Leptonic Decays of B and D Mesons from Lattice QCD

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[Fermilab Lattice and MILC]

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Motivation

- Precise values of decay constants of D and B systems are needed for
  - precise determination of $|V_{cs}|$ and $|V_{cd}|$
  - accurate Standard Model predictions for rare decays such as $B_s \to \mu^+\mu^-$ and also $B^+ \to \tau^+\nu_\tau$
  - resolve/sharpen tension between inclusive and exclusive $|V_{ub}|$

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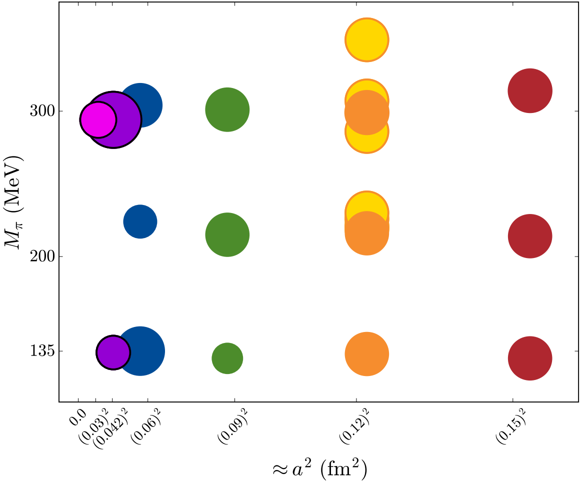
  - resolve/sharpen tension between inclusive and exclusive $|V_{ub}|$
- Challenges for Lattice-QCD simulation of heavy quarks
  - very small lattice spacings are needed to simulate the b quark ($am_b \ll 1$)
  - at small lattice spacings we reach to a regime where the distribution of the topological charge is not properly sampled

How we achieve high precision

- Gluon gauge-field configurations generated with the highly-improved staggered quark (HISQ) formulation for sea quarks
- HISQ formulation for all valence quarks
  Advantage: avoiding renormalization for partially conserved currents
- Calculation for several heavy quark masses ranging from m_c to m_b (following HPQCD)
- A merger of effective theories to carry out extrapolations to the physical point:
  - Heavy-quark effective theory (HQET) to treat the heavy-quark mass dependence
  - Chiral perturbation theory (ChPT) to treat the light-quark mass dependence
  - Symanzik effective theory (SET) to treat discretization errors
- We use ChPT to correct for incomplete sampling of topological charge
- High statistics: 24 gauge-field ensembles with approximate lattice spacing ranging from 0.03 to 0.15 fm, several values of the light quark masses, including physical values.

MILC HISQ ensembles with (2+1+1)-flavors of quarks



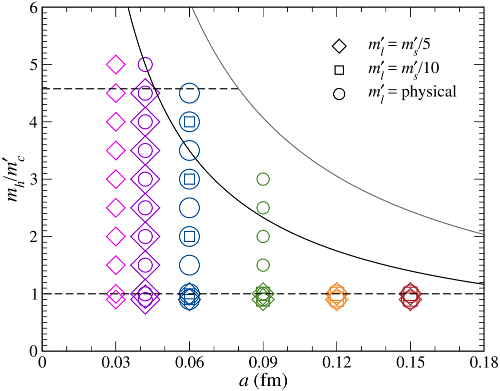
- 2+1+1 sea quarks
- 24 ensembles
- 5 physical m_π
- $m_\pi L > 3.2$
- down to 0.03 fm

MILC HISQ ensembles with (2+1+1)-flavors of quarks

- Ensembles with physical mass for the strange quark $(m'_s \approx m_s)$:

$\approx a$ (fm)	m'_l/m'_s	size	L (fm)	$M_\pi L$	M_π (MeV)
0.15	1/5	$16^3 \times 48$	2.38	3.8	314
0.15	1/10	$24^3 \times 48$	3.67	4.0	214
0.15	1/27	$32^3 \times 48$	4.83	3.2	130
0.12	1/5	$24^3 \times 64$	3.00	4.5	299
0.12	1/10	$24^3 \times 64$	2.89	3.2	221
0.12	1/10	$32^3 \times 64$	3.93	4.3	216
0.12	1/10	$40^3 \times 64$	4.95	5.4	214
0.12	1/27	$48^3 \times 64$	5.82	3.9	133
0.09	1/5	$32^3 \times 96$	2.95	4.5	301
0.09	1/10	$48^3 \times 96$	4.33	4.7	215
0.09	1/27	$64^3 \times 96$	5.62	3.7	130
0.06	1/5	$48^3 \times 144$	2.94	4.5	304
0.06	1/10	$64^3 \times 144$	3.79	4.3	224
0.06	1/27	$96^3 \times 192$	5.44	3.7	135
0.042	1/5	$64^3 \times 192$	2.91	4.34	294
0.042	1/27	$144^3 \times 288$	6.12	4.17	134
0.03	1/5	$96^3 \times 288$	3.25	4.84	294

