On exclusive hard processes with light-mesons

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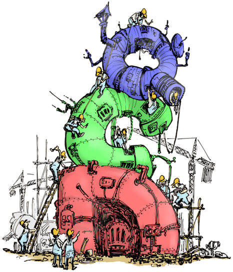


Getting to Grips with QCD
Paris, April 3rd, 2018

Outline

1. Prologue on exclusive processes with light-mesons
   - Basic ingredients of the pQCD approach
   - Status of the higher-order calculations
2. Light-meson form factors and distribution amplitudes
   - π transition form factor
   - η, η' transition form factors
   - $f_0(980)$ transition form factor
3. Summary and outlook

Introduction

Exclusive hard-scattering ($\exists$ large scale)

$\rightarrow$ factorization [ERBL '80] :

hard scattering amplitude	$=$	elementary hard-scattering amplitude	$\otimes$	hadron distribution amplitudes

Introduction

Exclusive hard-scattering ($\exists$ large scale)

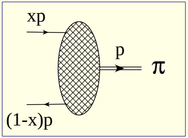
$\rightarrow$ factorization [ERBL '80] :

hard scattering amplitude	=	elementary hard-scattering amplitude	$\otimes$	hadron distribution amplitudes

- standard leading-twist hard-scattering picture:
  - $|\pi\rangle \rightarrow |q\bar{q}\rangle + |q\bar{q}g\rangle + \cdots$
  - collinear approximation:

    $p_q = x\, p, \quad p_{\bar{q}} = (1-x)\, p$

    ($0 < x < 1 \rightarrow$ longitudinal momentum fraction)



  - $m_q = m_{\bar{q}} = 0\,, m_\pi = 0$

Example: PHOTON-TO-PION TRANSITION FORM FACTOR $F_{\pi\gamma^{(*)}}$

$$\gamma^*(q_1, \mu) \quad \gamma^{(*)}(q_2, \nu) \quad \to \quad \pi(p) \ ,$$

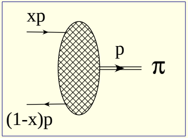
$$-q_1^2 = Q^2 \gg$$

in the standard hard-scattering picture:

$$F_{\pi\gamma}(Q^2) = T_H(x, Q^2, \mu_F^2) \otimes \Phi(x, \mu_F^2)$$

$$A(x) \otimes B(x) = \int_0^1 dx\, A(x) B(x)$$

$\mu_F^2 \cdots$ factorization scale



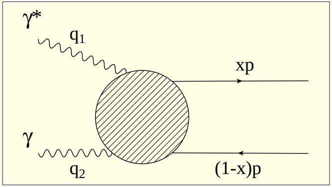
T_H ... *process–dependent*

(ELEMENTARY) HARD SCATTERING AMPLITUDE (HSA)

⇑

PQCD

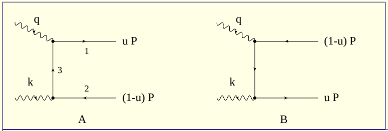
$\gamma^* \gamma \to q\bar{q}$



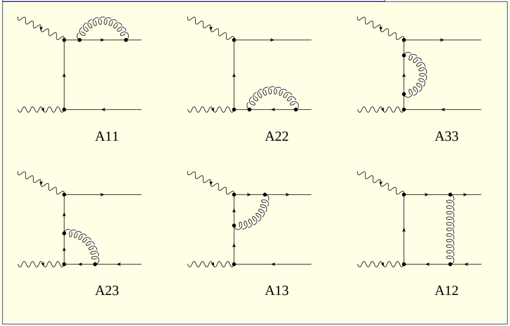
$$T_H(x,Q^2) = T_H^{(0)}(x,Q^2) + \frac{\alpha_S(\mu_R^2)}{4\pi} T_H^{(1)}(x,Q^2,\mu_F^2) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2} T_H^{(2)}(x,Q^2,\mu_F^2,\mu_R^2) + \cdots$$

μ_R^2 ... renormalization scale

leading order (LO)



next-to-leading order (NLO)



UV singularities $\rightarrow$ *coupling constant* (α_S) *renormalization* $\Rightarrow \mu_R^2$
collinear singularities $\rightarrow$ *factorization* $\Rightarrow \mu_F^2$

Φ . . . *process–independent* pion

DISTRIBUTION AMPLITUDE (DA)

defined in terms of the matrix elements of composite operators: $\langle 0|\Psi(-z)\gamma^{+}\gamma_5\Omega\Psi(z)|\pi\rangle$

$\Uparrow$

form: (nonperturbative) input at scale $\mu_0^2 \longrightarrow \Phi(x, \mu_0^2)$

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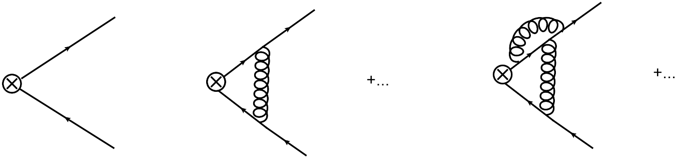
$\downarrow$



(LO)

(NLO)
LO evolution

(NNLO)
NLO evolution

Φ ... *process–independent* pion

DISTRIBUTION AMPLITUDE (DA)

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$\downarrow$

$$\Phi(x, \mu_F^2) = \underbrace{\phi_V(x, y, \mu_F^2, \mu_0^2)} \otimes \Phi(y, \mu_0^2)$$

$\downarrow$ *(resummation of* $(\alpha_S \ln(\mu_F^2/\mu_0^2))^n$ *terms)*

$$\begin{cases} \mu_F^2 \dfrac{\partial}{\partial \mu_F^2}\phi_V = V \otimes \phi_V \quad \cdots \text{ evolution equation} \\ \text{evolution kernel:} \quad V = \dfrac{\alpha_S(\mu_F^2)}{4\pi} V_1 + \dfrac{\alpha_S^2(\mu_F^2)}{(4\pi)} V_2 + \cdots \end{cases}$$

Solution of the DA evolution equation:

$$\Phi = \frac{f_\pi}{2\sqrt{2N_c}}\,\phi$$

$f_\pi = 0.131$ GeV . . . pion decay constant

$\int_0^1 dx\ \phi(x, \mu_F^2) = 1$

$$\phi(x, \mu_F^2) = 6x(1-x)\left[1 + \sum_{n=2}^{\infty}{}' B_n(\mu_F^2)\ C_n^{3/2}(2x-1)\right]$$

$C_n^{3/2}$ . . . Gegenbauer polynomials

$\longrightarrow$ eigenfunctions of the LO evolution equation

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$C_n^{3/2}$ ... Gegenbauer polynomials

$\longrightarrow$ eigenfunctions of the LO evolution equation

$$\begin{cases} B_n(\mu_F^2) = B_n^{LO}(\mu_F^2) + \dfrac{\alpha_S(\mu_F^2)}{4\pi} B_n^{NLO}(\mu_F^2) + \cdots \\ \qquad B_n^{LO}(\mu_F^2) = f(\mu_F^2, \mu_0^2, B_n(\mu_0^2)) \\ \qquad B_n^{NLO}(\mu_F^2) = g(\mu_F^2, \mu_0^2, B_{k(k\le n)}(\mu_0^2)) \end{cases}$$

