# Linear Accelerators

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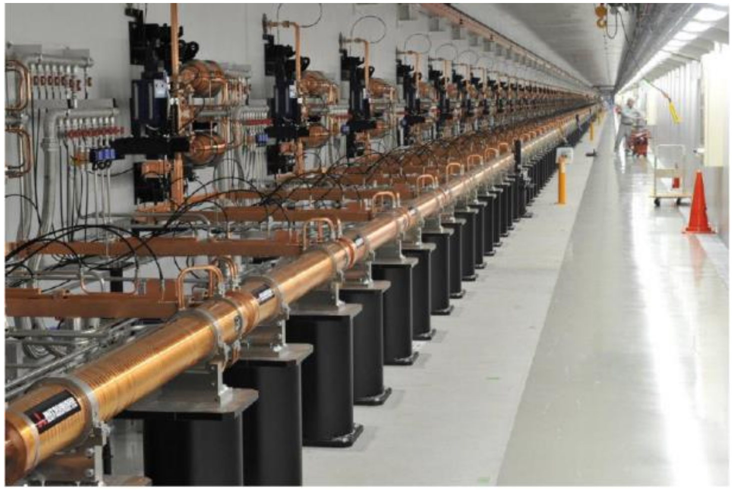

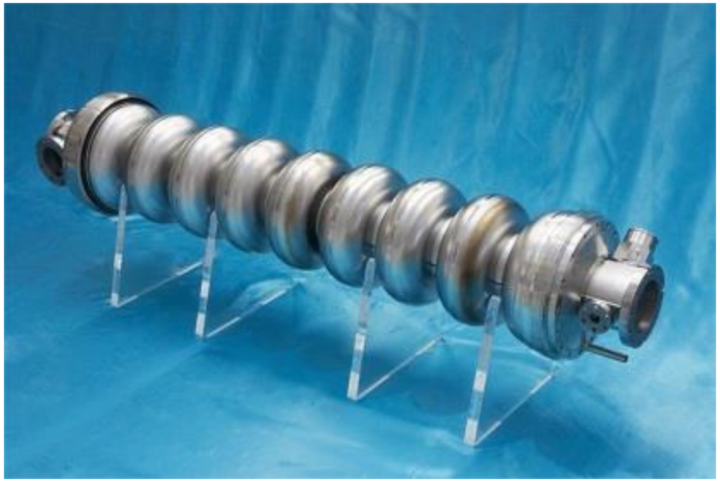

The CERN Accelerator School

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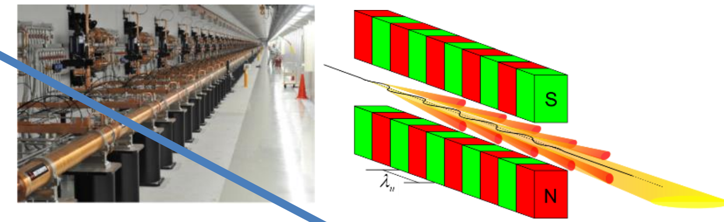
# LINAC APPLICATIONS

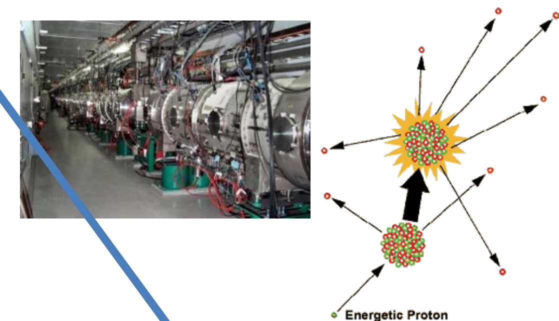
~$10^4$ LINACs operating around the world

## Free Electron Lasers

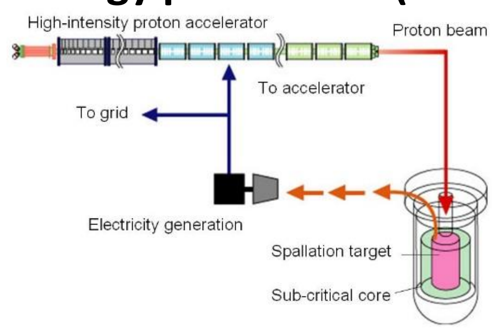



## Injectors for synchrotrons

## Spallation sources for neutron production



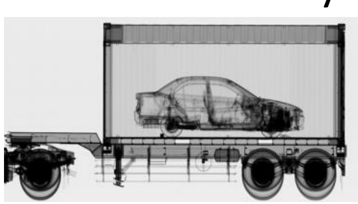

## Nuclear waste treatment and controlled fission for energy production (ADS)




## Medical applications: radiotherapy

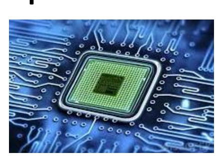
## Industrial applications

### National security



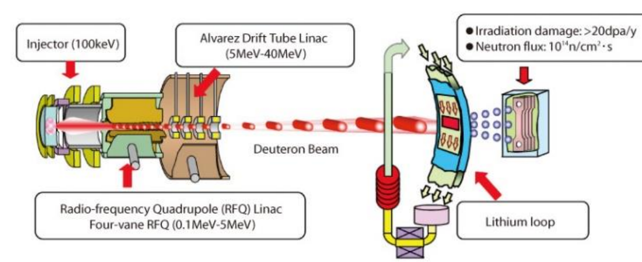
### Material treatment

### Ion implantation



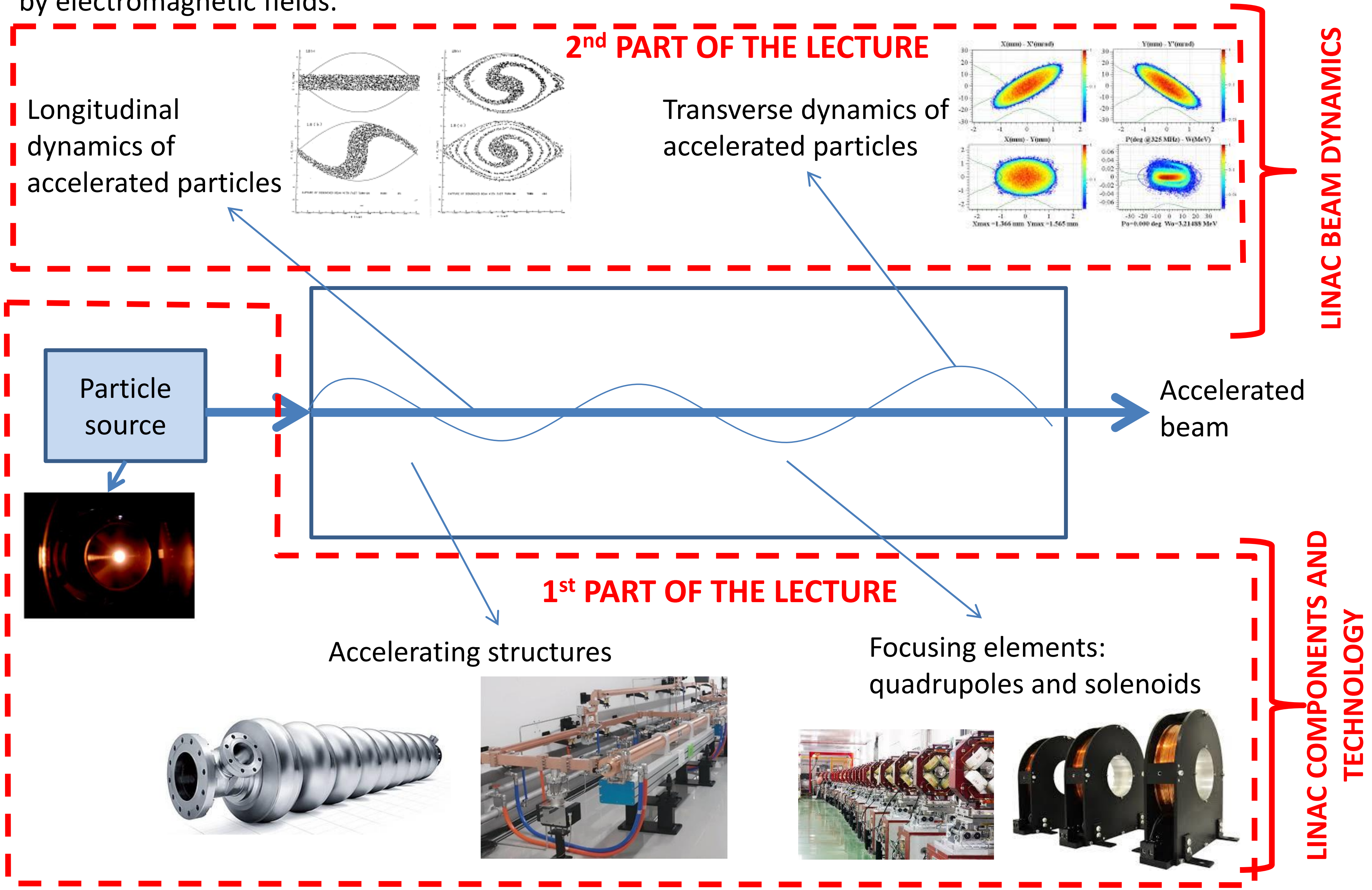
### Material/food sterilization

## Material testing for fusion nuclear reactors

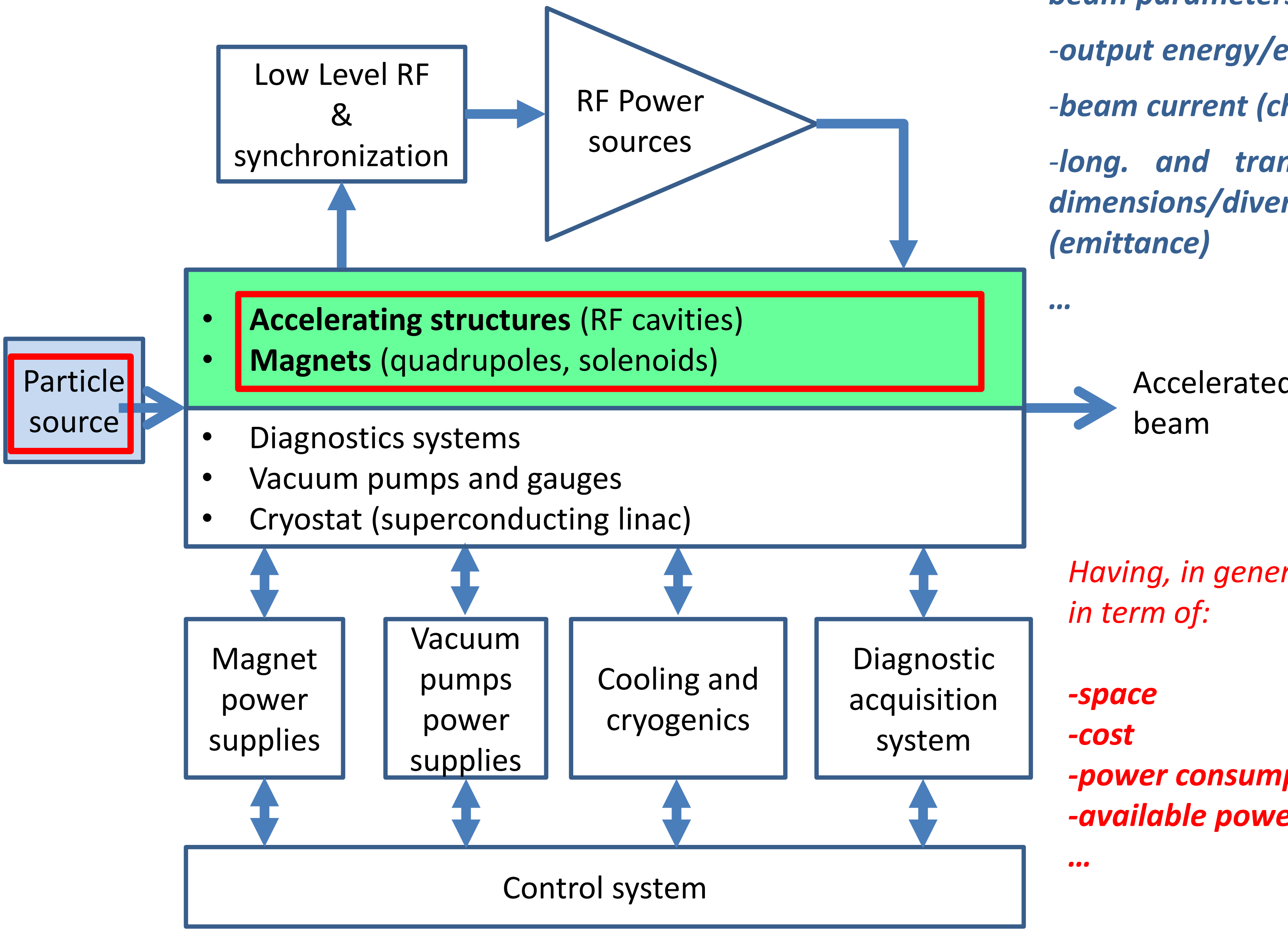


# LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.



# LINAC TECHNOLOGY COMPLEXITY




*The overhall LINAC has to be designed to obtain the* ***desired beam parameters*** *in term of:*

***-output energy/energy spread***

***-beam current (charge)***

***-long. and transverse beam dimensions/divergence (emittance)***

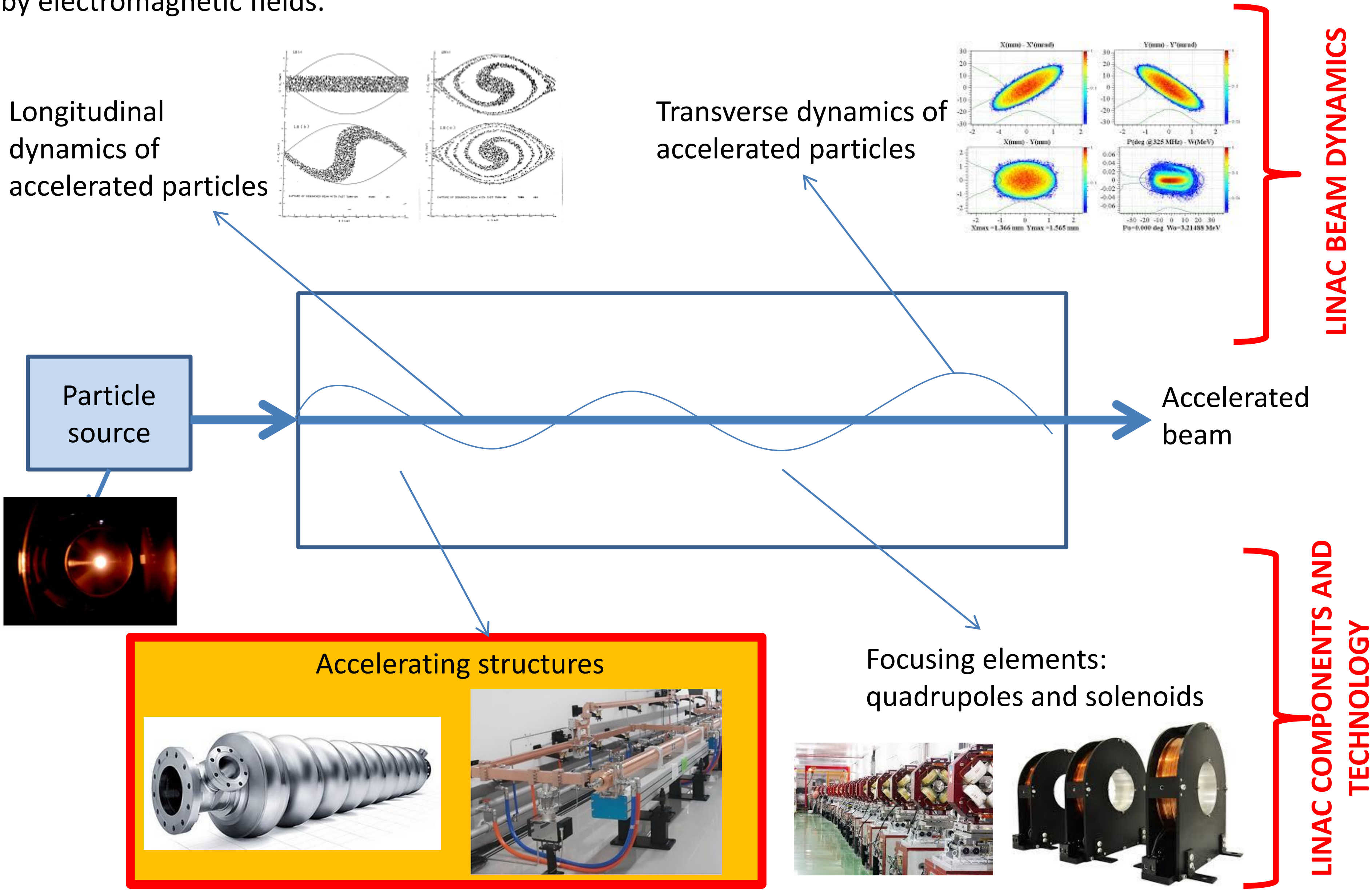
***…***

*Having, in general,* ***constraints*** *in term of:*

***-space***
***-cost***
***-power consumption***
***-available power sources***
***…***

# LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.



# LORENTZ FORCE: ACCELERATION AND FOCUSING

The basic equation that describes the acceleration/bending/focusing processes is the **Lorentz Force**.
Particles are **accelerated through electric** fields and are **bended and focused through magnetic** fields.

$\vec{p} = momentum$

$m = mass$

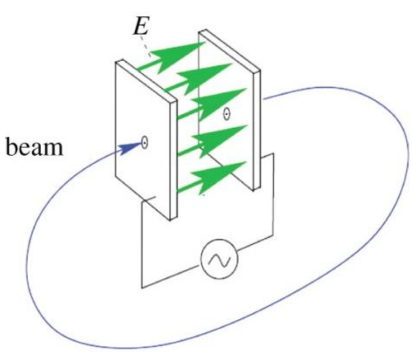
$\vec{v} = velocity$

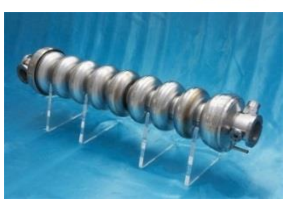
$q = charge$

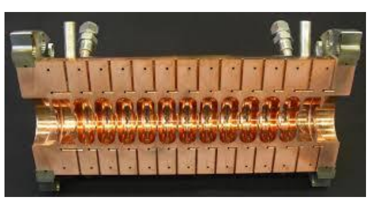
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

## ACCELERATION

To accelerate, we need a force in the direction of motion



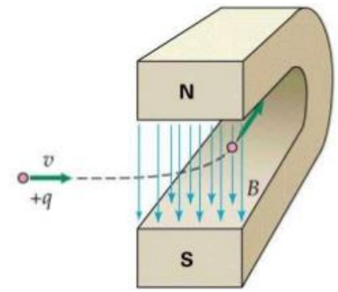



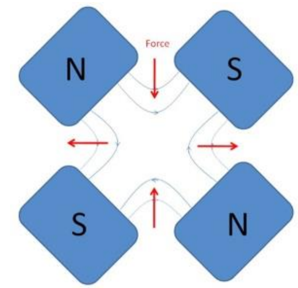


Longitudinal Dynamics

## BENDING AND FOCUSING

$2^{nd}$ term always perpendicular to motion => no energy gain

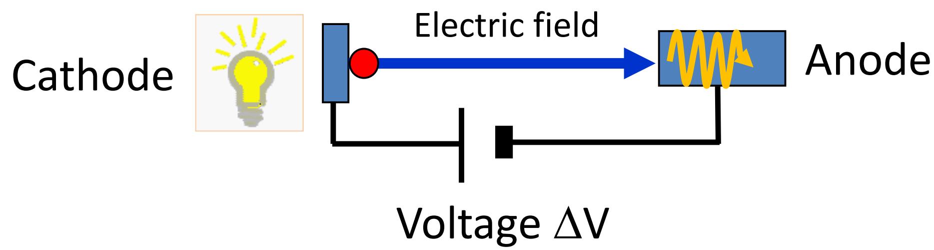



Transverse Dynamics

# ACCELERATION: SIMPLE CASE

The **first historical linear particle accelerator** was built by the Nobel prize Wilhelm Conrad Röntgen (1900). It consisted in a vacuum tube containing a cathode connected to the negative pole of a DC voltage generator. **Electrons emitted by the heated cathode** were accelerated while flowing to another electrode connected to the positive generator pole (anode). Collisions between the energetic electrons and the anode produced **X-rays**.




*Bertha Röntgen's Hand 8 Nov, 1895*

The **energy gained** by the electrons travelling from the cathode to the anode is equal to their charge multiplied the DC voltage between the two electrodes.

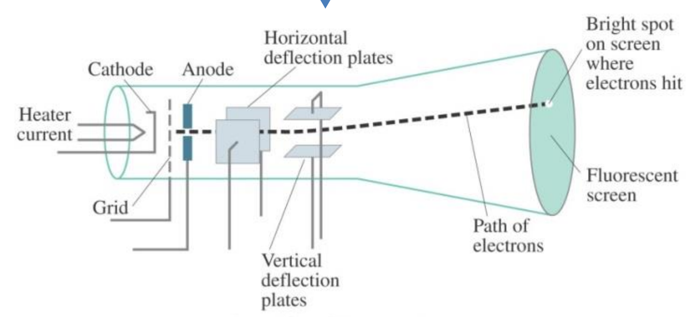
$$\frac{d\vec{p}}{dt} = q\vec{E} \quad \Rightarrow \quad \Delta E = q\Delta V$$

$$\vec{p} = momentum$$

$$q = charge$$

$$E = energy$$

**Particle energies are typically expressed in electron-volt** [eV], equal to the energy gained by 1 electron accelerated through an electrostatic potential of 1 volt:
1 eV=1.6x10$^{-19}$ J



# PARTICLE VELOCITY VS ENERGY: LIGHT AND HEAVY PARTICLES

Single particle
- rest mass $m_0$
- rest energy $E_0$ ($=m_0c^2$)
- total energy $E$
- relativistic mass $m$
- velocity $v$
- momentum $p$ ($=mv$)
- Kinetic energy $W=E-E_0$

Relativistic factor
$\beta = v/c$ (<1)

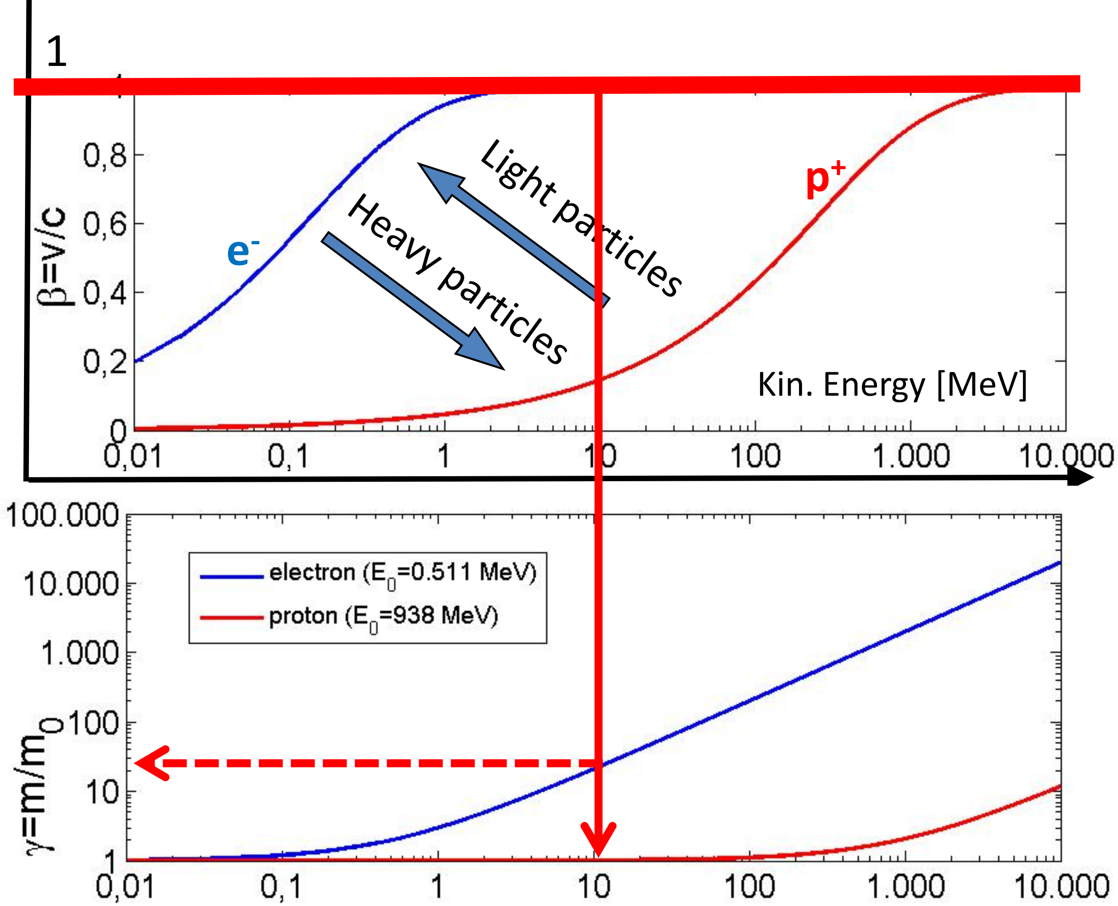
Relativistic factor
$\gamma = E/E_0$ ($\geq 1$)

+

$$E^2 = E_0^2 + p^2c^2$$

$$\begin{cases} \beta = \sqrt{1-1/\gamma^2} \\ \gamma = 1/\sqrt{1-\beta^2} \qquad (m = \gamma m_0) \\ W = (\gamma - 1)m_0c^2 \approx \frac{1}{2}m_0v^2 \quad \text{if } \beta \ll 1 \end{cases}$$

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\left(\frac{E_0}{E}\right)^2} = \sqrt{1-\left(\frac{E_0}{E_0+W}\right)^2}$$



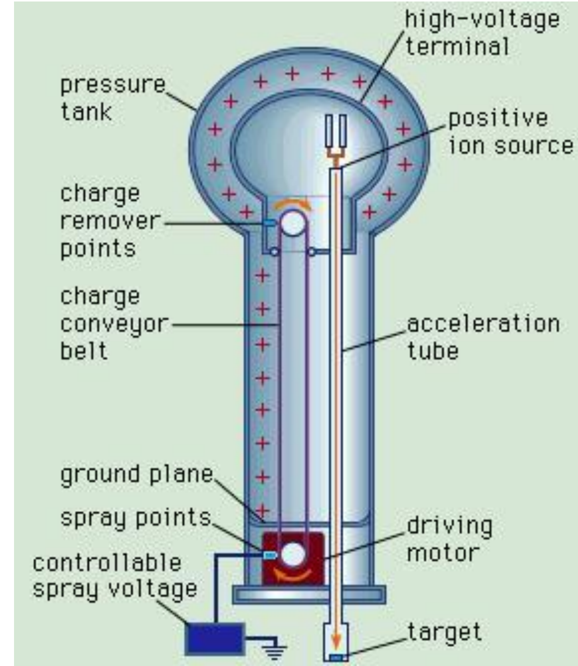

⇒**Light particles** (as **electrons**) are practically fully relativistic (β≅1, γ>>1) at relatively low energy and **reach a constant velocity** (~c). The acceleration process occurs at constant particle velocity

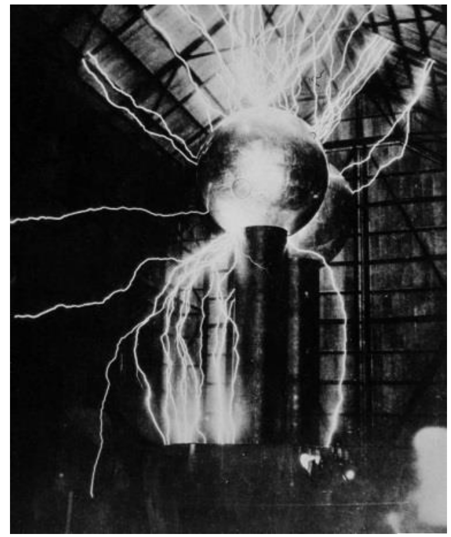
⇒**Heavy particles** (**protons and ions**) are typically weakly relativistic and **reach a constant velocity only at very high energy.** The velocity changes a lot during acceleration process.

⇒This implies **important differences** in the technical characteristics of the **accelerating structures**. In particular for **protons and ions** we need different types of accelerating structures, **optimized for different velocities** and/or the accelerating structure has to vary its geometry to take into account the velocity variation.

# ELECTROSTATIC ACCELERATORS

To increase the achievable maximum energy, Van de Graaff invented an electrostatic generator based on a **dielectric belt** transporting positive charges to an isolated electrode hosting an **ion source**. The positive ions generated in a large positive potential were accelerated toward ground by the static electric field.



## LIMITS OF ELECTROSTATIC ACCELERATORS

DC voltage as large as ~10 MV can be obtained (E~10 MeV). The main limit in the achievable voltage is the **breakdown** due to **insulation** problems.

## APPLICATIONS OF DC ACCELERATORS

DC particle accelerators are in operation worldwide, typically at V<15MV ($E_{max}$=15 MeV), I<100mA.

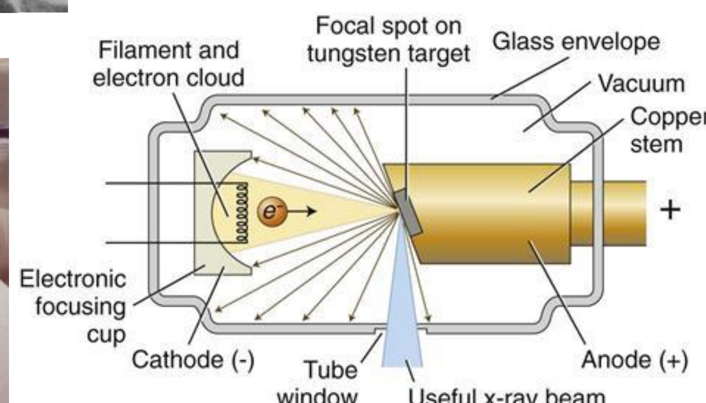
They are used for:

⇒ *material analysis*

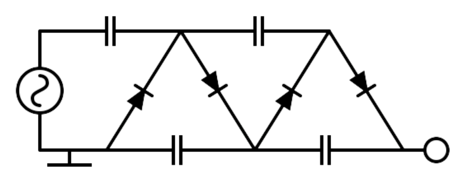
⇒ *X-ray production,*

⇒ *ion implantation for semiconductors*

⇒ *first stage of acceleration (particle sources)*





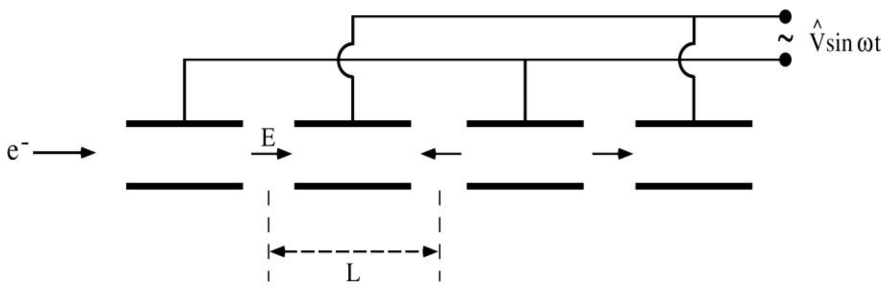


750 kV Cockcroft-Walton Linac2 injector at CERN from 1978 to 1992

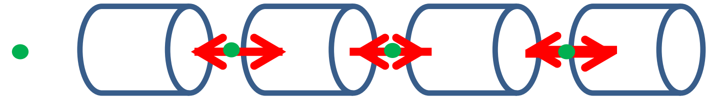
# RF ACCELERATORS : WIDERÖE "DRIFT TUBE LINAC" (DTL)

*(protons and ions)*

Basic idea: the particles are accelerated by the electric field in the gap between electrodes connected alternatively to the poles of an AC generator. This original idea of **Ising** (1924) was implemented by **Wideroe** (1927) who applied a sine-wave voltage to a sequence of **drift tubes**. The particles **do not experience any force while travelling inside the tubes** (equipotential regions) and are **accelerated across the gaps**. This kind of structure is called **Drift Tube LINAC (DTL).**



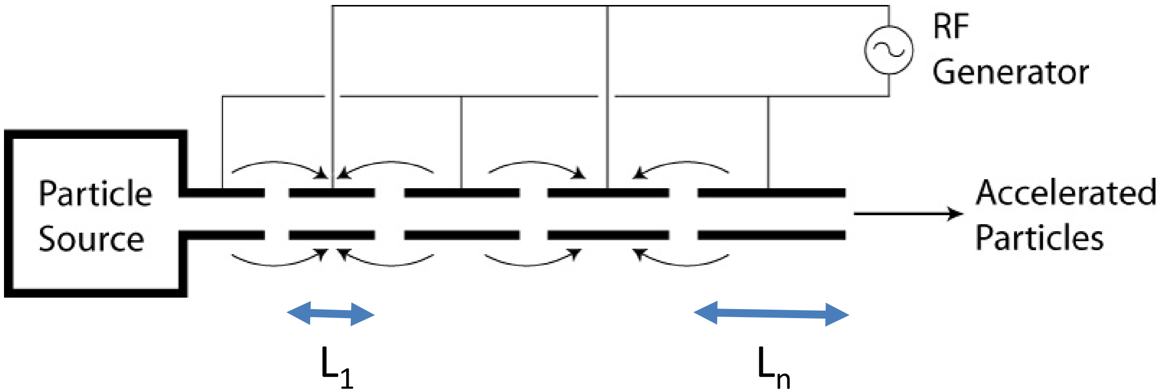




⇒If the **length of the tubes** increases with the particle velocity during the acceleration such that the time of flight is kept constant and equal to half of the RF period, the particles are subject to a **synchronous accelerating voltage** and experience an energy gain of $\Delta E=q\Delta V$ at each gap crossing.

