Syllabus and Conventions for General Courses organized by the CERN Accelerator School

Compiled by CAS Team

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Abstract

In this document we provide a combined syllabus for the general CAS courses ”Introduction to Accelerator Physics” and ”Advanced Accelerator Physics Course”. It is meant to give guidelines to the lecturers and to give an overview of the topics to the participants.
It is based on input from CAS committees, lectures and participants from previous schools.

BASIC CONSIDERATIONS

This syllabus was compiled based on a list of topics presented earlier (”Super Syllabus”) as well as the programme of previous schools for the General CAS Courses (Introductory and Advanced Course). It follows the advice and suggestions of CAS committee members, CAS lecturers and numerous comments from participants. It is an attempt to take into account the advancement and evolution of this field during the last 30 years. Furthermore to react to the demand for training of accelerator physicists.
It was discussed and endorsed at the CAS Advisory Committee and the combined Programme Committee.

- It is considered most important for beam dynamics (needed in all systems and must consistently follow a ”Red Thread”, if possible use same conventions)

  Mostly used and proposed: n-dimensional coordinate space (e.g. 2D) and 2n-dimensional phase space (but not 4D), ”D” only used for coordinate space

  For transverse coordinates: use x, y, **not** x, z to stay with right-handed coordinate system, otherwise mention explicitly

- 2D coordinate space (3D if required) should be used from the beginning and throughout

- The Introductory Course must lay down the foundations for the ”Advanced Course” which should build on these foundations

- Make sure we lecture at state of the art. Where appropriate, lectures should go beyond standard (i.e. not specialised) textbooks (since we have experts in various fields), in particular at the Advanced Course.

- Move away from ”synchrotron legacy”, emerged from first CAS on ”ppbar colliders” (very frequent request coming from participants)

  This was also expressed in the context of tutorials and afternoon courses

- Proper treatment of non-relativistic machines

- Always use SI units instead of cgs, otherwise specify explicitly and give reason

- Provide list of relevant bibliography

Introductory course

Split into: a) background material (to provide the foundation for following lectures, also to avoid unnecessary repetitions), b) introduction/overview to mainstream types of accelerators (linacs, cyclotrons and synchrotrons), c) particular problems, basic and advanced beam dynamics.

1 INTRODUCTION TO ACCELERATORS

Objective:

A brief overview of different types of accelerators and the transport of particles. Includes a short history of accelerators.

1.1 Types of accelerators and transport (very brief)

- **Beam lines**
- **Linear accelerators**
- **Circular accelerators**

1.2 A few basic conventions

- **Variables**
- **Basic parameters**
- **Units**

1.3 Some basics

- **Curved reference trajectory**
- **Magnetic rigidity**
- **Types of magnets: bending and quadrupole magnets**
- **Leptons versus Hadrons**
- **Basics of colliders (Luminosity, energy, ..)**

2 ELECTRO-MAGNETIC THEORY

Objective:

A short review of classical electrodynamics, derivations only where essential. Should be done considering the lectures on "Particle Motion in EM fields" and "Relativity and Kinematics of Particle Beams".

2.1 EM fields

2.2 Units

2.3 Maxwell's equations

- **Gauss, Ampere, Faraday: leading to Maxwell's equations**
- **Integral and differential forms**
- **Electro-magnetic potentials**
- **Boundary conditions: conductor, media with $\epsilon_r, \quad \mu_r \quad \neq \quad 1$**
- **Multipole fields in 2D**
- **Lorentz force**

2.4 EM waves

- **Wave guides and cavities**
- **EM-waves, solution and modes, (TE, TM, TEM)**
- **Propagation of waves, Poynting vector and energy**
- **Skin-depth and losses, cut-off frequency**
- **Group and phase velocity**

3 PARTICLE MOTION IN EM FIELDS

Objective:

Arrive at a description of particle motion in electromagnetic fields. Given that simulation and tracking codes play the central role in accelerator design and operation, the concepts should be formulated as an approach based on s-maps. Should allow the extension to non-linear maps in advanced courses. This approach allowing exact tracking, easy analysis and a map-based perturbation theory puts it above any other methods and should be introduced here as a concept. The physics for any type of magnets is contained in a map representing the element. For linear elements it can be done the classical way, a la K. Brown "Single Element Optics" in Handbook of accelerator Physics. For any (linear and non-linear elements) introduce a general view how they are obtained, glimpse at the concepts, but details left to advanced course. The maps for common machine elements should be given as input for the following lectures of the course.