Heavy-light pseudoscalar mesons



- We calculate decay constants for various light and heavy masses
  - light valence: $m_{u/d} \lesssim m_{\mathrm{x}} \lesssim m_s$
  - heavy valence: $m_c \lesssim m_h \lesssim m_b$
- omit $am_h > 0.9$ from fit
- omit 0.15 fm in base fit

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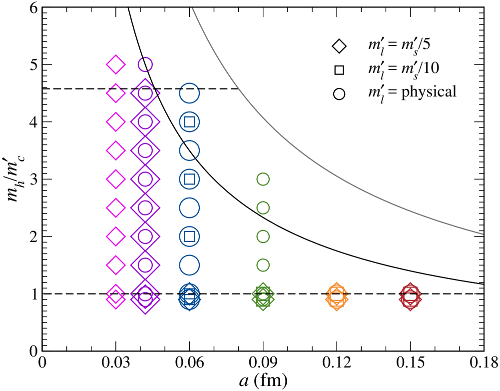

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We employ HMrPQASχPT to perform a combined correlated, multidimensional fit to ~ 500 data
$\Rightarrow$ reduces statistical errors
$\Rightarrow$ controls systematic errors of extrapolations

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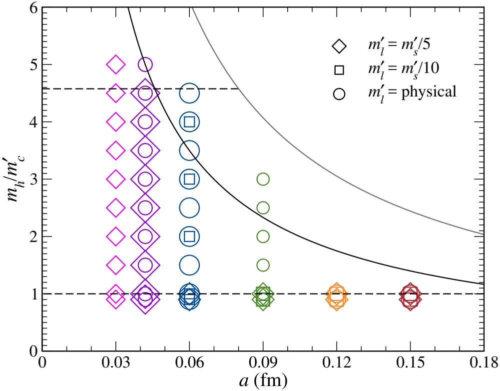

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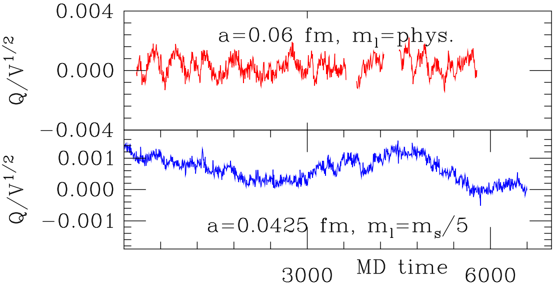
We first need to correct for incomplete sampling of topological charge at small lattice spacings

Incomplete sampling of topological charge

- Evolution of topological charge Q gets slower as lattice spacing $a \to 0$



The lower panel, for $a \approx 0.042$ fm, the time history is not well sampled

- Incomplete sampling of Q could lead to incorrect results/errors for physical quantities

$\Rightarrow$ We correct for this error and also account for it in our error budgets
[Bernard & Toussaint PRD 97, 074502 (2018), arXiv:1707.05430]

How we correct the incomplete sampling of Q

- Z at fixed QCD vacuum angle θ and topological susceptibility χ_t

$$\begin{aligned} Z(\theta) &\equiv \int \mathcal{O}A\,\mathcal{O}\bar{\psi}\,\mathcal{O}\psi\, e^{-S[A,\bar{\psi},\psi]}\, e^{-i\theta Q[A]} \\ \chi_t &\equiv -\frac{1}{V}\left(\frac{1}{Z}\frac{\partial^2 Z}{\partial\theta^2}\right)\bigg|_{\theta=0} = \frac{1}{V}\langle Q^2\rangle \end{aligned}$$

- Fourier transform on θ gets quantities at fixed Q

$$\begin{aligned} \tilde{Z}_Q &= \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta\, e^{i\theta Q}\, Z(\theta) \\ \tilde{G}_Q &= \langle \mathcal{O}_1\mathcal{O}_2\cdots\mathcal{O}_n\rangle_Q = \frac{1}{\tilde{Z}_Q}\frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta\, e^{i\theta Q}\, Z(\theta)G(\theta) \end{aligned}$$

with $G(\theta) = \langle \mathcal{O}_1\mathcal{O}_2\cdots\mathcal{O}_n\rangle_\theta$

