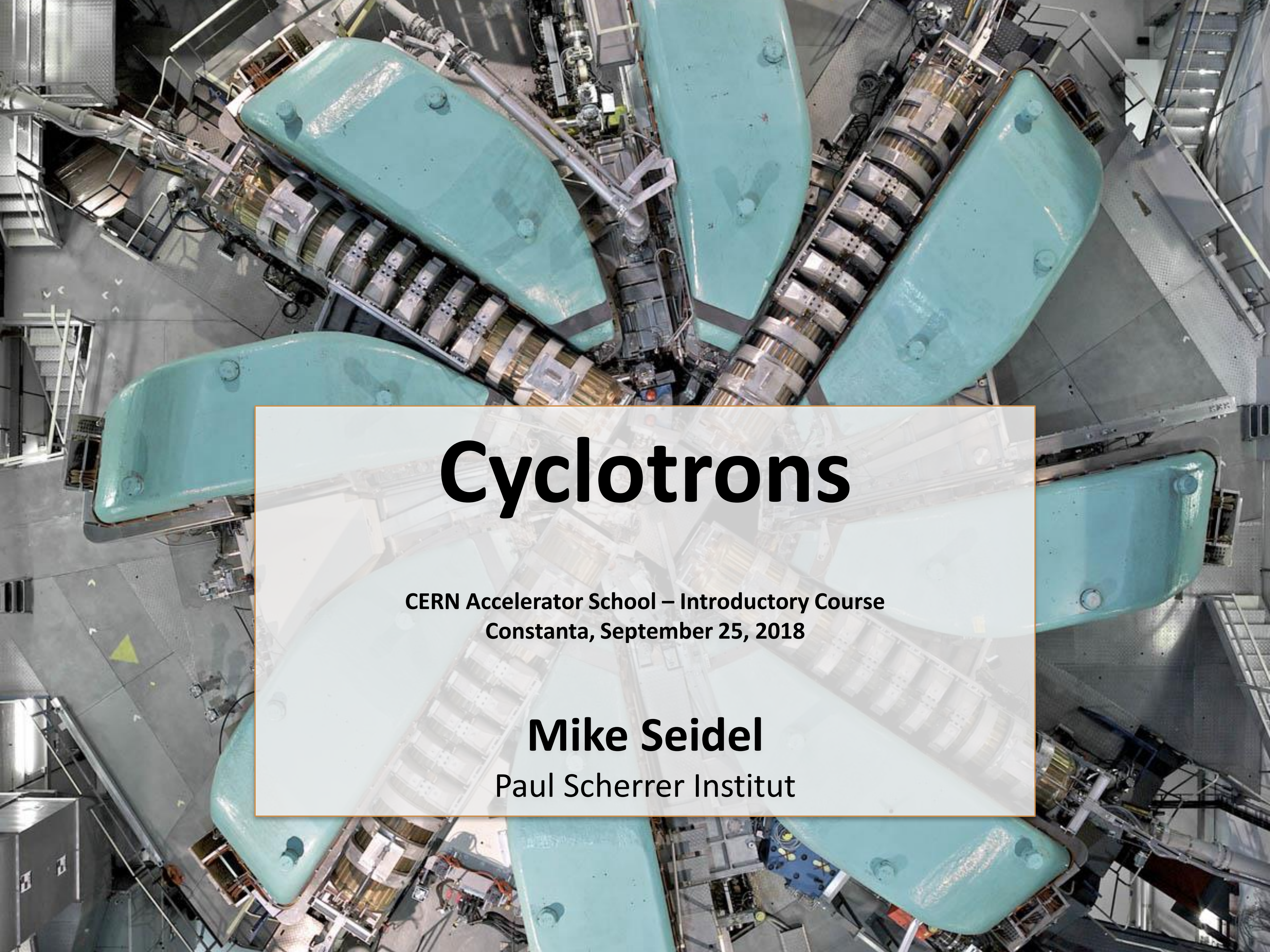


Cyclotrons

CERN Accelerator School – Introductory Course
Constanta, September 25, 2018

Mike Seidel
Paul Scherrer Institut

Cyclotrons - Outline

- the classical cyclotron

  history of the cyclotron, basic concepts and scalings, focusing, stepwidth, relativistic relations, classification of cyclotron-like accelerators

- synchro-cyclotrons

  concept, synchronous phase, example

- isochronous cyclotrons **($\rightarrow$ sector cyclotrons)**

  isochronous condition, focusing in Thomas-cyclotrons, spiral angle, classical extraction: pattern/stepwidth, transverse and longitudinal space charge

Part II

- cyclotron subsystems

  Injection/extraction schemes, RF systems/resonators, magnets, vacuum issues, instrumentation

- applications and examples of existing cyclotrons

  TRIUMF, RIKEN SRC, PSI Ring, PSI medical cyclotron

- discussion

  classification of circular accelerators, cyclotron vs. FFAG, Pro's and Con's of cyclotrons for different applications

The Classical Cyclotron

two capacitive electrodes „Dees", two gaps per turn

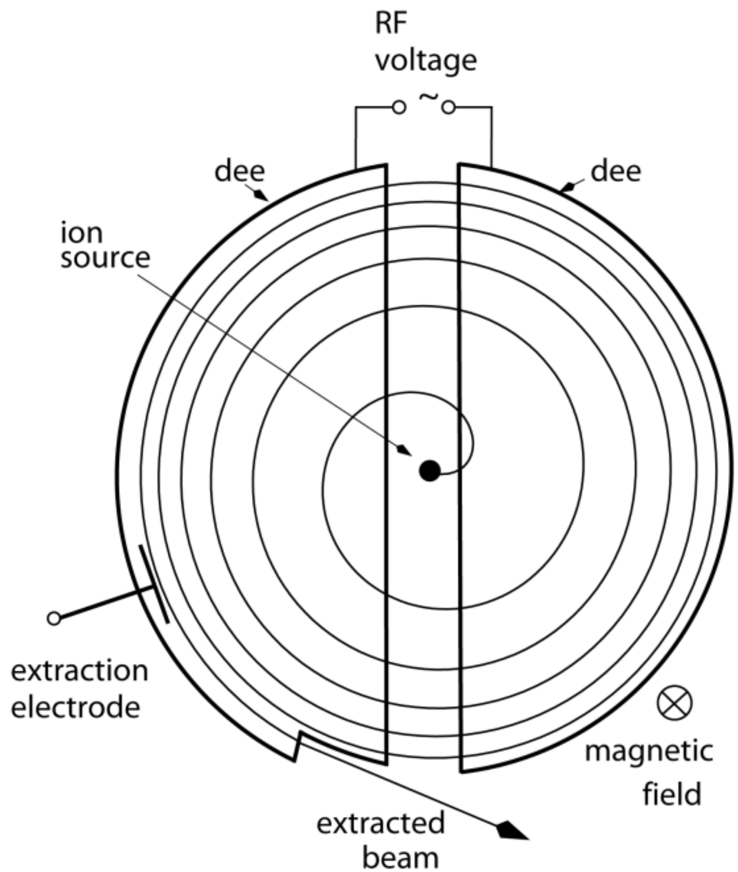
internal ion source

homogenous B field

constant revolution time

(for low energy, $\gamma \approx 1$)

$$\omega_c = \frac{eB_z}{m}$$



powerful concept:

- ➔ **simplicity, compactness**
- ➔ **continuous injection/extraction**
- ➔ **multiple usage of accelerating voltage**

some History …

first cyclotron: 1931, Berkeley

1kV gap-voltage 80keV Protons

Lawrence & Livingston, 27inch Zyklotron

Ernest Lawrence, Nobel Prize 1939
"for the invention and development of the cyclotron and for results obtained with it, especially with regard to artificial radioactive elements"

John Lawrence (center), 1940'ies
first medical applications: treating patients with neutrons generated in the 60inch cyclotron

PSI Ring Cyclotron & Crew

cyclotron frequency and *K* value

- **cyclotron frequency** (homogeneous) B-field:

$$\omega_c = \frac{eB}{\gamma m_0}$$

- **cyclotron *K*-value**:

→ *K* is the **kinetic energy reach** for protons **from bending strength** in non-relativistic approximation:

$$K = \frac{e^2}{2m_0}(B\rho)^2$$

→ *K* can be used to rescale the energy reach of protons to other charge-to-mass ratios:

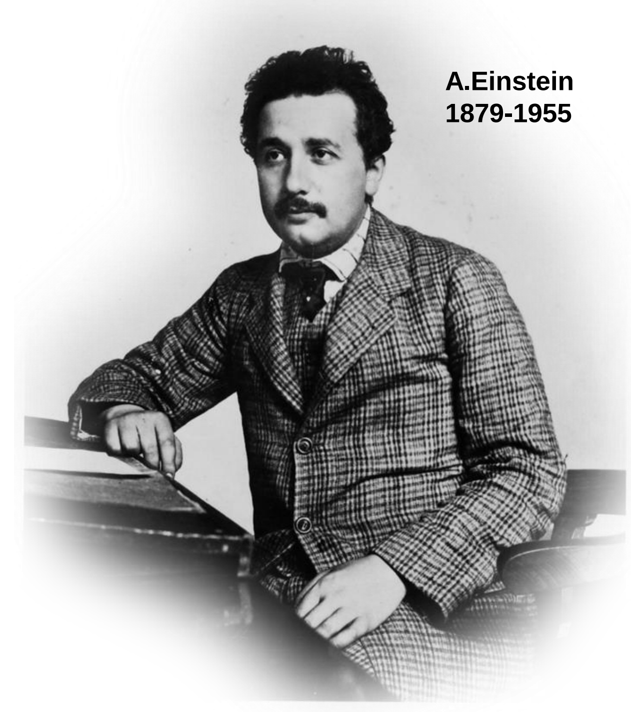
$$\frac{E_k}{A} = K\left(\frac{Q}{A}\right)^2$$

→ *K* in [MeV] is often used for naming cyclotrons

examples: K-130 cyclotron / Jyväskylä

cyclone C230 / IBA

relativistic quantities in the context of cyclotrons



energy

$$E = \gamma E_0$$

kinetic energy:

$$E_k = (\gamma - 1)E_0$$

velocity

$$v = \beta c$$

revolution time:

$$\tau = \frac{2\pi R}{\beta c}$$

momentum

$$p = \beta\gamma m_0 c$$

bending strength:

$$BR = \beta\gamma \frac{m_0 c}{e}$$

numerical example for protons

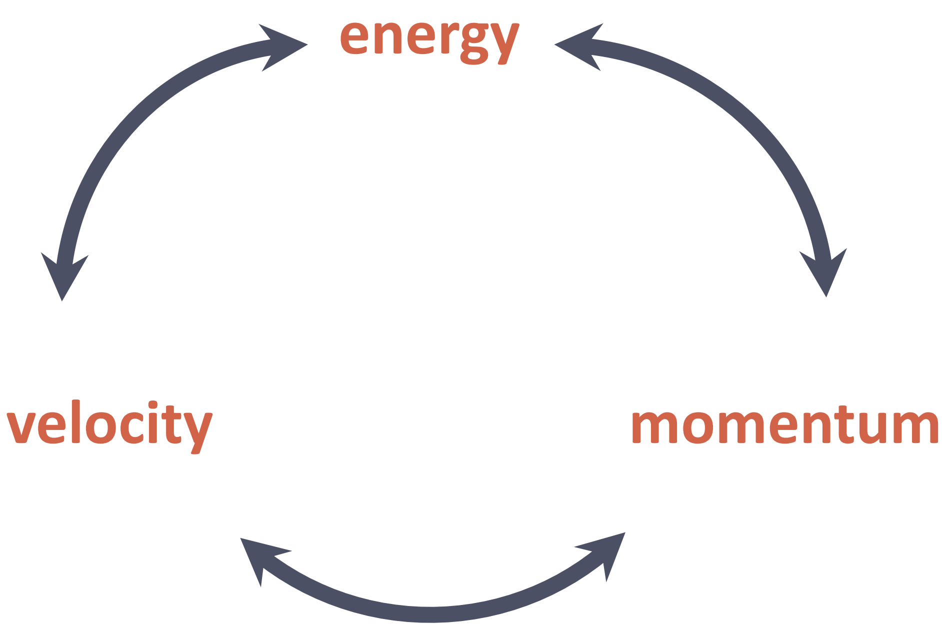
E_k [MeV]	γ	β	p [MeV/c]
590	1.63	0.79	1207

compare surface Muons:
p=29.8MeV/c $\rightarrow$ 40 times more sensitive than p_{590MeV} in same field

useful for calculations – differential relations

$$\frac{d\beta}{\beta} = \frac{1}{\gamma(\gamma+1)} \frac{dE_k}{E_k}$$

$$\frac{dE_k}{E_k} = \frac{\gamma+1}{\gamma} \frac{dp}{p}$$



$$\frac{dp}{p} = \gamma^2 \frac{d\beta}{\beta}$$

example: speed gain per turn in a cyclotron; comparison to classical $mv^2/2$

E_k	ΔE_k / turn	$\Delta\beta/\beta$
590MeV	3.4MeV	1.3‰
	classical calculation	(2.9‰)

cyclotron - isochronicity and scalings

continuous acceleration → revolution time should stay constant, though E_k, *R* vary

magnetic rigidity:

$$BR = \frac{p}{e} = \beta\gamma\frac{m_0 c}{e}$$

orbit radius from isochronicity:

$$R = \frac{c}{\omega_c}\beta = R_\infty \beta$$

deduced scaling of *B*:

$$R \propto \beta; BR \propto \beta\gamma \longrightarrow B(R) \propto \gamma(R)$$

thus, to keep the isochronous condition, *B* must be raised in proportion to $\gamma(R)$; this contradicts the focusing requirements!

→ technical solutions discussed under sector cyclotrons

field index

the field index describes the (normalized) radial slope of the bending field:

$$k = \frac{R}{B}\frac{dB}{dR} = \frac{\beta}{\gamma}\frac{d\gamma}{d\beta} = \gamma^2 - 1$$

from isochronous condition:
$B \propto \gamma$; $R \propto \beta$

→ **thus k > 0 (positive slope of field) to keep beam isochronous!**

cyclotron stepwidth classical (nonrelativistic, B const)

equation of motion for ideal centroid orbit *R*, → relation between **energy and radius**

centrifugal f. — Lorentz f.

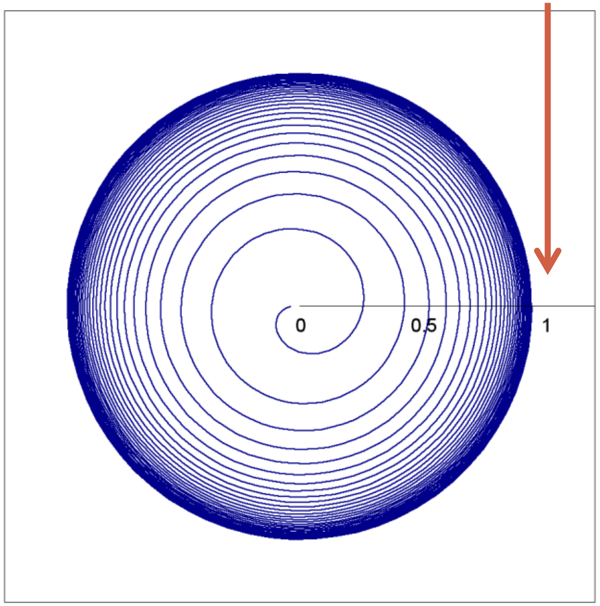
$$m\ddot{R} = m\frac{v^2}{R} - qvB_z = 0$$
$$qRB_z = \sqrt{2mE_k}$$
$$\frac{dR}{R} = \frac{1}{2}\frac{dE_k}{E_k}$$

"cyclotron language"

$$R_\infty = R/\beta$$

use: $\Delta E_k = \text{const}; B_z = \text{const}; E_k \propto R^2$

thus: $\Delta R \propto \frac{R}{E_k} \propto \frac{1}{R}$



radius increment per turn decreases with increasing radius
→ extraction becomes more and more difficult at higher energies

focusing in a classical cyclotron

centrifugal force mv^2/r

Lorentz force $qv \times B$

$$m\ddot{r} = mr\dot{\theta}^2 - qr\dot{\theta}B_z$$

focusing: consider small deviations *x* from beam orbit *R* (*r = R+x*):

$$\begin{aligned} \ddot{x} + \frac{q}{m} v B_z(R+x) - \frac{v^2}{R+x} &= 0, \\ \ddot{x} + \frac{q}{m} v \left(B_z(R) + \frac{\mathrm{d}B_z}{\mathrm{d}R} x \right) - \frac{v^2}{R} \left(1 - \frac{x}{R} \right) &= 0, \\ \ddot{x} + \omega_{\mathrm{c}}^2 (1+k) x &= 0. \end{aligned}$$

$$\text{using: } \omega_{\mathrm{c}} = qB_z/m = v/R,\ r\dot{\theta} \approx v, k = \tfrac{R}{B}\tfrac{dB}{dR}$$

