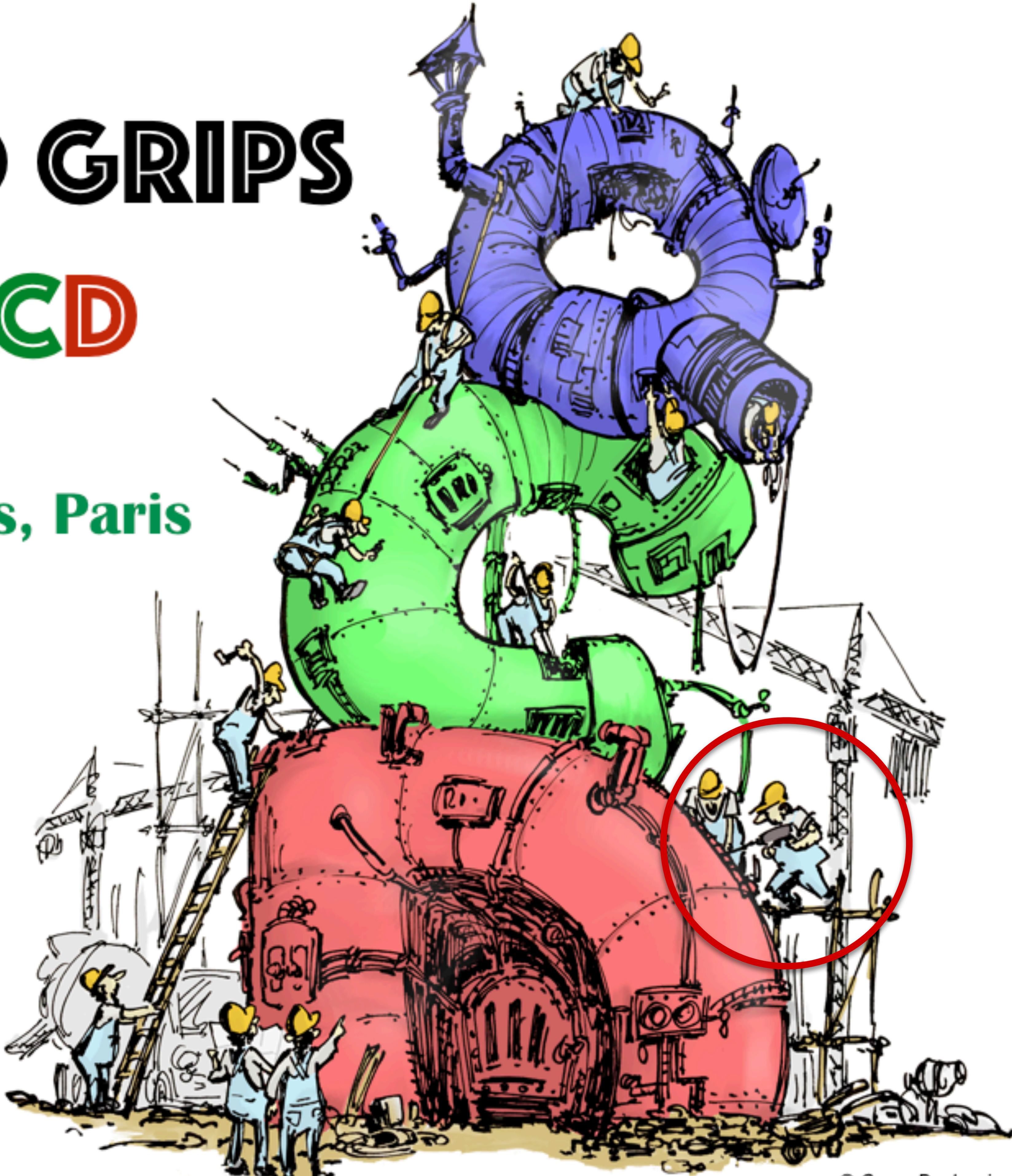
lattice QCD for precision flavour physics

