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Imperfections and Correction

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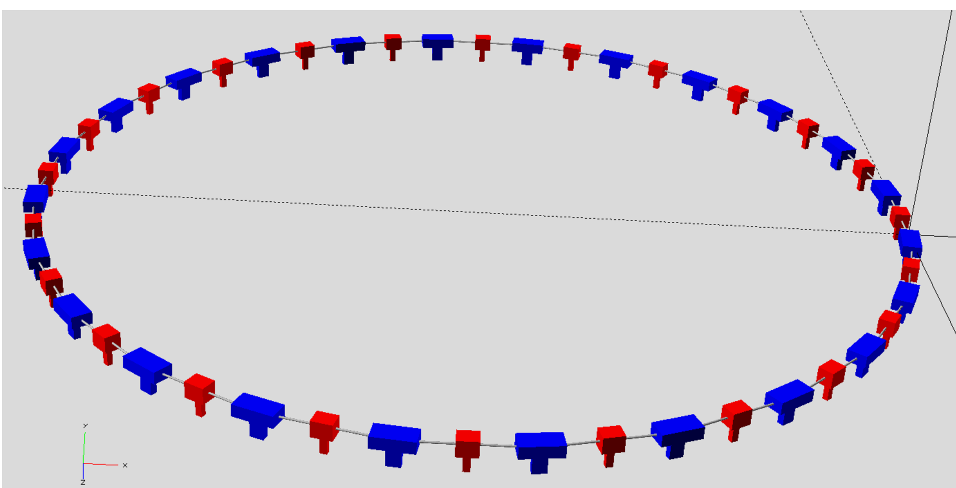
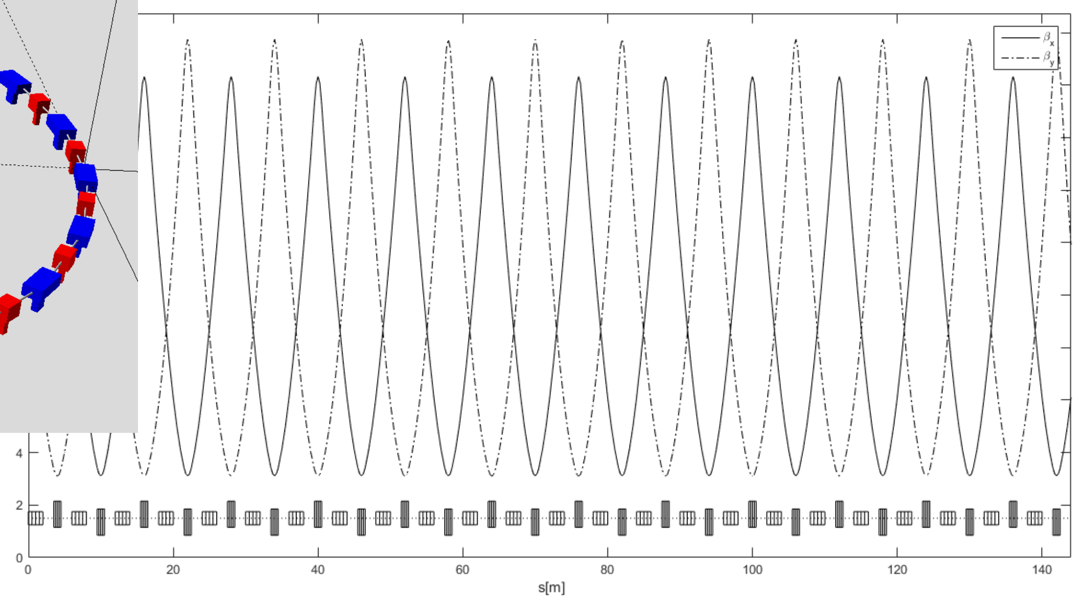
What is this talk about?

- First, you come up with lattice and design optics
  - nice and shiny beta functions
  - high periodicity → systematic errors cancel





But then...

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- ...the accelerator is built, and..
  - the magnets are not quite where they should be;
  - power supplies have calibration errors;
  - magnets have incorrects shims;
  - the diagnostics might have imperfections, too.
    - BPM
    - Screens

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Therefore...

- I talk about
  - things that can go wrong (courtesy of Mrs Murphy...) → Imperfections
  - how to figure out what is wrong → Diagnostics to use
  - and fix it → Corrections

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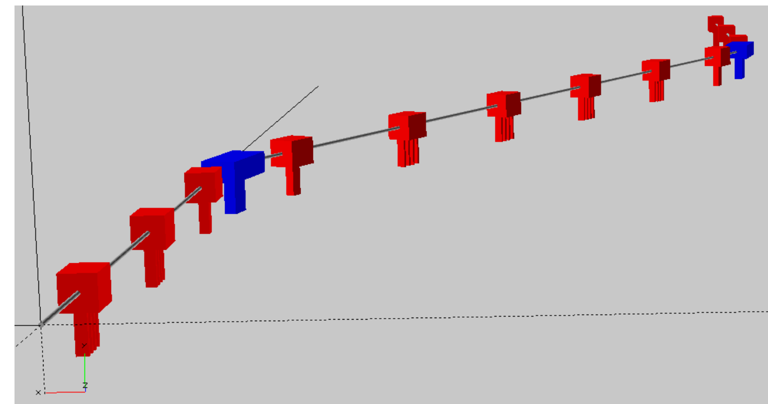
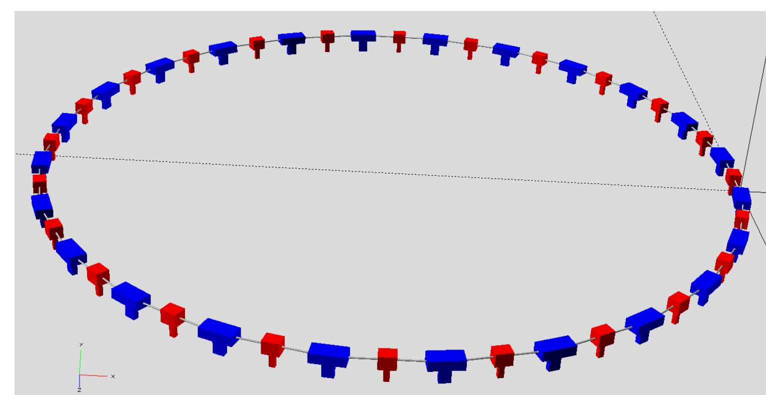
Outline

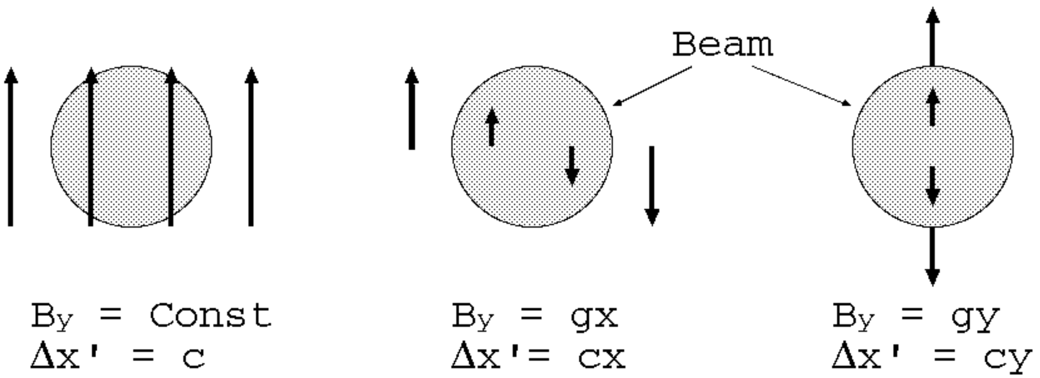
- Imperfections
- Straight systems
  - Beamlines and Linac
  - Imperfections and their corrections
- Rings
  - Imperfections and their corrections

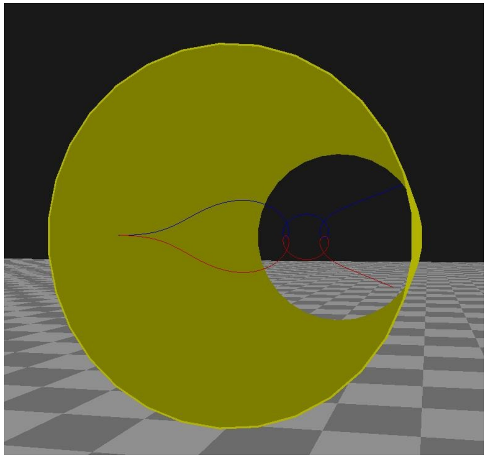




Part 1: Linear Imperfections

- Spoil the 'nice&shiny™' periodic magnet lattice
  - due to unwanted magnetic fields in the wrong place
- that's where the beam is
  - average: dipole kick
  - gradient: focussing
  - skew gradient: coupling
- Solenoidal fields
  - detector
  - electron cooler





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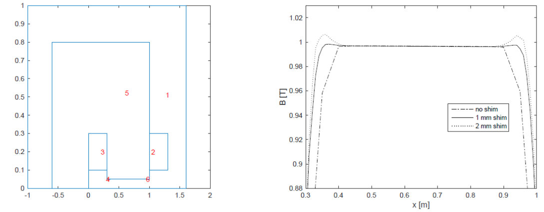
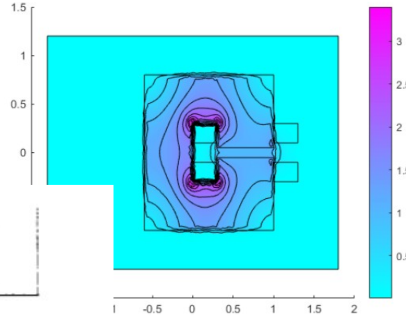
Sources of Imperfections

- Anything that is not in the design lattice
- Fringe fields and cross talk between magnets
- Saturation of magnets
- Power supply calibration and readback errors
- Wrong shims
- Earth magnetic field in low-energy beam lines
- Nickel layers in the wrong place
- Solenoids in detectors or coolers
- Weak focussing from undulators
- Tilt and roll angles of magnets
- Misaligned magnets (or beams)





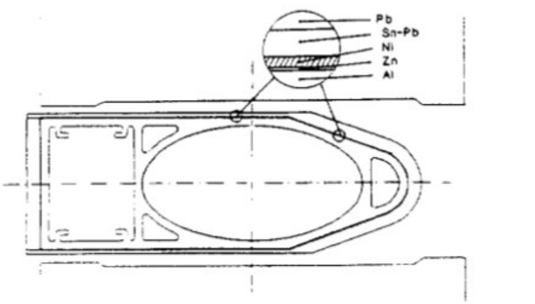


Figure 1: *The LEP dipole chamber and its nickel layer*

J. Billan et al., PAC 1993

Alignment

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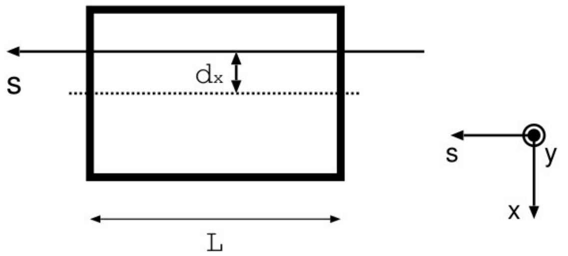
- How do you do it?
  - Magnets on tables
  - Fiducialization to pods
  - Triangulation
- How well can you do it?
  - 0.2-0.3 mm OK
  - <0.1 mm increasingly more difficult
  - more difficult in large installations
- Sub-micron for linear colliders → beam-based

Photo: R. Ruber, CTF3-TBTS

Misaligned Magnets

- Misalignment of linear elements



$$\begin{pmatrix} x_f \\ x'_f \end{pmatrix} = \begin{pmatrix} -d_x \\ 0 \end{pmatrix} + \tilde{R}\left[\begin{pmatrix} d_x \\ 0 \end{pmatrix} + \begin{pmatrix} x_i \\ x'_i \end{pmatrix}\right]$$

$$= \tilde{R}\begin{pmatrix} x_i \\ x'_i \end{pmatrix} + \left[\tilde{R} - 1\right]\begin{pmatrix} d_x \\ 0 \end{pmatrix} = \vec{q} + \tilde{R}\begin{pmatrix} x_i \\ x'_i \end{pmatrix}$$

- and for a thin quadrupole...

$$\vec{q} = \left[\tilde{R} - 1\right]\begin{pmatrix} d_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -\frac{1}{f} & 0 \end{pmatrix}\begin{pmatrix} d_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{d_x}{f} \end{pmatrix}$$

- An additional dipolar kick appears → **feed-down**

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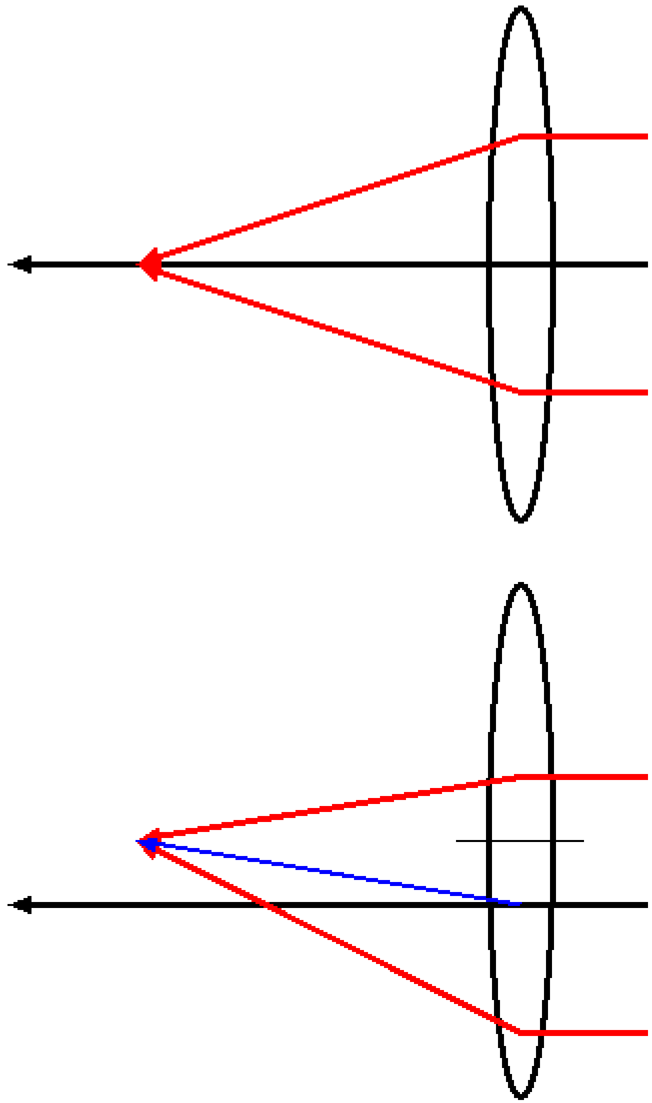
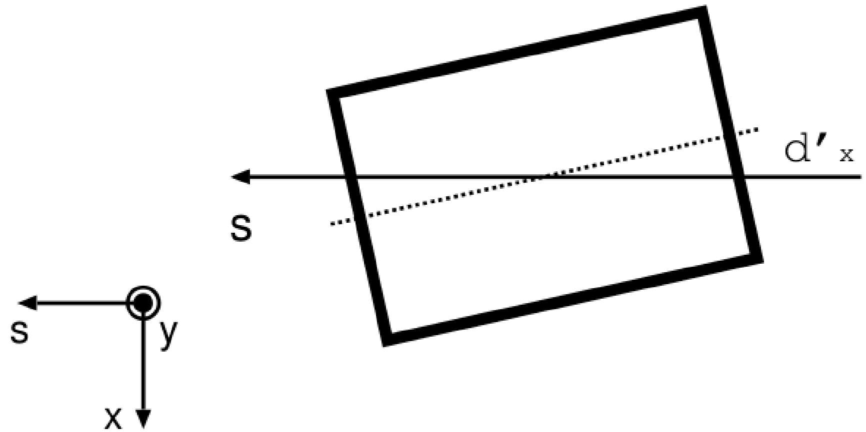
Misaligned quadupoles focus just as good as centered ones