$\overline{\text{MS}}$ factorization scheme (*D. Müller (1994)*)

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$\rightarrow$ eigenfunctions of the LO evolution equation

$$B_n(\mu_F^2) = B_n^{LO}(\mu_F^2) + \frac{\alpha_S(\mu_F^2)}{4\pi} B_n^{NLO}(\mu_F^2) + \cdots$$

$$B_n^{LO}(\mu_F^2) \;=\; B_n \left(\frac{\alpha_S(\mu_0^2)}{\alpha_S(\mu_F^2)}\right)^{\gamma_n/\beta_0} \qquad (\leq B_n)$$

Exclusive processes to NLO

$$\mathcal{M}(Q^2) = \mathcal{M}^{(0)}(Q^2) + \frac{\alpha_S(\mu_R^2)}{4\pi}\mathcal{M}^{(1)}(Q^2) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2}\mathcal{M}^{(2)}(Q^2, \mu_R^2) + \cdots$$

- higher-order corrections are important
  (stabilizing effect reducing the dependence of the predictions on the scales and schemes)
- only a handful of exclusive processes analyzed to NLO

♦ PHOTON-TO-π (η, η') TRANSITION FORM FACTOR

$$\gamma^* \quad \gamma \quad \to \quad \pi^0(\eta, \eta')$$

$$F_{\pi\gamma}(Q^2) = F_{\pi\gamma}^{(0)}(Q^2) + \frac{\alpha_S(\mu_R^2)}{4\pi} F_{\pi\gamma}^{(1)}(Q^2) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2} \left[\beta_0 \, F_{\pi\gamma}^{(2,\beta_0)}(Q^2, \mu_R^2) + \cdots\right] +$$

LO: (2 diagrams)

NLO: (12 one-loop diagrams)
Aguila, Chase (1981); Braaten (1983); Kadantseva, Mikhailov, Radyushkin (1986); Kroll, Passek-Kumerički (2003) [η, η': two-gluon states – 6 more diagrams]

β_0-proportional NNLO: (12 two-loop diagrams)
Melić, Nižić, Passek (2002)

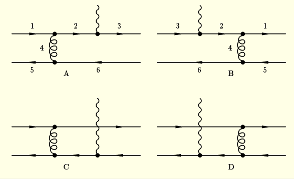
♦ PION ELECTROMAGNETIC FORM FACTOR

$$\gamma^* \quad \pi^{+(-)} \rightarrow \quad \pi^{+(-)}$$

$$F_\pi(Q^2) = \frac{\alpha_S(\mu_R^2)}{4\pi} F_\pi^{(1)}(Q^2) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2} F_\pi^{(2)}(Q^2, \mu_R^2) + \cdots$$

LO:

(4 diagrams)



$\gamma^*(q_1\bar{q}_2) \rightarrow (q_1\bar{q}_2)$

NLO:

(62 one-loop diagrams)

Field, Gupta, Otto, Chang (1981); Dittes, Radyushkin (1981); Sarmadi (1982); Khalmuradov, Radyushkin (1985); Bratten (1987); Kadantseva, Mikhailov, Radyushkin (1986); Melić, Nižić, Passek (1999)

♦ PION PAIR PRODUCTION

$\gamma \;\; \gamma \to \;\; \pi^+\pi^-$

$$\mathcal{M}(s,t) = \frac{\alpha_S(\mu_R^2)}{4\pi}\mathcal{M}^{(1)}(s,t) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2}\mathcal{M}^{(2)}(s,t,\mu_R^2) + \cdots$$

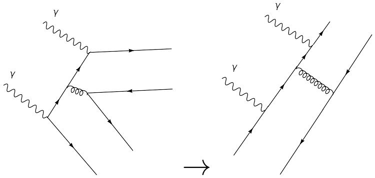
LO: (20 diagrams)

NLO: (454 one-loop diagrams)

Nižić (1987)

Duplančić, Nižić (2006)

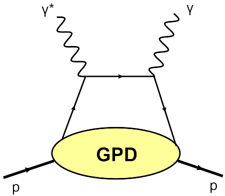
$\gamma\gamma \to (q_1\bar{q}_2)(q_2\bar{q}_1)$



NOTE: related (sub)processes with nucleons

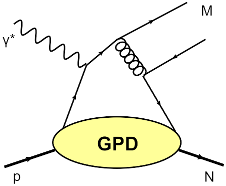
GPD... generalized parton distribution

Deeply virtual Compton scattering (DVCS)



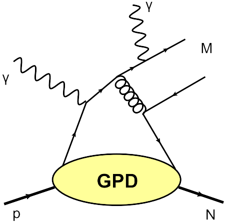
$$\boxed{\gamma^* p \to \gamma p}$$

Deeply virtual production of mesons (DVMP)



$$\boxed{\gamma^* p \to M p}$$

Deeply virtual production of photon-meson pair

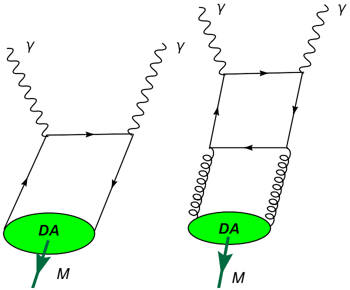


$$\boxed{\gamma p \to \gamma M p}$$

Elementary hard-scattering amplitudes: meson form factors vs. deeply virtual (DV) processes on nucleons

Meson transition form factor

$$\gamma^*\gamma \to (q\bar{q}),\ \gamma^*\gamma \to (gg)$$



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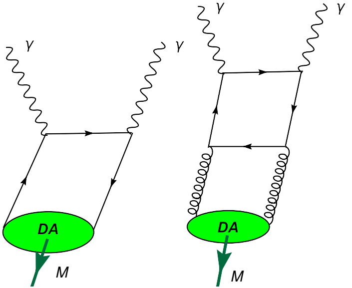
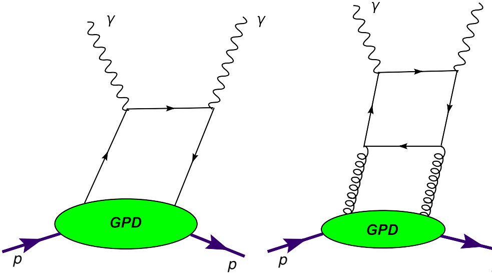


conf. NNLO: [Müller et al '02]

DVCS

$$\gamma^* q \to \gamma q,\ \gamma^* g \to \gamma g$$



NLO: [Ji et al, Belitsky et al, Mankiewicz et al, '97]

