\lambda_{+,0}(K_{L\mu 3})$	**2007**
	$\tau(K^\pm)$, BR(K_{Le3}), BR($K_{L\mu 3}$)	**2008**
NA48	**$\tau(K_S)$**	**2002**
	BR(K_{Le3}/2 tracks), $\lambda_+(K_{Le3})$	**2004**
	$\Gamma(K_{Se3}/K_{Le3})$, $\lambda_{+,0}(K_{L\mu 3})$	**2007**
NA48/2	**BR($K^+{}_{e3}/\pi^+\pi^0$), BR($K+{}_{\mu 3}/\pi^+\pi^0$)**	**2007**

Above data set used for 2010 FlaviaNet review (fits, averages, etc.)

Updated fit to K_S rate data

6 input measurements:

- **KLOE** BR $\pi^0\pi^0/\pi^+\pi^-$
- **KLOE** BR $\pi e\nu/\pi^+\pi^-$
- **NA48** $\Gamma(K_S \to \pi e\nu)/\Gamma(K_L \to \pi e\nu)$, τ_S
- **KLOE'11** τ_S
- **KTeV'11** τ_S

2 constraints:

- **Σ BR = 1**
- **BR(K_{e3})/BR($K_{\mu3}$) = 0.66492(137)**
  From ratio of phase-space integrals from current fit to dispersive $K_{\ell3}$ form factor parameters

Parameter	Value
BR($\pi^+\pi^-(\gamma)$)	69.20(5)%
BR($\pi^0\pi^0$)	30.69(5)%
BR(K_{e3})	7.05(8) × 10^{-4}
BR($K_{\mu3}$)	4.69(6) × 10^{-4}
τ_S	89.58(4) ps

χ^2/ndf = 0.20/3 (Prob = 98%)

ρ(BR($\pi^+\pi^-$), BR($\pi^0\pi^0$)) = –0.998
Little freedom in fit

Largest effect of **2011 τ_S data:**

FlaviaNet 2010
τ_S = 89.59(6) ps

→

Update
τ_S = 89.58(4) ps

Updated fit to K_L rate data

21 input measurements:

- **5 KTeV** ratios
- **NA48** BR(K_{e3}/2 track)
- **4 KLOE** BRs
  with dependence on τ_L
- **KLOE**, **NA48** BR($\pi^+\pi^-/K_{\ell 3}$)
- **KLOE**, **NA48** BR($\gamma\gamma/3\pi^0$)
- **BR($2\pi^0/\pi^+\pi^-$) from K_S fit, Re ε'/ε**
- **KLOE** τ_L from $3\pi^0$
- **Vosburgh '72** τ_L
- **KTeV** BR($\pi^+\pi^-\gamma/\pi^+\pi^-(\gamma)$)
- **E731, 2 KTeV** BR($\pi^+\pi^-\gamma_{\mathrm{DE}}/\pi^+\pi^-\gamma$)

1 constraint: Σ BR = 1

Parameter	Value	S
BR(K_{e3})	0.4056(9)	1.3
BR($K_{\mu 3}$)	0.2704(10)	1.5
BR($3\pi^0$)	0.1952(9)	1.2
BR($\pi^+\pi^-\pi^0$)	0.1254(6)	1.3
BR($\pi^+\pi^-(\gamma_{\mathrm{IB}})$)	$1.967(7) \times 10^{-3}$	1.1
BR($\pi^+\pi^-\gamma$)	$4.15(9) \times 10^{-5}$	1.6
BR($\pi^+\pi^-\gamma_{\mathrm{DE}}$)	$2.84(8) \times 10^{-5}$	1.3
BR($2\pi^0$)	$8.65(4) \times 10^{-4}$	1.4
BR($\gamma\gamma$)	$5.47(4) \times 10^{-4}$	1.1
τ_L	51.16(21) ns	1.1

χ^2/ndf = 19.8/12 (Prob = 7.0%)

Essentially same result as 2010 fit

Current PDG ('09): **37.4/17** (0.30%)

Updates: $K^{\pm}$ BRs and lifetimes

KLOE-2
PLB 738 (2014)

BR($\pi^+\pi^+\pi^-$) = 0.05565(31)(25) (0.7%)

- **No good measurements of BR($\pi^+\pi^+\pi^-$) in 2010 fit**
- Reconstruct 2 tracks in small fiducial volume near interaction region; evaluate missing mass for 3rd track
- Fully inclusive of radiation, but radiative corrections handled differently from other KLOE measurements
- Significant impact on value of BR($\mu\nu$) from fit
  Correlation between BR($\mu\nu$), BR($\pi^+\pi^+\pi^-$) = −0.75

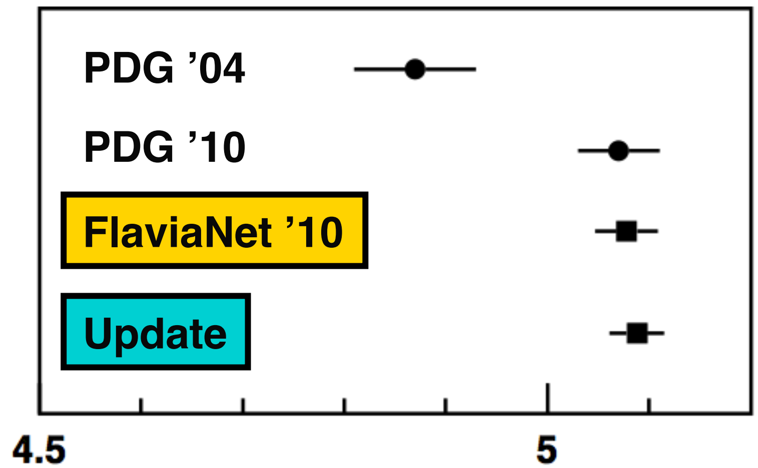
ISTRA+
PAN 77 (2014)

BR($K^-_{e3}/\pi^-\pi^0$) = 0.2423(15)(37) (1.6%)

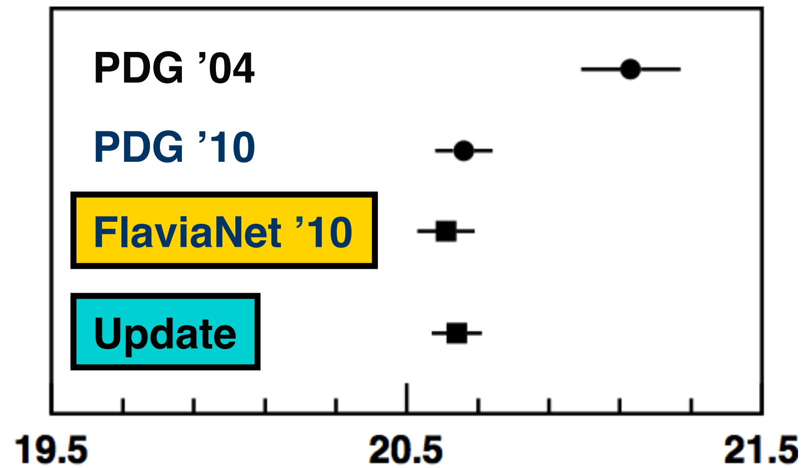
- Claimed to be fully inclusive for $K_{e3\gamma}$
  - No mention of radiative corrections
  - Many cuts, mainly topological
  - 3 different selections, at least 1 may be largely inclusive
- Included in PDG ’15 fit
- **Treated as preliminary here (not in $K^{\pm}$ BR fit)**

Updates: $K^{\pm}$ BRs and lifetimes

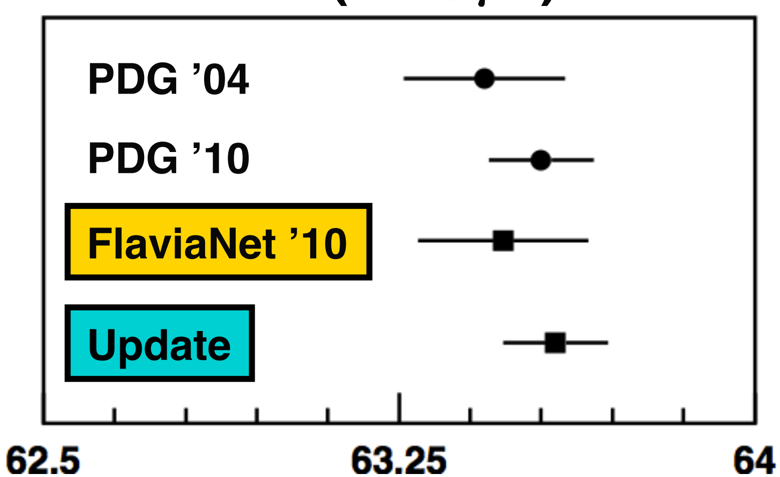
BR($K^{\pm} \to \pi^0 e\nu$)



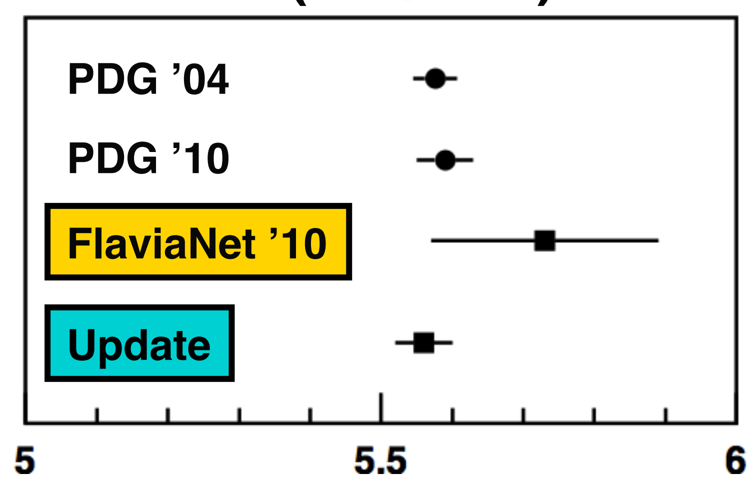
BR($K^{\pm} \to \pi\pi^0$)



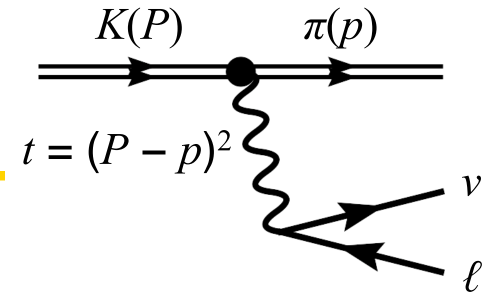
BR($K^{\pm} \to \mu\nu$)



BR($K^{\pm} \to \pi\pi\pi$)



2.3 Electroweak corrections



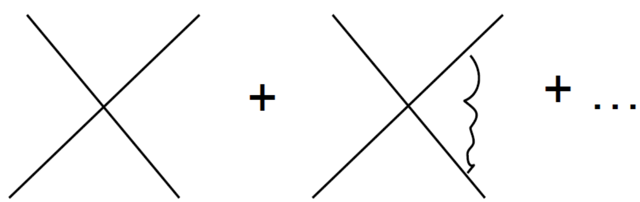
- Master formula for $K \to \pi l \nu_l$: K = {K^+,K^0}, l={e,μ}

$$\Gamma\left(K \to \pi l \nu[\gamma]\right) = Br(K_{l3}) * \tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} \boxed{S_{EW}^K} |V_{us}|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl} \left(1 + 2\Delta_{\mathrm{EM}}^{Kl} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

- S_{ew} : Short distance electroweak correction

$$S_{\mathrm{ew}} = 1 + \frac{2\alpha}{\pi}\left(1 + \frac{\alpha_s}{4\pi}\right)\log\frac{m_Z}{m_\rho} + O\left(\frac{\alpha\alpha_s}{\pi^2}\right) \Rightarrow S_{\mathrm{ew}} = 1.0232(3)$$

Sirlin'82



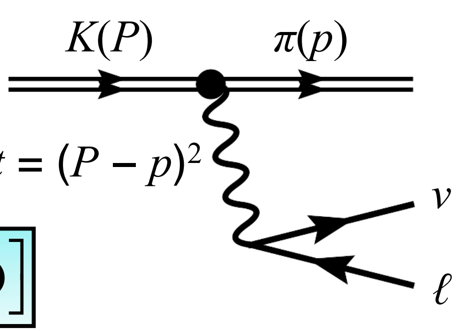
=> $G_\mu V_{ij} \sqrt{S_{\mathrm{ew}}}$

2.4 Kπ form factors

- Master formula for $K \to \pi l \nu_l$: K = {K^+,K^0}, l={e,μ}

$$\Gamma\left(K \to \pi l \nu[\gamma]\right) = Br(K_{l3}) * \tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K |V_{us}|^2 \boxed{f_+^{K^0\pi^-}(0)}^2 I_{Kl}\left(1 + 2\Delta_{\rm EM}^{Kl} + 2\Delta_{\rm SU(2)}^{K\pi}\right)$$

- *$f_+(0)$* : vector form factor at zero momentum transfer:



Hadronic matrix element:

$$\left\langle \pi^-(p) \right| \bar{s}\gamma_\mu u \left| K^0(P) \right\rangle = f_+^{K^0\pi^-}(0)\left[\left(P+p\right)_\mu \bar{f}_+^{K^0\pi^-}(t) + \left(P-p\right)_\mu \bar{f}_-^{K^0\pi^-}(t)\right]$$

$f_+(0)$ key hadronic quantity: In SU(3)$_V$ limit (mu=md=ms), CVC ⇨ $f_+(0) = 1$
Need to compute corrections in second order in SU(3) breaking

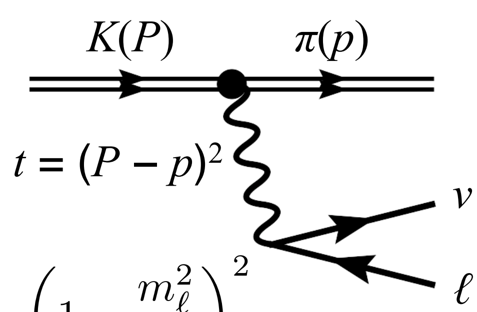
⇨ see later

2.4 Kπ form factors

- Master formula for K → πlν$_l$: K = {K$^+$,K^0}, l={e,μ}

$$\Gamma\left(K \to \pi l \nu[\gamma]\right) = Br(K_{l3})*\tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 \boxed{I_{Kl}} \left(1 + 2\Delta_{\mathrm{EM}}^{Kl} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

Hadronic matrix element:



$$\left\langle \pi^-(p)\right| \bar{s}\gamma_\mu u \left|K^0(P)\right\rangle = f_+^{K^0\pi^-}(0)\left[\left(P+p\right)_\mu \bar{f}_+^{K^0\pi^-}(t) + \left(P-p\right)_\mu \bar{f}_-^{K^0\pi^-}(t)\right]$$

- Phase space integrals: $$I_{K\ell} = \frac{2}{3}\int_{m_\ell^2}^{t_0} \frac{dt}{M_K^8}\,\bar{\lambda}^{3/2}\left(1+\frac{m_\ell^2}{2t}\right)\left(1-\frac{m_\ell^2}{2t}\right)^2 \times \left(\bar{f}_+^2(t) + \frac{3m_\ell^2\Delta_{K\pi}^2}{(2t+m_\ell^2)\bar{\lambda}}\bar{f}_0^2(t)\right),$$

- In K$_{e3}$ decays: only vector FF $\bar{f}_+^{K^0\pi^-}(t)$

- In K$_{\mu3}$ decays, also need the scalar FF $$\bar{f}_0(t) = \bar{f}_+(t) + \frac{t}{m_K^2 - m_\pi^2}\bar{f}_-(t)$$

- For V_{us}, need integral over phase space of squared matrix element: Parameterize form factors and fit distributions in t (or related variables)

$K_{\ell 3}$ form factors parametrizations

- Parametrizations based on Taylor expansion:

$$\bar{f}_{+,0}(t) = 1 + \lambda_{+,0}\left(\frac{t}{m^2_{\pi^\pm}}\right) \quad \text{or} \quad \bar{f}_{+,0}(t) = 1 + \lambda'_{+,0}\left(\frac{t}{m^2_{\pi^\pm}}\right) + \lambda''_{+,0}\left(\frac{t}{m^2_{\pi^\pm}}\right)^2$$

Very simple parametrization but limited in energy range and not physically motivated: many parameters and strong correlations between them

⇨ unstable fits

- Physically motivated parametrizations:
  - Pole parametrization

$$\bar{f}_{+,0}(t) = \left(\frac{M^2_{V,S}}{M^2_{V,S} - t}\right)$$

Well motivated for the vector (K* resonance)
But for the scalar M_S?

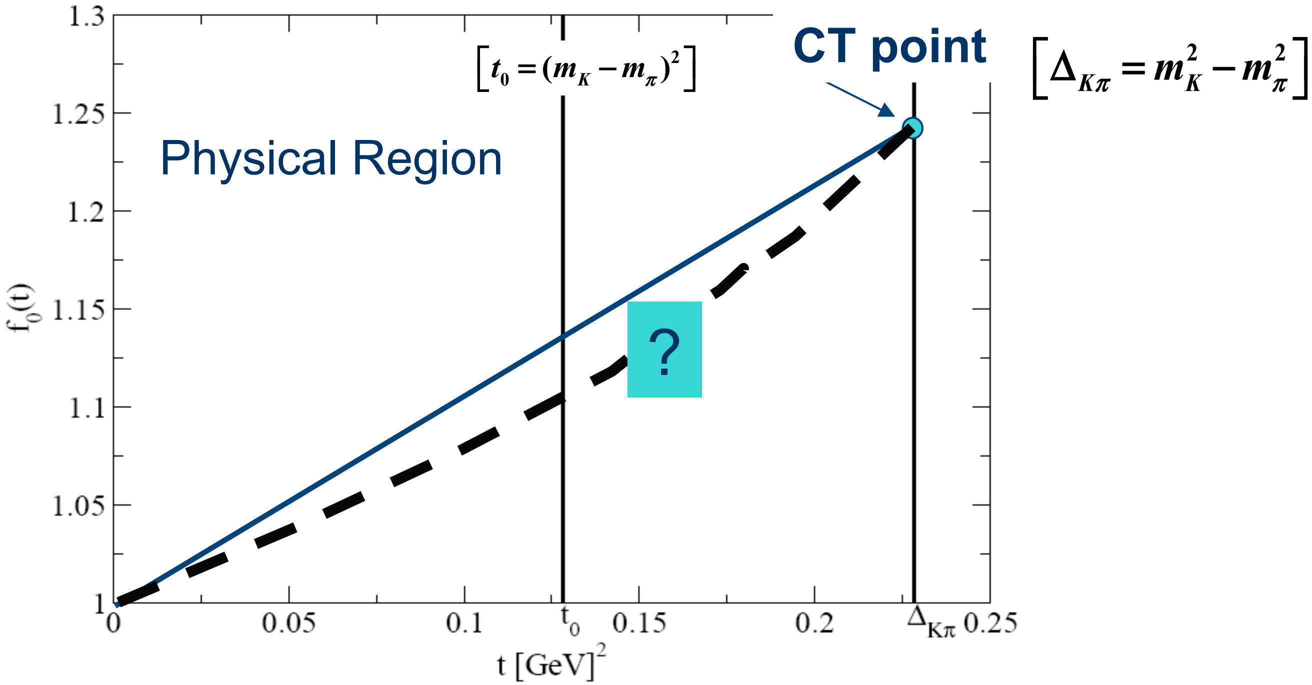
  - Dispersive parametrization

Bernard, Oertel, E.P., Stern'06,'09

$$\bar{f}_{+}(t) = \exp\left[\frac{t}{m^2_{\pi}}\left(\Lambda_{+} - H(t)\right)\right] \quad \text{and} \quad \bar{f}_{0}(t) = \exp\left[\frac{t}{m^2_K - m^2_{\pi}}\left(\ln C - G(t)\right)\right]$$

Dispersive representation for the form factors

- Take the Kπ rescattering into account *Bernard, Oertel, E.P., Stern'06, '09*
- Allow to determine the slope and *curvature* of the form factors: only 2 param.



- Use the CT theorem for the scalar FF ⇨ Write a twice substracted dispersion relation for ln f(t) at t=0 and at the CT point for the scalar FF
- Does it improve the agreement with data?