- Same focal length despite misalignment
- Lower ray is further away from the quad center and bent more
- Upper ray is closer to axis and is bent less
- But they kick the centroid of the beam

Tilted elements



- come in, step right and point left, go through, step right again and point right

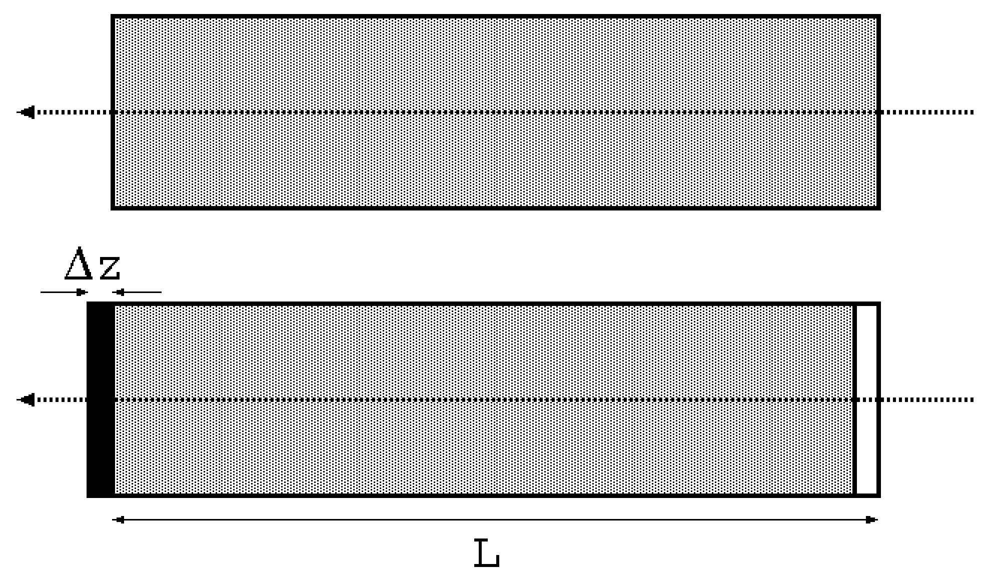
$$\begin{pmatrix} x_f \\ x'_f \end{pmatrix} = \begin{pmatrix} -d'_x L/2 \\ -d'_x \end{pmatrix} + \hat{R}\left[\begin{pmatrix} -d'_x L/2 \\ d'_x \end{pmatrix} + \begin{pmatrix} x_i \\ x'_i \end{pmatrix}\right]$$

$$= \hat{R}\begin{pmatrix} x_i \\ x'_i \end{pmatrix} + \left[\hat{R} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]\begin{pmatrix} -d'_x L/2 \\ d'_x \end{pmatrix} = \vec{q} + \tilde{R}\begin{pmatrix} x_i \\ x'_i \end{pmatrix}$$

- Again,normal transport and a constant vector

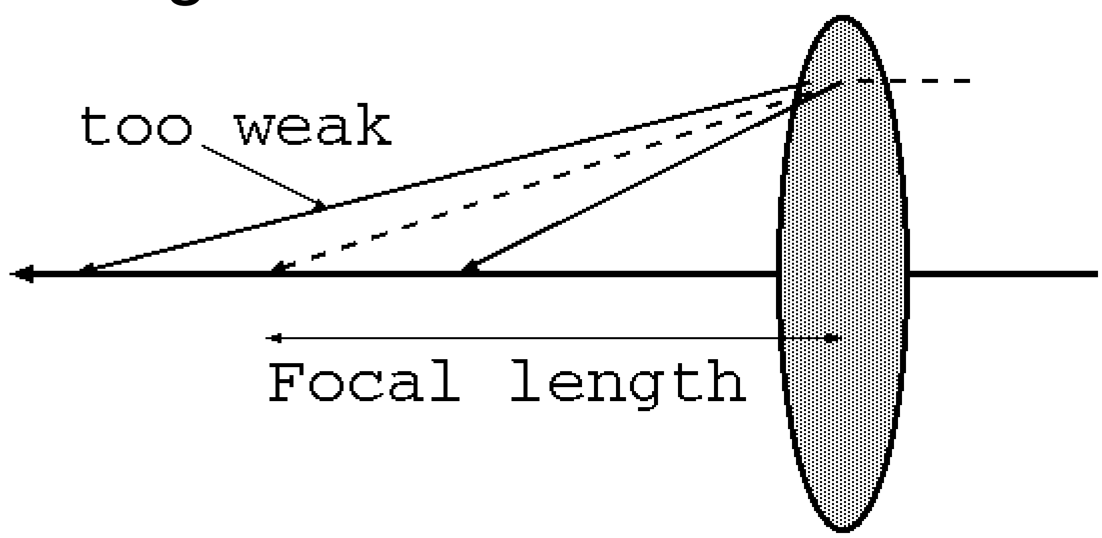
Longitudinally Shifted Elements

- Add a short positive element on one side and the negative on the other
- Dipole
  - kick on either side
- Quadrupoles
  - thin quadrupoles



Incorrectly powered Quadrupoles

- Focal length changes
  - beam matrix differs from the expected
  - beta functions change
  - in rings, the tune changes



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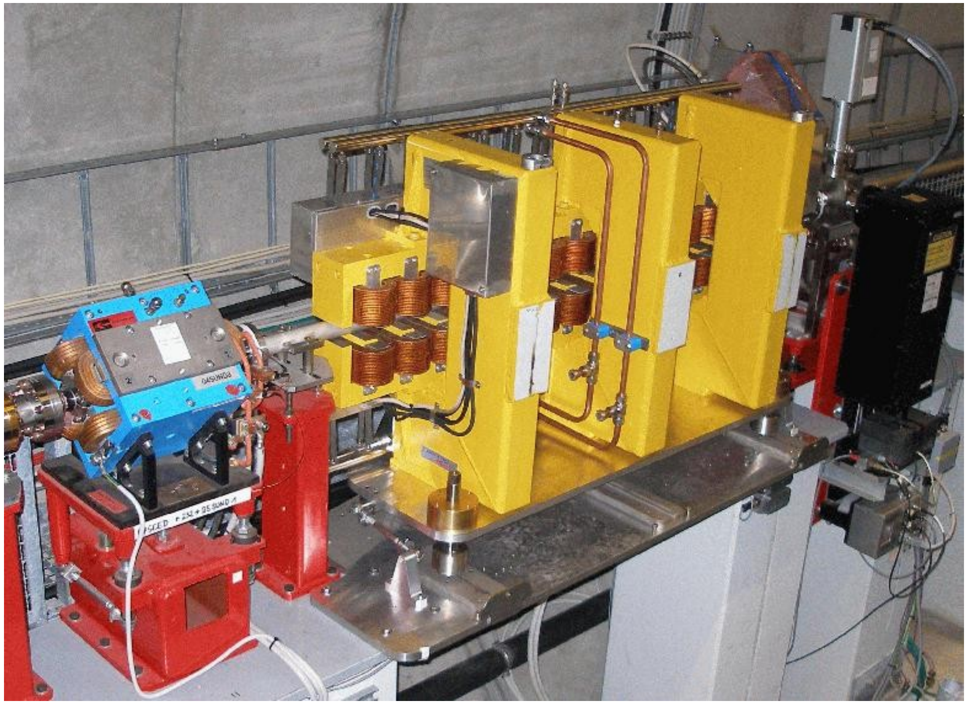
Undulators

- $B_y \sim \cos(2\pi s/\lambda_u) \rightarrow$ horizontal oscillations
- $\partial B_y/\partial s = \partial B_s/\partial y \rightarrow$ vertically changing B_s
- Focus vertically (only)
- Many Rbends
- weak effect $(l/\rho)^2$, but
- changing gap may
  - affect orbit
  - affect tune

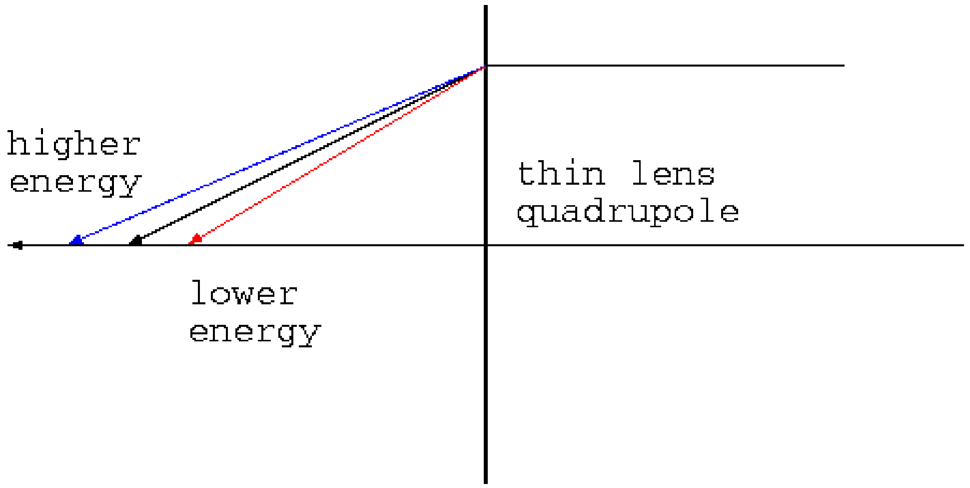


Dispersion

- Effect of magnetic fields on the beam (~*B/p*) with $p=p_0(1+\delta)$ is reduced by $1+\delta$
- Every dipole behaves as a spectrometer
  - separates the particles according to their momentum
  - even dipole correctors contribute
- In planar systems the vertical dispersion is by design zero
  - but rolled dipoles (and quadrupoles) make it non-zero.

Chromaticity

- Also quadrupolar fields are reduced by $1+\delta$
  - longitudinal location of the focal plane depends on momentum and enlarges the beam sizes at the IP
  - chromaticity $Q'=dQ/d\delta$
  - tune spread



Measuring Dispersion and Chromaticity

- Change the beam energy, in rings by changing the RF frequency
  - and look at orbit from BPMs (dispersion)
  - and measure the tune (chromaticity)
- In transfer line or linac change the energy of the injected beam
- Optionally, may scale all magnets with the same factor
  - all beam observables ~B/p

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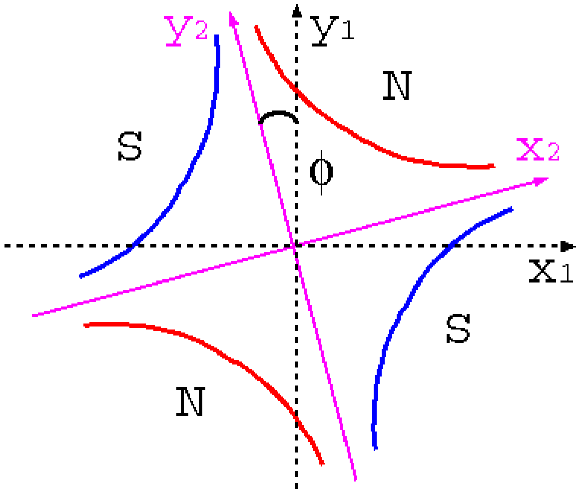
Rolled elements



- Coordinate rotation

$$\begin{pmatrix} x_2 \\ x_2' \\ y_2 \\ y_2' \end{pmatrix} = \begin{pmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & \cos\phi & 0 & \sin\phi \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & -\sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix}$$

- sandwich roll-left before the element and then roll-right after the element
- example quad to skew-quad (example, thin quad)

$$Q_s = R(-\pi/4) \begin{pmatrix} Q_f & 0_2 \\ 0_2 & Q_d \end{pmatrix} R(\pi/4) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1/f & 0 \\ 0 & 0 & 1 & 0 \\ 1/f & 0 & 0 & 1 \end{pmatrix}$$

- mixes the transverse planes → coupling

Reminder: Multipoles

- Magnet builder's view (b_m: upright, a_m: skew)

$$B_y + iB_x = B_0 \sum_{m=1}^{\infty} (b_m + ia_m) \left(\frac{x+iy}{R_0} \right)^{m-1}$$

- How the beam "sees" the fields

$$\Delta x' - i\Delta y' = \frac{(B_y + iB_x)L}{B\rho} = \sum_{n=0}^{\infty} \frac{k_n L}{n!} (x+iy)^n$$

- Multipole coefficients
  - real part: upright
  - imaginary part: skew

$$\frac{k_n L}{n!} = \frac{(B_0/R_0^n)L}{B\rho} (b_{n+1} + ia_{n+1})$$

$$k_n L = \frac{d^n B/dx^n}{B\rho} L$$

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Feed-down from displaced multipoles

- Kick from thin multipole $\Delta x' - i\Delta y' = \frac{k_n L}{n!}(x+iy)^n$
- and from a displaced multipole

$$\Delta x' - i\Delta y' = \frac{k_n L}{n!}(x+d_x+iy)^n$$
$$= \frac{k_n L}{n!}(x+iy)^n + \frac{k_n L}{n!}\sum_{k=0}^{n-1}\binom{n}{k} d_x^{n-k}(x+iy)^k$$

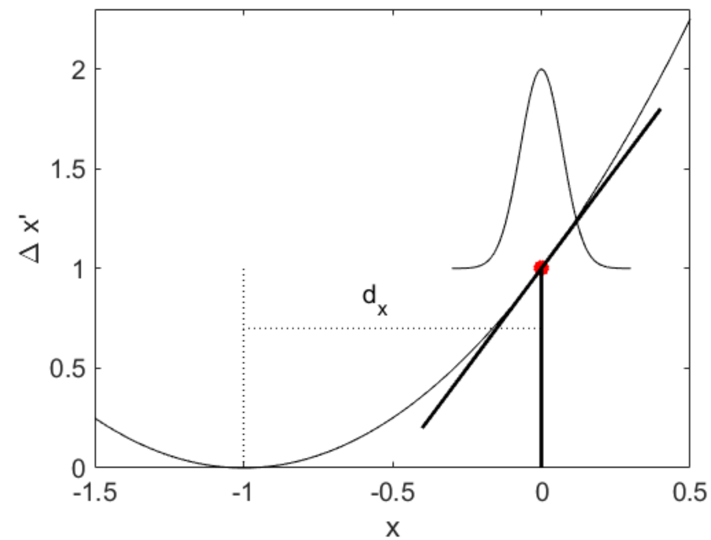
  - binomial expansion
- Displaced multipole still works as intended, but also generates all lower multipoles

Feed-down from sextupoles

- Horizontally misalinged

$$\Delta x' - i\Delta y' = \frac{k_2 L}{2}\left[(x+iy)^2 + 2d_x(x+iy) + d_x^2)\right]$$

  - additional quadrupolar and dipolar kicks



- Vertically misaligned

$$\Delta x' - i\Delta y' = (x+iy+id_y)^2 = \frac{k_2 L}{2}\left[(x+iy)^2 + 2id_y(x+iy) - d_y^2)\right]$$

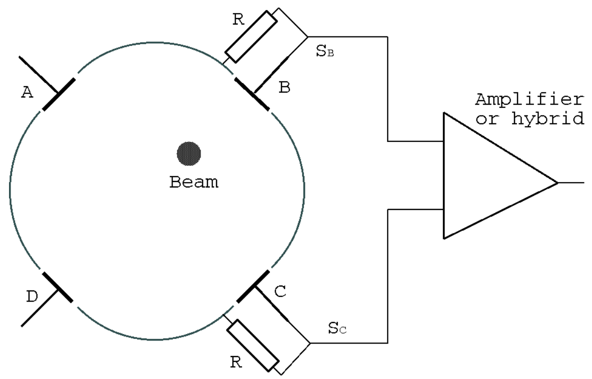
  - additional skew-quadrupolar and dipole kicks
  - vertical displacement in sextupoles causes coupling