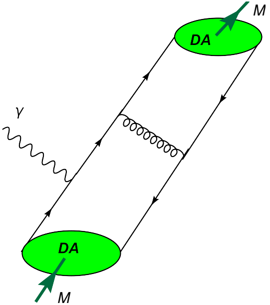
β_0 proportional NNLO: [Belitsky, Schäfer '98]

NNLO from conf. sym: [Müller '05, Kumerički et al. '07]

Elementary hard-scattering amplitudes: meson form factors vs. deeply virtual (DV) processes on nucleons

Meson em form factor

$$\gamma^*(q\bar{q}) \to (q\bar{q})$$



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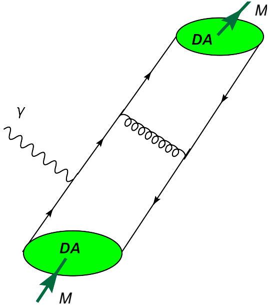
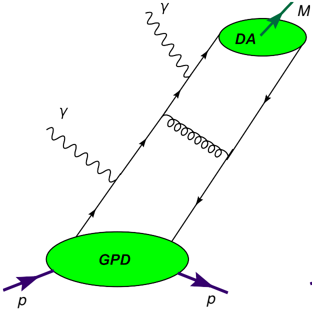
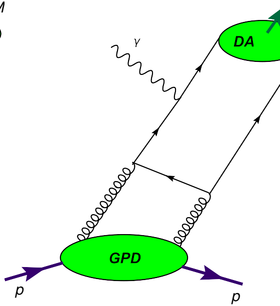


DVMP

$$\gamma^* q \to (q\bar{q})q, \; \gamma^* g \to (q\bar{q})g$$





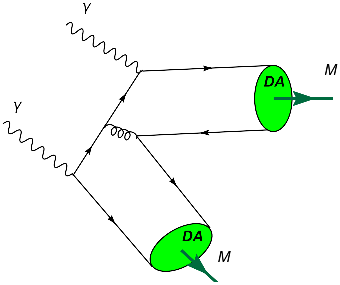
NLO: [Belitsky and Müller '01, Ivanov et al '04,]

[Duplančić, Müller, KPK '17]

Elementary hard-scattering amplitudes: meson form factors vs. deeply virtual (DV) processes on nucleons

Meson pair production

$$\gamma^*\gamma \to (q\bar{q})(q\bar{q})$$



Elementary hard-scattering amplitudes: meson form factors vs. deeply virtual (DV) processes on nucleons

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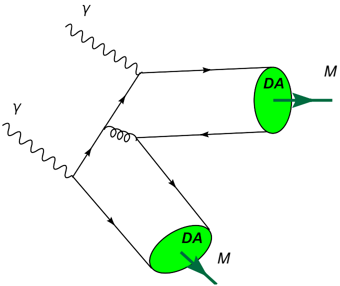
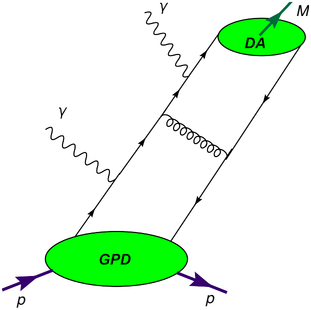


Photon-meson production

$$\gamma^* q \to \gamma q(q\bar{q})$$



LO V mesons: [Boussarie, Pire, Szymanowsky, Wallon '16]

LO PS mesons, NLO: *work in progress*

Pion transition form factor

$$F_{\pi\gamma}(Q^2) = F^{(0)}_{\pi\gamma}(Q^2) + \frac{\alpha_S(\mu_R^2)}{4\pi} F^{(1)}_{\pi\gamma}(Q^2) + \cdots$$

$\mu_R^2 = \mu_R^2(Q^2) \ldots$ renormalization scale

- power law $(1/Q^2)$ and logarithmic corrections

$F_{\pi\gamma}(Q^2 \to \infty) = \frac{\sqrt{2} f_\pi}{Q^2}$ $\Leftarrow$ perturbative QCD

$F_{\pi\gamma}(Q^2 \to 0) = \frac{\sqrt{2}}{(4\pi^2) f_\pi}$ $\Leftarrow$ $\Gamma(\pi^0 \to \gamma\gamma)$, axial anomaly

$f_\pi = 0.131$ GeV

Ancient history: DAs

DA input:

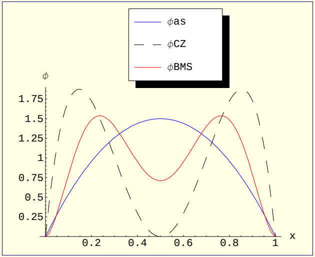
asymptotic function: $\phi_{as} \equiv \phi(x, \infty) = 6x(1-x)$

ϕ_{CZ} : $B_2(0.25\,\text{GeV}^2) = 2/3$

(Chernyak, Zhitnitsky (1984))

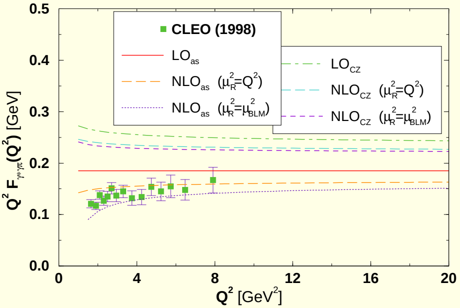
ϕ_{BMS} : $B_2(1\,\text{GeV}^2) = 0.188 \qquad B_4(1\,\text{GeV}^2) = -0.13$

(Bakulev, Mikhailov, Stefanis (2001))



Ancient history: pion transition form factor

Numerical predictions for $F_{\pi\gamma}(Q^2)$

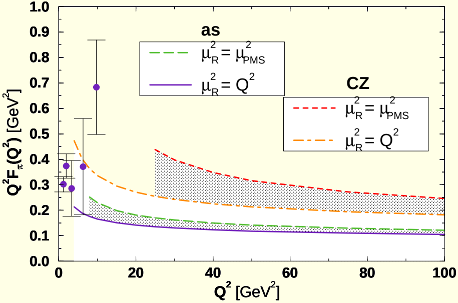


$(\mu_{BLM}^2)^{as} \approx Q^2/9\,,$ $\alpha_S \leq 0.5$ for $Q^2 > 4$ GeV^2 !