Leutwyler & Smilga 1992, Brower et al. 2003, Aoki et al. 2007, Dromard et al. 2015, and Bernard & Toussaint PRD 97, 074502 (2018), arXiv:1707.05430

How we correct the incomplete sampling of Q

- For large 4-dim volume V, we can do θ integrals by saddle point method

$$\tilde{G}_Q = G(\theta_s) + \frac{1}{2\langle Q^2\rangle} \left.\frac{\partial^2 G}{\partial \theta^2}\right|_{\theta=\theta_s} + \cdots \qquad \text{with } \theta_s = i\frac{Q}{\langle Q^2\rangle}$$

- This gives

$$\tilde{M}_Q = M\Big|_{\theta=0} + \frac{1}{2\langle Q^2\rangle} M'' \left(1 - \frac{Q^2}{\langle Q^2\rangle}\right) + \cdots$$

$$\tilde{f}_Q = f\Big|_{\theta=0} + \frac{1}{2\langle Q^2\rangle} f'' \left(1 - \frac{Q^2}{\langle Q^2\rangle}\right) + \cdots$$

- $M'' = \frac{\partial^2 M}{\partial \theta^2}|_{\theta=0}$ and $f'' = \frac{\partial^2 f}{\partial \theta^2}|_{\theta=0}$ are physical quantities

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- $M'' = \frac{\partial^2 M}{\partial\theta^2}|_{\theta=0}$ and $f'' = \frac{\partial^2 f}{\partial\theta^2}|_{\theta=0}$ are physical quantities
  - $\Rightarrow$ can get a theoretical handle on topological effects by calculating them in continuum, infinite volume, ChPT
- These methods allow us to estimate errors in M and f due to problems in our sampling of the topological-charge distribution, or even make corrections for poor sampling

Leutwyler & Smilga 1992, Brower et al. 2003, Aoki et al. 2007, Dromard et al. 2015,

and Bernard & Toussaint PRD 97, 074502 (2018), arXiv:1707.05430

How we correct the incomplete sampling of Q

- Adjust the raw results to account (in first approximation) for the incomplete sampling of Q in the small-a ensembles
  [Bernard & Toussaint PRD 97, 074502 (2018), arXiv:1707.05430]

	$m_l/m_s = 1/5$	m_l = physical
$\frac{\langle Q^2 \rangle_{\text{ens}}}{\langle Q^2 \rangle_{\text{ChPT}}}$	1.3	0.65
f_k/f_π	1.20508(0.00250)[-0.01271]	1.19680(0.00114)[0.00015]
aM_π	0.031147(0.000172)[-0.000707]	0.028964(0.000020)[0.000008]
af_D	0.048858(0.000261)[-0.000552]	0.045389(0.000245)[0.000006]
aM_D	0.409786(0.000391)[-0.000044]	0.400678(0.000258)[0.000001]
af_{D_s}	0.054828(0.000068)[-0.000001]	0.053582(0.000025)[0.000000]
aM_{D_s}	0.430966(0.000116)[-0.000004]	0.430966(0.000116)[-0.000004]

- Systematic error budget includes estimate of residual effects

Fit function for HQET-chiral-continuum extrapolation

- We use a cascade of EFTs to construct our fit functions
- We start from the following schematic form for decay constants of H_v mesons

$$\Phi_{H_x} \equiv f_{H_x}\sqrt{M_{H_x}} = C\ (1+\mathsf{SET})\ (1+\mathsf{HQET})\ (1+\mathsf{HMrAS}\chi\mathsf{PT})\left(\frac{m_c'}{m_c}\right)^{3/27}\tilde{\Phi}_0$$

- These terms correspond to different effective field theories

Symanzik Effective Theory (SET)

$$c_1\alpha_s\,(a\Lambda)^2+\cdots+c_3\alpha_s\,(am_h)^2+\cdots$$

Wilson coefficient C

$$\left[\alpha_s(M_{H_s})\right]^{-6/25}\Big(1+\mathsf{O}(\alpha_s)\Big)$$

HQET

$$k_1\left(\frac{\Lambda_{\mathsf{HQET}}}{M_{H_s}}\right)+k_2\left(\frac{\Lambda_{\mathsf{HQET}}}{M_{H_s}}\right)^2+\cdots$$