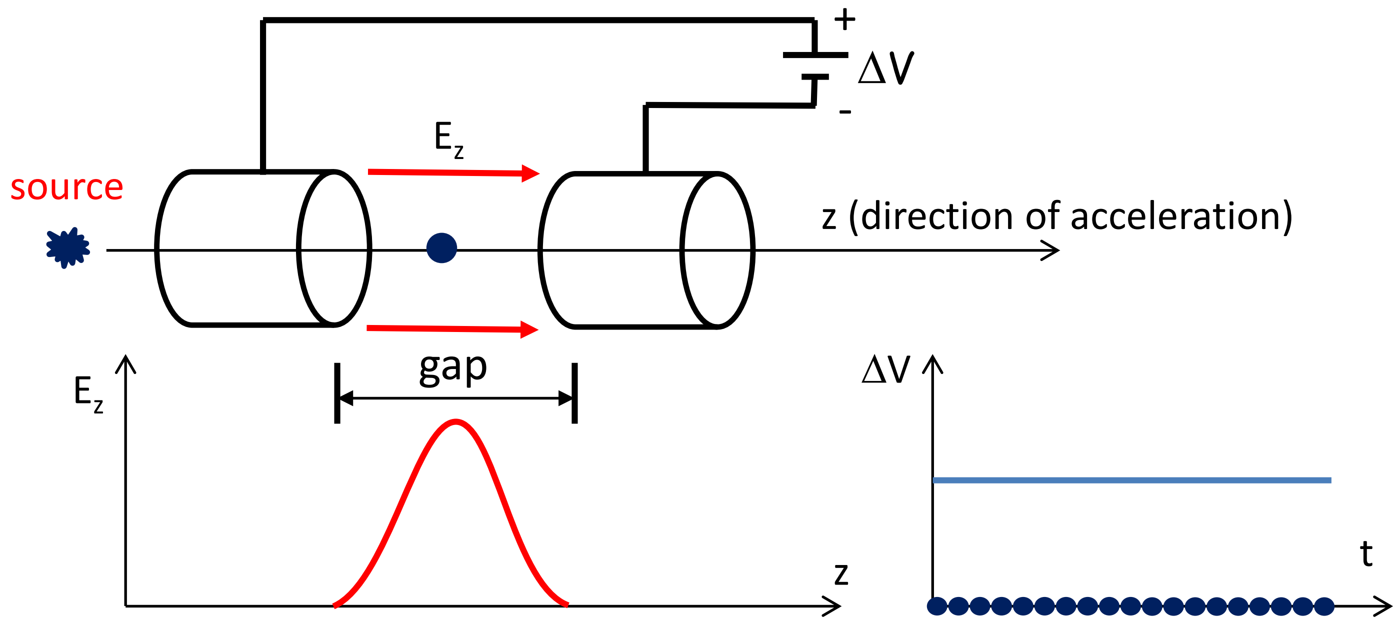
⇒In principle a single **RF generator** can be used to indefinitely accelerate a beam, **avoiding the breakdown limitation** affecting the electrostatic accelerators.

⇒The Wideroe LINAC is the **first RF LINAC**



# ACCELERATION: ENERGY GAIN

We consider the acceleration between two electrodes in DC.




$$E^2 = E_0^2 + p^2c^2 \Rightarrow 2EdE = 2pdpc^2 \Rightarrow dE = v\frac{mc^2}{E}dp \Rightarrow dE = vdp$$

$$\frac{dp}{dt} = qE_z \Rightarrow v\frac{dp}{dz} = qE_z \Rightarrow \boxed{\frac{dE}{dz} = qE_z} \quad \left(\text{and also } \frac{dW}{dz} = qE_z\right) \quad W=E-E_0$$

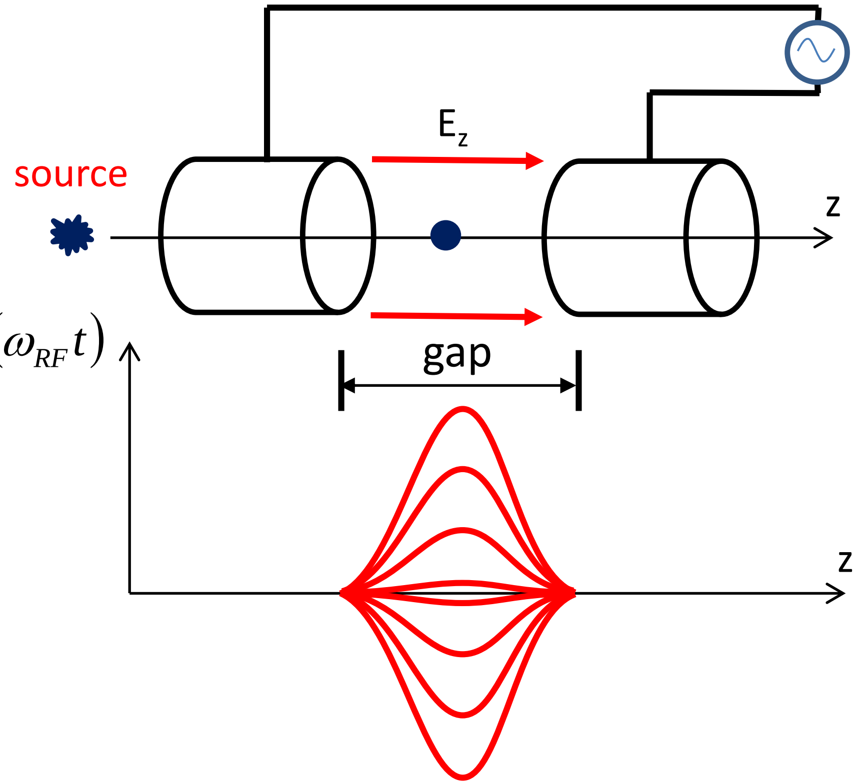
rate of energy gain per unit length

$$\Rightarrow \Delta E = \int_{gap} \frac{dE}{dz}dz = \int_{gap} qE_z dz \Rightarrow \boxed{\Delta E = q\Delta V}$$

energy gain per electrode

# RF ACCELERATION: BUNCHED BEAM

We consider now the acceleration between two electrodes fed by an RF generator

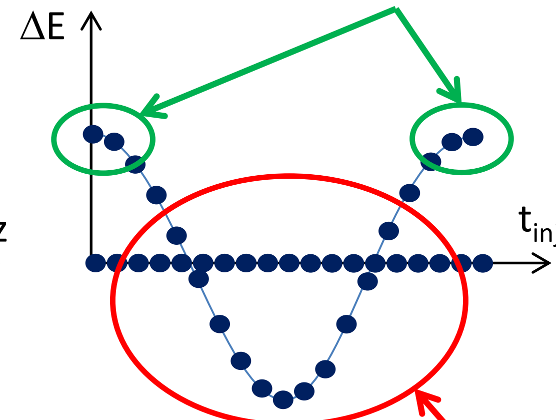



$$E_z(z,t) = E_{RF}(z)\cos(\omega_{RF} t)$$

$$\Delta V = V_{RF}\cos(\omega_{RF} t) \qquad \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}}$$

$$\Rightarrow \Delta E = q\hat{V}_{acc}\cos(\omega_{RF} t_{inj})$$

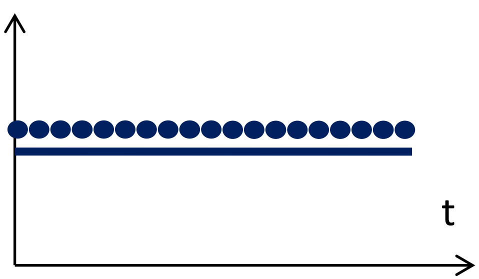
Only these particles are accelerated




These particles are not accelerated and basically are lost during the acceleration process

***DC acceleration***

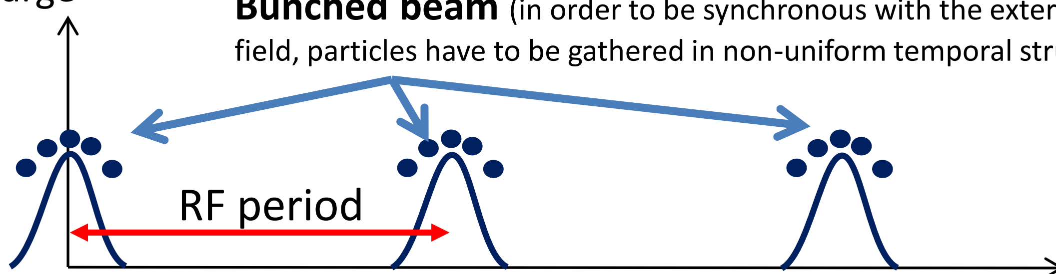
charge




***RF acceleration***

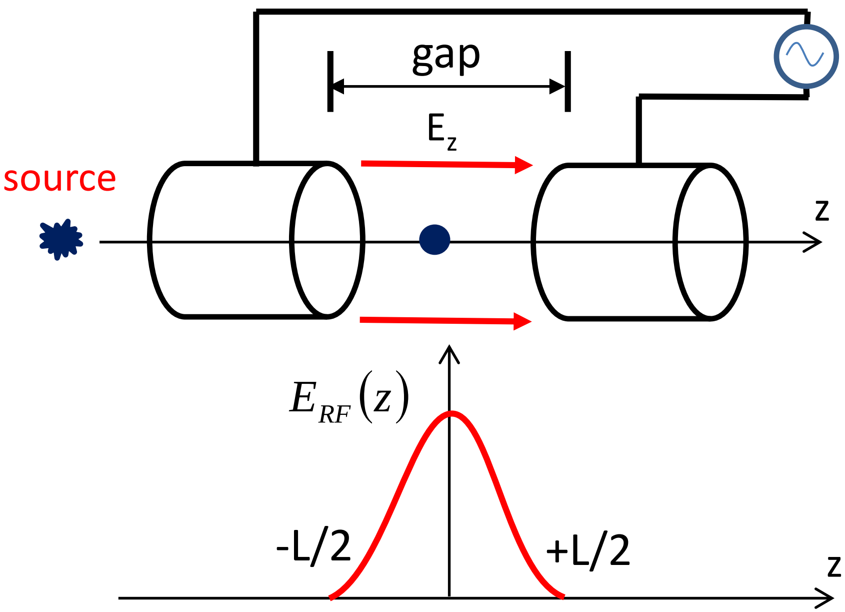
charge

**Bunched beam** (in order to be synchronous with the external AC field, particles have to be gathered in non-uniform temporal structure)



# RF ACCELERATION: ACCELERATING FIELD CALCULATION

We consider now the acceleration between two electrodes fed by an RF generator




$$\Delta V = V_{RF}\cos(\omega_{RF}t) \qquad \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}}$$

$$V_{RF} = \int_{gap} E_{RF}(z)dz \qquad E_z(z,t) = E_{RF}(z)\cos(\omega_{RF}t)$$

$$E_z(z,t)\Big|_{\substack{seen\\by\\particle}} = E_{RF}(z)\cos[\omega_{RF}(t+t_{inj})] = E_{RF}(z)\cos(\omega_{RF}t+\phi_{inj})$$

$$\phi_{inj} = \omega_{RF}t_{inj}$$

Hyp. of symmetric accelerating field

$$\Delta E = q\int_{gap} E_z(z,t)\Big|_{\substack{seen\\by\\particle}} dz = q\int_{-L/2}^{+L/2} E_{RF}(z)\cos\left(\omega_{RF}\frac{z}{v}+\phi_{inj}\right)dz = q\underbrace{\boxed{\int_{gap} E_{RF}(z)dz}}_{\substack{V_{RF}\\ \text{Peak gap voltage}}}\ \underbrace{\boxed{\frac{\int_{gap} E_{RF}(z)\cos\left(\omega_{RF}\frac{z}{v}\right)dz}{\int_{gap} E_{RF}(z)dz}}}_{\substack{T\\ \text{Transit time factor}}}\ \underbrace{\boxed{\cos(\phi_{inj})}}_{\text{Injection phase}}$$

$$\boxed{\Delta E = q\overbrace{V_{RF}T}^{\hat{V}_{acc}}\cos(\phi_{inj}) = \overbrace{q\hat{V}_{acc}\cos(\phi_{inj})}^{V_{acc}}}$$

$\hat{E}_{acc} = \hat{V}_{acc}/L$ Average accelerating field in the gap

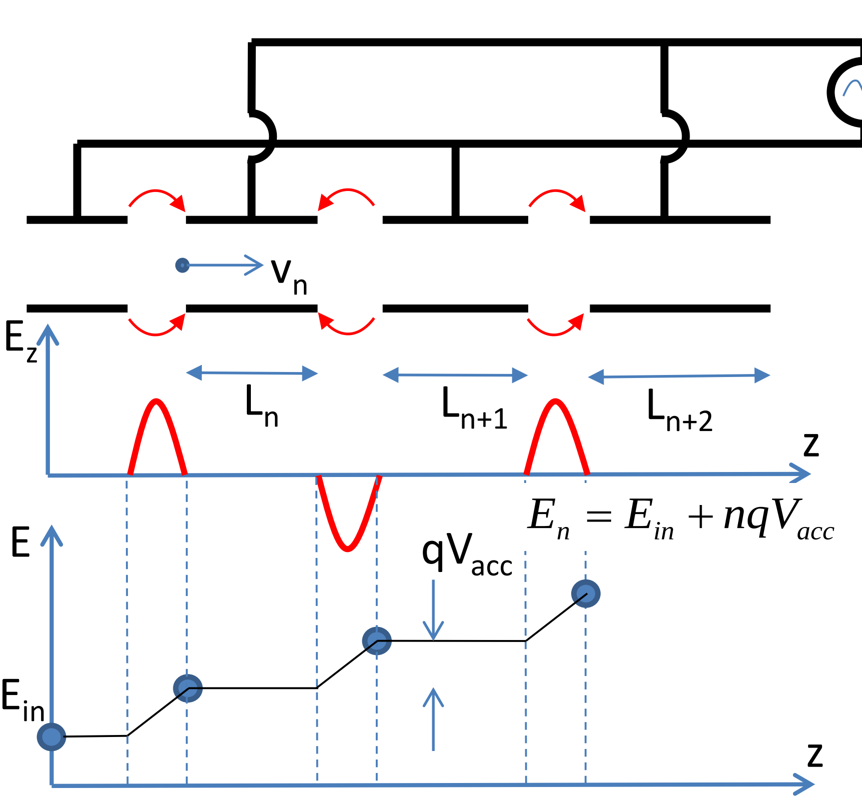
$E_{acc} = V_{acc}/L$ Average accelerating field seen by the particle

# DRIFT TUBE LENGTH AND FIELD SYNCHRONIZATION

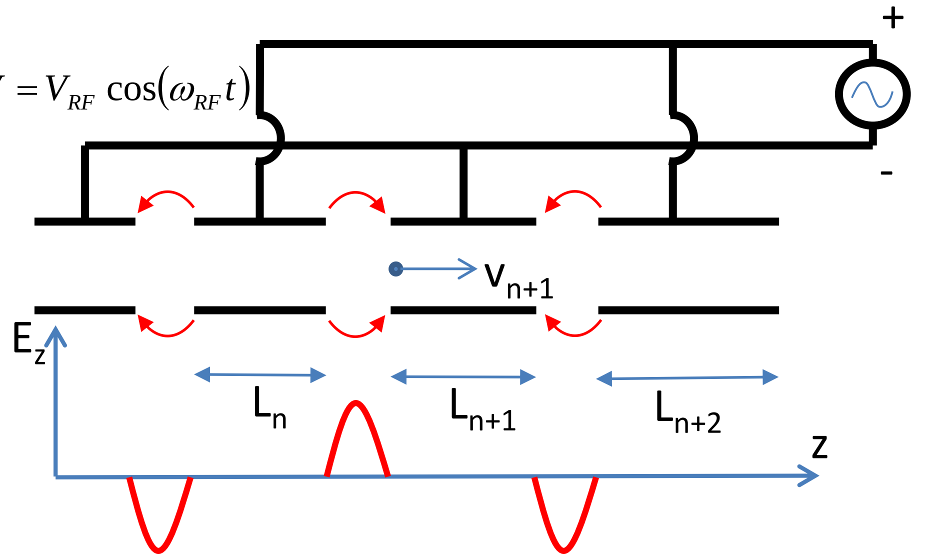
*(protons and ions or electrons at extremely low energy)*

If now we consider a DTL structure with an injected particle at an energy $E_{in}$, we have that at each gap the maximum energy gain is $\Delta E_n = qV_{acc}$ and the particle increase its velocity accordingly to the previous relativistic formulae.

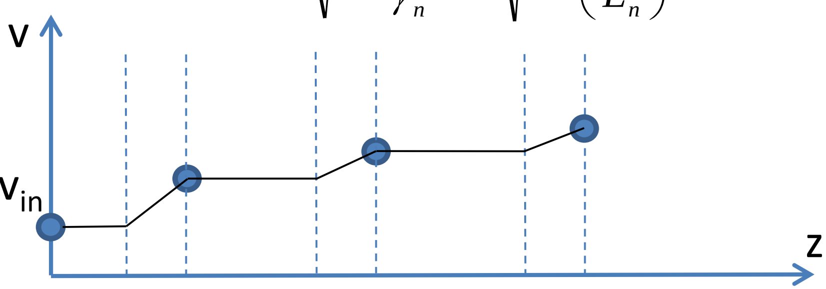
$$\Delta V = V_{RF}\cos(\omega_{RF} t)$$




$$E_n = E_{in} + nqV_{acc}$$




$$v_n = c\beta_n = c\sqrt{1-\frac{1}{\gamma_n^2}} = c\sqrt{1-\left(\frac{E_o}{E_n}\right)^2}$$




⇒ In order to be synchronous with the accelerating field at each gap the **length of the n-th drift tube** has to be $L_n$:

$$t_n = \frac{L_n}{v_n} = \frac{T_{RF}}{2} \Rightarrow L_n = \frac{1}{2}v_n T_{RF} = \frac{1}{2}\beta_n c T_{RF} \quad \boxed{L_n = \frac{1}{2}\beta_n \lambda_{RF}}$$

⇒The **energy gain per unit length** (i.e. the **average accelerating gradient**) is given by:

$$\boxed{\frac{\Delta E}{\Delta L} = \frac{qV_{acc}}{L_n} = \frac{2qV_{acc}}{\lambda_{RF}\beta_n}} \quad [\text{eV/m}]$$

# ACCELERATION WITH HIGH RF FREQUENCIES: RF CAVITIES

There are two important **consequences** of the previous obtained formulae:

$$\boxed{L_n = \frac{1}{2}\beta_n \lambda_{RF}}$$

The condition **$L_n << \lambda_{RF}$** (necessary to model the tube as an **equipotential region**) requires $\beta<<1$. $\Rightarrow$The Wideröe technique can **not be applied to relativistic particles**.

$$\boxed{\frac{\Delta E}{\Delta L} = \frac{qV_{acc}}{L_n} = qE_{acc} = \frac{2qV_{acc}}{\lambda_{RF}\beta_n}}$$

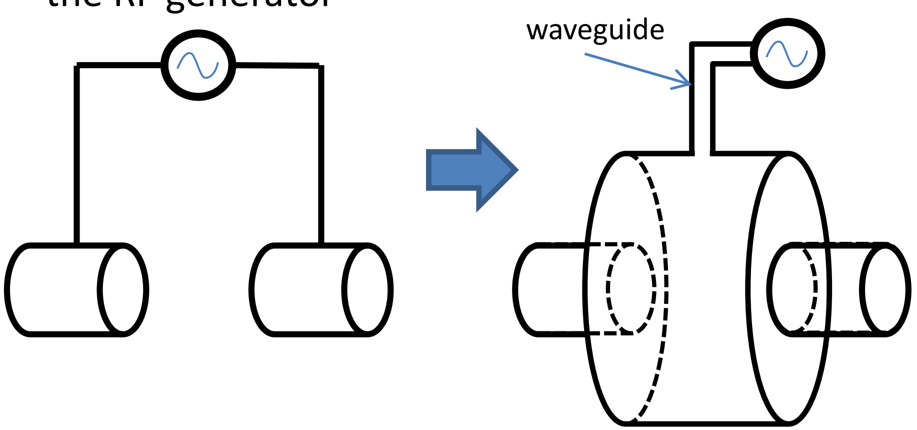
Moreover when particles get high velocities the drift spaces get longer and one looses on the efficiency. The **average accelerating gradient ($E_{RF}$ [V/m]) increase pushes towards small $\lambda_{RF}$** (high frequencies).

High frequency high power **sources** became available after the **2nd world war** pushed by **military** technology needs (such as **radar**). However, the concept of equipotential DT can not be applied at small $\lambda_{RF}$ and the **power lost by radiation is proportional to the RF frequency.**

**As a consequence** we must consider **accelerating structures different from drift tubes**.

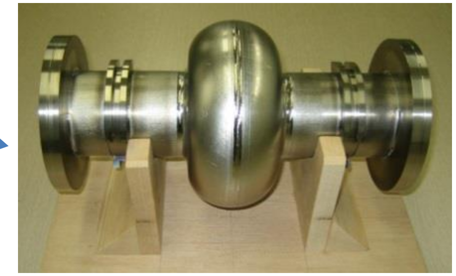
$\Rightarrow$The solution consists of **enclosing the system in a cavity** which resonant frequency matches the RF generator frequency.

$\Rightarrow$Each cavity can be independently powered from the RF generator

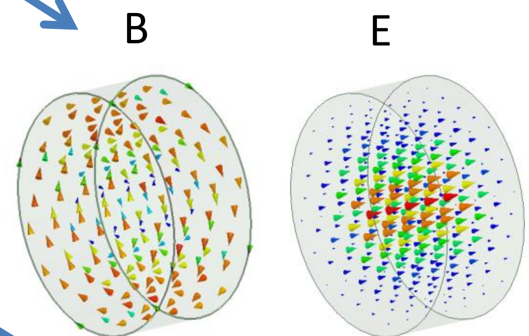


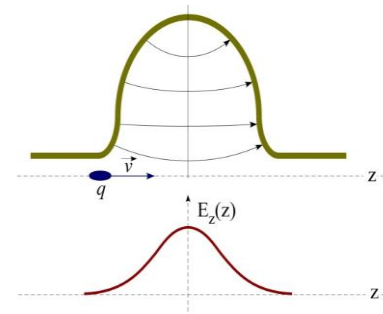
# RF CAVITIES

⇒High frequency RF accelerating fields are confined in **cavities**.

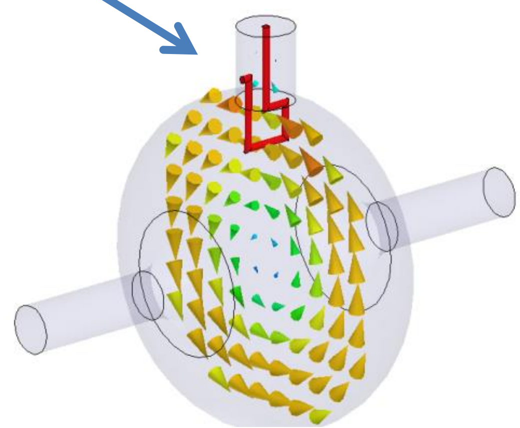


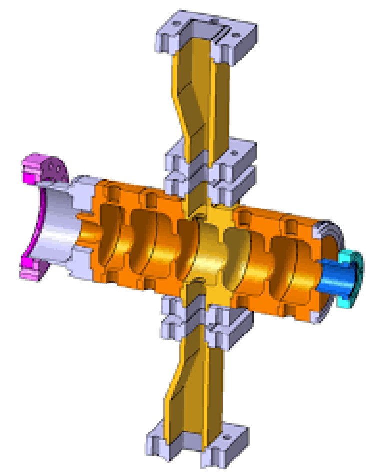
⇒The cavities are **metallic closed volumes** were the e.m fields has a particular spatial configuration (**resonant modes**) whose components, including the accelerating field $\mathbf{E_z}$, oscillate at some specific frequencies $\mathbf{f_{RF}}$ (resonant frequency) characteristic of the mode.



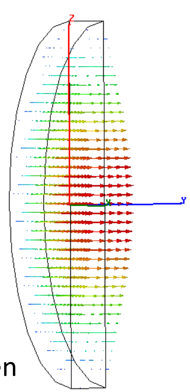

⇒The modes are excited by **RF generators** that are **coupled to the cavities** through waveguides, coaxial cables, etc…

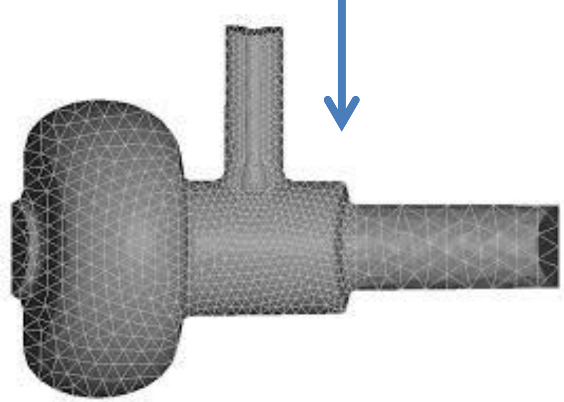




⇒The resonant modes are called **Standing Wave (SW) modes** (spatial fixed configuration, oscillating in time).

⇒The spatial and temporal field profiles in a cavity have to be computed (analytically or numerically) **by solving the Maxwell equations** with the proper boundary conditions.



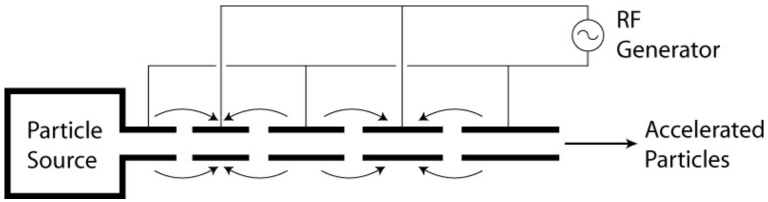


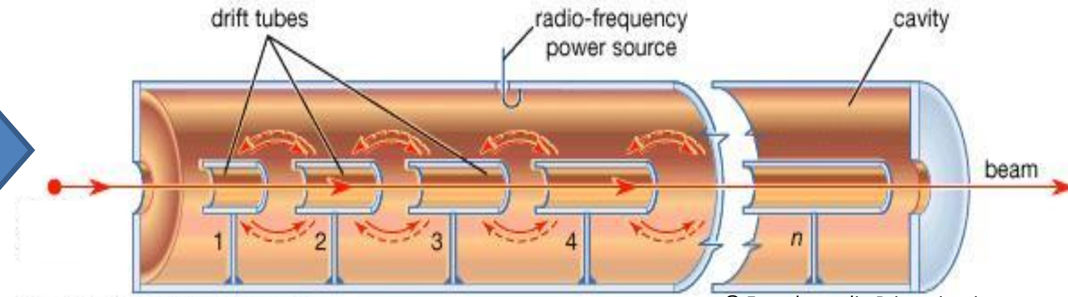
Courtesy E. Jensen

# ALVAREZ STRUCTURES

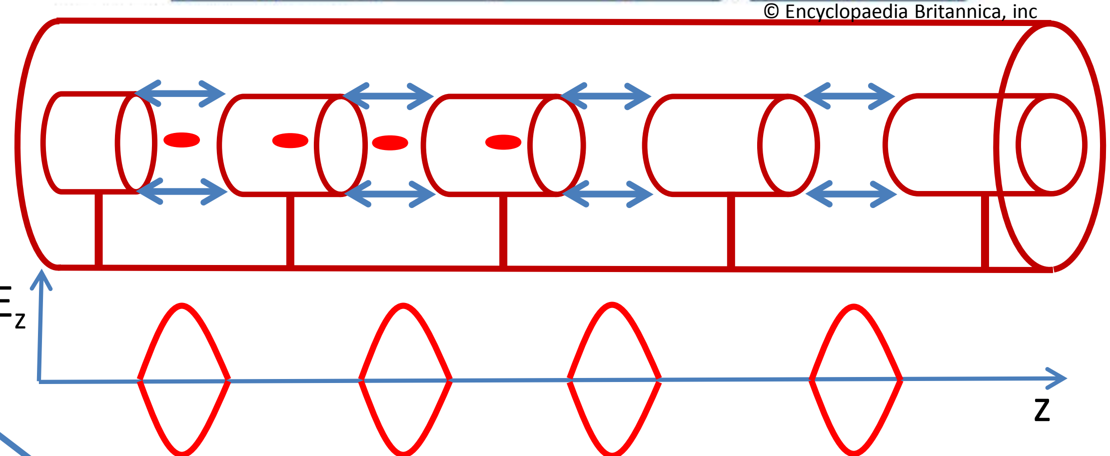
## *(protons and ions)*

**Alvarez's structure** can be described as a special DTL in which the electrodes are part of a **resonant macrostructure.**



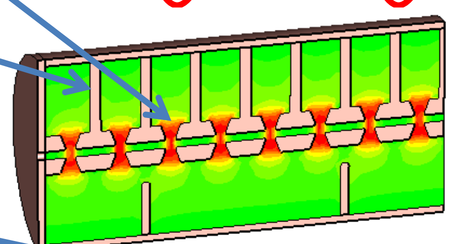


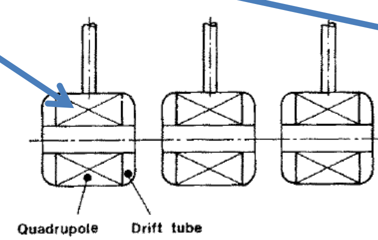

⇒The DTL operates in **0 mode** for **protons and ions** in the range $\beta$=0.05-0.5 ($f_{RF}$=50-400 MHz) 1-100 MeV;

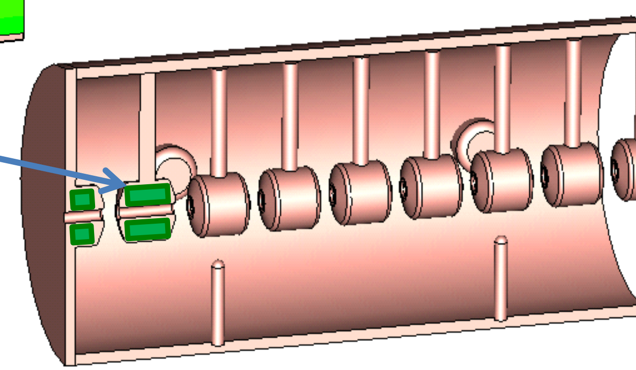
⇒The beam is inside the "**drift tubes**" when the electric field is decelerating. The **electric field** is concentrated between gaps;

⇒The drift tubes are suspended by **stems**;



⇒**Quadrupole** (for transverse focusing) can fit inside the drift tubes.



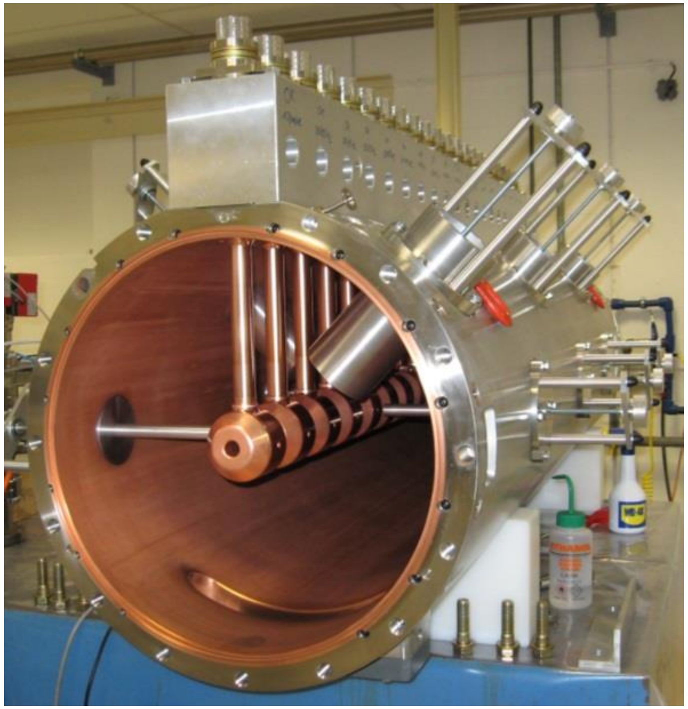



⇒In order to be synchronous with the accelerating field at each gap the **length of the n-th drift tube** $L_n$ has to be:

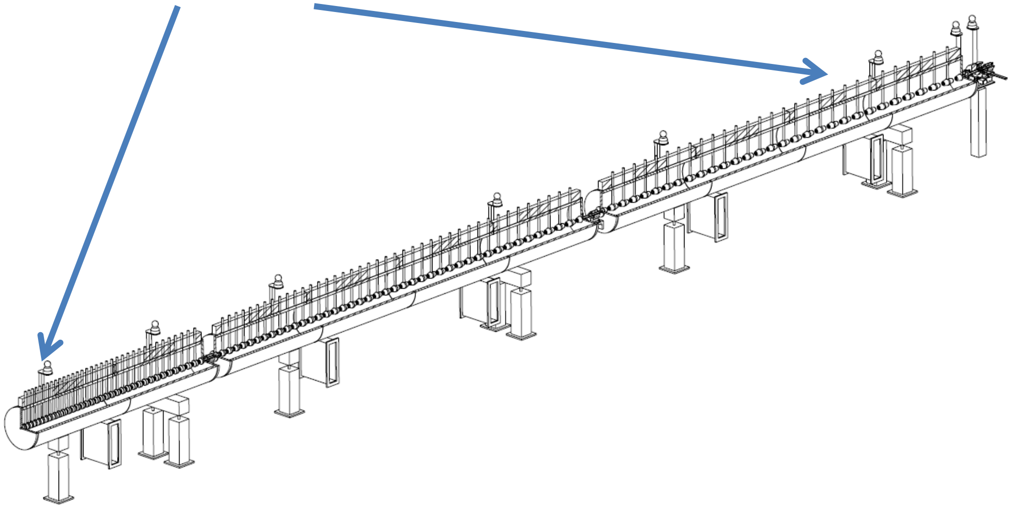
$$\boxed{L_n = \beta_n \lambda_{RF}}$$

# ALVAREZ STRUCTURES: EXAMPLES

CERN LINAC 2 tank 1:
200 MHz 7 m x 3 tanks, 1 m diameter, final energy 50 MeV.



CERN LINAC 4: 352 MHz frequency, Tank diameter 500 mm, 3 resonators (tanks), Length 19 m, 120 Drift Tubes, Energy: 3 MeV to 50 MeV, $\beta$=0.08 to 0.31 $\rightarrow$ cell length from 68mm to 264mm.