2.4 Dispersive representation for the form factors

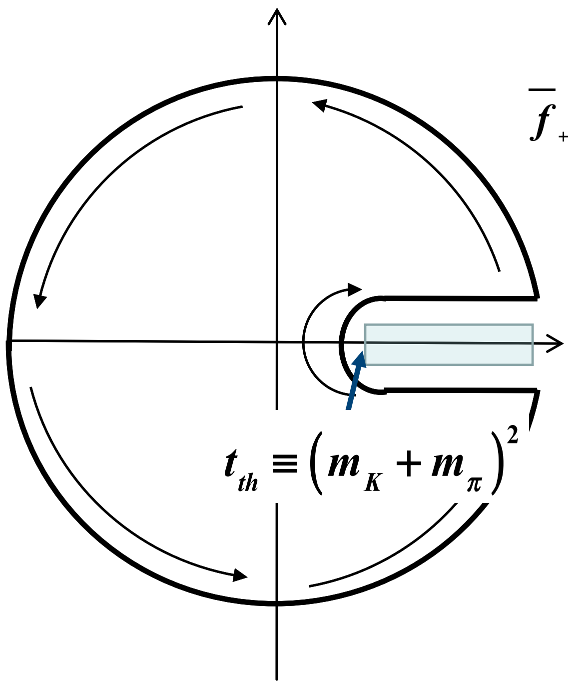
- Use dispersion relations to parametrize the FFs

Unitarity: $disc\left[\bar{f}_{0,+}(s)\right] \propto t_\ell^{I*}(s)\,\bar{f}_{0,+}(s)$ *Bernard, Oertel, E.P., Stern'06,'09*

- Omnès representation:

$$\bar{f}_0(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2}\left(\ln C - \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi}\int_{t_{th}}^{+\infty}\frac{ds}{s}\frac{\phi_0(s)}{(s-\Delta_{K\pi})(s-t-i\varepsilon)}\right)\right]$$

$$\bar{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + \frac{m_\pi^2 t}{\pi}\int_{t_{th}}^{+\infty}\frac{ds}{s^2}\frac{\phi_+(s)}{(s-t-i\varepsilon)}\right)\right]$$



$\phi_{+,0}(s)$: phase of the form factor

- $s < s_{in}$: $\phi_{+,0}(s) = \delta_{K\pi}(s)$

Kπ scattering phase

- $s \geq s_{in}$: $\phi_{+,0}(s)$ unknown

⇨ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \quad \left(\bar{f}_{+,0}(s) \to 1/s\right)$

Brodsky & Lepage

- A large error turns out in a small uncertainty in the physical region

Callan-Treiman Low Energy Theorem

Bernard, Oertel, E.P., Stern'06, '08

- Callan-Treiman theorem:

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K \left|V^{us}\right|}{F_\pi \left|V^{ud}\right|} \frac{1}{f_+(0)\left|V^{us}\right|} \left|V^{ud}\right|}_{\text{Very precisely known from Br(Kl2/πl2), Γ(Ke3) and } |V_{ud}|} r + \Delta_{CT}$$

$\Delta_{K\pi} = m_K^2 - m_\pi^2$

$\Rightarrow$ $\boxed{B_{\exp} = 1.2446(41)}$

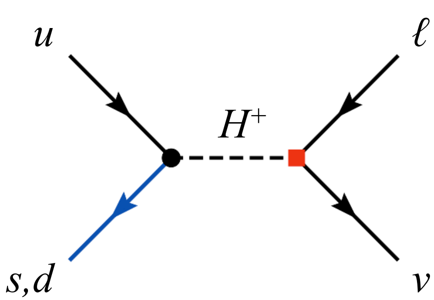
- In the Standard Model : $\boxed{r = 1}$ $\left(\ln C_{SM} = 0.2141(73)\right)$

$\Delta_{CT} = (-3.5 \pm 8).10^{-3}$

NLO value + large error bars in agreement with
Bijnens&Ghorbani'07
Kastner & Neufeld'08

- In presence of new physics, new couplings : $\boxed{r \neq 1}$

- Ex:



2.4 $K_{\ell 3}$ form factor data

- Form-factor parameter measurements in FlaviaNet 2010 fit:

  K_L: KTeV, KLOE, NA48 (K_{e3} only)

  K^-: ISTRA+

- Even if not in the original publications, all experiments have:
  - Obtained results for Taylor, pole, and dispersive parameterizations
  - Supplied parameter correlation coefficients

New measurements:

NA48/2
1808.09041

$2.3 \times 10^6\ K^{\pm}_{\mu 3}$
$4.4 \times 10^6\ K^{\pm}_{e3}$

Updating 2012 preliminary
See talk by M. Piccini

K^+ and K^- simultaneously acquired in dedicated minimum-bias run

Taylor, pole, and dispersive fits with complete investigation of systematics

OKA
JETPL 107 (2018)

$5.25 \times 10^6\ K^{+}_{e3}$

Described as preliminary

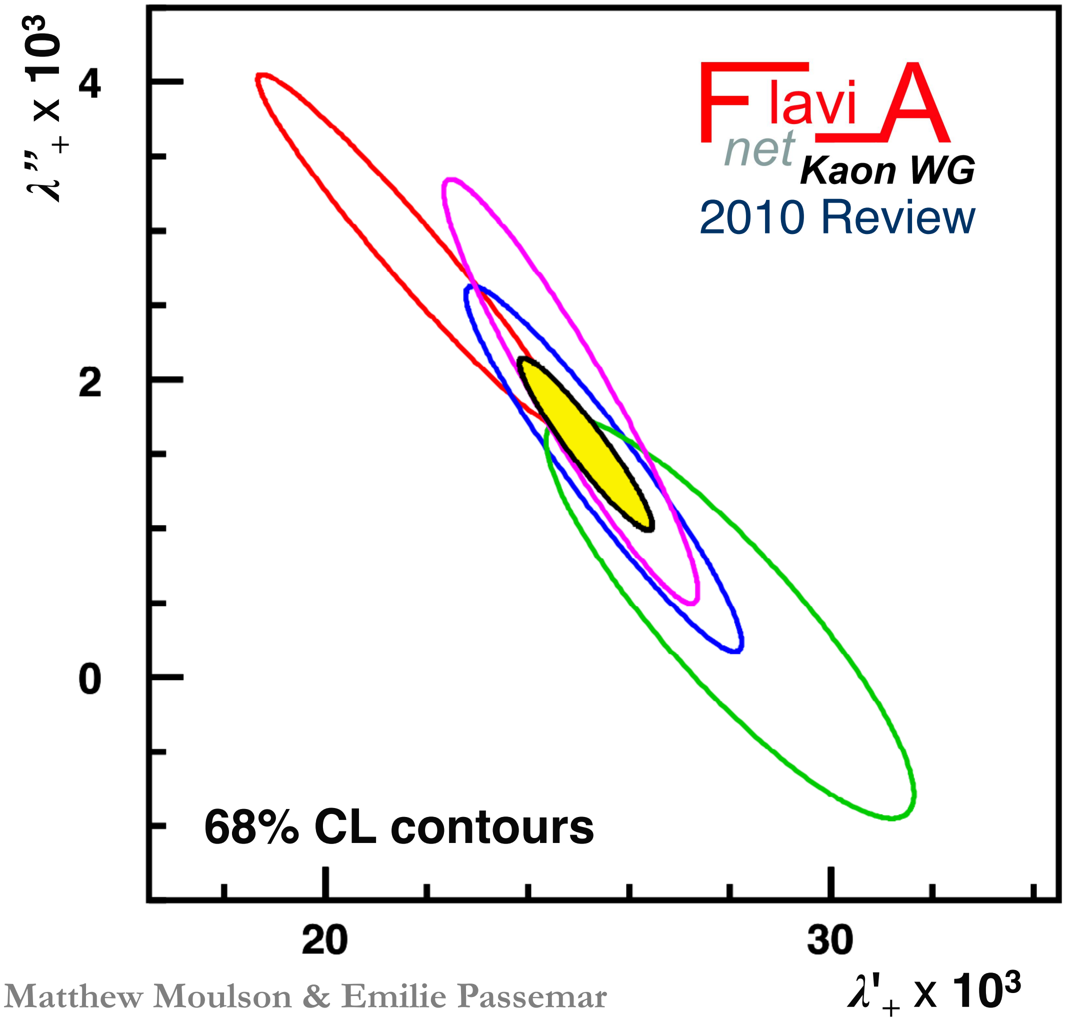
Extraordinarily high precision claimed, esp. for λ_+', λ_+''

Rudimentary discussion of systematics

Not yet included in updated K_{e3} fit

2.4 Fit to K_{e3} form-factor slopes: 2010

Slopes from KTeV KLOE ISTRA+ NA48 2010 fit



Slope parameters x 10^3

λ'_+ = **25.15 ± 0.87**

λ''_+ = **1.57 ± 0.38**

$\rho(\lambda'_+, \lambda''_+)$ = −0.941

χ^2/ndf = 5.3/6 (51%)

Excellent compatibility

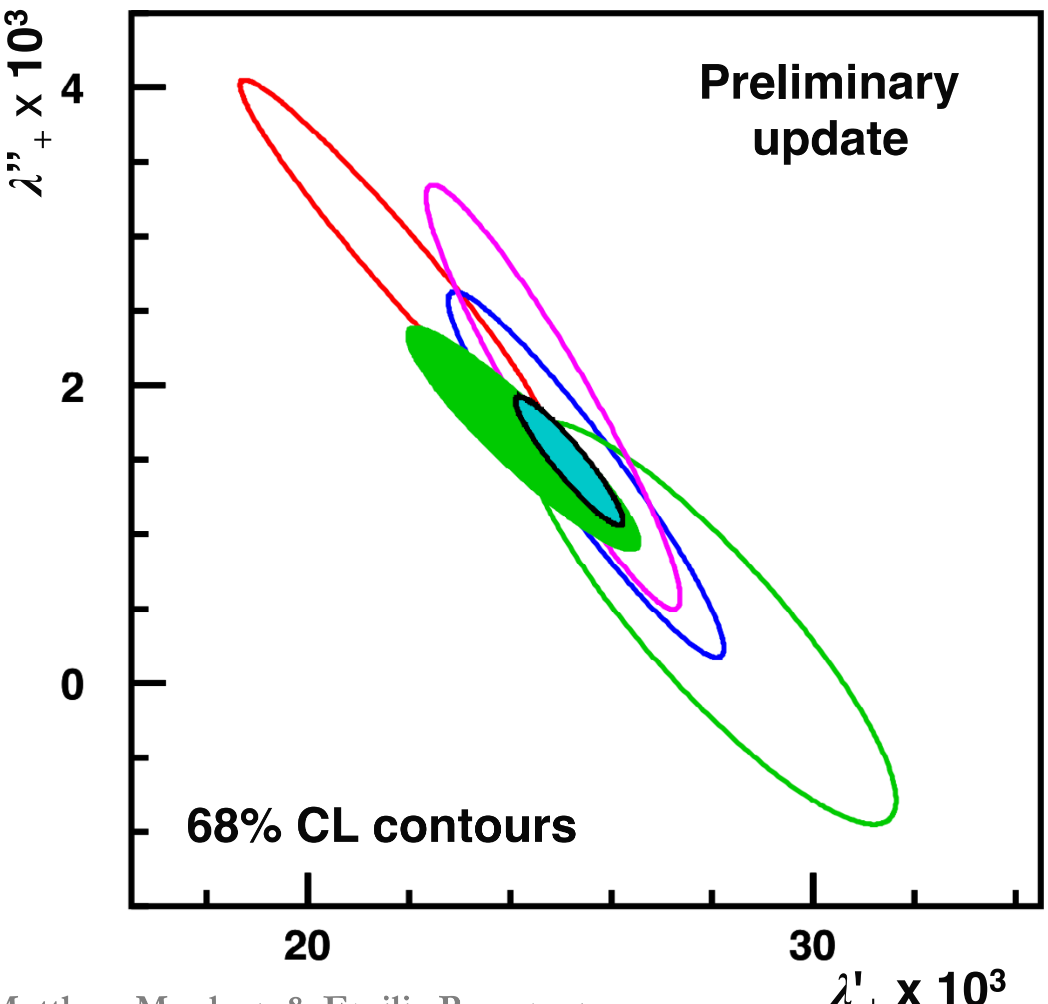
Significance of $\lambda''_+ > 4\sigma$

$I(K^0_{e3})$ = **0.15463(21)**

$I(K^+_{e3})$ = **0.15900(22)**

2.4 Fit to K_{e3} form-factor slopes: Update

Slopes from KTeV | KLOE | ISTRA+ | NA48 | NA48/2 | Update



Slope parameters x 10^3

$$\lambda'_+ = \mathbf{25.17 \pm 0.70}$$

$$\lambda'_+ = \mathbf{1.49 \pm 0.29}$$

$$\rho(\lambda'_+, \lambda''_+) = -0.929$$

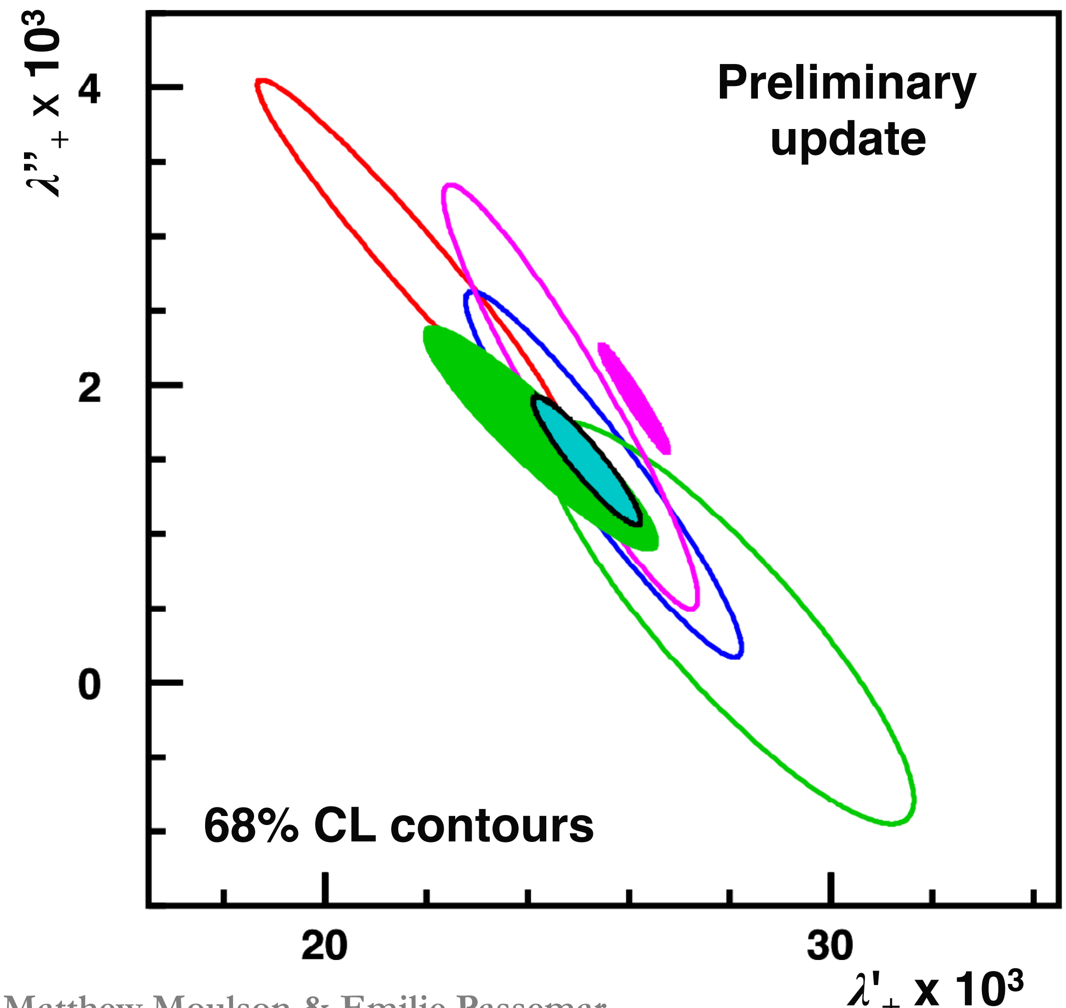
$$\chi^2/\text{ndf} = 6.4/10\ (61\%)$$

Excellent compatibility
Very small change in λ'_+

$I(K^0_{e3})$ = **0.15463(21)**
$I(K^+_{e3})$ = **0.15900(22)**

2.4 Fit to K_{e3} form-factor slopes: Update

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **NA48/2** **Update**



Slope parameters x 10^3

$$\lambda'_+ = \mathbf{25.17 \pm 0.70}$$

$$\lambda'_+ = \mathbf{1.49 \pm 0.29}$$

$$\rho(\lambda'_+, \lambda''_+) = -0.929$$

$$\chi^2/\text{ndf} = 6.4/10\ (61\%)$$

OKA
JETPL 107 (2018)

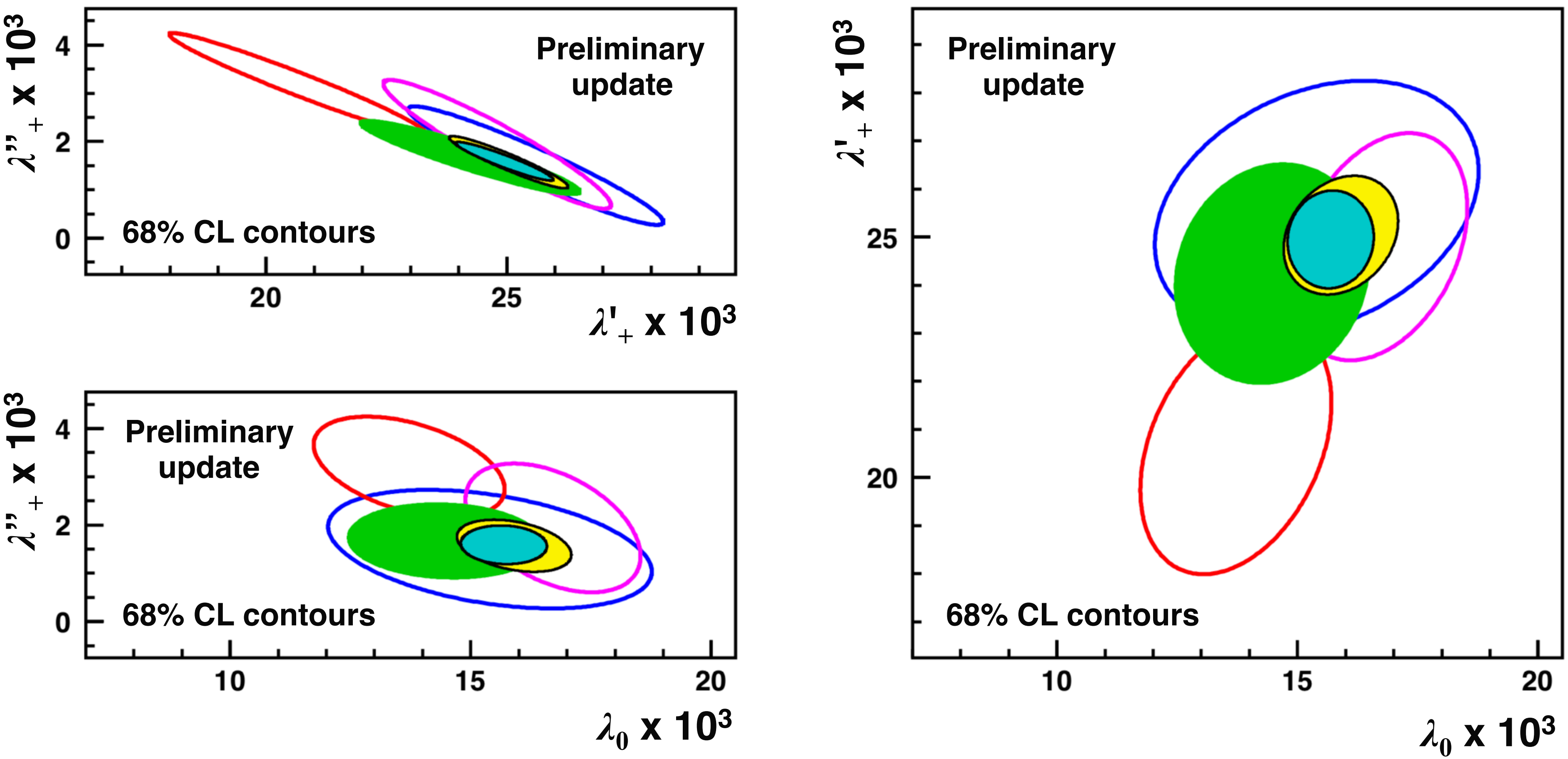
Not included in the fit

- Stated as preliminary
- If included: χ^2/ndf → 45/10 ($P \sim 10^{-6}$)

2.4 Fits to K_{e3} + $K_{\mu3}$ form-factor slopes: Update

KTeV | KLOE | ISTRA+ | NA48/2   2010 fit | Update

NA48 K_{e3} data included in fits but not shown



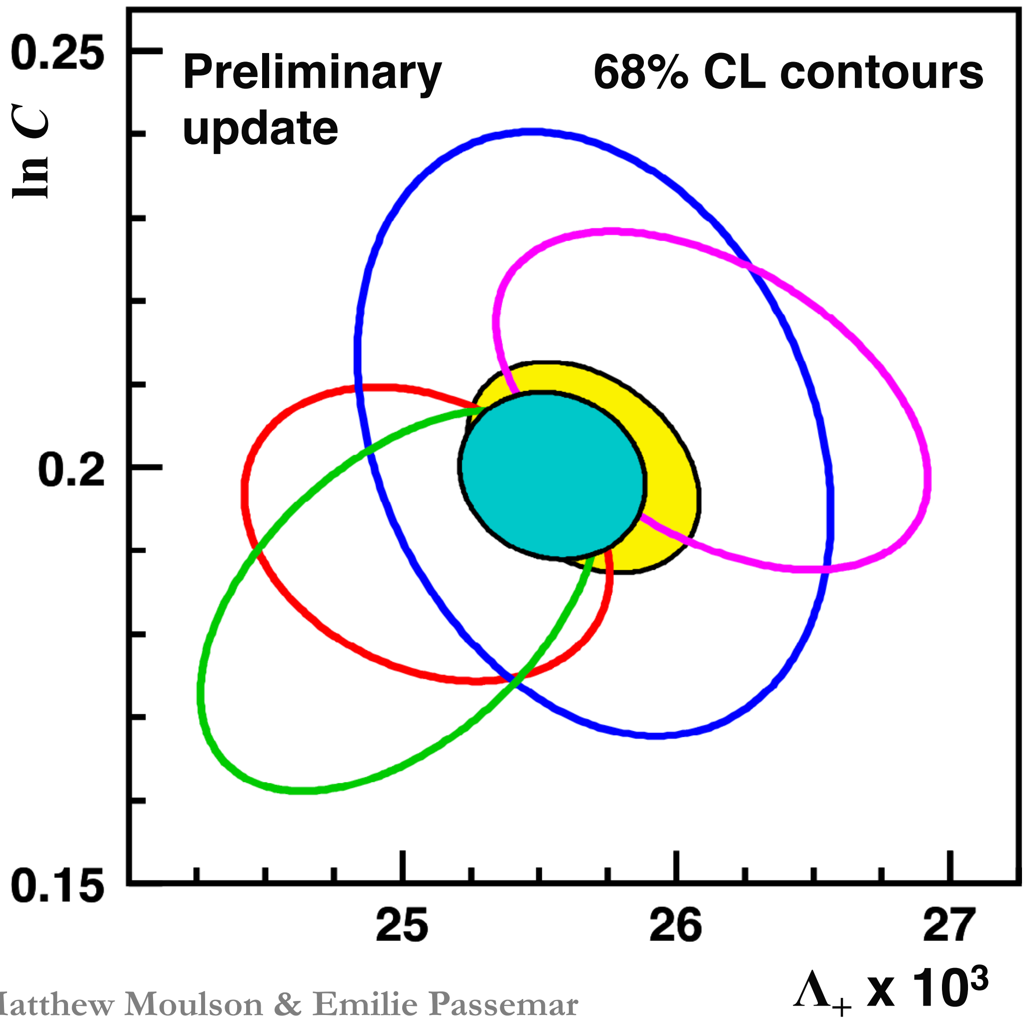
2010: χ^2 = 12.1/8 (P = 14.5%) | Update: χ^2 = 13.4/11 (P = 26.8%)

2.4 Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

2010 fit **Update**



$\Lambda_+ \times 10^3$	=	**25.55 ± 0.38**
$\ln C$	=	**0.1992(78)**
$\rho(\Lambda_+, \ln C)$	=	−0.110
χ^2/ndf	=	7.5/7 (38%)

Integrals

Mode	Update	2010
K^0_{e3}	**0.15470(15)**	0.15476(18)
K^+_{e3}	**0.15915(15)**	0.15922(18)
$K^0_{\mu 3}$	**0.10247(15)**	0.10253(16)
$K^+_{\mu 3}$	**0.10553(16)**	0.10559(17)

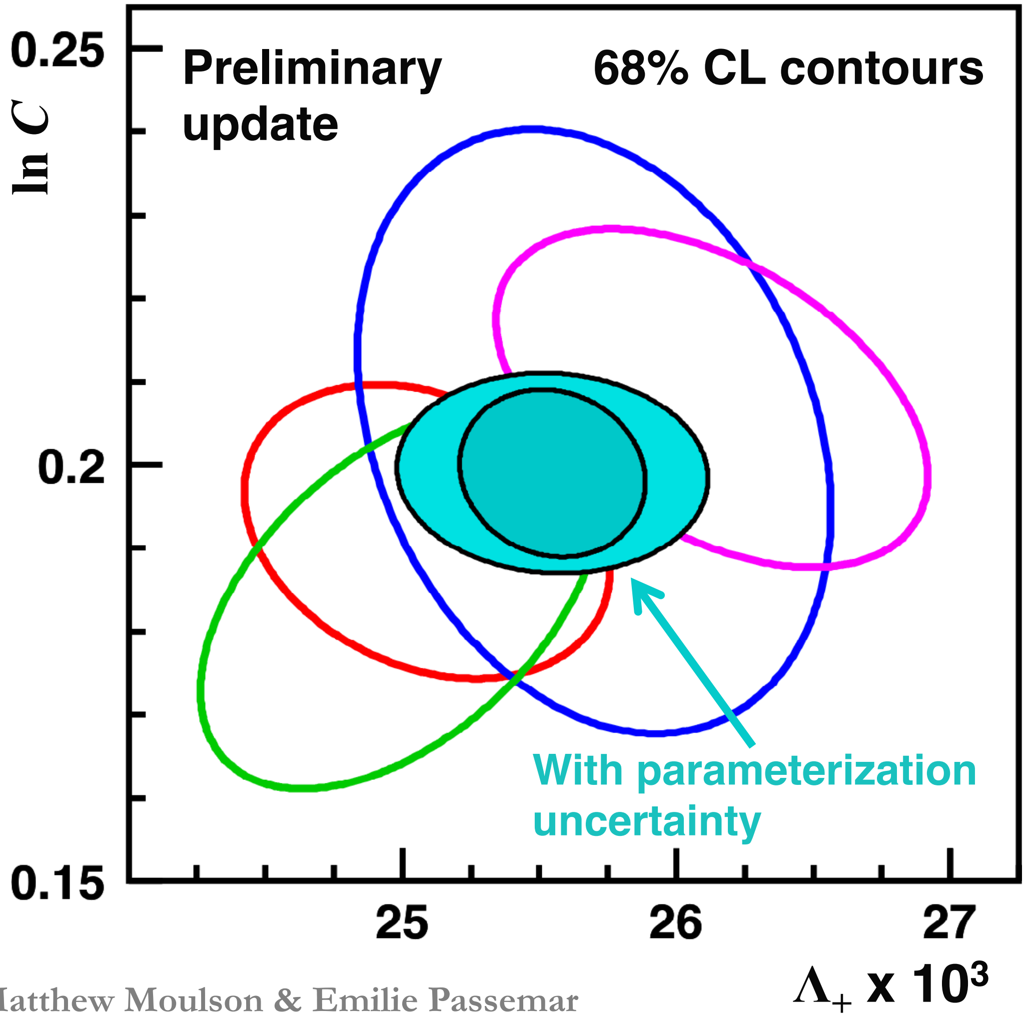
Only tiny changes in central values

2.4 Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from KTeV KLOE ISTRA+ NA48/2

