Status of $\bar{B} \to D^* \ell \bar{\nu}$ semileptonic decay and $|V_{cb}|$

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September 18^{th}, 2018

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Outline

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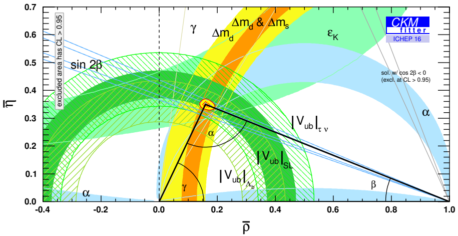
Introduction: The $|V_{cb}|$ CKM matrix element

- Precision test of the standard model, looking into new physics
- CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Determination	$\lvert V_{cb}\rvert\ (\cdot 10^{-3})$
Exclusive	39.2 ± 0.7
Inclusive	42.5 ± 0.9

FLAG '17, HFLAV '17

- Apparent 2σ tension between inclusive and exclusive determinations
- Forthcoming experiments (LHCb, Belle-II) aim to reduce the uncertainty in the determination of the CKM matrix elements

Introduction: The $|V_{cb}|$ CKM matrix element

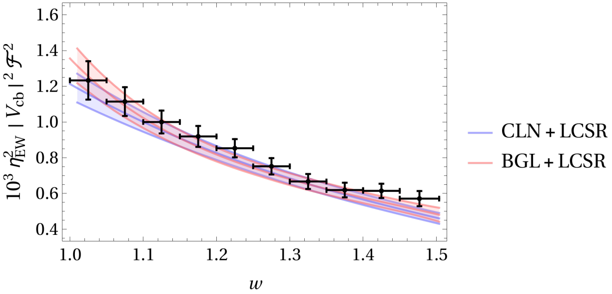
$$\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right) = \frac{G_F^2 m_B^5}{48\pi^2}\left|V_{cb}\right|^2\left(w^2-1\right)^{\frac{1}{2}}P(w)\left|\eta_{ew}\mathcal{F}(w)\right|^2$$

- Experiments measure the decay rate as a function of $w = v_{D^*}\cdot v_B$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental measurements
- Lattice calculations measure the form factors and reconstruct the whole $\mathcal{F}$ function
  - $\lim_{m_Q\to\infty}\mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- A fit of the form factor to a theory-motivated function (parametrization) allows one to extract V_{cb} from experimental data
- Caprini-Lellouch-Neubert (CLN) *Nucl. Phys.* **B**530 (1998) 153-181

$$F(w) = F(1) - \rho^2 z + cz^2, \quad \text{with } c = f(\rho), \quad z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$$

Introduction: The $|V_{cb}|$ CKM matrix element

- Relies on some strong assumptions
- Tightly constrains $F(w)$: only one independent parameter



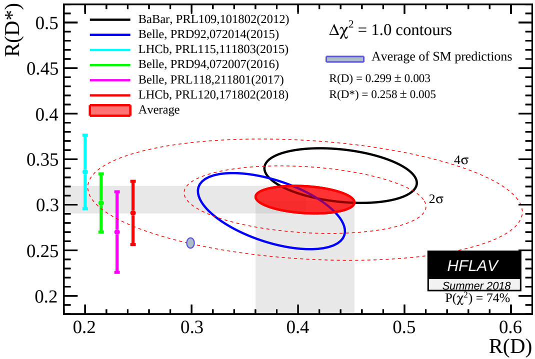
Bigi, Gambino and Schacht, *Phys. Lett.* **B** *769 (2017) 441-445* using Belle data at non-zero recoil and lattice data at zero recoil

- Our current understanding is that CLN might underestimate the slope at low recoil
- Current discrepancy might be an artifact
- A lattice QCD calculation at $w \gtrsim 1$ is urgently needed to settle the issue

Introduction: The $|V_{cb}|$ CKM matrix element

Tensions in lepton universality

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current 4σ tension with the SM