# HIGH β CAVITIES: CYLINDRICAL STRUCTURES

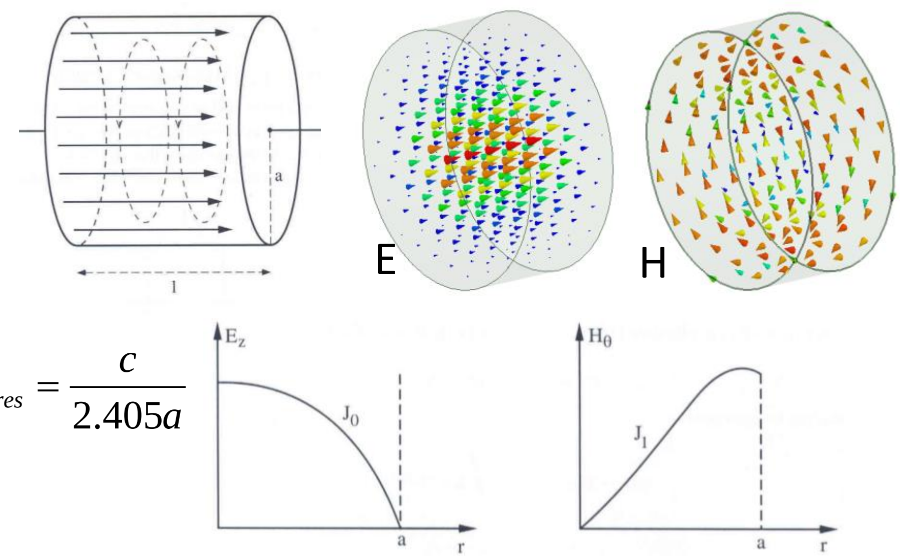
*(electrons or protons and ions at high energy)*

⇒When the β of the particles increases (>0.5) one has to use **higher RF frequencies** (>400-500 MHz) to increase the accelerating gradient per unit length

⇒the **DTL structures became less efficient** (effective accelerating voltage per unit length for a given RF power);

**Cylindrical single or multiple cavities** working on the **$TM_{010}$-like** mode are used

For a **pure cylindrical structure** (also called **pillbox cavity**) the first accelerating mode (i.e. with non zero longitudinal electric field on axis) is the **$TM_{010}$ mode**. It has a well known analytical solution from Maxwell equation.

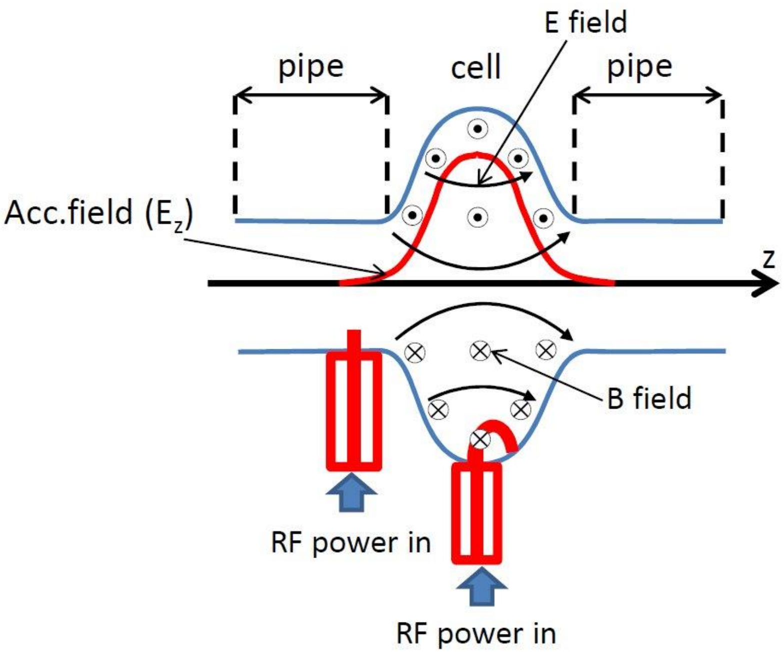



$$f_{res} = \frac{c}{2.405a}$$

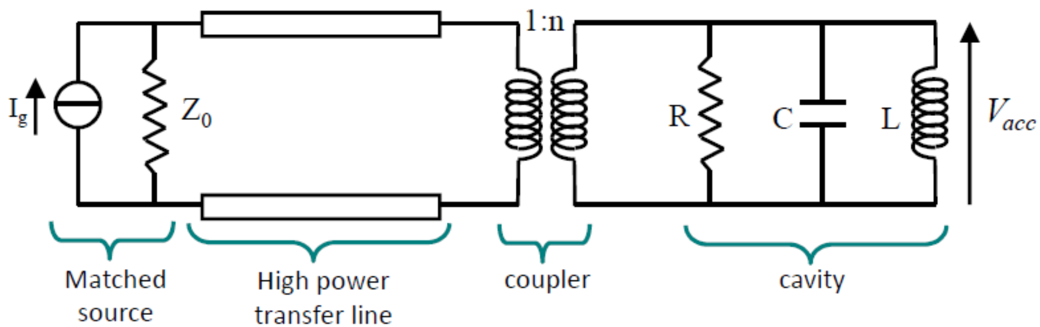
$$E_z = AJ_0\left(2.405\frac{r}{a}\right)\cos(\omega_{RF}t) \qquad H_\theta = A\frac{1}{Z_0}J_1\left(2.405\frac{r}{a}\right)\sin(\omega_{RF}t)$$

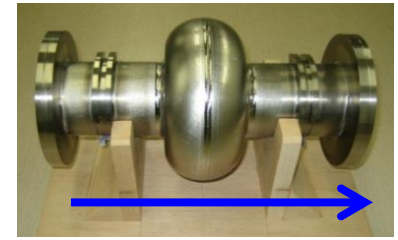
## Real cylindrical cavity

($TM_{010}$-like mode because of the shape and presence of beam tubes)



# SW CAVITIES PARAMETERS: R, Q



$V_{acc}$

***ACCELERATING VOLTAGE ($V_{acc}$)***

***DISSIPATED POWER ($P_{diss}$)***

***STORED ENERGY (W)***

## *SHUNT IMPEDANCE*

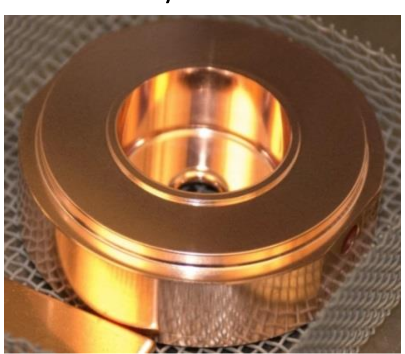
The shunt impedance is the parameter that qualifies the **efficiency of an accelerating mode**. The higher is its value, the larger is the obtainable accelerating voltage for a given power. Traditionally, it is the quantity to optimize in order to **maximize the accelerating field for a given dissipated power**:

$$R = \frac{\hat{V}_{acc}^2}{P_{diss}} \quad [\Omega]$$

***SHUNT IMPEDANCE PER UNIT LENGTH***

$$r = \frac{\left(\hat{V}_{acc}/L\right)^2}{P_{diss}/L} = \frac{\hat{E}_{acc}^2}{p_{diss}} \quad [\Omega/m]$$

NC cavity R~1MΩ



SC cavity R~1TΩ

## *QUALITY FACTOR*

$$Q = \omega_{RF} \frac{W}{P_{diss}}$$

NC cavity Q~$10^4$

SC cavity Q~$10^{10}$

***Example:***

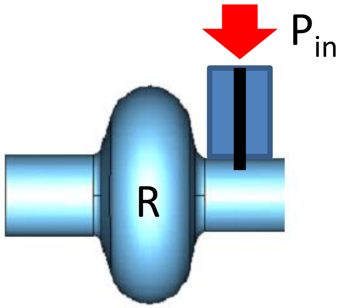
R~1MΩ

$P_{diss}$=1 MW

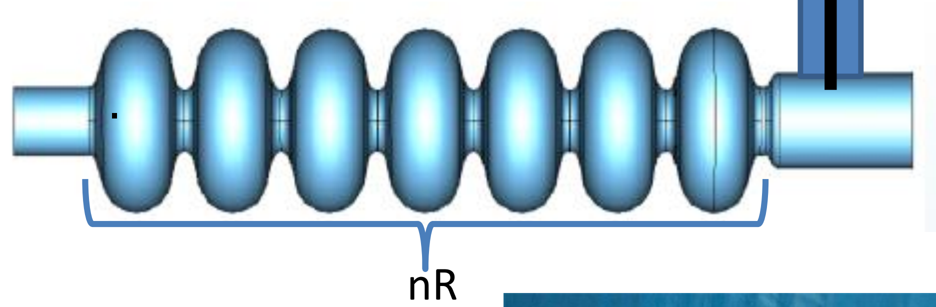
$V_{acc}$=1MV

For a cavity working at 1 GHz with a structure length of 10 cm we have an average accelerating field of 10 MV/m

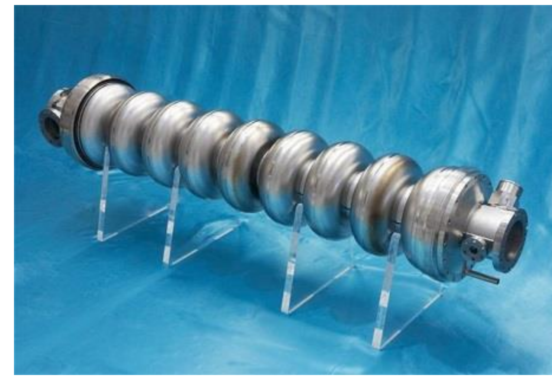
# MULTI-CELL SW CAVITIES




***(electrons or protons and ions at high energy)***




- In a multi-cell structure there is **one RF input coupler**. As a consequence the **total number of RF sources is reduced**, with a **simplification of the layout and reduction of the costs;**
- The **shunt impedance is n time** the impedance of a single cavity
- They are **more complicated** to fabricate than single cell cavities;
- The fields of adjacent cells couple through the cell **irises** and/or through properly designed coupling **slots**.



# MULTI-CELL SW CAVITIES: π MODE STRUCTURES

*(electrons or protons and ions at high energy)*

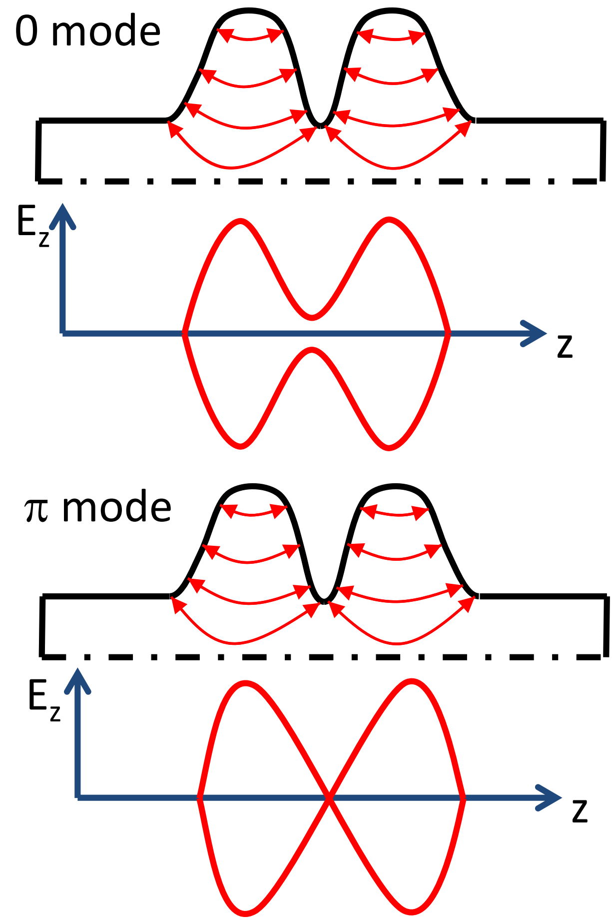
- The N-cell structure behaves like a system composed by **N coupled oscillators** with **N coupled multi-cell resonant modes**.

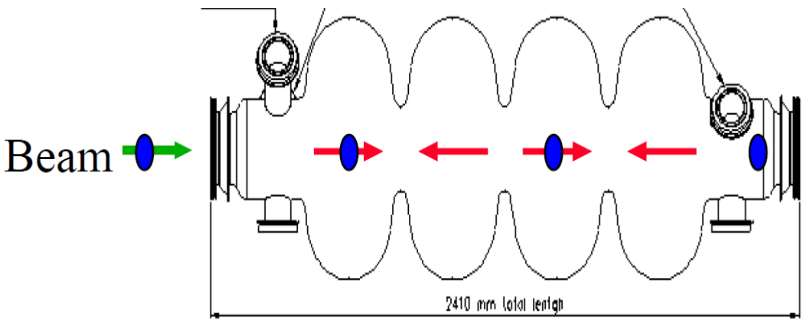
- The modes are characterized by a cell-to-cell phase advance given by:

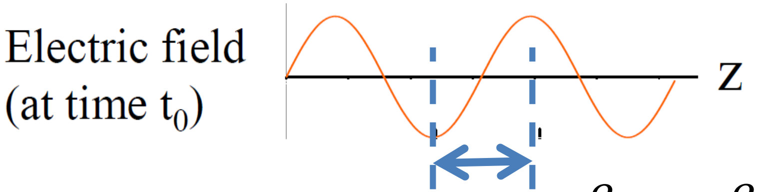
$$\Delta\phi_n = \frac{n\pi}{N-1} \qquad n = 0,1,\ldots,N-1$$

- The multi cell mode generally used for acceleration is the **π, π/2 and 0 mode** (DTL as example operate in the 0 mode).

- In this case as done for the DTL structures the cell length has to be chosen in order to synchronize the accelerating field with the particle traveling into the structure at a certain velocity



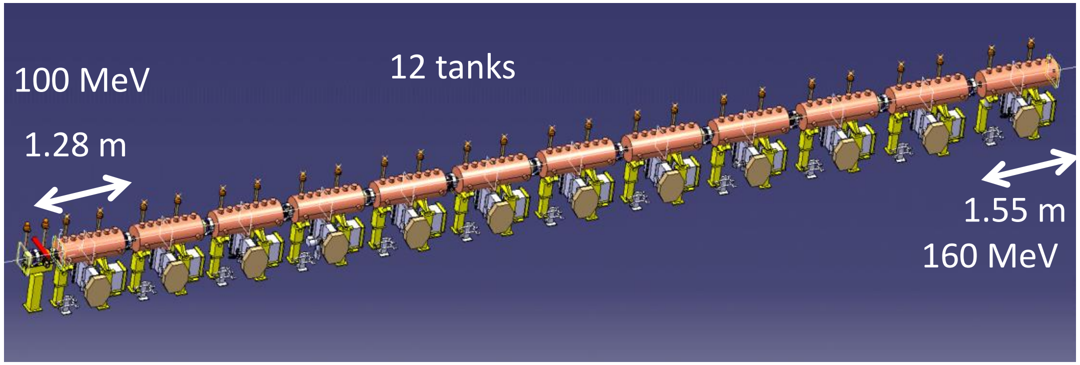

$$d = \frac{\beta c}{2 f_{RF}} = \frac{\beta \lambda_{RF}}{2}$$

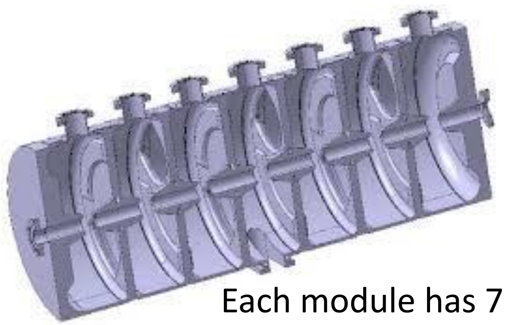
⇒For **ions and protons** the cell length has to be increased and the linac will be made of a sequence of different accelerating structures matched to the ion/proton velocity.

⇒For **electron**, β=1, $d=\lambda_{RF}/2$ and the linac will be made of an injector followed by a series of identical accelerating structures, with cells all the same length.

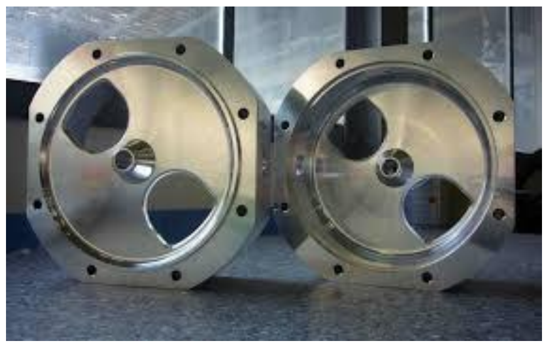
# π MODE STRUCTURES: EXAMPLES

***LINAC 4 (CERN) PIMS (PI Mode Structure) for protons: $f_{RF}$=352 MHz, $\beta$>0.4***



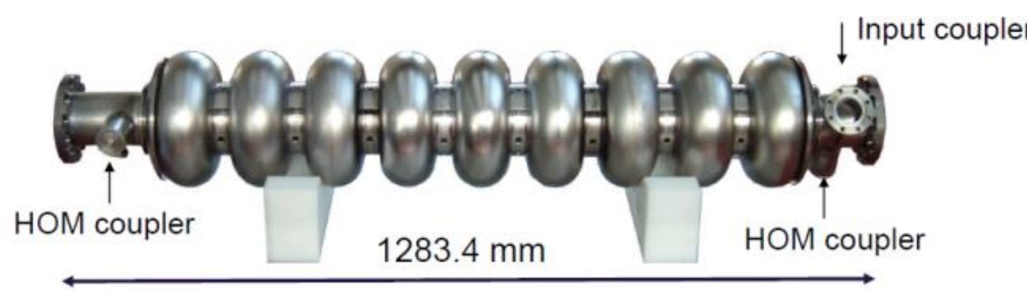
Each module has 7 identical cells



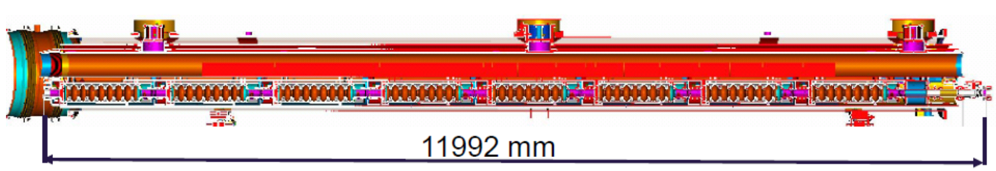
***European XFEL (Desy): electrons***

**800 accelerating cavities**

**1.3 GHz / 23.6 MV/m**




Cryomodule housing: 8 cavities, quadrupole and BPM




All identical β=1
Superconducting cavities

# MULTI-CELL SW CAVITIES: $\pi/2$ MODE STRUCTURES

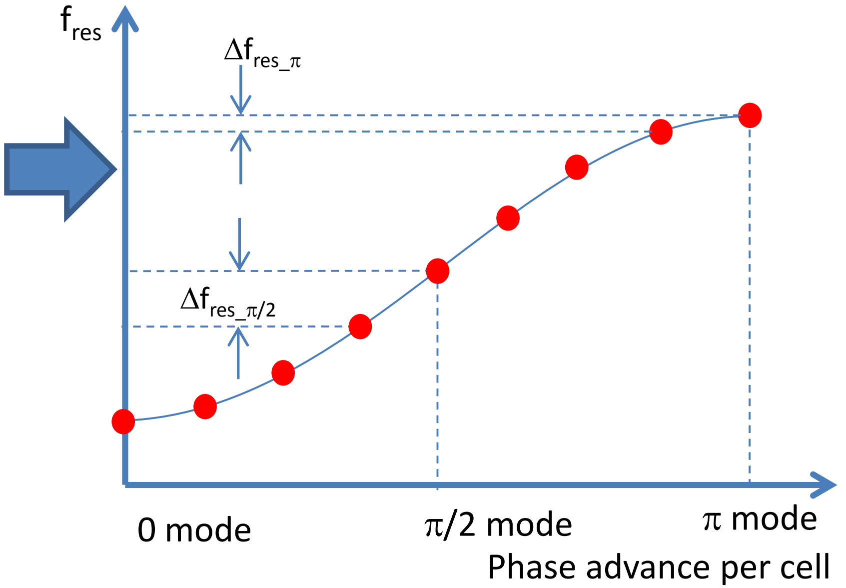
*(electrons or protons and ions at high energy)*

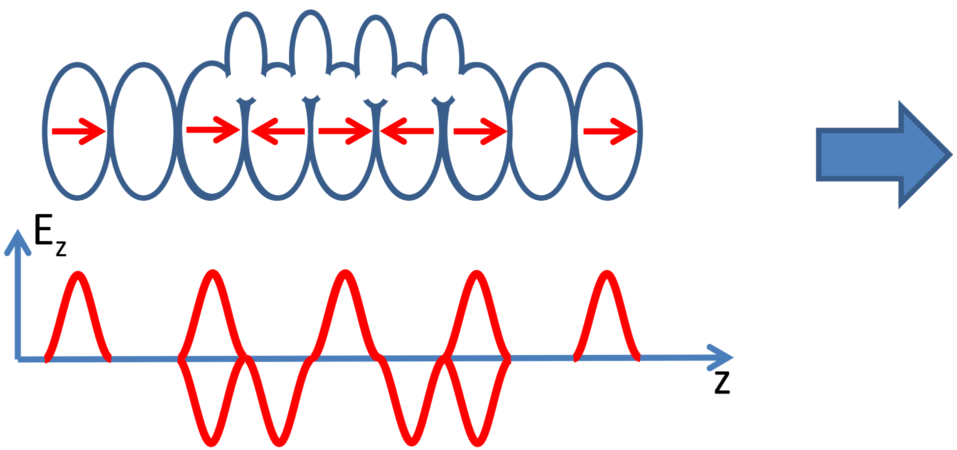
⇒It is possible to demonstrate that **over a certain number of cavities** (>10) working on the $\pi$ mode, the **overlap between adjacent modes** can be a problem (as example the field uniformity due to machining errors is difficult to tune).

⇒The criticality of a working mode depend on the **frequency separation between the working mode and the adjacent mode**

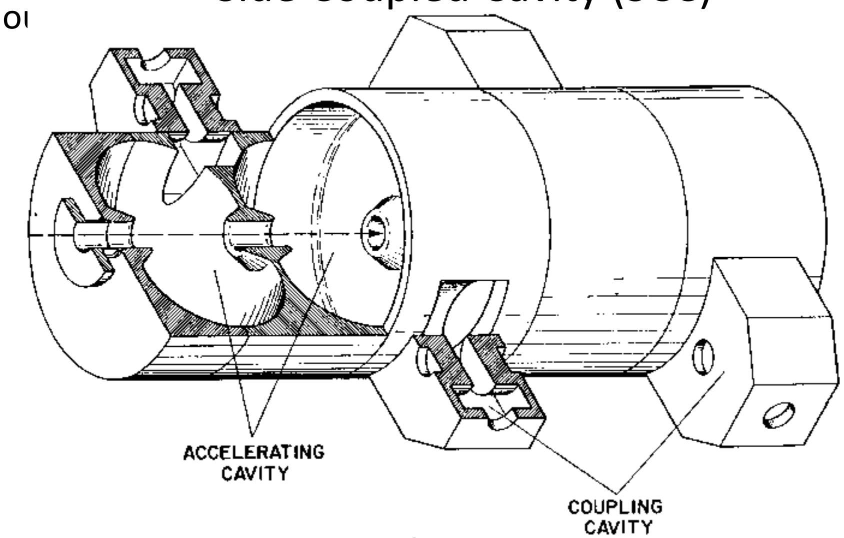
⇒the **$\pi/2$ mode from this point of view is the most stable mode**. For this mode it is possible to demonstrate that the accelerating field is zero every two cells. For this reason the empty cells are put of axis and coupling slots are opened from the accelerating cells to the empty cells.

⇒this allow to increase the number of cells to >20-30 withou problems



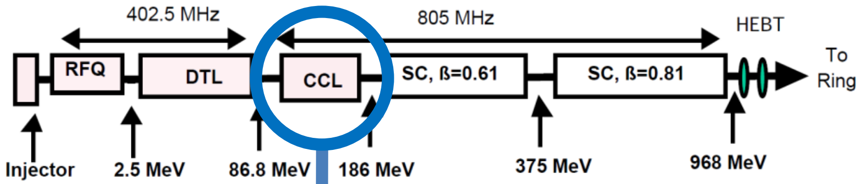

Side Coupled Cavity (SCC)



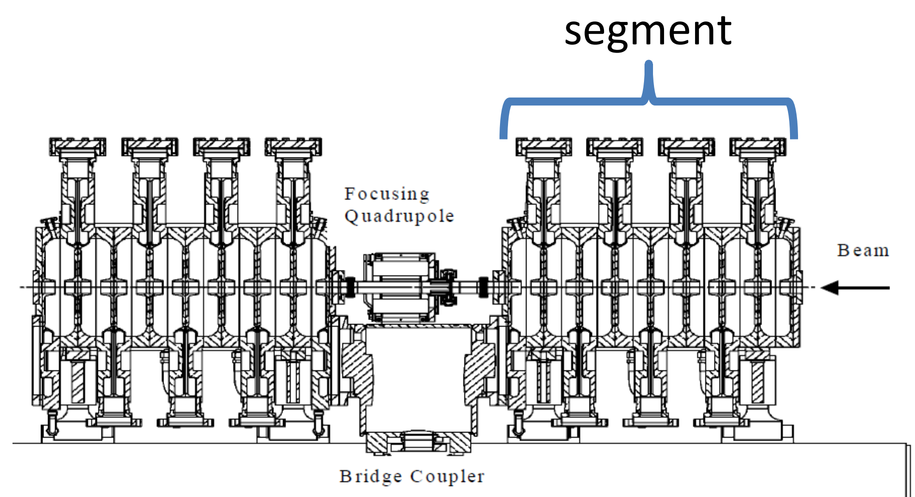

$f_{RF}$=800 - 3000 MHz for proton ($\beta$=0.5-1) and electrons

# SCC STRUCTURES: EXAMPLES

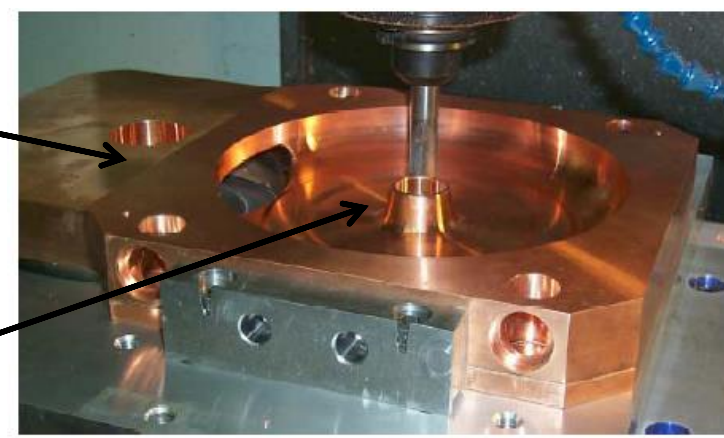
Spallation Neutron Source Coupled Cavity Linac (protons)




4 modules, each containing 12 accelerator segments CCL and 11 bridge couplers. The CCL section is a RF Linac, operating at **805 MHz** that accelerates the beam **from 87 to 186 MeV** and has a physical installed length of slightly over **55 meters.**



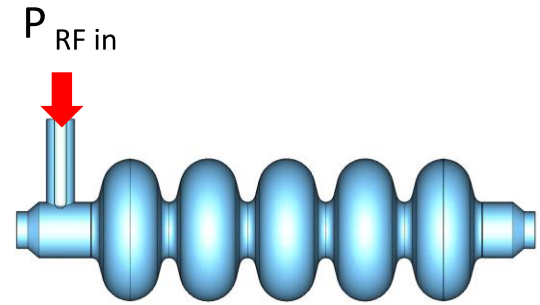

Coupling cell



Accelerating cell

# TRAVELLING WAVE (TW) STRUCTURES

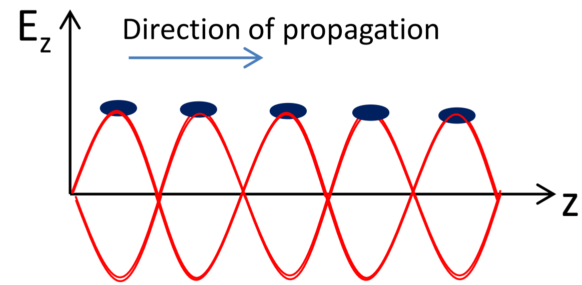
## *(electrons)*




⇒To accelerate charged particles, the electromagnetic field must have an **electric field along the direction of propagation of the particle**.

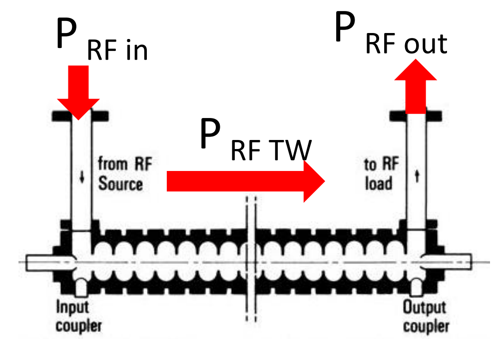
⇒The field has to be synchronous with the particle velocity.

⇒Up to now we have analyzed the cases standing **standing wave (SW)** structures in which the field has basically a given profile and oscillate in time (as example in DTL or **resonant cavities operating on the** $TM_{010}$-like).

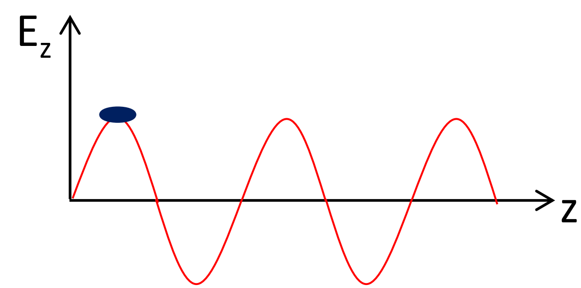



$$E_z(z,t) = \underbrace{E_{RF}(z)}_{\textit{field profile}}\overbrace{\cos(\omega_{RF}t)}^{\textit{Time oscillation}}$$

⇒There is another possibility to accelerate particles: using a **travelling wave (TW)** structure in which the RF wave is **co-propagating** with the beam with a **phase velocity equal to the beam velocity**.




⇒Typically these structures **are used for electrons** because in this case the **phase velocity can be constant** all over the structure and equal to c. On the other hand it is difficult to modulate the phase velocity itself very quickly for a low β particle that changes its velocity during acceleration.

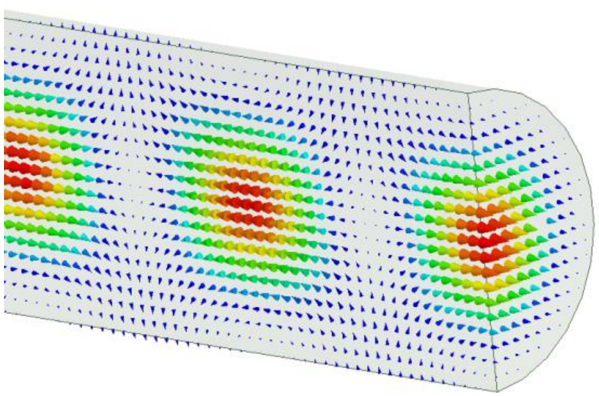


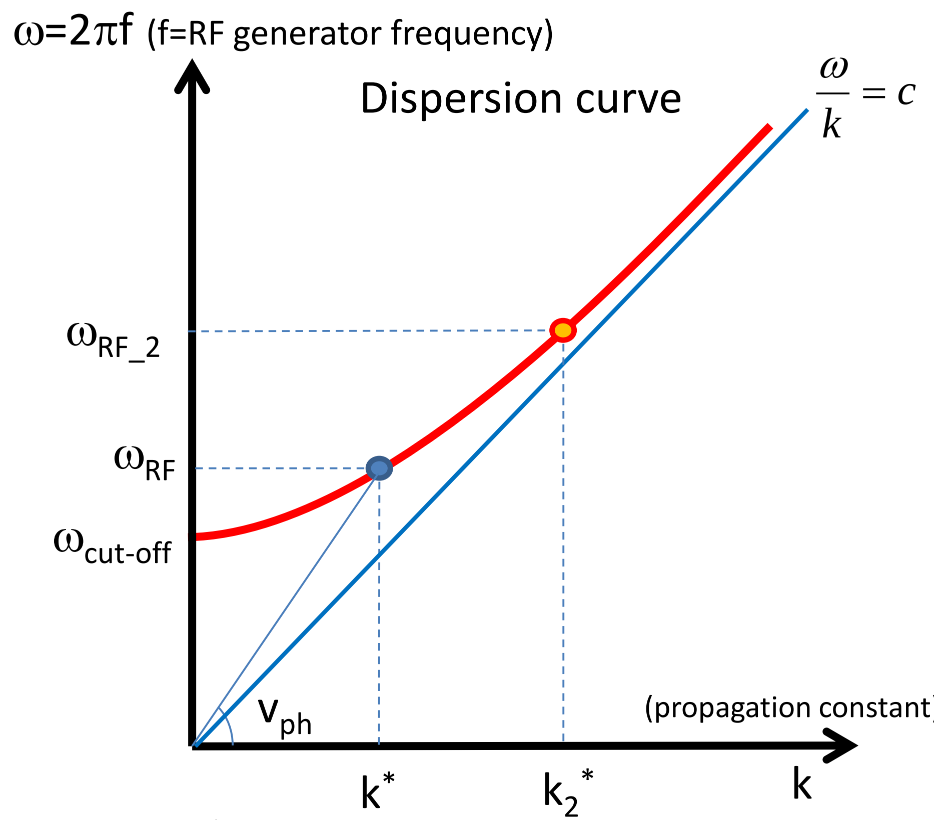
# TW CAVITIES: CIRCULAR WAVEGUIDE AND DISPERSION CURVE

***(electrons)***

In **TW structures** an e.m. wave with **$E_z \neq 0$** travel together with the beam in a special guide in which the **phase velocity of the wave matches the particle velocity (v)**. In this case the beam absorbs energy from the wave and it is **continuously accelerated**.