Effect of the sea charm quark

$$\frac{\Lambda^{(3)}_{\mathsf{QCD}}(m_c')}{\Lambda^{(3)}_{\mathsf{QCD}}(m_c)}\approx\left(\frac{m_c'}{m_c}\right)^{2/27}$$

HMrPQASχPT at NLO

chiral non-analytic terms
$+L_x m_x + L_\mathsf{s}(2m_l'+m_s') + L_a a^2$

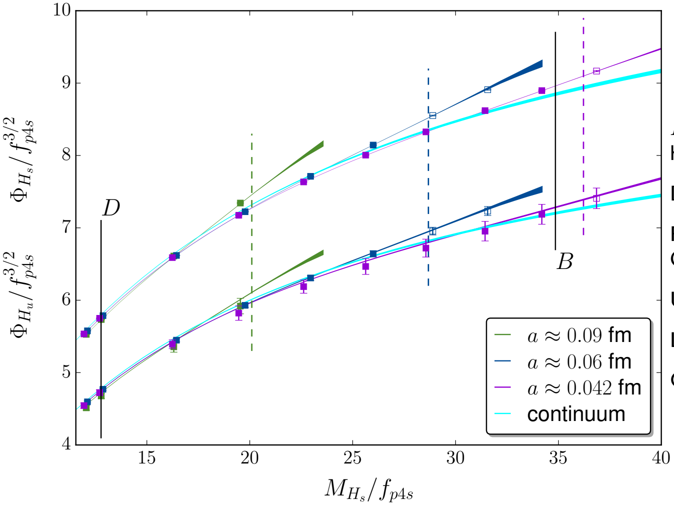
Chiral terms contain effects of

- taste splittings in “pion” masses & new logs.
- hyperfine and flavor splittings
- finite lattice volume

Fit function for HQET-chiral-continuum extrapolation

- For higher order effects in ChPT, HQET and SET we include analytic terms as polynomials in dimensionless, "natural" expansion parameters:
  - Light-quark and gluon discretization effects: $(a\Lambda)^2$ with $\Lambda = 600$ MeV
  - Heavy-quark discretization effects: $(2am_h/\pi)^2$
  - Light quark mass effects: $B_0 m_q/(4\pi^2 f_\pi^2)$
  - Heavy quark mass effects: $\Lambda_{\mathsf{HQET}}/M_{H_s}$ with $\Lambda_{\mathsf{HQET}} = 800$ MeV
- The coefficients of the polynomials are fit parameters; expected to be of size 1; to be conservative we use the prior value of 0 ± 1.5
- Altogether we have 60 fit parameters and 492 data points in our base fit

A snapshot of the fit and data



M_{Hs} as proxy of heavy-quark mass m_h

Dashed lines: $am_h = 0.9$

Filled Symbols: included
Open Symbols: excluded

Upper curves: $m_x = m_s$

Lower curves: $m_x = m_u$

Cyan bands: extrap. to cont.

492 lattice data points; 60 parameters; χ^2/d.o.f $= 466/432$; $p = 0.12$

Results for decay constants

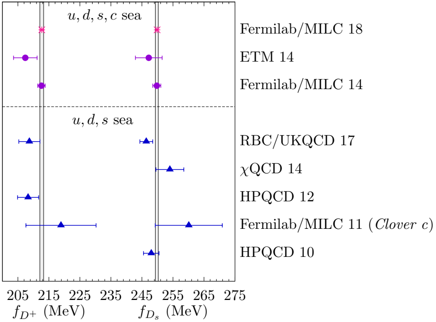
Results for decay constants [see arXiv:1712.09262 and wait for the update]