Introduction: The weak decay $\bar{B} \to D^* \ell \bar{\nu}$

- Form factors

$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu)| \mathcal{V}^\mu \left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\, m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *} \varepsilon^{\mu\nu}_{\rho\sigma} v_B^\rho v_{D^*}^\sigma h_V(w)$$

$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu)| \mathcal{A}^\mu \left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\, m_{D^*}}} = \frac{i}{2}\epsilon^{\nu *} \left[g^{\mu\nu}\left(1+w\right) h_{A_1}(w) - v_B^\nu \left(v_B^\mu h_{A_2}(w) + v_{D^*}^\mu h_{A_3}(w)\right)\right]$$

- Playing with the polarization/momentum of the D^* we can calculate the different h_X form factors
- From the differential decay rate and the form factors (encoded in $\mathcal{F}(w)$) we can extract V_{cb}

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_B^5}{4\pi^3} r^3 (1-r^2)(w^2-1)^{\frac{1}{2}} \left|\eta_{EW}\right|^2 \left|V_{cb}\right|^2 \chi(w) \left|\mathcal{F}(w)\right|^2$$

Introduction: The weak decay $\bar{B} \to D^* \ell \bar{\nu}$

- Helicity amplitudes

$$H_\pm = \sqrt{m_B\, m_{D^*}}(w+1)\left(h_{A_1}(w) \mp \sqrt{\frac{w-1}{w+1}} h_V(w)\right)$$

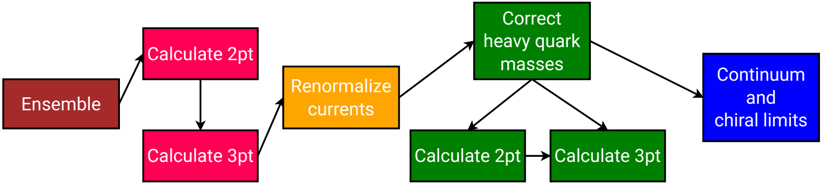
$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1)m_B\left[(w-r)h_{A_1}(w) + (w-1)\left(r\, h_{A_2}(w) + h_{A_3}(w)\right)\right]/\sqrt{q^2}$$

$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)h_{A_1}(w) + (wr-1)h_{A_2}(w) + (r-w)h_{A_3}(w)\right]$$

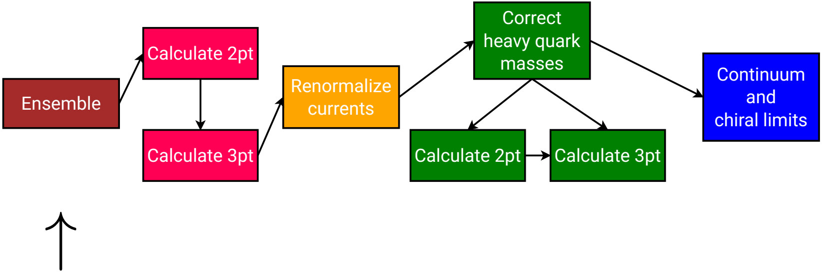
- Form factor in terms of the helicity amplitudes

$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12 m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

A lattice calculation



A lattice calculation



- Usually ensembles are generated and analyzed separatedly
- Takes large amounts of supercomputing time
- Need small runs at different values of the parameters to tune the quark masses
  - The tuning of the quark masses is a big project on its own right

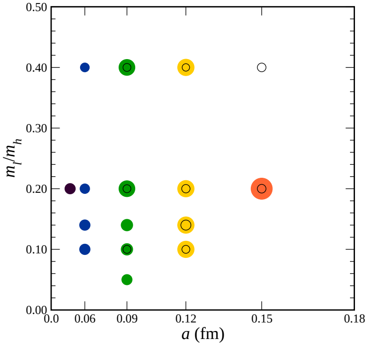
Available data and simulations

- Using 15 $N_f = 2+1$ MILC ensembles of sea asqtad (**a-sq**uared **tad**pole, an improved action) quarks