Carlos Pena

GETTING TO GRIPS WITH QCD, Paris, 05/04/2018

GETTING TO GRIPS WITH QCD

Campus des Cordeliers, Paris
4-6 April 2018



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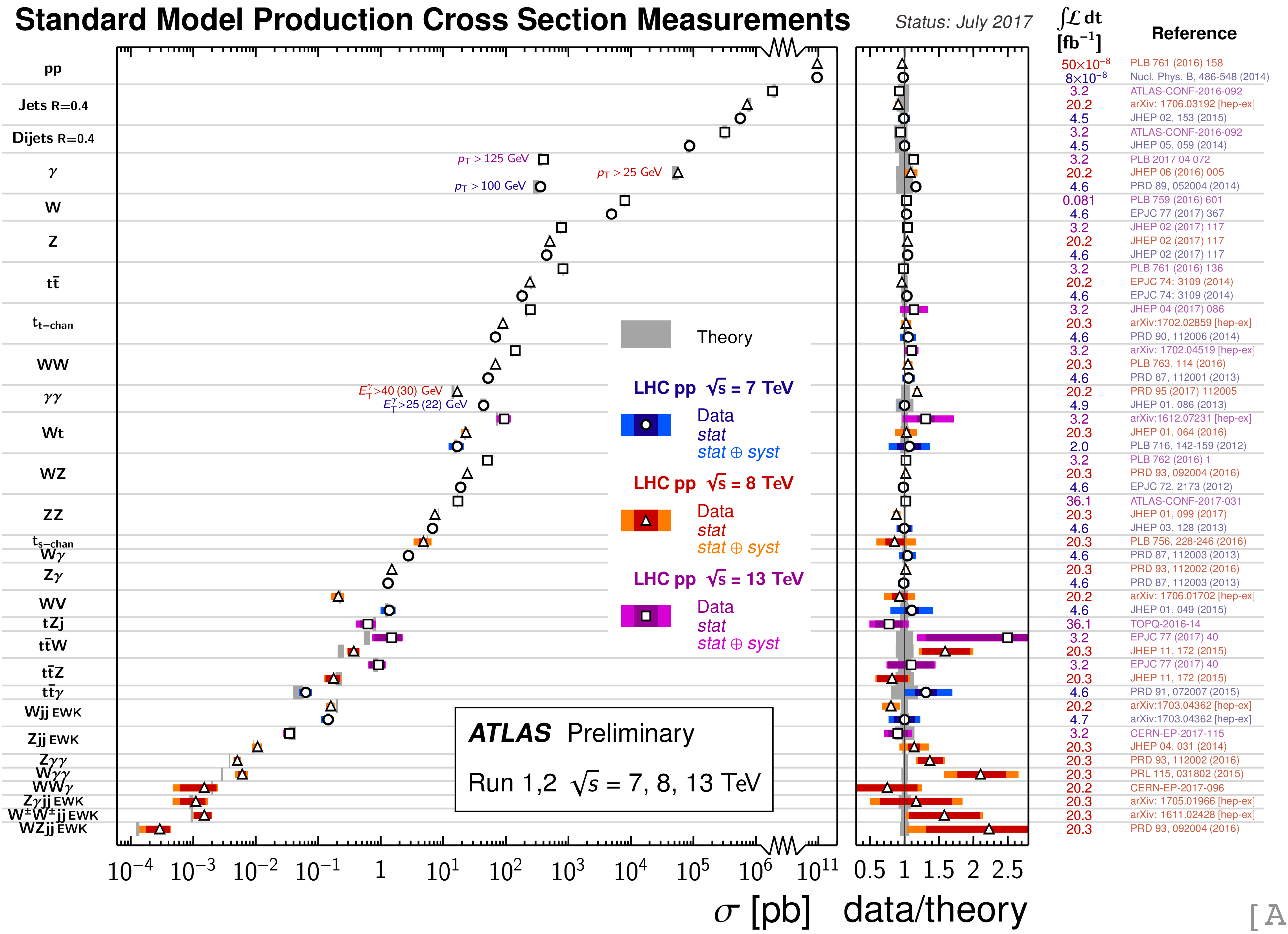
GDR QCD

INTENSITY frontier GDR-InF

why we care

- energy frontier has revealed the/a BEH + barren (?) land
  - exquisite control of SM predictions needed to dig up possible new Physics
  - hadronic sector: α_s, quark masses, …