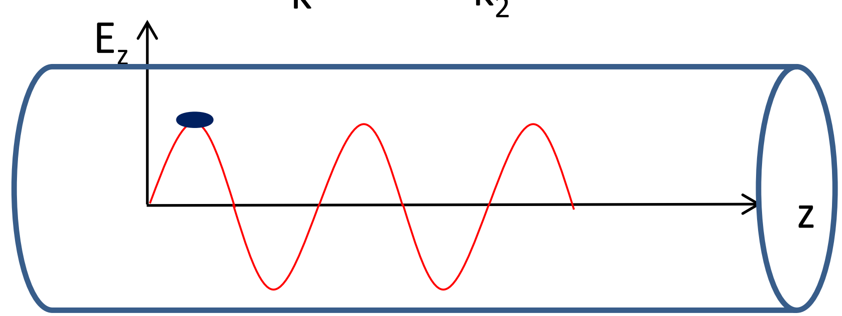
***CIRCULAR WAVEGUIDE***






As example if we consider a simple circular waveguide the first propagating mode with **$E_z \neq 0$** is the $TM_{01}$ mode. Nevertheless by solving the wave equation it turns out that an e.m. wave propagating in this **constant cross section waveguide** will **never be synchronous with a particle beam** since the **phase velocity is always larger than the speed of light c**.

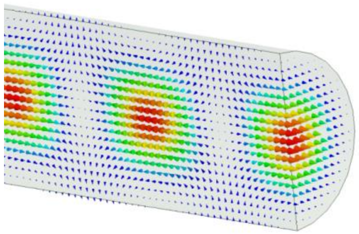
$$E_z\big|_{TM_{01}} = \underbrace{E_0(r)}_{J_0\left(\frac{p_{01}}{a} r\right)} \cos\left(\omega_{RF} t - k^* z\right) \quad \Rightarrow \quad v_{ph} = \frac{\omega_{RF}}{k^*} > c$$



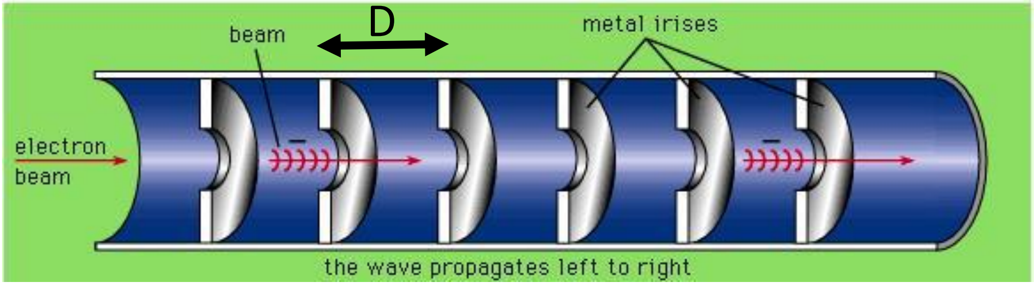
# TW CAVITIES: IRIS LOADED STRUCTURES
*(electrons)*

In order to **slow-down the wave phase velocity, iris-loaded periodic structure have to be used**.

***CIRCULAR WAVEGUIDE***



***IRIS LOADED STRUCTURE***

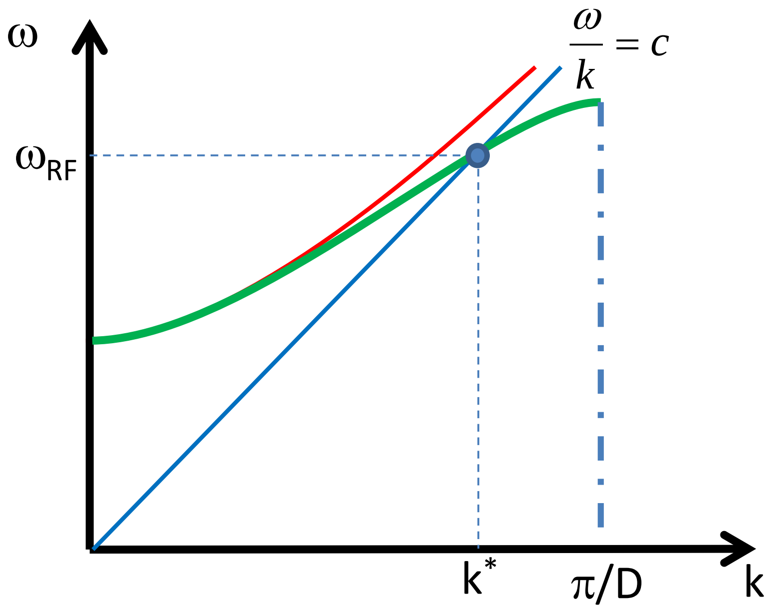





MODE $TM_{01}$ → MODE $TM_{01}$-like

$$E_z\big|_{TM_{01}} = E_0(r)\cos(\omega_{RF}t - k^* z) \longrightarrow E_z\big|_{TM_{01}-like} = \overbrace{\hat{E}_{acc}(r,z)}^{\text{Periodic in z of period D}}\cos(\omega_{RF}t - k^* z)$$

⇒The field in this kind of structures is that of a special wave travelling within a spatial periodic profile.

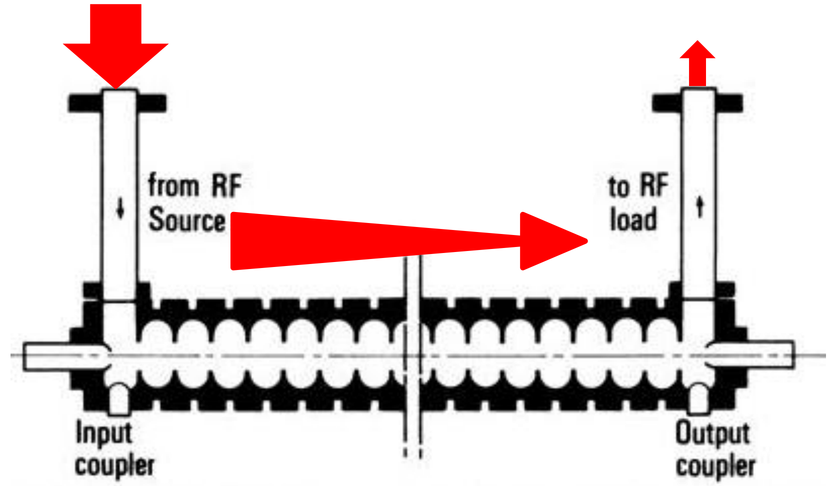


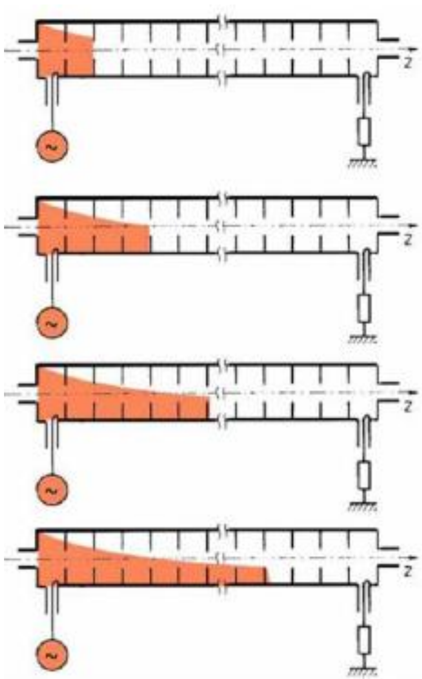

⇒The structure can be designed to have the **phase velocity equal to the speed of the particles**.

⇒This allows **acceleration over large distances** (few meters, hundred of cells) with just an input coupler and a relatively **simple geometry**.

# APPENDIX: TW CONSTANT GRADIENT STRUCTURES
## *(electrons)*

In a TW structure, the **RF power enters** into the cavity through an **input coupler**, flows (travels) through the cavity in the same direction as the beam and an **output coupler at the end** of the structure is connected to a **matched power load**.
If there is no beam, the input power reduced by the cavity losses goes to the power load where it is dissipated.
In the presence of a large beam current, however, a fraction of the TW power is transferred to the beam.

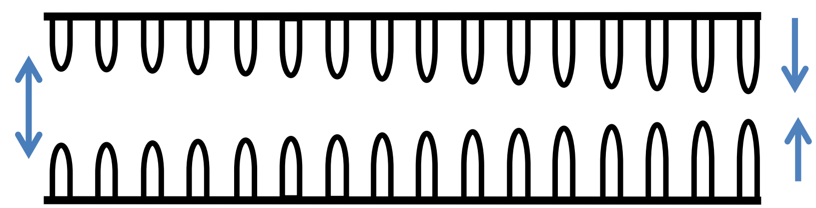


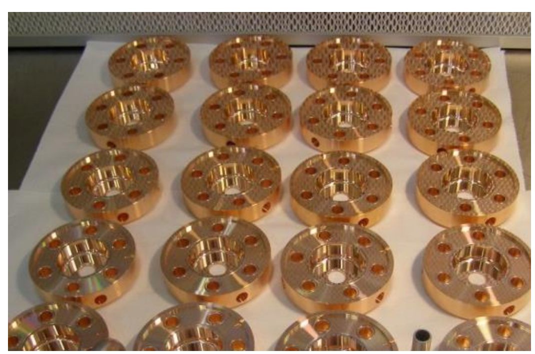



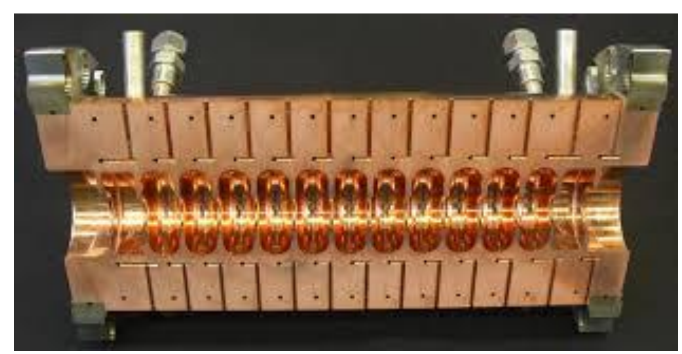
In a purely periodic structure, made by a sequence of **identical cells** (also called "**constant impedance structure**"), the RF power flux and the intensity of the accelerating field decay exponentially along the structure :

$$\hat{E}_{acc}(z) = E_0 e^{-\alpha z}$$

It is possible to demonstrate that, in order to keep the **accelerating field constant** along the structure, the **iris apertures have to decrease along the structure.**

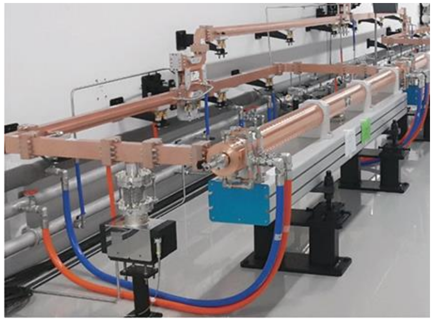






# LINAC TECHNOLOGY

# ACCELERATING CAVITY TECHNOLOGY



⇒The structures are powered by RF generators (like **klystrons**).

⇒The cavities (and the related LINAC technology) can be of different material:

- **copper** for **normal conducting (NC, both SW than TW)** cavities;
- **Niobium** for **superconducting cavities (SC, SW)**;

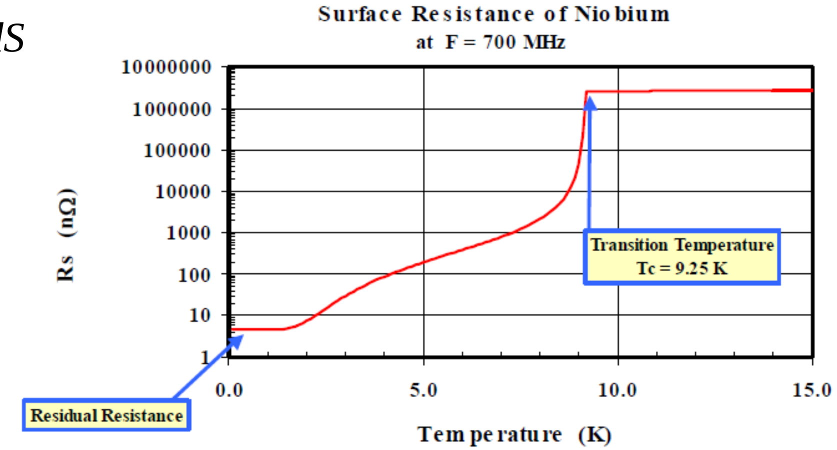
⇒We can choose between NC or the SC technology depending on the required performances in term of:

- **accelerating gradient** (MV/m);
- **RF pulse length** (how many bunches we can contemporary accelerate);
- **Duty cycle**: pulsed operation (i.e. 10-100 Hz) or continuous wave (CW) operation;
- **Average beam current**.
- …

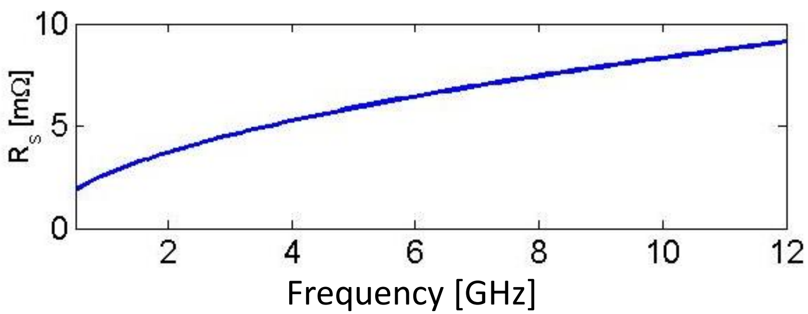
Dissipated power into the cavity walls is related to the surface currents

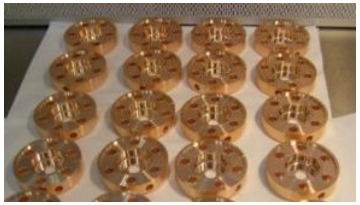
$$P_{diss} = \int_{\substack{cavity\\ wall}} \overbrace{\frac{1}{2} R_s H_{tan}^2}^{\text{power density}} dS$$

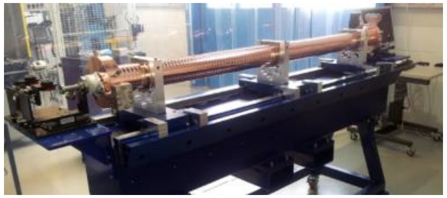
## NIOBIUM




## COPPER



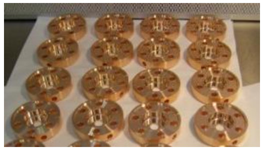





**Between copper and Niobium there is a factor $10^5$-$10^6$**

# APPENDIX: NC AND SC MATERIALS

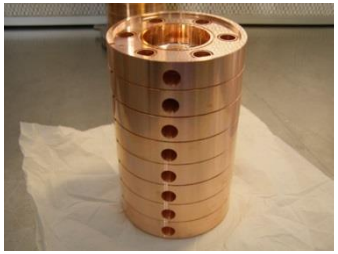
## *NC: COPPER*

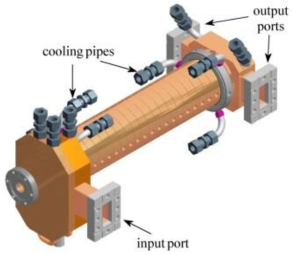


The most widely used NC metal for RF structures is **OFHC copper** (Oxigen free high conductivity) for several reasons:

1) Easy to machine (good achievable roughness at the few nm level)
2) Easy to braze/weld
3) Easy to find at relatively low cost
4) Very good electrical (and thermal) conductivity
5) Low SEY (multipacting phenomena)
6) Good performances at high accelerating gradient

-**Higher dissipation**
-**Pulsed** operation
-**Higher peak accelerating gradient** (up to 50-100 MV/m)
-**Standard cleaning** procedures for the cavity fabrication
-**Cooling** of dissipated power with pipes






## *SC: NIOBIUM*

The **most common material for SC cavities is Nb** because:

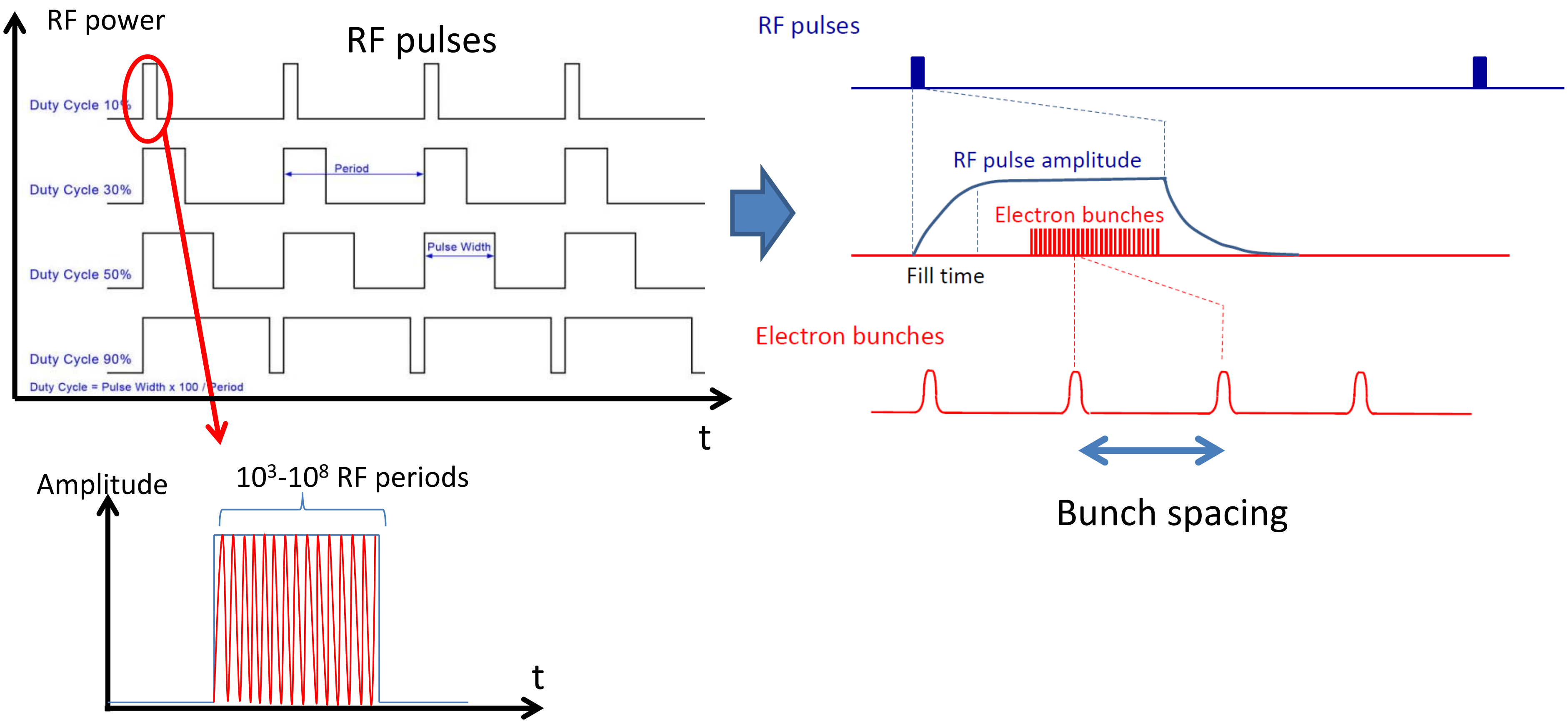
1) Nb has a relatively **high transition temperature** (Tc=9.25 K).
2) SC can be destroyed by magnetic field greater than a critical field $H_c \Rightarrow$ Pure Nb has a **relatively high critical magnetic field Hc**=170-180 mT.
3) It is chemically inert
4) It can be **machined and deep-drawn**
5) It is available as bulk and sheet material in any size, fabricated by forging and rolling….

**-lower dissipation**
-Allow **continuous operation**
-**lower peak accelerating gradient** (max 30-40 MV/m)
-**Special cleaning procedures** for the cavity fabrication
-They need a **cryostat** to reach the SC temperature of few K

# RF STRUCTURE AND BEAM STRUCTURE

The “**beam structure**” in a LINAC is directly related to the “**RF structure**”. There are basically two possible type of operations:

- CW (continuous wave) ⇒ allow, in principle, to operate with a continuous beam
- PULSED OPARATION ⇒ there are RF pulses at a certain repetition rate (**Duty Cycle (DC)=pulsed width/period**)



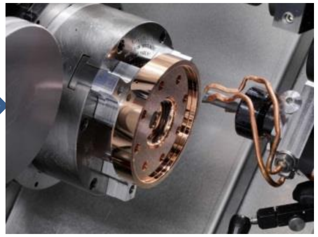

⇒Because of the very low power dissipation and low RF power required to achieve a certain accelerating voltage the **SC structures allow operation at very high Duty Cycle (DC) up to a CW operation with high gradient (>20 MV/m)**.

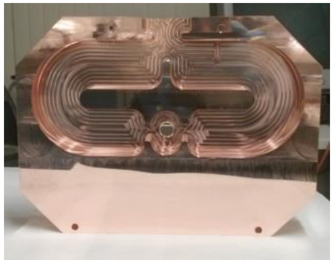
⇒On the other hand **NC structures can operate in pulsed mode** at very low DC with **higher peak field** (TW structures can >50-80 MV/m peak field).

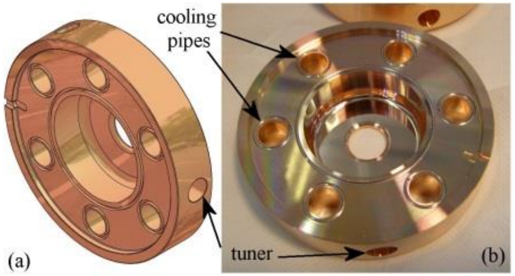
⇒NC structures can also operate in CW but at very low gradient because of the dissipated power.

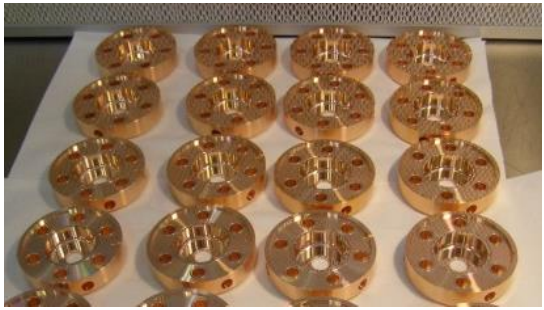
# APPENDIX: FABRICATION PROCESS TW STRUCTURES

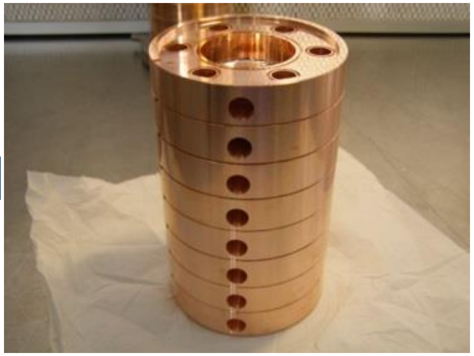
The cells and couplers are fabricated with milling machines and lathes starting from **OFHC forged or laminated copper** with precisions that can be of the order of few um and surface roughness <50 nm.



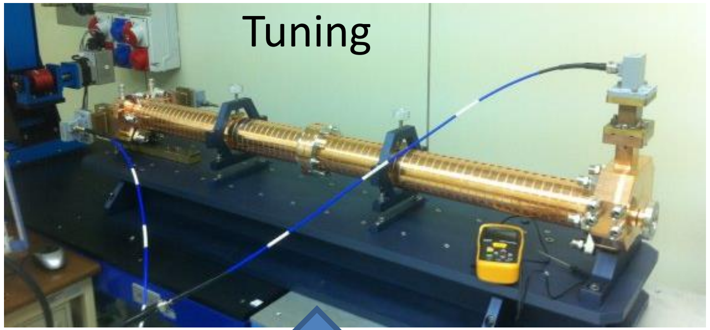




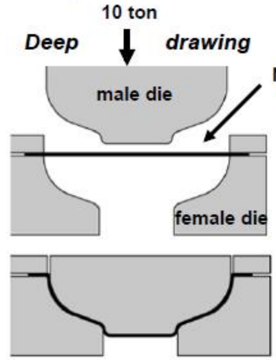


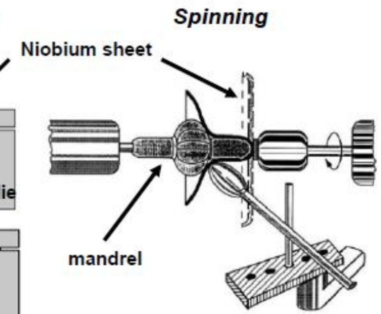
The cells are then piled up and **brazed** together in vacuum or hydrogen furnace using different alloys at different temperatures (700-1000 C) and/or in different steps.



# APPENDIX: FABRICATION PROCESS SC SW STRUCTURES

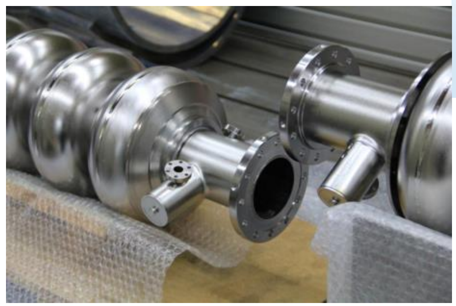
**Nb** is available as **bulk and sheet material** in any size, fabricated by forging and rolling. **High Purity Nb** is made by **electron beam melting** under good vacuum.
The most common fabrication techniques for the cavities are to **deep draw or spin half-cells**.
**Alternative techniques** are: hydroforming, spinning an entire cavity out of single sheet or tube and Nb sputtering

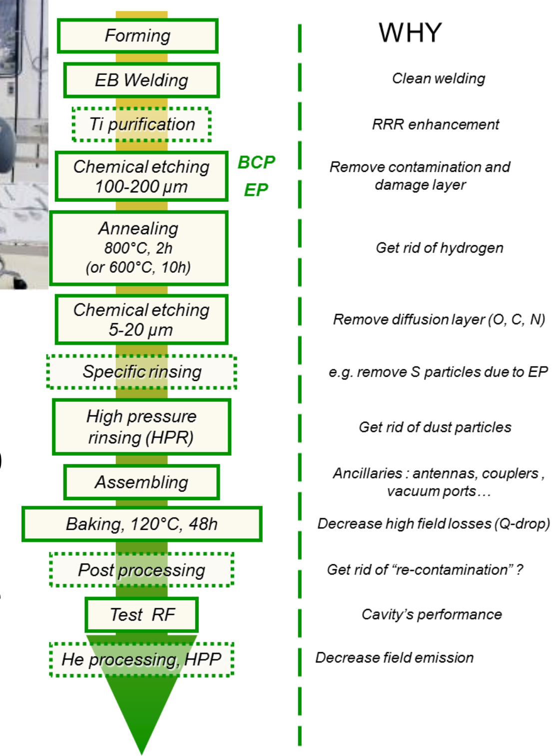


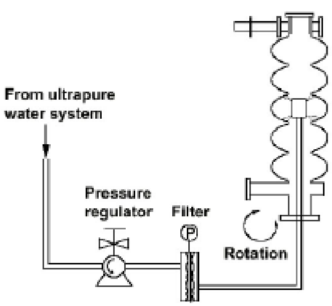





After forming the parts are **electron beam welded** together





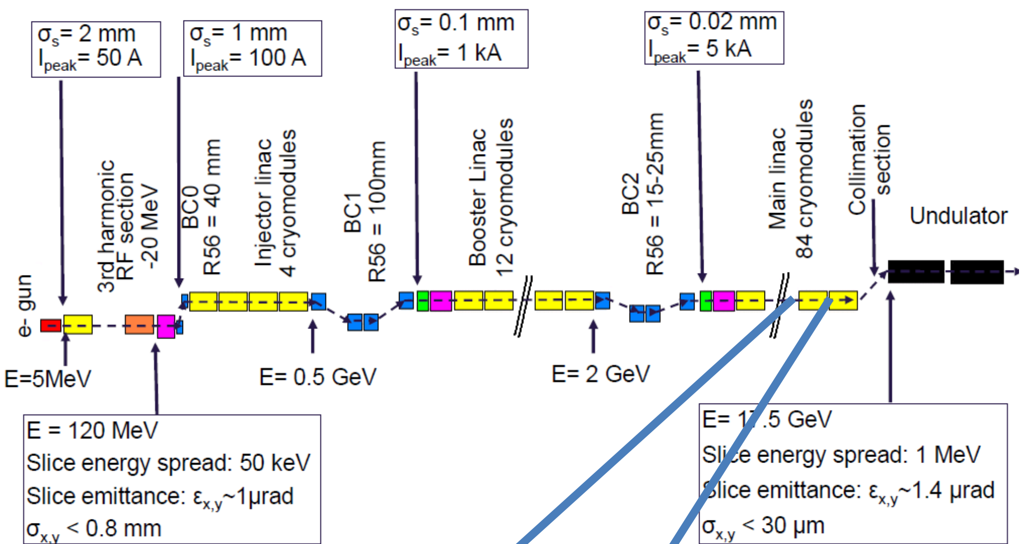




## *CAVITY TREATMENT*

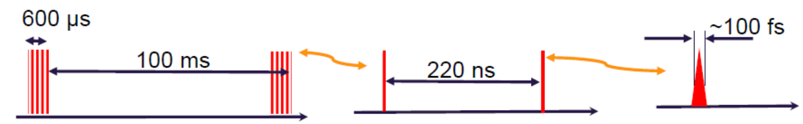
The cavity treatment after the welding is quite complicated and require several steps between:

- buffered **chemical polishing** (BCP), **electropolishing** and etching to remove surface damaged layers of the order of 100 µm
- **rinsed with ultraclean water** also at high pressure (100 bar)
- **Thermal treatments** up to >1000 C to diffuse $H_2$ out of the material increasing the Nb purity (RRR)
- **high-temperature treatment** with Ti getter (post-purification)
- RF tuning

# EXAMPLES: EUROPEAN XFEL

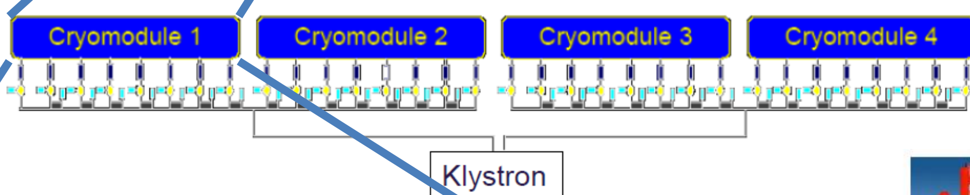



| | | |
|---|---|---|
| Nominal Energy | GeV | 17.5 |
| Beam pulse length | ms | 0.60 |
| Repetition rate | Hz | 10 |
| Max. # of bunches per pulse | | 2700 |
| Min. bunch spacing | ns | 220 |
| Bunch charge | nC | 1 |
| Bunch length, $\sigma_z$ | µm | < 20 |
| Emittance (slice) at undulator | µrad | < 1.4 |
| Energy spread (slice) at undulator | MeV | 1 |

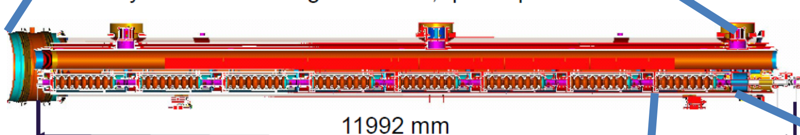



## 101 cryomodules in total

RF- system: 25 RF units. The unit = 4 cryomodules + RF-power source (klystron)




Cryomodule housing: 8 cavities, quadrupole and BPM

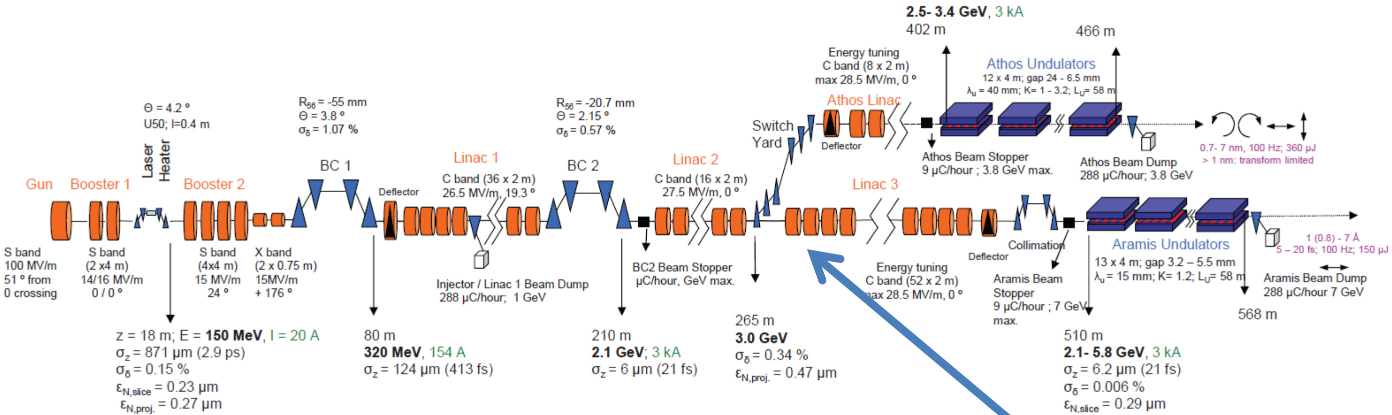


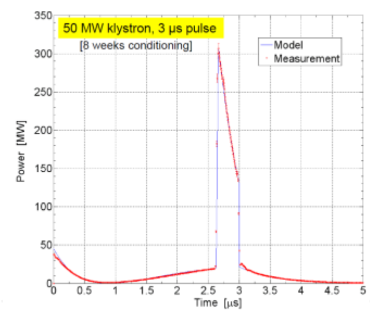

**25 RF stations**
**5.2 MW each**

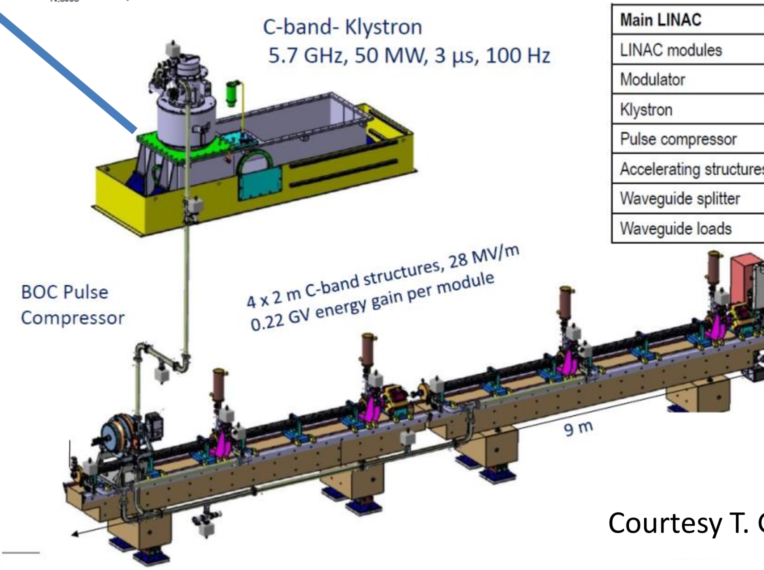
**800 accelerating cavities**

**1.3 GHz / 23.6 MV/m**

# EXAMPLE: SWISSFEL LINAC (PSI)

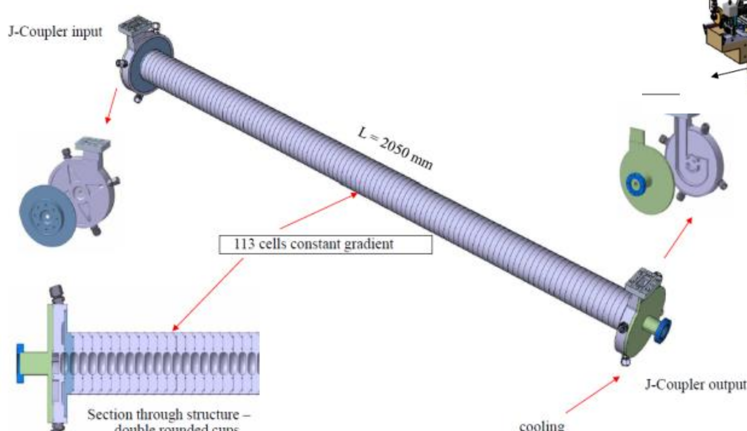


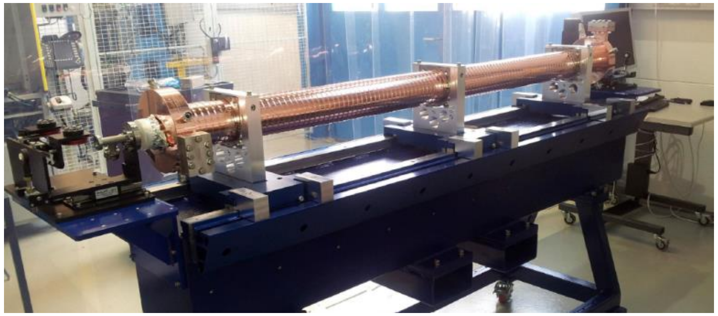




| Main LINAC | # |
|---|---|
| LINAC modules | 26 |
| Modulator | 26 |
| Klystron | 26 |
| Pulse compressor | 26 |
| Accelerating structures | 104 |
| Waveguide splitter | 78 |
| Waveguide loads | 104 |