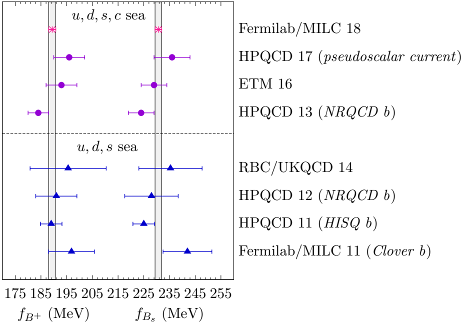
$$f_{D^0} = 211.6(0.3)_{\text{stat}}(0.5)_{\text{syst}}(0.2)_{f_\pi,\text{PDG}}[0.2]_{\text{EM scheme}}\ \text{MeV}$$
$$f_{D^+} = 212.7(0.3)_{\text{stat}}(0.4)_{\text{syst}}(0.2)_{f_\pi,\text{PDG}}[0.2]_{\text{EM scheme}}\ \text{MeV}$$
$$f_{D_s} = 249.9(0.3)_{\text{stat}}(0.2)_{\text{syst}}(0.2)_{f_\pi,\text{PDG}}[0.2]_{\text{EM scheme}}\ \text{MeV}$$
$$f_{B^+} = 189.4(0.8)_{\text{stat}}(1.1)_{\text{syst}}(0.3)_{f_\pi,\text{PDG}}[0.1]_{\text{EM scheme}}\ \text{MeV}$$
$$f_{B^0} = 190.5(0.8)_{\text{stat}}(1.0)_{\text{syst}}(0.3)_{f_\pi,\text{PDG}}[0.1]_{\text{EM scheme}}\ \text{MeV}$$
$$f_{B_s} = 230.7(0.8)_{\text{stat}}(1.0)_{\text{syst}}(0.2)_{f_\pi,\text{PDG}}[0.2]_{\text{EM scheme}}\ \text{MeV}$$

The systematic error includes

- systematic errors in calculation of scale setting quantities and tuned quark masses
- continuum extrapolation
- finite volume
- uncertainty in adjustment for incomplete sampling of topological charge
- contamination from higher order states in 2-point correlators
- EM contribution to meson masses that are used to fix the quark masses (Decay constants are pure-QCD quantities; EM contributions to the relation between decay constants and physical decay rates are not included here by definition but would be relevant for phenomenology)

Comparison with previous 3 and 4 flavor calculations





The magenta points and gray bands show our results

Unitarity of CKM matrix

⇒ Using our D^+ and D_s meson decay constants and the PDG values for products of decay constants and CKM factors we obtain

$$|V_{cd}|_{\mathsf{SM},\, f_D} = 0.2151(6)_{f_D}(49)_{\rm expt}(6)_{\rm EM}$$
$$|V_{cs}|_{\mathsf{SM},\, f_{D_s}} = 1.000(2)_{f_{D_s}}(16)_{\rm expt}(3)_{\rm EM}$$

- "EM" denotes the error due to unknown structure-dependent electromagnetic corrections
- The uncertainties from the decay constants are an order of magnitude smaller than those from experiment

⇒ Test of the unitarity of the second row of the CKM matrix

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1.0 = 0.049(2)_{|V_{cd}|}(32)_{|V_{cs}|}(0)_{|V_{cb}|}$$

(with $|V_{cb}| = 41.40(77) \times 10^{-3}$ from a weighted average of inclusive and exclusive semileptonic B decays)

Rare leptonic decays $B_q \to \mu^+\mu^-$

$\Rightarrow$ Using our B^0 and B_s meson decay constants, we obtain

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\mathsf{SM}} = 3.64(4)_{f_{B_s}}(8)_{\mathsf{CKM}}(7)_{\mathsf{other}} \times 10^{-9}$$
$$\overline{\mathcal{B}}(B^0 \to \mu^+\mu^-)_{\mathsf{SM}} = 1.00(1)_{f_{B^0}}(2)_{\mathsf{CKM}}(2)_{\mathsf{other}} \times 10^{-11}$$

with the largest errors from the CKM elements $|V_{ts}|$ and $|V_{td}|$

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with the largest errors from the CKM elements $|V_{ts}|$ and $|V_{td}|$

- The recent experimental measurements are

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\mathsf{ATLAS}} = 0.9(^{+1.1}_{-0.8}) \times 10^{-9}$$ [arXiv:1604.04263]

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\mathsf{LHCb}} = 3.0(0.6)(^{+0.3}_{-0.2}) \times 10^{-9}$$ [arXiv:1703.05747]

and

$$\overline{\mathcal{B}}(B^0 \to \mu^+\mu^-)_{\mathsf{ATLAS}} < 3.4 \times 10^{-10}$$ [arXiv:1604.04263]

$$\overline{\mathcal{B}}(B^0 \to \mu^+\mu^-)_{\mathsf{LHCb}} < 4.2 \times 10^{-10}$$ [arXiv:1703.05747]

$|V_{ub}|$ and $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$

- Combing our f_{B^+} with the expt. average for $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$ yields

$$|V_{ub}| = 4.07(3)_{f_{B^+}}(37)_{\text{expt}} \times 10^{-3}$$

with ∼10% uncertainty (mainly from the error on the measured decay width)