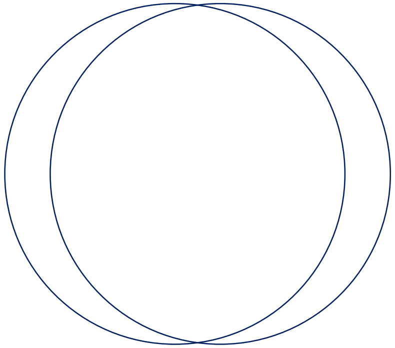
betatron tunes in cyclotrons

thus in radial plane:

$$\omega_r = \omega_c\sqrt{1+k} = \omega_c\nu_r$$
$$\nu_r = \sqrt{1+k}$$
$$\approx \gamma$$

using isochronicity condition

note: simple case for *k = 0*: $\nu_r = 1$
(one circular orbit oscillates w.r.t the other)



using Maxwell to relate B_z and B_R:

$$\text{rot}\,\vec{B} = \frac{dB_R}{dz} - \frac{dB_z}{dR} = 0$$

in vertical plane:

$$\nu_z = \sqrt{-k}$$

k<0 to obtain vertical focus.

thus: in classical cyclotron *k < 0* required for vert. focus;
however this violates isochronous condition $k = \gamma^2 - 1 > 0$

naming conventions of cyclotrons …

1.) resonant acceleration

- limit energy / ignore problem [classical cyclotron]
- frequency is varied [synchro- cyclotron]
- avg. field slope positive [isochronous cyclotron]

2.) transverse focusing

- negative field slope [classical cyclotron]
- focusing by flutter, spiral angle [AVF-/Thomas-/sector cyclotron]

classification of cyclotron like accelerators

classical cyclotron
[B(θ) = const]

↓

Thomas cyclotron
[Azimuthally Varying Field,
e.g. $B(\theta) \propto b+\cos(3\theta)$, one pol]

↓

separated sector cyclotron
[separated magnets, resonators]

synchro-cyclotron
[varying RF frequency]

Fixed Focus Alternating Gradient Accelerator FFAG
[varying RF, strong focusing]

complexity →

high intensity

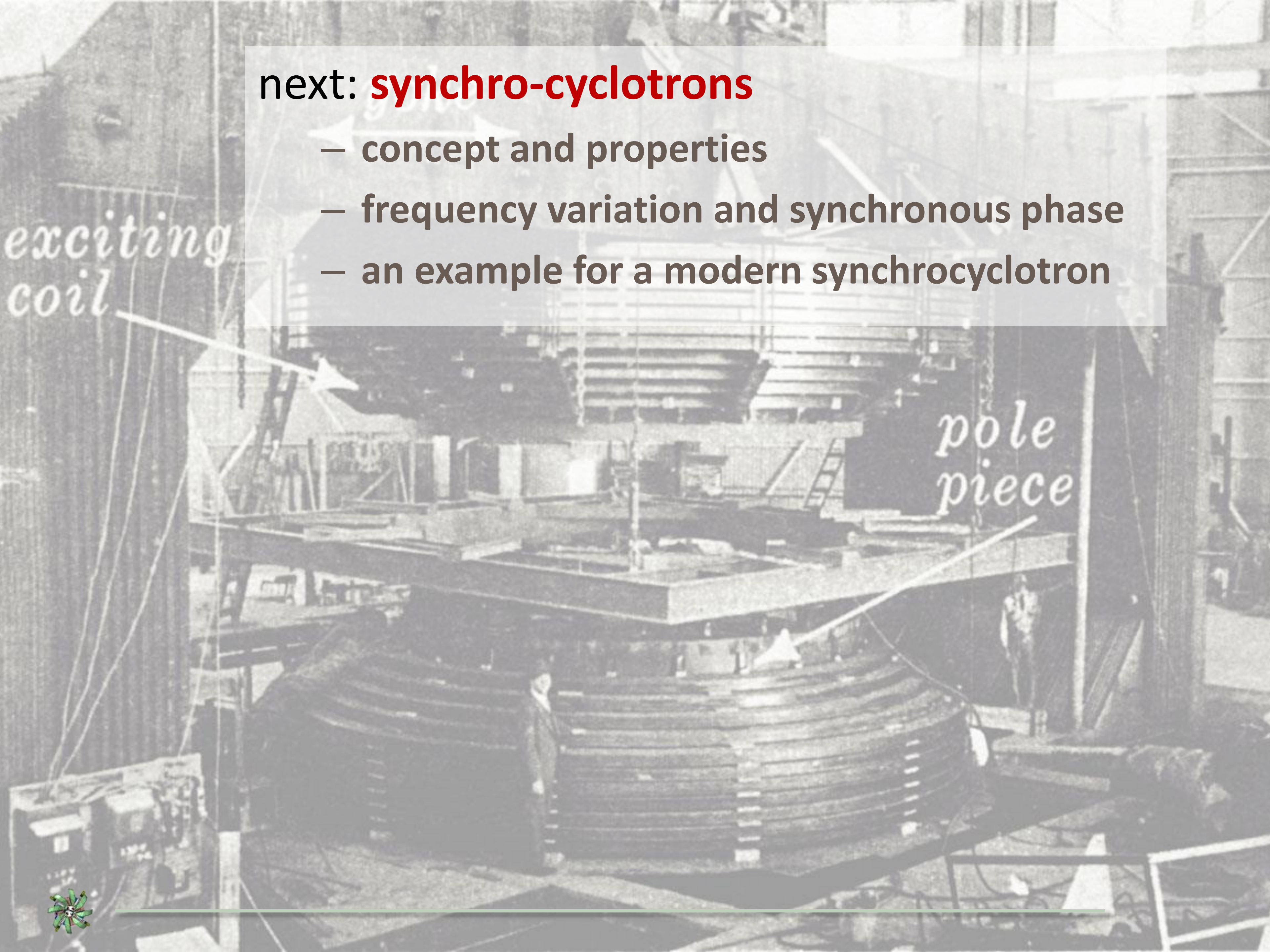
high energy

compact machine

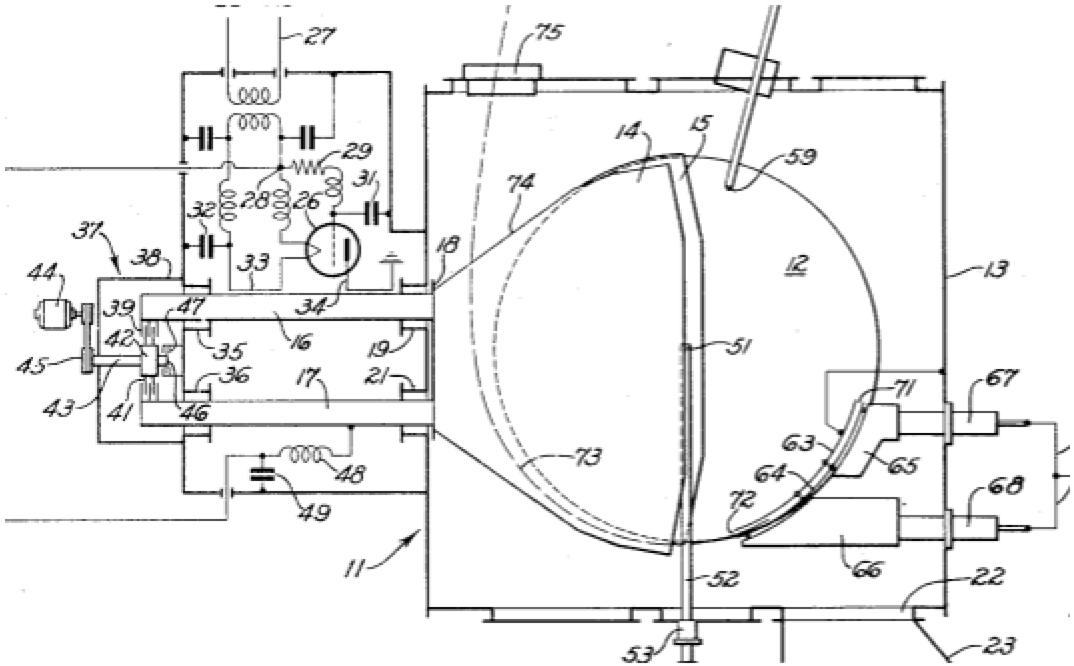
AVF concept – harmonic pole shaping,
electron model, Richardson et al (1950),
courtesy of Lawrence Berkeley National Laboratory

next: **synchro-cyclotrons**

- **concept and properties**
- **frequency variation and synchronous phase**
- **an example for a modern synchrocyclotron**