NA48 K_{e3} data included in fits but not shown

2010 fit | Update



$\Lambda_+ \times 10^3$	=	**25.55 ± 0.38**
$\ln C$	=	**0.1992(78)**
$\rho(\Lambda_+, \ln C)$	=	−0.110
χ^2/ndf	=	7.5/7 (38%)

Fit results include common uncertainty from $H(t)$, $G(t)$:

$\sigma_{\text{param}}(\Lambda_+) = 0.3 \times 10^{-3}$

$\sigma_{\text{param}}(\ln C) = 0.0040$

KTeV, Bernard et al.'09

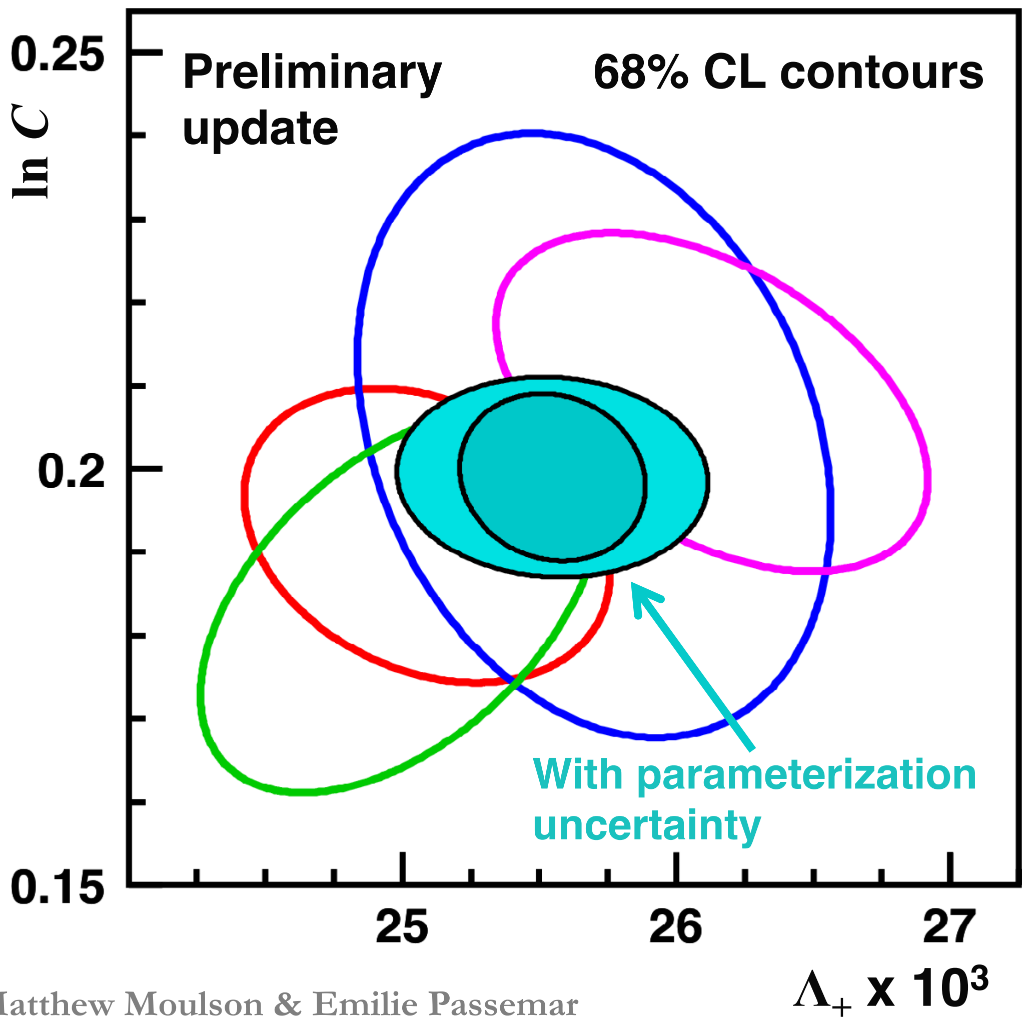
Confidence ellipses shown **without** common uncertainty (except as indicated)

2.4 Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from KTeV KLOE ISTRA+ NA48/2

NA48 K_{e3} data included in fits but not shown

2010 fit | Update



$\Lambda_+ \times 10^3$	=	**25.55 ± 0.38**
$\ln C$	=	**0.1992(78)**
$\rho(\Lambda_+, \ln C)$	=	−0.110
χ^2/ndf	=	7.5/7 (38%)

Fit results include common uncertainty from $H(t)$, $G(t)$.

Without common uncertainty:

$\sigma(\Lambda_+)$	**(0.38 → 0.22) x 10^{-3}**
$\sigma(\ln C)$	**0.0078 → 0.0067**
$\sigma(K_{e3}$ int)	**0.10% → 0.09%**
$\sigma(K_{\mu 3}$ int)	**0.15% → 0.11%**

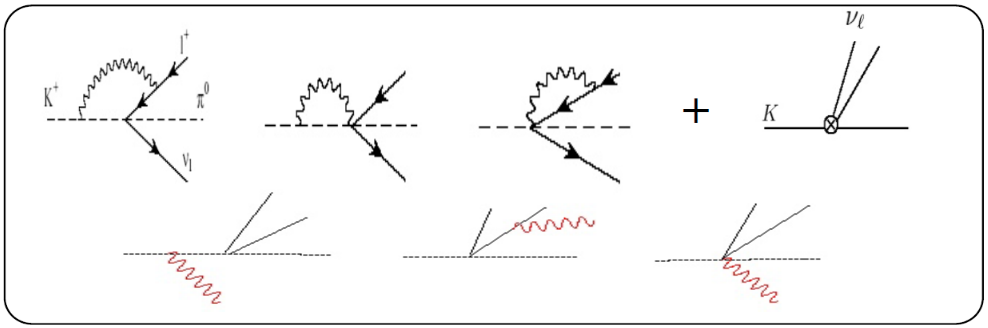
2.5 Long distance electromagnetic corrections

- Master formula for K → πlν$_l$: K = {K$^+$,K^0}, l={e,μ}

$$\Gamma\left(K \to \pi l \nu[\gamma]\right) = Br(K_{l3}) * \tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K |V_{us}|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl}\left(1 + \boxed{2\Delta_{\mathrm{EM}}^{Kl}} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

- Long distance EM corrections: $\delta_{\mathrm{EM}}^{Kl}$

Cirigliano, Giannotti, Neufeld'08



↔

Mode	$\delta_{\mathrm{EM}}^{K\ell}$ (%)
K_{e3}^0	0.495 ± 0.110
$K_{e3}^{\pm}$	0.050 ± 0.125
$K_{\mu3}^0$	0.700 ± 0.110
$K_{\mu3}^{\pm}$	0.008 ± 0.125

- ChPT to O(p^{2}e^2)
- Fully inclusive prescription for real photons
- Uncertainties: LECs (100%)

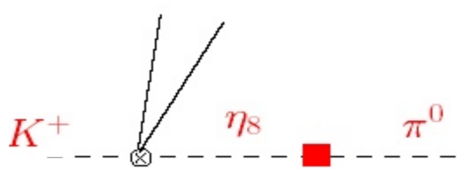
2.6 Isospin breaking corrections

- Master formula for K $\rightarrow$ $\pi l \nu_l$: K = {K^+,K^0}, l={e,μ}

$$\Gamma\left(K\to\pi l\nu\left[\gamma\right]\right)=Br(K_{l3})*\tau=C_K^2\frac{G_F^2 m_K^5}{192\pi^3}S_{EW}^K\left|V_{us}\right|^2\left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl}\left(1+2\Delta_{\rm EM}^{Kl}+\boxed{2\Delta_{\rm SU(2)}^{K\pi}}\right)$$

- Isospin breaking corrections: $\Delta_{\rm SU(2)}^{K\pi}$

$$\Delta_{\rm SU(2)}^{K\pi}=\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)}-1$$



+ IB in one loop graphs + CT

Gasser&Leutwyler'85

- ➢ In ChPT at O(p^4) :

$$\Delta_{\rm SU(2)}^{K\pi}=\frac{3}{4}\frac{1}{Q^2}\left[\frac{\bar{m}_K^2}{\bar{m}_\pi^2}+\frac{\chi_{p^4}}{2}\left(1+\frac{m_s}{\hat{m}}\right)\right]$$

$$Q^2\equiv\frac{m_s^2-\hat{m}^2}{m_d^2-m_u^2}$$

$$\left[\hat{m}\equiv\frac{m_u+m_d}{2}\right]$$

2.5 Isospin breaking corrections

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$= \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1+\frac{m_s}{\hat{m}}\right)\right] \qquad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

$\chi_p{}^4 = 0.252$

NLO in strong interaction

O(e^2p^2) term $\varepsilon_{\mathrm{EM}}{}^{(4)} \sim 10^{-6}$

$=$ **+2.61(17)%** **Calculated using**

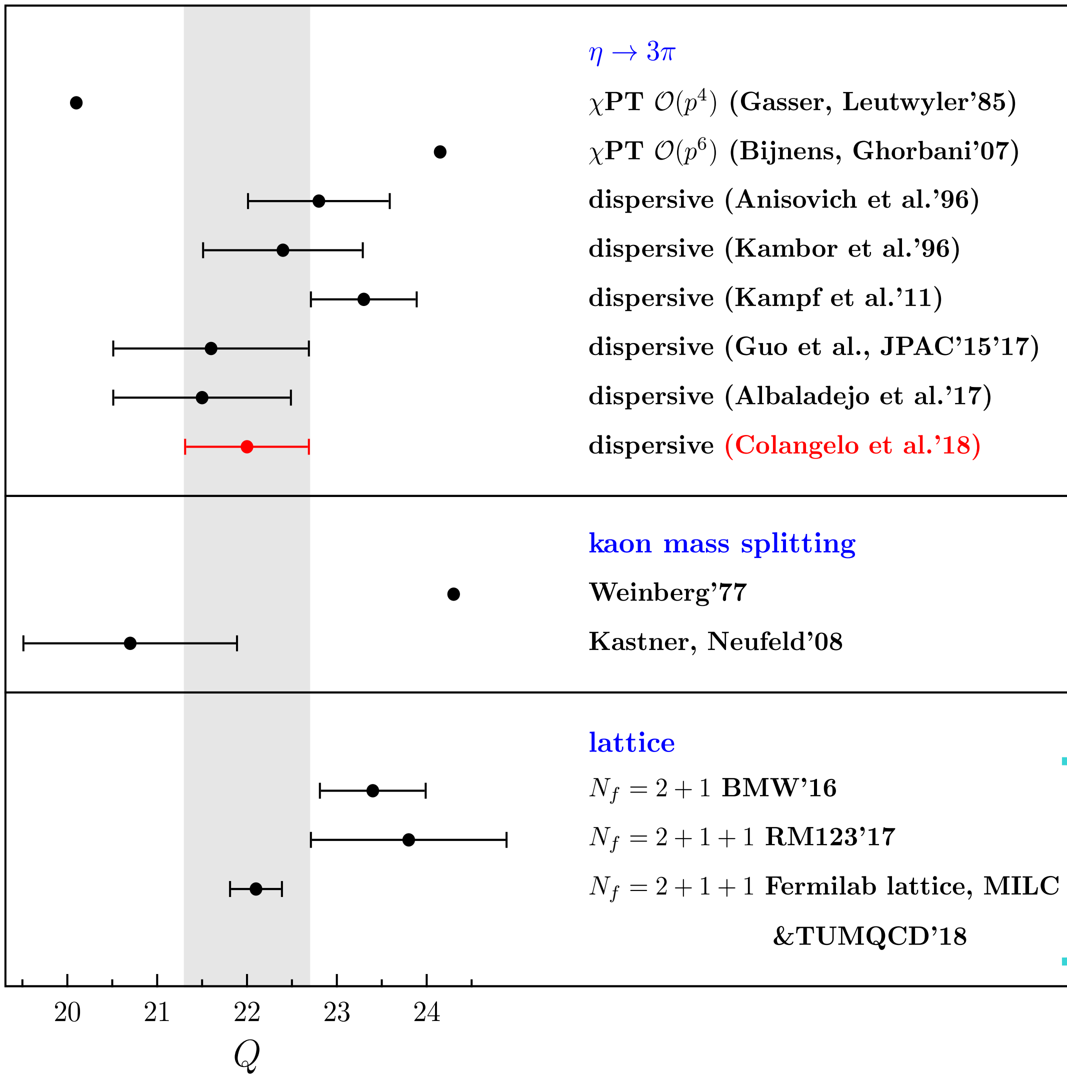
$Q = 22.1(7)$	$M_K = 494.2(3)$	Isopsin-limit
$m_s/\hat{m} = 27.43(13)(27)$	$M_\pi = 134.8(3)$	meson masses

- Calculation scheme of Kastner & Neufeld ’08, Cirigliano et al. ’02
- LECs from Bijnens & Ecker ’14

Test by evaluating V_{us} from $K^\pm$ and K^0 data with **no** corrections:
Equality of V_{us} values would require $\mathbf{\Delta^{SU(2)} = 2.82(38)\%}$

2.5 Isospin breaking corrections

Previous to new results on Q, uncertainty on $\Delta^{SU(2)}$ leading contributor to uncertainty on V_{us} from $K^{\pm}$ decays **—can it be reduced?**



Continuing progress + systematic review of existing results for light-quark masses may help

Recent dispersion relation analyses of $\eta \to 3\pi$ Dalitz plot

e.g. Colangelo, Lanz, Leutwyler, E.P'18

1.6 fb^{-1} KLOE '04 -'05 data

Continuing progress on lattice

E.g. **BMW '16** PRL 117

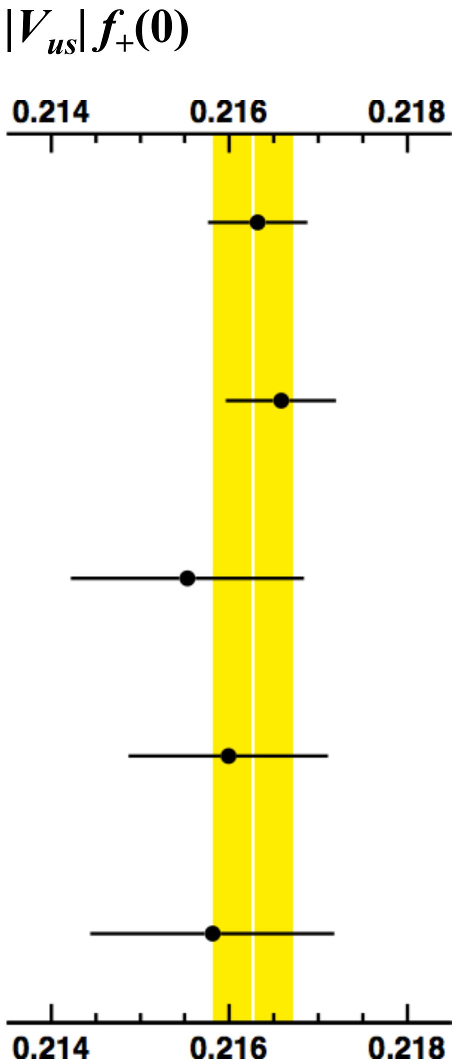
N_f = 2+1 QCD, 5sp, m_π phys

Partially quenched QED

$Q = 23.4(4)_{st}(3)_{sy}(4)_{QED}$

but some tension

2.6 $|V_{us}| f_+(0)$ from world data: 2010

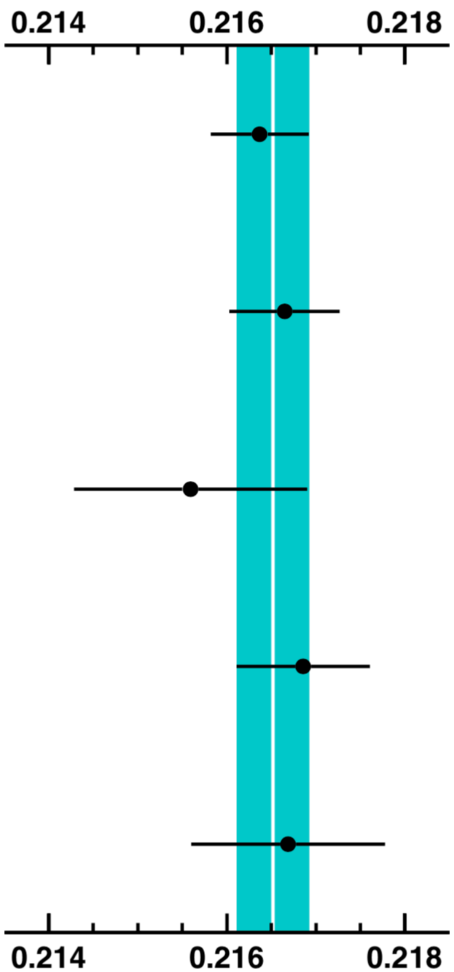


			Approx. contrib. to % err from:			
		% err	**BR**	τ	Δ	**Int**
$K_L e3$	**0.2163(6)**	**0.26**	0.09	0.20	0.11	0.06
$K_L \mu 3$	**0.2166(6)**	**0.29**	0.15	0.18	0.11	0.08
$K_S e3$	**0.2155(13)**	**0.61**	0.60	0.03	0.11	0.06
$K^{\pm} e3$	**0.2160(11)**	**0.52**	0.31	0.09	0.40	0.06
$K^{\pm} \mu 3$	**0.2158(14)**	**0.63**	0.47	0.08	0.39	0.08

Average: $|V_{us}| f_+(0)$ = 0.2163(5) **χ^2/ndf = 0.77/4 (94%)**

2.6 $|V_{us}| f_+(0)$ from world data: Update

$|V_{us}| f_+(0)$



Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
$K_L e3$	**0.2164(6)**	**0.26**	0.09	0.20	0.11	0.05
$K_L \mu 3$	**0.2167(6)**	**0.29**	0.15	0.18	0.11	0.07
$K_S e3$	**0.2156(13)**	**0.61**	0.60	0.02	0.11	0.05
$K^\pm e3$	**0.2169(8)**	**0.35**	0.27	0.06	0.21	0.05
$K^\pm \mu 3$	**0.2167(11)**	**0.50**	0.45	0.06	0.21	0.07

Average: $|V_{us}| f_+(0)$ = 0.21652(41) **χ^2/ndf = 0.98/4 (91%)**

2.7 Determination of $f_+(0)$

- SU(3) breaking in $f_+(0)$
  - CVC + Ademollo-Gatto theorem: $f_+^{K^0\pi^-}(0) - 1 = O\left((m_s - m_u)^2\right)$

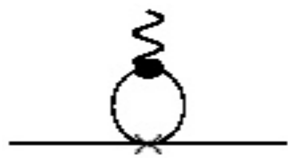
$$\boxed{f_+^{K^0\pi^-}(0) = 1 + \underbrace{f_{p^4}}_{O(m_q)} + \underbrace{f_{p^6}}_{O(m_q^2)} + \ldots}$$

chiral expansion

- f_{p4}:

Gasser & Leutwyler'85

→ One loop graph :



→ First order in m_q, 2nd order in $(m_s - m_u)$ ⇒ $f_{p^4} \sim \dfrac{(m_s - m_u)^2}{m_s}$

→ No local operators, UV finite, free of uncertainties

⇒ $\boxed{f_{p^4} = -0.0227}$

2.7 Determination of $f_+(0)$

- SU(3) breaking in $f_+(0)$
  - CVC + Ademollo-Gatto theorem: $f_+^{K^0\pi^-}(0) - 1 = O\left((m_s - m_u)^2\right)$

$$\boxed{f_+^{K^0\pi^-}(0) = 1 + \underbrace{f_{p^4}}_{O(m_q)} + \underbrace{f_{p^6}}_{O(m_q^2)} + \dots}$$

chiral expansion

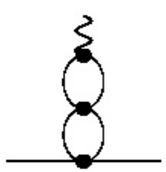
Bijnens & Talavera'02

  - f_{p6}:

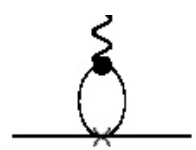
$$\boxed{f_{p^6}} = f_{p^6}^{2-\text{loops}}(\mu) + f_{p^6}^{\text{L}_\text{i}\times\text{loop}}(\mu) + f_{p^6}^{\text{tree}}(\mu)$$

$$\boxed{f_{p^6}^{2-\text{loops}}(M_\rho) \;=\; 0.0113} \qquad \boxed{f_{p^6}^{L_i\times\text{loop}}(M_\rho) \;=\; -0.0020}$$

$$\boxed{8\frac{(M_K^2 - M_\pi^2)^2}{F_\pi^2}\left[\frac{(L_5^r(M_\rho))^2}{F_\pi^2} - C_{12}^r(M_\rho) - C_{34}^r(M_\rho)\right]}$$

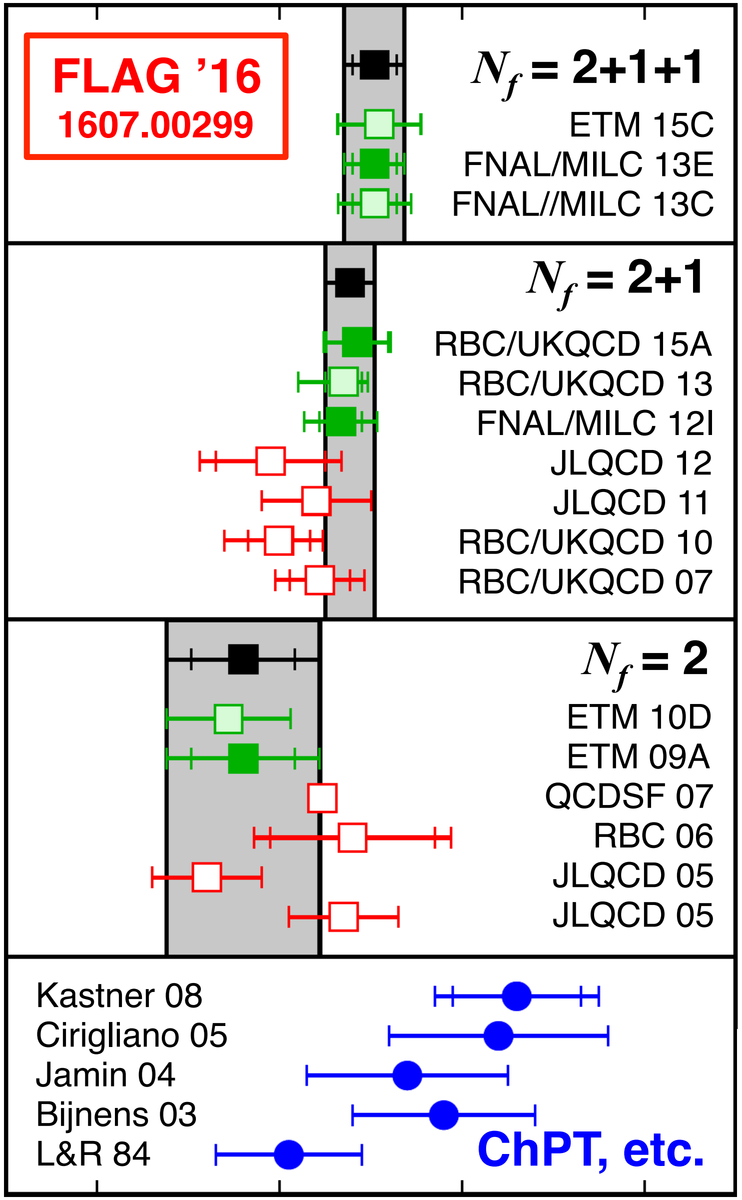


Large positive
chiral loop cont.