$(\mu_V^2)^{as} \approx Q^2/2$

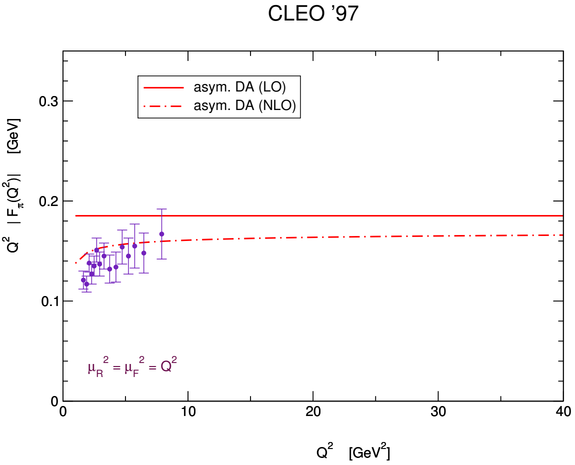
Ancient history: pion electromagnetic form factor

Numerical predictions for $F_\pi(Q^2)$



- $\mu_R^2 = Q^2$: NLO corrections large ($< 30(50)\%$ for $Q^2 > 500(10)$ GeV²)
- $\mu_R^2 = (\mu_{BLM}^2)^{as} \approx Q^2/106$: very small scale ! $\Rightarrow \alpha_S$ large

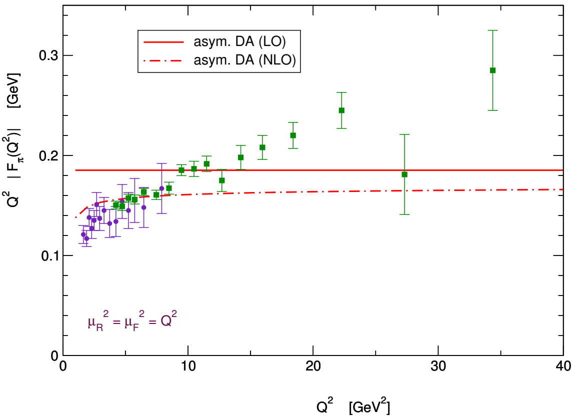
Experimental situation for pion tff



CLEO '97 (purple)

Experimental situation for pion tff

CLEO '97, BABAR '09

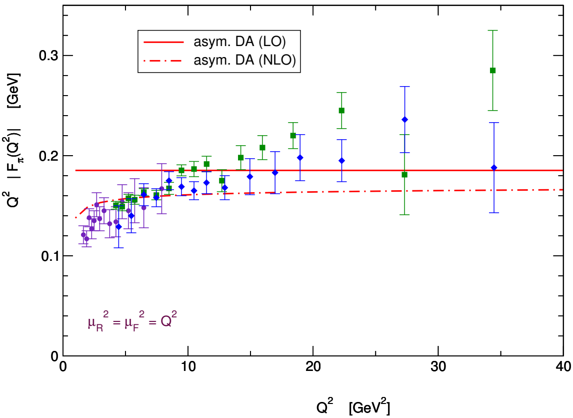


CLEO '97 (purple)

BABAR '09 (green)

Experimental situation for pion tff

CLEO '97, BABAR '09, BELLE '12

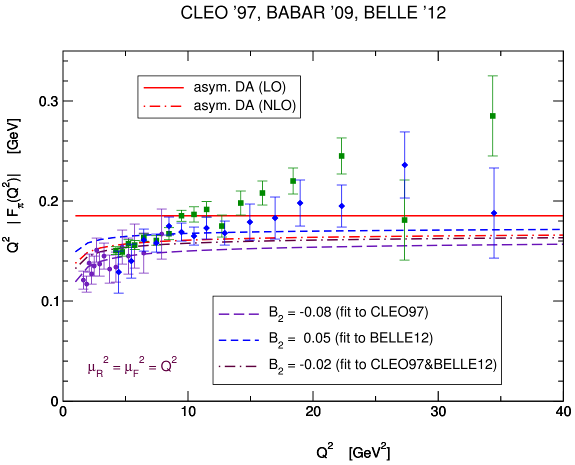


CLEO '97 (purple)

BABAR '09 (green)

BELLE '12 (blue)

Example fits

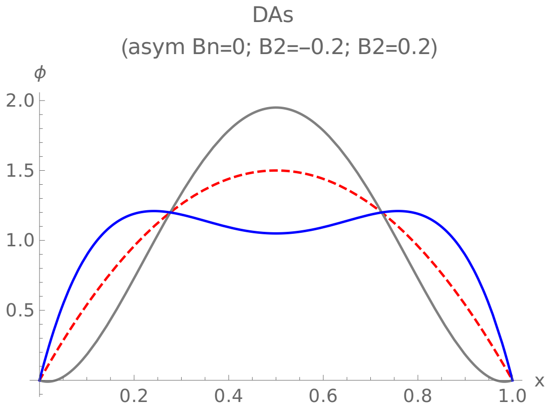


CLEO '97 (purple)

BABAR '09 (green)

BELLE '12 (blue)

DAs



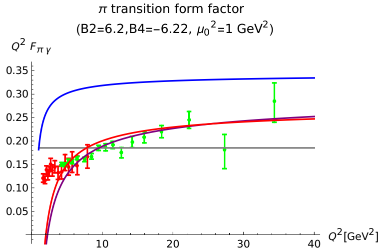
$B_i = 0$ (asym.) ... red

$B_2(1\text{GeV}^2) = -0.2$ ... gray

$B_2(1\text{GeV}^2) = 0.2$ ... blue

What about BaBar '09 data?

Example:

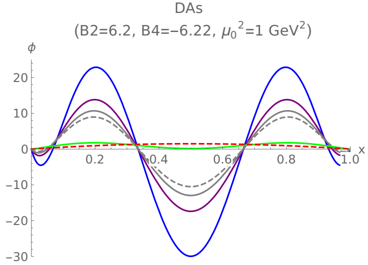


$B_2(1\text{GeV}^2) = 6.2$

$B_4(1\text{GeV}^2) = -6.22$

LO (blue)

NLO (red, purple)



$\phi(1\text{GeV}^2)\ldots$ blue

$\phi(10\text{GeV}^2)\ldots$ purple

$\phi(100\text{GeV}^2)\ldots$ grey

$\phi(1000\text{GeV}^2)\ldots$ dashed grey

$\phi(Q^2 \to \infty) = \phi_{\text{asy}}\ldots$ dashed red

On literature...

Impact of tff results on literature:

- Round 1:

  a number of papers trying to accommodate BABAR '09 results, eg.:

  - flat DA [Radyuskin '09, Polyakov '09]
  - DA from light cone sum rules with $B_4 > B_2$ and large B_6 [Agaev, Braun, Offen, Porkert '10],
  - DA in modified approach [Kroll '11]
  - ...

  or not [Brodsky, Cao, Teramond '11] ...

- Round 2: no definitive proof (neither from experimental nor theoretical side) but BELLE '12 results favoured in the literature

On fits...

How much can the fits tell us?

$$Q^2 F_{\pi\gamma}(Q^2) = 6 f_\pi C_\pi \left[1 + \sum_{n=2}^{\infty}{}' B_n(Q^2) + \frac{\alpha_S(Q^2)}{\pi} \left(-1.667 + \sum_{n=2}^{\infty}{}' T_n^{NLO}\, B_n(Q^2) \right) \right]$$

On fits...