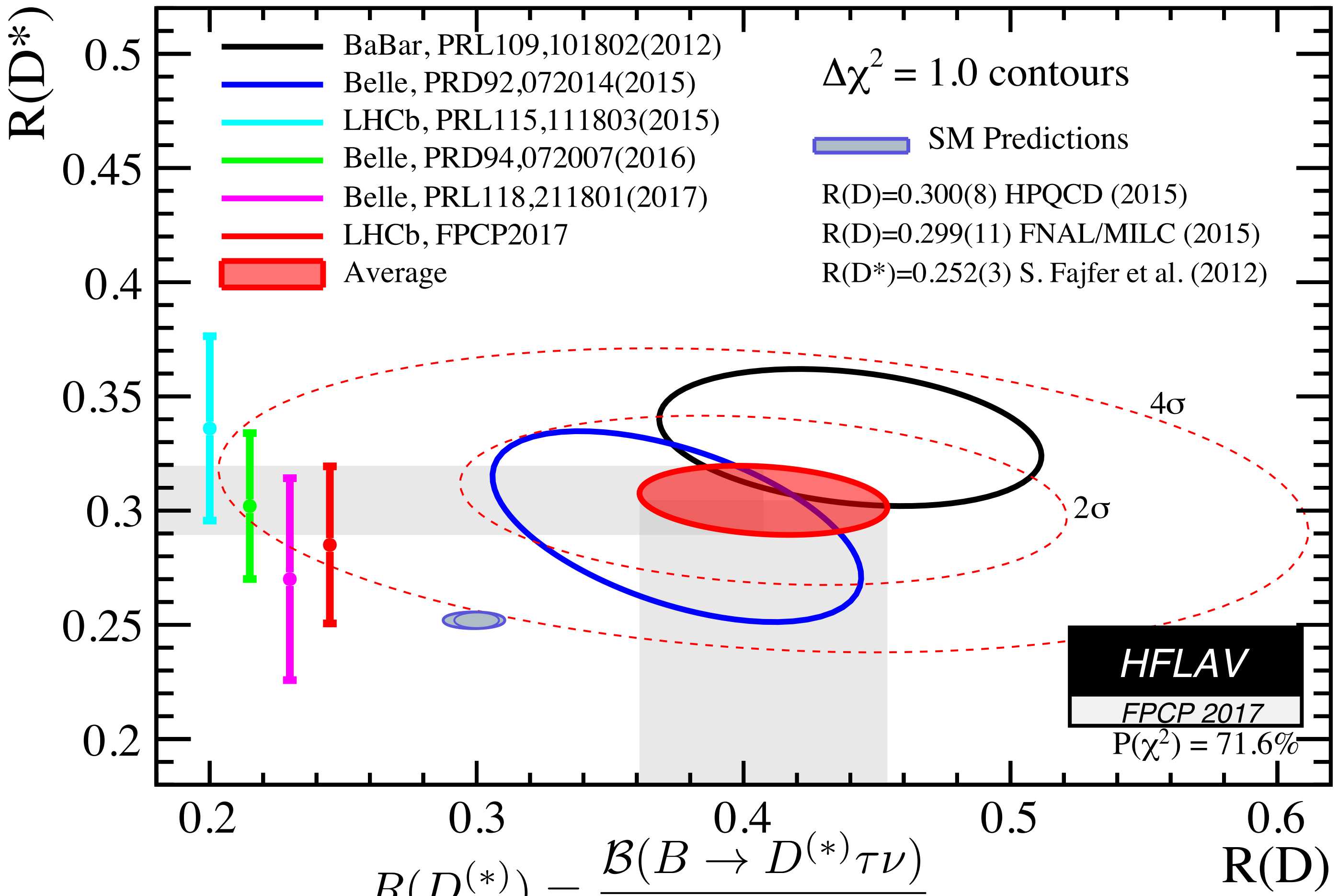


[ATLAS 2017]

why we care

- energy frontier has revealed the/a BEH + barren (?) land
  - exquisite control of SM predictions needed to dig up possible new Physics
  - hadronic sector: α_s, quark masses, ...

- intensity frontier
  - land of opportunity (LHCb, Belle II, (g-2) programme, nEDM, ...)
  - strong interaction effects key to attain necessary precision



$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\mu\nu)}$$

why we care

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  - hadronic sector: α_s, quark masses, ...