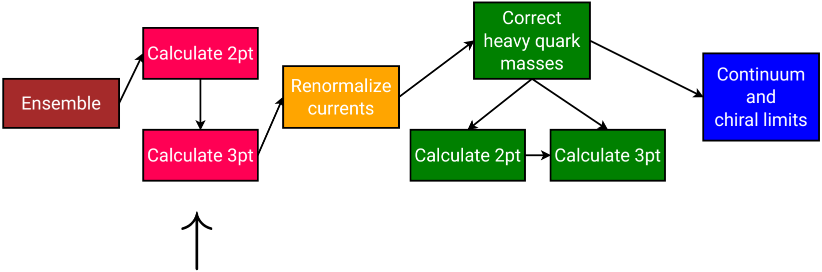
A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, Phys. Rev. D**55**, 3933 (1997)

- The heavy quarks are treated using the Fermilab action

A. Bazavov *et al.*, MILC/Fermilab col., Rev. Mod. Phys. **82** 1349 (2010)



A lattice calculation



- Design states on the lattice that represent the right particles
  - Deal with superposition of states and contamination
- Use actions that allows us to deal with heavy quarks $m_q \sim 1/a$

Dealing with excited states

- Ansatz for a $N+N$ two-point fit:

$$C_{2pt}(t) = \sum_{i=0,2,4\ldots}^{2N-1} \left[\underbrace{Z_i \left(e^{-E_i t} + e^{-E_i (T-t)} \right)}_{\text{Non-oscillating}} + \underbrace{(-1)^t Z_{i+1} \left(e^{-E_i t} + e^{-E_i (T-t)} \right)}_{\text{Oscillating}} \right]$$

- Z_i are the overlap factors, required for the three-point fits
- Different energy levels + smearing deals with contamination (max $N=3$)
- Distinguish between different orientations of p^μ with respect to ϵ^μ

- We fit ratios of 3pt:

$$\bar{R}(t,T) = R \left(1 + A e^{-\Delta E_X t} + B e^{-\Delta E_Y (T-t)} \right)$$

- The oscillating states are suppressed through a clever weighted average

$$\bar{R}(t,T) = \frac{1}{2} R(t,T) + \frac{1}{4} R(t,T+1) + \frac{1}{4} R(t+1,T+1)$$

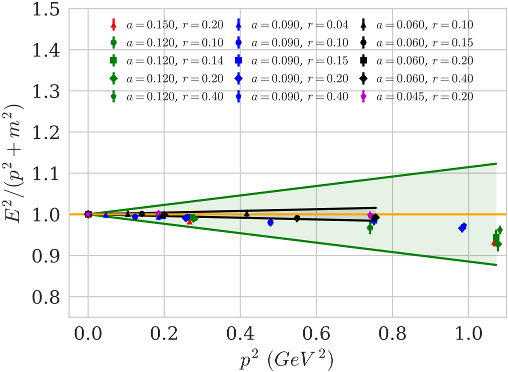
We use the same fit range in physical units for the same quantities across ensembles

Dealing with heavy quarks

- Either use a non-relativistic action or deal with discretization effects
- The Fermilab action uses tree-level matching, discretization errors $O(\alpha\, m)$

$$a^2E^2(p_\mu) = (am_1)^2 + \frac{m_1}{m_2}(\mathbf{p}a)^2 + \frac{1}{4}\left[\frac{1}{(am_2)^2} - \frac{am_1}{(am_4)^3}\right](a^2\mathbf{p}^2)^2 - \frac{am_1 w_4}{3}\sum_{i=1}^{3}(ap_i)^4 + O(p_i^6)$$

- As long as the discretization errors are under control, this is all right
- In the Fermilab action we interpret the kinetic mass am_2 as the particle mass