How much can the fits tell us?

$$\stackrel{t=\alpha_S(Q^2)}{=} \quad 6 f_\pi C_\pi \left[1 + \sum_{n=2}^{\infty}{}' B_n(\mu_0^2) \left(\frac{t}{t_0}\right)^{-\gamma_n/\beta_0} + \frac{t}{\pi} \left(-1.667 + \sum_{n=2}^{\infty}{}' T_n^{NLO} \, B_n(\mu_0^2) \left(\frac{t}{t_0}\right)^{-\gamma_n/\beta_0} \right) \right]$$

- fractional polynomial in $t = \alpha_S(Q^2)$ variable
- $0.4 > t > 0.2(0.26)$ for $2 < Q^2 < 34(8)$GeV^2

$\Rightarrow$ large correlations for B_ns

$$\Downarrow$$

- one (effective) coefficient (B_2) to be determined

On fits...

How much can the fits tell us?

We can rewrite the tff contribution in terms of a linear fit:

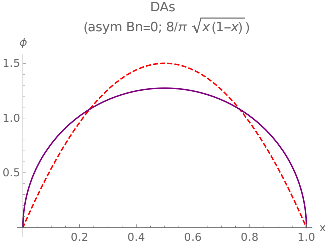
$$F(X(t)) = B_2 + B_4\, X(t)$$

data range $(0.9 < X(t) < 1.2)$ is narrow and away from the ordinate

$\Rightarrow$ strong correlation between B_2 and B_4

Beyond the truncation of the Gegenbauer series

- from AdS/QCD [Brodsky, Teramond '08]:



$B_i = 0$ (asym.) ...red

$\phi_\pi = \frac{8}{\pi}\sqrt{x(1-x)}$ ...purple

- summing the series...

Inspiration by DVCS calculation (conformal moment series representation) [D. Müller '05, talk by K. Kumerički]

- factorization formula in momentum fraction space (x-space):

$$F_{M\gamma}(Q^2;\mu_R^2) = \int \mathrm{d}x \;\; T_H(x, Q^2, \mu_R^2, \mu_F^2)\; \Phi(x, \mu_F^2)$$

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- ...in terms of conformal moments (n-space)

(analogous to Mellin moments in DIS: $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$):

$$= \sum_{n=0}^{\infty} T_n(Q^2,\mu_R^2,\mu_F^2)\; B_n(\mu_F^2)$$

$$T_n(Q^2,\mu_R^2,\mu_F^2)) = \int_0^1 x(1-x) C_n^{3/2}(2x-1)\; T_H(x,Q^2,\mu_R^2,\mu_F^2)$$

Inspiration by DVCS calculation (conformal moment series representation) [D. Müller '05, talk by K. Kumerički]

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$$= \sum_{n=0}^{\infty} T_n(Q^2, \mu_R^2, \mu_F^2) \; B_n(\mu_F^2)$$

$$T_n(Q^2, \mu_R^2, \mu_F^2)) = \int_0^1 x(1-x) C_n^{3/2}(2x-1) \; T_H(x, Q^2, \mu_R^2, \mu_F^2)$$

- series summed using Mellin-Barnes integral over complex n:

$$= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dn \left[i + \tan\left(\frac{\pi n}{2}\right)\right] T_n(Q^2, \mu_R^2, \mu_F^2) \; B_n(\mu_F^2)$$

$\Rightarrow$ complete summation (inclusion of NLO evolution easier...) but modeling of meson DA coefficients $B_n(\mu_F^2)$ needed

η, η' transition form factors

Valence Fock components of $M = \eta, \eta'$:

$$|q\bar{q}_8\rangle = |(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}\rangle \quad \text{(flavour-octet)}$$

$$\left\{ \begin{array}{l} |q\bar{q}_1\rangle = |(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}\rangle \quad \text{(flavour-singlet)} \\ |gg\rangle \end{array} \right.$$

Novel features:

1. flavour-mixing (singlet-octet mixing)
2. $|gg\rangle$ states contribute

   $\Rightarrow$ mixing of $q\bar{q}_1$ and gg DAs under evolution

1. flavour-mixing [review Feldman '00]

$\downarrow$

simplest possibility: to take particle dependence and the flavour-mixing to be solely embedded in the decay constants f_M^i

- $\boxed{\Phi_{Mi} = f_M^i \, \phi_i}$ $i \in \{1, 8\}$
- the decay constants parametrized as

$$f_\eta^8 = f_8 \cos\theta_8 \qquad f_\eta^1 = -f_1 \sin\theta_1$$

$$f_{\eta'}^8 = f_8 \sin\theta_8 \qquad f_{\eta'}^1 = f_1 \cos\theta_1$$

[Leutwyler '98, Feldman, Kroll, Stech, '98,'99]

2. $|gg\rangle$ states contribute

$\Rightarrow$ mixing of $q\bar{q}_1$ and gg DAs under evolution

$$(\Phi_{M1} \equiv \Phi_{Mq}) \quad (\Phi_{Mg})$$

$$\downarrow$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix} = \begin{pmatrix} V_{qq} & V_{qg} \\ V_{gq} & V_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix}$$

$$\Phi_{Mq} = f_{M1}\,\phi_{Mq}\,, \qquad \Phi_{Mg} = f_{M1}\,\phi_{Mg}$$

$$\int_0^1 dx\,\phi_{Mq}(x,\mu_F^2) = 1 \quad \Leftarrow \quad \phi_{Mq}(x,\mu_F^2) = \phi_{Mq}(1-x,\mu_F^2)$$

$$\int_0^1 dx\,\phi_{Mg}(x,\mu_F^2) = 0 \quad \Leftarrow \quad \phi_{Mg}(x,\mu_F^2) = -\phi_{Mg}(1-x,\mu_F^2)$$

$$\phi_{Mq}(x,\mu_F^2) = 6x(1-x)\left[1 + \sum_{n=2}^{\infty}{}' B^q_{Mn}(\mu_F^2)\; C_n^{3/2}(2x-1)\right]$$

$$\phi_{Mg}(x,\mu_F^2) = x^2(1-x)^2 \sum_{n=2}^{\infty}{}' B^g_{Mn}(\mu_F^2)\; C_{n-1}^{5/2}(2x-1)$$

$$B^q_{Mn}(\mu_F^2) = f\left(\underline{B^q_{Mn}(\mu_0^2)}, \underline{B^g_{Mn}(\mu_0^2)};\ \alpha_S(\mu_F^2),\ \gamma_n^{ij}\right)$$

$$B^g_{Mn}(\mu_F^2) = g\left(\underline{B^g_{Mn}(\mu_0^2)}, \underline{B^q_{Mn}(\mu_0^2)};\ \alpha_S(\mu_F^2),\ \gamma_n^{ij}\right)$$