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  - strong interaction effects key to attain necessary precision

- this talk: focus on hadronic flavour physics, especially B decay
  - leptonic
  - tree-level semileptonic + R(D) + CKM
  - rare semileptonic, mixing

why we care

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- this talk: focus on hadronic flavour physics, especially B decay

`[G Salerno (next talk), D Mohler: charm physics]`

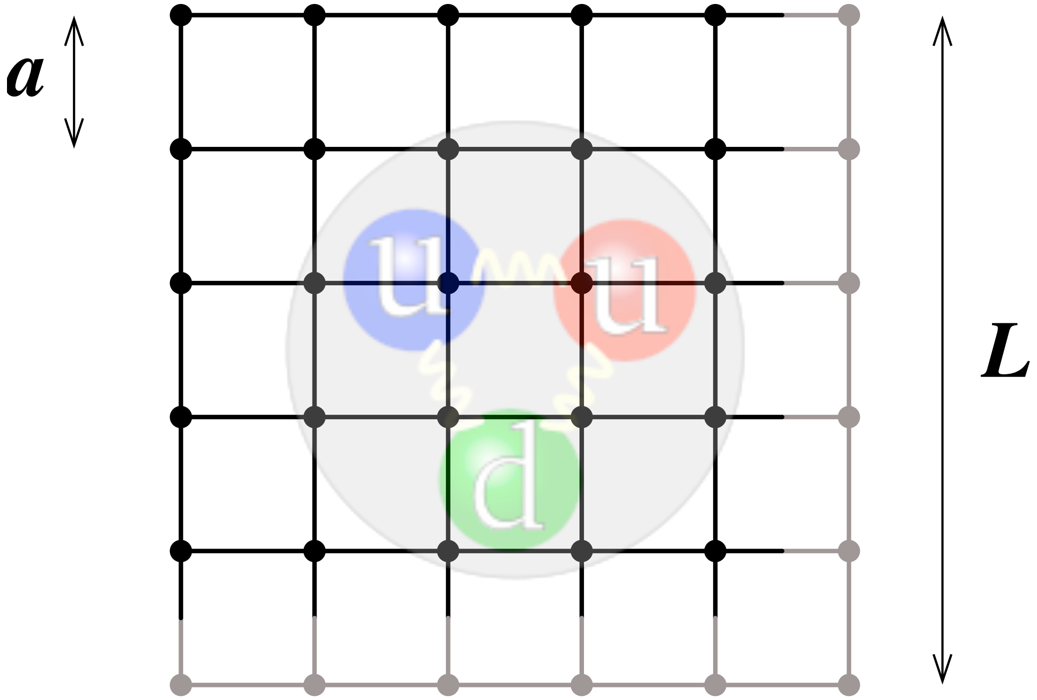
`[(`g`-2) talks by G Colangelo and A Gérardin]`
`[QED corrections by F Sanfilippo]`
`[meson DA by F Hutzler]`

`[related B-physics talks by M Fedele, P Colangelo, A Bharucha, D Leljak]`

lattice QCD

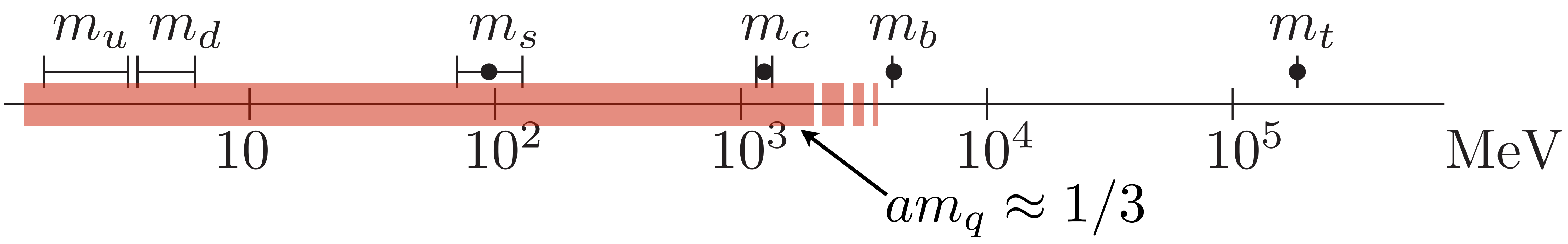
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2g^2}\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \sum_{q=1}^{N_\mathrm{f}} \bar{\psi}_q \left[i\not{D} - m_q\right]\psi_q + \underbrace{\frac{i\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left[F_{\mu\nu}F_{\rho\sigma}\right]}_{\not{C}P}$$