Detrimental effects

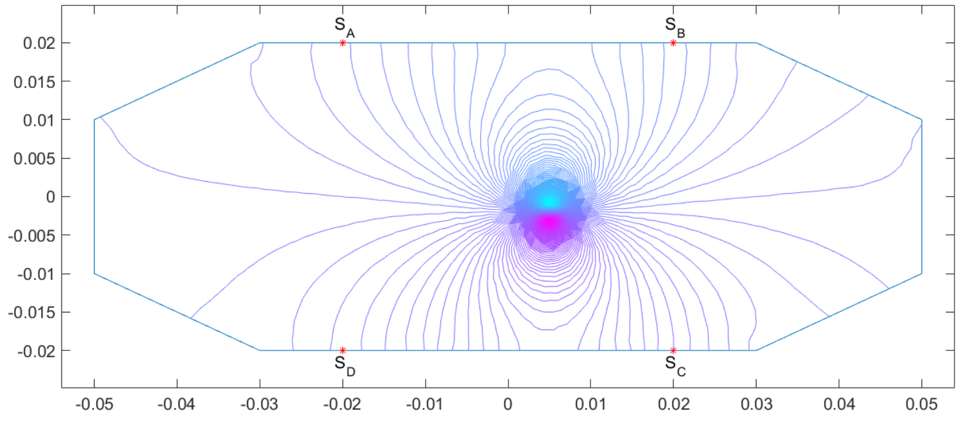
- Dipole fields cause beam to be in wrong place
  - losses, bad if you have a multi-MJ beam
  - Background in the experiments (Coll+LS)
- Gradients change the beam size, this spoils
  - Luminosity, if you work on a collider
  - Coherence, if you work on a light source
- Breaks the symmetry of the optics of a ring
  - more resonances
  - reduces dynamic aperture
- Need observables to figure out what's wrong

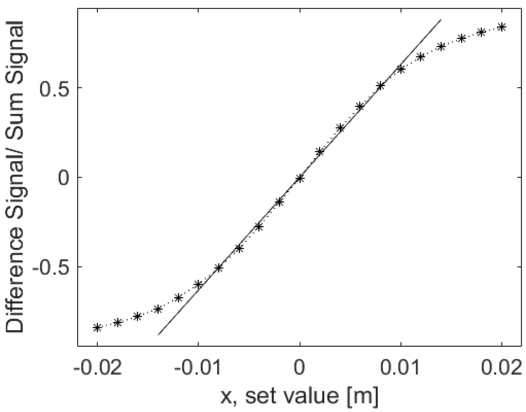
Beam Position Monitors and their Imperfections

- Transverse offset
- (Longitudinal position)
- Electrical offset
- Scale error



$$x = k_x \frac{(S_A + S_D) - (S_B + S_C)}{S_A + S_B + S_C + S_D}$$





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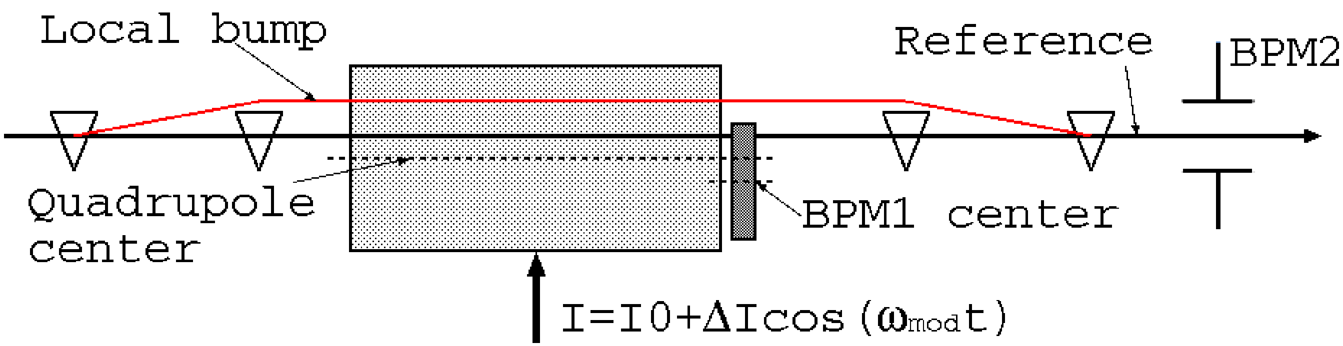
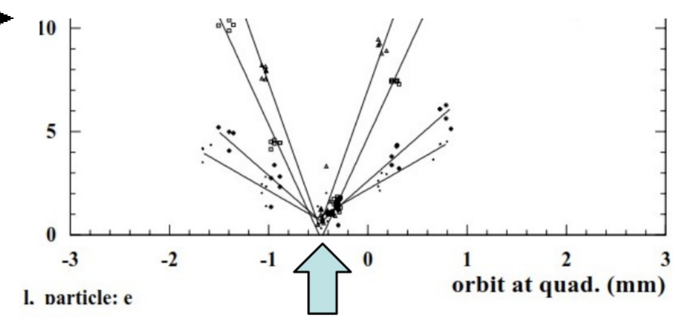
Find offsets with K-modulation

- BPM+Quadrupole are often mounted on the same support





- Modulate gradient of quadrupole
  - Kick from quadrupole $\Theta = dx/f(\omega)$ is also modulated
  - Observe on BPM2 and minimize signal by moving beam with a bump → quadrupole center
  - Reading of BPM1 gives BPM1 offset rel. to Quad

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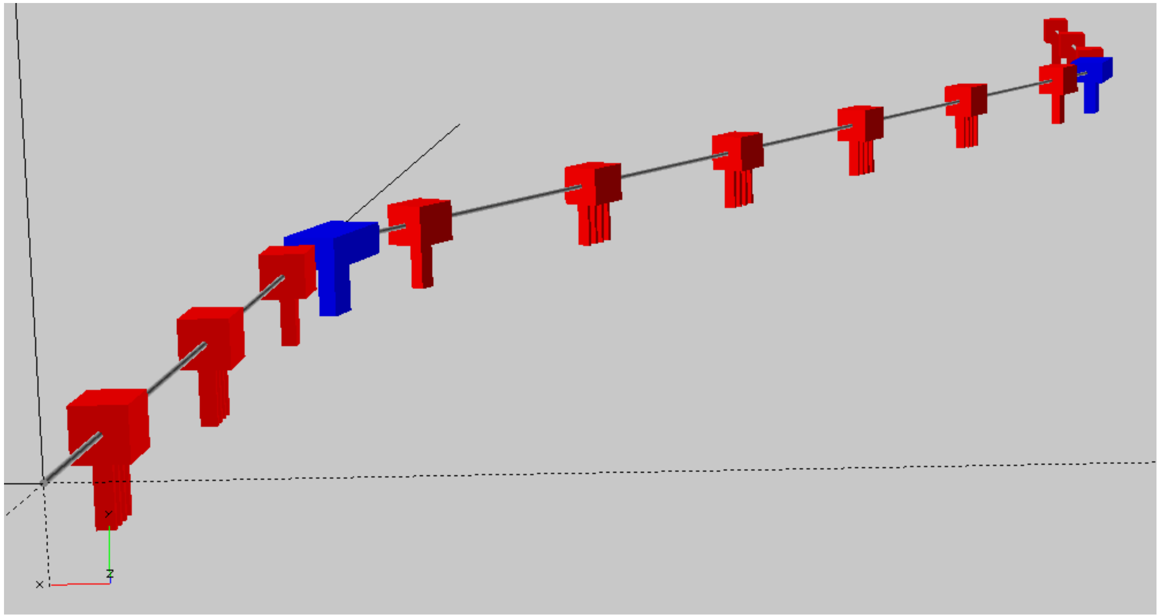
Screens et al. and their Bugs

- Transverse position
- Scale errors from the optical system
  - place fiducial marks on the screen
- Looking at an angle
- Depth of focus limitations, especially at large magnification levels
- Burnt-out spots on fluorescent screens
- Non-linear response of screen and saturation

Imperfections and their Correction in Beam Lines or Linacs

- Dipole errors
- Gradient errors
- Skew-gradient errors
- Filamentation



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Transfer matrices in Linacs

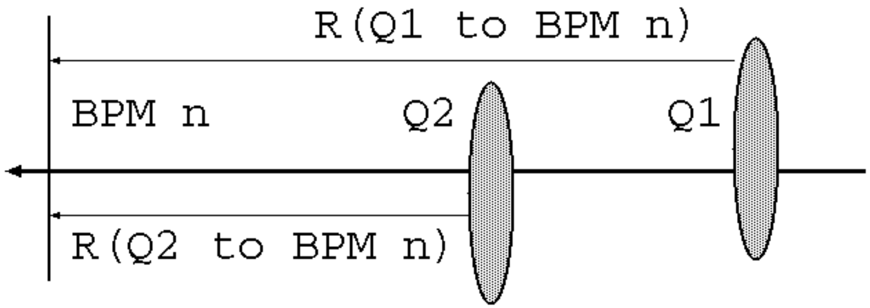
- Just a reminder...
- The beam energy at the location for the kick and the observation point may be different.
- Adiabatic damping
  - transverse momentum p_x is constant
  - longitudinal momentum p_s increases
  - $x'=p_x/p_s$ scales with $p_s = \beta\gamma\, mc$
- R_{12} then scales with $(\beta\gamma)_{kick}/(\beta\gamma)_{look}$

Beam lines: Dipole errors

- Each misaligned element with label *k* may add a misalignment dipole-kick $\vec{q}_k$

$$\begin{aligned}\vec{x}_n &= R_n \cdots (\vec{q}_{k+1} + R_{k+1})(\vec{q}_k + R_k) \cdots (\vec{q}_1 + R_1)\vec{x}_0 \\ &= R_n \cdots R_1 \vec{x}_0 + \sum_{j=1}^{n-1} (R_n \cdots R_{j+1})\vec{q}_j\end{aligned}$$

- Simple interpretation
  - at the look-point (BPM) *n* all perturbing kicks are added with the transfer matrix from kick to end



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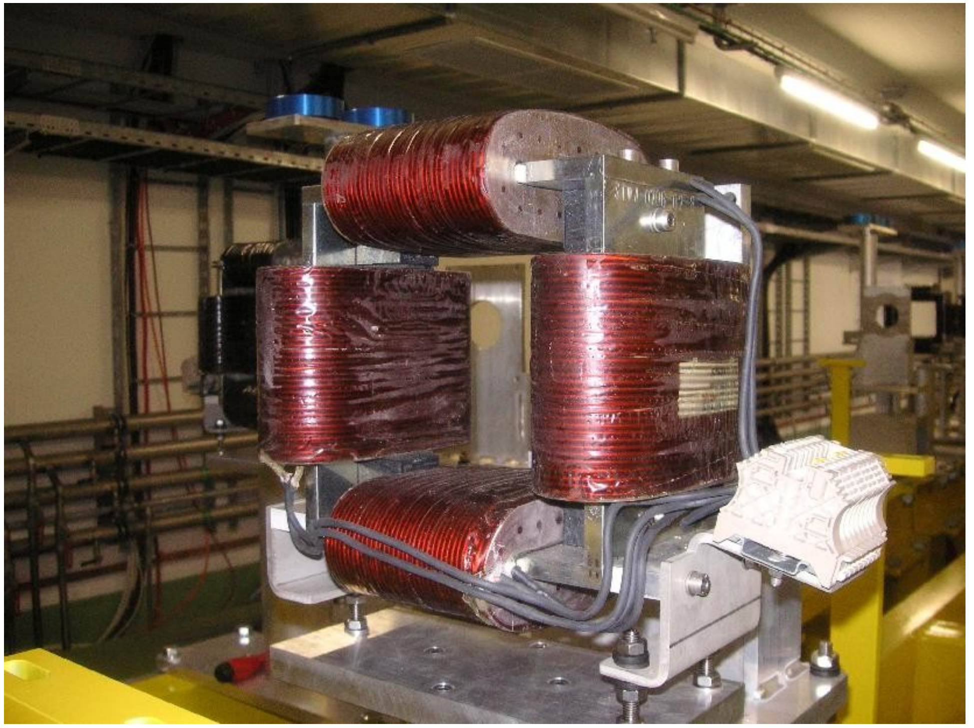
Correct with orbit correctors

- small dipole magnet, here for both planes (steerer for CTF3-TBTS)
- affects the beam like any other error



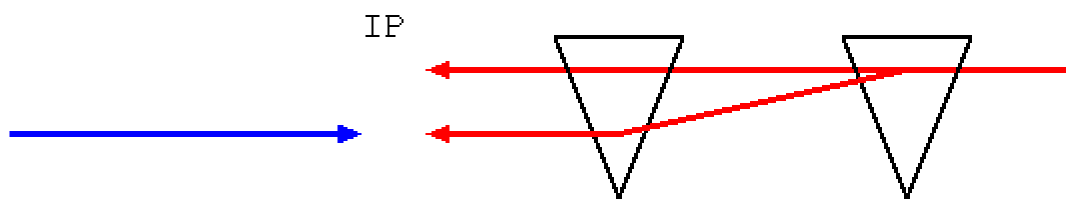
$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 0 \\ \theta \end{pmatrix} + \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

$$\vec{x}_1 = \vec{q} + \tilde{R}\vec{x}_0$$

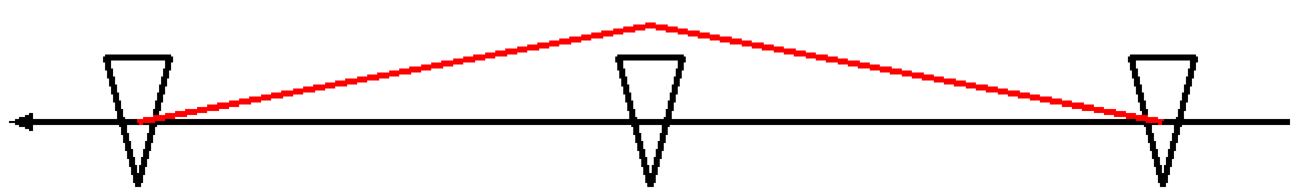
- treat just as additional misalignment

Local trajectory Bumps

- Occasionally a particular displacement or angle of the orbit at a given point might be required
- Displace orbit at IP to bring beams into collision



- or a slight excursion (3-bump)



- Differential changes ('by' not 'to')

Trajectory knob

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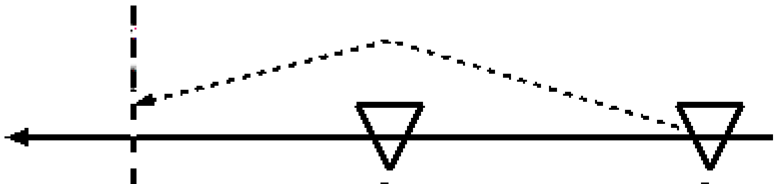
- Change position and angle at reference point



- Remember that kicks add up with TM from source to observation or reference point

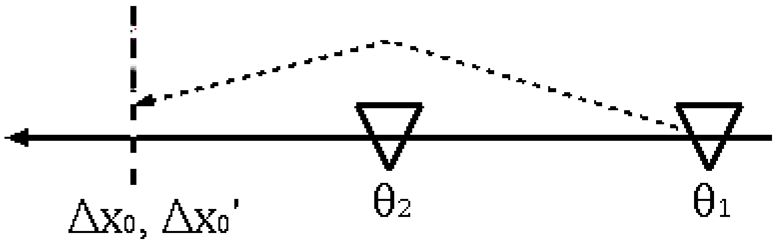
$$\begin{pmatrix} \Delta x_0 \\ \Delta x_0' \end{pmatrix} = \begin{pmatrix} R_{12}^{01} & R_{12}^{02} \\ R_{22}^{01} & R_{22}^{02} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

- and the **columns of the inverse matrix** are the knobs

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} R_{12}^{01} & R_{12}^{02} \\ R_{22}^{01} & R_{22}^{02} \end{pmatrix}^{-1} \begin{pmatrix} \Delta x_0 \\ \Delta x_0' \end{pmatrix}$$

A trivial example

- Two steering magnets with drift between them



$$R^{02} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad R^{01} = \begin{pmatrix} 1 & 2L \\ 0 & 1 \end{pmatrix}$$

- Response matrix

$$\begin{pmatrix} \Delta x_0 \\ \Delta x_0' \end{pmatrix} = \begin{pmatrix} R_{12}^{01} & R_{12}^{02} \\ R_{22}^{01} & R_{22}^{02} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 2L & L \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