$\mu_0^2 \ldots$ initial scale

$$F_{M\gamma}(Q^2) = F^{8}_{M\gamma}(Q^2) + F^{1g}_{M\gamma}(Q^2)$$

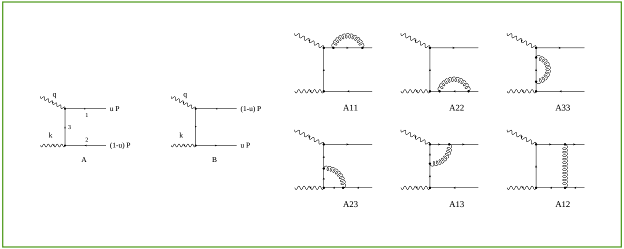
$$(|q\bar{q}_8\rangle) \qquad (|q\bar{q}_1\rangle, |gg\rangle)$$

$$F^{1g}_{M\gamma}(Q^2) = \left(T_{H,q\bar{q}}(x,Q^2,\mu_F^2) \quad T_{H,gg}(x,Q^2,\mu_F^2)\right) \otimes \begin{pmatrix} \Phi_{Mq}(x,\mu_F^2) \\ \Phi_{Mg}(x,\mu_F^2) \end{pmatrix}$$

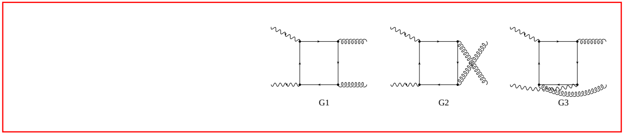
$$\gamma^*\gamma \to q\bar{q}_1 \quad \Rightarrow \quad T_{q\bar{q}}$$

$$\gamma^*\gamma \to gg \quad \Rightarrow \quad T_{gg}$$

$T_{q\bar{q}}$ LO NLO



T_{gg}



On fits...

How much can the fits tell us?

- We can rewrite the flavour-octet contribution in terms of a linear fit:

$$F_8(X_8(t)) = B_2^8 + B_4^8\ X_8(t)$$

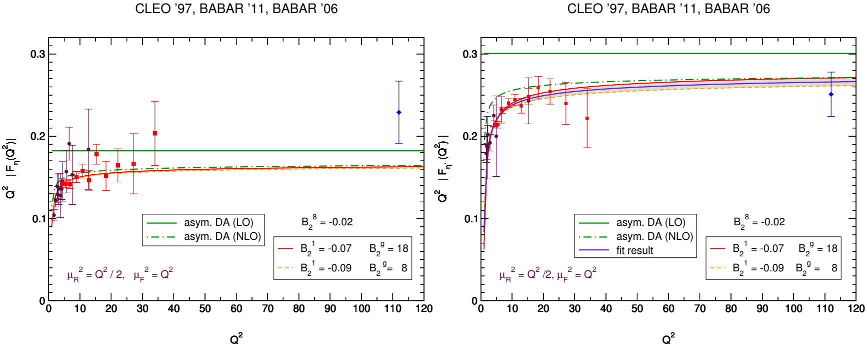
data range $0.9 < X_8(t) < 1.2$ is narrow and away from the ordinate
$\Rightarrow$ strong correlation between B_2^8 and B_4^8,
only effective B_2^8 to be determined

- We can rewrite the flavour-singlet contribution in terms of a linear fit:

$$F_1(X_1(t)) = B_2^q + B_2^g\ X_1(t)$$

data range $-0.0029 < X_1(t) < 0.032$ (for $2 < Q^2 < 34\text{Gev}^2$) is near the ordinate $\Rightarrow$ moderate correlation between B_2^q and B_2^g,
both B_2^q and B_2^g to be determined

Experimental situation and fits for η and η' tff



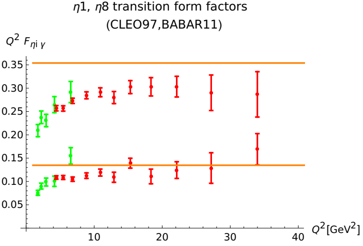
CLEO '97 (purple)

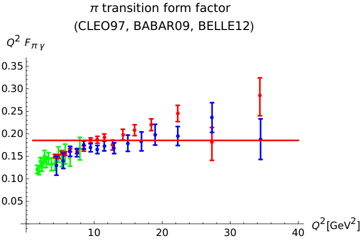
BABAR '11 (red)

BABAR '06 (blue) . . . timelike! (not used in fits)

[Kroll, KPK '13]

Flavour singlet and flavour nonsinglet analysis

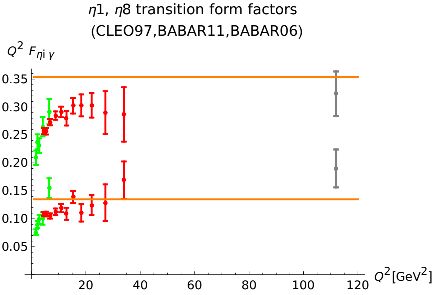




CLEO '97 (green)

BABAR '09,
BABAR '11 (red)

BELLE '12 (blue)



CLEO '97 (green)

BABAR '11 (red)

BELLE '06 (gray)

. . . timelike!

Scalar transition form factor

- BELLE 2015 data on the scalar $f_0(980)$ meson transition form factor

- [Kroll '16]:
  - $f_0(980)$ considered as mainly $s\bar{s}$ state
  - LO contribution in modified approach with quark transverse moment effects included
  - $\gamma^*\gamma^* \to M$ discussed

    $\downarrow$

    NOTE: theoretically clean case which provides information on separate B_n depending on the difference of photon virtualities; experimentally difficult

- we discuss here the contribution up to NLO in standard hard-scattering approach with gluons included (NLO taken from DVCS)

- PSEUDOSCALAR CASE:

$$\begin{cases} \phi_{Pq}(x,\mu_F^2) = 6x(1-x)\left[1 + \sum_{n=2}^{\infty}{}' B_{Pn}^{q}(\mu_F^2)\, C_n^{3/2}(2x-1)\right] \\ \phi_{Pg}(x,\mu_F^2) = 30x^2(1-x)^2 \sum_{n=2}^{\infty}{}' B_{Pn}^{g}(\mu_F^2)\, C_{n-1}^{5/2}(2x-1) \end{cases}$$

- SCALAR CASE:

$$\begin{cases} \phi_{Sq}(x,\mu_F^2) = 6x(1-x)\left[B_{S1}^{q}(\mu_F^2)\, C_1^{3/2}(\ldots) + \sum_{n=3}^{\infty}{}' B_{Sn}^{q}(\mu_F^2)\, C_n^{3/2}(\ldots)\right] \\ \phi_{Sg}(x,\mu_F^2) = 30x^2(1-x)^2 \left[B_{S1}^{g}(\mu_F^2) + \sum_{n=3}^{\infty}{}' B_{Sn}^{g}(\mu_F^2)\, C_{n-1}^{5/2}(2x-1)\right] \end{cases}$$

PSEUDOSCALAR CASE:

$$\begin{cases} \phi_{Pq}(x,\mu_F^2) = 6x(1-x)\left[1 + \sum_{n=2}^{\infty}{}' B_{Pn}^{q}(\mu_F^2)\, C_n^{3/2}(2x-1)\right] \\ \phi_{Pg}(x,\mu_F^2) = 30x^2(1-x)^2 \sum_{n=2}^{\infty}{}' B_{Pn}^{g}(\mu_F^2)\, C_{n-1}^{5/2}(2x-1) \end{cases}$$

$$\begin{matrix} \int_0^1 dx\ \phi_{Pq}(x,\mu_F^2) = 1 \\ \int_0^1 dx\ \phi_{Pg}(x,\mu_F^2) = 0 \end{matrix}\ , \quad \mu_F^2 \to \infty: \begin{matrix} \phi_{Pq} \to 6x(1-x) \\ \phi_{Pg} \to 0 \end{matrix} \qquad \gamma_0^{qq} = 0!$$