Synchrocyclotron -concept



first proposal by Mc.Millan, Berkeley

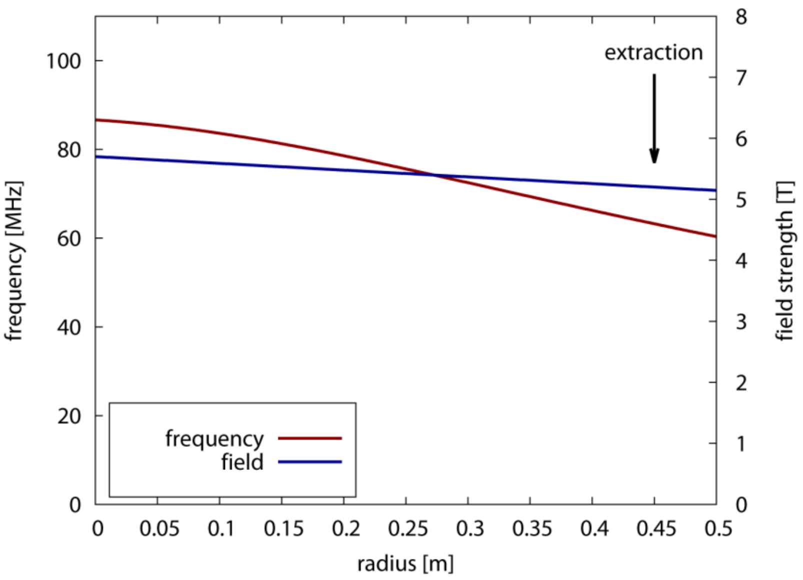
- accelerating frequency is variable, is reduced during acceleration
- negative field index (= negative slope) ensures sufficient focusing
- operation is pulsed, thus avg. intensity is low
- bending field constant in time, thus rep. rate high, e.g. 1kHz

Synchrocyclotron continued

advantages	disadvantages
- high energies possible (≥1Gev) - focusing by field gradient, no complicated flutter required → thus compact magnet - only RF is cycled, fast repetition as compared to synchrotron	- low intensity, at least factor 100 less than CW cyclotron - complicated RF control required - weak focusing, large beam

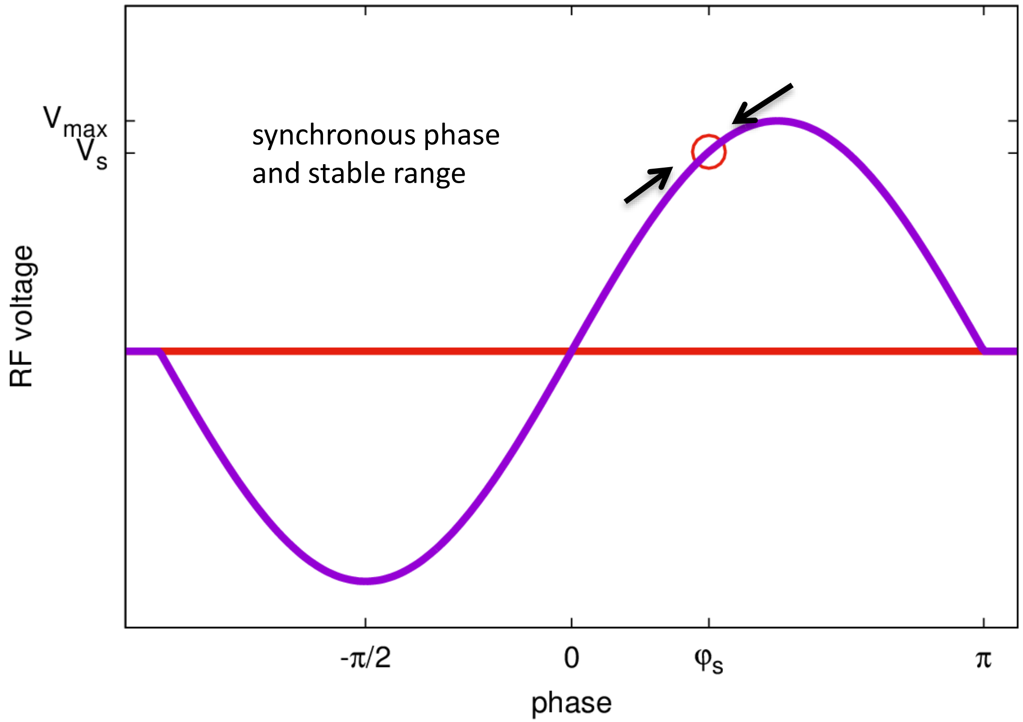
numerical example
field and frequency vs. radius:

- 230MeV p, strong field
- RF curve must be programmed in some way



Synchrocyclotron and synchronous phase

- internal source generates continuous beam; only a fraction is captured by RF wave in a phase range around a synchronous particle
- in comparison to a synchrotron the "storage time" is short, thus in practice no synchrotron oscillations



relation of
energy gain per turn and
rate of frequency change

$$\frac{qU_0 N \cos\varphi_s}{E_k + E_0} = -\frac{2\pi}{\omega^2}\frac{d\omega}{dt}$$

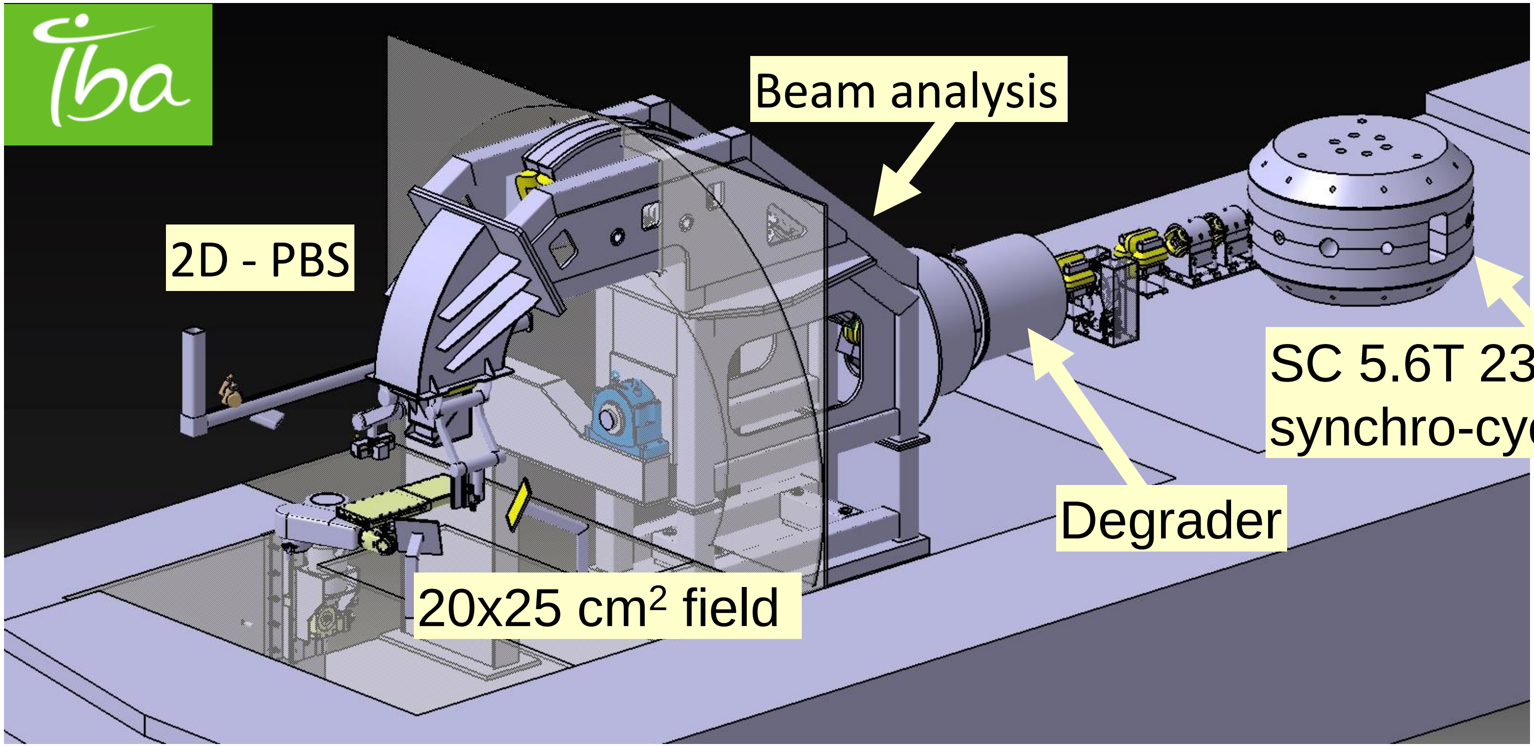
A modern synchrocyclotron for medical application – IBA S2C2

→ at the same energy synchrocyclotrons can be build more compact and with lower cost than sector cyclotrons; however, the achievable current is significantly lower

energy	230 MeV
current	20 nA
dimensions	∅2.5 m x 2 m
weight	< 50 t
extraction radius	0.45 m
s.c. coil strength	5.6 Tesla
RF frequency	90…60 MHz
repetition rate	1 kHz