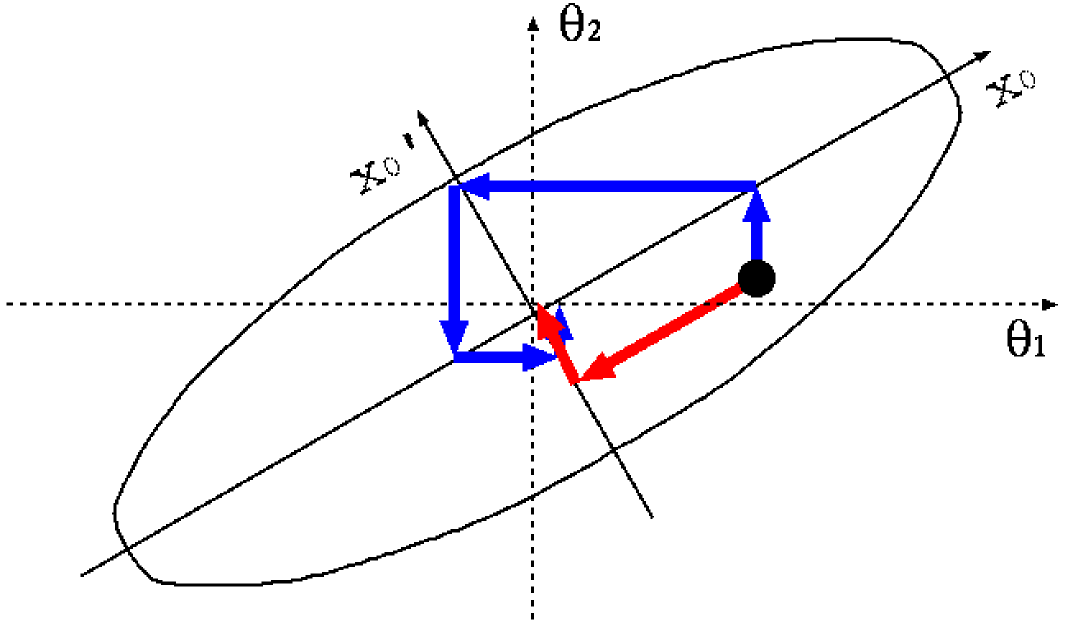
- Knobs

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \frac{1}{L} \begin{pmatrix} 1 & -L \\ -1 & 2L \end{pmatrix} \begin{pmatrix} \Delta x_0 \\ \Delta x_0' \end{pmatrix} \longrightarrow \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \frac{1}{L} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Delta x_0$$

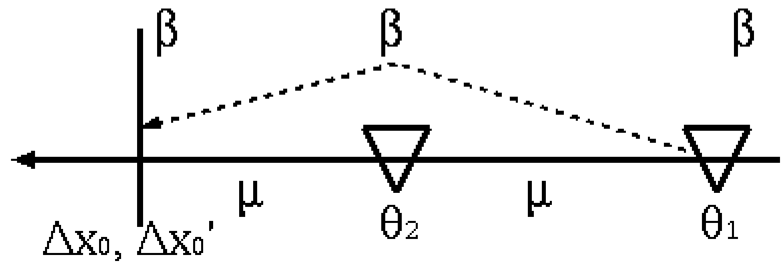
Almost common sense!

Remark about Orthogonality

- Knobs are orthogonal
- Optimize one parameter without screwing up the other(s).
  - Faster convergence
  - Enables heuristic optimization
  - Deterministic
- Use physics rather than hardware parameters



Optimality



- How “good” are the knobs?
- Position knob is ill-defined for L→0.

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \frac{1}{L}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \Delta x_0$$

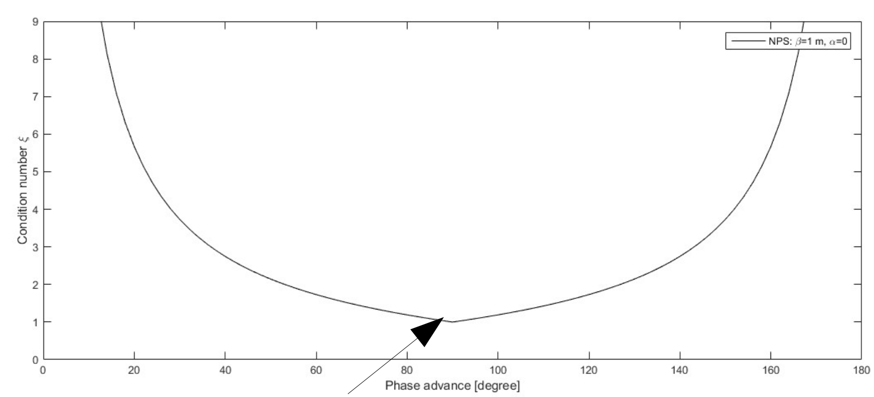
- Matrix inversion can fail → condition number

$$\xi = \frac{\lambda_{max}}{\lambda_{min}}$$

  - ξ=1: All parameters controlled equally well
- Consider beamline with betas equal and α=0 at steerers and observation point

$$\begin{pmatrix} \Delta x_0 \\ \Delta x_0' \end{pmatrix} = \begin{pmatrix} R_{12}^{01} & R_{12}^{02} \\ R_{22}^{01} & R_{22}^{02} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

$$\begin{pmatrix} \Delta x/\sqrt{\beta} \\ \sqrt{\beta}\Delta x' \end{pmatrix} = \begin{pmatrix} \sin 2\mu & \sin\mu \\ \cos 2\mu & \cos\mu \end{pmatrix} \begin{pmatrix} \sqrt{\beta}\theta_1 \\ \sqrt{\beta}\theta_2 \end{pmatrix}$$



90°, common sense!

- Generally applicable

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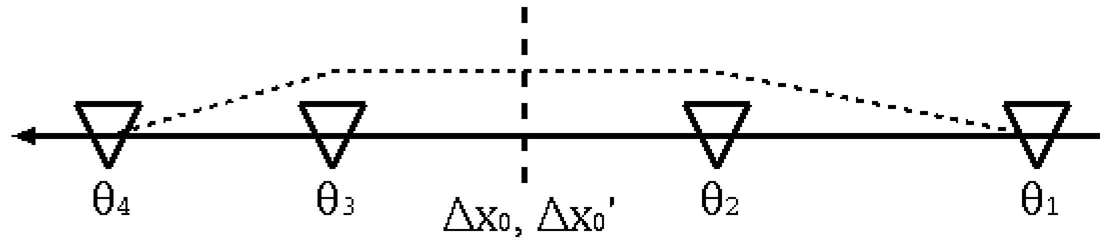
4-Bump



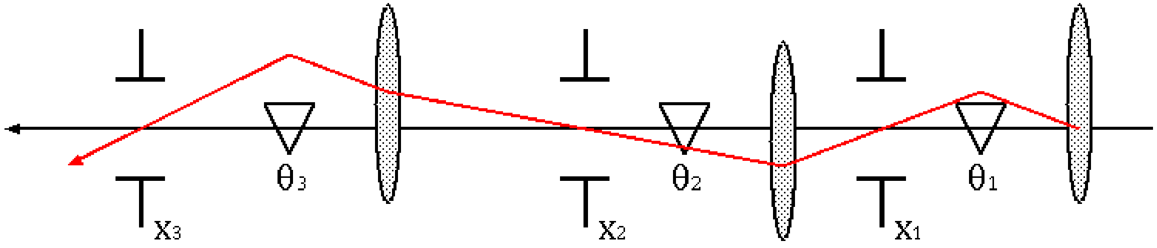
- Use four steerers to adjust angle and position at a center point and then flatten orbit downstream of the last steerer.

$$\begin{pmatrix} x_0 \\ x_0' \\ x_f = 0 \\ x_f' = 0 \end{pmatrix} = \begin{pmatrix} R_{12}^{01} & R_{12}^{02} & 0 & 0 \\ R_{22}^{01} & R_{22}^{02} & 0 & 0 \\ R_{12}^{f1} & R_{12}^{f2} & R_{12}^{f3} & R_{12}^{f4} \\ R_{22}^{f1} & R_{22}^{f2} & R_{22}^{f3} & R_{22}^{f4} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_3 \end{pmatrix}$$

- Invert matrix and express thetas as a function of the constraints x_0 and x_0'
- Gives the required steering excitations θ_j as a function of x_0 and x_0' $\rightarrow$ Multiknob

Orbit Correction in Beamline #1

- Observe the orbit on beam-position monitors
- and correct with steering dipoles
- How much do we have to change the steering magnets in order to compensate the observed orbit either to zero or some other 'golden orbit'.
- In beam line the effect of a corrector on the downstream orbit is given by transfer matrix element R_{12}
- One-to-one steering



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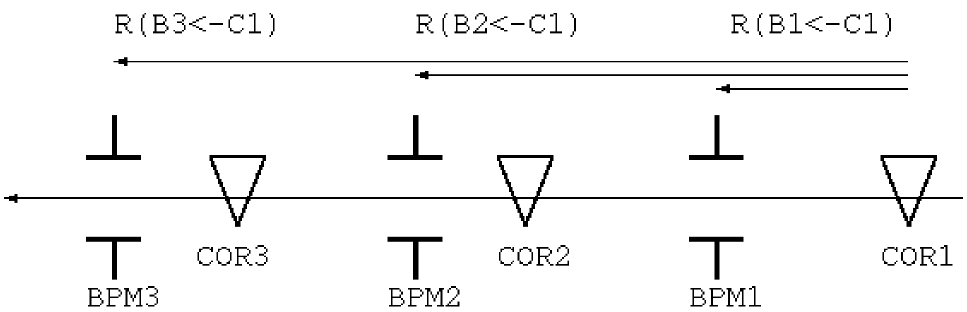
Orbit correction in a Beamline #2



$$\begin{pmatrix} -x_1 \\ -x_2 \\ -x_3 \end{pmatrix} = \begin{pmatrix} R_{12}^{11} & 0 & 0 \\ R_{12}^{21} & R_{12}^{21} & 0 \\ R_{12}^{31} & R_{12}^{32} & R_{12}^{33} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$$

- Observed beam positions x_1, x_2, and x_3
- Only downstream BPM can be affected
- Linear algebra problem to **invert matrix** and find required corrector excitations θ_j to produce negative of observed x_i
- Include BPM errors by left-multiplying the equation with

$$\bar{\Lambda} = \mathrm{diag}\left(\frac{1}{\sigma_1}, \ldots, \frac{1}{\sigma_n}\right)$$

This weigths each BPM measurement by its inverse error. Good BPMs are trusted more!

How to get the response matrix?

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- With the computer (MADX or other code)
  - tables of transfer matrix elements
  - but it is based on a model and somewhat idealized
  - no BPM or COR scale errors known
- Experimentally by measuring difference orbits
  - record reference orbit $\vec{x}_0$
  - change steering magnet $\Delta\theta_j$
  - record changed orbit $\vec{x}_j$
  - Build response matrix one column at a time

$$A = \left(\frac{\vec{x}_1 - \vec{x}_0}{\Delta\theta_1} \, , \quad \frac{\vec{x}_2 - \vec{x}_0}{\Delta\theta_2} \, , \quad \cdots \right)$$

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Inversion Algorithms for $-x=A\theta$

- A is an n x m matrix, n BPM and m correctors
- $n=m$ and matrix A is non-degenerate:

$$\vec{\theta} = -A^{-1}\vec{x}$$

- $m<n$: too few correctors, least squares $\chi^2 = |-\vec{x} - A\vec{\theta}|^2$

$$\vec{\theta} = -(A^t A)^{-1} A^t \vec{x}$$

- MICADO: pick the most effective, fix orbit, the next effective, fix residual orbit, the next...
  - good for large rings with many BPM and COR
- $m>n$ or degenerate: singular-value dec. (SVD)

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Digression on SVD

- Singular Value Decomposition $A = O\Lambda U^t$
  - may need to zero-pad
  - U is orthogonal, a coordinate rotation
  - Λ is diagonal, it stretches the coordinates by λ_i
  - O is orthogonal and rotates, but differently
- If A is symmetric → eigenvalue decomposition
- Inversion is trivial $\text{"}A^{-1}\text{"} = U\Lambda^{-1}O^t$
  - invert only in sub-space where you can if *λ ≠ 0*
  - and set projection onto degenerate subspace to zero "1/0 = 0" (see *Numerical Recipes* for a discussion)

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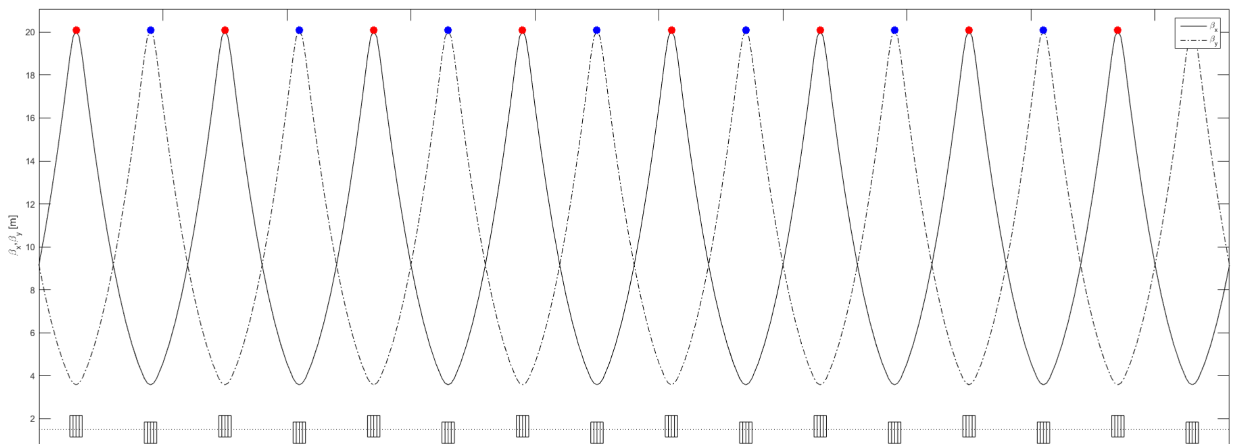
Comment on Matrix Inversion

- Many correction problems can be brought into a generic form, if you
  - pretend you know the excitation of all controllers (think correctors)
  - determine the response matrix (expt. or numerically)

    $C_{ij} = \partial \text{Observable}_i / \partial \text{Controller}_j$
  - to predict the changes of the observables (think BPM)
- Then invert the response matrix to detemine the controller values required to change the observable by some value.