LECs not fixed by chiral symmetry:
quark model, large-Nc estimates, *LQCD*

2.7 Determination of $f_+(0)$



FLAG'16 averages:

N_f = 2+1 $\quad f_+(0) = 0.9677(27)$
Uncorrelated average of:
RBC/UKQCD 15A: DWF, $m_\pi \to 139$ MeV
FNAL/MILC 12I: HISQ, $m_\pi \sim 300$ MeV

N_f = 2+1+1 $\quad f_+(0) = 0.9704(32)$
FNAL/MILC 13E: HISQ, $m_\pi \to 135$ MeV

Recent updates:

N_f = 2+1 $\quad f_+(0) = 0.9636(^{+62}_{-65})$ PRD96 (2017)
JLQCD: Overlap, $m_\pi \to 300$ MeV
Exact chiral symmetry, one lattice spacing

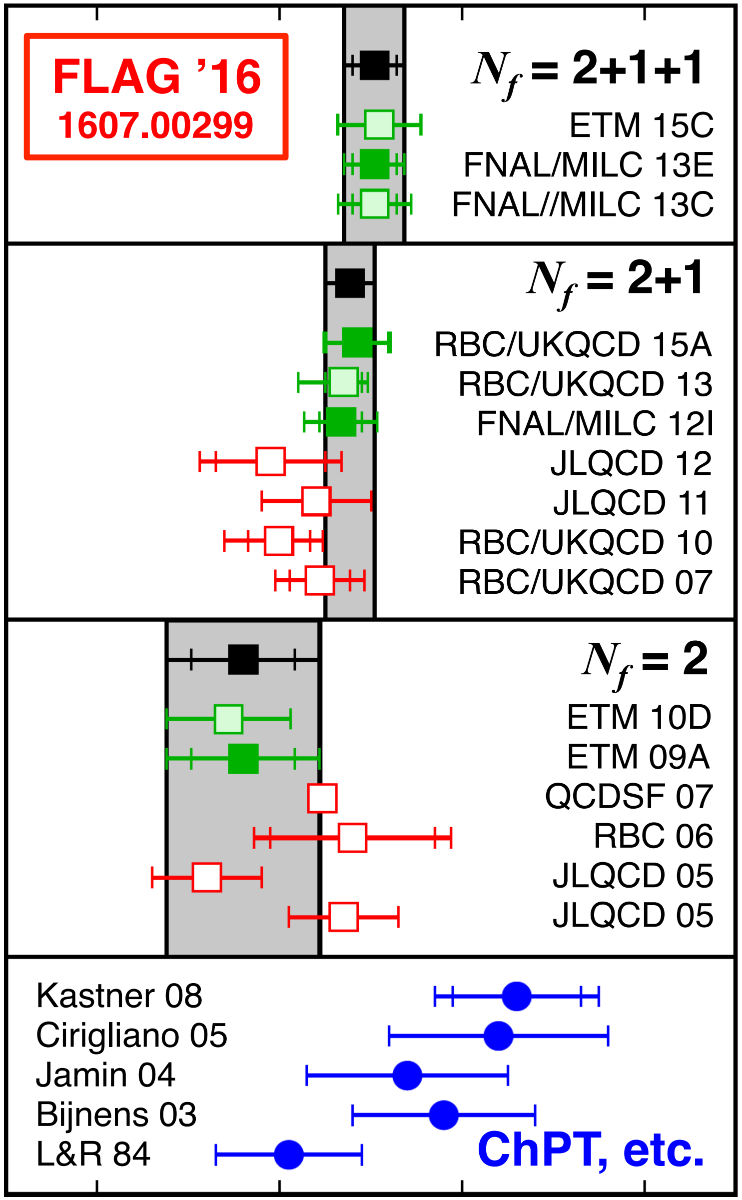
N_f = 2+1+1 $\quad f_+(0) = 0.9709(44)(9)(11)$ PRD 93 (2016)
ETM 16: TwMW, 3sp, $m_\pi \to 210$ MeV
Full q^2 dependence of f_+, f_0

$f_+(0) = 0.9696(15)(11)$ 1809.02827
FNAL/MILC update 13E

ChPT:

N_f = 2+1 $\quad f_+(0) = 0.970(8)$ Chiral Dynamics 15
Ecker 15: According to Bijnens 03
New LECs from Bijnens, Ecker 14

2.7 Determination of $f_+(0)$



FLAG'16 averages:

N_f = **2+1** $f_+(0)$ = **0.9677(27)**

Uncorrelated average of:

RBC/UKQCD 15A: DWF, $m_\pi \to 139$ MeV

FNAL/MILC 12I: HISQ, $m_\pi \sim 300$ MeV

N_f = **2+1+1** $f_+(0)$ = **0.9704(32)**

FNAL/MILC 13E: HISQ, $m_\pi \to 135$ MeV

Our averages:

N_f = **2+1** $f_+(0)$ = **0.9677(27)**

FLAG average, Nov 2016 update JLQCD17 not included because only 1 lattice spacing usedF

RBC/UKQCD15A 0.9685(34)(14)

FNAL/MILC12I 0.9667(23)(33)

N_f = **2+1+1** $f_+(0)$ = **0.9709(44)(9)(11)**

FNAL/MILC18 preliminary replaces FNAL/MILC13E in FLAG average

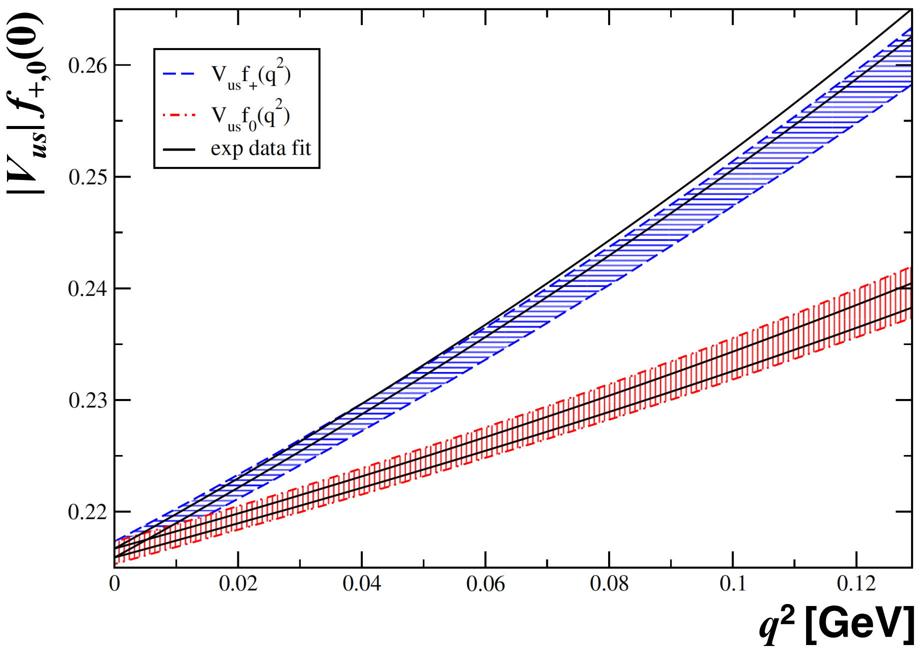
FNAL/MILC18 0.9696(15)(11)

ETM16 $0.9709(44)(9)(11)_{ext}$

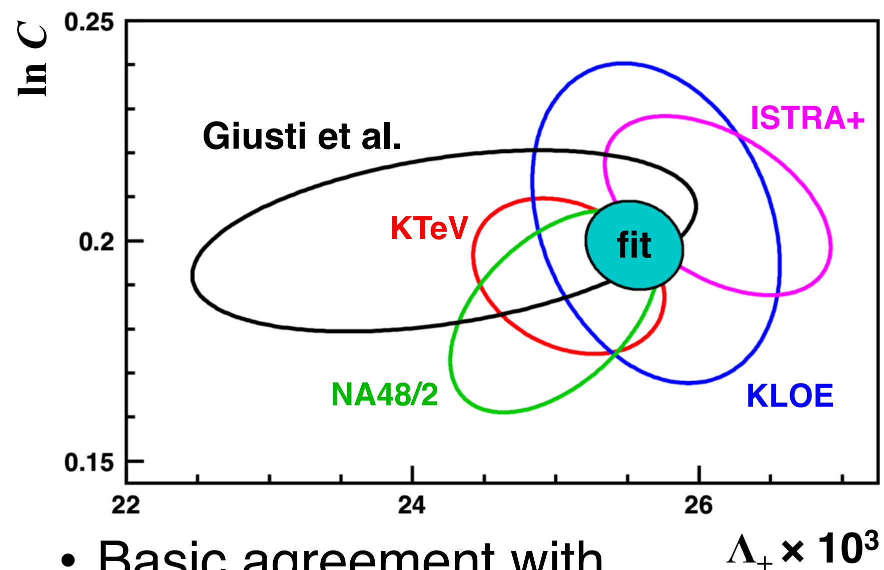
q^2 dependence of $K\pi$ form factors

ETM
PRD 93 (2016)

N_f **= 2+1+1** Twisted-mass Wilson fermions
3 lattice spacings, smallest $m_\pi \to$ 210 MeV
Results for full q^2 dependence of f_+, f_0



$f_+(0)$ = 0.9709(44)$_{st}$(9)$_{sy}$(11)$_{ext}$



Fit synthetic data points with dispersive parameterization

Λ_+ = 24.22(1.16) × 10^{-3}
ln C = 0.1998(138)

$\rho(\Lambda_+, f_+(0)) = -0.228$
$\rho(\ln C, f_+(0)) = -0.719$
$\rho(\Lambda_+, \ln C) = +0.376$

- Basic agreement with experimental results
- Confirms basic correctness of lattice calculations for $f_+(0)$
- In the near future FF parameters will be obtained on lattice?

2.8 $|V_{us}|(K_{\ell 3})$ and $|V_{ud}|(0^+ \to 0^+)$: Update

Hardy & Towner, CIPANP ’18
$|V_{ud}|$ = 0.97420(21)

World data set very robust

- 14 transitions with compatible measurements at 0.1% precision or better

From FlaviaNet 2010 $K_{\ell 3}$ analysis

$|V_{us}|\, f_+(0)$ = 0.2163(5)	**$|V_{us}|$ = 0.2254(13)**
with $f_+(0)$ = 0.959(5)	**with $|V_{ud}|$ = 0.97425(22)**

Δ_{CKM} = +0.0000(8)

Update with $|V_{us}|\, f_+(0)$ = 0.21652(41) and $|V_{ud}|$ = 0.97420(21)

N_f = 2+1
$f_+(0)$ = 0.9677(27)

V_{us} = 0.22375(43)$_{\text{exp}}$(62)$_{\text{lat}}$

Δ_{CKM} = −0.00085(19)$_{\text{exp}}$(28)$_{\text{lat}}$(41)$_{ud}$ = −1.6σ

N_f = 2+1+1
$f_+(0)$ = 0.9698(17)

V_{us} = 0.22326(43)$_{\text{exp}}$(39)$_{\text{lat}}$

Δ_{CKM} = −0.00107(19)$_{\text{exp}}$(17)$_{\text{lat}}$(41)$_{ud}$ = −2.2σ

1.5-2σ inconsistency with unitarity first seen with 2014-era lattice results

Relative to 2014 slightly better agreement between N_f = 2+1 and N_f = 2+1+1

V_{ud} from $0^+ \rightarrow 0^+$

$$\frac{1}{t} = \frac{G_\mu^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} f(Q)\,(1+RC) \longrightarrow f\,t\,(1+RC) = \frac{2984.48(5)\ s}{|V_{ud}|^2}$$

$$(1+RC) = (1-\delta_C)\,(1+\delta_R)\,(1+\Delta_C)$$

$$\langle f|\tau_+|i\rangle = \sqrt{2}\,(1-\delta_C/2)$$

Coulomb distortion
of wave-functions

$$\delta_C \sim 0.5\%$$

Towner-Hardy
Ormand-Brown

Nucleus-dependent
rad. corr.
(Z, E^{max} ,nuclear structure)

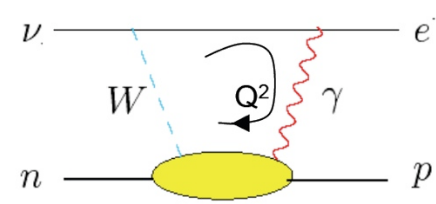
$$\delta_R \sim 1.5\%$$

Sirlin-Zucchini '86
Jaus-Rasche '87

Nucleus-independent
short distance rad. corr.

$$\Delta_R \sim 2.4\%$$

Marciano-Sirlin '06

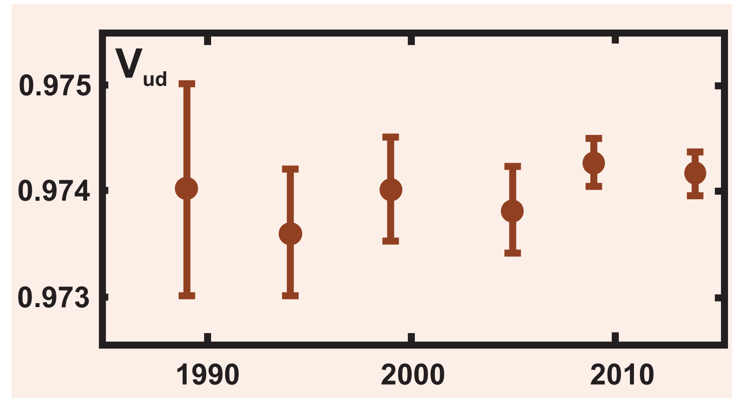


V_{ud} from $0^+ \rightarrow 0^+$

In 2010 :

- Survey of 150 measurements of 13 different $0^+ \rightarrow 0^+$ β decays
- 27 new *ft* measurements including Penning-trap measurements for QEC
- Some old measurements dropped
- Improved EW radiative corrections *Marciano & Sirlin'06*
- New *SU*(2)-breaking corrections *Towner & Hardy'08*

Evolution of V_{ud} over years



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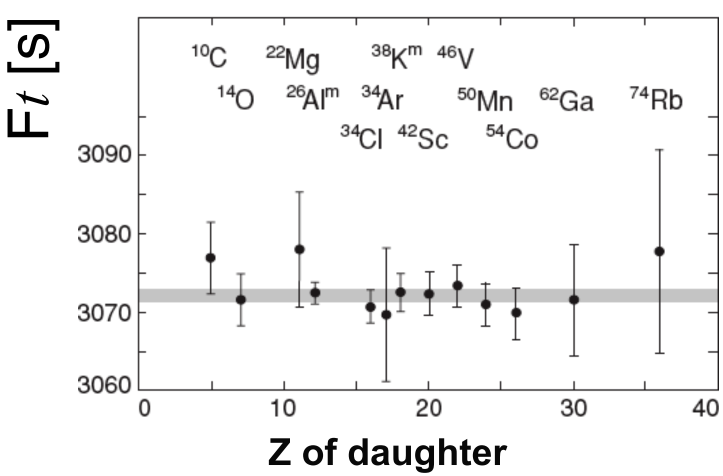
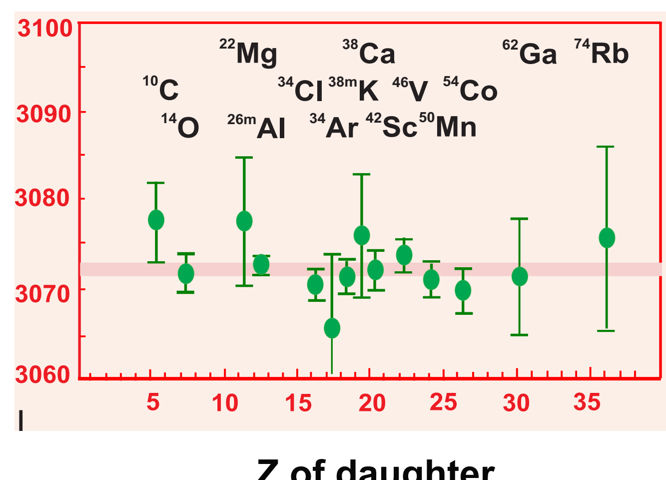
Since then 24 new measurements, critical review of IB correction and test of CVC





Current Status of V_{ud} determination

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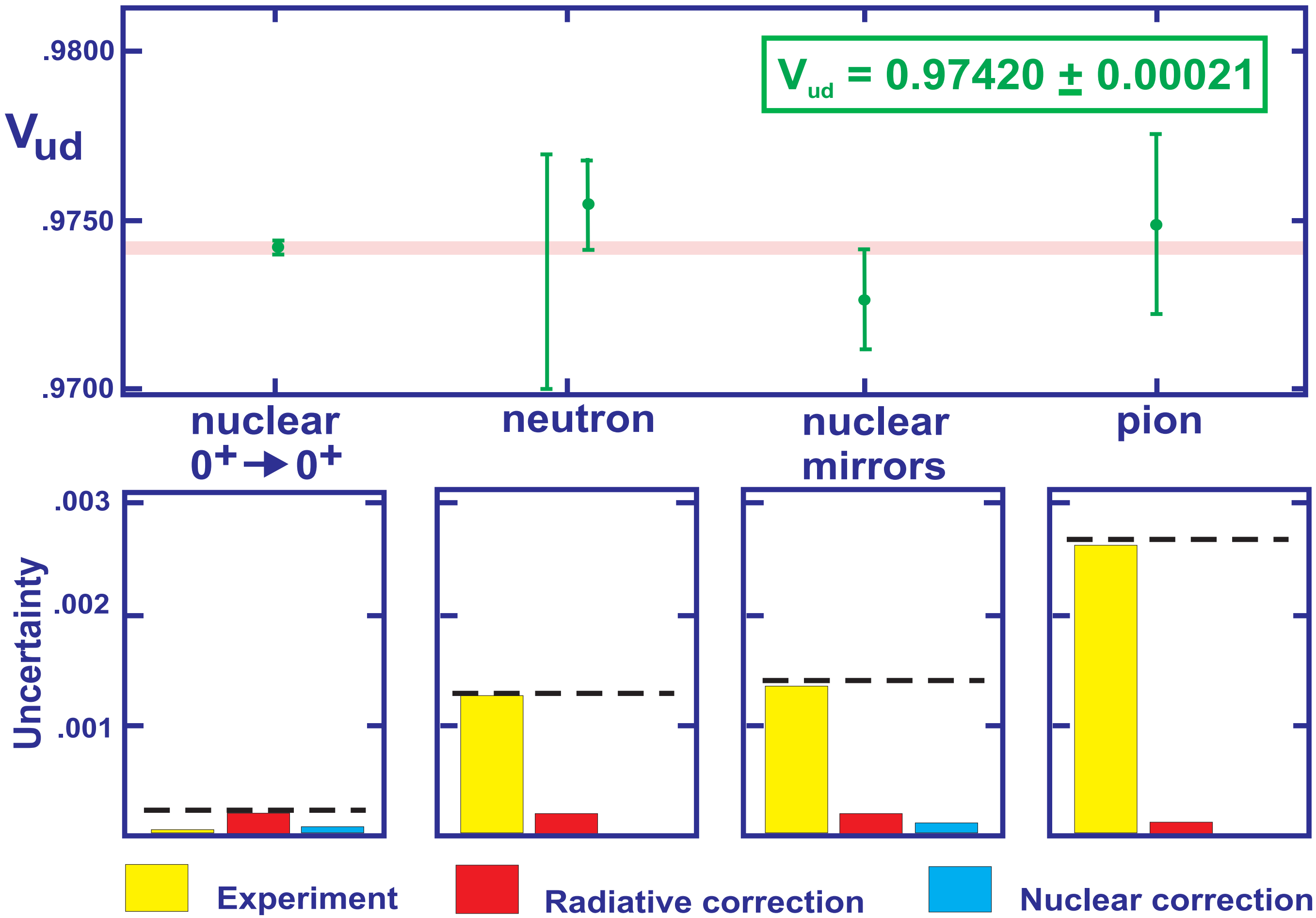


2.8 $|V_{us}|(K_{\ell 3})$ and $|V_{ud}|(0^+ \to 0^+)$: Update

Seng, Gorchtein, Patel & Ramsey-Musolf
arXiv:1807.10197

$|V_{ud}|$ = 0.97366(15)

–1.5σ shift in V_{ud}

New calculation γW-box contribution to universal radiative correction using dispersion relations and DIS structure functions

- Contribution possibly already in part included in structure-dependent radiative corrections
- **Needs verification!**

Update with $|V_{us}|\, f_+(0)$ = 0.21652(41) and $|V_{ud}|$ = 0.97366(15)

Choice of $f_+(0)$		V_{us}	$\Delta_{\mathrm{CKM}} = V_{ud}^2 + V_{us}^2 - 1$	
N_f = 2+1	0.9677(27)	**0.2238(8)**	**–0.0019(5)**	= –4.2σ
N_f = 2+1+1	0.9698(17)	**0.2233(6)**	**–0.0021(4)**	= –5.4σ

If correct, 4-5σ unitarity violation in first row!