List of three-point function ratios

Calculated three-point functions

$$\frac{\langle D^*(p)|\,\mathbf{V}\,|D^*(0)\rangle}{\langle D^*(p)|\,V_4\,|D^*(0)\rangle} \to x_f, \qquad w = \frac{1+x_f^2}{1-x_f^2}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle \left\langle \bar{B}(0)\right|\mathbf{A}\left|D^*(p_\perp,\varepsilon_\parallel)\right\rangle^*}{\langle D^*(0)|\,V_4\,|D^*(0)\rangle \left\langle \bar{B}(0)\right| V_4 \left|\bar{B}(0)\right\rangle} \to R_{A_1}, \qquad h_{A_1} = \left(1-x_f^2\right) R_{A_1}^{\frac{1}{2}}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\perp)\right|\mathbf{V}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to X_V, \qquad h_V = \frac{2}{\sqrt{w^2-1}} R_{A_1} X_V$$

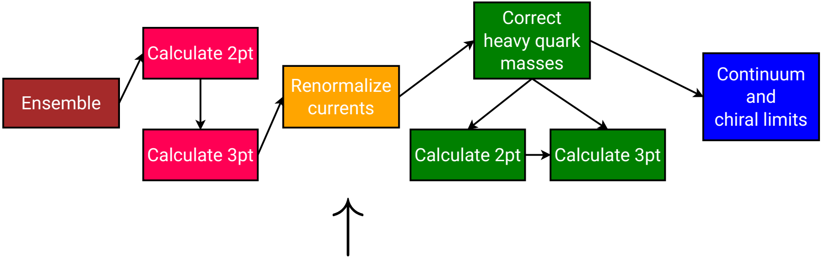
$$\frac{\left\langle D^*(p_\parallel,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_1, \qquad h_{A_3} = \frac{2}{w^2-1} R_{A_1}\left(w - R_1\right)$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right| A_4 \left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_0,$$

$$h_{A_2} = \frac{2}{w^2-1} R_{A_1}\left(wR_1 - \sqrt{w^2-1}R_0 - 1\right)$$

* Phys.Rev. **D**66, 01503 (2002)

A lattice calculation

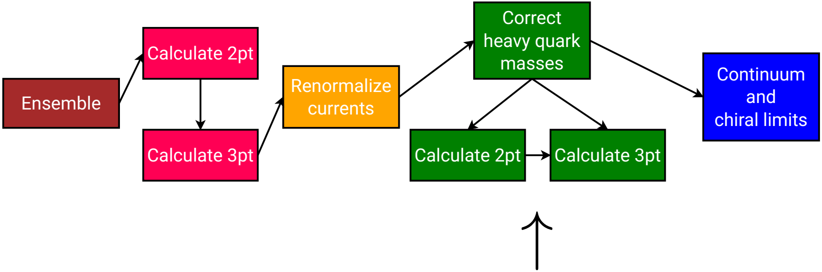


- The lattice breaks many symmetries that are restored in the continuum $\Longrightarrow$ currents require renormalization
- The calculation of renormalization factors can be a computationally expensive project on its own
- Our ratios in the three-point cancel most of the renormalization factors

$$\rho^2_{A_i} = \frac{Z_{A^i_{cb}} Z_{A^i_{bc}}}{Z_{V^4_{cc}} Z_{V^4_{bb}}}$$

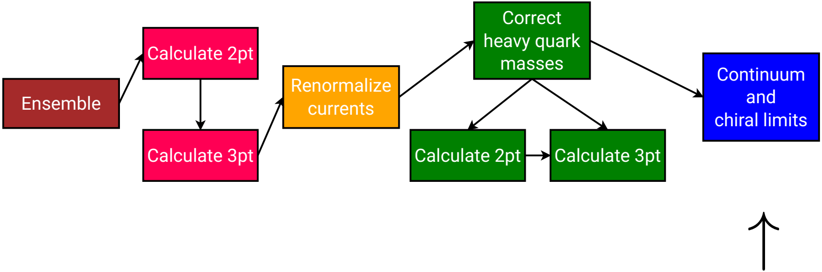
- The ρ's are **BLINDED** and computed perturbatively at one-loop for $w = 1$
- Dependence on w and further corrections are added as estimated errors (small, $\rho \sim 1$)

A lattice calculation



- The initial tuning of the heavy quark masses is corrected when large ensembles are available
- A few extra ensembles are generated at slightly different values of the heavy quark masses to compute a correction
- This correction is computed at a particular a and m_π, and it is applied to all the ensembles
- The corrections are small, usually below the statistical error