3.1 Map approach to particle motion - general concept

- **Variables for beam dynamics**
- **Map definition and conventions**
- **Maps for circular machines**
- **Linear maps**
- **Non-linear maps**

3.2 Linear maps - matrix formulation 2D and 3D

- **Reference system**
- **Maps for linear elements from basic equation of motion**
- **Matrix description of thick linear elements, including skewed magnets**
- **Thin lens approximation**
- **Matrix description in 3D**

3.3 Non-linear maps

- General approach to compute linear and non-linear maps for machine elements

  Hamiltonian description of EM-fields, sketch of principle

- Outlook to advanced school, reference to ”Non-linear Dynamics” this school

3.4 Examples: main types of magnets

- Drift space
- Bending magnets
- Quadrupoles
- Sextupoles
- Solenoids

4 RELATIVITY AND KINEMATICS OF PARTICLE BEAMS

Objective:

Introduce special relativity, main emphasis on applications in beam dynamics. Provide the mathematical tools.
A general concept into systems of particles and description of the properties of the system. Should provide the framework for consistent use in following lectures (mainly to avoid duplicated and/or inconsistent definitions and conventions).

4.1 Special Relativity

- Principles
- Inertial and non-inertial systems
- Lorentz transformation
- Time dilation and Lorentz contraction
- Four-vectors and applications in accelerators
- Fields of moving charges
- Maxwell’s equation in invariant form (2D)

4.2 Systems of particles

- Phase space versus trace space and configuration space, degrees of freedom
- Invariants
- Poincare Integral Invariants
- Liouville theorem
- Emittance, as valid for any accelerator, including sources
- Vlasov and Boltzmann equations

5 LINEAR ACCELERATORS

Objective:

Overview of the basic principles of Linear Accelerators and the required components and structures. An introduction to the beam dynamics should cover the main aspects.

5.1 Acceleration principles

- EM forces
- Accelerating structures
- RF frequencies

5.2 Beam dynamics

- Bunched and unbunched beams
- Beam dynamics in linear structures
- Phase stability and acceptance
- Equations of motion
- Relativistic effects

6 TRANSVERSE LINEAR BEAM DYNAMICS FOR SYNCHROTRONS AND BEAM LINES, ALL IN 2D

Objective:

Starting from the linear transfer maps (i.e. matrices) describe the first-order dynamics in beam lines and synchrotrons. Define as a sequence of linear machine components and derive the basic quantities, global quantities (e.g. tune, chromaticity) and local (Twiss parameters, dispersion). Assume no coupling between the planes, but treat the problem in 2D.

6.1 Linear motion in one pass (linear) structures

- **Choice of independent variable, limitations of applicability**
- **Transformation of coordinates in two dimensional phase space**
- **Optical parameters (2D)**

6.2 Linear motion in multi pass (circular) structures

- **Weak and strong focusing, FODO cells**
- **One-turn matrix**
- **Closed orbit, reference orbit, physical meaning**
- **Beam stability condition**
- **Optical parameters (2D), tune optical functions, etc.**
- **Action-Angle variables (to keep it simple: can be done in 1D)**
- **Periodic focusing systems, Floquet theorem and Hill's equation**
- **Linear Courant-Snyder invariants**
- **Beam dynamics with acceleration**

6.3 Dynamics with off momentum particles

- **Dispersion in rings**
- **Momentum compaction**
- **Transition energy**
- **Chromaticity: origin, advantages and disadvantages (correction later)**

7 LONGITUDINAL BEAM DYNAMICS IN CIRCULAR ACCELERATORS

Objective:

Introduce the acceleration principles, strong emphasis on beam dynamics and required structures.

7.1 Acceleration principles

- **Electrostatic and time varying fields**
- **Phase stability and energy gain**
- **Transit time factor**

7.2 Accelerating structures

- **Basic overview**
- **Reference to later lectures on RF systems**

7.3 Synchrotron oscillations

- **Equation of motion**
- **Stationary and accelerating buckets, fixed points and separatrix**
- **Matching of buckets**

7.4 RF systems

- **Waves in wave guides and modes in cavities**
- **Types of cavities**
- **Structures for relativistic beams**
- **Structures for non-relativistic beams**
- **Stand wave and travelling wave structures**
- **Higher Order Modes**
- **Shunt impedance, transit time factor, quality factor, filling time**
- **Power and coupling to cavities**

8 LINEAR IMPERFECTIONS

(Closely related to lecture on Linear Beam optics)

8.1 Field errors

- **Dipole and quadrupole errors**
- **How to correct it, means and procedure**

8.2 Alignment errors

- **Feed down**
- **Dipole and quadrupole errors**
- **How to correct it, means and procedure**