Calculation is attracting interest and requires better understanding

3. V_{us}/V_{ud} from $K_{\ell 2}/\pi_{\ell 2}$ decays

3.1 Master formula for V_{us}/V_{ud} from $K_{\ell 2}/\pi_{\ell 2}$ decays

- From K_{l2}/π_{l2}:

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\mathrm{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from theory:

Cirigliano, Neufeld ’11

δ_{EM} **= –0.0069(17)**

Long-distance EM corrections

$\delta_{SU(2)}$ **= –0.0043(5)(11)**

Strong isospin breaking

$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

Lattice: f_K/f_π

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

Inputs from experiment:

Updated $K^\pm$ BR fit:

BR($K^\pm_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ **= 12.384(15) ns**

PDG:

BR($\pi^\pm_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ **= 26.033(5) ns**

$\boxed{|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm} = 0.27599(37)}$

No $SU(2)$-breaking correction

3.2 Electromagnetic corrections

Giusti et al.
PRL 120 (2018)

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- N_f = 2+1+1 Twisted-mass Wilson fermions

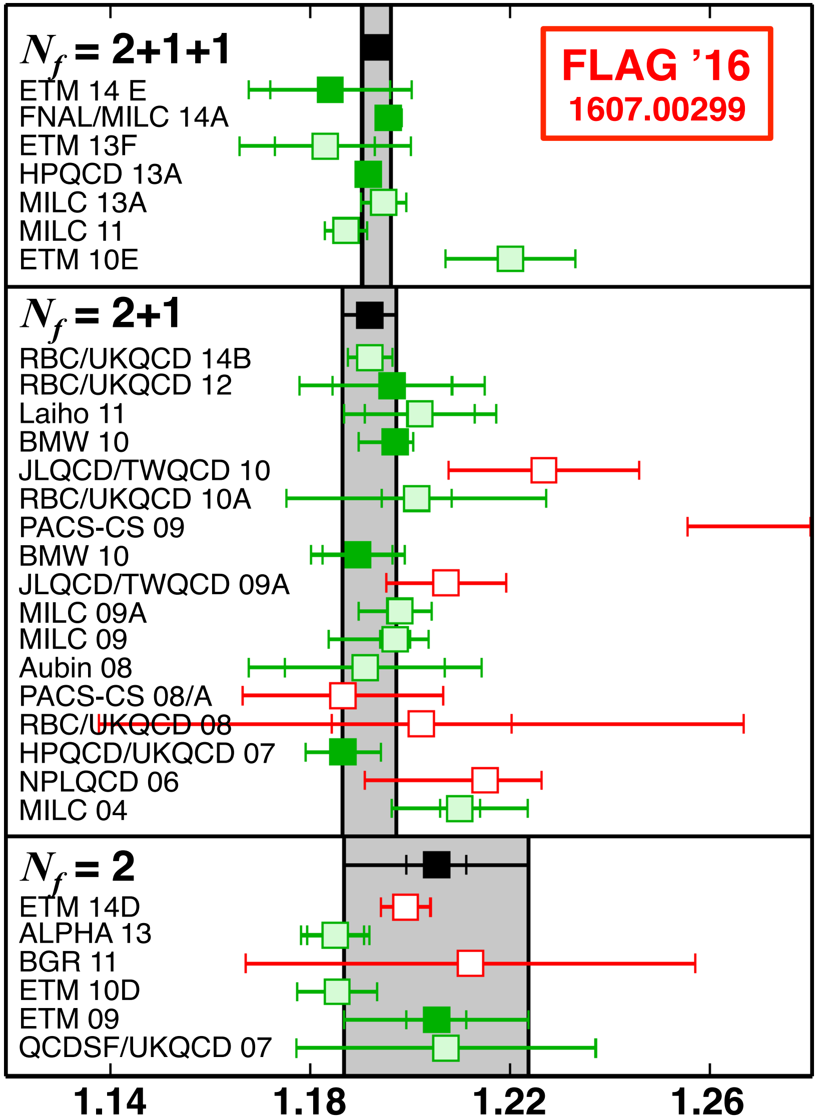
$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0122(16)$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld ’11:

$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)$

3.3 Lattice results for f_K/f_π



FLAG '16 averages:

N_f = 2+1 $f_{K\pm}/f_{\pi\pm}$ **= 1.192(5)**

Unchanged from FLAG '13 average

N_f = 2+1+1 $f_{K\pm}/f_{\pi\pm}$ **= 1.1933(29)**

ETM 14E: TwM, 3sp, m_π = 210-450 MeV

FNAL/MILC 14A: HISQ, 4sp, m_π phys

Updates MILC 13A

HPQCD 13A: HISQ, 3sp, m_π phys,

Same ensembles as FNAL/MILC 14A

Recent updates:

N_f = 2+1 f_K/f_π = **1.1945(45)**

RBC/UKQCD '14: DWF, m_π = 139 MeV

f_K and f_π separately (isospin limit)

Recently published

$f_{K\pm}/f_{\pi\pm}$ = **1.1978(28)**

BMW '16: Clover, 5sp, $m_\pi \to$ 139 MeV

3.3 Lattice results for f_K/f_π

Our updates of FLAG averages for results without $SU(2)$-breaking

N_f = 2+1+1 **f_K/f_π = 1.1960(40)**

HPQCD13A	1.1948(15)(18)	Correlated uncertainties
FNAL/MILC14A*	$1.1983(^{+28}_{-21})$	
ETM14E	1.188(15)	← Uncorrelated uncertainty

N_f = 2+1 **f_K/f_π = 1.1927(38)**

HPQCD/UKQCD07	1.198(7)
RBC/UKQCD14B	1.1945(45)
BMW16	1.182(10)(26)

* Corrected using Cirigliano, Neufeld '11 with updated values:

$$\frac{f_{K^\pm}}{f_{\pi^\pm}} = \frac{f_K}{f_\pi}\sqrt{1+\delta_{SU(2)}} \qquad \delta_{SU(2)} \approx \frac{3}{4R}\left[-\frac{4}{3}(f_K/f_\pi - 1) + \frac{2}{3(4\pi)^2 f_0^2}\left(M_K^2 - M_\pi^2 - M_\pi^2 \ln\frac{M_K^2}{M_\pi^2}\right)\right]$$

R = 34.4(2.1) Colangelo et al. '18 M_K = 494.2(3) FLAG '17

3.3 Lattice results for f_K/f_π

$|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm}$ = 0.27599(37) and $|V_{ud}|$ = 0.97420(21)

$\delta_{SU(2)} + \delta_{\mathrm{EM}}$ = –0.0122(16) from Giusti et al. ’18

N_f = 2+1

$f_{K\pm}/f_{\pi\pm}$ = 1.1927(38)

$V_{us} = 0.22604(29)_{\mathrm{exp}}(72)_{\mathrm{lat}}(05)_{ud}$

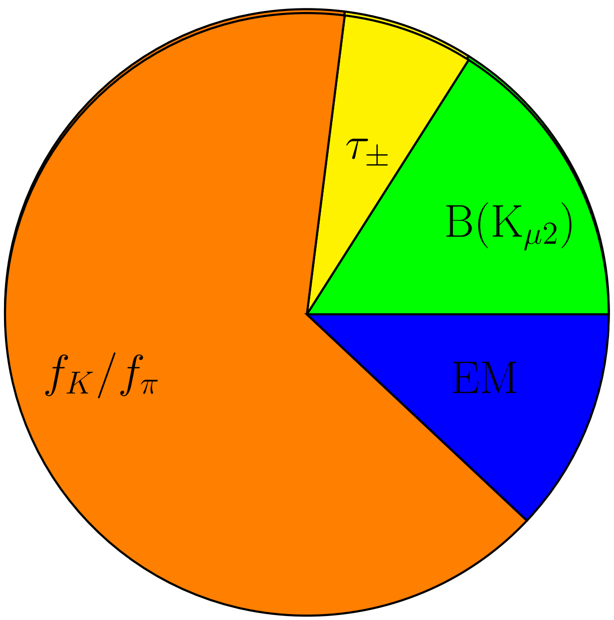
$\Delta_{\mathrm{CKM}} = +0.00018(13)_{\mathrm{exp}}(33)_{\mathrm{lat}}(43)_{ud}$ $= +0.3\sigma$

N_f = 2+1+1

$f_{K\pm}/f_{\pi\pm}$ = 1.1960(40)

$V_{us} = 0.22542(29)_{\mathrm{exp}}(75)_{\mathrm{lat}}(05)_{ud}$

$\Delta_{\mathrm{CKM}} = -0.00011(13)_{\mathrm{exp}}(34)_{\mathrm{lat}}(43)_{ud}$ $= -0.2\sigma$



$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2}\left(1 - \frac{1}{2}\delta_{\mathrm{EM}} - \frac{1}{2}\delta_{SU(2)}\right)$$

3.3 Lattice results for f_K/f_π

$|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm} = 0.27599(37)$ and $|V_{ud}| = 0.97420(21)$

$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0122(16)$ from Giusti et al. '18

$N_f = 2+1$

$f_{K\pm}/f_{\pi\pm} = 1.1927(38)$

$V_{us} = 0.22604(29)_{\mathrm{exp}}(72)_{\mathrm{lat}}(05)_{ud}$

$\Delta_{\mathrm{CKM}} = +0.00018(13)_{\mathrm{exp}}(33)_{\mathrm{lat}}(43)_{ud}$ $= +0.3\sigma$

$N_f = 2+1+1$

$f_{K\pm}/f_{\pi\pm} = 1.1960(40)$

$V_{us} = 0.22542(29)_{\mathrm{exp}}(75)_{\mathrm{lat}}(05)_{ud}$

$\Delta_{\mathrm{CKM}} = -0.00011(13)_{\mathrm{exp}}(34)_{\mathrm{lat}}(43)_{ud}$ $= -0.2\sigma$

$K_{\ell 2}$ results give better agreement with unitarity via V_{ud} than $K_{\ell 3}$ results (-2σ)

Exercise:

- Assume $|V_{ud}|$, $|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm}$, and $f_{K\pm}/f_{\pi\pm}$ all correct
- In $K_{\ell 3}$ does the discrepancy arise from data or from lattice results for $f_+(0)$?

4. V_{us} and Unitarity of the CKM matrix

4.1 Looking for New Physics with K_{l2} and K_{l3}

- Callan-Treiman theorem:

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K |V_{us}|}{F_\pi |V_{ud}|} \frac{1}{f_+(0)|V_{us}|} |V_{ud}|}_{\text{Very precisely known from Br(Kl2/}\pi\text{l2), }\Gamma\text{(Ke3) and }|V_{ud}|} r + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

$$B_{\exp} = 1.2446(41)$$

- In the Standard Model : $\boxed{r = 1}$ $\quad (\ln C_{SM} = 0.2141(73))$ $\quad \Delta_{CT} = (-3.5 \pm 8).10^{-3}$

- In presence of new physics, new couplings : $\boxed{r \neq 1}$

NLO value + large error bars in agreement with

Experiment K_{e3}+$K_{\mu3}$	ln C
NA48'07 ($K_{\mu3}$ alone)	0.144(14)
KLOE'08	0.204(25)
KTeV'10	0.192(12)
NA48/2, previous talk	0.184(15)

4.1 Form factors & the Callan-Treiman relation

Callan-Treiman relation:

$$\tilde{f}_0(t_{\rm CT}) = \frac{f_K}{f_\pi}\frac{1}{f_+(0)} + \Delta_{\rm CT}$$

$t_{\rm CT} = m_K^2 - m_\pi^2$

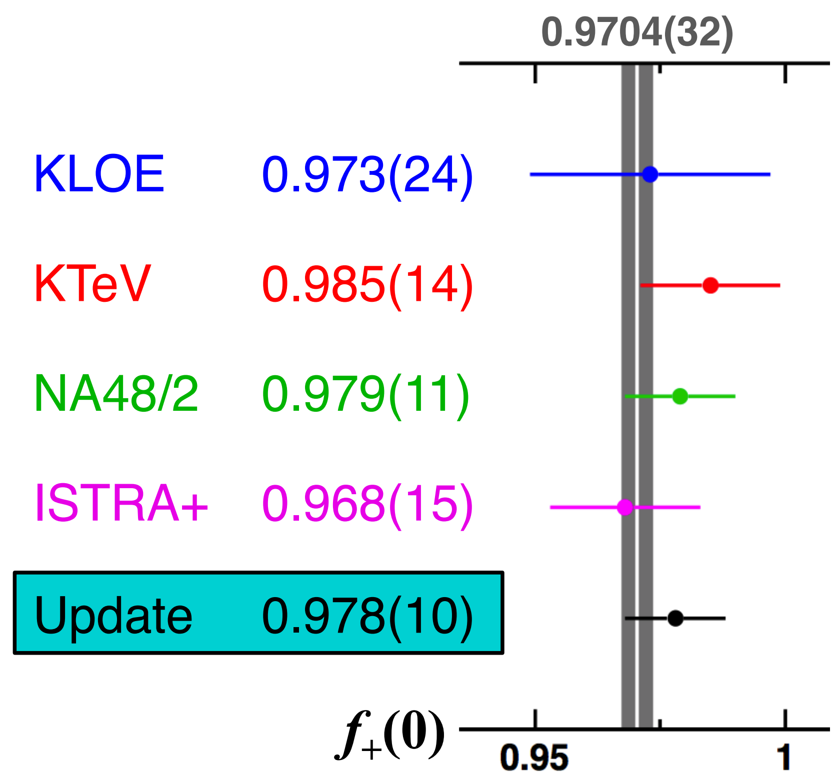
$\Delta_{\rm CT} = (-3.5 \pm 0.8) \times 10^{-3}$ ~$O(m_u, m_d)$

Gasser, Leutwyler '85

Dispersive representation: $f_0(t_{\rm CT}) \equiv C$

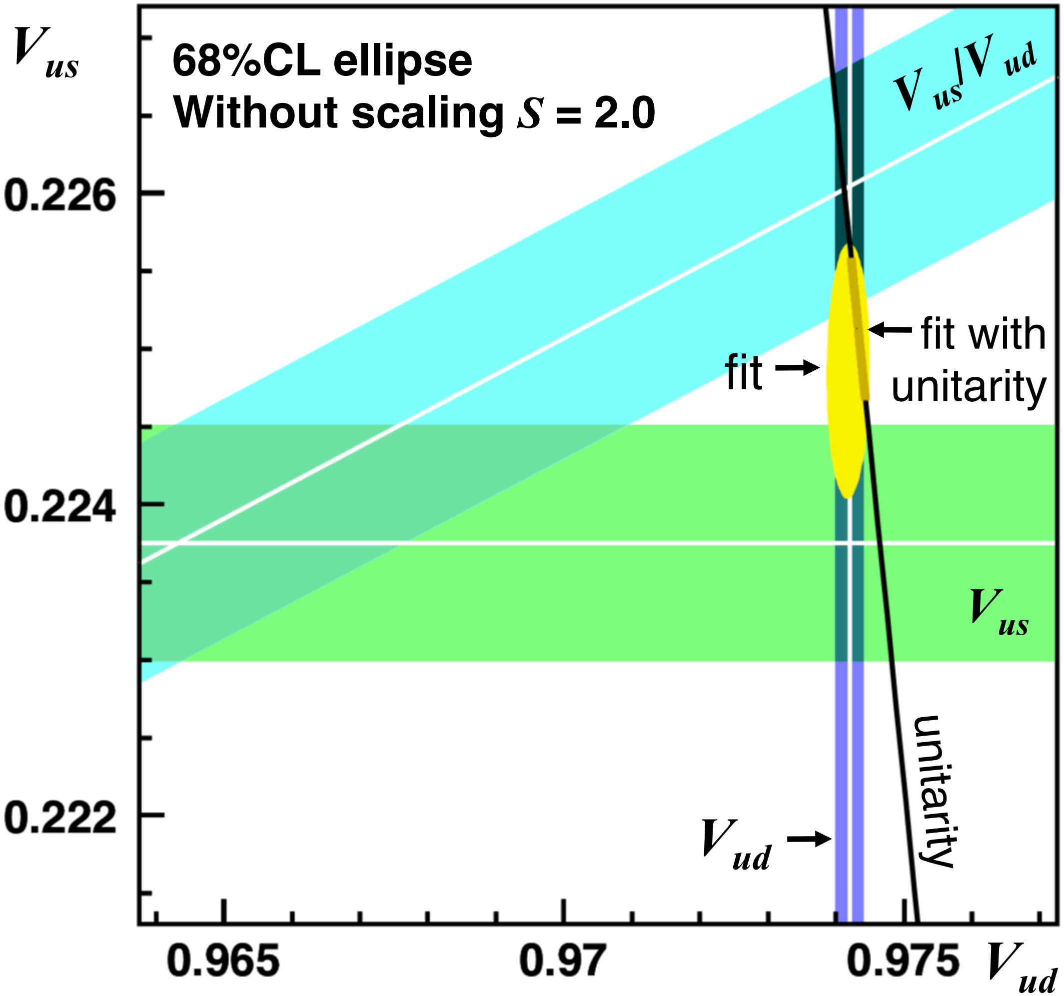
Use ChPT & form-factor data to test N_f = 2+1+1 lattice consistency:

- Use lattice reference value
  f_K/f_π = **1.1933(29)**
- Obtain $f_+(0)$ corresponding to each result for $\ln C$
- Compare to lattice reference value
  $f_+(0)$ = **0.9704(32)**
- Basic consistency **(0.7σ)** between lattice values for f_K/f_π and $f_+(0)$ and measurements of $\ln C$
- **Uses no experimental information on decay widths**



4.2 V_{us} and CKM unitarity: All data, N_f=2+1

N_f = 2+1: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
$f_+(0)$ = 0.9677(27), f_K/f_π = 1.1927(38)



$|V_{ud}|$ = 0.97420(21)
$|V_{us}|$ = 0.2238(8)
$|V_{us}|/|V_{ud}|$ = 0.2320(8)

Fit results, no constraint

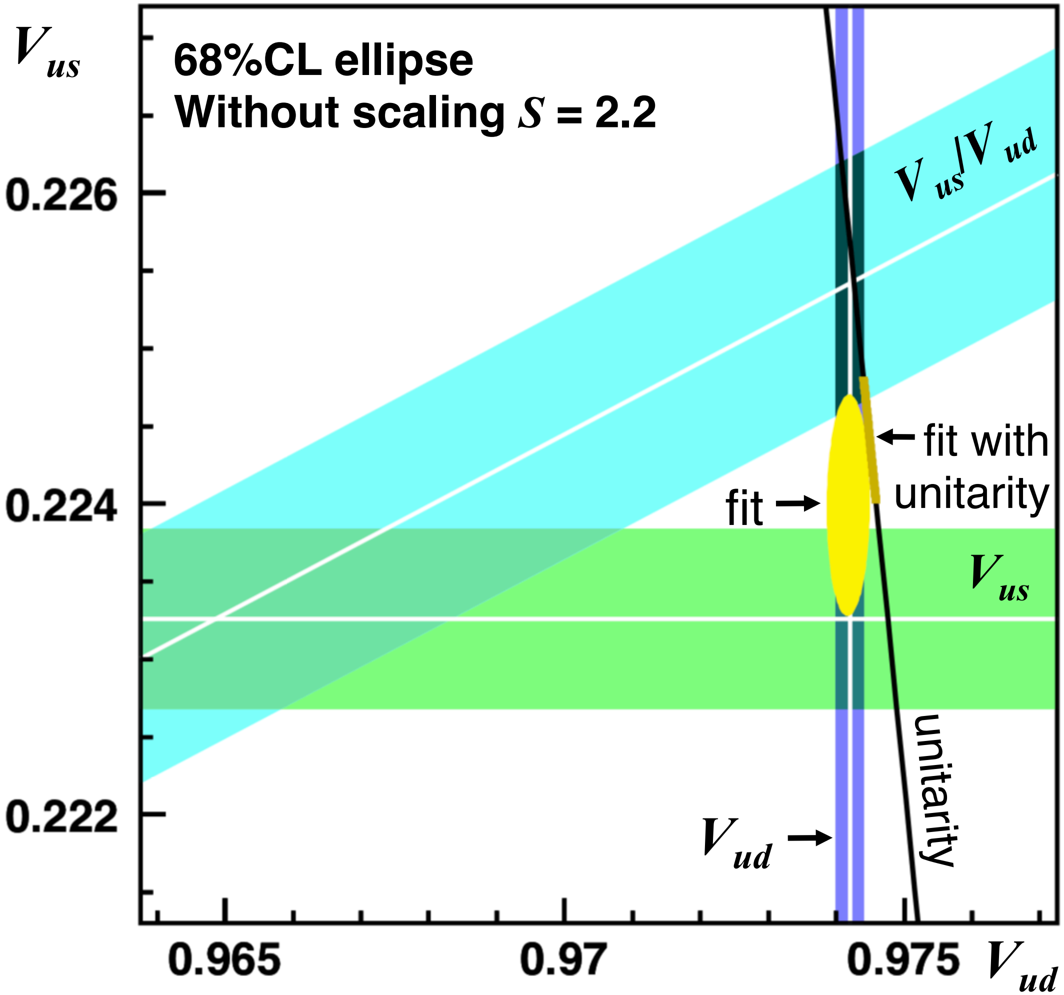
V_{ud} = **0.97418(21)**
V_{us} = **0.2249(5)**
χ^2/ndf = 4.5/1 (3.5%)
Δ_{CKM} = **−0.0004(5)**
−0.8σ

With scale factor S = 2.1

V_{ud} = 0.97418(45)
V_{us} = 0.2249(12)

4.2 V_{us} and CKM unitarity: All data, N_f=2+1+1

N_f **= 2+1+1:** Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
$f_+(0)$ **= 0.9698(17),** f_K/f_π **= 1.1960(40)**



$|V_{ud}|$ = 0.97420(21)
$|V_{us}|$ = 0.2233(6)
$|V_{us}|/|V_{ud}|$ = 0.2314(8)

Fit results, no constraint

V_{ud} = **0.97418(21)**
V_{us} = **0.2240(5)**
χ^2/ndf = 4.7/1 (3.1%)
Δ_{CKM} = **–0.0008(5)**
–1.7σ

With scale factor S **= 2.2**

V_{ud} **= 0.97418(46)**
V_{us} **= 0.2240(10)**

5. Conclusion and outlook

5.1 Preliminary conclusions

Experimental results

$|V_{us}|\, f_+(0) = 0.21654(41)$

$|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm} = 0.27599(37)$

With $N_f = 2+1+1$ lattice inputs

$V_{ud} = 0.97418(21)$ ±0.02%

$V_{us} = 0.2246(5)$ ±0.22%

$\Delta_{\mathrm{CKM}} = -0.0005(5) = -1.1\sigma$

Good agreement with unitarity for $K_{\ell 2}$

Previous excellent consistency for $K_{\ell 3}$ no longer observed

- Change occurred after 2014-era more precise evaluations of $f_+(0)$
- Experimental results for $K_{\ell 3}$ have changed little since 2010

Are residual systematics in the data and/or calculations becoming important as stated uncertainties shrink?

- Evaluation of $|V_{us}|\, f_+(0)$ from $K_{\ell 3}$ data set based on some creaky BR fits, but errors are scaled and consistency between modes is good (K_L, K_S, $K^\pm$)
- Lots of redundancy in $K_{\ell 3}$ data set. Adding or eliminating individual measurements doesn't change $|V_{us}|\, f_+(0)$ much.

5.2 Prospects for new measurements

NA48/2

NA62

Can measure BRs and form-factor parameters for K^+

NA48/2 (2003-2004) recently measured $K_{\ell 3}$ form factors

NA62-RK (2007) has O(10M) $K_{\ell 3}$ decays

NA62 has O(few M) K_{e3} from minimum bias runs (2015-16)

Relative to NA48/2, NA62 has

- Better particle identification π/μ
- Better systematics for t reconstruction:
  - full beam tracking, better σ_p in spectrometer

ISTRA+

OKA

Fixed target experiment at U-70 (Protvino), like ISTRA+

- New beamline with RF-separated K^+ beam

Can measure BRs and form-factor parameters

- Need more analysis of systematics for K_{e3} form factors

Runs from 2010-2013: ~17M K^+_{e3} events

- Additional runs in 2016-2018; more planned in future

5.2 Prospects for new measurements

KLOE

KLOE-2

Can measure all observables: BRs, τs, FFs: $K^\pm$, K_L, K_S

5.5 fb^{-1} of data from KLOE-2 running (2015-2018)

- +2 fb^{-1} of original KLOE data not yet analyzed for V_{us}

Measurements that can be improved with KLOE-2 statistics:

- K_S BRs ($K_S \to \pi e \nu$, but also $K_S \to \pi \mu \nu$)
  See e.g. KLOE-2 measurement of A_S 1806.08654
  70k $K_S \to \pi e \nu$ decays
- $K^\pm$, K_L form factors (particularly $K_{\ell 3}$), K_L mean life?