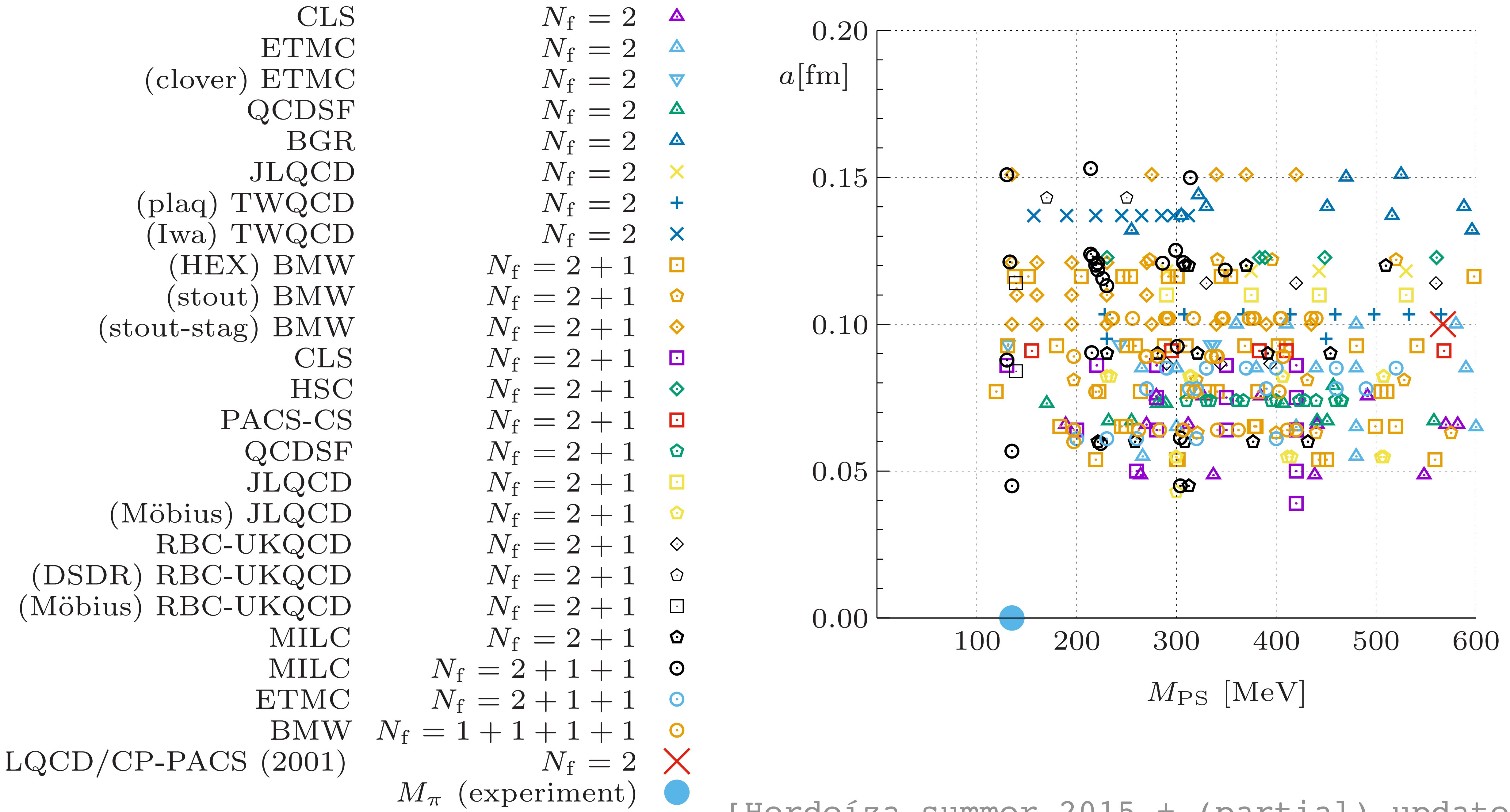
first-principles approach = control all systematic uncertainties