8.3 Coupling of horizontal and vertical planes

- **Origin**
- **How to correct it, means and procedure**

9 ELECTRON BEAM DYNAMICS - SYNCHROTRONS AND LIGHT SOURCES

Objective:

**For the whole theme: properties of synchrotron radiation and impact (negative and positive effects, applications).
Principles of synchrotron radiation, main parameter, e.g. spectra. If time permits, incoherent and coherent radiation.
Beam dynamics with radiation together with principles and properties of light sources.**

9.1 Synchrotron radiation

- **Fields of moving and accelerated particles**
- **Classical versus quantum radiation**
- **Power and typical frequency**
- **Energy loss**

9.2 Beam dynamics with synchrotron radiation

- Radiation Damping, linear versus circular machines
- Damping times and damping partition numbers
- Emittance, equilibrium and growth
- Radiation integrals

10 NON-LINEAR BEAM DYNAMICS

Objective:
Mostly phenomenology and qualitative. Quantitative treatment and details in Advanced Level Schools, Outlook to lectures at the "Advanced Level Course".

10.1 Sources of non-linearities

- Introduce key concepts with practical examples rather than abstract definitions
- Chromaticity correction with sextupoles
- Multipole expansion of magnetic fields
- Other sources of non-linear effects, beam-beam, space charge

10.2 Phenomenology

- Representation of non-linear effects
- Tune shift
- Resonances
- Amplitude detuning, decoherence
- Structure of phase space, islands etc.
- Applications (extraction etc.)

10.3 Outlook: tools and techniques, (like: A. Wolski, 2014)

- Representation of non-linear elements
- Analysis techniques

11 COLLECTIVE AND MULTI PARTICLE EFFECTS

Objective:

Introduction to multi particle effects and interaction with the environment, concepts of impedance and wake fields. Should give a general overview over different types of collective effects. Mostly phenomenology, details in Advanced Level School, should follow after non-linear dynamics lectures.

11.1 Direct Space Charge and Image charges

- **Origin**
- **Cover both linear and circular machines**

11.2 Impedance and wake fields

- **Definition and properties**
- **Complex impedance**
- **Broad Band and Resistive Wall Impedances**
- **Narrow Band Impedance and resonators**
- **Longitudinal and transverse Impedance, Panowski-Wenzel Theorem**
- **Time and Frequency Domain**
- **Effect on beams, complex tune shift, heating**
- **Impedance measurements (bench and beam)**
- **Calculation of Impedances**

11.3 Beam instabilities

- **Bunched and unbunched beams**
- **Origin of instabilities**
- **Simplified Models**
- **Instabilities in linacs**
- **Single and multi particle instabilities**
- **Longitudinal and Transverse instabilities**
- **Cures**

Advanced course

12 RECAP LECTURES

Objective:

These background lectures should recapitulate the lectures on the main themes of the Introductory Course and serve as basics for the lectures at the Advanced Course.

12.1 General introduction: kinematics

- Electromagnetism
- Relativity
- Particle motion in electromagnetic fields

12.2 Transverse Dynamics

Should cover the introductory lectures on: Transverse Linear beam Optics, Linear Imperfections and Phenomenology of Non-linear effects.

- Linear Optics, including chromaticity
- Linear Imperfections, including Coupling
- Basic phenomenology of non-linear dynamics

12.3 Longitudinal Dynamics

Should cover the introductory lectures on: Longitudinal Beam Dynamics in Linear and Circular Accelerators, RF Systems.

- Acceleration methods and structures, Cavity basics, transit time factor
- Phase stability
- Linac versus circular accelerator
- Transition, synchrotron oscillations
- Equations of motion, buckets
- Phase stability

13 INTRODUCTION TO COURSES

13.1 RF Measurements

13.2 Beam Measurements

13.2.1 Introduction to Beam Instrumentation

13.2.2 Introduction to Beam Diagnostics

13.3 Optics design

13.3.1 Lattice cells

- FODO cells
- Separated and combined function FD cells
- Basic parameters, tune, optical parameters, chromaticity
- Choice of lattice parameters
- Linear imperfections
- Other cells, reason and basics

13.3.2 Insertions

- Dispersion Suppressors
- Low β insertions
- Matching optical functions

14 NON-LINEAR DYNAMICS (rings, but all concepts applicable to single pass machines)

Objective:
Introduce contemporary methods and tools for the analysis of non-linear effects. Demonstrate first the concepts in the linear cases and extend to non-linear dynamics. Key elements are: symplecticity, maps and matrices, analysis of maps, normal forms. It includes the use of Lie methods and Truncated Power Series Algebra. The latter with a hands-on simulation program.