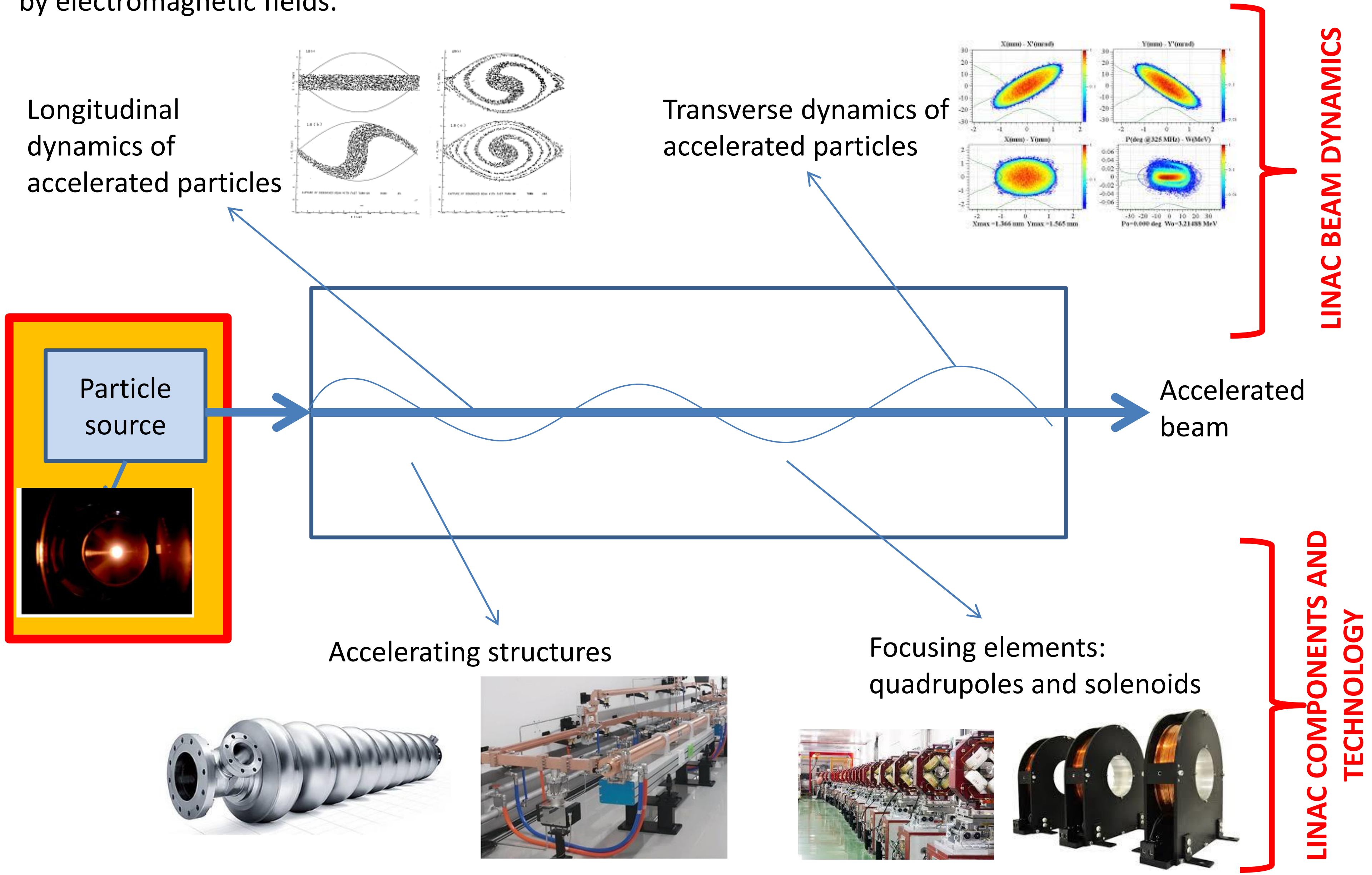
Courtesy T. Garvey

# LINAC: BASIC DEFINITION AND MAIN COMPONENTS

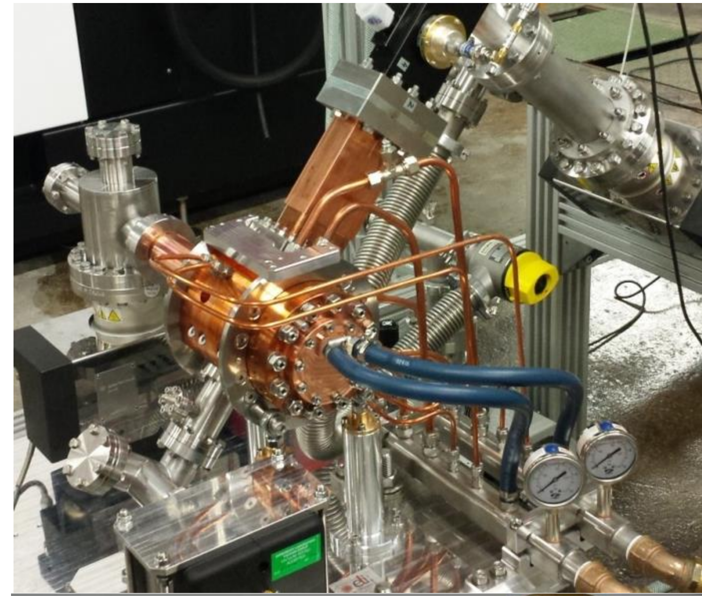
LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.

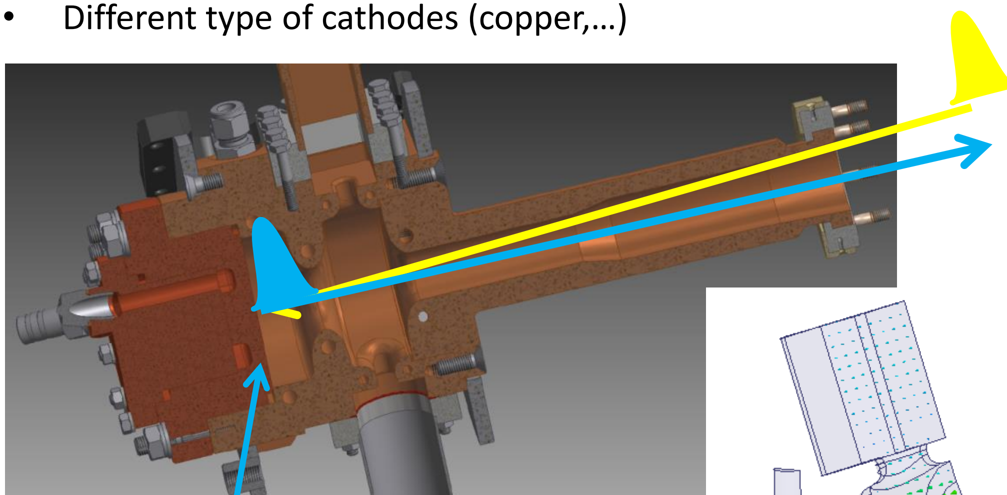


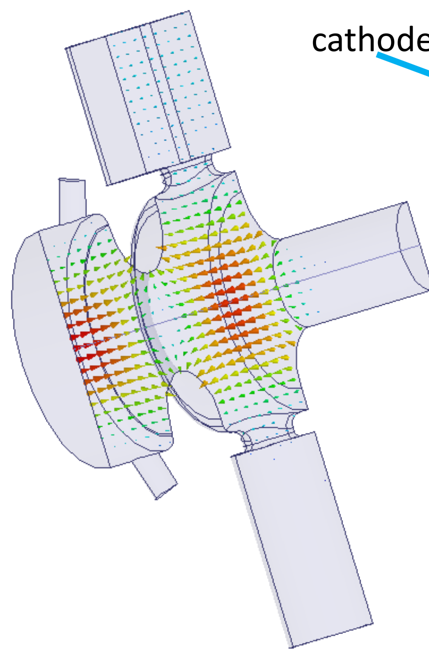
# ELECTRON SOURCES: RF PHOTO-GUNS

**RF guns are used in the first stage of electron beam generation in FEL** and acceleration.

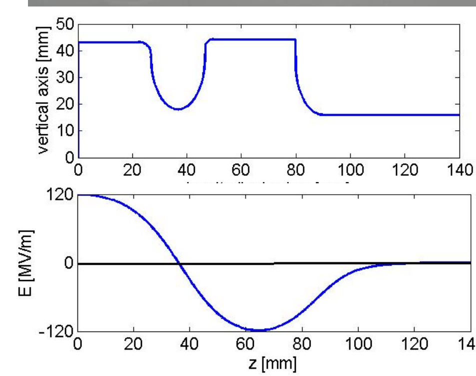
- Multi cell: typically 2-3 cells
- SW $\pi$ mode cavities
- operate in the range of 60-120 MV/m cathode peak accelerating field with up to 10 MW input power.
- Typically in L-band- S-band (1-3 GHz) at 10-100 Hz.
- Single or multi bunch (L-band)
- Different type of cathodes (copper,…)





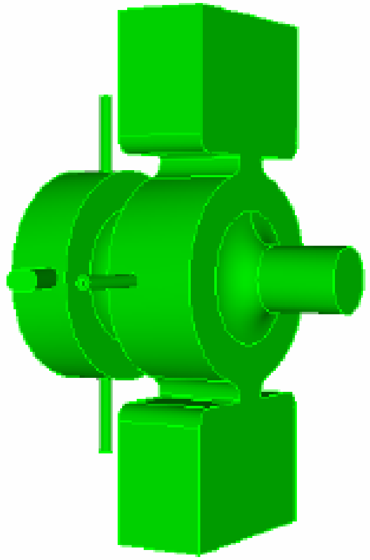


cathode




The electrons are emitted on the **cathode** through a laser that hit the surface. They are then accelerated trough the electric field that has a longitudinal component on axis $TM_{010}$.

# RF PHOTO-GUNS: EXAMPLES

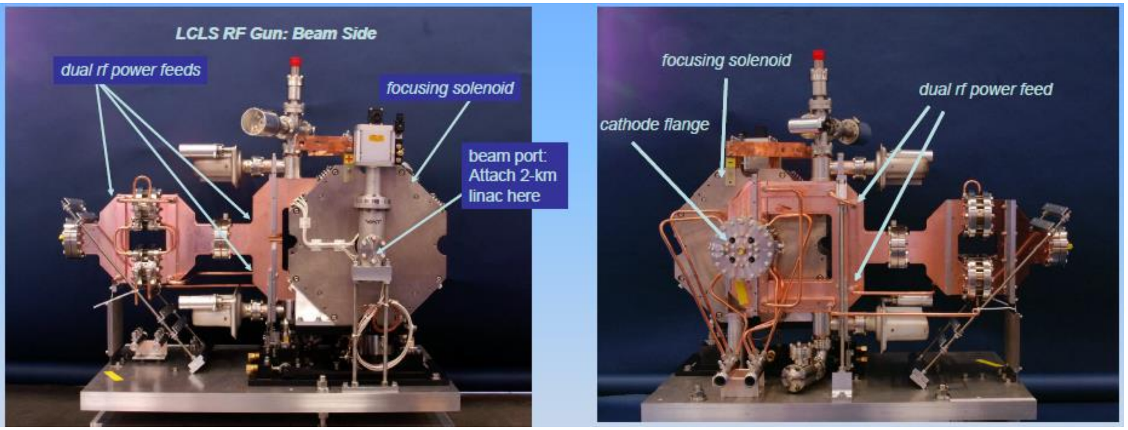


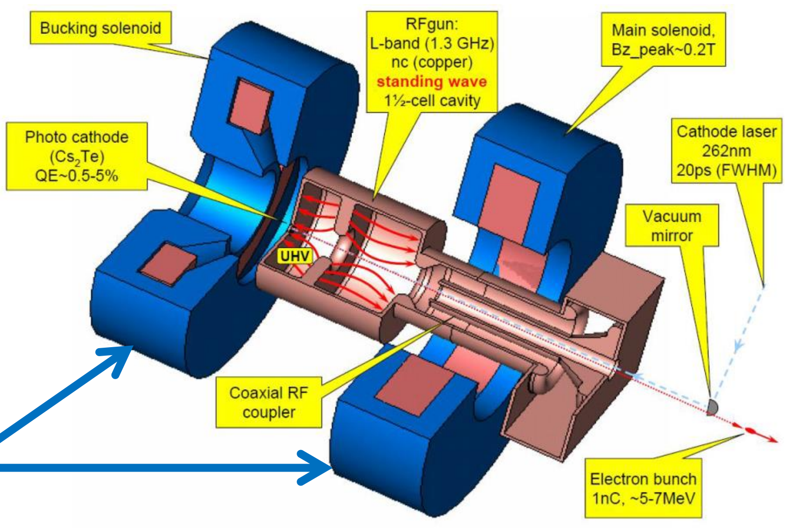
**LCLS**

Frequency = 2,856 MHz
Gradient = 120 MV/m
Exit energy = 6 MeV
Copper photocathode
RF pulse length ~2 μs
Bunch repetition rate = 120 Hz
Norm. rms emittance
0.4 mm·mrad at 250 pC

**PITZ L-band Gun**

Frequency = 1,300 MHz
Gradient = up to 60 MV/m
Exit energy = 6.5 MeV
Rep. rate 10 Hz
$Cs_2Te$ photocathode
RF pulse length ~1 ms
800 bunches per macropulse
Normalized rms emittance
1 nC 0.70 mm·mrad
0.1 nC 0.21 mm·mrad

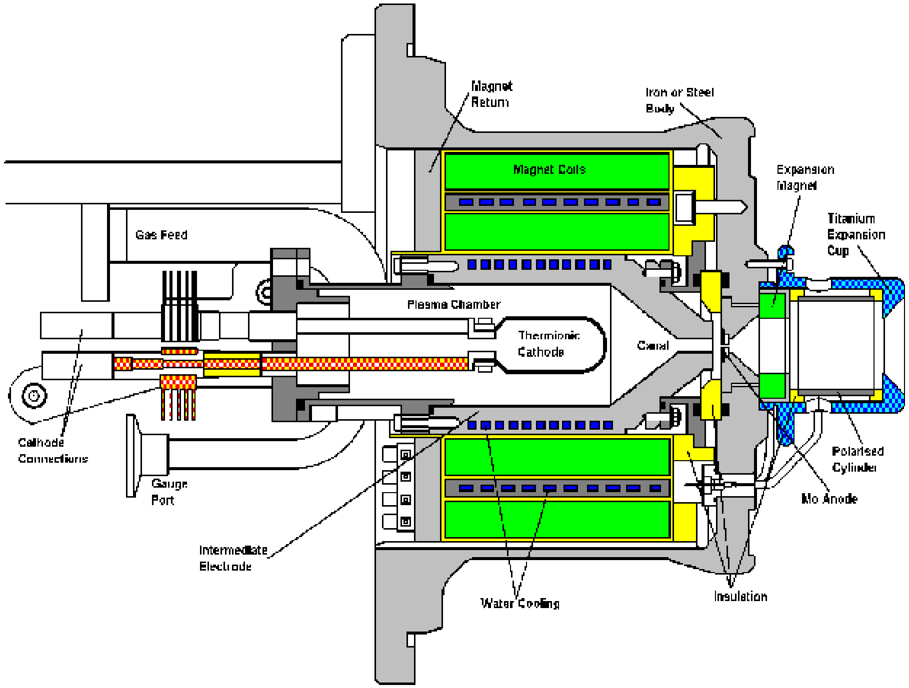


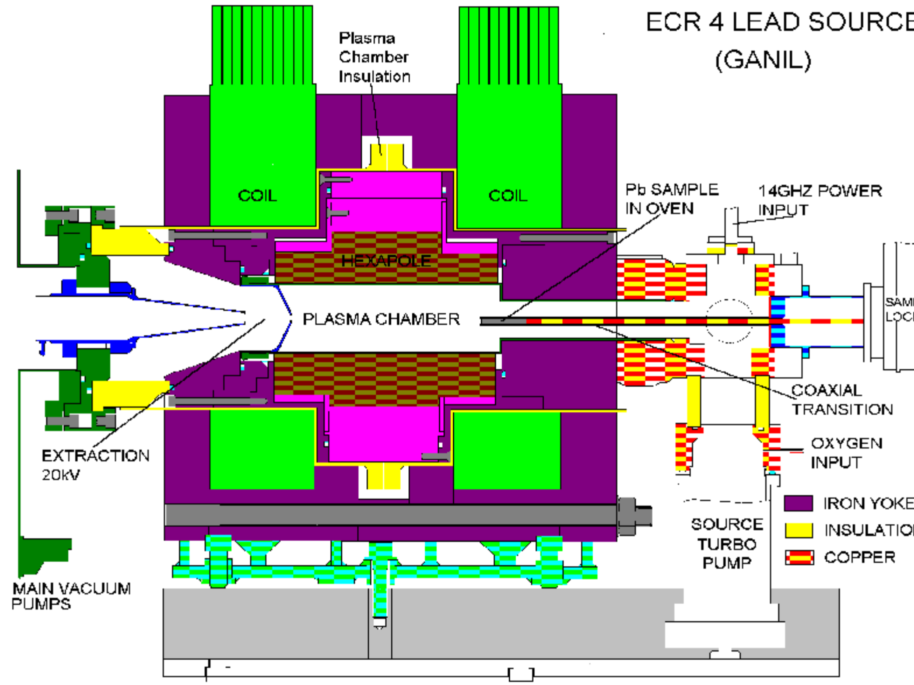

Solenoids field are used to compensate the space charge effects in low energy guns. The configuration is shown in the picture

# ION SOURCES

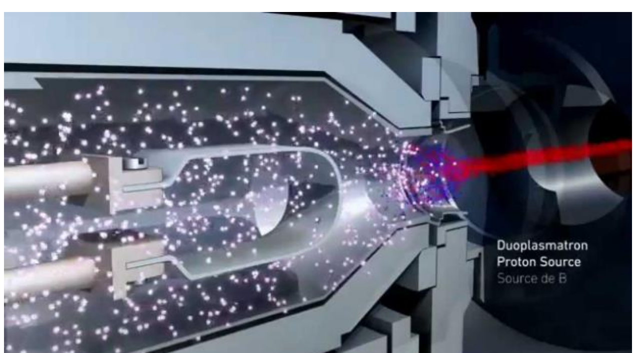
**Basic principle**: create a plasma and optimize its conditions (heating, confinement and loss mechanisms) to produce the desired ion type. Remove ions from the plasma via an aperture and a strong electric field.

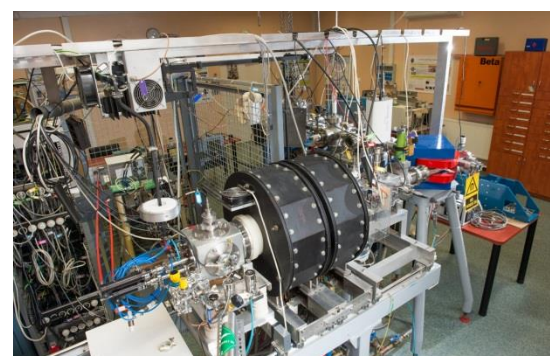



CERN Duoplasmatron proton Source




Electron Cyclotron Resonance (ECR) ECR



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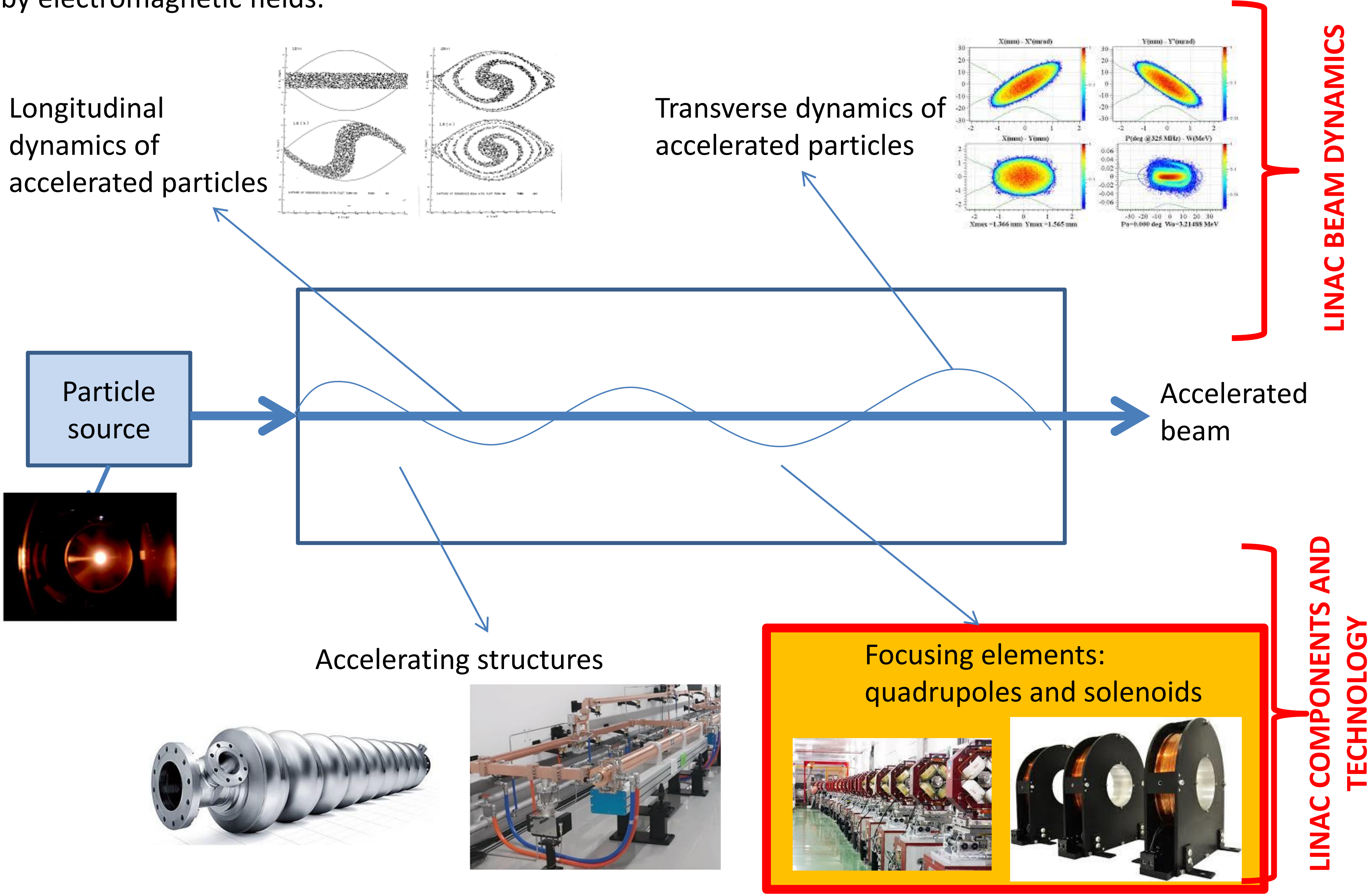


# LORENTZ FORCE: ACCELERATION AND FOCUSING

Particles are accelerated through electric field and are bended and focalized through magnetic field. The basic equation that describe the acceleration/bending /focusing processes is the **Lorentz Force**.

$\vec{p} = momentum$

$m = mass$

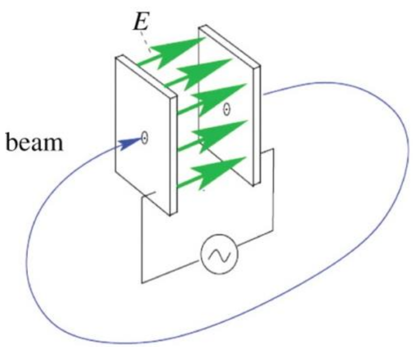
$\vec{v} = velocity$

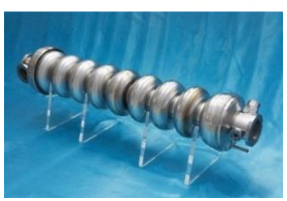
$q = charge$

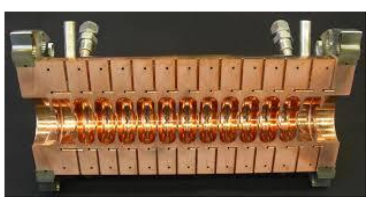
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

## ACCELERATION

To accelerate, we need a force in the direction of motion



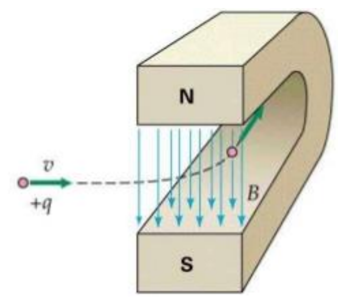



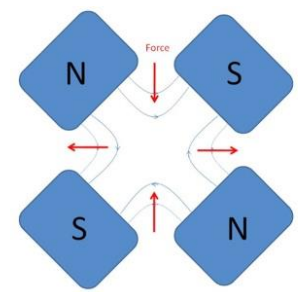


Longitudinal Dynamics

## BENDING AND FOCUSING

2nd term always perpendicular to motion => no energy gain




Transverse Dynamics

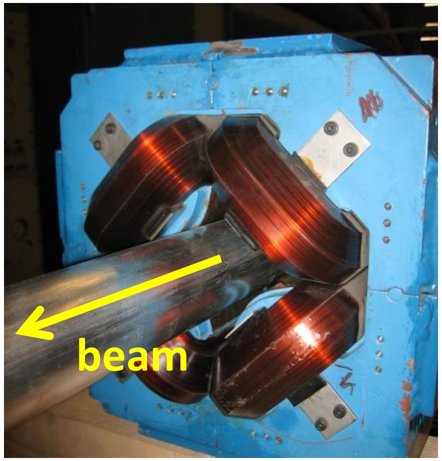
# MAGNETIC QUADRUPOLE

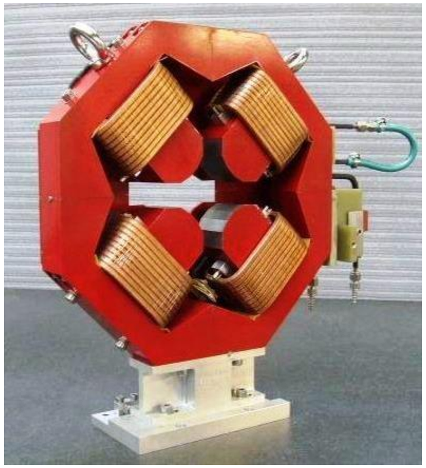
Quadrupoles are used to **focalize the beam in the transverse plane**. It is a **4 poles magnet**:

$\Rightarrow$**B=0 in the center of the quadrupole**

$\Rightarrow$The **B intensity increases linearly** with the off-axis displacement.

$\Rightarrow$If the quadrupole is **focusing in one plane is defocusing in the other plane**



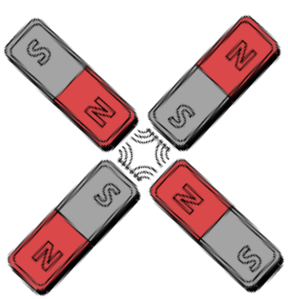



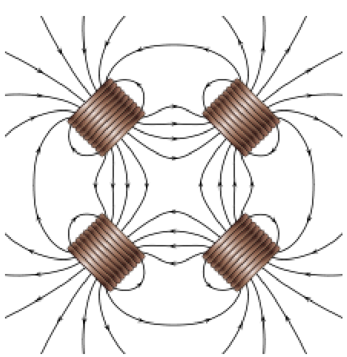
$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases} \Rightarrow \begin{cases} F_y = qvG \cdot y \\ F_x = -qvG \cdot x \end{cases}$$

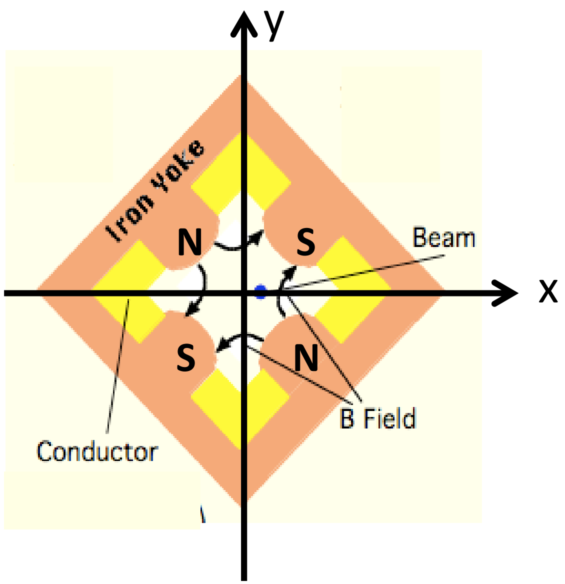
$$G = \text{quadrupole gradient} \left[ \frac{T}{m} \right]$$

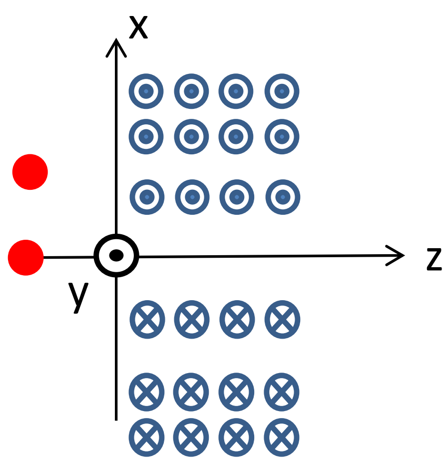
Electromagnetic quadrupoles G <50-100 T/m

$$\frac{F_B}{F_E} = v \Rightarrow \begin{cases} F_B(1T) = F_E\left(300\,\frac{MV}{m}\right) @\, \beta = 1 \\ F_B(1T) = F_E\left(3\,\frac{MV}{m}\right) @\, \beta = 0.01 \end{cases}$$



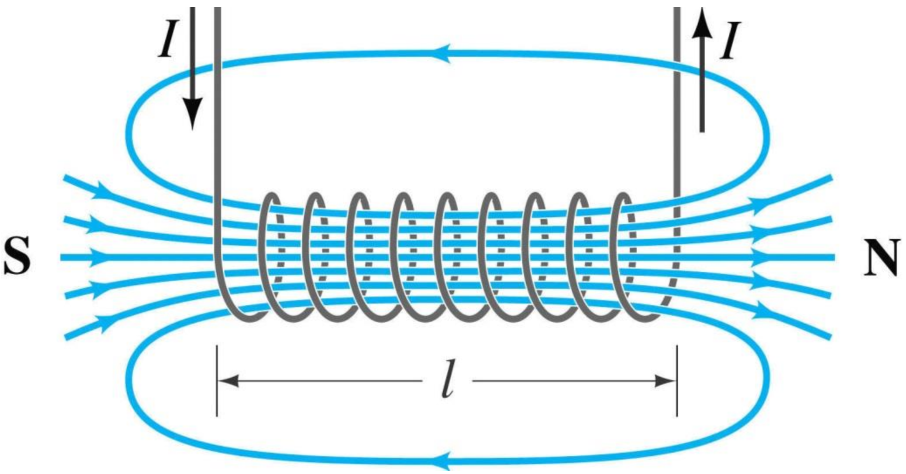



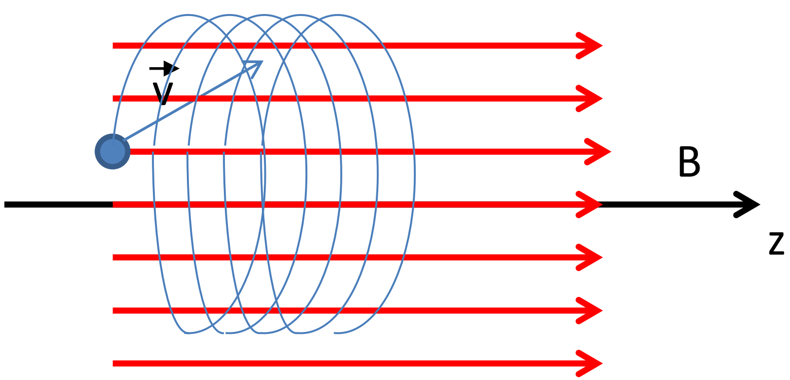


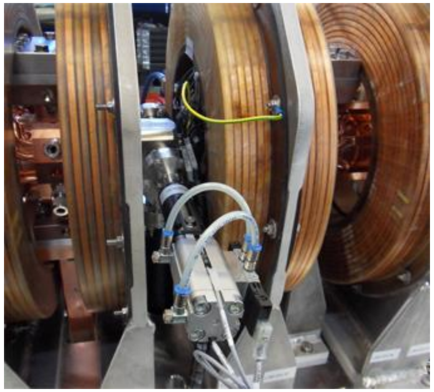



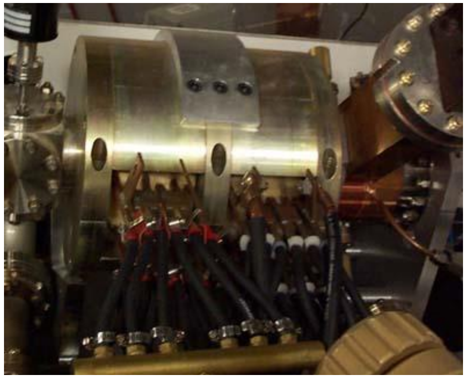
# SOLENOID

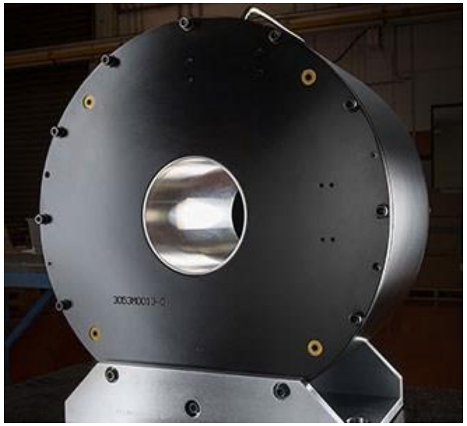
Also solenoids can be used for focalization of beams (in particular electron beams).