LHCb

Proven capability to measure K_S decays to muons

- 10^{13} K_S/fb^{-1} produced
- EPJC 77 (2017): BR($K_S \to \mu\mu$) < 1.0 × 10^{-9} 95%CL
  Limited by hardware trigger efficiency ($\varepsilon_{\text{trig}} \sim 1\%$)

Can LHCb measure BR($K_S \to \pi \mu \nu$) to < 1% in Run III?

- Would require dedicated software HLT line

$K_S \to \pi \mu \nu$ never yet measured – a new channel for V_{us}

- τ_S known to 0.04% (vs 0.41% for τ_L, 0.12% for $\tau_\pm$)

5.2 Prospects for new measurements

KEK-246

TREK E36

Primary focus is BR($K_{e2}/K_{\mu2}$) to 0.25%

+ Invisible heavy neutrino searches

+ T violation in $K_{\mu3}$ (as E06)

Upgraded KEK-246 setup, moved to J-PARC

- Stopped K^+ in active target
- Toroidal spectrometer surrounding target
- e/μ particle ID by time of flight, Cerenkov counters, lead-glass calorimetry

KEK-246 measured BR($K_{\mu3}/K_{e3}$) and K_{e3} FF, so TREK could potentially measure at least some BRs and FFs of interest for V_{us}

5.3 Progress on V_{us} from kaons: Final notes

- **$K_{\ell 3}$ FFs do not directly contribute significantly to uncertainty on V_{us}**
  - However, uncertainties on high-statistics BR ratio measurements may be so low that FFs become a major systematic
    - e.g. BR($K_{\mu 3}/\pi\pi^0$), BR($K_{\mu 3}/K_{e3}$)
- **Uncertainties from parameterization of $K\pi$ phase shift data now limit precision for $K_{\ell 3}$ FFs and phase space integrals**
  - Better parameterization will require old data to be re-fit!
  - Imperative for future averages that experiments publish full FF data so that it can be re-fit as parameterizations improve
  - Direct lattice calculation of $K_{\ell 3}$ FFs may help
- **For $K^{\pm}$, normalization BRs have significant uncertainties**
  - Effect of any precise new BR($K_{e3}/\pi\pi^0$) results will limited by uncertainty on BR($\pi\pi^0$)
  - Very important to measure absolute BRs or ratios involving BRs of other modes, e.g. $\pi\pi^0/\mu\nu$, $\pi\pi\pi/\pi\pi^0$, $\pi\pi\pi/\mu\nu$

5.4 Summary and conclusions

Experimental results from kaons		With $\|V_{ud}\|(0^+ \to 0^+)$ and N_f = 2+1+1 lattice
$\|V_{us}\| f_+(0)$ = 0.21652(41)	▶	$\Delta_{\mathrm{CKM}} = -0.0011(5) = -2.2\sigma$
$\|V_{us}/V_{ud}\| \times f_{K\pm}/f_{\pi\pm}$ = 0.27599(37)	▶	$\Delta_{\mathrm{CKM}} = -0.0001(6) = -0.2\sigma$

2σ inconsistency between $K_{\ell 3}$ and $K_{\ell 2}$ results for V_{us}

- **$K_{\ell 2}$ result shows good agreement with unitarity and Vud**
- **$K_{\ell 3}$ result 2σ smaller than expected from unitarity and Vud**
  - Change occurred after 2014-era more precise evaluations of $f_+(0)$
  - Experimental result for $|V_{us}| f_+(0)$ has changed little since 2010

Continuing to see impressive progress on the lattice

- Not only $f_+(0)$ and $f_{K\pm}/f_{\pi\pm}$, but also full t-dependence of FFs, EM corrections, etc.

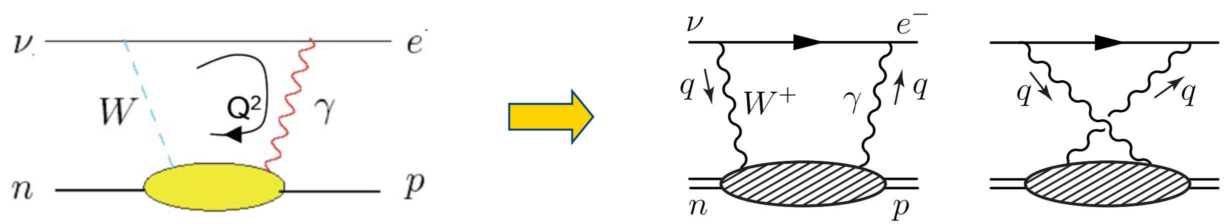
Good prospects for new round of measurements to reduce uncertainty on $|V_{us}| f_+(0)$ from current 0.18% to ~0.12% within next few years:

NA62, OKA, KLOE-2, LHCb, TREK…

6. Back-up

New Radiative Corrections for free neutron

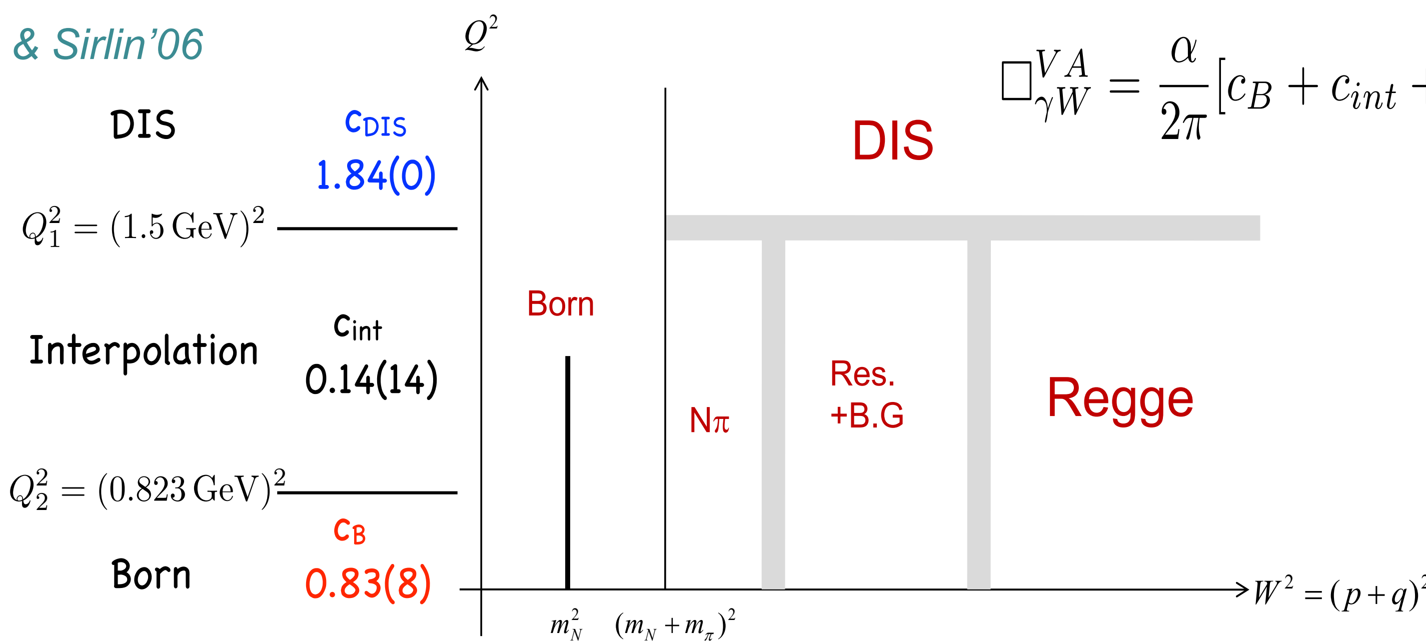
Gorchtein@CIPANP'18



$$\Box_{\gamma W}^{VA\,(0)} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2}F_3^{(0)}(\nu,Q^2)$$

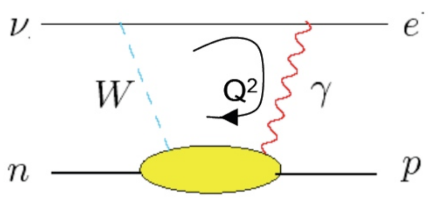
Marciano & Sirlin'06

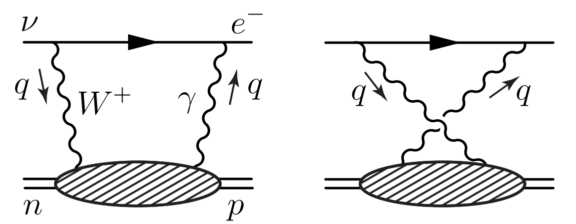
$$\Box_{\gamma W}^{VA} = \frac{\alpha}{2\pi}[c_B + c_{int} + c_{DIS}]$$



New Radiative Corrections for free neutron

Gorshtein@CIPANP'18





$$\boxed{\Box^{VA\,(0)}_{\gamma W} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2} F_3^{(0)}(\nu,Q^2)}$$

Marciano & Sirlin'06
$$\Box^{VA}_{\gamma W} = \frac{\alpha}{2\pi}[c_B + c_{int} + c_{DIS}] = \frac{\alpha}{2\pi}[0.83(8) + 0.14(14) + 1.84(0)]$$

$$\Box^{MS}_{\gamma W} = \frac{\alpha}{2\pi} 2.79(17) = 3.24(20)\times 10^{-3}$$

New evaluation: *Seng, Gorchtein, Patel & Ramsey-Musolf arXiv:1807.10197*

$$\Box^{VA}_{\gamma W} = \frac{\alpha}{2\pi}[c_B + c_{piN} + c_{\text{Res}} + c_{\text{Regge}} + c_{DIS}] = \frac{\alpha}{2\pi}[0.91(5) + 0.044(5) + 0.01(1) + 0.238(14) + 1.84(0)]$$

$$\boxed{\Box^{\text{New}}_{\gamma W} = \frac{\alpha}{2\pi} 3.03(5) = 3.51(6)\times 10^{-3}}$$ ⇒ V_{ud} from free n: ~1σ smaller

2.6 $K_{\ell 3}$ data and lepton universality

- For each state of kaon charge, evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\text{obs}}}{(R_{\mu e})_{\text{SM}}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \cdot \frac{I_{e3}\,(1+\delta_{e3})}{I_{\mu 3}\,(1+\delta_{\mu 3})} = \frac{[|V_{us}|\,f_+(0)]^2_{\mu 3,\,\text{obs}}}{[|V_{us}|\,f_+(0)]^2_{e3,\,\text{obs}}} = \frac{g_\mu^2}{g_e^2}$$

Modes	2004 BRs*,†	Current†
K_L	**1.054(14)**	**1.003(5)**
$K^\pm$	**1.014(12)**	**0.999(9)**
Avg	**1.030(9)**	**1.002(5)**

← Was 0.998(9) for 2010

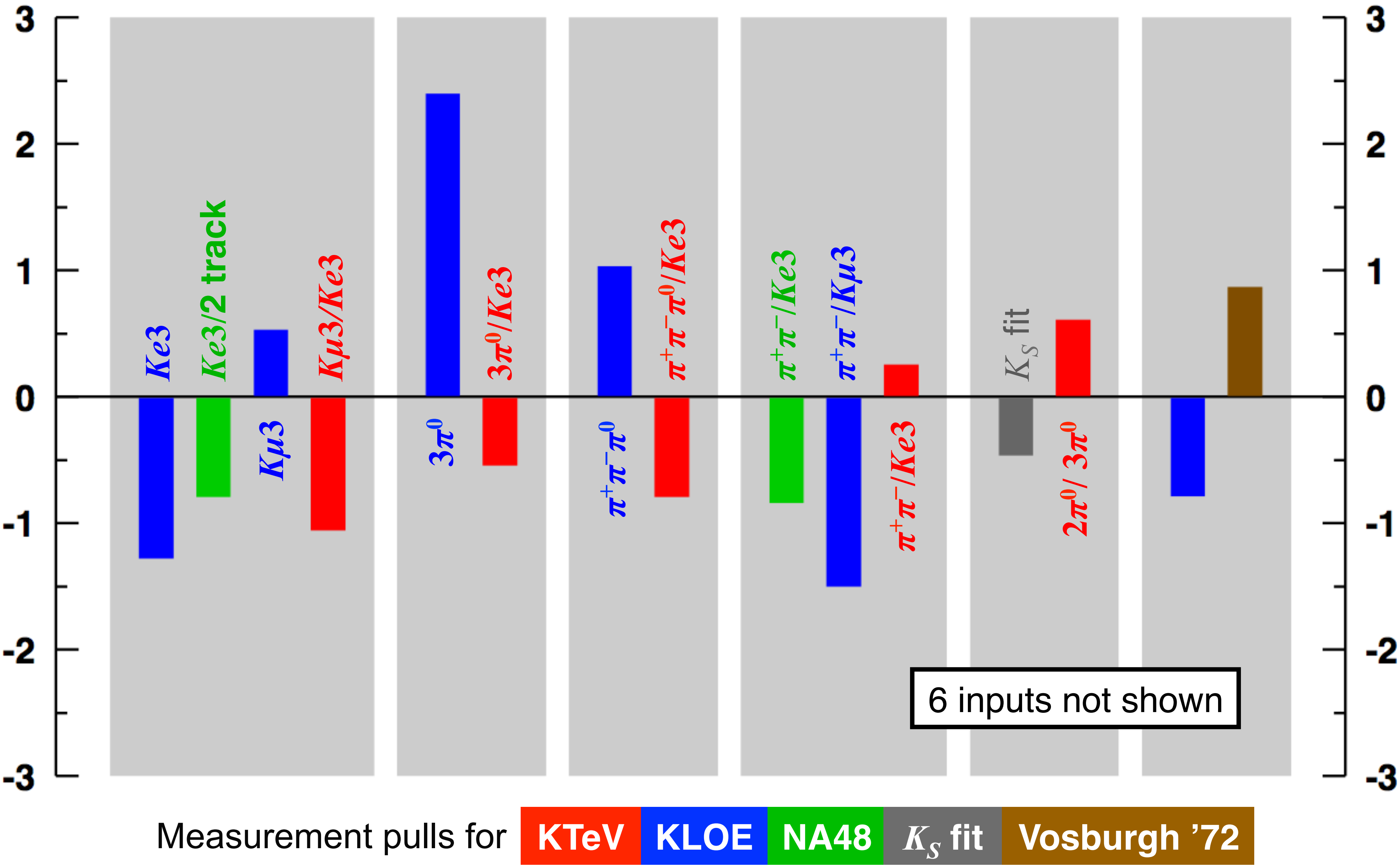
*Assuming current values for form-factor parameters and Δ^{EM} $^\dagger K_S$ not included

- Compare to other precise tests:

$\pi \to \ell\nu$ **$(r_{\mu e})$ = 1.0020(19)**
PDG '16 with PIENU '15 result

$\tau \to \ell\nu\nu$ **$(r_{\mu e})$ = 1.0038(28)**
HFLAV May '17 web update

Comparison: K_L fit result vs. input data



Updated fit to $K^{\pm}$ rate data

17 input measurements:

- **3 old τ values in PDG**
- **KLOE τ**
- **KLOE BR $\mu\nu$, $\pi\pi^0$**
- **KLOE BR K_{e3}, $K_{\mu3}$**
  with dependence on τ
- **NA48/2 BR $K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$**
- **E865 BR K_{e3}/KDal**
- **3 old BR $\pi\pi^0/\mu\nu$**
- **KEK-246 $K_{\mu3}/K_{e3}$**
- **KLOE BR $\pi\pi\pi$, $\pi\pi^0\pi^0$**
- (Bisi '65 BR $\pi\pi^0\pi^0/\pi\pi\pi$ removed)

1 constraint: Σ BR = 1

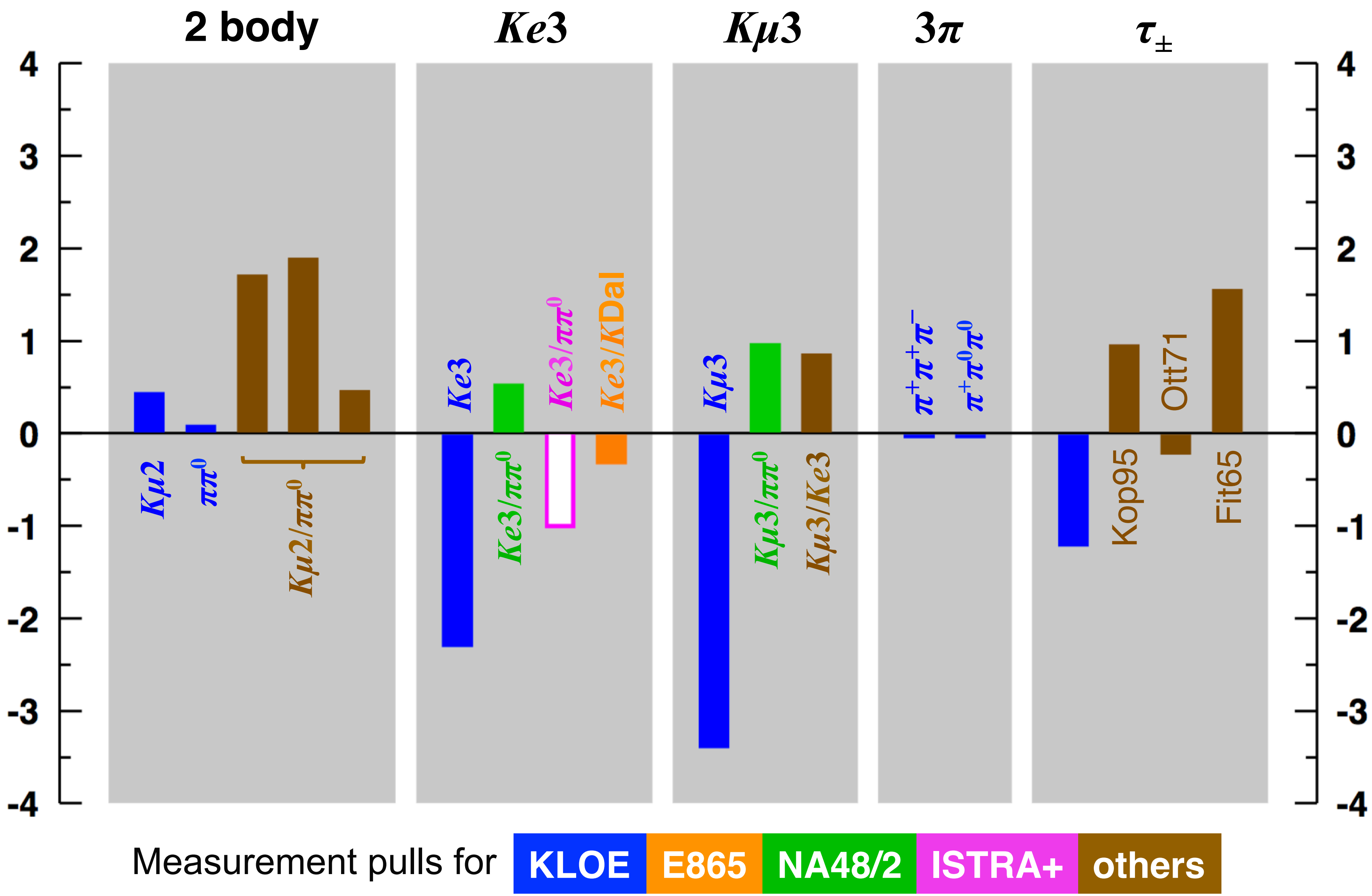
Much more selective than PDG fit
PDG '16: 35 inputs, 8 parameters

Parameter	Value	S
BR($\mu\nu$)	63.58(11)%	1.1
BR($\pi\pi^0$)	20.64(7)%	1.1
BR($\pi\pi\pi$)	5.56(4)%	1.0
BR(K_{e3})	5.088(27)%	1.2
BR($K_{\mu3}$)	3.366(30)%	1.9
BR($\pi\pi^0\pi^0$)	1.764(25)%	1.0
$\tau_{\pm}$	12.384(15) ns	1.2

χ^2/ndf = 25.5/11 (Prob = 0.78%)
compare PDG '16: 53/28 (0.26%)

With **ISTRA+ '14 BR($K^{-}_{e3}/\pi^{-}\pi^{0}$)**
- **BR(K_{e3}) = 5.083(27)%**
- Negligible changes in other parameters, fit quality

Comparison: $K^{\pm}$ fit result vs. input data



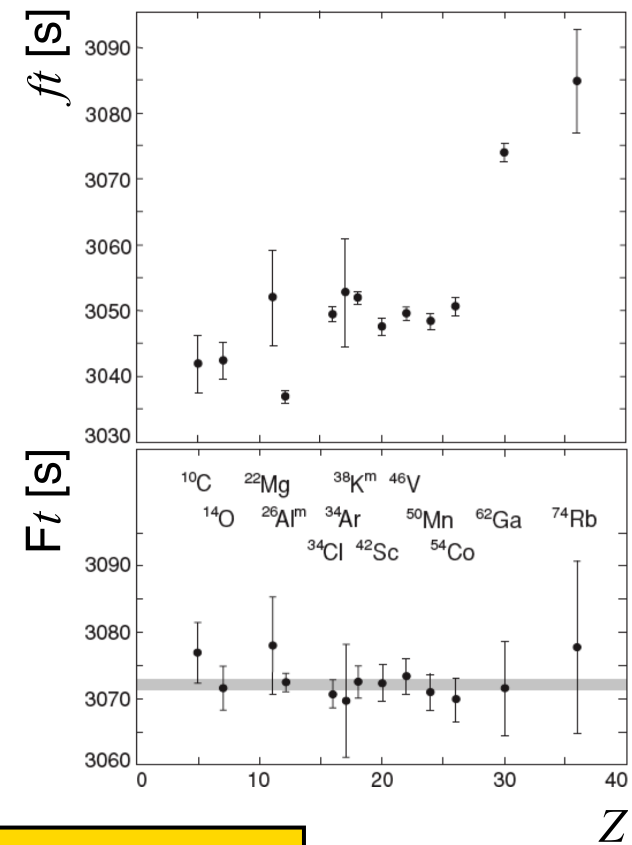
$|V_{us}|(K_{\ell 3})$ and $|V_{ud}|(0^+ \to 0^+)$: 2010

Moulson@CKM2014

$|V_{us}| f_+(0)$ = 0.2163(5) $\xrightarrow{f_+(0)\,=\,0.959(5)}$ **$|V_{us}|$ = 0.2254(13)**

Hardy & Towner '10
$|V_{ud}|$ = 0.97425(22)