14.1 Methods and tools

- **Conceptual and formal tools for beam dynamics, maps, normal forms and analysis**
  - **Demonstrate the concepts in the linear cases**
  - **Maps and matrices, analysis of maps, normal forms**
  - **Symplecticity**
- **Action-angle variables (as a general concept)**
- **Generalization: extend linear → non-linear dynamics**
- **Higher order maps, Taylor maps, thin lenses**
- **Numerical integrators, Symplectic integrators**
- **Basics of Hamiltonian treatment, Poisson brackets**
- **Lie operators and Lie Transformations**
- **Non-linear normal forms**
- **Truncated Power Series Algebra (TPSA), simple examples and usage, demonstration with small program**

14.2 Phenomenology, applications and examples

- **Non-linear imperfections**
- **Phase space and canonical variables, Poincare section and invariants, effect of imperfections**
- **Dynamic Aperture, definition and evaluation**
- **Non-linear generalized Courant-Snyder invariants**
- **Normal form analysis of non-linear effects (sextupoles, octupoles, beam-beam)**
- **Canonical Perturbation Theory**
- **Non-linear detuning, application (example e.g. octupole, NF)**
- **Islands and island width , away and near resonance**
- **Chaotic motion and diffusion, Lyapunov exponent, Chirikov criterion**
- **Tools (e.g. FMA, tracking)**

15 INSTABILITIES AND COLLECTIVE EFFECTS

(similar to introductory course, but quantitative treatment)

15.1 Impedance and space charge

- **Direct and image space charge**
- **Envelope equation, KV distribution**
- **Recapitulation of impedances, wake fields, cavities**
- **Coherent and incoherent frequency shifts**
- **Neutralisation**

15.2 Beam instabilities

- **Origin of instabilities**
- **Bunched and unbunched beams, coupled bunch instabilities**
- **Simplified Models**
- **Single and multi particle instabilities**
- **Longitudinal and Transverse instabilities**
- **Microwave instability**
- **Robinson instability and Negative Mass instability**
- **Sacherer's integral equation**
- **Longitudinal and transverse Head-Tail instability and beam break up**
- **Transverse Mode Coupling Instability**
- **Bunch lengthening**
- **Cures**

15.3 Other collective effects

- **Intra Beam Scattering**
- **Electron Cloud and Ion trapping**

15.4 Instabilities in LINACS

- **Fields of moving charges**
- **Wake potentials, off-centred beams and transitions**
- **Effect of wake fields, transverse and longitudinal**
- **BNS damping**

15.5 Beam-beam effects

- **Beam-beam forces (2D)**
- **Head-on and long range beam-beam interactions**
- **Tune shift and amplitude detuning, footprints**
- **Beam-beam limit (lepton machines mainly)**
- **Coherent Beam-beam effects**
- **Beam-beam compensation**

15.6 Landau Damping

- **Principles**
- **Landau Damping in Plasmas**
- **Landau Damping in particle beam**
- **Dispersion integrals**
- **Response to excitation and Beam transfer Functions**
- **Stability diagrams**
- **Applications in accelerators, use and control of stability diagrams**

Proposed conventions

All variables, units and constants used in lectures (present and past)

VARIABLES AND SYMBOLS

Transverse coordinates	x, y
Four-vector	(x, y, z, ct)
Reference momentum, energy	p_0, E_0
Relativistic beta and gamma function	β_r, γ_r
Scalar and vector potentials	$\Phi, \vec{A}$
Charge density	$\rho(x, y, z)$
Particle Charge	q
Current density	$\vec{J}$
Displacement current	$\vec{D}$
Magnetic induction	$\vec{B}$
Magnetic and electric fields	$\vec{H}, \vec{E}$
Tunes	Q_x, Q_y, Q_s
Linear Chromaticity	Q'_x, Q'_y
Betatron Amplitude Functions	β_x, β_y
Alpha Courant-Snyder Functions	α_x, α_y
Gamma Courant-Snyder Functions	γ_x, γ_y
Luminosity	**L**
Circumference	**C**
Action (transverse)	J_x, J_y
Angle	Φ_x, Φ_y
Damping partitions	j_x, j_y, j_ϵ
Transverse and longitudinal emittances	$\epsilon_x, \epsilon_y, \epsilon_z$
Transverse emittances ϵ_x, ϵ_y	$< J_x >, < J_y >$
Transverse beam size (r.m.s.)	σ_x, σ_y
Longitudinal beam size (r.m.s.)	σ_z
Bunch duration	$\sigma_t = \frac{\sigma_z}{\beta_r c}$
Energy spread (r.m.s.)	$\sigma_\delta = \frac{\delta E}{E_0}$
Path along reference orbit	**s**
Phase space variables	x, p_x, y, p_y
Angular deviation	$x' = \frac{dx}{ds}, y' = \frac{dy}{ds}$
Phase space density distribution	$\Psi(x_1..x_{3N}, p_1, ..p_{3N})$
Horizontal and vertical betatron phase	μ_x, μ_y
Bunch intensity	N_b
Bunch current	I_b
Number of bunches per beam	n_b