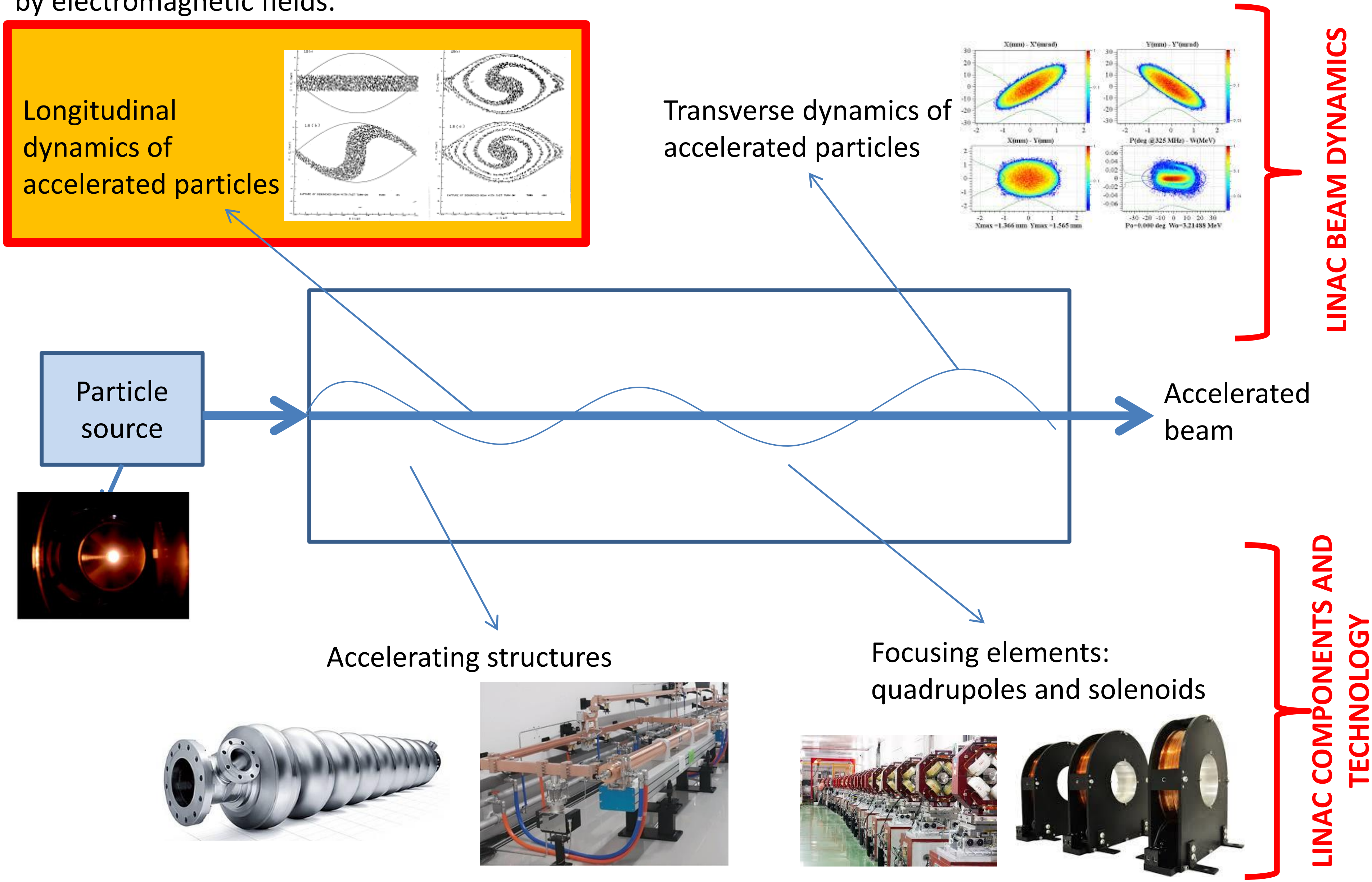


Useful for beam emittance compensation at low energies

Particles that enter into a solenoidal field with a transverse component of the velocity (divergence) start to **spiralize describing circular trajectories** .

# LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.

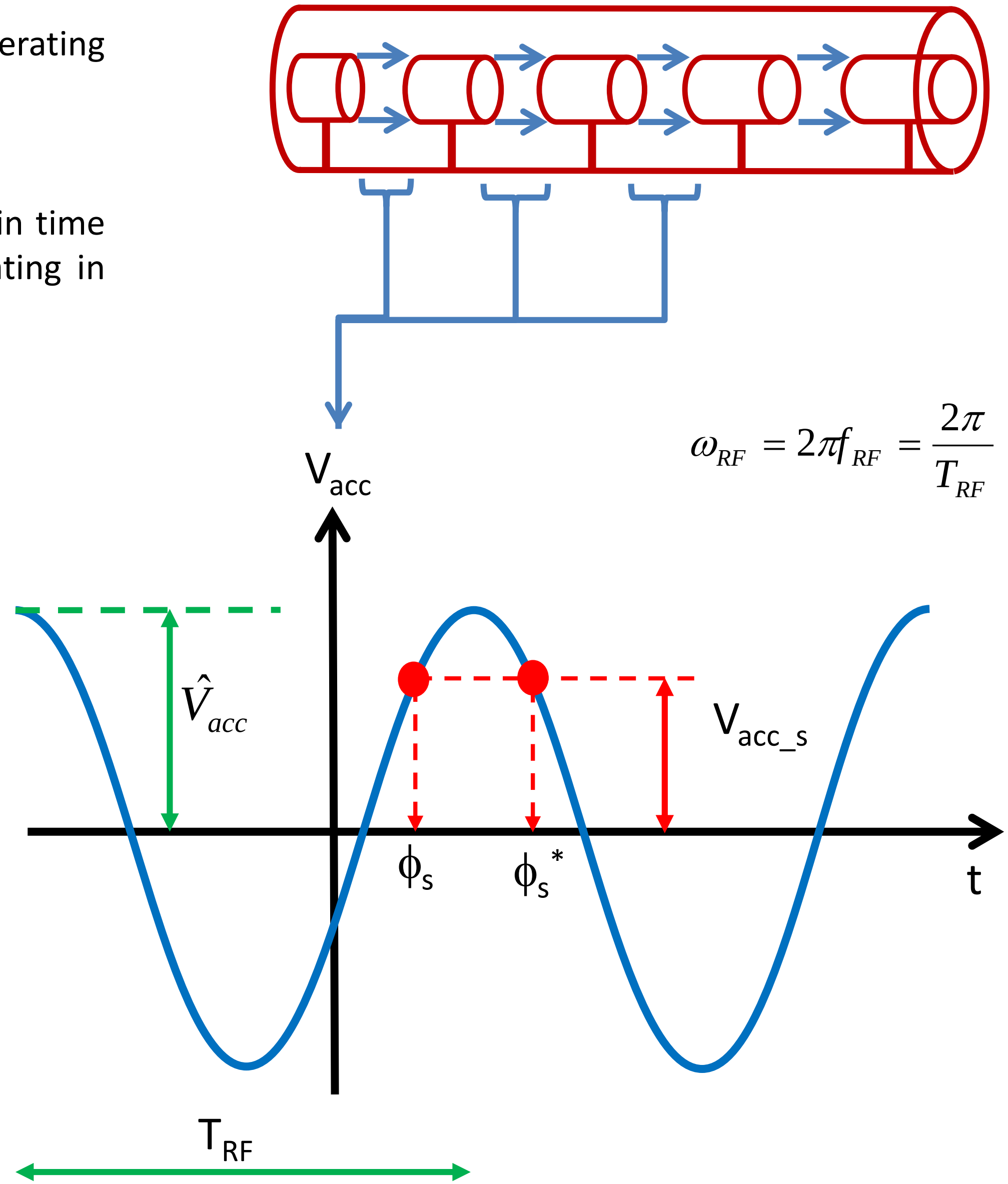


# SYNCHRONOUS PARTICLE/PHASE

⇒Let us consider a **SW linac structure** made by accelerating **gaps** (like in DTL) or **cavities**.

⇒In **each gap we have an accelerating field** oscillating in time and an integrated accelerating voltage ($V_{acc}$) still oscillating in time than can be expressed as:

$$V_{acc} = \hat{V}_{acc}\cos\left(\omega_{RF}t + \theta\right)$$




⇒Let's assume that **the "perfect" synchronism condition is fulfilled for a phase $\phi_s$** (called ***synchronous phase***)**.** This means that a particle (called ***synchronous particle***) entering in a gap with a phase $\phi_s$ ($\phi_s=\omega_{RF}t_s$) with respect to the RF voltage receive a **energy gain** (and a consequent change in velocity) that allow entering in the subsequent gap with the **same phase $\phi_s$** and so on.

⇒for this particle the energy gain in each gap is:

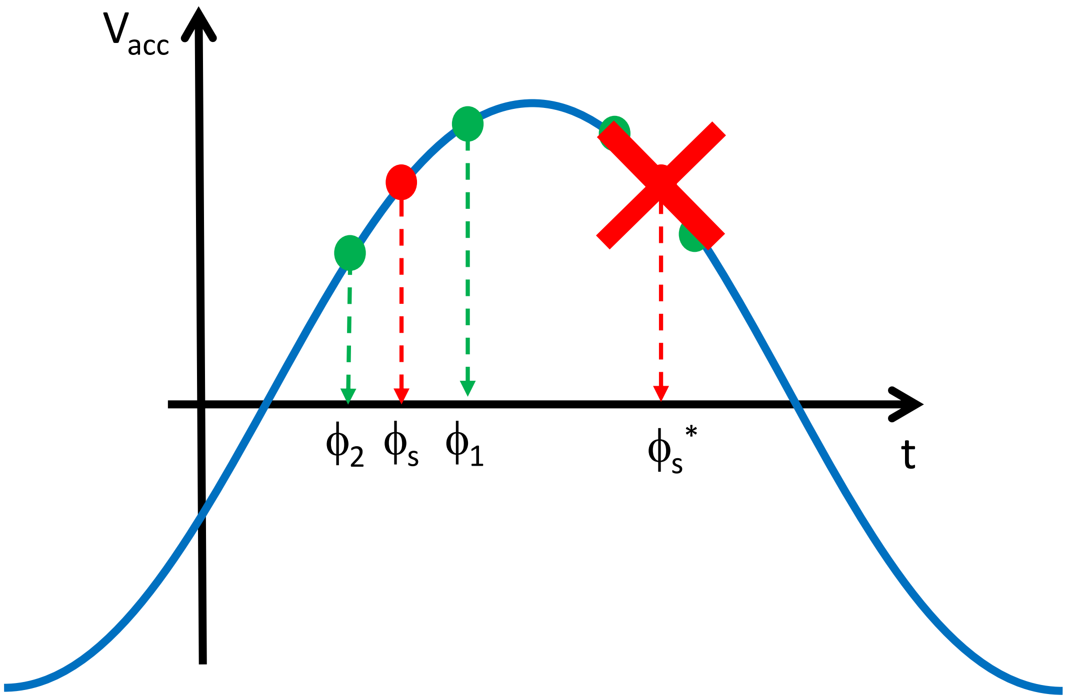
$$\Delta E = q\underbrace{\hat{V}_{acc}\cos\left(\phi_s + \theta\right)}_{V_{acc\_s}} = qV_{acc\_s}$$

⇒obviously both $\phi_s$ and $\phi_s^*$ are synchronous phases.

# PRINCIPLE OF PHASE STABILITY

## *(protons and ions or electrons at extremely low energy)*

⇒Let us consider now the first synchronous phase $\phi_s$ (on the positive slope of the RF voltage). If we consider **another particle** "near" to the synchronous one **that arrives later in the gap** ($t_1>t_s$, $\phi_1>\phi_s$), it will see an higher voltage, it will gain an higher energy and an higher velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be shorter, partially **compensating its initial delay**.




⇒**Similarly** if we consider another particle "near" to the synchronous one that arrives before in the gap ($t_1<t_s$, $\phi_1<\phi_s$), it will see a smaller voltage, it will gain a smaller energy and a smaller velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be longer, compensating the initial advantage.

⇒The choice of the synchronous phase in the positive slope of the RF voltage provides longitudinal focusing of the beam: **phase stability principle**.

⇒**On the contrary** if we consider now the synchronous particle at phase $\phi_s^*$ and another particle "near" to the synchronous one that arrives later or before in the gap, it will receive an energy gain that will increase further its distance form the synchronous one

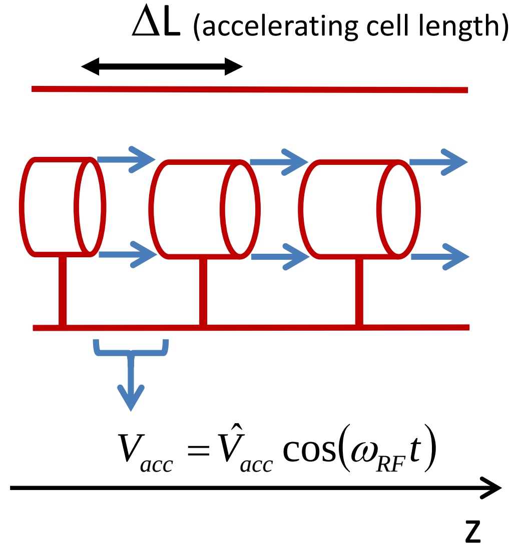
⇒The synchronous phase on the negative slope of the RF voltage is, on the contrary, **unstable**

⇒Relying on particle velocity variations, **longitudinal focusing does not work for fully relativistic beams** (electrons). In this case acceleration "on crest" is more convenient.

# ENERGY-PHASE EQUATIONS (1/2)

## *(protons and ions or electrons at extremely low energy)*

In order to study the **longitudinal dynamics in a LINAC**, the following variables are used, which describe the generic particle **phase** (time of arrival) and **energy with respect to the synchronous particle**:




Arrival time (phase) of a **generic particle** at a certain gap (or cavity)

Arrival time (phase) of the **synchronous particle** at a certain gap (or cavity)

$$\begin{cases} \varphi = \phi - \phi_s = \omega_{RF}(t - t_s) \\ w = E - E_s \end{cases}$$

Energy of the **synchronous particle** at a certain position along the linac

Energy of a **generic particle** at a certain position along the linac

The **energy gain per cell (one gap + tube in case of a DTL)** of a generic particle and of a synchronous particle are (we put θ=0 in the generic expression of the accelerating voltage just for simplicity ):

$$\begin{cases} \Delta E_s = q\hat{V}_{acc}\cos\phi_s \\ \Delta E = q\hat{V}_{acc}\cos\phi = q\hat{V}_{acc}\cos(\phi_s + \varphi) \end{cases}$$

subtracting

$$\Delta w = \Delta E - \Delta E_s = q\hat{V}_{acc}\left[\cos(\phi_s + \varphi) - \cos\phi_s\right]$$

Dividing by the accelerating cell length ΔL and assuming that:

$$\frac{\hat{V}_{acc}}{\Delta L} = \hat{E}_{acc}$$

Average accelerating field over the cell (or accelerating gradient)

$$\frac{\Delta w}{\Delta L} = q\hat{E}_{acc}\left[\cos(\phi_s + \varphi) - \cos\phi_s\right]$$

Approximating

$$\frac{\Delta w}{\Delta L} \approx \frac{dw}{dz}$$

$$\boxed{\frac{dw}{dz} = q\hat{E}_{acc}\left[\cos(\phi_s + \varphi) - \cos\phi_s\right]}$$

# ENERGY-PHASE EQUATIONS (2/2)

*(protons and ions or electrons at extremely low energy)*

On the other hand we have that the **phase variation per cell** of a generic particle and of a synchronous particle are:

$$\begin{cases} \Delta\phi_s = \omega_{RF}\Delta t_s \\ \Delta\phi = \omega_{RF}\Delta t \end{cases}$$

$\Delta$t is basically the time of flight between two accelerating cells

subtracting

$$\Delta\varphi = \omega_{RF}\left(\Delta t - \Delta t_s\right)$$

Dividing by the accelerating cell length $\Delta$L

v, $v_s$ are the average particles velocities

$$\frac{\Delta\varphi}{\Delta L} = \omega_{RF}\left(\frac{\Delta t}{\Delta L} - \frac{\Delta t_s}{\Delta L}\right) = \omega_{RF}\left(\frac{1}{v} - \frac{1}{v_s}\right) \underbrace{\cong}_{MAT} -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w$$

Approximating

$$\frac{\Delta\varphi}{\Delta L} \cong \frac{d\varphi}{dz}$$

This system of coupled (non linear) differential equations **describe the motion of a non synchronous particles** in the longitudinal plane with respect to the synchronous one.

$$\begin{cases} \dfrac{d\varphi}{dz} = -\dfrac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w \\ \dfrac{dw}{dz} = q\hat{E}_{acc}\left[\cos(\phi_s + \varphi) - \cos\phi_s\right] \end{cases}$$

*MAT*

$$\omega_{RF}\left(\frac{1}{v} - \frac{1}{v_s}\right) = \omega_{RF}\left(\frac{v_s - v}{vv_s}\right) \underbrace{\cong}_{\substack{vv_s \cong v_s^2 \\ v - v_s \cong \Delta v}} -\frac{\omega_{RF}}{v_s^2}\Delta v = -\frac{\omega_{RF}}{c}\frac{\Delta\beta}{\beta_s^2} \quad \text{remembering that} \quad \beta = \sqrt{1 - 1/\gamma^2} \Rightarrow \beta d\beta = d\gamma/\gamma^3 \Rightarrow -\frac{\omega_{RF}}{c}\frac{\Delta\beta}{\beta_s^2} \cong -\frac{\omega_{RF}}{c}\frac{\Delta\gamma}{\beta_s^3\gamma_s^3} = -\frac{\omega_{RF}}{c}\frac{\overbrace{\Delta E}^{w}}{E_0\beta_s^3\gamma_s^3}$$

# SMALL AMPLITUDE ENERGY-PHASE OSCILLATIONS

***(protons and ions or electrons at extremely low energy)***

$$\begin{cases} \dfrac{dw}{dz} = q\hat{E}_{acc}\left[\cos(\phi_s+\varphi)-\cos\phi_s\right] \\[4em] \dfrac{d\varphi}{dz} = -\dfrac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w \end{cases}$$

Assuming small oscillations around the synchronous particle that allow to approximate $\cos(\phi_s+\varphi)-\cos\phi_s \cong \varphi\sin\phi_s$

Deriving both terms with respect to z and assuming an **adiabatic acceleration** process i.e. a particle energy and speed variations that allow to consider $\dfrac{d\left(\dfrac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\right)}{dz}w << \dfrac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\dfrac{dw}{dz}$

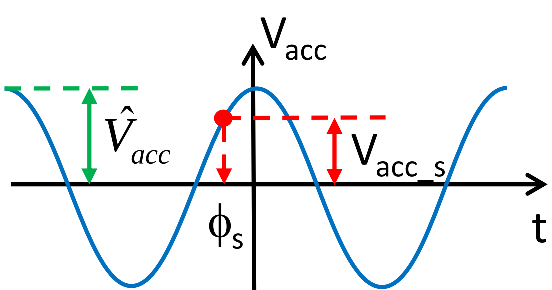
$$\frac{d^2\varphi}{dz^2} = -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\frac{dw}{dz}$$

$$\boxed{\frac{d^2\varphi}{dz^2} + \underbrace{q\frac{\omega_{RF}\hat{E}_{acc}\sin(-\phi_s)}{cE_0\beta_s^3\gamma_s^3}}_{\Omega_s^2}\varphi = 0}$$

**harmonic oscillator equation**

$\Rightarrow$The condition to have stable longitudinal oscillations and acceleration at the same time is:

$$\left.\begin{array}{l} \Omega_s^2 > 0 \Rightarrow \sin(-\phi_s) > 0 \\ V_{acc} > 0 \Rightarrow \cos\phi_s > 0 \end{array}\right\} \Rightarrow -\frac{\pi}{2} < \phi_s < 0$$




if we accelerate on the rising part of the positive RF wave we have a **longitudinal force keeping the beam bunched** around the synchronous phase.

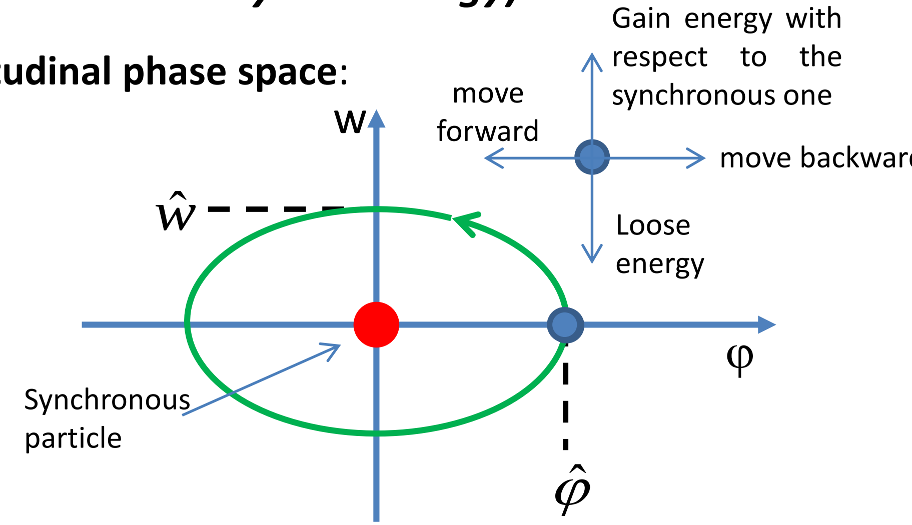
$$\begin{cases} \varphi = \hat{\varphi}\cos(\Omega_s z) \\ w = \hat{w}\sin(\Omega_s z) \end{cases}$$

# ENERGY-PHASE OSCILLATIONS IN PHASE SPACE

*(protons and ions or electrons at extremely low energy)*

The energy-phase oscillations can be drawn in the **longitudinal phase space**:

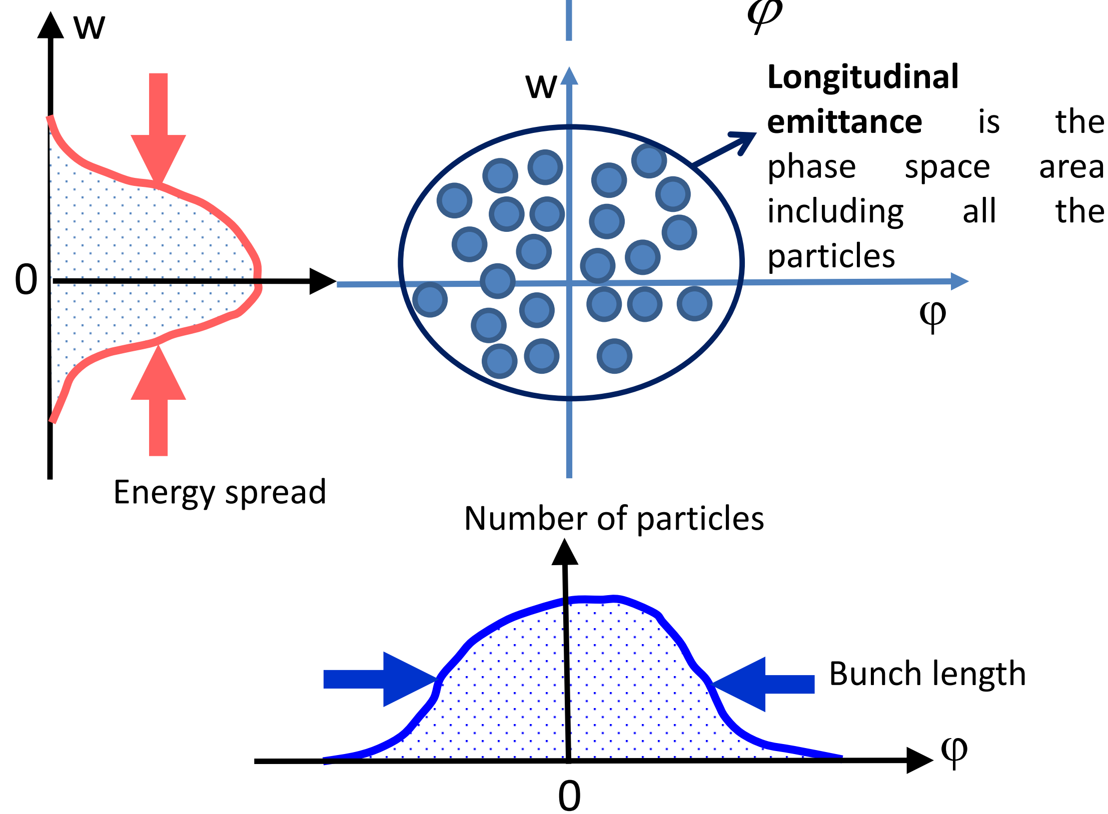
$$\begin{cases} \varphi = \hat{\varphi}\cos(\Omega_s z) \\ w = \hat{w}\sin(\Omega_s z) \end{cases}$$




⇒The trajectory of a generic particle in the longitudinal phase space is an **ellipse**.

⇒The **maximum energy deviation** is reached at φ=0 while the **maximum phase excursion** corresponds to w=0.

⇒the bunch occupies an area in the longitudinal phase space called **longitudinal emittance** and the projections of the bunch in the energy and phase planes give the **energy spread** and the **bunch length**.



# APPENDIX: LARGE OSCILLATIONS AND SEPARATRIX

To study the longitudinal dynamics **at large oscillations**, we have to consider the **non linear system of differential equations** without approximations. By neglecting particle energy and speed variations along the LINAC (**adiabatic acceleration**) it is possible to easily obtain the following relation between w and φ that is the **Hamiltonian of the system** related to the total particle energy:

$$\frac{1}{2}\left(\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\right)^2 w^2 + \frac{\omega_{RF}\, q\hat{E}_{acc}}{cE_0\beta_s^3\gamma_s^3}\left[\sin(\phi_s+\varphi)-\varphi\cos\phi_s-\sin(\phi_s)\right] = \text{const} = \text{H}$$

⇒**For each H we have different trajectories** in the longitudinal phase space

⇒the oscillations are **stable** within a region bounded by a special curve called **separatrix**: its equation is:

$$\frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w^2 + q\hat{E}_{acc}\left[\sin(\phi_s+\varphi)-(2\varphi_s+\varphi)\cos\phi_s+\sin(\phi_s)\right]=0$$

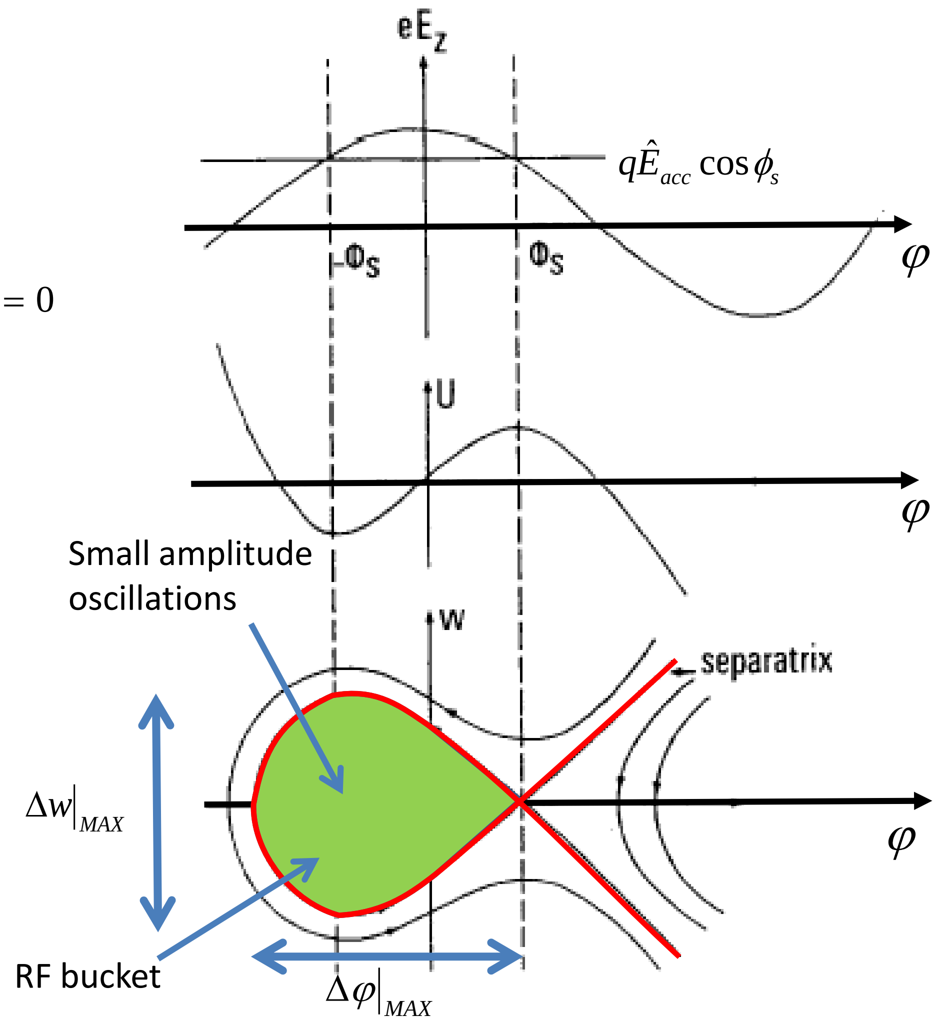
⇒the region inside the separatrix is called **RF bucket**. The dimensions of the bucket shrinks to zero if $\phi_s$=0.

⇒trajectories outside the RF buckets are **unstable**.

⇒we can define the **RF acceptance** as the maximum extension in phase and energy that we can accept in an accelerator:

$$\Delta\varphi\big|_{MAX} \cong 3\phi_s$$

$$\Delta w\big|_{MAX} = \pm 2\left[\frac{qcE_o\beta_s^3\gamma_s^3\hat{E}_{acc}(\phi_s\cos\phi_s-\sin\phi_s)}{\omega_{RF}}\right]^{\frac{1}{2}}$$



# APPENDIX: SEPARATRIX EQUATION

To study the longitudinal dynamics at large oscillations, we have to consider the non linear system of differential equations without approximations. By neglecting particle energy and speed variations along the LINAC we obtain:

$$\frac{d^2\varphi}{dz^2} = -\frac{\omega_{RF} q\hat{E}_{acc}}{cE_0\beta_s^3\gamma_s^3}\left[\cos(\phi_s+\varphi)-\cos\phi_s\right] = F$$

The restoring force ***F*** can not be considered purely elastic anymore and may by derived from a **potential function** according to the usual definition:

$$U = -\int_0^{\varphi} F d\varphi' = \frac{\omega_{RF}\, q\hat{E}_{acc}}{cE_0\beta_s^3\gamma_s^3}\left[\sin(\phi_s+\varphi)-\varphi\cos\phi_s-\sin(\phi_s)\right]$$

With few simple passages we obtain an “energy conservation”–like law:

$$\frac{d}{dz}\left[\left(\frac{d\varphi}{dz}\right)^2\right] = 2\frac{d\varphi}{dz}\frac{d^2\varphi}{dz^2} = 2\frac{d\varphi}{dz}\cdot\left(-\frac{dU}{d\varphi}\right) = -2\frac{d}{dz}U \Rightarrow \frac{d}{dz}\left[\left(\frac{d\varphi}{dz}\right)^2 + 2U\right] = 0 \;\Rightarrow\; \frac{1}{2}\left(\frac{d\varphi}{dz}\right)^2 + U = \text{cost}$$

$$\frac{d\varphi}{dz} = -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w$$

$$\frac{1}{2}\left(\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\right)^2 w^2 + \frac{\omega_{RF}\, q\hat{E}_{acc}}{cE_0\beta_s^3\gamma_s^3}\left[\sin(\phi_s+\varphi)-\varphi\cos\phi_s-\sin(\phi_s)\right] = \text{const} = \text{H}$$

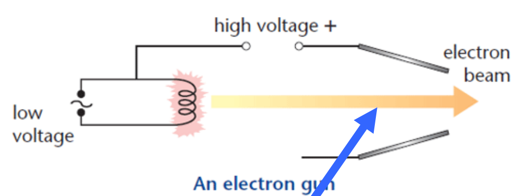
# LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS

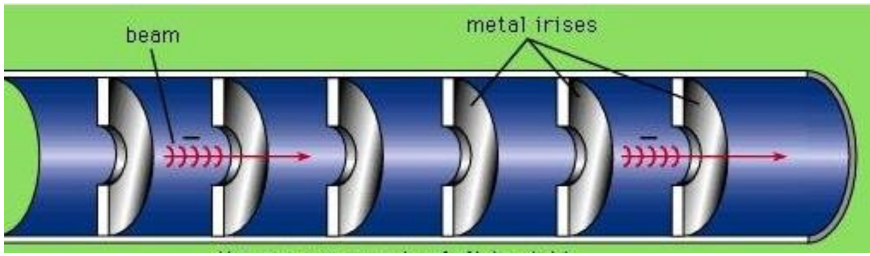
From previous formulae it is clear that there is **no motion in the longitudinal phase plane for ultrarelativistic particles** ($\gamma$>>1).