- Survey of 150 measurements of 13 different $0^+ \to 0^+$ β decays
- 27 new ft measurements including Penning-trap measurements for Q_{EC}
- Some old measurements dropped
- Improved EW radiative corrections [Marciano & Sirlin '06]
- New $SU(2)$-breaking corrections [Towner & Hardy '08]



$$\Delta_{\mathrm{CKM}} = V_{ud}^2 + V_{us}^2 - 1 = +0.0000(8)$$

Exact compatibility with unitarity

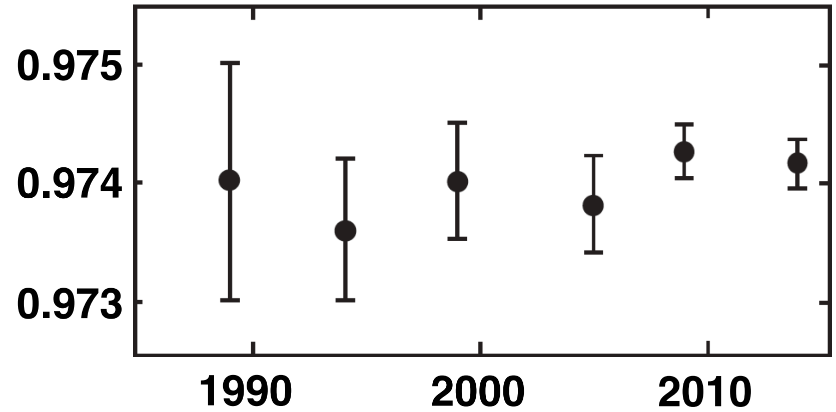
$|V_{us}|(K_{\ell 3})$ and $|V_{ud}|(0^+ \to 0^+)$: Update

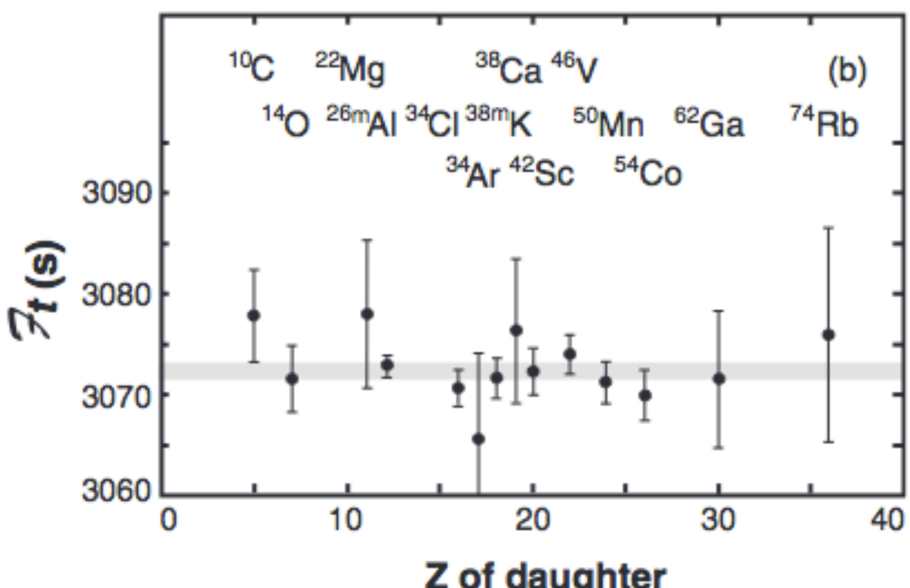
M. Moulson@CKM'16

Hardy, CKM '16 preliminary
$|V_{ud}| = 0.97420(21)$

- 24 new measurements
- Critical review of IB correction schemes as per PRC 82 (2010)
- Rejection of results with IB corrections giving results in conflict with CVC

V_{ud} vs analysis year
Courtesy of J. Hardy



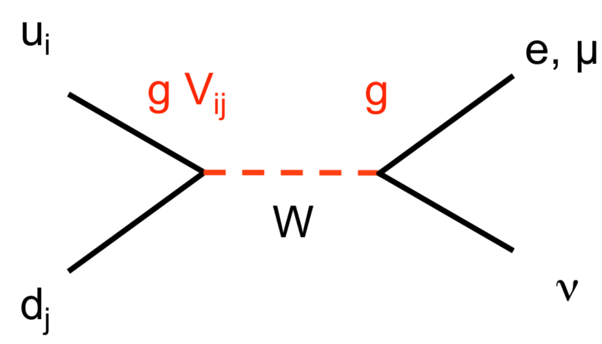


Hardy & Towner'15

1.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \bar{\nu}_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \bar{\nu}_e$	$K \to l \nu_l$



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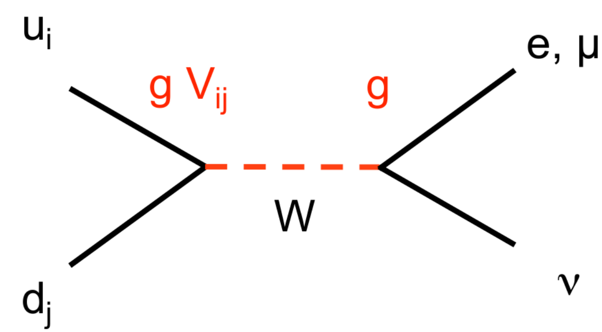

- These are the *golden modes* to extract V_{ud} and V_{us}
  - Only the *vector current* contributes $\langle A(p_A)|\bar{q}^i \gamma_\mu q^j|B(p_B)\rangle$
  - Normalization known in SU(2) [SU(3)] symmetry limit
  - Corrections start at 2nd order in SU(2) [SU(3)] breaking

Ademollo & Gato, Berhands & Sirlin

- Currently the most precise determination of V_{ud} and V_{us}

⇨ V_{ud} (*0.02 %*) and V_{us} (*0.5 %*)

1.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \bar{\nu}_e$	$\pi \to l \nu_l$
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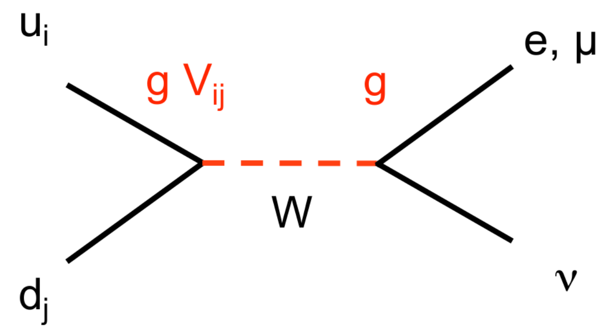

- Both *V* and *A* currents contribute: need experimental input on <A> (e.g. β-asymmetry)

- Free of nuclear structure uncertainties

- Probe different combinations of BSM operators

1.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \bar{\nu}_e$	$\pi \to l \nu_l$
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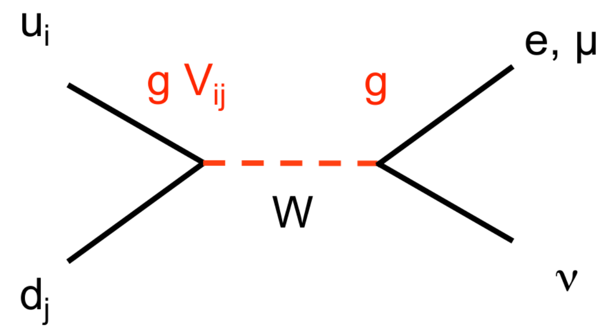


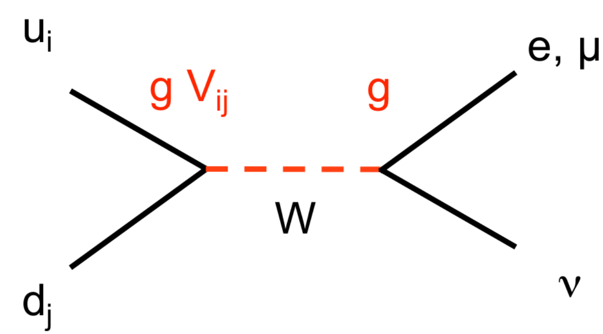
- K_{l2}/π_{l2}
  - Only the *axial current* contributes
  - Need to know the decay constants F_K, F_π ⇒ *Lattice QCD*
  - Probe different BSM operators than from the vector case

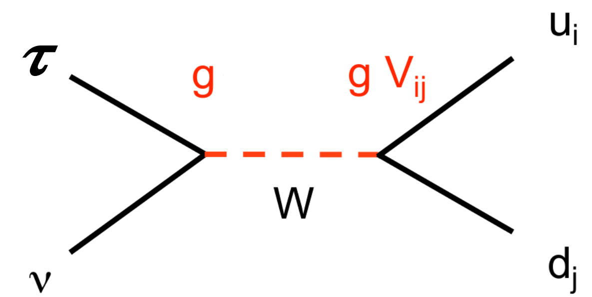
- Input on F_K / F_π ⇒ V_{us}/V_{ud} very precisely

1.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \rightarrow 0^+$ $\pi^\pm \rightarrow \pi^0 e \nu_e$	$n \rightarrow p e \bar{\nu}_e$	$\pi \rightarrow l \nu_l$
V_{us}	$K \rightarrow \pi l \nu_l$	$\Lambda \rightarrow p e \bar{\nu}_e$	$K \rightarrow l \nu_l$





- From τ decays (crossed channel) + test of LU

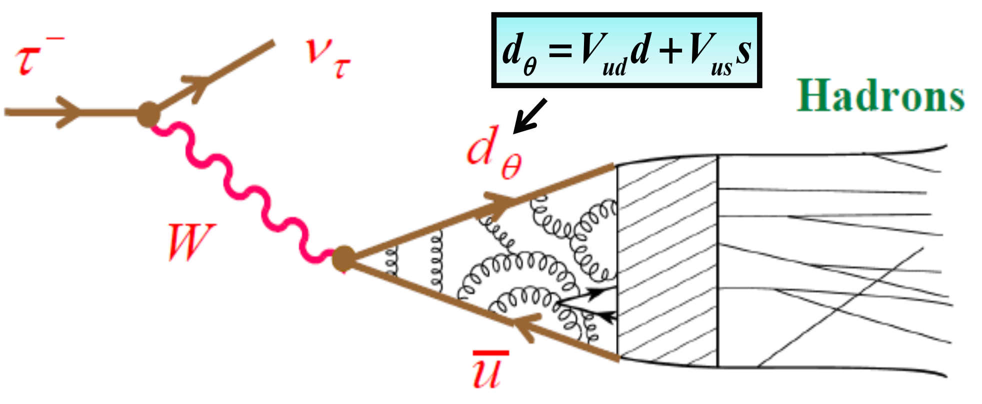
V_{ud}	$\tau \rightarrow \pi\pi\nu_\tau$		$\tau \rightarrow \pi\nu_\tau$	$\tau \rightarrow h_{NS}\nu_\tau$
V_{us}	$\tau \rightarrow K\pi\nu_\tau$		$\tau \rightarrow K\nu_\tau$	$\tau \rightarrow h_S\nu_\tau$ (inclusive)

Paths to V_{ud} and V_{us}

- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

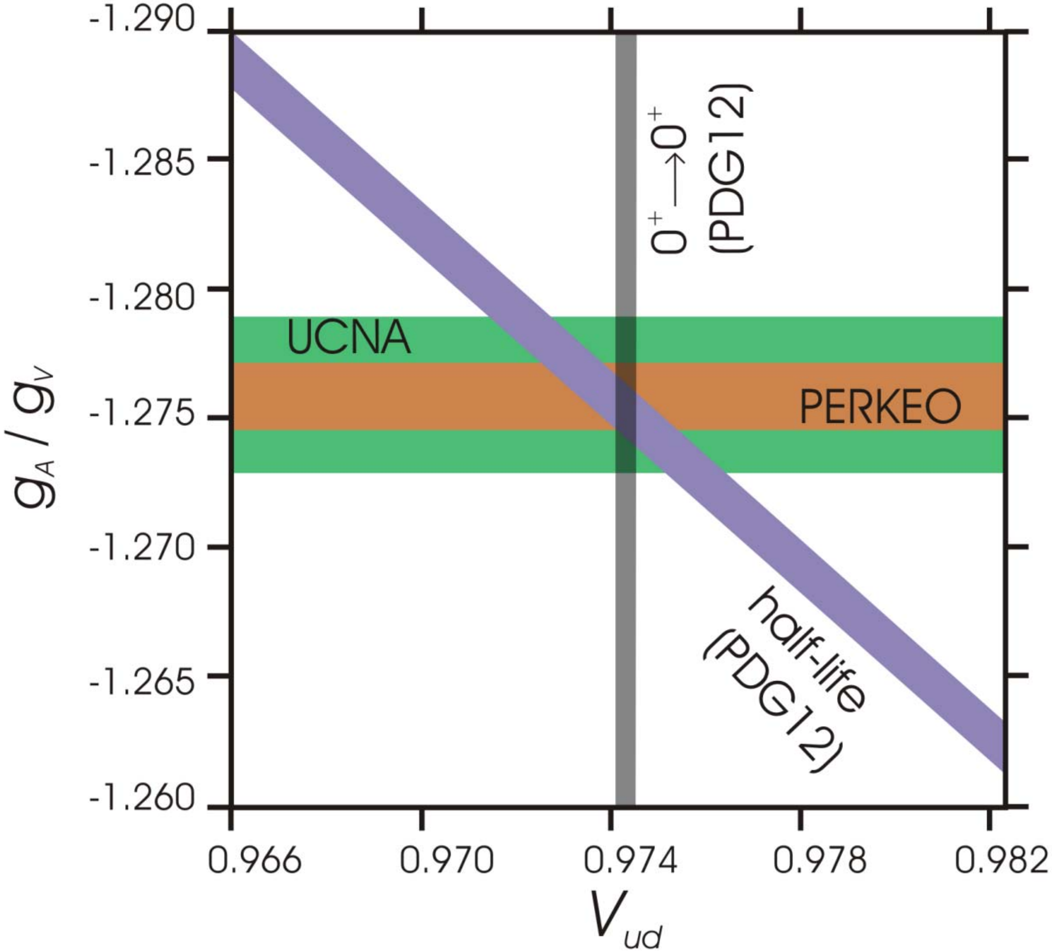
- Use OPE to calculate inclusive BRs
- Information from exclusive modes too



Extraction of Vud

- See also V_{ud} extraction from neutron decay

Mendenhall et al.'13



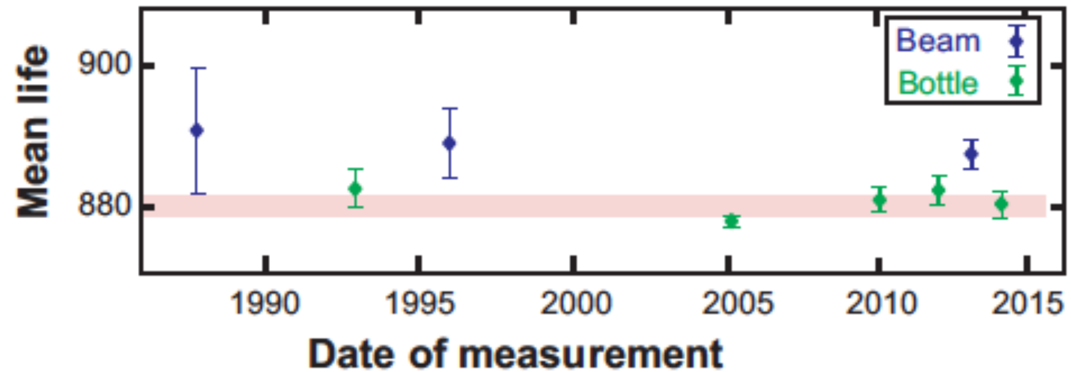
Extraction of V_{ud}

Mean life:

τ **= 880.2 ± 1.0 s**

χ^2/N **= 3.7**

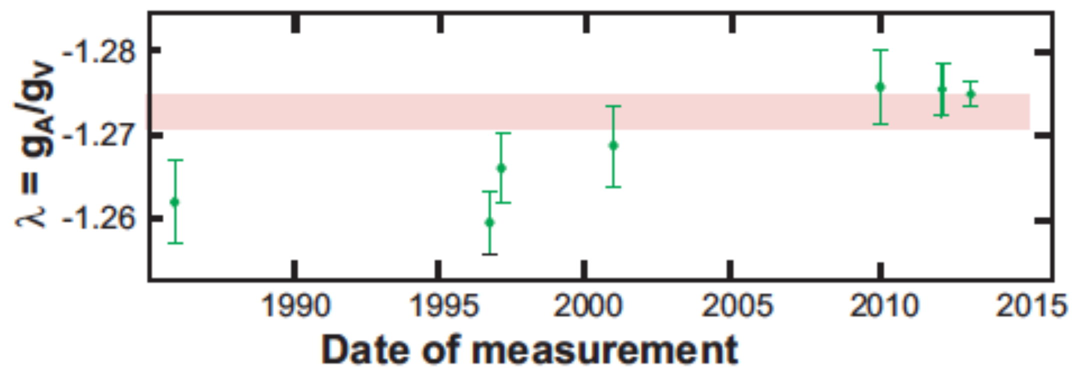
Beam: 888.1 ± 2.0 s
Bottle: 879.6 ± 0.7 s



β asymmetry:

λ **= -1.2725 ± 0.0020**

χ^2/N **= 4.1**



$\boxed{V_{ud} = 0.9757 \pm 0.0014}$

Beam-bottle span
$0.9701 \leq V_{ud} \leq 0.9767$

nuclear $0^+ \to 0^+$
$V_{ud} = 0.9742 \pm 0.0002$

4. Implication of Cabibbo universality tests for new physics

4.1 Looking for New Physics using Δ_{CKM}

- Δ_{CKM} a constraining quantity: $\boxed{\Delta_{CKM} = 1 - \left(|V_{ud}|^2 + |V_{us}|^2\right)}$

$$\boxed{\frac{1-|V_{uD}|^2}{1-|U_{\mu N}|^2} \cdot \frac{1}{1+BR^{\mu}_{\text{exotic}}} \cdot \frac{[G_F^{(\beta)}]^2}{[G_F^{(\mu)}]^2} = 1 + \Delta_{CKM}}$$

Heavy fermion mixing

Exotic muon decays

Gauge universality violations

$$\boxed{\begin{array}{l} |V_{uD}| \leq 0.03 \\ |U_{\mu N}| \leq 0.03 \end{array}}$$

95% C.L.

$$\boxed{\begin{array}{c} BR^{\mu}_{\text{exotic}} < 0.001 \\ \text{95\% C.L.} \end{array}}$$

Stronger than direct limits

$$BR(\mu^+ \to e^+ \bar{\nu}_e \nu_\mu) < 0.012$$

Constraints on TeV scale SM extensions

4.1 Looking for New Physics using Δ_{CKM}

- Effective Theory approach:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

- Δ_{CKM} a constraining quantity:

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$$

4.1 Looking for New Physics using $\Delta_{\rm CKM}$

from: SJ, talk at NA62 Handbook workshop 2009

	Operator / Observable	$K^+\to\pi^+\nu\bar\nu$	$K_L\to\pi^0\nu\bar\nu$	$K_L\to\pi^0\ell^+\ell^-$	$K_L\to\ell^+\ell^-$	$K^+\to\ell^+\nu$	$P_T(K^+\to\pi^0\mu^+\nu)$	$\Delta_{\rm CKM}$	ϵ'/ϵ	ϵ_K	in MSSM?
$O_{lq}^{(1)}$	$(\bar D_L\gamma^\mu S_L)(\bar L_L\gamma_\mu L_L)$	✓	✓	✓	hs	–	–	–	–	–	✓
$O_{lq}^{(3)}$	$(\bar D_L\gamma^\mu\sigma^i S_L)(\bar L_L\gamma_\mu\sigma^i L_L)$	✓	✓	✓	hs	hs	✓	✓	–	–	✓
O_{qe}	$(\bar D_L\gamma^\mu S_L)(\bar l_R\gamma_\mu l_R)$	–	–	✓	hs	–	–	–	–	–	small
O_{ld}	$(\bar d_R\gamma^\mu s_R)(\bar L_L\gamma_\mu L_L)$	✓	✓	✓	hs	–	–	–	–	–	small
O_{ed}	$(\bar d_R\gamma^\mu s_R)(\bar l_R\gamma_\mu l_R)$	–	–	✓	hs	–	–	–	–	–	small
$O_{lq}^\dagger$	$(\bar u_R S_L)\cdot(\bar l_R L_L)$	–	–	–	–	✓	✓	✓	–	–	tiny
$(O_{lq}^t)^\dagger$	$(\bar u_R\sigma_{\mu\nu} S_L)\cdot(\bar l_R\sigma^{\mu\nu} L_L)$	–	–	–	–	–	?	?	–	–	tiny
O_{qde}	$(\bar d_R S_L)(\bar L_L l_R)$	–	–	✓	✓	–	–	–	–	–	tiny
$O_{qde}^\dagger$	$(\bar D_L s_R)(\bar l_R L_L)$	–	–	✓	✓	✓	✓	✓	–	–	large $\tan\beta$
$O_{\varphi q}^{(1)}$	$(\bar D_L\gamma^\mu S_L)(H^\dagger D_\mu H)$	✓	✓	✓	hs	–	–	–	✓	(✓)	✓
$O_{\varphi q}^{(3)}$	$(\bar D_L\gamma^\mu\sigma^i S_L)(H^\dagger D_\mu\sigma^i H)$	✓	✓	✓	hs	hs	✓	✓	✓	(✓)	✓
$O_{\varphi d}$	$(\bar d_R\gamma^\mu s_R)(H^\dagger D_\mu H)$	✓	✓	✓	hs	–	–	–	✓	(✓)	large $\tan\beta$ (non-MFV)

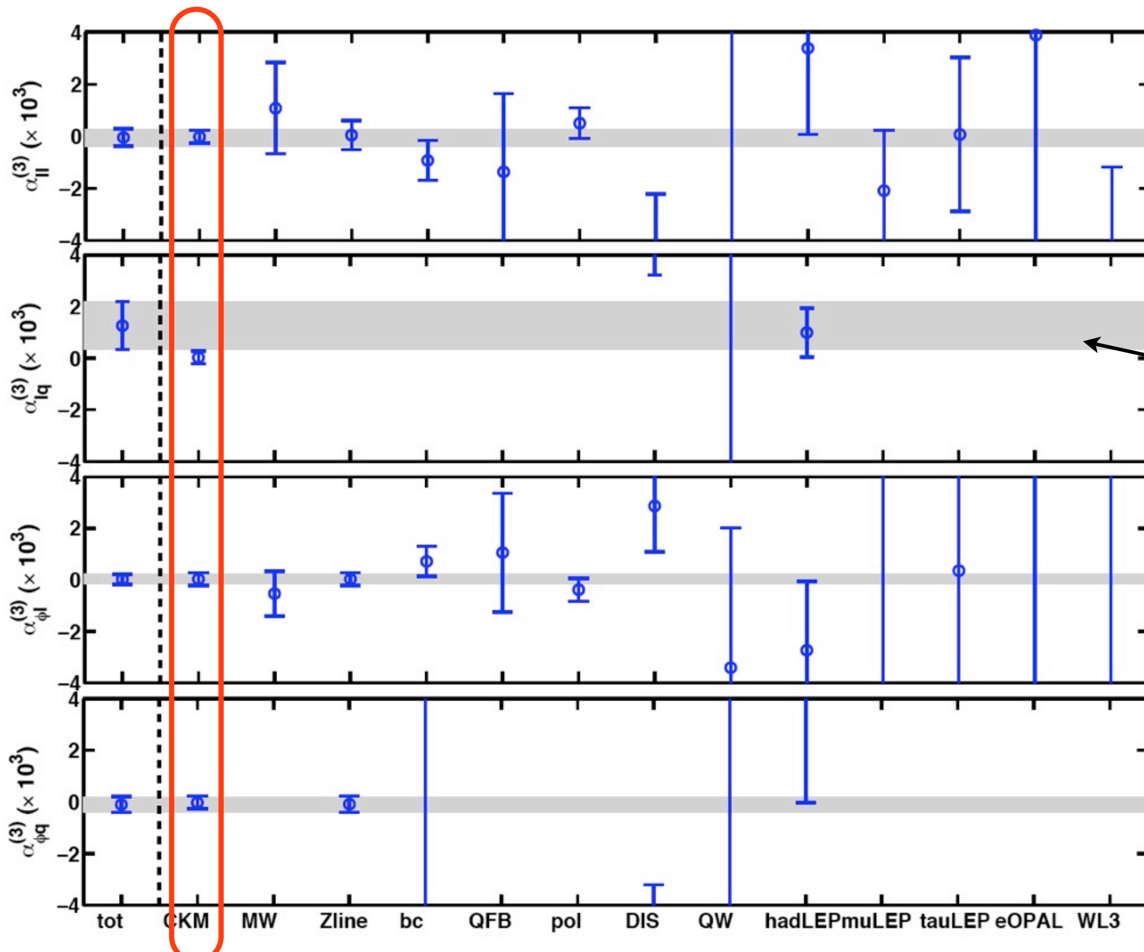
4.1 Looking for New Physics using Δ_{CKM}