SCALAR CASE:

$$\begin{cases} \phi_{Sq}(x,\mu_F^2) = 6x(1-x)\left[B_{S1}^{q}(\mu_F^2)\, C_1^{3/2}(\ldots) + \sum_{n=3}^{\infty}{}' B_{Sn}^{q}(\mu_F^2)\, C_n^{3/2}(\ldots)\right] \\ \phi_{Sg}(x,\mu_F^2) = 30x^2(1-x)^2 \left[B_{S1}^{g}(\mu_F^2) + \sum{}' B_{Sn}^{g}(\mu_F^2)\, C_{n-1}^{5/2}(2x-1)\right] \end{cases}$$

$$\begin{matrix} \int_0^1 dx\ \phi_{Sq}(x,\mu_F^2) = 0 \\ \int_0^1 dx\ \phi_{Sg}(x,\mu_F^2) = B_{S1}^{g}(\mu_F^2) \end{matrix}\ , \quad \mu_F^2 \to \infty: \begin{matrix} B_{S1}^{q} \to B_{S1}^{+} \\ B_{S1}^{g} \to \rho_1^{+} B_{S1}^{+} \end{matrix} \qquad \gamma_1^{+} = 0!$$

- SCALAR CASE:

$$\begin{cases} \phi_{Sq}(x,\mu_F^2) = 6x(1-x)\left[B^q_{S1}(\mu_F^2)\, C_1^{3/2}(\dots) + \sum_{n=3}^{\infty}{}' B^q_{Sn}(\mu_F^2)\, C_n^{3/2}(\dots)\right] \\ \phi_{Sg}(x,\mu_F^2) = 30x^2(1-x)^2\left[B^g_{S1}(\mu_F^2) + \sum_{n=3}^{\infty}{}' B^g_{Sn}(\mu_F^2)\, C_{n-1}^{5/2}(2x-1)\right] \end{cases}$$

$$\int_0^1 dx\ \phi_{Sq}(x,\mu_F^2) = 0 \qquad \int_0^1 dx\ \phi_{Sg}(x,\mu_F^2) = B^g_{S1}(\mu_F^2)\,, \quad \mu_F^2 \to \infty: \ B^q_{S1} \to B^+_{S1} \quad B^g_{S1} \to \rho_1^+ B^+_{S1} \qquad \gamma_1^+ = 0!$$

$$B^q_{Mn}(\mu_F^2) = B^+_{Mn}(\mu_F^2) + \rho_n^-\, B^-_{Mn}(\mu_F^2)$$

$$B^g_{Mn}(\mu_F^2) = \rho_n^+\, B^+_{Mn}(\mu_F^2) + B^-_{Mn}(\mu_F^2)$$

$$B^\pm_{Mn}(\mu_F^2) = B^\pm_{Mn}(\mu_0^2)\left(\frac{\alpha_S(\mu_0^2)}{\alpha_S(\mu_F^2)}\right)^{\gamma_n^\pm/\beta_0} \qquad \gamma^\pm_{n>1} < 0$$

$$B^+_{S1}(\mu_F^2) = B^+_{S1} \qquad (\rho_1^+ > 1)$$

$\gamma_1^{qq} < 0$ and for octet (flavour-nonsinglet) meson $B^q_{S1} \to 0$ for $\mu_F^2 \to \infty$

SCALAR CASE:

$$\begin{cases} \phi_{Sq}(x,\mu_F^2) = 6x(1-x)\left[B^q_{S1}(\mu_F^2)\, C_1^{3/2}(\ldots) + \sum_{n=3}^{\infty}{}' B^q_{Sn}(\mu_F^2)\, C_n^{3/2}(\ldots)\right] \\ \phi_{Sg}(x,\mu_F^2) = 30x^2(1-x)^2\left[B^g_{S1}(\mu_F^2) + \sum_{n=3}^{\infty}{}' B^g_{Sn}(\mu_F^2)\, C_{n-1}^{5/2}(2x-1)\right] \end{cases}$$

$$\int_0^1 dx\ \phi_{Sq}(x,\mu_F^2) = 0, \quad \int_0^1 dx\ \phi_{Sg}(x,\mu_F^2) = B^g_{S1}(\mu_F^2)\ , \qquad \mu_F^2 \to \infty:\ \begin{array}{l} B^q_{S1} \to B^+_{S1} \\ B^g_{S1} \to \rho_1^+ B^+_{S1} \end{array} \qquad \gamma_1^+ = 0!$$

$$B^q_{Mn}(\mu_F^2) = B^+_{Mn}(\mu_F^2) + \rho_n^-\, B^-_{Mn}(\mu_F^2)$$

$$B^g_{Mn}(\mu_F^2) = \rho_n^+\, B^+_{Mn}(\mu_F^2) + B^-_{Mn}(\mu_F^2)$$

$$B^{\pm}_{Mn}(\mu_F^2) = B^{\pm}_{Mn}(\mu_0^2)\left(\frac{\alpha_S(\mu_0^2)}{\alpha_S(\mu_F^2)}\right)^{\gamma_n^{\pm}/\beta_0} \qquad \gamma^{\pm}_{n>1} < 0$$

$\Rightarrow$ natural choice (decay constant unknown):

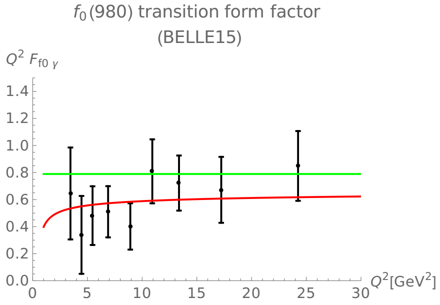
$$\begin{array}{l} B^+_{S1} = 1/\rho_1^+ = 5/6 \\ B^-_{S1}(\mu_F^2) = 0 \end{array} \Rightarrow \begin{array}{l} B^q_{S1}(\mu_F^2) = 1/\rho_1^+ = 5/6 \\ B^g_{S1}(\mu_F^2) = 1 \end{array}$$

$$
Q^2 F_{M\gamma}^{1(\underline{8})}(Q^2) = 6\, f_M^{1(\underline{8})}\, C_{1(\underline{8})} \left[\begin{array}{c} \underline{1 + \sum_{n=2}^{\infty}{}' B_n^q(Q^2)} \quad \to \text{PS} \\ \underline{B_1^q(Q^2) + \sum_{n=3}^{\infty}{}' B_n^q(Q^2)} \quad \to \text{S} \end{array} \right.
$$

$$
+ \frac{\alpha_S(\mu_R^2)}{\pi} \left(\begin{array}{c} \underline{-1.667 + \sum_{n=2}^{\infty}{}' T_{NLO,n}^q\, B_n^q(Q^2)} \\ \underline{-0.037\, B_1^q(Q^2) + \sum_{n=3}^{\infty}{}' T_{NLO,n}^q\, B_n^q(Q^2)} \end{array} \right)
$$

$$
\left. + \frac{\alpha_S(\mu_R^2)}{\pi} \left(\begin{array}{c} 0 + \sum_{n=2}^{\infty}{}' T_{NLO,n}^g\, B_n^g(Q^2) \\ -2.655\, B_1^g(Q^2) + \sum_{n=3}^{\infty}{}' T_{NLO,n}^g\, B_n^g(Q^2) \end{array} \right) \right]
$$