Feb/2013, courtesy: P.Verbruggen,IBA

compact treatment facility using the high field synchro-cyclotron



- required area: 24x13.5m^2 (is small)
- 2-dim pencil beam scanning

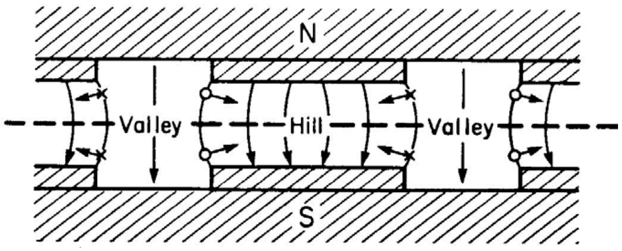
- next: **isochronous- / sector cyclotrons**
  - focusing and AVF vs. separated sector cyclotron
  - how to keep isochronicity
  - extraction: pattern/stepwidth
  - RF acceleration
  - transv./long. space charge

focusing in sector cyclotrons

hill / valley variation of magnetic field (Thomas focusing) makes it possible to design cyclotrons for higher energies

Flutter factor:

$$F^2 = \frac{\overline{B_z^2} - \overline{B_z}^2}{\overline{B_z}^2}$$

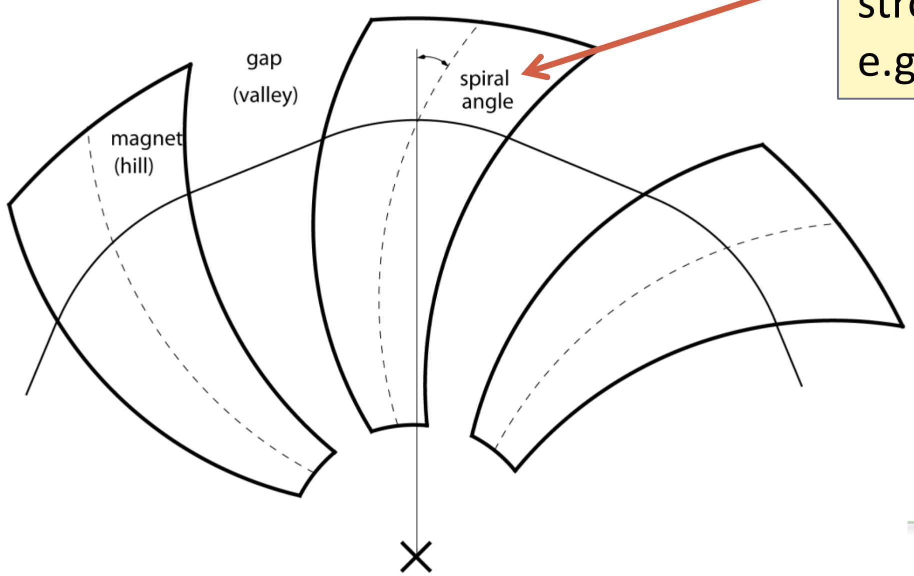


[illustration of focusing at edges]

with flutter and additional spiral angle of bending field:

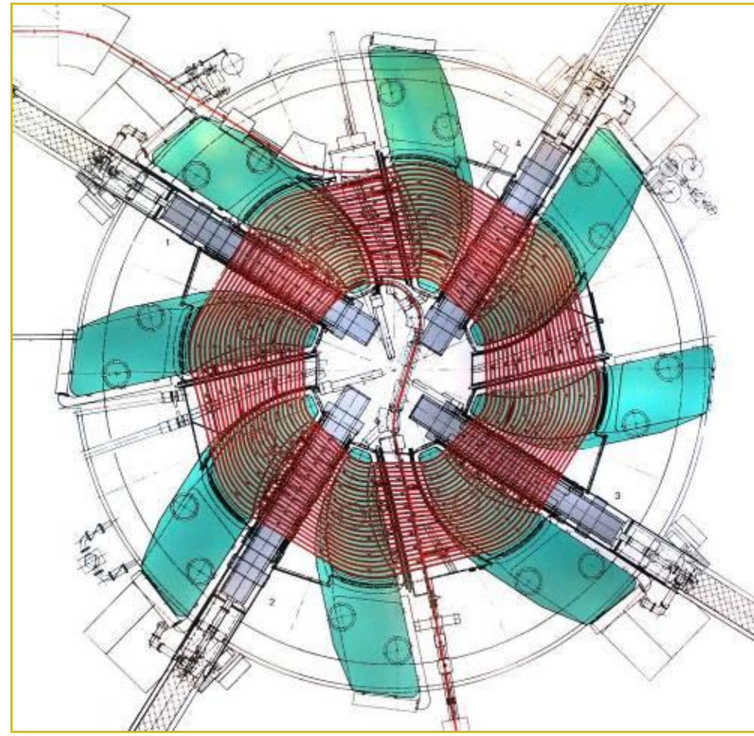
$$\nu_z^2 = -\frac{R}{B_z}\frac{dB_z}{dR} + F^2(1 + 2\tan^2\delta)$$

strong term
e.g.: δ=27°:   2tan$^2\delta$ = 1.0



Azimuthally Varying Field vs. Separated Sector Cyclotrons

PSI/Varian comet: 250MeV sc. medical cyclotron



PSI Ring cyclotron

- **AVF = single pole with shaping**
- often **spiral poles** used
- **internal source** possible
- **D-type RF electrodes**, rel. low energy gain
- **compact**, cost effective
- depicted Varian cyclotron: 80% extraction efficiency; **not suited for high power**

- **modular layout,** larger cyclotrons possible, sector magnets, box resonators, stronger focusing, injection/extraction in straight sections
- **external injection** required, i.e. pre-accelerator
- **box-resonators** (high voltage gain)
- high **extraction efficiency** possible:
  e.g. PSI: 99.98% = $(1 - 2 \cdot 10^{-4})$

three methods to raise the average magnetic field with γ

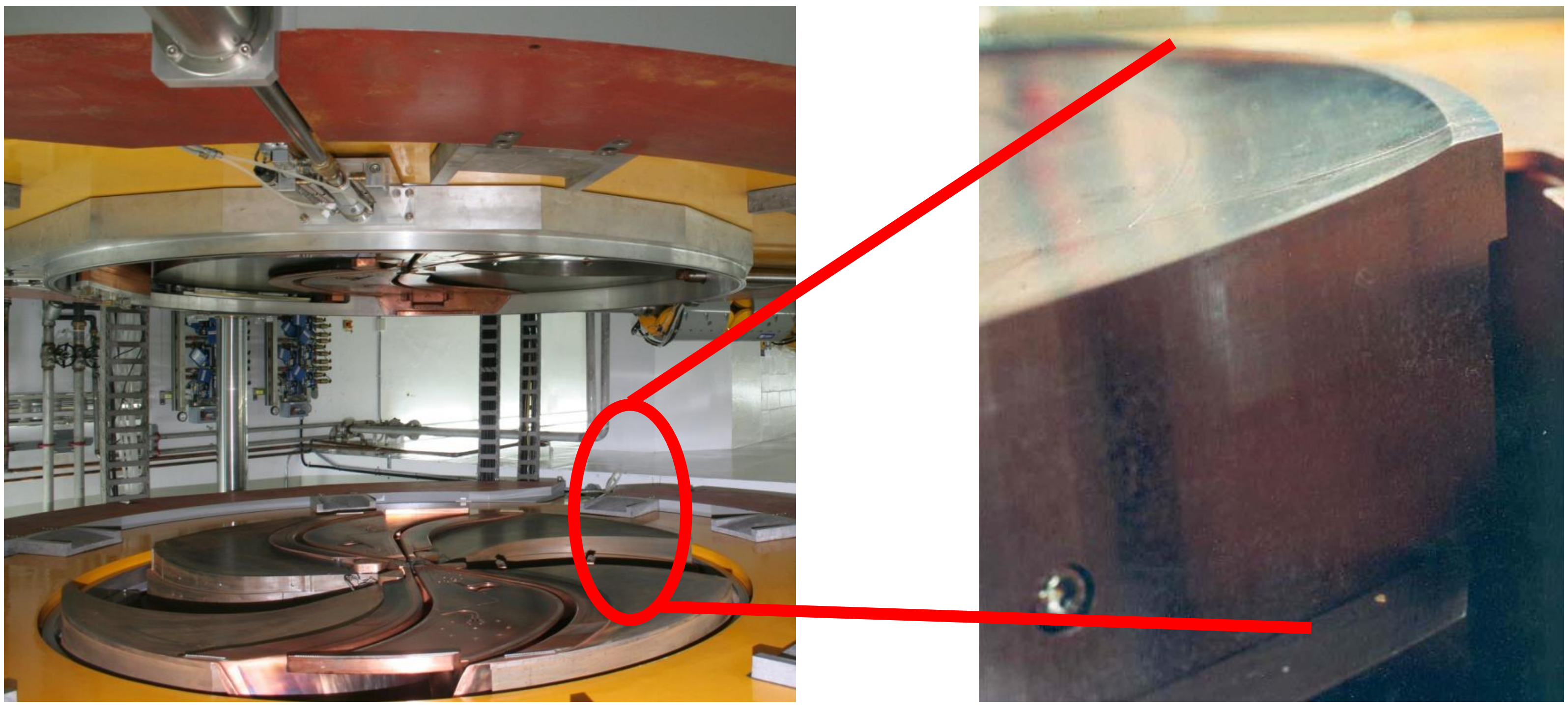
remember:

$$\begin{aligned} \text{rev.time}: \quad R &\propto \beta \\ \text{momentum}: \quad BR &\propto \beta\gamma \\ \text{thus}: \quad B &\propto \gamma \end{aligned}$$