⇒This is the case of **electrons** whose **velocity is always close to speed of light c** even at low energies.

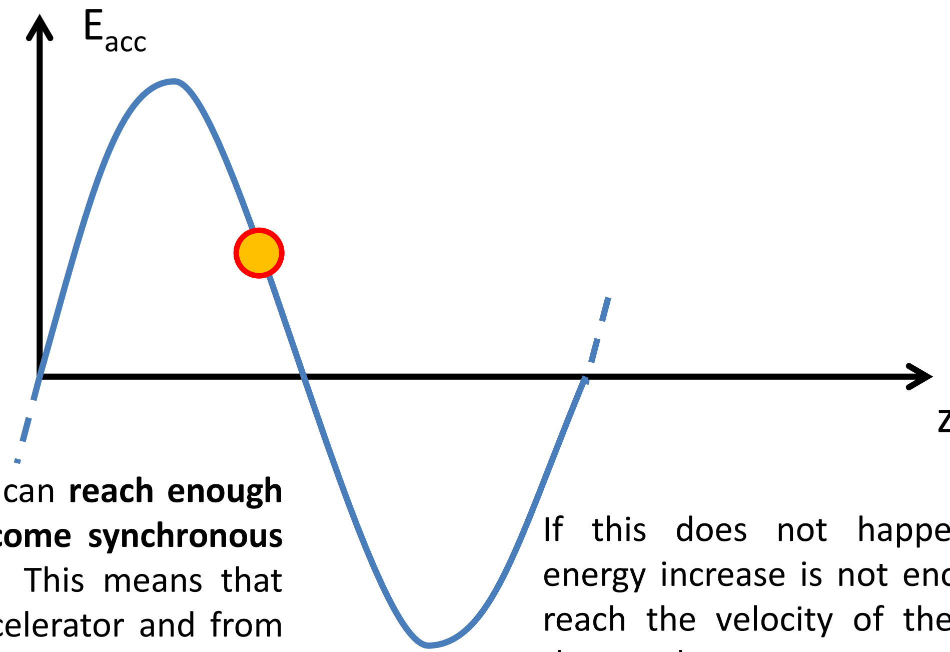
⇒Accelerating structures are designed to provide an accelerating field synchronous with particles moving at v=c. like **TW structures with phase velocity equal to c**.

It is interesting to analyze what happen if we **inject an electron beam produced by a cathode (at low energy) directly in a TW structure** (with $v_{ph}$=c) and the conditions that allow to **capture** the beam (this is equivalent to consider instead of a TW structure a SW designed to accelerate ultrarelativistic particles at v=c).




Particles enter the structure with velocity **v<c** and, initially, they are **not synchronous with the accelerating field** and there is a so called slippage.




After a certain distance they can **reach enough energy (and velocity) to become synchronous** with the accelerating wave. This means that they are captured by the accelerator and from this point they are stably accelerated.

If this does not happen (the energy increase is not enough to reach the velocity of the wave) they are lost

# LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: PHASE SPLIPPAGE

The **accelerating field** of a TW structure can be expressed by

$$E_{acc} = \hat{E}_{acc}\cos\underbrace{\left(\omega_{RF}t - kz\right)}_{\phi(z,t)}$$

The **equation of motion of a particle with a position z at time t accelerated by the TW** is then

$$\frac{d}{dt}(mv) = q\hat{E}_{acc}\cos\phi(z,t) \;\Rightarrow\; m_0c\frac{d}{dt}(\gamma\beta) = m_0c\gamma^3\frac{d\beta}{dt} = q\hat{E}_{acc}\cos\phi$$

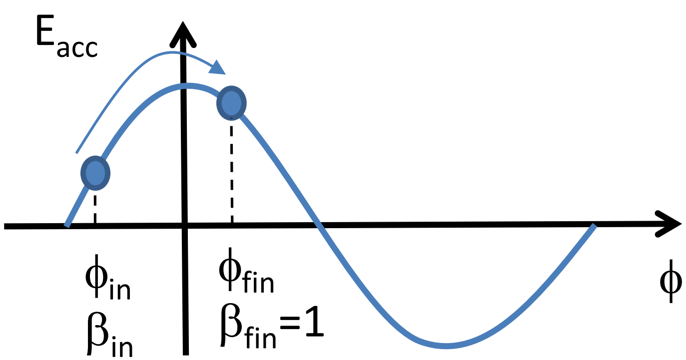
It is useful to find which is the relation between β and ϕ

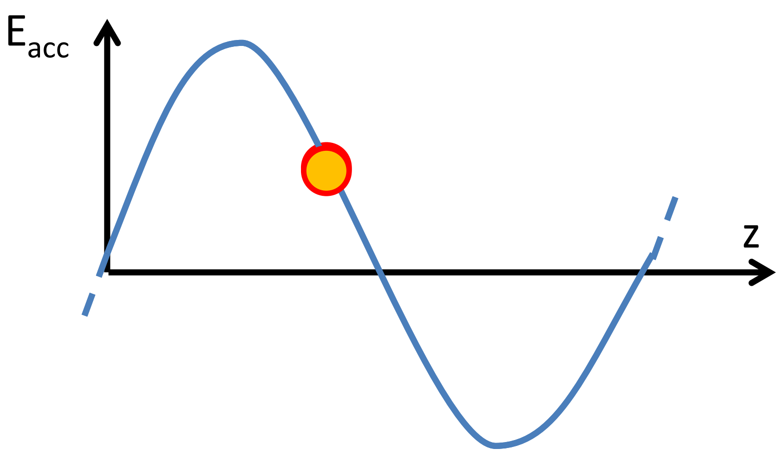
$$\sin\phi_{fin} = \sin\phi_{in} + \frac{2\pi E_0}{\lambda_{RF}q\hat{E}_{acc}}\left(\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}} - \sqrt{\frac{1-\beta_{fin}}{1+\beta_{fin}}}\right)$$

Suppose that the particle reach asymptotically the value $\beta_{fin}$=1 we have:

$$\sin\phi_{fin} = \sin\phi_{in} + \frac{2\pi m_0c^2}{\lambda_{RF}q\hat{E}_{acc}}\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}}$$

$$\sin\phi_{fin} > \sin\phi_{in} \Rightarrow \phi_{fin} > \phi_{in}$$



# LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: CAPTURE ACCELERATING FIELD

For a given injection energy ($\beta_{in}$) and phase ($\phi_{in}$) we can find which is the accelerating field ($E_{acc}$) that is necessary to have the completely relativistic beam at phase fin (that is necessary to **capture the beam at phase fin**)

$$\Rightarrow \quad \hat{E}_{acc} = \frac{2\pi E_0}{\lambda_{RF} q \left(\sin\phi_{fin} - \sin\phi_{in}\right)} \sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}}$$

Example:
$E_{in}$ = *50 keV*, (kinetic energy), $\phi_{in}$ =-$\pi$/2,
$\phi_{\infty}$ =0$\Rightarrow$ $\gamma_{in}\approx 1.1$; $\beta_{in}\approx 0.41$
$f_{RF}$= *2856 MHz*$\Rightarrow$$\lambda_{RF}\approx$ *10.5 cm*

*We obtain* $E_{acc}\cong$20MV/m;

The **minimum value of the electric field ($E_{acc}$) that allow to capture a beam**. Obviously this correspond to an injection phase
$\phi_{in}$ =-$\pi$/2 and $\phi_{\infty}$=$\pi$/2.

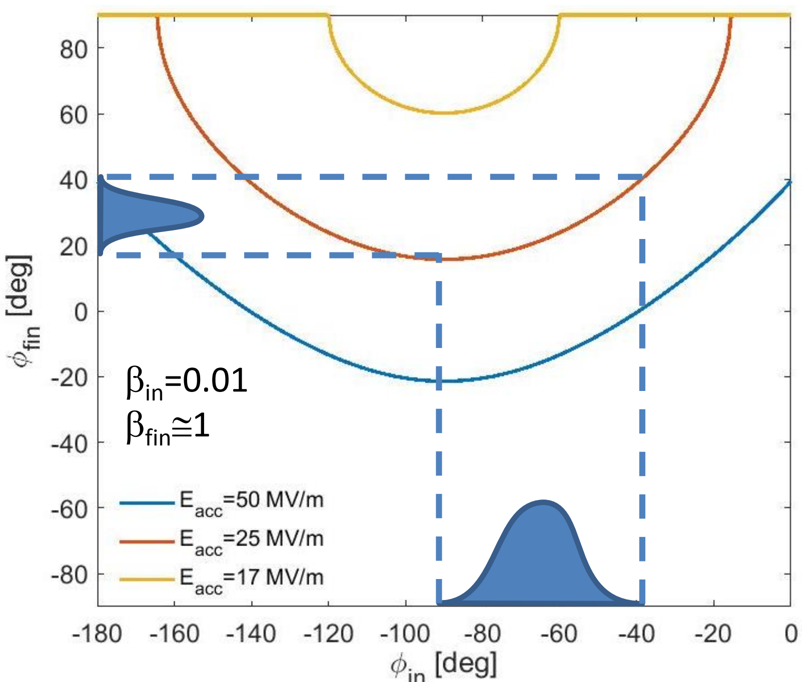
$$\Rightarrow \quad \hat{E}_{acc\_MIN} = \frac{\pi E_0}{\lambda_{RF} q} \sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}}$$

Example: For the previous case we obtain: $E_{acc_min}\cong$10MV/m;

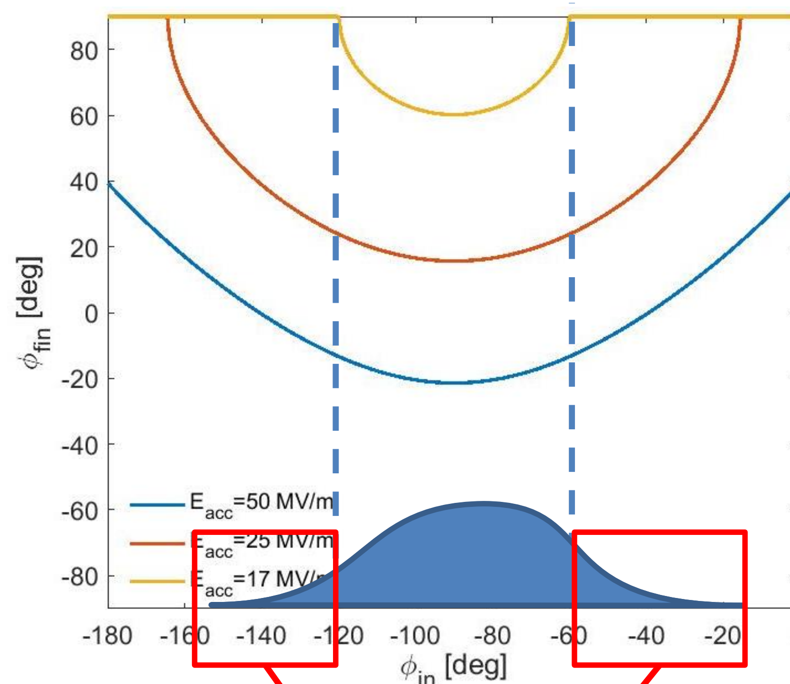
# LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: BUNCH COMPRESSION AND CAPTURE EFFICIENCY

During the capture process, as the injected beam moves up to the crest, the beam is also bunched, which is caused by **velocity modulation** (velocity bunching). This mechanism can be used to compress the electron bunches (FEL applications).

## BUNCH COMPRESSION

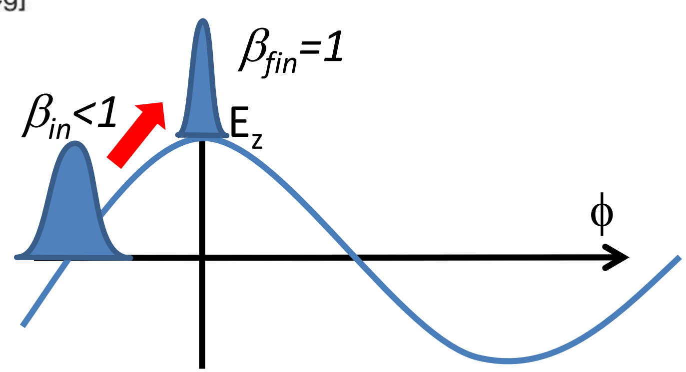



## CAPTURE EFFICIENCY




Bunch length variation

$$\Delta\phi_{fin} = \Delta\phi_{in}\frac{\cos\phi_{in}}{\cos\phi_{fin}}$$

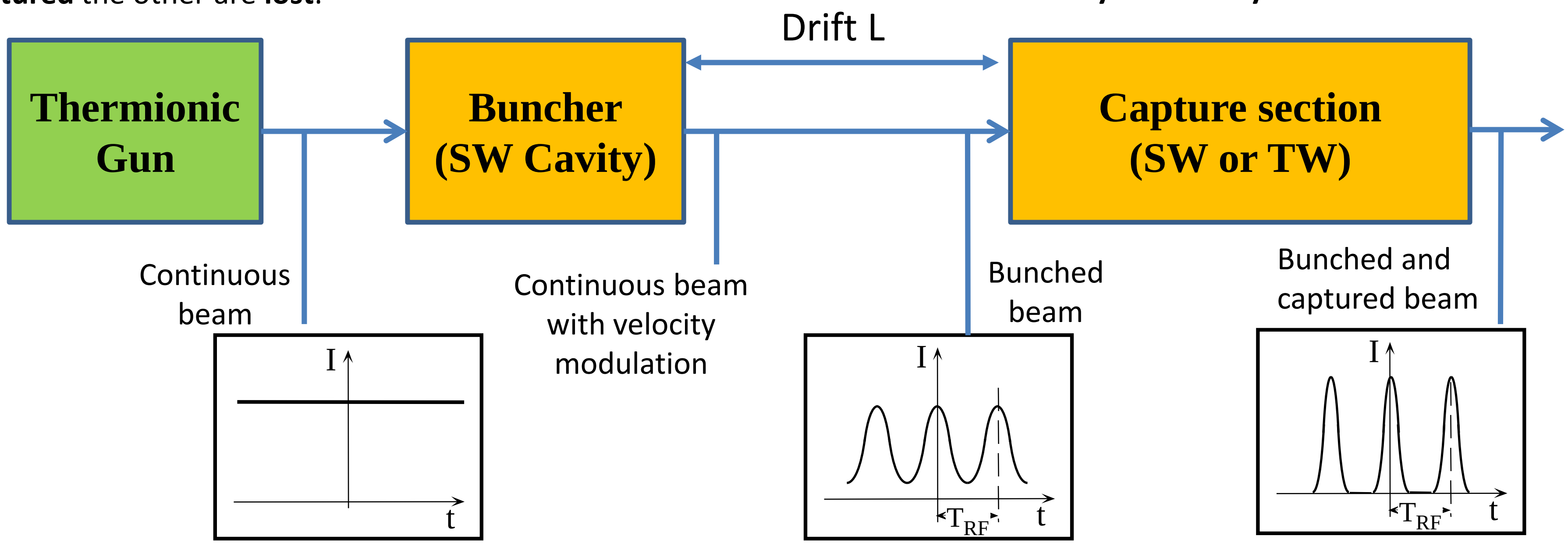



These particle are lost during the capture process

# BUNCHER AND CAPTURE SECTIONS
## *(electrons)*

**Once the capture condition $E_{RF}>E_{RF\_MIN}$ is fulfilled** the fundamental equation of previous slide sets the **ranges of the injection phases $\phi_{in}$ actually accepted**. Particles whose injection phases are within this range can be **captured** the other are **lost**.

In order **to increase the capture efficiency** of a traveling wave section, **pre-bunchers** are often used. They are SW cavities aimed at **pre-forming particle bunches gathering particles continuously emitted by a source.**




⇒**Bunching** is obtained by modulating the energy (and therefore the velocity) of a continuous beam using the longitudinal E-field of a SW cavity. After a certain **drift space** the **velocity modulation is converted in a density charge modulation**. The density modulation depletes the regions corresponding to injection phase values incompatible with the capture process

⇒A TW accelerating structure (**capture section**) is placed at an **optimal distance from the pre-buncher**, to capture a large fraction of the charge and accelerate it till relativistic energies. The **amount of charge lost is drastically reduced**, while the capture section provide also further beam bunching.

# APPENDIX: PHASE SPLIPPAGE CALCULATIONS

The **accelerating field** of a TW structure can be expressed by

$$E_{acc} = \hat{E}_{acc}\cos\underbrace{\left(\omega_{RF}t - kz\right)}_{\phi(z,t)}$$

The **equation of motion of a particle with a position z at time t accelerated by the TW** is then

$$\frac{d}{dt}(mv) = q\hat{E}_{acc}\cos\phi(z,t) \;\Rightarrow\; m_0c\frac{d}{dt}(\gamma\beta) = m_0c\gamma^3\frac{d\beta}{dt} = q\hat{E}_{acc}\cos\phi$$

This is **the phase of the TW wave seen by the particle at a certain time t and position z**

The **phase motion during acceleration** is then

$$\frac{d\phi}{dt} = \omega_{RF} - k\frac{dz}{dt} = \omega_{RF} - \frac{\omega_{RF}}{c}\frac{dz}{dt} \;\Rightarrow\; \frac{d\phi}{dt} = \omega_{RF}\left(1-\beta\right) > 0$$

The phase of the wave "seen" by the particle always increase because $\beta<1$.

It is useful to find which is the relation between $\beta$ and $\phi$

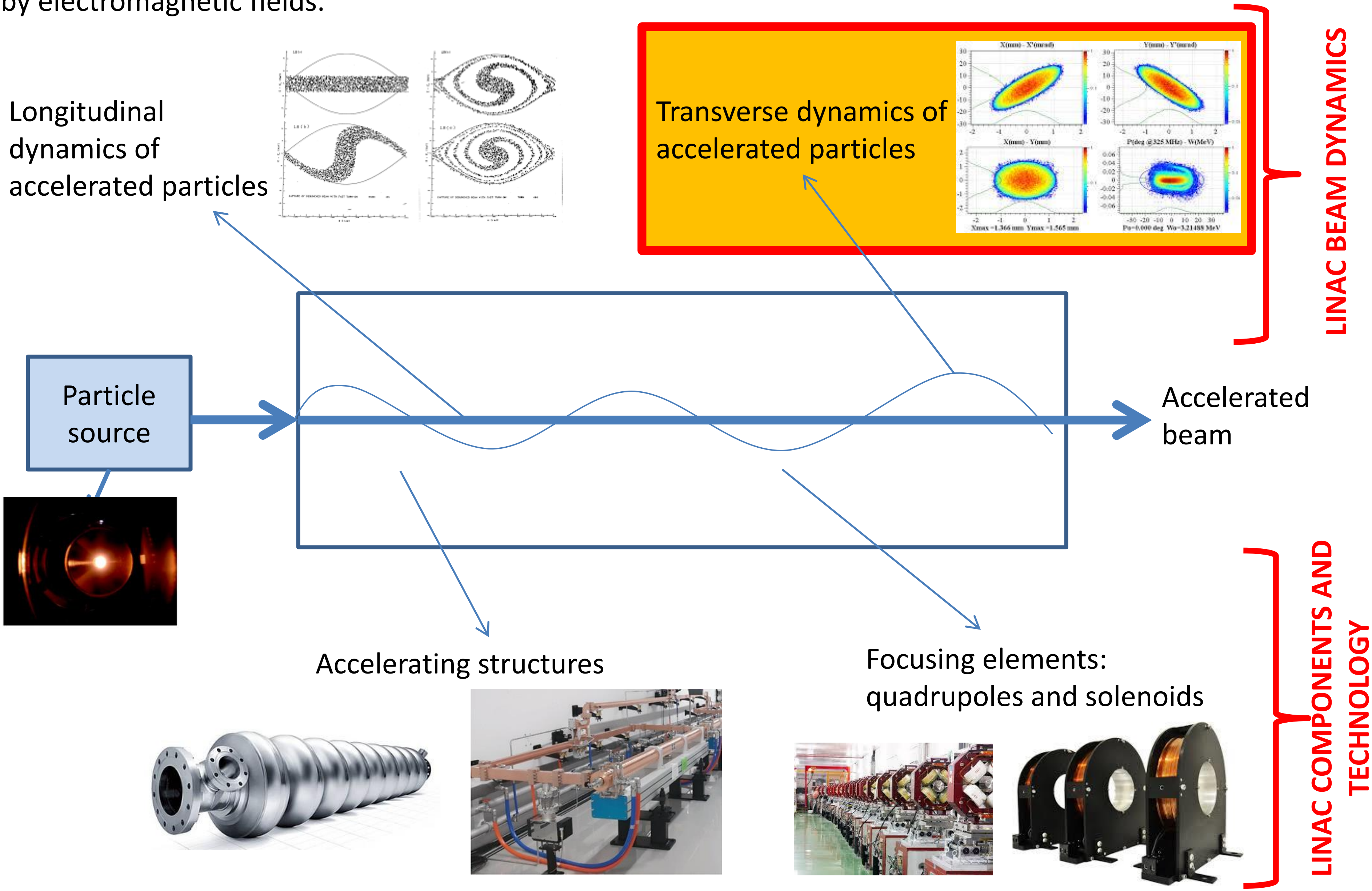
$$m_0c\gamma^3\frac{d\beta}{dt} = m_0c\gamma^3\frac{d\beta}{d\phi}\frac{d\phi}{dt} = \frac{1}{(1+\beta)\sqrt{1-\beta^2}}\frac{d\beta}{d\phi} = \frac{q\hat{E}_{acc}}{\omega_{RF}m_0c}\cos\phi \Rightarrow \frac{1}{(1+\beta)\sqrt{1-\beta^2}}d\beta = \frac{q\hat{E}_{acc}}{\omega_{RF}m_0c}\cos\phi d\phi$$

We can integrate both sides between the initial and the final condition to find the dependence of phase on velocity during the acceleration process (using, as example, the new variable $\beta=\cos\alpha$)

$$\sin\phi_{fin} = \sin\phi_{in} + \frac{2\pi\, m_0c^2}{\lambda_{RF}q\hat{E}_{acc}}\left(\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}} - \sqrt{\frac{1-\beta_{fin}}{1+\beta_{fin}}}\right)$$

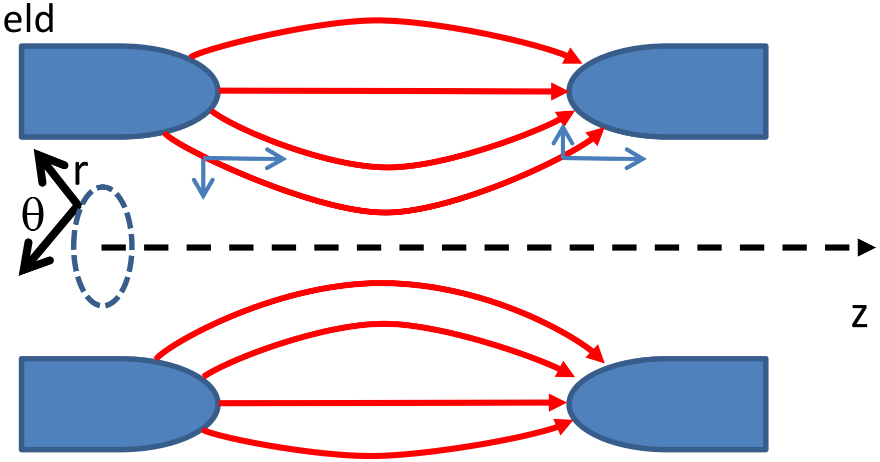
# LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.



# RF TRANSVERSE FORCES

The **RF fields act on the transverse beam dynamics** because of the transverse components of the E and B field




$$E_z(z,t)=E_{RF}(z)\cos(\omega_{RF}t+\phi)$$

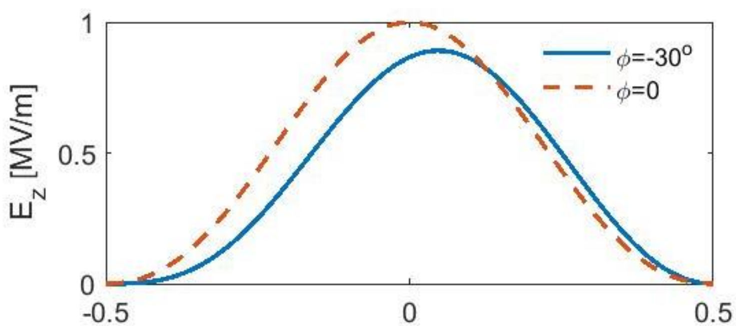
⇒According to Maxwell equations the **divergence of the field is zero** and this implies that in traversing one accelerating gap there is a focusing/defocusing term

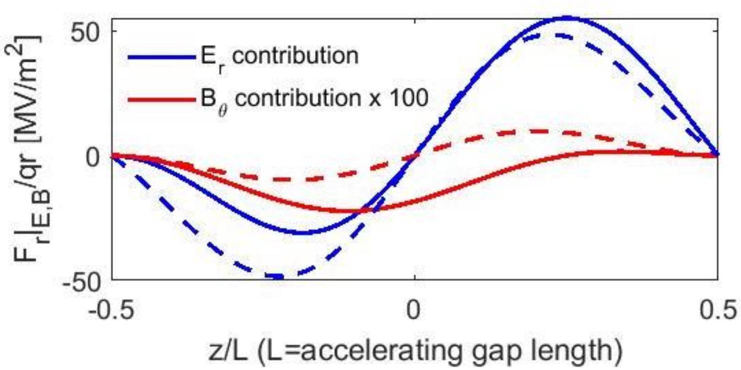
$$\begin{array}{l}\nabla\vec{E}=0\\ \nabla\times\vec{B}=\frac{1}{c^2}\vec{E}\end{array}\Rightarrow\begin{cases}E_r=-\frac{r}{2}\frac{\partial E_z}{\partial z}\\ B_\theta=\frac{r}{2c^2}\frac{\partial E_z}{\partial t}\end{cases}$$

$$F_r=q(E_r-vB_\theta)=-q\frac{r}{2}\left(\frac{\partial E_z}{\partial z}-\frac{\beta}{c}\frac{\partial E_z}{\partial t}\right)$$

$$F_r\big|_E=-q\frac{r}{2}\frac{\partial E_{RF}(z)}{\partial z}\cos\left(\omega_{RF}\frac{z}{\beta c}+\phi\right)$$

$$F_r\big|_B=q\frac{r}{2}\omega_{RF}\frac{\beta}{c}E_{RF}(z)\sin\left(\omega_{RF}\frac{z}{\beta c}+\phi\right)$$




$f_{RF}$=350 MHz
β=0.1
L=3cm

# RF DEFOCUSING/FOCUSING

From previous formulae it is possible to calculate the **transverse momentum increase** due to the RF transverse forces. Assuming that the velocity and position changes over the gap are small we obtain to the first order:

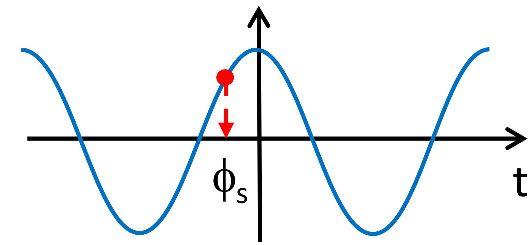
Transverse momentum increase

Defocusing force since sinϕ<0

Gap length

Defocusing effect

$$\Delta p_r = \int_{-L/2}^{+L/2} F_r \frac{dz}{\beta c} = -\frac{\pi q \hat{E}_{acc} L \sin\phi}{c\gamma^2\beta^2\lambda_{RF}} r$$




⇒ transverse **defocusing scales as ~1/γ²** and **disappears at relativistic regime (electrons)**

⇒ At relativistic regime (**electrons**), moreover, we have, in general, **ϕ=0 for maximum acceleration** and this completely cancel the defocusing effect

⇒ Also in the **non relativistic regime** for a correct evaluation of the defocusing effect we have to:

⇒ take into account the **velocity change across the accelerating gap**

⇒ the **transverse beam dimensions changes across the gap** (with a general reduction of the transverse beam dimensions due to the focusing in the first part)

Both effects give a **reduction of the defocusing force**

# RF FOCUSING IN ELECTRON LINACS

**-RF defocusing is negligible in electron linacs**.
-There is a second order effect due to the non-synchronous harmonics of the accelerating field that give a **focusing effect**. These harmonics generate a **ponderomotive force i.e.** a force in an inhomogeneous oscillating electromagnetic field.

## MECHANISM

The Lorentz force is linear with the particle dispacement

$$F_r = -q\frac{r}{2}\left(\frac{\partial E_z}{\partial z} + \frac{\beta}{c}\frac{\partial E_z}{\partial t}\right)$$

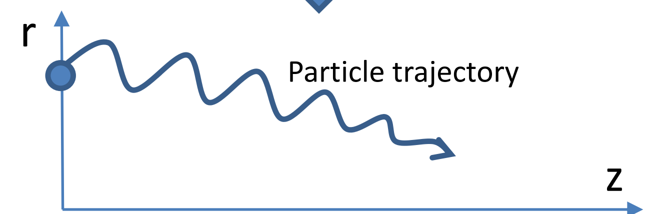
$\Longrightarrow$

Transverse equation of motion

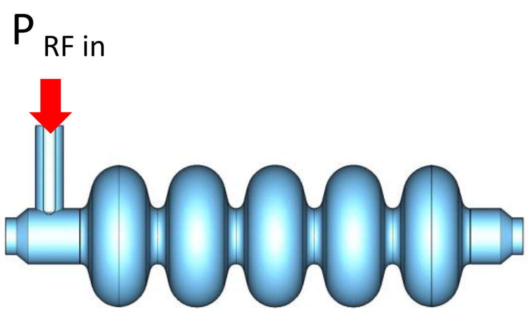
$$\ddot{r} = \frac{1}{\gamma m_0} r \sum_n a_n \cos(n\omega t)$$

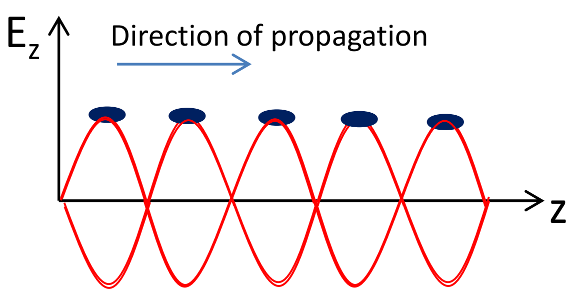
RF harmonics

This generate a global focusing force

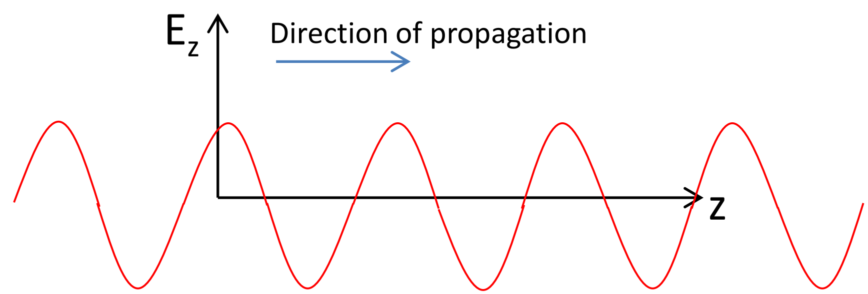



## NON-SYNCHRONOUS RF HARMONICS: SIMPLE CASE OF SW STRUCTURE




Is equivalent to the **superposition of two counterpropagating TW waves**




The **forward wave only contribute to the acceleration** (and does not give transverse effect).
The **backward wave** does not contribute to the acceleration but generates an **oscillating transverse force** (ponderomotive force)

**Average focusing force**

$$\overline{F}_r = -r\frac{\left(q\hat{E}_{acc}\right)^2}{8\gamma m_0 c^2}\eta(\phi)$$

With accelerating gradients of few tens of MV/m can easily reach the level of MV/m²

# COLLECTIVE EFFECTS: SPACE CHARGE AND WAKEFIELDS

Collective effects are all effects related to the number of particles and they can play a crucial role in the longitudinal and transverse beam dynamics

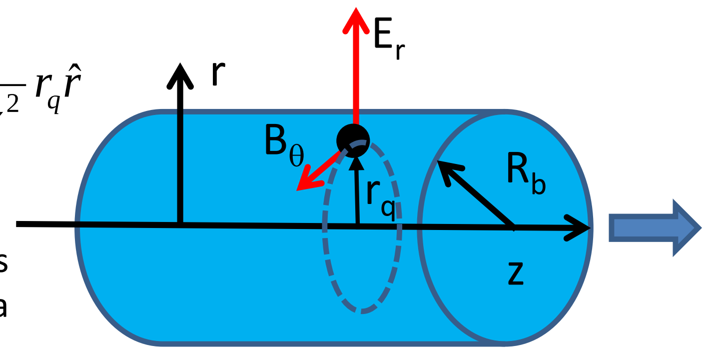
## *SPACE CHARGE*

⇒ **Effect of Coulomb repulsion between particles (space charge).**

⇒ These effects cannot be neglected especially at **low energy and at high current** because the space charge forces **scales as $1/\gamma^2$ and with the current I.**

EXAMPLE: Uniform and infinite cylinder of charge moving along z

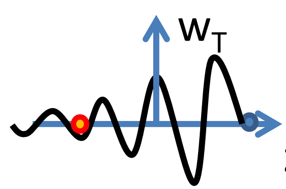
$$\vec{F}_{SC} = q\frac{I}{2\pi\varepsilon_0 R_b^2 \beta c \gamma^2} r_q \hat{r}$$

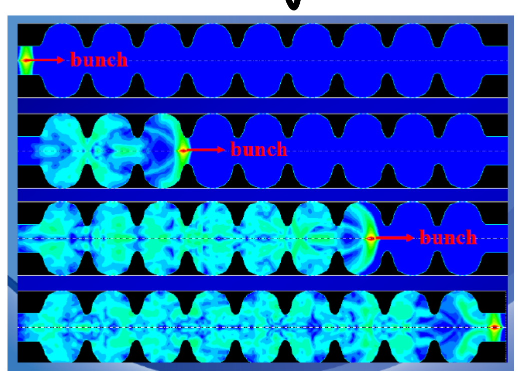



In this particular case it is linear but in general it is a **non-linear force**

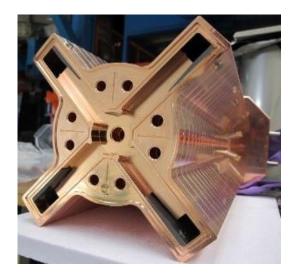
## *WAKEFIELDS*

The other effects are due to the **wakefield**. The passage of bunches through accelerating structures excites electromagnetic **field**. This field can have longitudinal and transverse components and, interacting with subsequent bunches (long range wakefield), **can affect the longitudinal and the transverse beam dynamics**. In particular the **transverse wakefields**, can drive an instability along the train called **multibunch beam break up** (BBU).