Cirigliano, Gonzalez-Alonso & Jenkins'09

- Δ_{CKM} is sensitive to 4 fermion operators:

2) What is the strength of Δ_{CKM} constraint? Same level or better than Z-pole observables (effective scale Λ > 11 TeV @ 90% CL)

$$\hat{\alpha}_X = \frac{v^2}{\Lambda_X^2}$$



$O_{lq}^{(3)}$

Dramatic improvement over LEP2 and APV

Deviations as large as Δ_{CKM} ~0.01 could be blamed on this operator

4.2 Looking for New Physics with K_{l2} and K_{l3}

- Callan-Treiman theorem:

Bernard, Oertel, E.P., Stern'06, '08

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K |V^{us}|}{F_\pi |V^{ud}|} \frac{1}{f_+(0)|V^{us}|} |V^{ud}|}_{\text{Very precisely known}} r + \Delta_{CT}$$

$\Delta_{K\pi} = m_K^2 - m_\pi^2$

Very precisely known from Br(Kl2/πl2), Γ(Ke3) and $|V_{ud}|$

$\Rightarrow$ $\boxed{B_{exp} = 1.2446(41)}$

- In the Standard Model : $\boxed{r = 1}$ $\left(\ln C_{SM} = 0.2141(73)\right)$ $\Delta_{CT} = (-3.5 \pm 8).10^{-3}$

- In presence of new physics, new couplings : $\boxed{r \neq 1}$

NLO value + large error bars in agreement with

Bijnens&Ghorbani'07
Kastner & Neufeld'08

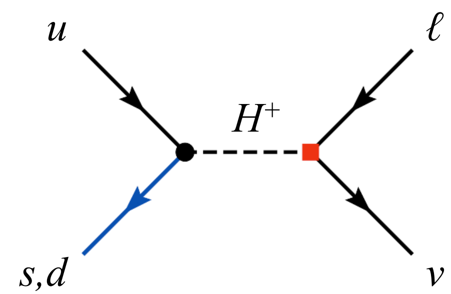
Experiment K_{e3}+$K_{\mu3}$	ln C
NA48'07 **($K_{\mu3}$ alone)**	**0.144(14)**
KLOE'08	**0.204(25)**
KTeV'10	**0.192(12)**
NA48 **(preliminary)**	**?**

4.3 Constraints on 2HDM

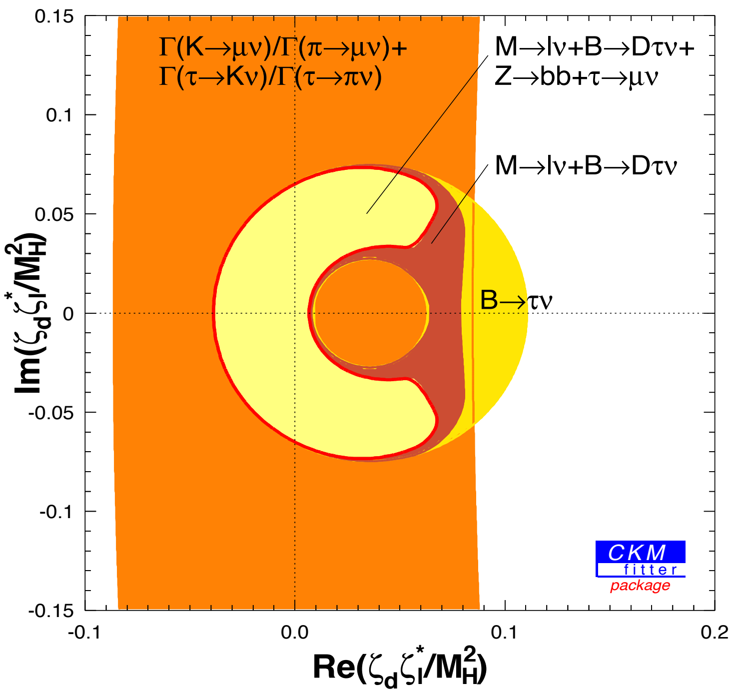
- Ex: Constraints on the aligned 2-Higgs-doublet model:

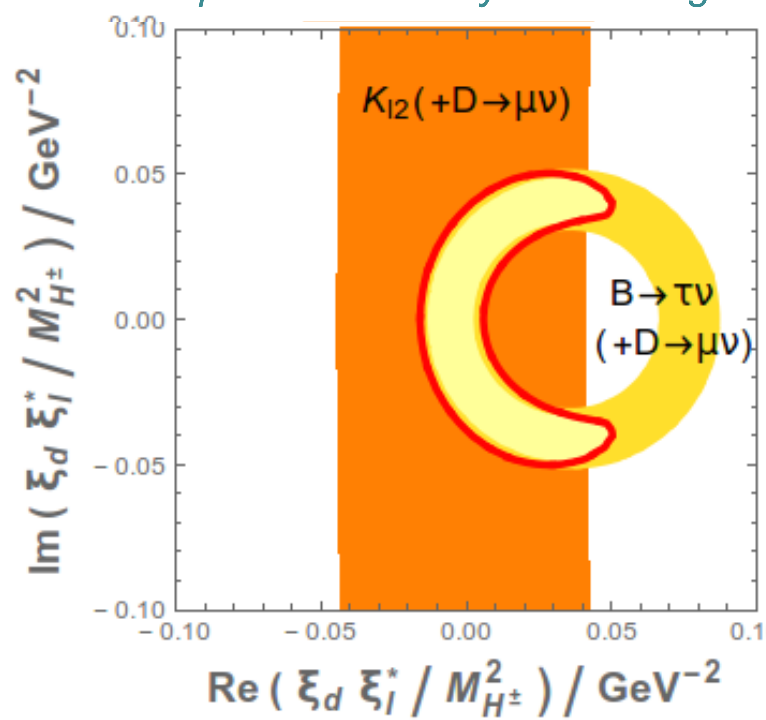
Jung, Pich, Tuzon'10
Pich@HQL'12

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v} H^+ \Big\{ \bar{u} \left[\varsigma_d V_{\mathrm{CKM}} M_d \mathcal{P}_R - \varsigma_u M_u^\dagger V_{\mathrm{CKM}} \mathcal{P}_L \right] d + \varsigma_l \left(\bar{\nu} M_l \mathcal{P}_R l \right) \Big\} + \mathrm{h.c.}$$



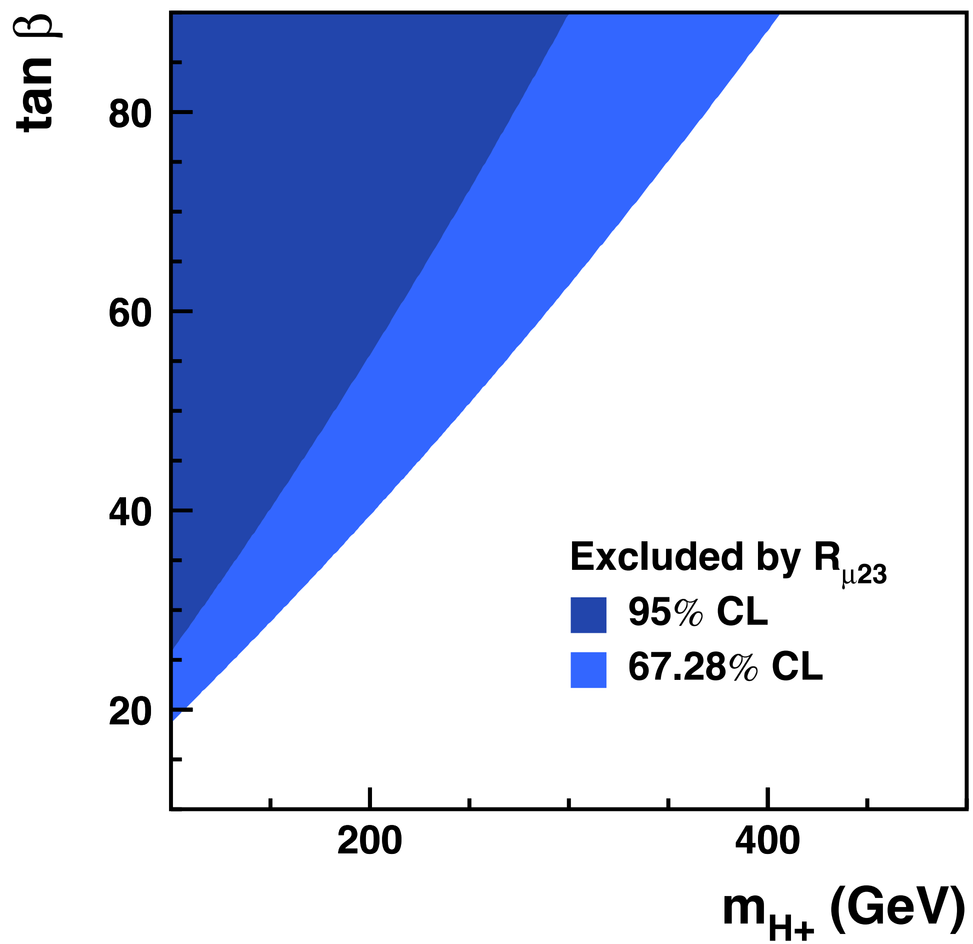
Update: Courtesy of M. Jung





4.3 Constraints on charged Higgs

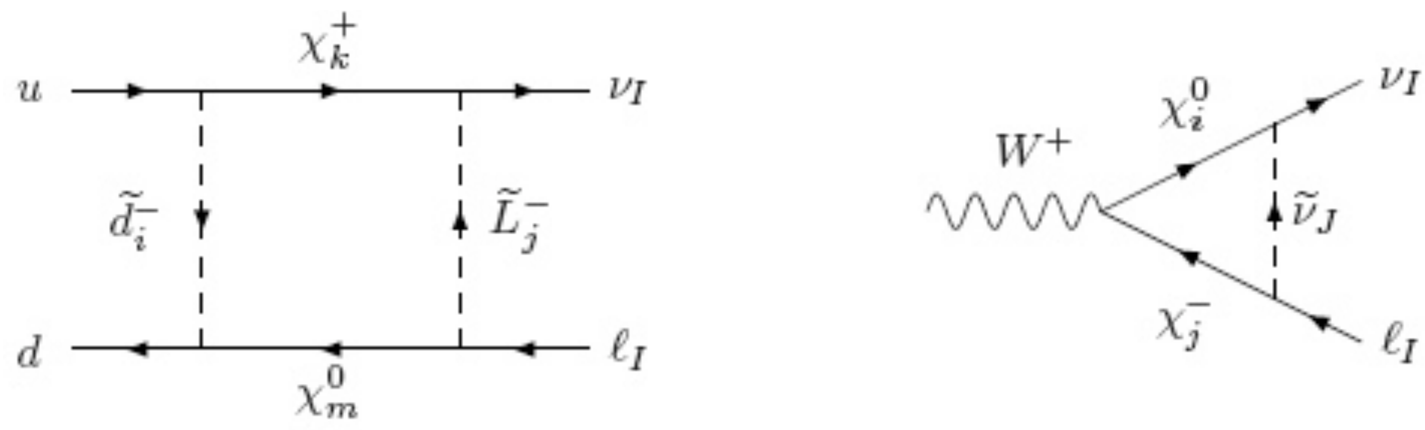
Antonelli et al.'10



4.4 Universality and SUSY

- MSSM: box and vertex corrections induce non-universal corrections to the V-A CC operators

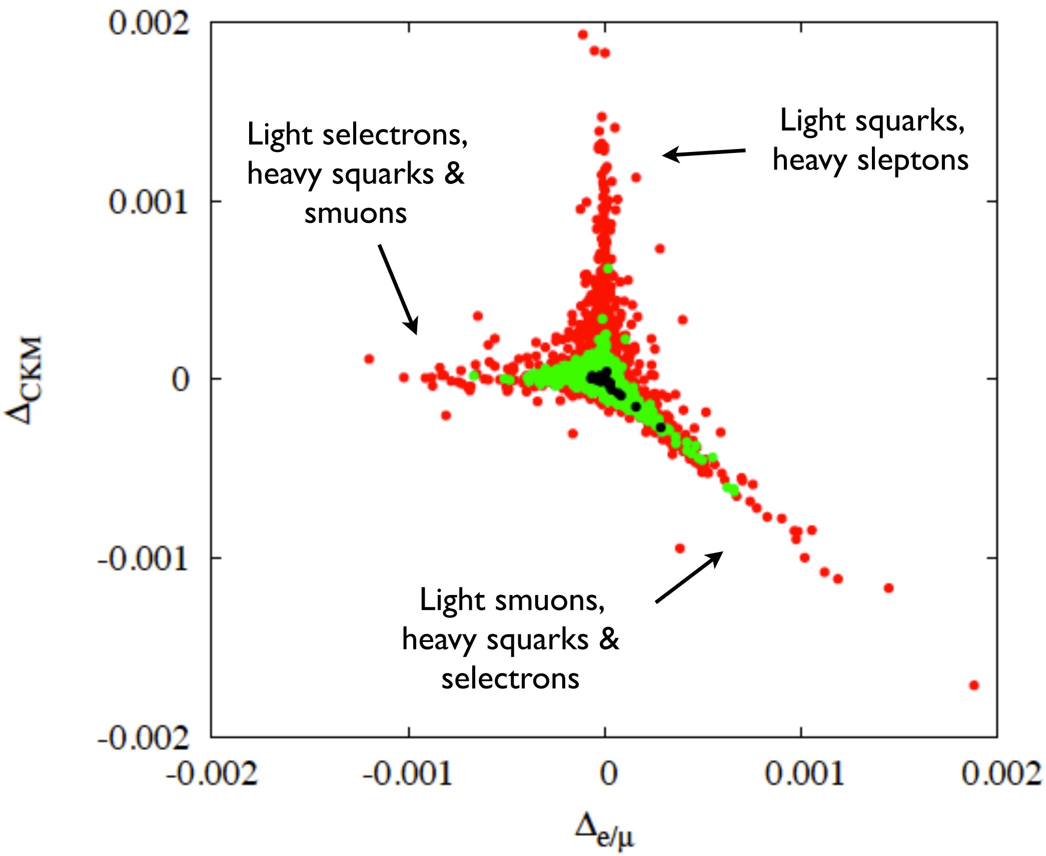
- S,P,T operators suppressed by insertions of Yukawa couplings



Barbieri et al '85, Hagiwara-Matsumoto-Yamada '95
Ramsey-Musolf Kurylov '01
Bauman, Erler, Ramsey-Musolf '12

4.4 Universality and SUSY

Bauman, Erler, Ramsey-Musolf '12



Before EW + LHC constraints

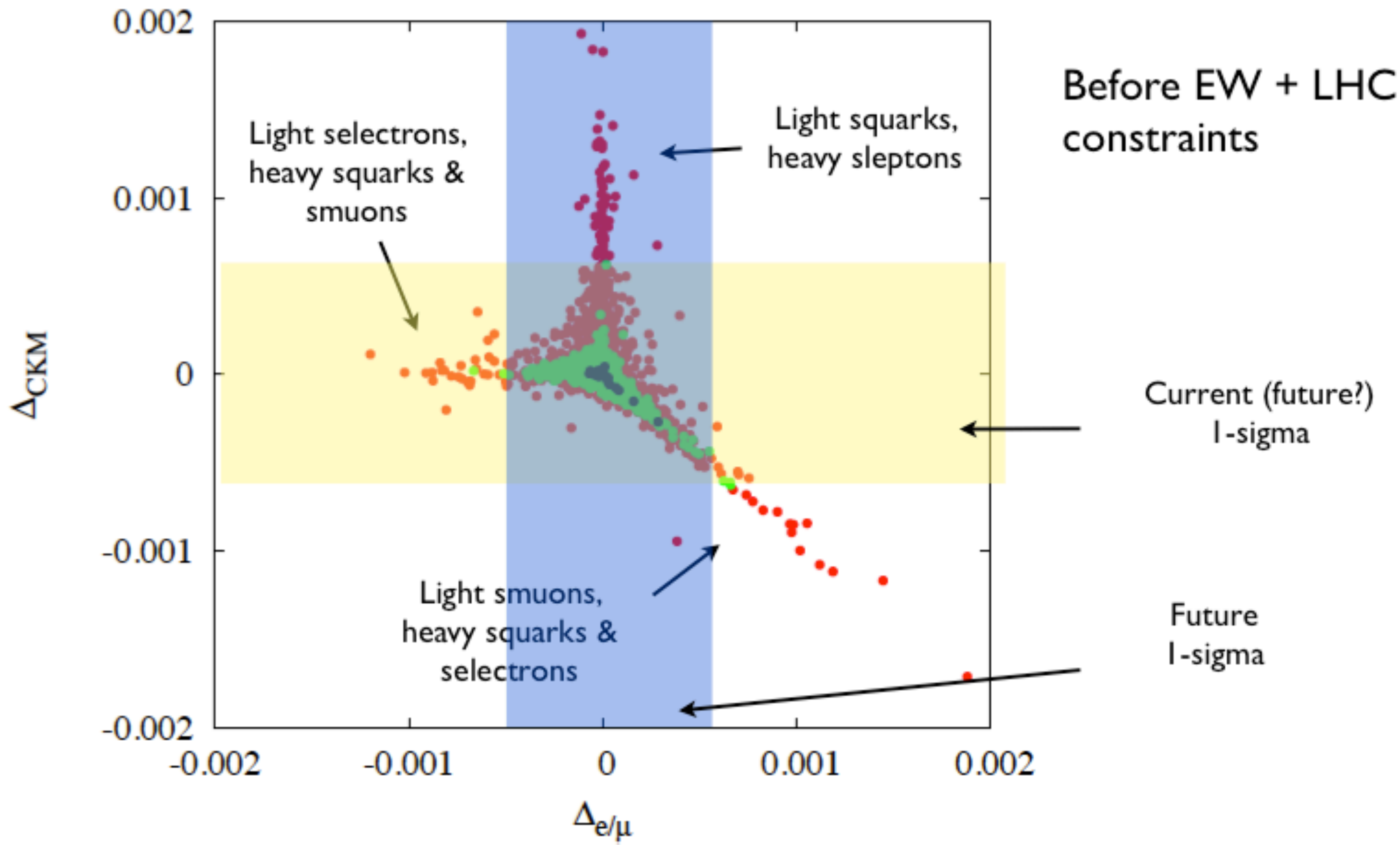
$$[G_F]_e/[G_F]_\mu = 1 + \Delta_{e/\mu}$$

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1 + \Delta_{\rm CKM}$$

- Interesting correlation between Cabibbo universality and lepton universality: information on sfermion spectrum
- Essentially squark-slepton and selectron-smuon universality

4.4 Universality and SUSY

Bauman, Erler, Ramsey-Musolf '12

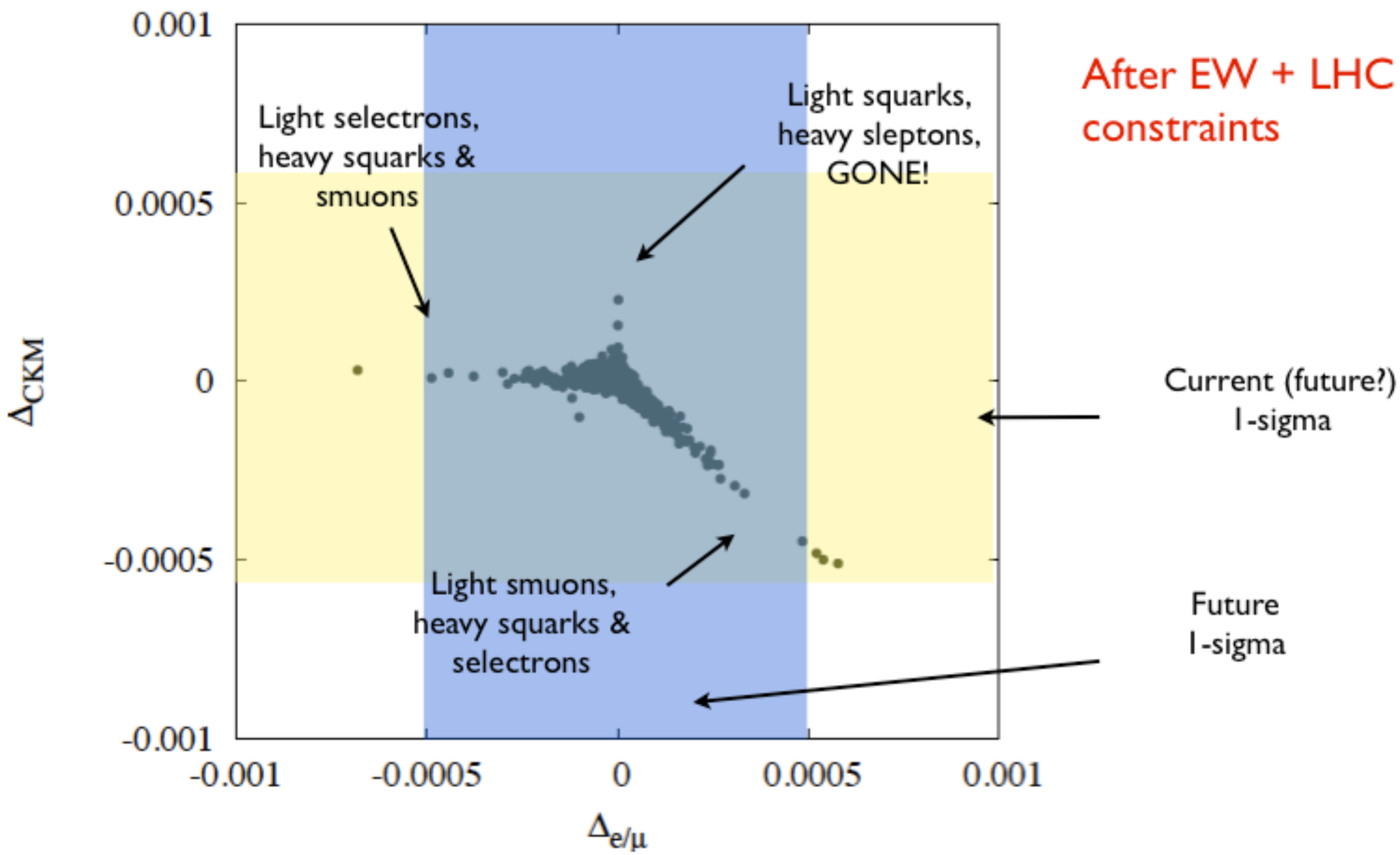


- Interesting correlation between Cabibbo universality and lepton universality: information on sfermion spectrum

- Essentially squark-slepton and selectron-smuon universality

4.4 Universality and SUSY

Bauman, Erler, Ramsey-Musolf '12



- Effects in the MSSM are small.
- Probing MSSM parameter space requires improved precision