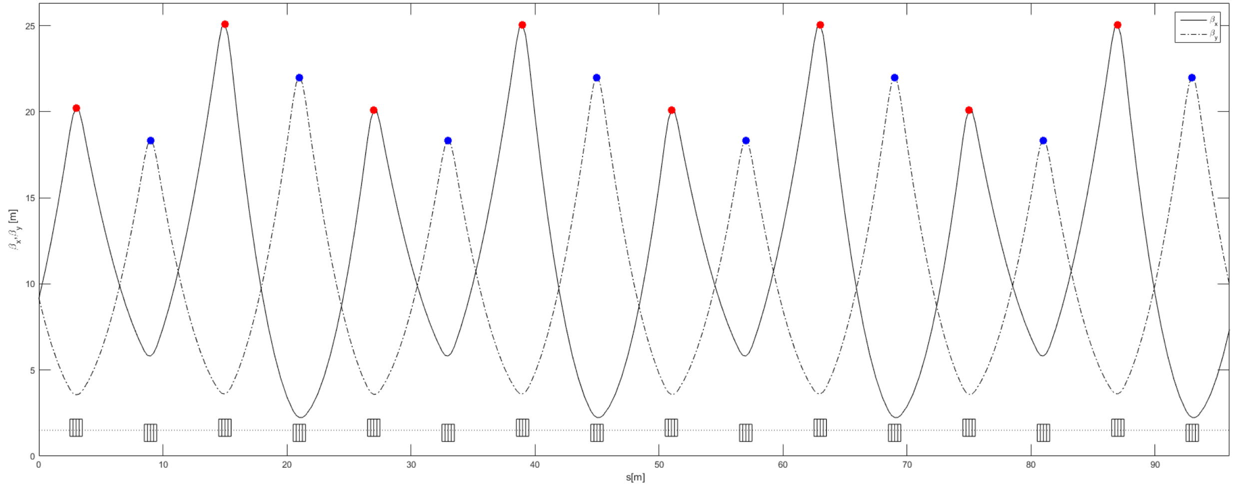
Effect of gradient errors

Eigth 90° FODO cells, first quad 10% too low



Unperturbed lattice

Nice and repetetive beta functions



Repeats after 2 cells or 2 x 90°

Beta-function "beats"

Injection into following beam line or ring is compromised

Beamlines: Gradient errors

- Gradient errors cause the beam matrix or beta functions β to differ from their design values $\hat{\beta}$
- Downstream beam size

$$\bar{\sigma}_x^2 = \varepsilon\bar{\beta}\left[B_{mag} + \sqrt{B_{mag}^2 - 1}\cos(2\mu - \varphi)\right]$$

  - enlarged effective emittance, beta-beat oscillations with twice the betatron phase advance μ
- This is called mismatch and is quantified by

$$B_{mag} = \frac{1}{2}\left[\left(\frac{\hat{\beta}}{\beta} + \frac{\beta}{\hat{\beta}}\right) + \beta\hat{\beta}\left(\frac{\alpha}{\beta} - \frac{\hat{\alpha}}{\hat{\beta}}\right)^2\right]$$

- For a single thin quad we have $B_{mag} = 1 + \frac{\hat{\beta}^2}{2f^2}$

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Filamentation #1

- What happens when we inject a mismatched beam into a ring with chromaticity Q' ?

$$\sigma_n^2 = \varepsilon\bar{\beta}\left[B_{mag} + \sqrt{B_{mag}^2 - 1}\ \cos(4\pi n(Q + Q'\delta) - \varphi)\right]$$

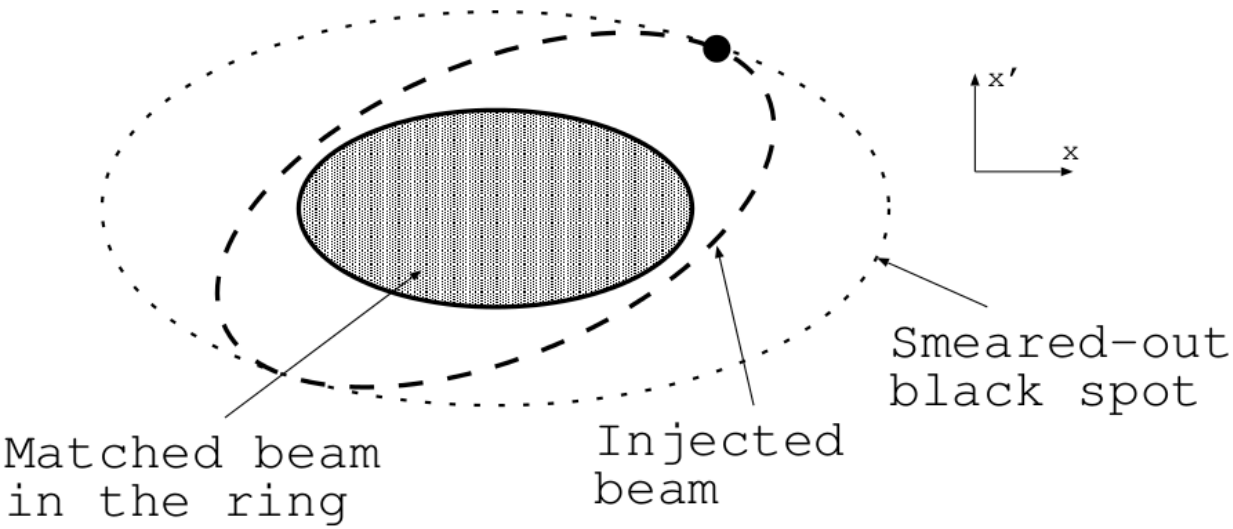
  - with momentum distribution $\psi(\delta) = \frac{1}{\sqrt{2\pi}\sigma_\delta} e^{-\delta^2/2\sigma_\delta^2}$

- Averaging over δ gives

$$\sigma_n^2 = \varepsilon\bar{\beta}\left[B_{mag} + e^{-2(2\pi Q'\sigma_\delta)^2 n^2}\sqrt{B_{mag}^2 - 1}\ \cos(4\pi n Q - \varphi)\right]$$

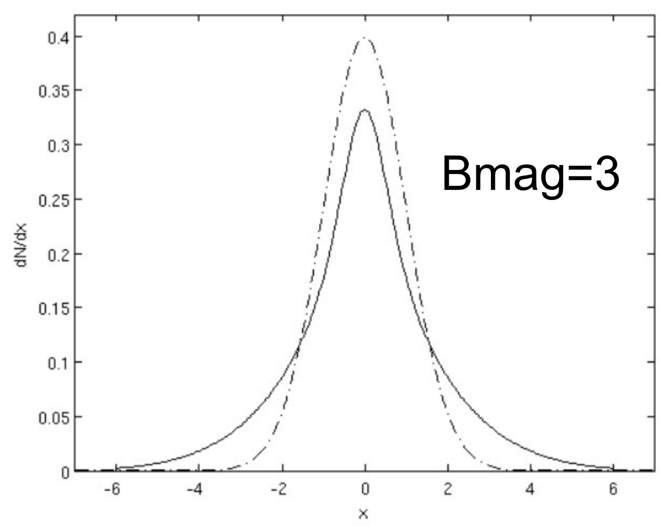
- 'Damps' with $\exp(-n^2)$ and leaves an increased beam size (by Bmag).

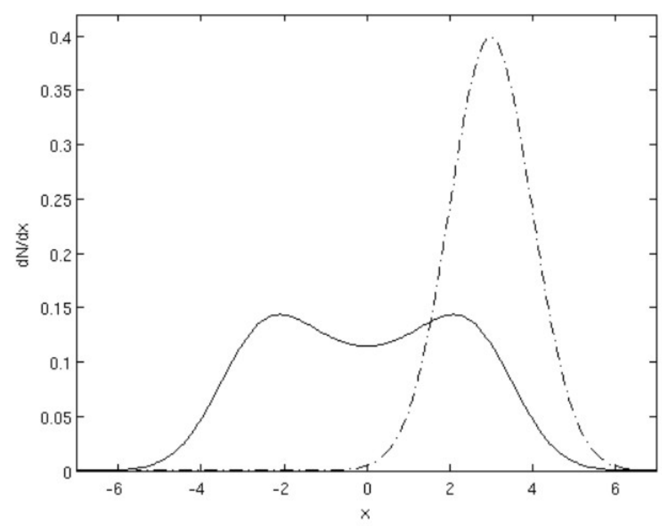
Filamentation #2



Injecting with transverse offset also leads to filamentation

Final distribution is not Gaussian





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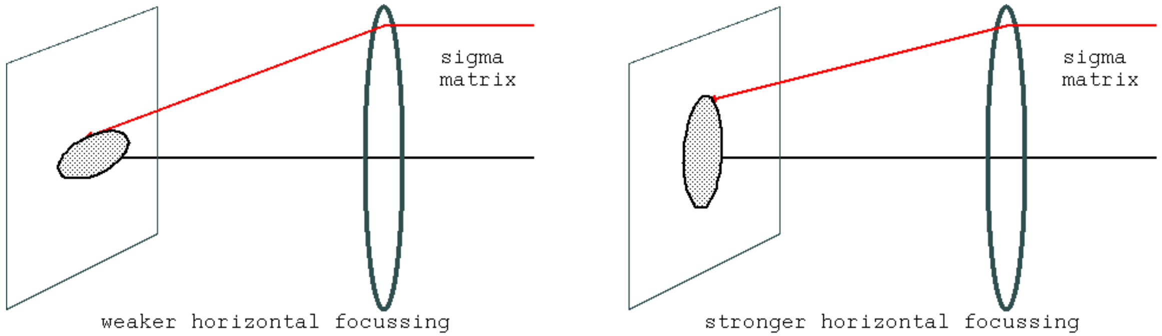
Measuring Beam Matrices



$$\bar{\sigma} = R(f)\ \sigma\ R(f)^t$$

Vary quadrupole and observe changes on a screen, usually one plane at a time

- Beam size on screen depends on quad setting

$$\bar{\sigma}_x^2 = \bar{\sigma}_{11} = R_{11}^2\sigma_{11} + 2R_{11}R_{12}\sigma_{12} + R_{12}^2\sigma_{22}$$

- where R=R(f), use several measurement and solve for the three sigma matrix elements

$$\varepsilon_x^2 = \sigma_{11}\sigma_{22} - \sigma_{12}^2 \qquad \beta_x = \sigma_{11}/\varepsilon_x \qquad \alpha_x = -\sigma_{12}/\varepsilon_x$$

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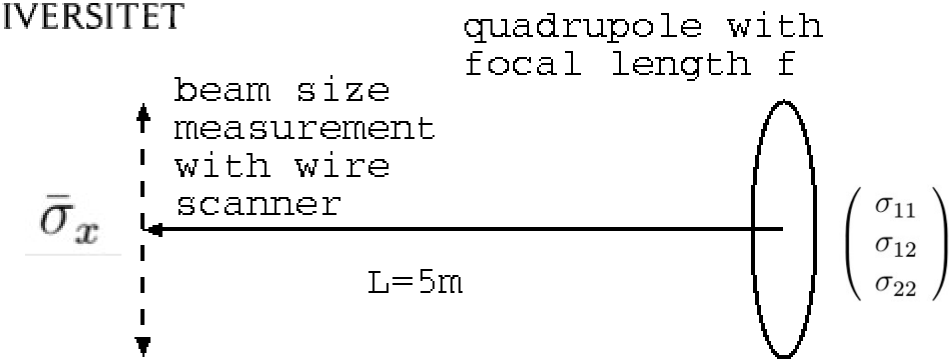
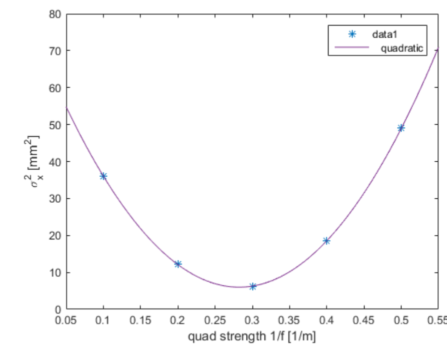
A worked example: Quad scan



$1/f$ [1/m]	$\bar{\sigma}_x$ [mm]
0.1	6.0
0.2	3.5
0.3	2.5
0.4	4.3
0.5	7.0



- Transfer matrix

$$R = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1-l/f & l \\ -1/f & 1 \end{pmatrix}$$

- Relate unknown beam matrix to measurements

$$\begin{aligned} \bar{\sigma}_x^2 &= R_{11}^2\sigma_{11} + 2R_{11}R_{12}\sigma_{12} + R_{12}^2\sigma_{22} \\ &= (1-l/f)^2\sigma_{11} + 2l(1-l/f)\sigma_{12} + l^2\sigma_{22} \\ &= \left(\frac{l}{f}\right)^2 \sigma_{11} - \left(\frac{l}{f}\right)(2\sigma_{11} + 2l\sigma_{12}) + (\sigma_{11} + 2l\sigma_{12} + l^2\sigma_{22}) \end{aligned}$$

- Indeed a parabola in *l/f*

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Quad scan #2

- Build matrix of the type *y=Ax*
  - and with error bars $\Sigma_k=2\sigma_k\Delta\sigma_k$

$$\begin{pmatrix} \bar{\sigma}^2_{x,1} \\ \bar{\sigma}^2_{x,2} \\ \bar{\sigma}^2_{x,3} \\ \bar{\sigma}^2_{x,4} \\ \bar{\sigma}^2_{x,5} \end{pmatrix} = \begin{pmatrix} (1-l/f_1)^2 & 2l(1-l/f_1) & l^2 \\ (1-l/f_2)^2 & 2l(1-l/f_2) & l^2 \\ (1-l/f_3)^2 & 2l(1-l/f_3) & l^2 \\ (1-l/f_4)^2 & 2l(1-l/f_4) & l^2 \\ (1-l/f_5)^2 & 2l(1-l/f_5) & l^2 \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{22} \end{pmatrix}$$

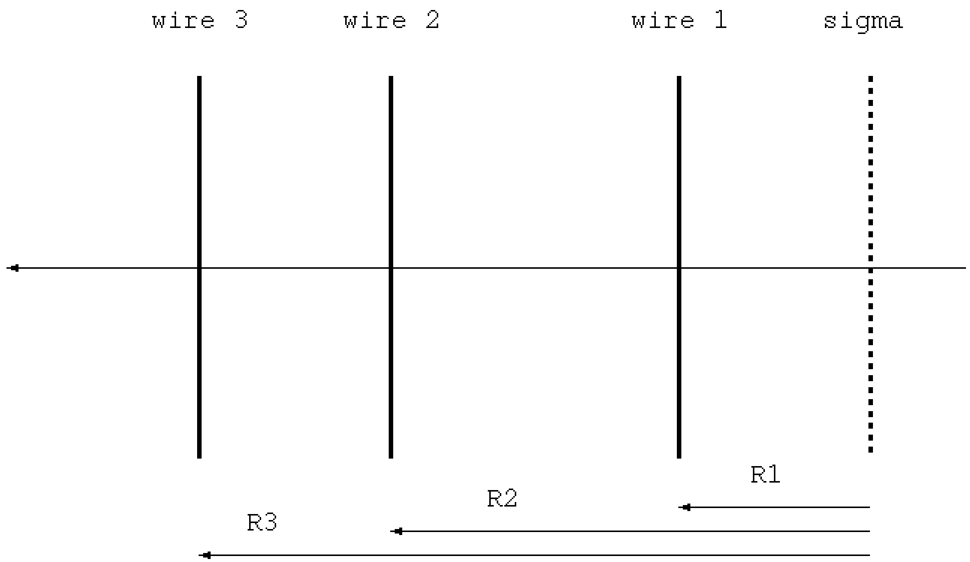
$$\begin{pmatrix} \frac{\bar{\sigma}^2_{x,1}}{\Sigma_1} \\ \frac{\bar{\sigma}^2_{x,2}}{\Sigma_2} \\ \frac{\bar{\sigma}^2_{x,3}}{\Sigma_3} \\ \frac{\bar{\sigma}^2_{x,4}}{\Sigma_4} \\ \frac{\bar{\sigma}^2_{x,5}}{\Sigma_5} \end{pmatrix} = \begin{pmatrix} \frac{(1-l/f_1)^2}{\Sigma_1} & \frac{2l(1-l/f_1)}{\Sigma_1} & \frac{l^2}{\Sigma_1} \\ \frac{(1-l/f_2)^2}{\Sigma_2} & \frac{2l(1-l/f_1)}{\Sigma_2} & \frac{l^2}{\Sigma_2} \\ \frac{(1-l/f_3)^2}{\Sigma_3} & \frac{2l(1-l/f_1)}{\Sigma_3} & \frac{l^2}{\Sigma_3} \\ \frac{(1-l/f_4)^2}{\Sigma_4} & \frac{2l(1-l/f_1)}{\Sigma_4} & \frac{l^2}{\Sigma_4} \\ \frac{(1-l/f_5)^2}{\Sigma_5} & \frac{2l(1-l/f_1)}{\Sigma_5} & \frac{l^2}{\Sigma_5} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{22} \end{pmatrix}$$

- Solve by least-squares pseudo-inverse

$$x=(A^tA)^{-1}A^ty$$

- with the covariance matrix $Cov=(A^tA)^{-1}$
  - diagonal elements are square of error bars of fit parameter x

Or use several wire scanners



$$
\begin{aligned}
\sigma_1^2 &= (R^1)_{11}^2\sigma_{11} + 2R_{11}^1R_{12}^1\sigma_{12} + (R^1)_{12}^2\sigma_{22}\\
\sigma_2^2 &= (R^2)_{11}^2\sigma_{11} + 2R_{11}^2R_{12}^2\sigma_{12} + (R^2)_{12}^2\sigma_{22}\\
\sigma_3^2 &= (R^3)_{11}^2\sigma_{11} + 2R_{11}^3R_{12}^3\sigma_{12} + (R^3)_{12}^2\sigma_{22}
\end{aligned}
$$