⇒ in agreement with $|V_{ub}|$ from both inclusive and exclusive semileptonic decays

- Combing our f_{B^+} and $|V_{ub}|$ from our calculation of the $B \to \pi\ell\nu$ form factor [arXiv:1503.07839], we obtain for the SM branching ratio

$$\mathcal{B}(B^+ \to \tau^+\nu_\tau) = 0.876(13)_{f_{B^+}}(75)_{V_{ub}}(2)_{\text{other}} \times 10^{-4},$$

⇒ in agreement with the experimental average $\mathcal{B}(B^+ \to \tau^+\nu_\tau) = 1.06(20) \times 10^{-4}$ [see arXiv:1509.02220]

Conclusion

- We presented the most precise lattice-QCD calculations to-date of the leptonic decay constants of heavy-light pseudoscalar mesons with charm and bottom quarks
- The determinations of $|V_{cd}|$ and $|V_{cs}|$ from leptonic D decays enabled us to test the unitarity of the second row of the CKM matrix at the few-percent level
  $\Rightarrow$ compatible with three-generation CKM unitarity within 1.5σ
- The theoretical uncertainty in our value for $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$ is more than ten times smaller than recent experimental measurements
- Our prediction for $\overline{\mathcal{B}}(B^0 \to \mu^+\mu^-)$ is below present experimental limits
- Our value for $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$ is in agreement with the experimental average
- In all quantities the uncertainty from decay constants of D and B systems is less than other significant sources of uncertainty (by a factor of 2 or so)

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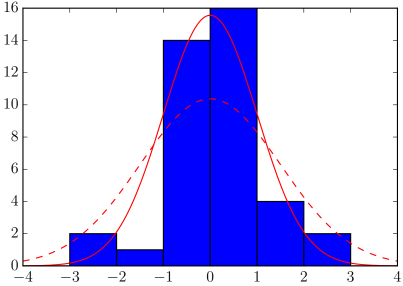
Thanks for your attention!

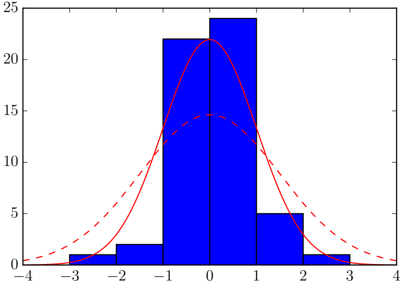
back-up slides

Scale setting and calculating tuned quark masses

- Scale setting for chiral analysis is done using f_{p4s}
  (the decay constant of a fiducial pseudoscalar meson with both valence masses equal to $m_{p4s} \equiv 0.4 m_s$)
- The physical value of f_{p4s} is set from f_π
- This method yields a precise determination of both the lattice spacing a and the quark mass am_{p4s} (and in turn $m_s = 2.5 m_{p4s}$)
- The values of f_{p4s} and quark mass ratio m_s/m_l are determined by analyzing light-light data from the same ensembles
  $\Rightarrow$ Various systematic errors (such as FV, EM, continuum extrapolation *etc*) in estimate of f_{p4s} and tuned quark masses must be incorporated to our estimate of uncertainties

Distribution of fit posteriors

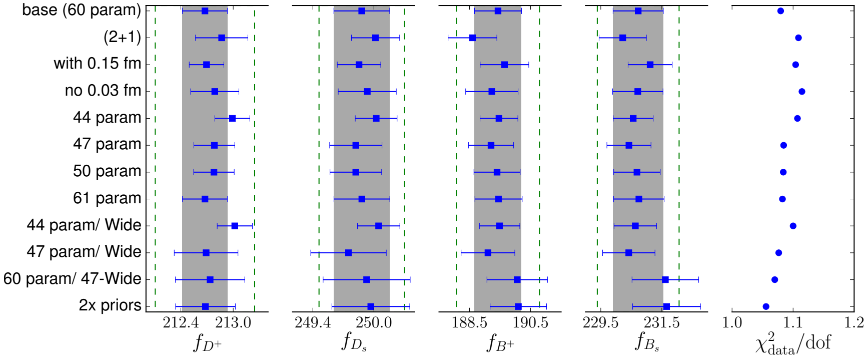




Left: distribution of fit posteriors in a fit with 44 parameters and essentially no prior constraints

Right: distribution of fit posteriors in the base fit for parameters constrained with priors 0 ± 1.5

Stability



Stability plot showing the sensitivity to different choices of lattice data and fit models. The error bars show only the statistical errors.