Revolution frequency	f
Angular Revolution frequency	$\omega = 2\pi f$
Angular RF frequency	ω_{rf}
RF Voltage	V_{rf}
Quality Factor	Q
Harmonic number h	$h = \omega_{rf}/\omega$
Bending radius	ρ
Gamma at transition	γ_t
Machine slip factor	$\eta = \alpha - \dfrac{1}{\gamma^2}$
Transit time	τ
Particle RF phase (modulo 2π)	ϕ
Synchronous phase	ϕ_s
Energy loss per turn	U_0
Momentum compaction	α_c
Dispersion functions	D_x, D_y
Critical frequency	ω_c
Longitudinal Wake Potential	$W_{\|}$ **[V/C]**
Transverse Wake Potential	$W_{\perp}$ **[V/C m]**
Longitudinal Coupling Impedance	$Z_{\|}$ **[Ω]**
Transverse Coupling Impedance	$Z_{\perp}$ **[Ω/m]**
Longitudinal Loss Factor	$\kappa_{\|}$ **[V/C]**
Transverse Loss Factor	$\kappa_{\perp}$ **[V/C m]**
Beam-beam parameter	ξ_x, ξ_y
Beam-beam tune shift	$\Delta Q_x, \Delta Q_y$

UNITS

Variable	unit
Energy	eV (≡ 1.6021773 10^{-19} J)
Power	W = J/s (≡ 1.356 ft lb/s)
Momentum	eV/c
Mass	eV/c^2
Transverse emittances	m
Longitudinal emittance	eVs
Luminosity	cm^{-2} s^{-1}
Electric charge	C
Resistance	Ohm, Ω
Conductance	Siemens, S = $\frac{1}{\Omega}$
Conductivity	S/m
Capacitance	Farad, F
Inductance	Henry, H
Impedances	MΩ
Loss factor	V/pC
Dose	Gray, Gy
Activity	Bequerel, Bq

DEFINITIONS

k_1	$\frac{1}{B\rho}\frac{dB_y}{dx}$
k_2	$\frac{1}{B\rho}\frac{d^2B_y}{dx^2}$
k_n	$\frac{1}{B\rho}\frac{d^nB_y}{dx^n}$
Transverse emittances ϵ_x, ϵ_y	$< J_x >, < J_y >$
Normalised transverse emittances	$\epsilon_n = \epsilon\gamma_r\beta_r$
Longitudinal emittance ϵ_s	$4\pi\sigma_t\sigma_\delta E_0$
Action invariant (here x**)**	$J_x = \frac{1}{2}(\gamma x^2 + 2\alpha_x x p_x + \beta_x p_x^2)$
(Avoid calling it emittance)	
Area of phase space ellipse	$\pi \cdot J$
Multipole expansion	$B_y + iB_x = \sum_{n=1}^{\infty} C_n(x+iy)^{n-1}$

CONSTANTS FOR CALCULATIONS

Pemittivity (vacuum)	8.854187817 10^{-12} Farad/m
Permeability (exact, vacuum)	4π 10^{-7} Henry/m
Electron charge	1.6021773 10^{-19} C
Speed of light (exact)	2.99792458 10^{8} m/s
Classical electron radius	$r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} = 2.817940910^{-15}$ m
Classical proton radius	$r_p = \frac{e^2}{4\pi\epsilon_0 m_p c^2} = 1.53469810^{-18}$ m
C_γ	8.858$10^{-5}$ m/GeV^3
Reduced Planck constant $\hbar = \frac{h}{2\pi}$	1.054572 10^{-34} Js = 6.582122 10^{-16} eV s
Compton wavelength λ_C	2.4 10^{-12} m
Electron rest mass	0.511 MeV/c^2
Proton rest mass	0.9382723 GeV/c^2