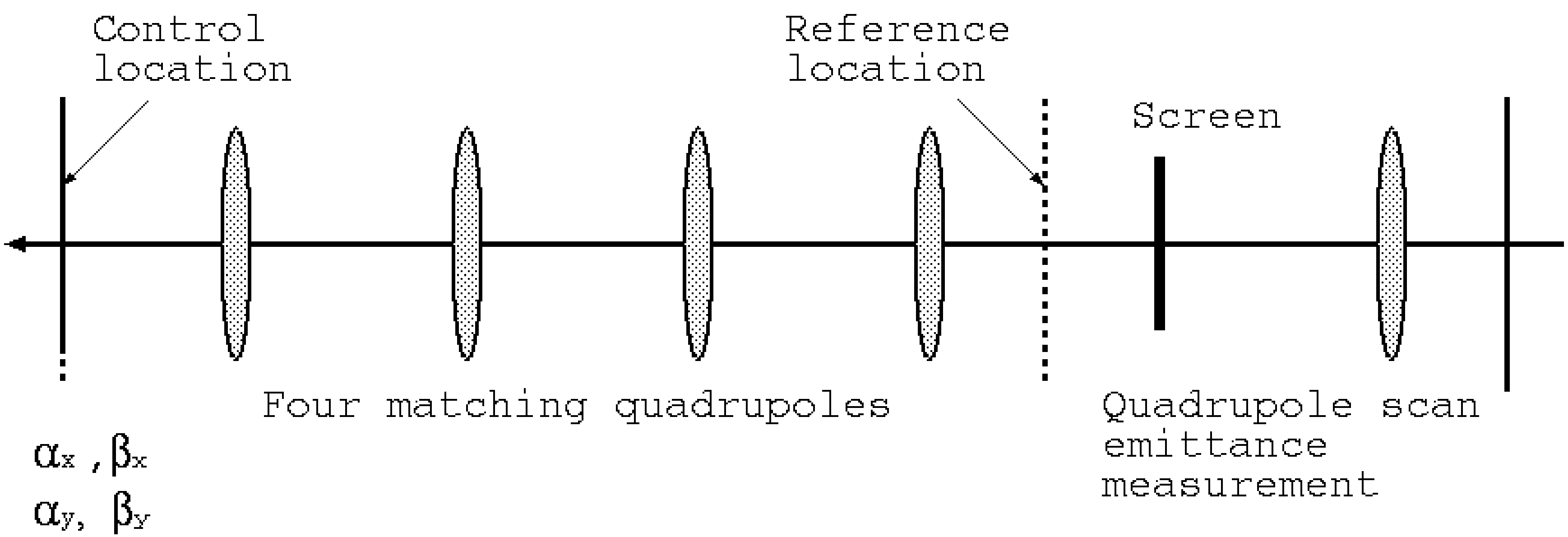
$$
\begin{pmatrix}\sigma_1^2\\ \sigma_2^2\\ \sigma_3^2\end{pmatrix} = \begin{pmatrix}(R^1)_{11}^2 & 2R_{11}^1R_{12}^1 & (R^1)_{12}^2\\ (R^2)_{11}^2 & 2R_{11}^2R_{12}^2 & (R^2)_{12}^2\\ (R^3)_{11}^2 & 2R_{11}^3R_{12}^3 & (R^3)_{12}^2\end{pmatrix}\begin{pmatrix}\sigma_{11}\\ \sigma_{12}\\ \sigma_{22}\end{pmatrix}
$$

- $(A^t A)^{-1}A^t$ - gymnastics with error bar estimates
- Derive emittance in same way, once σ_{ij} is found by inversion
- Can use several more wire scanners which allows χ^2 calculation for goodness-of-fit estimate

Beam matrix a.k.a. Beta match

$$\gamma_x = \frac{1+\alpha_x^2}{\beta_x}$$

- Uncoupled beam matrix $\varepsilon_x \begin{pmatrix} \beta_x & -\alpha_x \\ -\alpha_x & \gamma_x \end{pmatrix}$
  - need four quadrupoles to adjust $\alpha_x, \beta_x, \alpha_y,$ and β_y
  - non-linear optimizer (MADX matching module)



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Waist knob

- Finding quad-excitations to match beta functions (or sigma matrix) is a non-linear problem
- and depends on the incoming beam matrix.
- Tricky, but one sometimes can build knobs, based on the design optics, to correct some observable
  - conceptually: linearizing around a working point
- Example:
  - IP-waist knob
  - $d\alpha_x/dQ_{1,2}$ and $d\alpha_y/dQ_{1,2}$

Beamlines: Skew-gradient errors

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- Transfer matrix for a skew-quadrupole

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1/f & 0 \\ 0 & 0 & 1 & 0 \\ 1/f & 0 & 0 & 1 \end{pmatrix}$$

- Vertical part of the sigma-matrix after skew quad

$$\begin{pmatrix} \hat{\sigma}_{33} & \hat{\sigma}_{34} \\ \hat{\sigma}_{34} & \hat{\sigma}_{44} \end{pmatrix} = \begin{pmatrix} \sigma_{33} & \sigma_{34} \\ \sigma_{34} & \sigma_{44} + \sigma_{11}/f^2 \end{pmatrix}$$

- *Projected emittance* after skew quadrupole

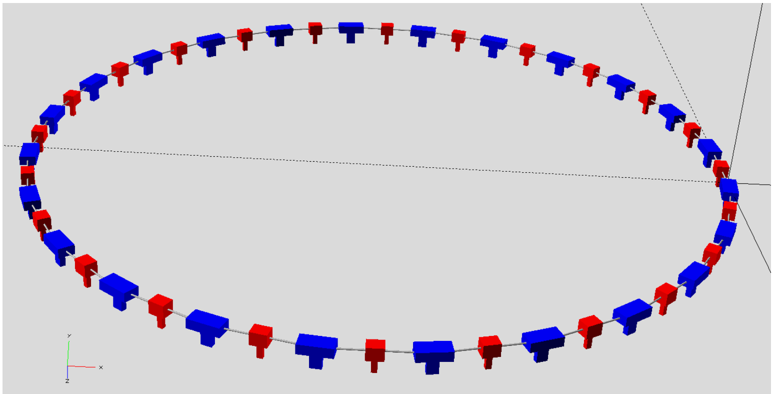
$$\hat{\varepsilon}_y^2 = \varepsilon_y^2 + \frac{\sigma_{11}\sigma_{33}}{f^2} = \varepsilon_y^2 \left(1 + \frac{\varepsilon_x}{\varepsilon_y}\frac{\beta_x\beta_y}{f^2}\right)$$

  - Problem with flat beams. Increases with ratio $\varepsilon_x/\varepsilon_y$ and is proportional to both beta functions.

- Problem in Final-Focus Systems with flat beams. Solenoid fields needs compensation.

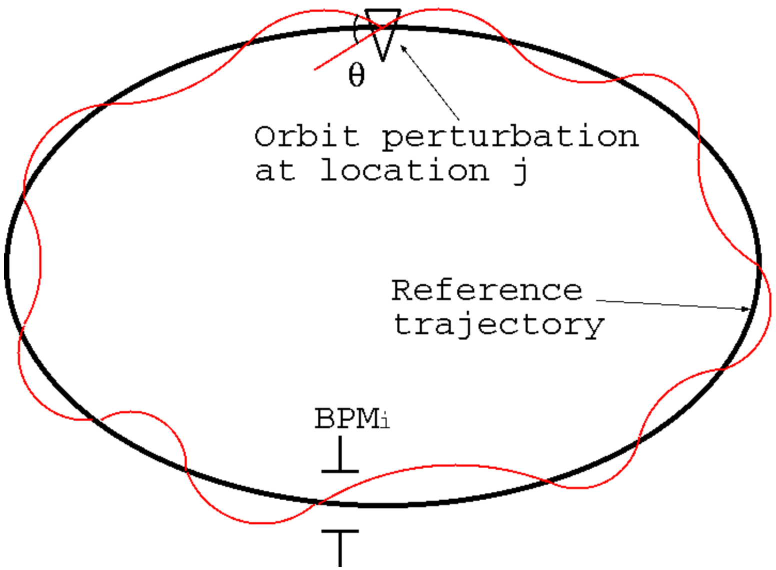
Imperfections in a Ring

- Effect of a localized kick on orbit
- Effect of a localized gradient error
- Effect of a skew gradient error
- Stop-bands and resonances



Dipole errors in a Ring

- Beam bites its tail → periodic boundary conditions → closed orbit



- Orbit after perturbation at *j*

$$\vec{x}_j = R^{jj}\vec{x}_j + \vec{q}_j$$

$$\vec{x}_j = (1 - R^{jj})^{-1}\vec{q}_j$$

- Propagate to BPM i

$$\vec{x}_i = R^{ij}\vec{x}_j = R^{ij}(1 - R^{jj})^{-1}\vec{q}_j = C^{ij}\vec{q}_j$$

- Response coefficients $C^{ij} = R^{ij}(1 - R^{jj})^{-1}$
  - just like transfer matrix in beam line, but with built-in closed-orbit constraint.

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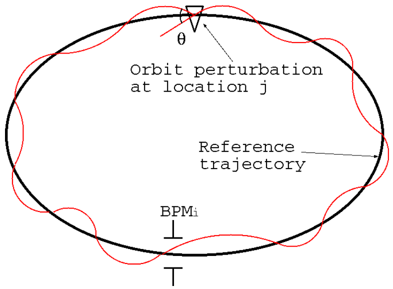
Response coefficients



- Express transfer-matrices through beta functions

$$\begin{pmatrix} x \\ x' \end{pmatrix}_j = \begin{pmatrix} \cos(2\pi Q) & \beta_j \sin(2\pi Q) \\ -\sin(2\pi Q)/\beta_j & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_j + \begin{pmatrix} 0 \\ \theta \end{pmatrix}$$

- Solve for closed orbit

$$\begin{pmatrix} x \\ x' \end{pmatrix}_j = \frac{\theta}{2} \begin{pmatrix} \beta_j \cot(\pi Q) \\ 1 \end{pmatrix}$$

- Transfer matrix to BPM i

$$R^{ij} = \begin{pmatrix} \sqrt{\beta_i} & 0 \\ -\alpha_i/\sqrt{\beta_i} & 1/\sqrt{\beta_i} \end{pmatrix} \begin{pmatrix} \cos\mu_{ij} & \sin\mu_{ij} \\ -\sin\mu_{ij} & \cos\mu_{ij} \end{pmatrix} \begin{pmatrix} 1/\sqrt{\beta_j} & 0 \\ 0 & \sqrt{\beta_j} \end{pmatrix}$$

- Response coefficient

$$x_i = \left[\frac{\sqrt{\beta_i \beta_j}}{2\sin(\pi Q)} \cos(\mu_{ij} - \pi Q) \right] \theta$$

Divergencies at integer tunes

$$C_{12}^{ij} = \frac{\partial BPM_i(x)}{\partial COR_j(x')}$$

Quadrupole alignment amplification factor

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- Consider randomly displaced quadrupoles

$$\theta_j = d_j/f \qquad \langle d_j \rangle = 0 \qquad \langle d_j d_k \rangle = \sigma_d^2 \delta_{jk}$$

- Incoherently (RMS) add all contributions

$$\begin{aligned}\langle x_i^2 \rangle &= \langle \left[\sum_j \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi Q} \cos(\mu_{ij} - \pi Q) \frac{d_j}{f_j} \right] \left[\sum_k \frac{\sqrt{\beta_i \beta_k}}{2 \sin \pi Q} \cos(\mu_{ik} - \pi Q) \frac{d_k}{f_k} \right] \rangle \\ &= \sum_j \frac{\beta_i \beta_j}{(2 \sin \pi Q)^2} \cos^2(\mu_{ij} - \pi Q) \frac{\sigma_d^2}{f_j^2}\end{aligned}$$

- Misalignment amplification factor $\sqrt{\langle x_i^2 \rangle} \approx \sqrt{N_q} \frac{\bar{\beta}/\bar{f}}{2\sqrt{2} \sin \pi Q} \sigma_d$
  - large rings with large N_q are sensitive
  - such as LHC and FCC

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Response Coefficients with RF

- Radio-frequency system constrains the revolution time

$$\frac{\Delta T}{T} = \frac{\Delta C}{C} - \frac{\Delta v}{v} = \left(\alpha - \frac{1}{\gamma^2}\right)\delta$$

- but a horizontal kick causes a horizontal closed orbit distortion which causes the circumference to change by $\Delta C = D_x\theta_x$ (6x6 TM is symplectic, and if uncoupled: $R_{16}=R_{52}$)
- Since RF fixes the revolution frequency the momentum of the particle has to adjust to $\delta = -D_j\theta/\eta C$
- ...and the particle moves on a dispersion trajectory
- Complete response coefficient between BPM_i and dipole error or COR_j

$$C_{12}^{ij} = \left[\frac{\sqrt{\beta_i\beta_j}}{2\sin(\pi Q)}\cos(\mu_{ij} - \pi Q) - \frac{D_i D_j}{\eta C}\right]$$

Orbit Correction in a Ring

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- Every steering magnet affects every BPM
  - orbit response coefficients and matrix $C^{ij} = R^{ij}(1 - R^{jj})^{-1}$
- Compensate measured positions x_i by inverting

$$\begin{pmatrix} -x_1 \\ -x_2 \\ \vdots \\ -x_m \end{pmatrix} = \begin{pmatrix} C_{12}^{11} & C_{12}^{12} & \dots & C_{12}^{1n} \\ C_{12}^{21} & C_{12}^{22} & \dots & C_{12}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{12}^{m1} & C_{12}^{m2} & \dots & C_{12}^{mn} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{pmatrix}$$

- and also in the vertical plane
- left-multiply with diagonal BPM error matrix $\bar{\Lambda} = \mathrm{diag}\left(\frac{1}{\sigma_1}, \dots, \frac{1}{\sigma_n}\right)$
- use either calculated or measured response matrix
- inversion with pseudo-inverse, MICADO, or SVD