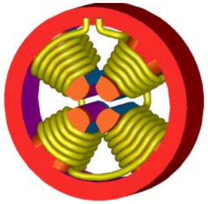

Several approaches are used to absorb these field from the structures like **loops** couplers, **waveguides**, Beam pipe **absorbers**



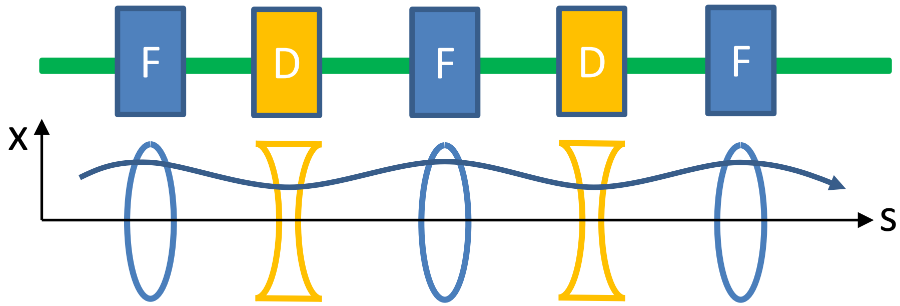
# MAGNETIC FOCUSING AND CONTROL OF THE TRANSVERSE DYNAMICS

⇒**Defocusing RF forces, space charge** or the natural divergence (emittance) of the beam need to be **compensated** and controlled by **focusing forces**.

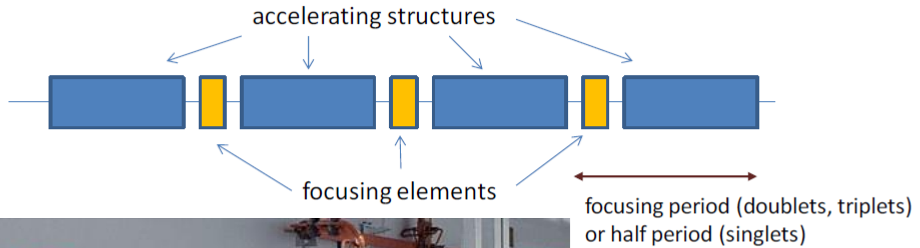
This is provided by **quadrupoles** along the beam line.
At low energies also **solenoids** can be used



⇒Quadrupoles are focusing in one plane and defocusing on the other. A global focalization is provides by **alternating quadrupoles** with opposite signs




⇒In a linac one **alternates accelerating sections with focusing sections**.

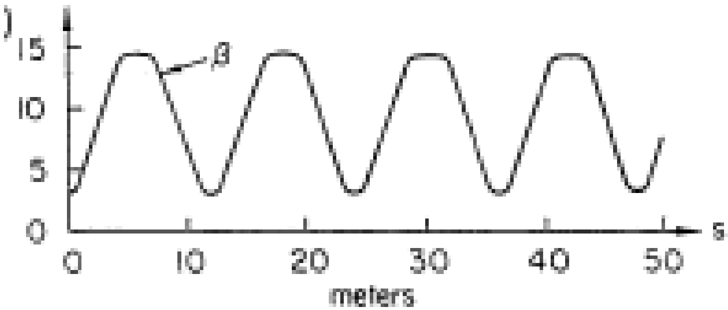



⇒The type of magnetic configuration and magnets type/distance depend on the type of particles/energies/beam parameters we want to achieve.

# TRANSVERSE OSCILLATIONS AND BEAM ENVELOPE

Due to the **alternating quadrupole focusing system** each particle perform transverse oscillations along the LINAC.

The **equation of motion in the transverse plane** is of the type:



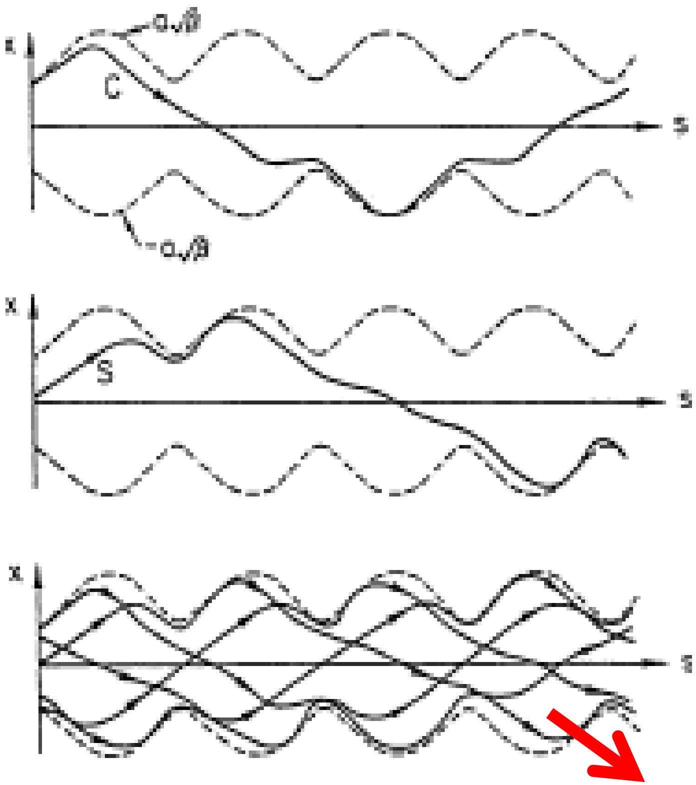

**Focusing period**= length after which the structure is repeated (usually as Nβλ).

Term depending on the magnetic configuration

RF defocusing/focusing term

$$\frac{d^2x}{ds^2}+\underbrace{\left[\kappa^2(s)+k_{RF}^2(s)\right]}_{K^2(s)}x-F_{SC}=0$$

Space charge term




The **single particle trajectory is a pseudo-sinusoid** described by the equation:

$$x(s)=\sqrt{\varepsilon_o\beta(s)}\cos\left[\int_{s_o}^{s}\frac{ds}{\beta(s)}+\phi_0\right]$$

Characteristic function (Twiss β-function [m]) that depend on the magnetic and RF configuration

Depend on the initial conditions of the particle

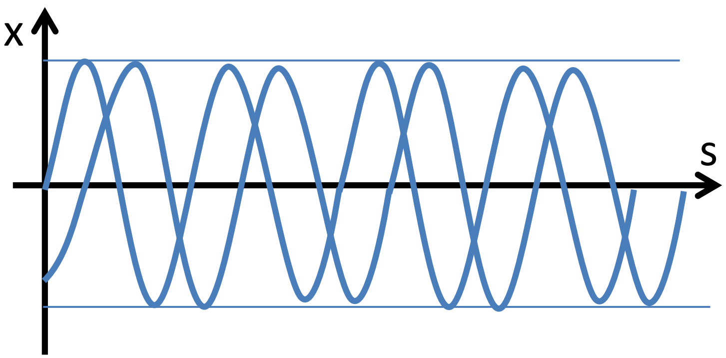
$$\sigma=\int_D\frac{ds}{\beta(s)}\approx\frac{D}{\langle\beta\rangle}$$

**Transverse phase advance per period. For stability should be 0<σ<π**

The final transverse beam dimensions ($\sigma_{x,y}(s)$) varies along the linac and are contained within an **envelope**

# SMOOTH APPROXIMATION OF TRANSVERSE OSCILLATIONS

⇒In case of "**smooth approximation**" of the LINAC (we consider an average effect of the quadrupoles and RF) we obtain a simple harmonic motion along s of the type ($\beta$ is constant):




$$x(s) = \sqrt{\varepsilon_o}\sqrt{1/K_0}\cos(K_0 s + \phi_0)$$

Considering prontons or ions

Phase advance per unit length ($\sigma$/D)

$$K_0 = \sqrt{\left(\frac{qGl}{2m_0 c\gamma\beta}\right)^2 - \frac{\pi q\hat{E}_{acc}\sin(-\phi)}{m_0 c^2 \lambda_{RF}(\gamma\beta)^3}}$$

Magnetic focusing elements

RF defocusing term

*G=quadrupole gradient [T/m]*
*l=quadrupole length*

NB: the RF defocusing term $\propto$f sets a higher limit to the working frequency

If we consider also the SC contribution in the simple caso of an **ellipsoidal beam** (linear space charges) we obtain:

$$K_0 = \sqrt{\left(\frac{qGl}{2m_0 c\gamma\beta}\right)^2 - \frac{\pi q\hat{E}_{acc}\sin(-\phi)}{m_0 c^2 \lambda_{RF}(\gamma\beta)^3} - \frac{3Z_0 qI\lambda_{RF}(1-f)}{8\pi m_0 c^2 \beta^2\gamma^3 r_x r_y r_z}}$$

Space charge term

I= beam current
$r_{x,y,z}$=ellipsoid semi-axis
f= form factor (0<f<1)
$Z_0$=free space impedance (377 Ω)

For ultrarelativistic **electrons** RF defocusing and space charge disappear and the external focusing is required to control the emittance and to stabilize the beam against instabilities.

# GENERAL CONSIDERATIONS ON LINAC OPTICS DESIGN (1/2)

***PROTONS AND IONS***

⇒ Beam dynamics dominated by **space charge and RF defocusing forces**

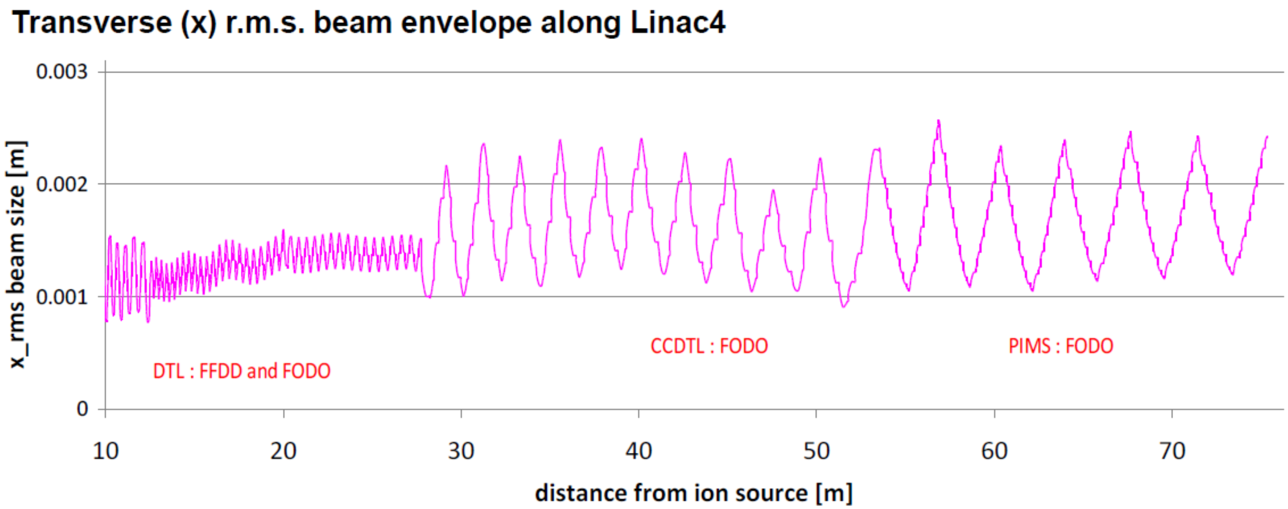
⇒ Focusing is usually provided by **quadrupoles**

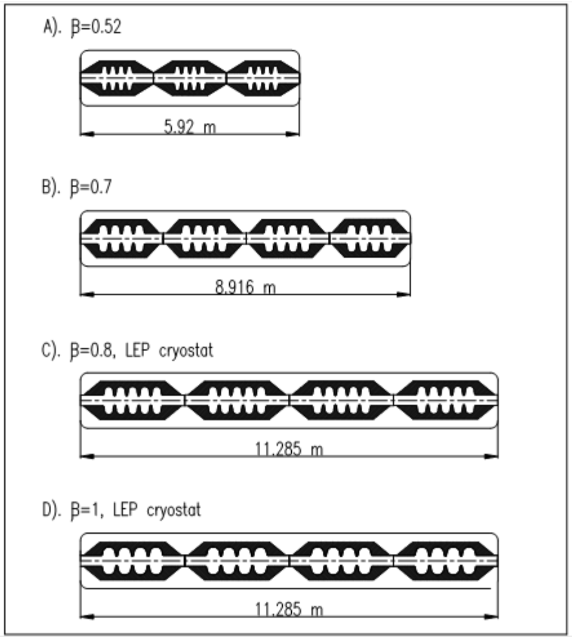
⇒ Phase advance per period ($\sigma$) should be, in general, in the range 30-80 deg, this means that, at low energy, we need a strong focusing term (**short quadrupole distance and high quadrupole gradient**) to compensate for the defocusing, but the limited space ($\beta\lambda$) limits the achievable G and beam current

⇒ **As $\beta$ increases, the distance between focusing elements can increase** ($\beta\lambda$ in the DTL goes from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV), and can be increased to 4-10$\beta\lambda$ at higher energy (>40 MeV).

⇒ A linac is made of a **sequence of structures, matched to the beam velocity**, and where the length of the focusing period increases with energy. As $\beta$ increases, longitudinal phase error between cells of identical length becomes small and we can have **short sequences of identical cells** (lower construction costs).

⇒ Keep sufficient safety **margin between beam radius and aperture**



# GENERAL CONSIDERATIONS ON LINAC OPTICS DESIGN (2/2)

## *ELECTRONS*

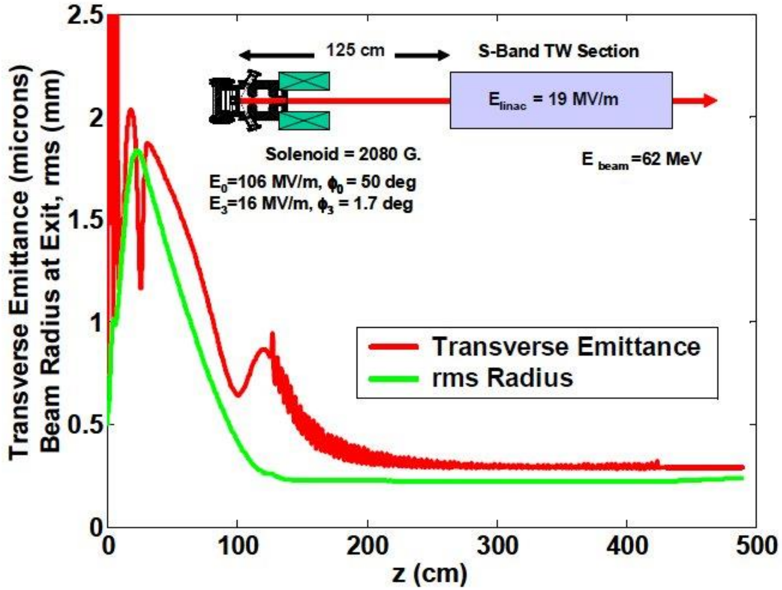
⇒ **Space charge only at low energy and/or high peak current**: below 10-20 MeV (injector) the beam dynamics optimization has to include emittance compensation schemes with, typically solenoids;

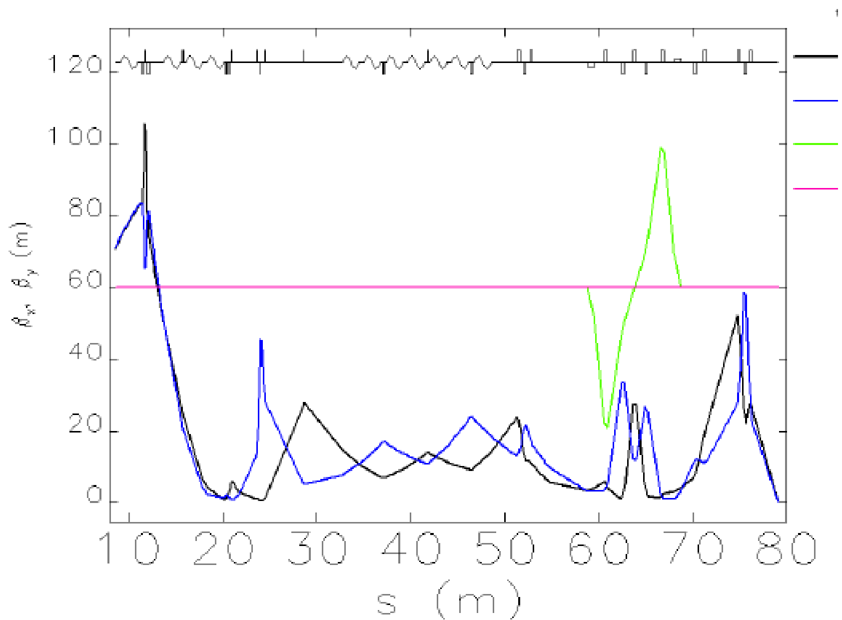
⇒ At higher **energies no space charge and no RF defocusing effects** occur but we have **RF focusing due to the ponderomotive force: focusing periods up to several meters**

⇒ Optics design has to take into account **longitudinal and transverse wakefields** (due to the higher frequencies used for acceleration) that can cause energy spread increase, head tail oscillations, multi-bunch instabilities

⇒ Longitudinal bunch compressors schemes based on magnets and chicanes have to take into account, for short bunches, the interaction between the beam and the emitted synchrotron radiation (**Coherent Synchrotron Radiation effects**)

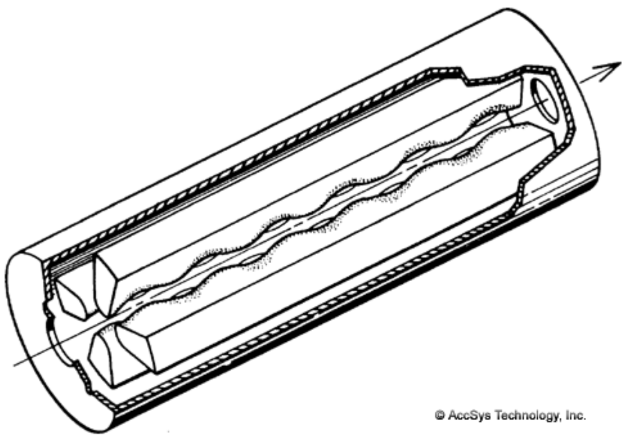
⇒ All these effects are important especially in LINACs for **FEL that requires extremely good beam qualities**

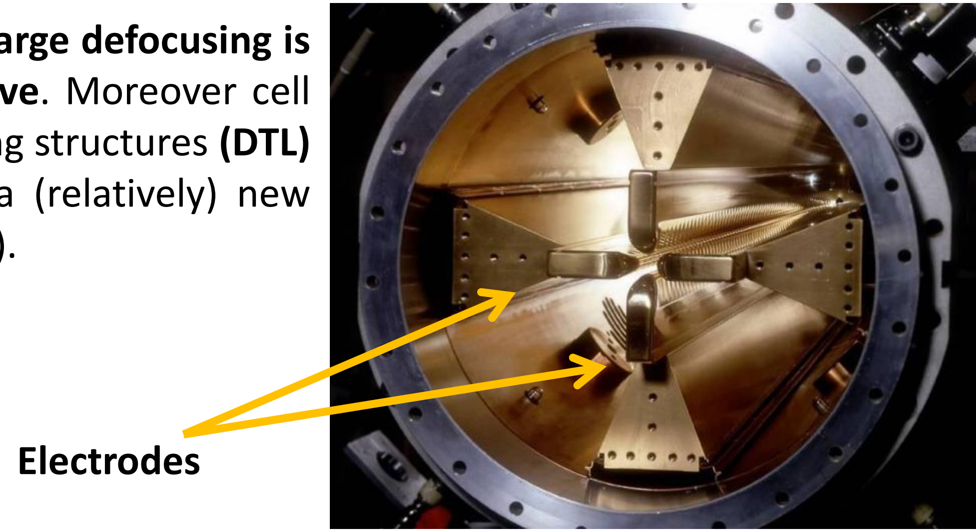


# RADIO FREQUENCY QUADRUPOLES (RFQ)

At low proton (or ion) energies (β~0.01), **space charge defocusing is high and quadrupole focusing is not very effective**. Moreover cell length becomes small and conventional accelerating structures **(DTL) are very inefficient**. At this energies it is used a (relatively) new structure, the **Radio Frequency Quadrupole** (1970).

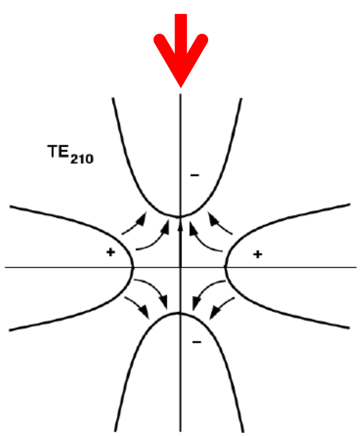


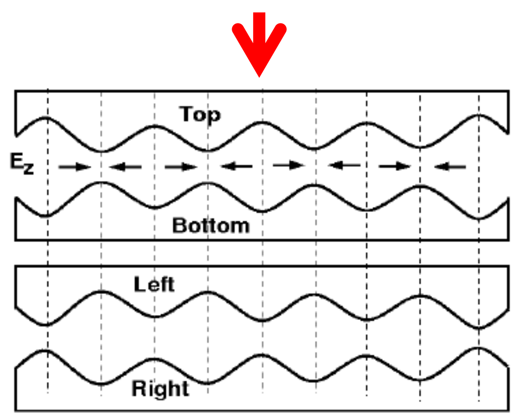

Courtesy M. Vretenar

These structures allow to simultaneously provide:

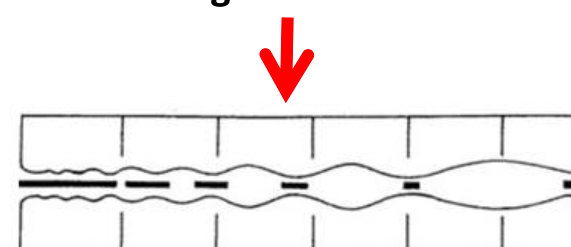
**Transverse Focusing**




**Acceleration**



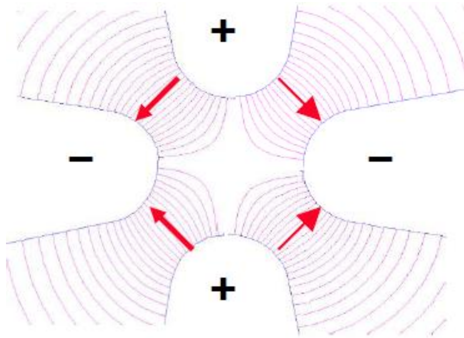

**Bunching of the beam**

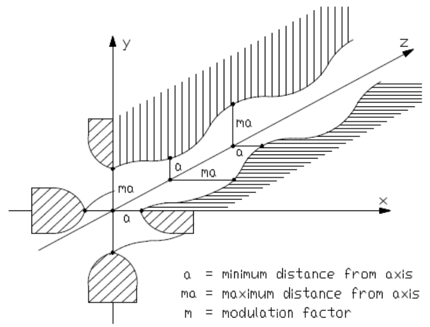


# RFQ: PROPERTIES

***1-Focusing***

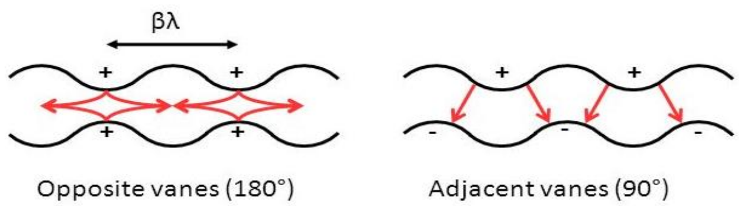
The resonating mode of the cavity (between the four electrodes) is a **focusing mode**: **Quadrupole mode ($TE_{210}$)**. The alternating voltage on the electrodes produces an **alternating focusing channel** with the period of the RF (**electric focusing** does not depend on the velocity and is ideal at low $\beta$)



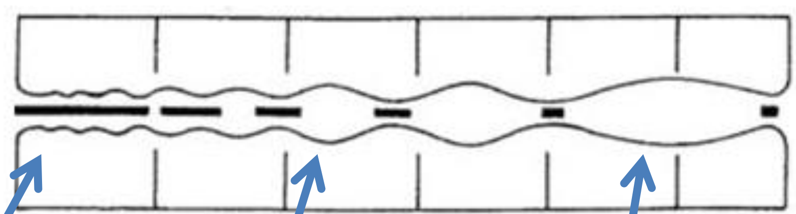

***2-Acceleration***

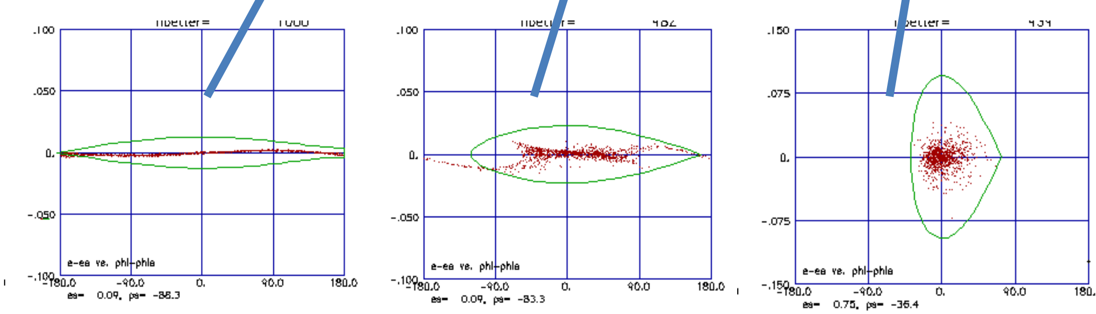
The vanes have a **longitudinal modulation** with period = $\beta\lambda_{RF}$ this creates a **longitudinal component of the electric field** that accelerate the beam (the modulation corresponds exactly to a series of RF gaps).

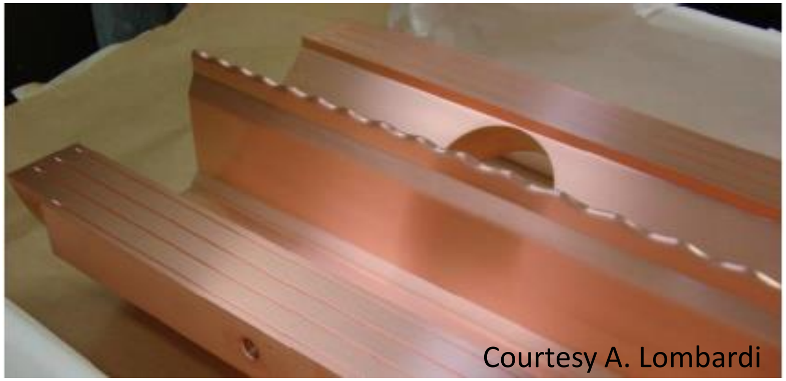



**3-Bunching**

The modulation period (distance between maxima) can be slightly adjusted to change the phase of the beam inside the RFQ cells, and the amplitude of the modulation can be changed to change the accelerating gradient. One can start at -90° phase (linac) with some **bunching cells**, progressively **bunch the beam** (adiabatic bunching channel), and only in the last cells switch on the **acceleration**.





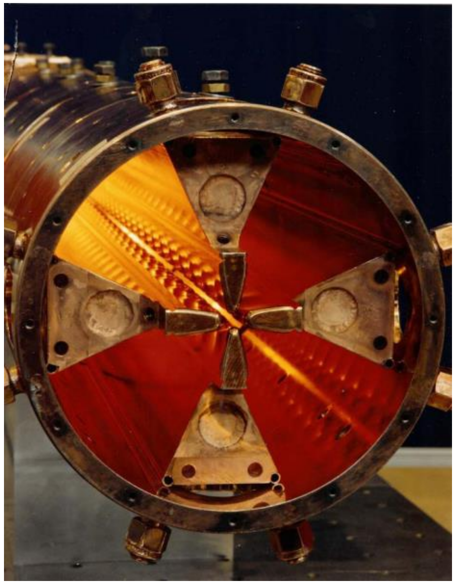



Courtesy A. Lombardi

The RFQ is the only linear accelerator that can accept a low energy continuous beam.

Courtesy M. Vretenar and A. Lombardi

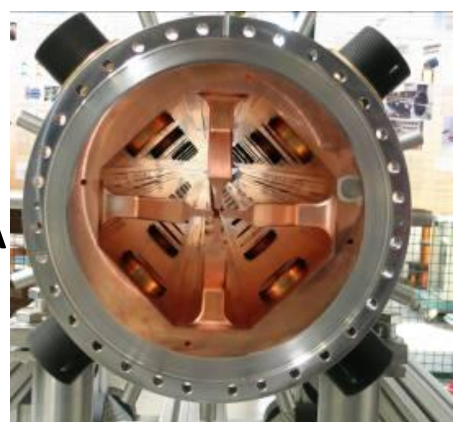
# RFQ: EXAMPLES



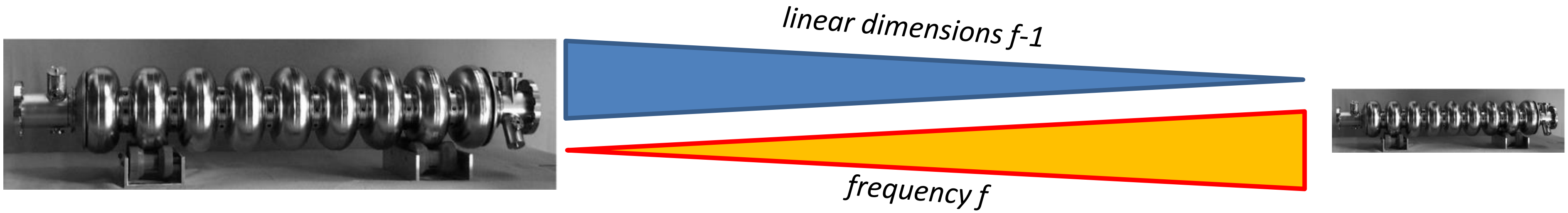
The 1st 4-vane RFQ, Los Alamos
1980: 100 KeV - 650 KeV, 30 mA , 425 MHz


The CERN Linac4 RFQ
45 keV – 3 MeV, 3 m
80 mA H-, max. 10% duty cycle

TRASCO @ INFN Legnaro
Energy In: 80 keV
Energy Out: 5 MeV
Frequency 352.2 MHz
Proton Current (CW) 30 mA



# APPENDIX: THE CHOICE OF THE FREQUENCY




| **Structure dimensions** | **Scales with 1/f** |
|---|---|
| Shunt impedance (efficiency) per unit length r | **NC structures** r increases and this push to adopt **higher frequencies** $\propto f^{1/2}$ |
| | **SC structures** the power losses increases with $f^2$ and, as a consequence, r scales with 1/f this push to adopt **lower frequencies** |
| Power sources | At very high frequencies (>10 GHz) **power sources** are commercially not available or expensive |
| Mechanical realization | Cavity fabrication at very high frequency requires **higher precision** but, on the other hand, at low frequencies one needs more material and **larger machines**/brazing oven |
| Bunch length | **short bunches** are easier with higher f (FEL) |
| RF defocusing (ion linacs) | Increases with frequency ($\propto$ f) |
| Cell length ($\beta\lambda$RF) | 1/f |
| Wakefields | more critical at high frequency ($w_{//} \propto f^{2}$, $w_{\perp} \propto f^{3}$) |

$\Rightarrow$**Higher frequencies** are economically convenient (shorter, less RF power, higher gradients possible) but the limitation comes from **mechanical precision** (tight tolerances are expensive!) and **beam dynamics** for ion linacs.

$\Rightarrow$**Electron** linacs tend to use higher frequencies (1-12 GHz) than ion linacs.
**SW SC: 500 MHz-1500 MHz**
**TW NC: 3 GHz-12 GHz**

$\Rightarrow$**Proton** linacs use lower frequencies (100-800 MHz), increasing with energy (ex.: 350–700 MHz): compromise between focusing, cost and size.
**Heavy ion** linacs tend to use low frequencies (30-200 MHz),

# THE CHOICE OF THE ACCELERATING STRUCTURE

In general **the choice of the accelerating structure depends on**:

$\Rightarrow$ **Particle type**: mass, charge, energy
$\Rightarrow$ **Beam current**
$\Rightarrow$ **Duty cycle** (pulsed, CW)
$\Rightarrow$ **Frequency**
$\Rightarrow$ **Cost** of fabrication and of operation

Moreover **a given accelerating structure has also a curve of efficiency** (shunt impedance) with respect to the particle energies and the choice of one structure with respect to another one depends also on this.

As example a very general scheme is given in the Table (absolutely not exhaustive).

| Cavity Type | β Range | Frequency | Particles |
|---|---|---|---|
| **RFQ** | 0.01– 0.1 | 40-500 MHz | Protons, Ions |
| **DTL** | 0.05 – 0.5 | 100-400 MHz | Protons, Ions |
| **SCL** | 0.5 – 1 | 600 MHz-3 GHz | Protons, Electrons |
| **SC Elliptical** | > 0.5-0.7 | 350 MHz-3 GHz | Protons, Electrons |
| **TW** | 1 | 3-12 GHz | Electrons |

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