1.) broader hills (poles) with radius

2.) **decrease pole gap with radius**

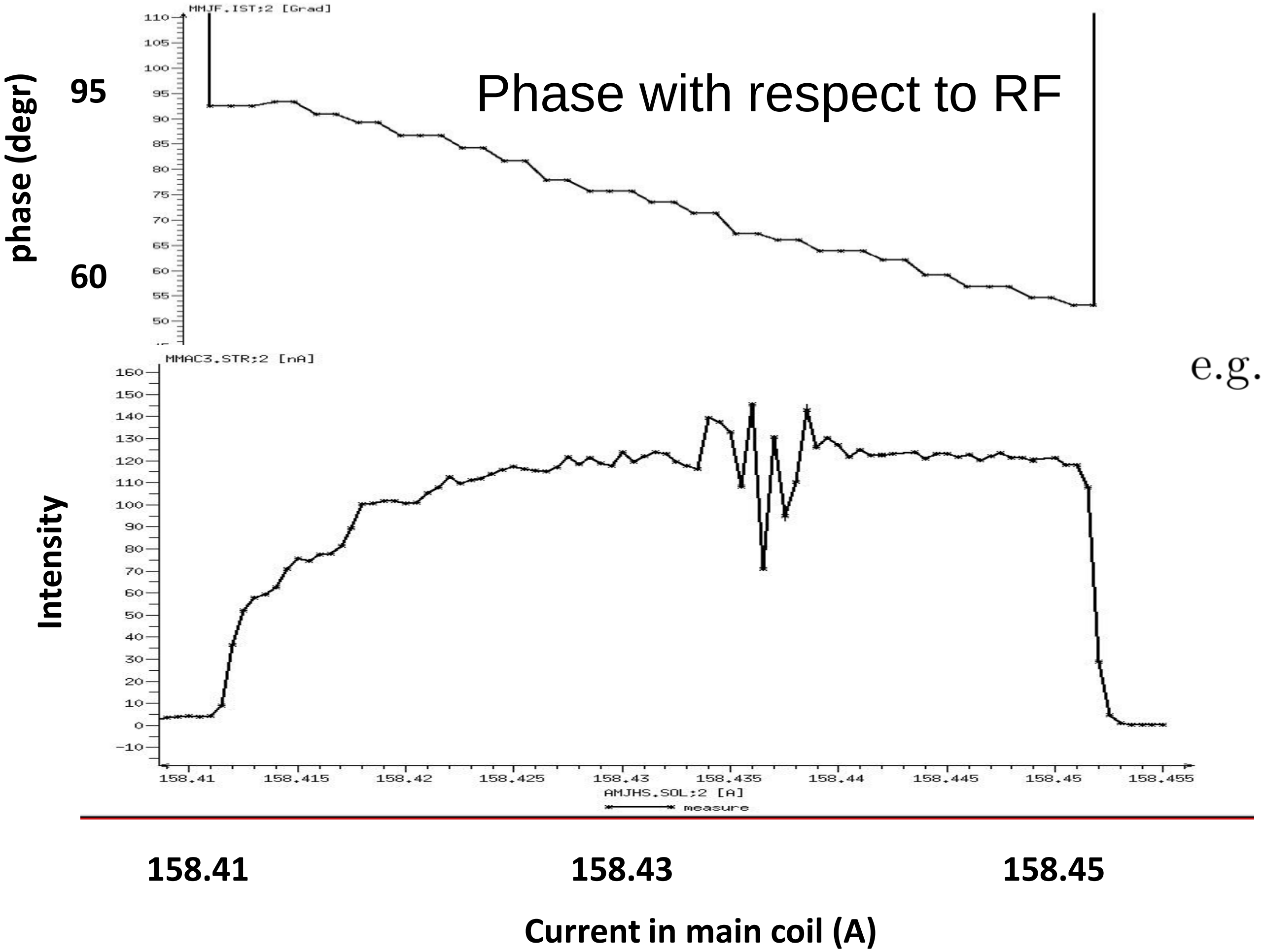
3.) s.c. coil arrangement to enhance field at large radius (in addition to iron dominated field)



(photo: S. Zaremba, IBA)

field stability is critical for isochronicity

example: medical Comet cyclotron (PSI)



$$\Delta\phi_{RF} \propto n_{\text{turn}} \frac{\Delta B}{B}$$

$$\text{e.g.}: \; n_{\text{turn}} = 600$$

derivation of (relativistic) turn separation in a cyclotron

starting point: bending strength
→ compute total log.differential
→ use field index $k = R/B \cdot dB/dR$

$$BR = \sqrt{\gamma^2 - 1}\,\frac{m_0 c}{e}$$

$$\frac{dB}{B} + \frac{dR}{R} = \frac{\gamma\, d\gamma}{\gamma^2 - 1}$$

$$\frac{dR}{d\gamma} = \frac{\gamma R}{\gamma^2 - 1}\,\frac{1}{1+k}$$

radius change per turn

$$\frac{dR}{dn_t} = \frac{dR}{d\gamma}\,\frac{d\gamma}{dn_t}$$

[U_t = energy gain per turn]

$$= \frac{U_t}{m_0 c^2}\,\frac{\gamma R}{(\gamma^2 - 1)(1+k)}$$

} isochronicity not conserved (last turns)

$$= \frac{U_t}{m_0 c^2}\,\frac{R}{(\gamma^2 - 1)\gamma}$$

} isochronicity conserved (general scaling)

turn separation - discussion

for clean extraction a large stepwidth (turn separation) is of utmost importance; in the PSI Ring most efforts were directed towards maximizing the turn separation

general scaling at extraction:

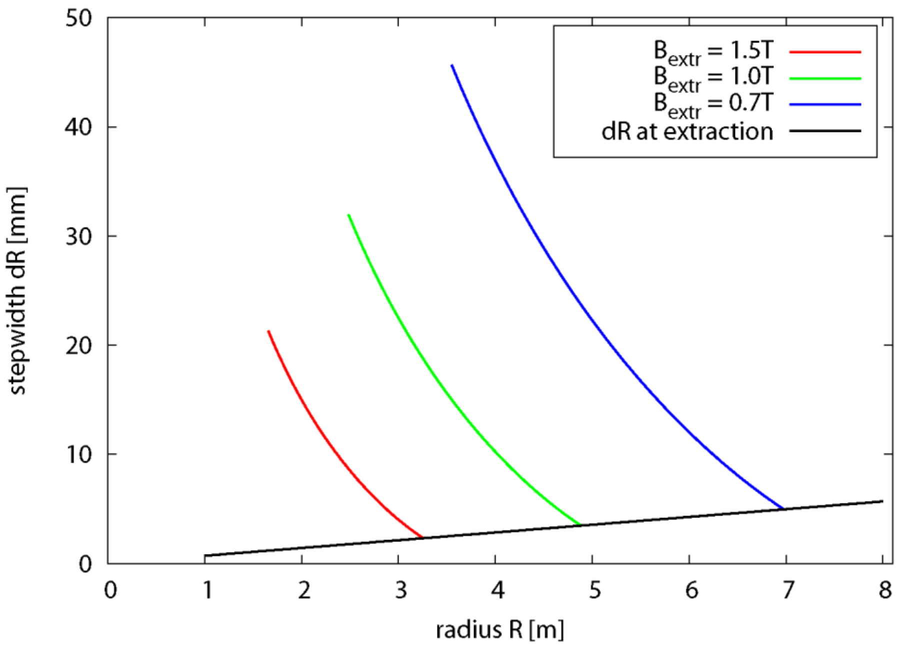
$$\Delta R(R_{\rm extr}) = \frac{U_t}{m_0 c^2} \frac{R_{\rm extr}}{(\gamma^2 - 1)\gamma}$$

desirable:
- limited energy (< 1GeV)
- large radius R_{extr}
- high energy gain U_t

scaling during acceleration:

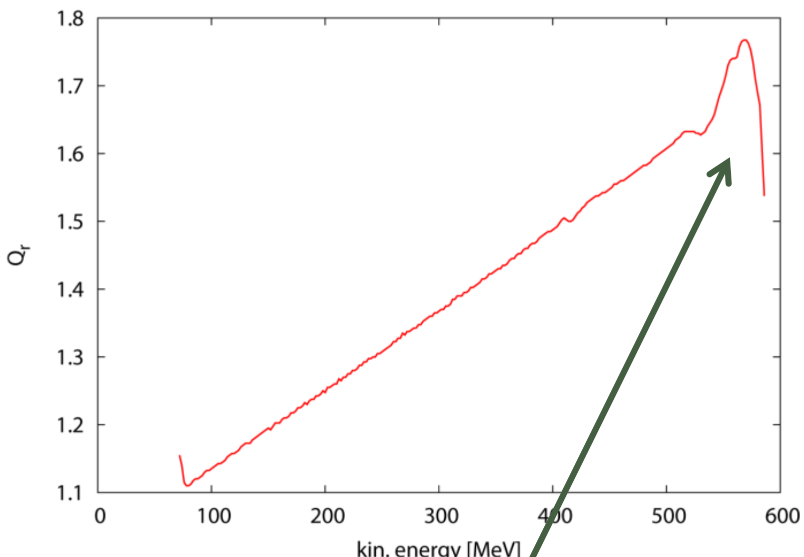
$$\frac{dR}{dn_t} \approx \frac{U_t}{m_0 c^2} \frac{R}{\beta^2} \rightarrow \Delta R(R) \propto \frac{1}{R}$$

illustration:
stepwidth vs. radius in cyclotrons of different sizes but same energy;
100MeV inj $\rightarrow$ 800MeV extr



extraction with off-center orbits

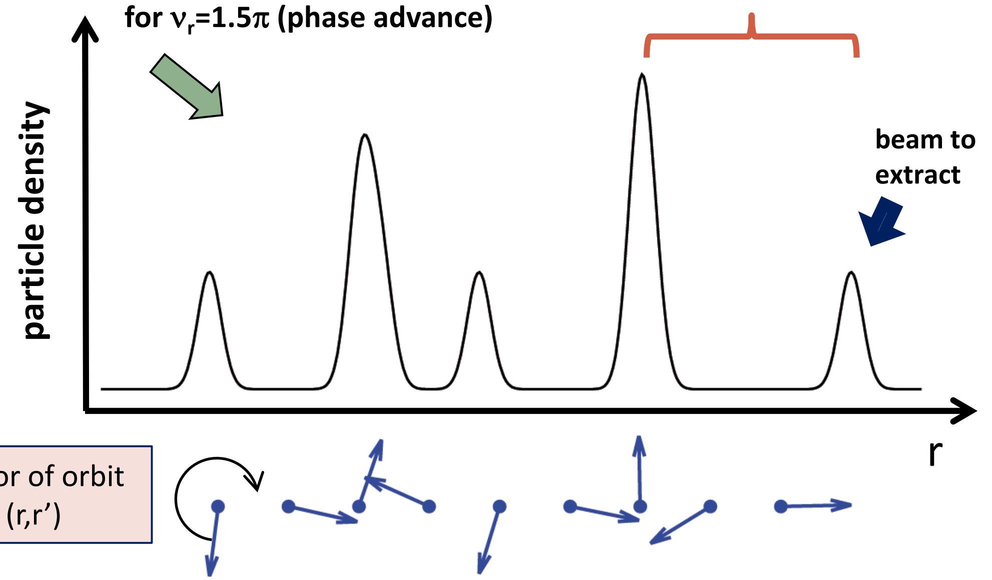
betatron oscillations around the “closed orbit” can be used to increase the radial stepwidth by a factor 3 !



radial tune vs. energy (PSI Ring)
typically $\nu_r \approx \gamma$ during acceleration;
but decrease in outer fringe field

without orbit oscillations: stepwidth from E_k-gain (PSI: 6mm)

with orbit oscillations: extraction gap; up to 3 x stepwidth possible for ν_r=1.5π (phase advance)



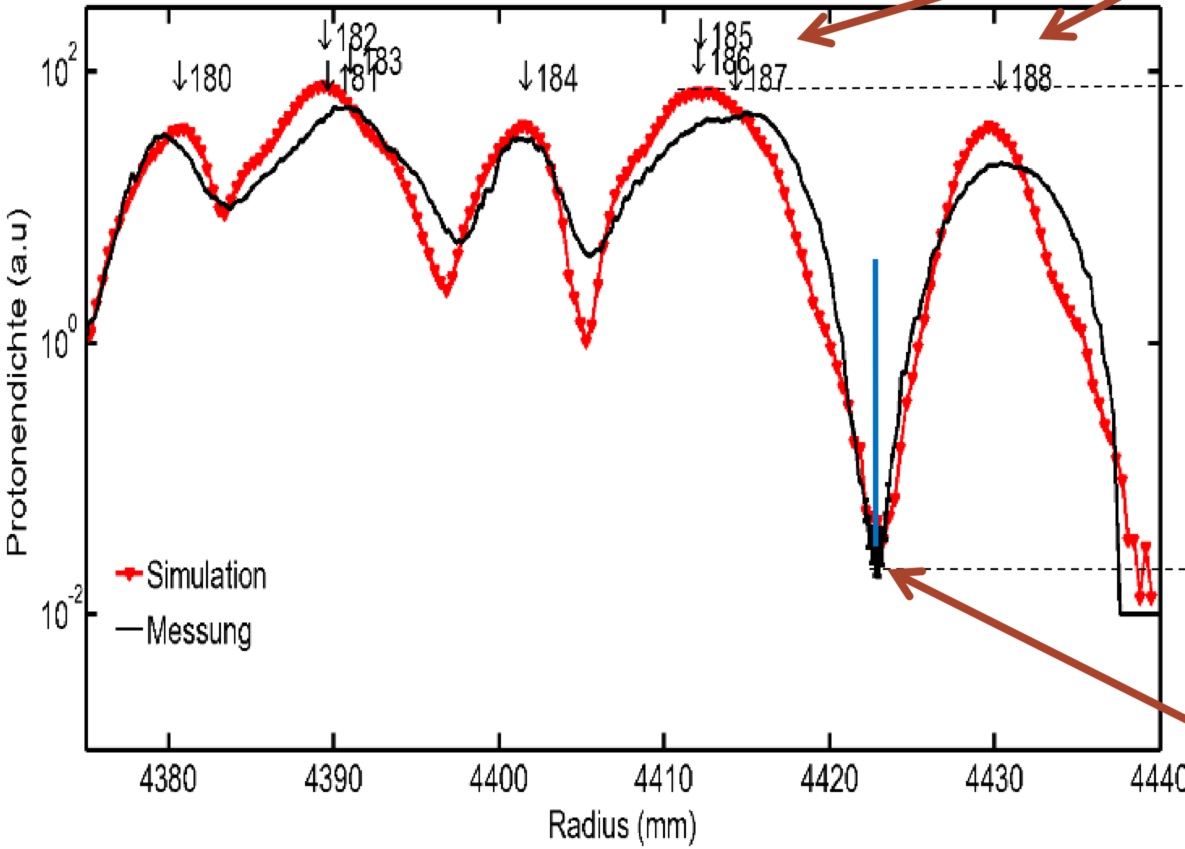
phase vector of orbit oscillations (r,r’)

extraction profile measured at PSI Ring Cyclotron

red: tracking simulation [OPAL]
black: measurement

turn numbers

from simulation



dynamic range:
factor 2.000 in
particle density

position of extraction septum

d=50µm

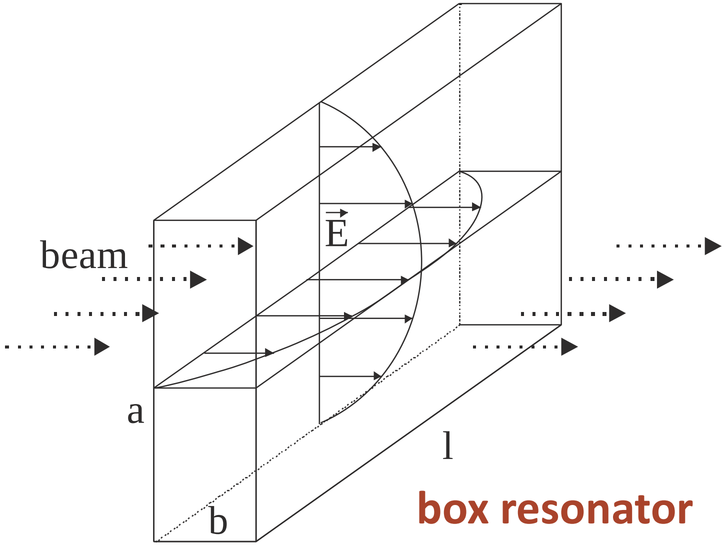
[Y.Bi et al]