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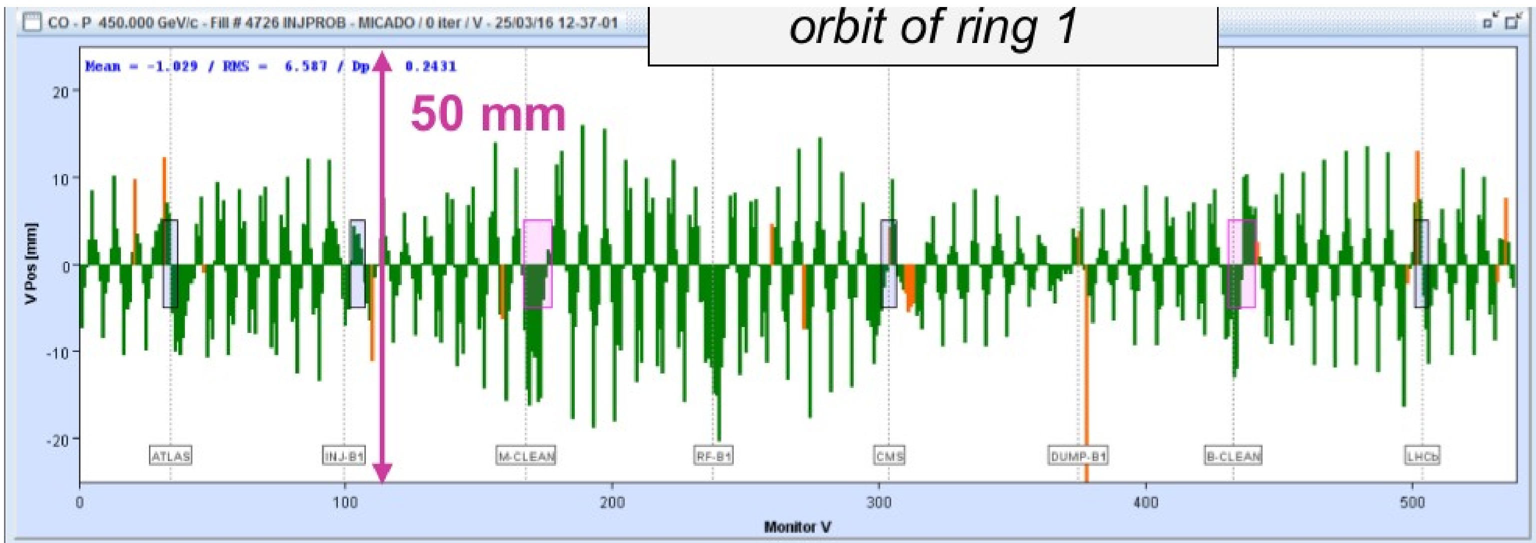
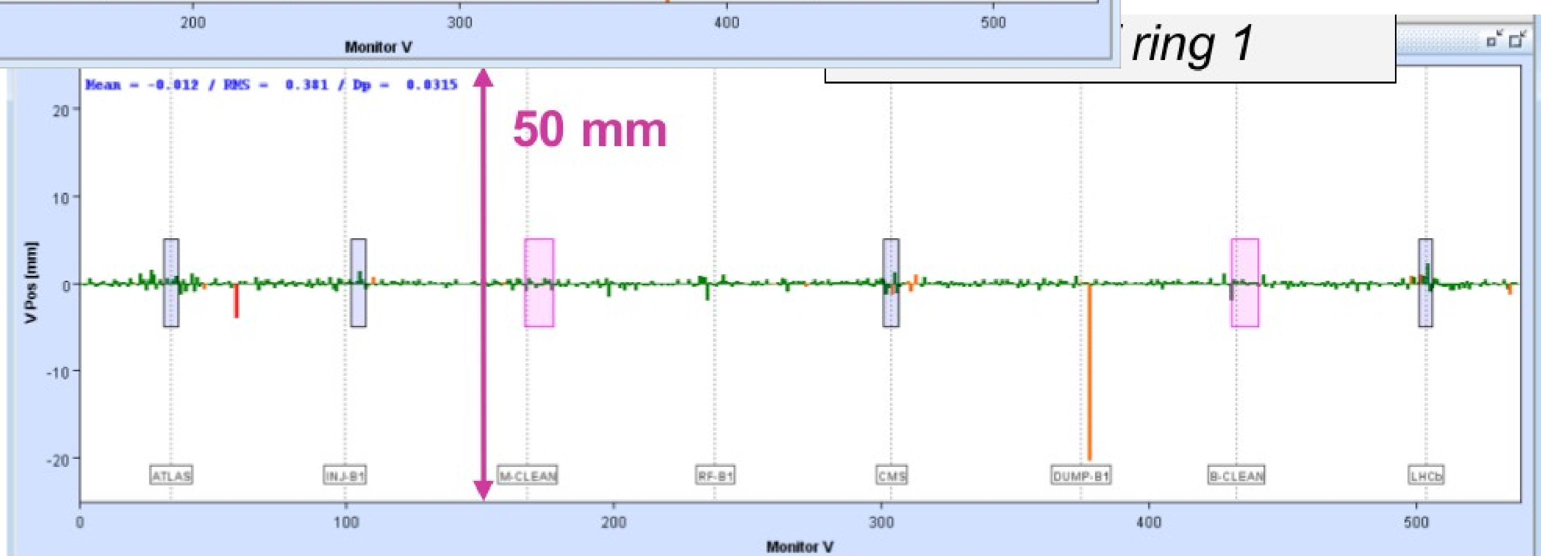
Example: orbit correction

Vertical orbit in LHC, before and after correction

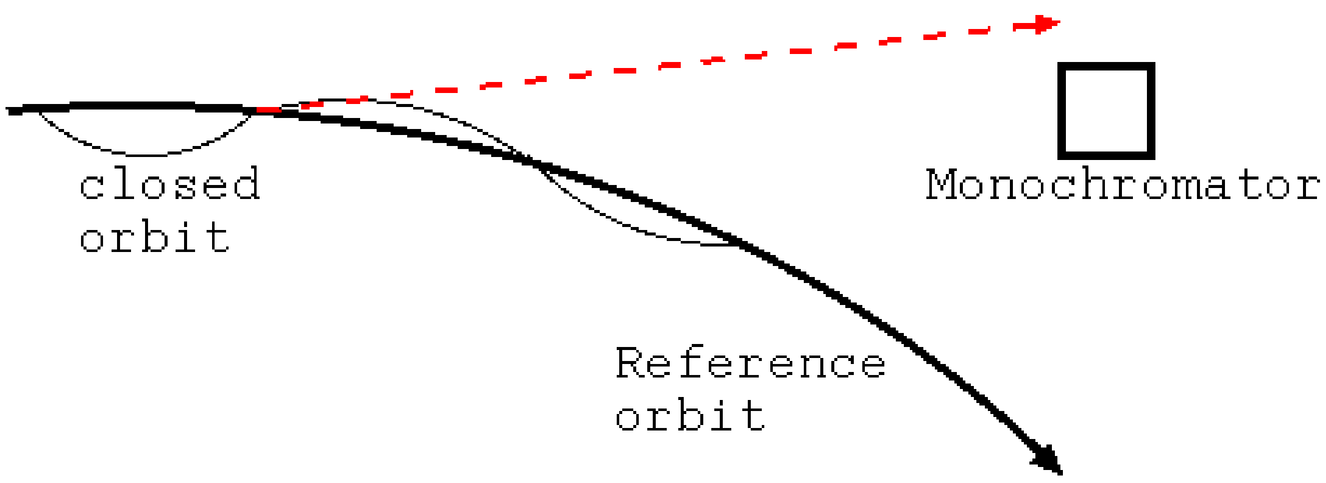




J. Wenninger
CAS, 2018

Steering synchrotron beam lines

- steer synchrotron light beam onto experiment
- fix angle at source point
- incorporate in orbit correction by +L,vBPM,-L



Dispersion-free steering

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- Steering magnets are small dipoles and also affect the dispersion (in ring and linac) besides the orbit.
- Take into account with dispersion response matrix $S_{ij}=dD_i/d\theta_j=d^2x_i/d\delta d\theta_j$ $\quad (D_i=dx_i/d\delta)$
  - Either numerically or from measurments
- Simultaneously correct orbit and dispersion
  - weight with Σs
  - more constraints
  - same number of correctors

$$\begin{pmatrix} \vdots \\ x_i/\Sigma_i \\ \vdots \\ D_i/\hat{\Sigma}_i \\ \vdots \end{pmatrix} = \begin{pmatrix} C_{ij}/\Sigma_i \\ S_{ij}/\hat{\Sigma}_i \end{pmatrix} \begin{pmatrix} \vdots \\ \theta_j \\ \vdots \end{pmatrix}$$

Gradient Errors in a Ring

- Add a gradient error (modelled as a thin quad) to a ring with $\mu=2\pi Q$

$$R_Q R = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\frac{1+\alpha^2}{\beta}\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

$$= \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -(\cos\mu + \alpha\sin\mu)/f + \gamma\sin\mu & \cos\mu - \alpha\sin\mu - (\beta/f)\sin\mu \end{pmatrix}$$

- Trace gives the perturbed tune $\bar{Q} = Q + \Delta Q$

$$2\cos(2\pi(Q+\Delta Q)) = 2\cos(2\pi Q) - \frac{\beta}{f}\sin(2\pi Q)$$

- and if β/f is small: the tune-shift is $\Delta Q \approx \dfrac{\beta}{4\pi f}$
- Gradient errors change the tune!

Changes of the beta function and stop bands

- From R_{12} get the change in the beta function

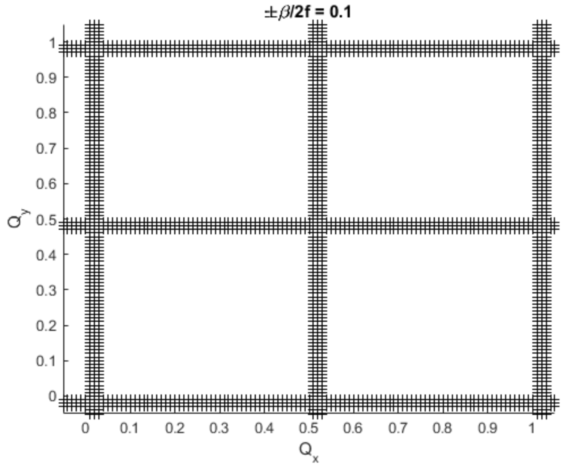
$$\bar{\beta} = \frac{\beta \sin(2\pi Q)}{\sin(2\pi(Q+\Delta Q))} \approx \beta\left[1 + 2\pi\Delta Q \cot(2\pi Q)\right]$$

$$\frac{\Delta\beta}{\beta} = 2\pi\Delta Q \cot(2\pi Q) \approx \frac{\beta}{2f}\cot(2\pi Q)$$

- Divergencies at half-integer values of the tune
- Stability requires

$$\left|\cos(2\pi Q) - \frac{\beta}{2f}\sin(2\pi Q)\right| \leq 1$$

  - stop-band width



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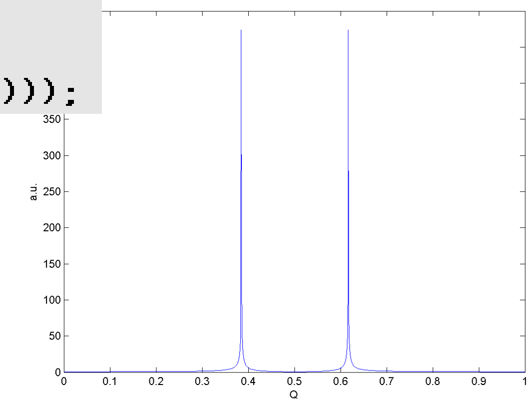
Measuring the Tune

- Kick beam and look at BPM difference-signal on spectrum analyzer
  - and divide frequency by revolution frequency gives fractional part of the tune
- Turn by turn BPM recordings and FFT
  - is it Q or 1-Q?
  - change QF and

    see which way the tune moves
- PLL in LHC: Beam is band-pass, tickle it, and detect synchronously

```
Qx=0.616
n=1:1:1024;
x=sin(2*pi*Qx*n);
plot(n/1024,abs(fft(x)));
```



Tune Correction

- Use a variable quadrupole with $1/f = \Delta k_1 l$
- Changes both Q_x and Q_y $\quad \Delta Q_x = \frac{\beta_{1x}}{4\pi f_1} \quad \text{and} \quad \Delta Q_y = -\frac{\beta_{1y}}{4\pi f_1}$
- Use two independent quadrupoles

$$\begin{aligned} \Delta Q_x &= \frac{\beta_{1x}}{4\pi f_1} + \frac{\beta_{2x}}{4\pi f_2} \\ \Delta Q_y &= -\frac{\beta_{1y}}{4\pi f_1} - \frac{\beta_{2y}}{4\pi f_2} \end{aligned} \qquad \begin{pmatrix} \Delta Q_x \\ \Delta Q_y \end{pmatrix} = \frac{1}{4\pi} \begin{pmatrix} \beta_{1x} & \beta_{2x} \\ -\beta_{1y} & -\beta_{2y} \end{pmatrix} \begin{pmatrix} 1/f_1 \\ 1/f_2 \end{pmatrix}$$

- Solve by inversion

$$\begin{pmatrix} 1/f_1 \\ 1/f_2 \end{pmatrix} = \frac{-4\pi}{\beta_{1x}\beta_{2y} - \beta_{2x}\beta_{1y}} \begin{pmatrix} -\beta_{2y} & -\beta_{2x} \\ \beta_{1y} & \beta_{1x} \end{pmatrix} \begin{pmatrix} \Delta Q_x \\ \Delta Q_y \end{pmatrix}$$

- Quads on same power supply → sum of betas

Measuring beta functions

- Change quadrupole and observe tune variation

$$\Delta Q_x = \frac{\beta_{1x}}{4\pi f_1} \quad \text{and} \quad \Delta Q_y = -\frac{\beta_{1y}}{4\pi f_1}$$

- Need independent power supplies
  - or piggy-back boost supply
  - or a shunt resistor
- May get sums of betas in quads-on-the-same-power-supply.

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Model Calibration

- Compare measured $\hat{C}^{ij}$ orbit response matrix to computer model C^{ij}
  - enormous amount of data 2 x N_{bpm} x N_{cor}
- and blame the difference on quad gradients g_k or other parameters p_l
  - much fewer fit-parameters N_{quad} and N_{para}

$$\hat{C}^{ij} - C^{ij} = \sum_k \frac{\partial C^{ij}}{\partial g_k} \Delta g_k + \sum_l \frac{\partial C^{ij}}{\partial p_l} \Delta p_l$$

- First used in SPEAR and later perfected in NSLS → LOCO

Model Calibration #2

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- Normally the parameters p_l are BPM and corrector scale errors
  - fit for N_{quad} gradients and 2 x (N_{bpm}+N_{cor}) scales

$$\hat{C}^{ij} - C^{ij} = \sum_k \frac{\partial C^{ij}}{\partial g_k} \Delta g_k + C^{ij} \Delta x^i - C^{ij} \Delta y^j$$

- Determine derivatives $\partial C^{ij} / \partial g_k$ numerically by changing a gradient and re-calculating all response coefficients, then taking differences
- BPM-cor degeneracy → SVD needed to invert
- Converges, if χ^2/DOF is close to unity

micro-LOCO

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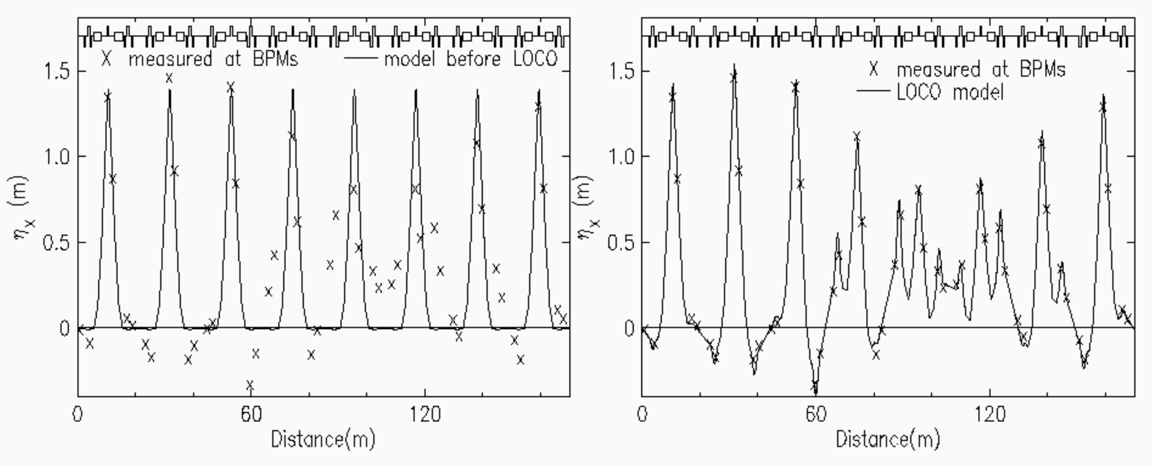
- 2 Quads, 2 BPM, 2 COR, only horizontal "C_{12}"
  - ill-defined, but useful to see the structure of matrix
  - gradient errors Δg, BPM scales Δx, corrector scales Δy
- Blame difference on Δg,Δx,Δy

$$C^{ij} = R^{ij}(1-R^{jj})^{-1}$$

$$\begin{pmatrix} \hat{C}^{11} - C^{11} \\ \hat{C}^{21} - C^{21} \\ \hat{C}^{12} - C^{12} \\ \hat{C}^{22} - C^{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial C^{11}}{\partial g_1} & \frac{\partial C^{11}}{\partial g_2} & C^{11} & 0 & -C^{11} & 0 \\ \frac{\partial C^{21}}{\partial g_1} & \frac{\partial C^{21}}{\partial g_2} & 0 & C^{21} & -C^{21} & 0 \\ \frac{\partial C^{12}}{\partial g_1} & \frac{\partial C^{12}}{\partial g_2} & C^{12} & 0 & 0 & -C^{12} \\ \frac{\partial C^{22}}{\partial g_1} & \frac{\partial C^{22}}{\partial g_2} & 0 & C^{22} & 0 & -C^{22} \end{pmatrix} \begin{pmatrix} \Delta g_1 \\ \Delta g_2 \\ \Delta x_1 \\ \Delta x_2 \\ \Delta y_1 \\ \Delta y_2 \end{pmatrix}$$