Why you shouldn't use our lattice data before it's ready



- Our data is computed at a finite a and non-physical m_π
- Heavy quark discretization errors, although under control, can be large
- We rely on effective theories that describe the behavior of our form factors with a, m_π

The chiral-continuum limit

- Functional form in χPT up to NNLO explicitly known

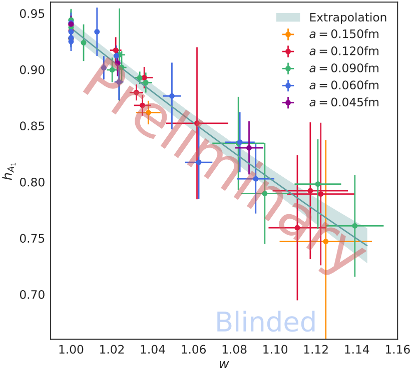
$$h_{A_1}(w) = \underbrace{1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g^2_{D^*-D\pi}}{48\pi^2 f_\pi^2 r_1^2}\text{logs}_{\text{SU3}}(a, m_l, m_s, \Lambda_{QCD})}_{\text{NLO }\chi\text{PT} + \text{HQET}} -$$

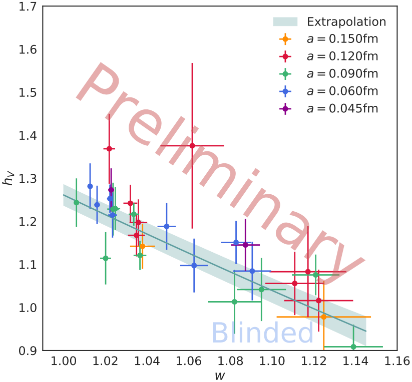
$$\underbrace{\rho^2(w-1) + k(w-1)^2}_{w\text{ dependence}} \underbrace{+c_1 x_l + c_2 x_l^2 + c_{a1} x_{a^2} + c_{a2} x_{a^2}^2 + c_{a,m} x_l x_{a^2}}_{\text{NNLO }\chi\text{PT}}$$

with

$$x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \qquad x_{a^2} = \left(\frac{a}{4\pi f_\pi r_1^2}\right)^2$$

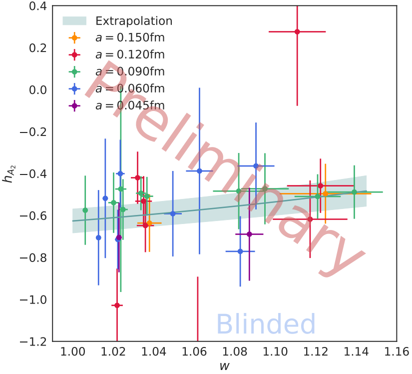
The chiral-continuum limit

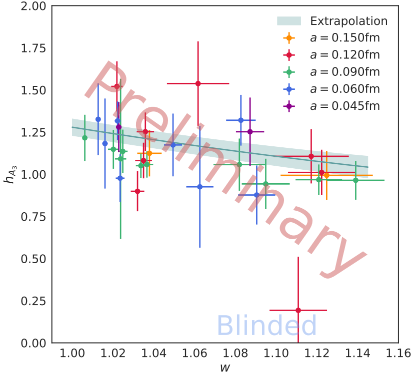




- Preliminary blinded results

The chiral-continuum limit





- Preliminary blinded results

z-Expansion

- Conformal transformation

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

- Kinematic range $w_{\text{Min}} = 1 \to z_{\text{Min}} = 0$, $w_{\text{Max}} = \frac{1+r^2}{2r} \to z_{\text{Max}} = \left(\frac{\sqrt{r}-1}{\sqrt{r}+1}\right)^2$
- Use BGL expansion (less constrained than CLN)

$$f_X(z) = \frac{1}{\phi_{f_X} B_{f_X}} \sum_j k_j z^j$$

- B_{f_X} Blaschke factors, includes contributions from the poles in the kinematic range
- ϕ_{f_X} is called *outer function* and must be computed for each form factor

z-Expansion

- The expansion is performed on different (more convenient) form factors

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w) h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- BGL unitarity constraints