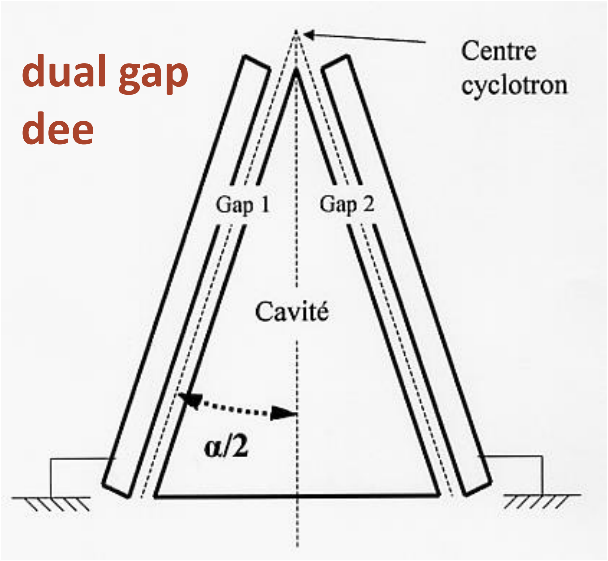
RF acceleration

- acceleration is realized in the classical way using 2 or 4 “Dees”
- or by box resonators in separated sector cyclotrons
- frequencies typically around 50…100MHz, harmonic numbers h = 1…10
- voltages 100kV…1MV per device

RF frequency can be a multiple of the cyclotron frequency:

$$\omega_{\mathrm{RF}} = h \cdot \omega_c$$

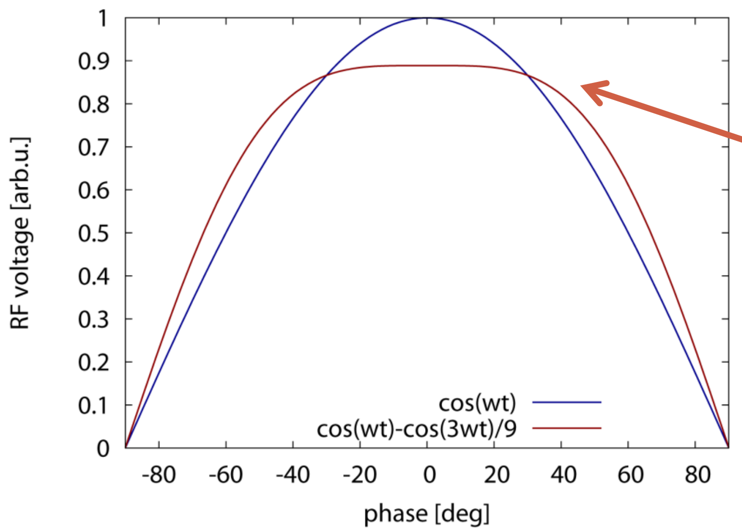




RF and Flattop Resonator

for high intensities it is necessary to flatten the RF field over the bunch length
→ use 3rd harmonic cavity to generate a flat field (over time)

optimum condition: $U_{\text{tot}} = \cos\omega t - \frac{1}{9}\cos 3\omega t$



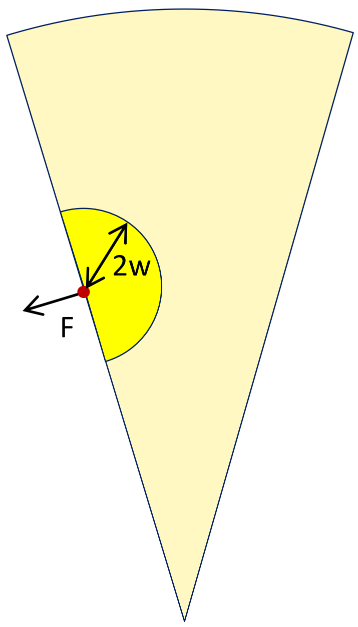
broader flat region for bunch

longitudinal space charge

sector model (W.Joho, 1981):

→ accumulated energy spread transforms into transverse tails

- consider rotating uniform sectors of charge (overlapping turns)
- test particle "sees" only fraction of sector due to shielding of vacuum chamber with gap height 2*w*



two factors are proportional to the number of turns:

1) the charge density in the sector
2) the time span the force acts

$$\Delta U_{sc} = \frac{8}{3} e I_p Z_0 \ln\left(4\frac{w}{a}\right) \cdot \frac{n_{\rm max}^2}{\beta_{\rm max}} \approx 2.800\Omega \cdot e I_p \cdot \frac{n_{\rm max}^2}{\beta_{\rm max}}$$

derivation see: [High Intensity Aspects of Cyclotrons, ECPM-2012, PSI]

in addition:

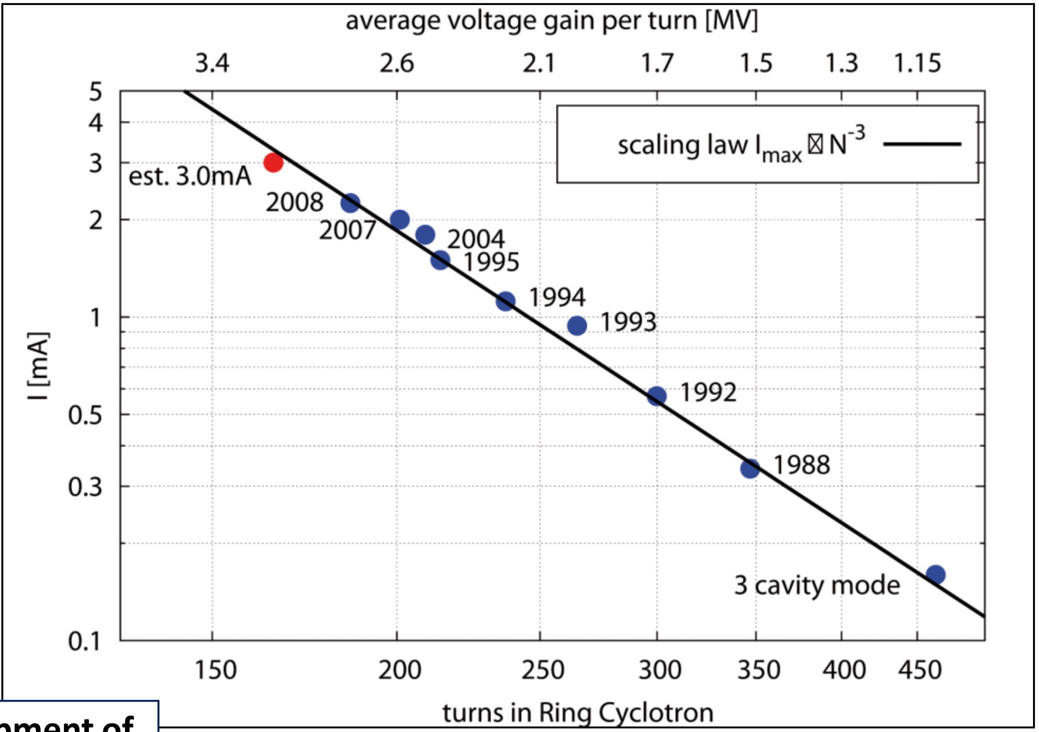
3) the inverse of turn separation at extraction: $\frac{1}{\Delta R_{\rm extr}} \propto n_{\rm max}$

► thus the attainable current at constant losses scales as n_{max}^{-3}

longitudinal space charge; evidence for third power law

- at PSI the maximum attainable current indeed scales with the third power of the turn number
- maximum energy gain per turn is of utmost importance in this type of high intensity cyclotron

→ with constant losses at the extraction electrode the maximum attainable current indeed scales as: $I_{\max} \propto {n_t}^{-3}$



historical development of current and turn numbers in PSI Ring Cyclotron

transverse space charge

with overlapping turns use current sheet model!

vertical force from space charge: $F_y \quad = \quad \frac{n_v e^2}{\epsilon_0 \gamma^2} \cdot y, \;\; n_v = \frac{N}{(2\pi)^{\frac{3}{2}} \sigma_y D_f R \Delta R}$

[constant charge density, $D_f = I_{avg}/I_{peak}$]

focusing force: $F_y \quad = \quad -\gamma m_0 \omega_c^2 \nu_{y0}^2 \cdot y$

thus, eqn. of motion: $\ddot{y} + \left(\omega_c^2 \nu_{y0}^2 - \frac{n_v e^2}{\epsilon_0 m_0 \gamma^3} \right) y = 0$

→ equating space charge and focusing force delivers an **intensity limit for loss of focusing**!

tune shift from forces:

$$\Delta\nu_y \quad \approx \quad -n_v \frac{2\pi r_p R^2}{\beta^2 \gamma^3 \nu_{y0}}$$

$$\approx \quad -\sqrt{2\pi} \frac{r_p R}{e \beta c \nu_{y0} \sigma_z} \frac{m_0 c^2}{U_t} \; I_{\text{avg}}$$

Outlook: Cyclotrons II

- cyclotron subsystems

  extraction schemes, RF systems/resonators, magnets, vacuum issues, instrumentation

- applications and examples of existing cyclotrons

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- discussion

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