Experience

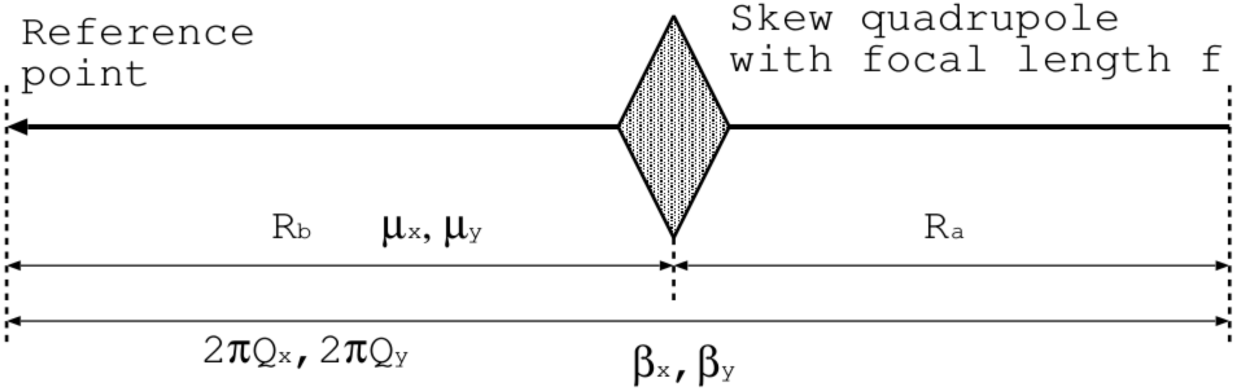
- SPEAR: could explain measured tunes to within $4x10^{-3}$ from quadrupole settings which had percent errors (J. Corbett, M. Lee, VZ, PAC93).
- NSLS: LOCO, $\Delta\beta/\beta = 10^{-3}$, dispersion fixed, emittance factor 2 improved (J. Safranek, NIMA 388, 1997)



- and practically every light source since then uses it

Skew-gradient Errors in a Ring

- Consider a single skew-quad in a ring



- with 4 x 4 transfer matrix (in NPS)

$$\tilde{S} = \begin{pmatrix} 1_2 & \tilde{Q} \\ \tilde{Q} & 1_2 \end{pmatrix} \qquad \tilde{Q} = \begin{pmatrix} 0 & 0 \\ \sqrt{\beta_x \beta_y}/f & 0 \end{pmatrix}$$

- and move the perturbation to a reference point

$$\hat{R} = R_b \tilde{S} R_a = \left(R_b \tilde{S} R_b^{-1} \right) \left(R_b R_a \right)$$

Skew-gradient in Ring #2

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- Transfer matrix in normalized phase space is rotation, calculate the moved TM

$$R_b\tilde{S}R_b^{-1} = \begin{pmatrix} O_x & 0 \\ 0 & O_y \end{pmatrix}\begin{pmatrix} 1_2 & \tilde{Q} \\ \tilde{Q} & 1_2 \end{pmatrix}\begin{pmatrix} O_x^t & 0 \\ 0 & O_y^t \end{pmatrix} = \begin{pmatrix} 1_2 & O_x\tilde{Q}O_y^{-1} \\ O_y\tilde{Q}O_x^{-1} & 1_2 \end{pmatrix}$$

$$\begin{aligned} O_x\tilde{Q}O_y^{-1} &= \frac{\sqrt{\beta_x\beta_y}}{f}\begin{pmatrix} \sin\mu_x\cos\mu_y & -\sin\mu_x\sin\mu_y \\ \cos\mu_x\cos\mu_y & -\cos\mu_x\sin\mu_y \end{pmatrix} \\ &= \frac{\sqrt{\beta_x\beta_y}}{2f}\begin{pmatrix} \sin(\mu_x-\mu_y)+\sin(\mu_x+\mu_y) & -\cos(\mu_x-\mu_y)+\sin(\mu_x+\mu_y) \\ \cos(\mu_x-\mu_y)+\cos(\mu_x+\mu_y) & \sin(\mu_x-\mu_y)-\sin(\mu_x+\mu_y) \end{pmatrix} \end{aligned}$$

4!

- If many weak skew quads, combine their effect

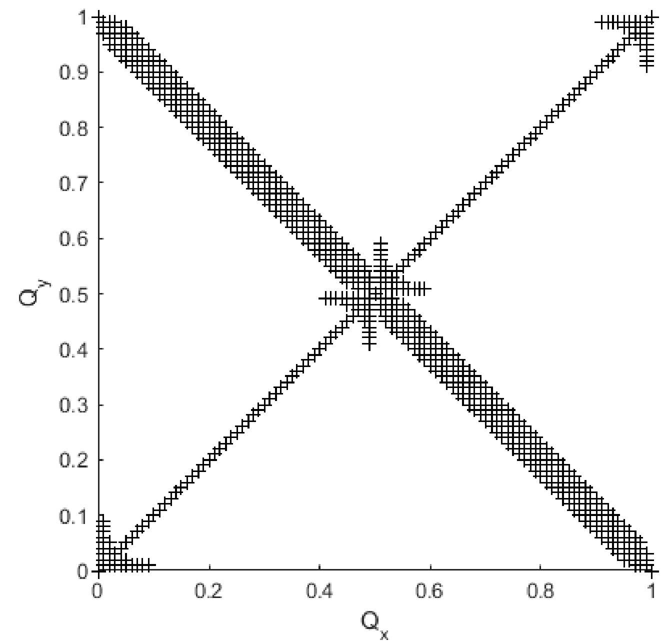
$$\hat{R} = (1+\tilde{P}_1)(1+\tilde{P}_2)\cdots R_0 \approx (1+\tilde{P}_1+\tilde{P}_2+\cdots)R_0$$

- Combinations of sines and cosines may add coherently

$$F_\pm = \sum_j \frac{\beta_{x,j}\beta_{y,j}}{2f_j}e^{i(\mu_{x,j}\pm\mu_{y,j})}$$

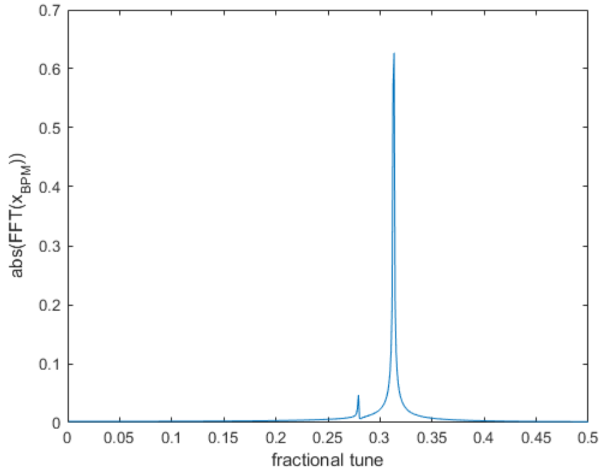
Skew-gradient stop bands

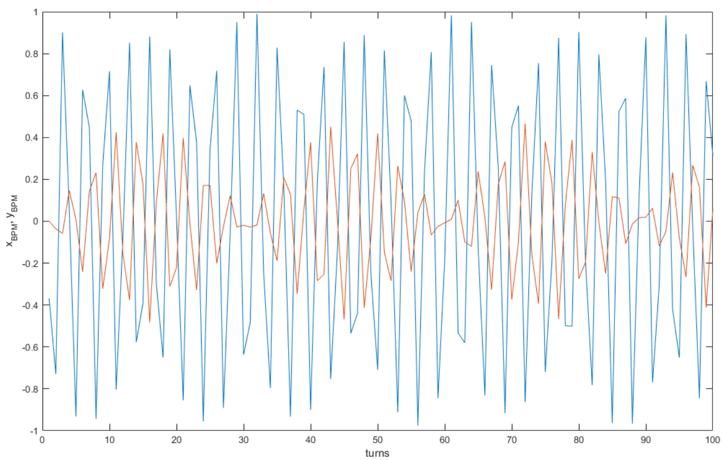
- Why are skew-gradient errors bad?
  - they also add stop bands along the diagonals
- Ring with single skew
  - with strength $\sqrt{\beta_x \beta_y}/f = 0.2$
- Calculate the eigen-tunes
  - Edwards-Teng algorithm
- for each pair Q_x,Q_y
- make cross if unstable
  - complex or NAN in Matlab



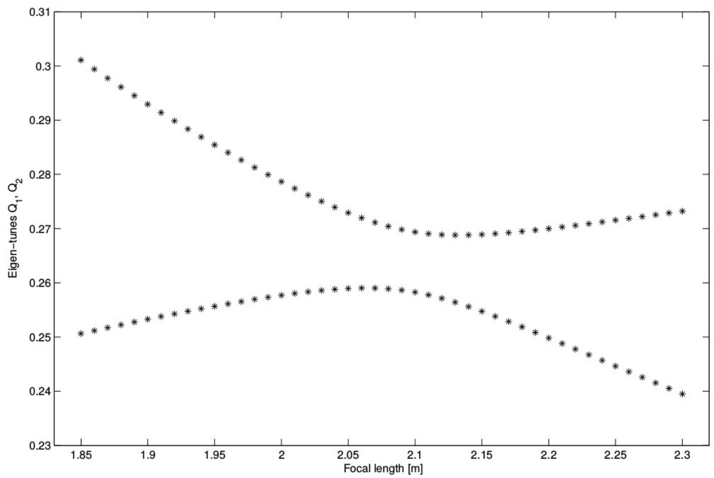
Measuring Coupling

- BPM turn-by-turn data cross talk, beating





- Closest tune
  - try to make the tunes equal with an upright quad
  - measure tunes
  - coupling 'repels' the tunes



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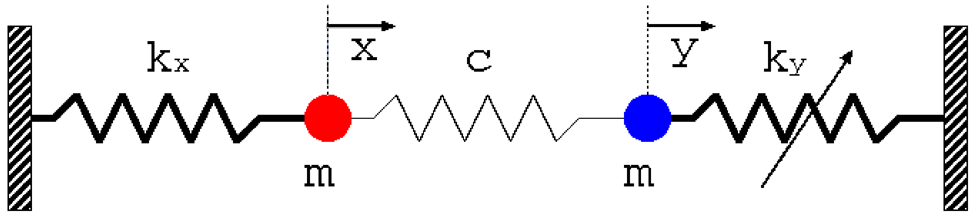
Coupling: mechanical analogy



- Two weakly coupled oscillators: simple to find equations of motion

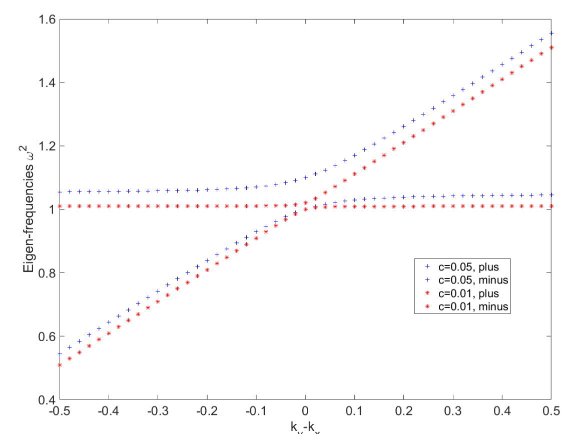
$$0 = m\ddot{x} + (k_x + c)x - cy$$
$$0 = m\ddot{y} + (k_y + c)y - cx$$

- and eigen-frequencies

$$\omega^2 = \frac{k_x + k_y + 2c}{2m} \pm \sqrt{\left(\frac{k_x - k_y}{2m}\right)^2 + \frac{c^2}{m^2}}$$

- Minimum tune separation
- Excite one, get beating

Translation for accelerator physicists:
x → horiz. betatr. osc.
y → vert. betatr. osc.
$k_x/m \rightarrow Q_x^2$
$k_y/m \rightarrow Q_y^2$ (adj.)
c/m → coupling source



Coupling correction

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- Use a single skew-quad if that is all you have to minimize the closest tune.
- Otherwise build knobs for the four resonance-driving terms with normalized skew gradients

$$\begin{pmatrix} \mathrm{Re}(F_-) \\ \mathrm{Im}(F_-) \\ \mathrm{Re}(F_+) \\ \mathrm{Im}(F_+) \end{pmatrix} = \begin{pmatrix} \cos(\mu_{x1}-\mu_{y1}) & \dots & \cos(\mu_{x4}-\mu_{y4}) \\ \sin(\mu_{x1}-\mu_{y1}) & \dots & \sin(\mu_{x4}-\mu_{y4}) \\ \cos(\mu_{x1}+\mu_{y1}) & \dots & \cos(\mu_{x4}+\mu_{y4}) \\ \sin(\mu_{x1}+\mu_{y1}) & \dots & \sin(\mu_{x4}+\mu_{y4}) \end{pmatrix} \begin{pmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa_4 \end{pmatrix}$$

$$F_\pm = \sum_j \frac{\beta_{x,j}\beta_{y,j}}{2f_j} e^{i(\mu_{x,j}\pm\mu_{y,j})}$$

$$\kappa_i = \sqrt{\beta_{xi}\beta_{yi}}/2f_i$$

- and empirically minimize each RDT,
  - often F_- (if tunes are close) is sufficient
- Choose phases *μ* to make the condition number of the matrix as close to unity as possible.

Measuring Chromaticity Q'

- Chromaticity is the momentum-dependence of the tunes: $Q = Q_0 + Q'\delta$
- Force the momentum to change by changing the RF frequency. The beam follows, because synchrotron oscillations are stable.

$$-\frac{\Delta f_{rf}}{f_{rf}} = \frac{\Delta T}{T} = \eta\delta = \left(\alpha - \frac{1}{\gamma^2}\right)\delta \qquad \rightarrow \qquad \delta = -\frac{1}{\eta}\frac{\Delta f_{rf}}{f_{rf}}$$

- Plot tune change ΔQ versus $\Delta f_{rf}/f_{rf}$. The slope is proportional to (1/chromaticity Q') [can also use PLL]

$$Q' = \frac{\Delta Q}{\delta} = -\eta\frac{\Delta Q}{\Delta f_{rf}/f_{rf}}$$

Chromaticity correction

- Need **controllable and momentum-dependent quadrupole** to compensate or at least change the natural chromaticity Q'=dQ/dδ.
- Momentum dependent feed-down: Use sextupole with dispersion, replace d_x by $D_x\delta$

$$\Delta x' - i\Delta y' = \frac{k_2 L}{2}\left[(x+iy)^2 + 2D_x\delta(x+iy) + D_x^2\delta^2)\right]$$

- Linear (quadrupolar) term with effective focal length that is momentum dependent

$$\frac{1}{f_\delta} = k_2 L D_x \delta$$

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Chromaticity correction #2

- Momentum-dependent tune shifts

$$\Delta Q_x = \frac{k_2 L D_x \beta_x}{4\pi}\delta \qquad \Delta Q_y = -\frac{k_2 L D_x \beta_y}{4\pi}\delta$$

- Build correction matrix in the same way as for the tune correction for ΔQ'=ΔQ/δ

$$\begin{pmatrix} \Delta Q'_x \\ \Delta Q'_y \end{pmatrix} = \frac{1}{4\pi}\begin{pmatrix} D_{1x}\beta_{1x} & D_{2x}\beta_{2x} \\ -D_{1x}\beta_{1y} & -D_{2x}\beta_{2y} \end{pmatrix}\begin{pmatrix} (k_2 L)_1 \\ (k_2 L)_2 \end{pmatrix}$$

- Invert to find sextupole excitations $k_2 L$ that add chromaticities to partially compensate the natural

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Bloopers

- LEP vacuum pipe soldering
- Beer bottle in LEP
- Stand-up metal-piece in magnet
- Shielding in SLC DR

- These non-standard 'imperfections' are very difficult to identify, but it is good to keep in mind that even such odd-balls occur.