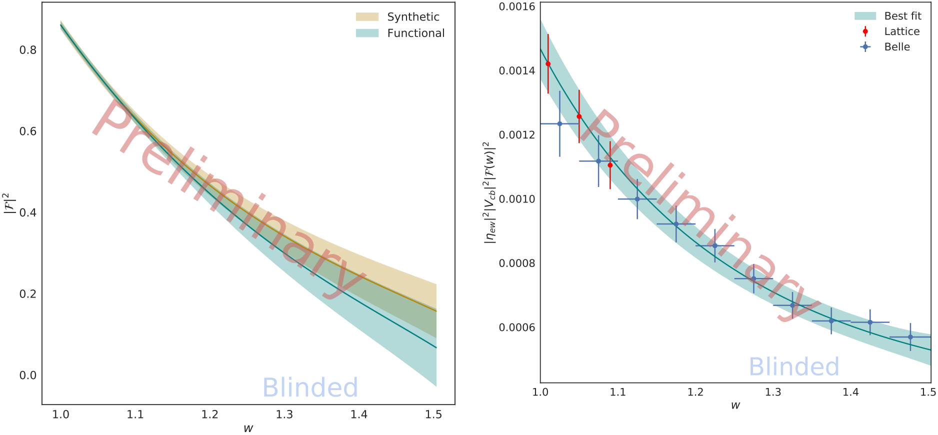
$$\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1$$

Phys.Lett. **B**769, 441 (2017)

Phys.Lett. **B**771, 359 (2017)

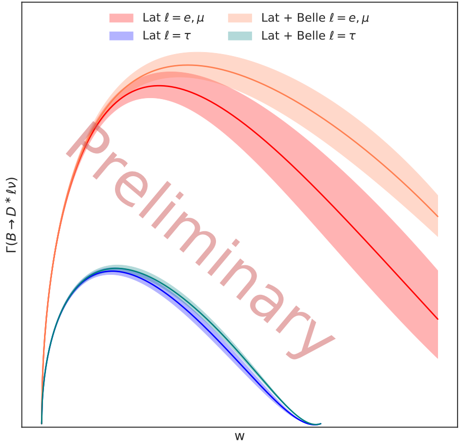
Lattice result and joint fit

- For the preliminary joint fit we use Belle data + synthetic data for the form factor at three recoil values



A word on $R(D^*)$

- Pure lattice QCD prediction of $R(D^*)$
- Probably underestimating errors (prepared last week)
- Lattice very reliable up to the point we have lattice data
- Sensitive to the slope in $\mathcal{F}$



What to expect

- Errors might not be improved compared to previous lattice estimations
- The final result might help reduce the tension inclusive/exclusive
  - The main new information of this analysis won't come from the zero-recoil value, but from the slope
- Main source of errors of our form factor seems to be discretization errors (to be confirmed in error budget)
- Main source of uncertainty in a preliminary V_{cb} joint fit currently comes from experimental data
  - Might be able to improve final result using more experimental datasets
- $R(D^*)$ is work in progress and we haven't computed it using all form factors coming from joint fits with experimental and lattice data

The future

- Current analysis is on track to be finished before the end of the year
  - Final revision before freezing the results (crosscheck)
  - Error budget
  - Unblinding
  - $R(D^*)$ might come a bit later (constrain the form factor, use experimental data, etc)

- Well established roadmap to reduce errors in our calculation
  - Light HISQ quarks + heavy Fermilab quarks aim to reduce mainly chiral fit errors
  - Light HISQ quarks + heavy HISQ quarks aim to reduce discretization and renormalization errors
  - New runs measure other interesting quantities (i.e. the tensor form factor)
- This roadmap is to be followed in other processes involving other CKM matrix elements, and has been in decay constants: see Komijani talk, Th 15:30