$f_0(980)$ transition form factor (experiment and fits)



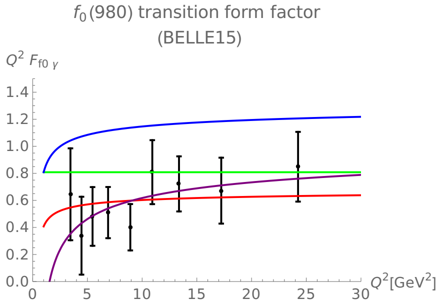
BELLE '15 (black)

LO (green)

NLO (red)

$f_1 = 0.41 \pm 0.05$ GeV
$B_q(\mu^2) = 5/6,\ B_g(\mu^2) = 1$

$f_0(980)$ transition form factor (experiment and fits)



BELLE '15 (black)

LO (green)

NLO (red)

$f_1 = 0.41 \pm 0.05$ GeV

$B_q(\mu^2) = 5/6,\ B_g(\mu^2) = 1$

LO (blue)

NLO (purple)

$f_1 = 0.42 \pm 0.05$ GeV

$B_q(\mu^2) = 5/6$

$B_g(1\text{GeV}^2) = 2.82 \pm 1.66$

More informations on light-meson DAs

- Higher twist contributions:
  ex. twist-3 contributions crucial for wide-angle photoproduction of pions [talk by Peter Kroll]
- DA coefficients from QCD sum rules
- Lattice results [talk by Fabian Hutzler]
- ...

Conclusions, outlook...

- at the moment two contradicting sets of experimental data exist for the simplest exclusive process (pion transition form factor $\gamma^*\gamma \to \pi$)
- the question of the form of pion DA can be thus considered still open
- fits inconclusive and cannot be improved significantly but should be tested on other processes (pion em form factor)
- $\gamma^*\gamma^* \to M$ offer more info but experimentally difficult
- η, η' transition form factor reexamined and results on the scalar meson $f_0(980)$ obtained
- a number of recent proposals for pion DA in the literature
- BELLE II data (2018?) expected to shed more light

Conclusions, outlook...

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Thank you!

Factorization scale dependence

$$F_{\pi\gamma}(Q^2) = T_H(x, Q^2, \mu_F^2) \otimes \Phi(x, \mu_F^2)$$

DISTRIBUTION AMPLITUDE DEPENDENCE ON μ_F^2: $\mu_F^2 \frac{\partial}{\partial \mu_F^2} \Phi = V \otimes \Phi$

$$\Phi(x, \mu_F^2) = \phi_V(x, y, \mu_F^2, \mu_0^2) \otimes \Phi(y, \mu_0^2)$$

T_H DEPENDENCE ON μ_F^2: $\mu_F^2 \frac{\partial}{\partial \mu_F^2} T_H = -T_H \otimes V$

$$T_H(x, Q^2, \mu_F^2) = T_H(x, Q^2, Q^2) \otimes \phi_V(x, y, Q^2, \mu_F^2)$$

$$\boxed{\phi_V(x, y, Q^2, \mu_F^2) \otimes \phi_V(x, y, \mu_F^2, \mu_0^2) = \phi_V(x, y, Q^2, \mu_0^2)}$$

resummation of $(\alpha_S \ln(\mu_F^2/\mu_0^2))^n$ terms in Φ and $(\alpha_S \ln(Q^2/\mu_F^2))^n$ terms in T_H $\Leftrightarrow$ resummation of $(\alpha_S \ln(Q^2/\mu_0^2))^n$

$\rightarrow$ no residual dependence on μ_F^2 in finite order calculations !
(for practical purposes, $\mu_F^2 = Q^2$ the simplest intermediate choice)

$\rightarrow$ but, we are now left with the ambiguity in choosing the characteristic scale (Q^2) of the process up to which the $\ln$ terms are resummed ! (problems: few scales processes etc.)

Renormalization scale choices

RENORMALIZATION SCALE SETTINGS:

- characteristic scale of the process: $\mu_R^2 = Q^2$
- FAC (fastest apparent convergence): $\mathcal{M}^{(2)}(Q^2, \mu_R^2) = 0$
  (Grunberg (1980))
- PMS (principle of minimum sensitivity): $\dfrac{d\mathcal{M}_{\text{finite order}}(Q^2, \mu_R^2)}{d\mu_R^2} = 0$
  (Stevenson (1981))
- BLM scheme: *(Brodsky, Lepage, Mackenzie (1983))*
  - vacuum polarization effects from the β function resummed into $\alpha_S(\mu_R^2)$

$$\mathcal{M}^{(2)}(Q^2, \mu_R^2) = \beta_0 \underbrace{\mathcal{M}^{(2,\beta_0)}(Q^2, \mu_R^2)}_{=0} + \mathcal{M}^{(2,\text{rest})}(Q^2)$$

♦ α_V scheme: $\alpha_{\overline{MS}}(\mu_{BLM}^2) = \alpha_V(\mu_V^2)\left(1 + \frac{\alpha_V(\mu_V^2)}{4\pi}\frac{8C_A}{3} + \cdots\right)$ *(Brodsky, Ji, Pang, Robertson (1998))*

$\alpha_V(\mu_V^2)\ldots$ defined from the heavy-quark potential $V(\mu_V^2)$

$\mu_V^2 = e^{5/3}\,\mu_{BLM}^2 \ldots$ expresses the mean gluon virtuality of the exchanged gluons

On fits...

How much can the fits tell us?

- We can rewrite the flavour-octet contribution in terms of a linear fit:

$$F_8(X_8(t)) = B_2^8 + B_4^8\ X_8(t)$$

data range $0.9 < X_8(t) < 1.2$ is narrow and away from the ordinate
$\Rightarrow$ strong correlation between B_2^8 and B_4^8,
only effective B_2^8 to be determined

- We can rewrite the flavour-singlet contribution in terms of a linear fit:

$$F_1(X_1(t)) = B_2^q + B_2^g\ X_1(t)$$

data range $-0.0029 < X_1(t) < 0.032$ (for $2 < Q^2 < 34\text{Gev}^2$) is near the ordinate $\Rightarrow$ moderate correlation between B_2^q and B_2^g,
both B_2^q and B_2^g to be determined