- spacetime = Euclidean lattice
- allows to define path integral rigorously *and* compute it via Monte Carlo methods
- QCD recovered by removing cutoffs at physical kinematics

physics reach

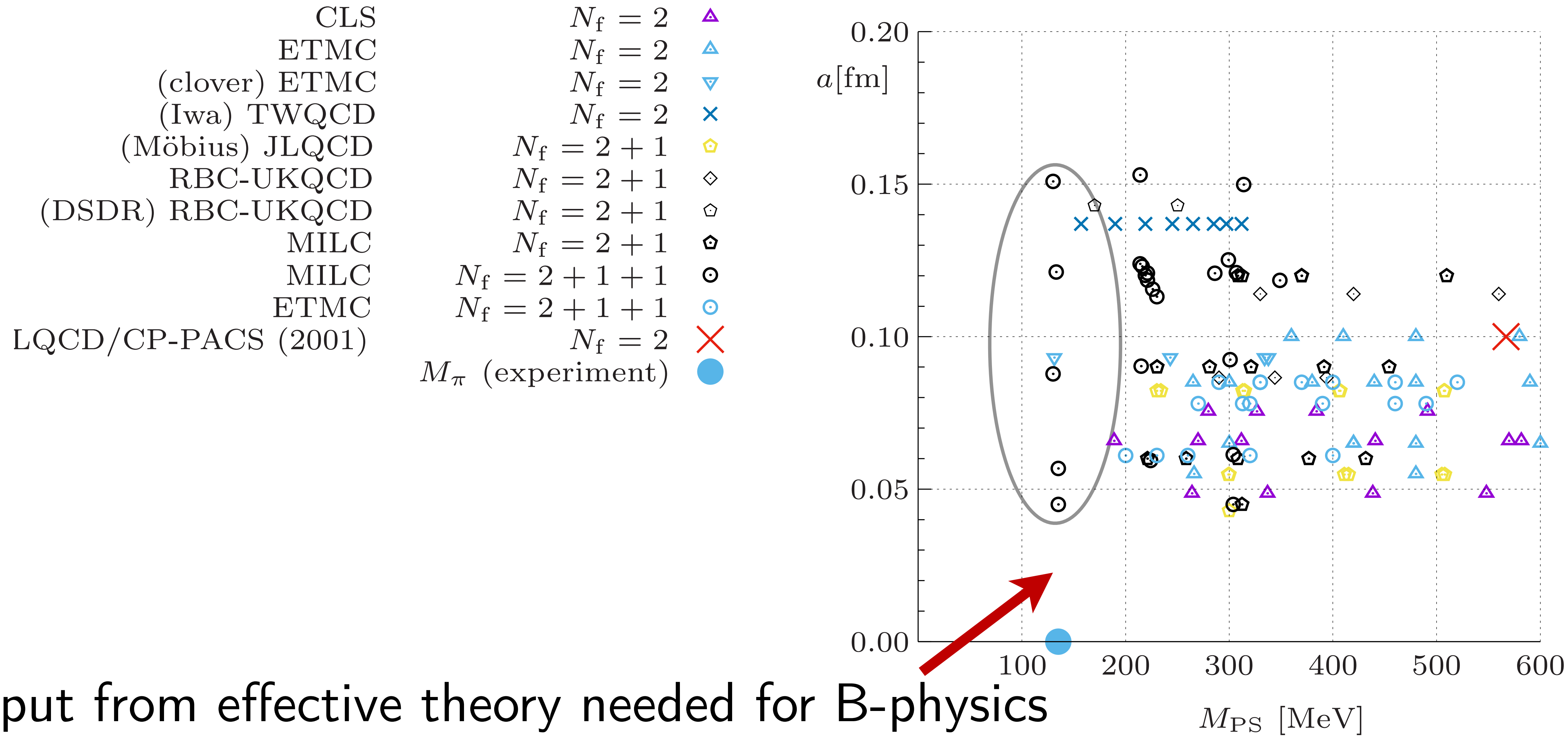




[Herdoíza summer 2015 + (partial) updates]

physics reach

- charm physics directly accessible for some time now
- fraction of available ensembles used for HQ physics still limited

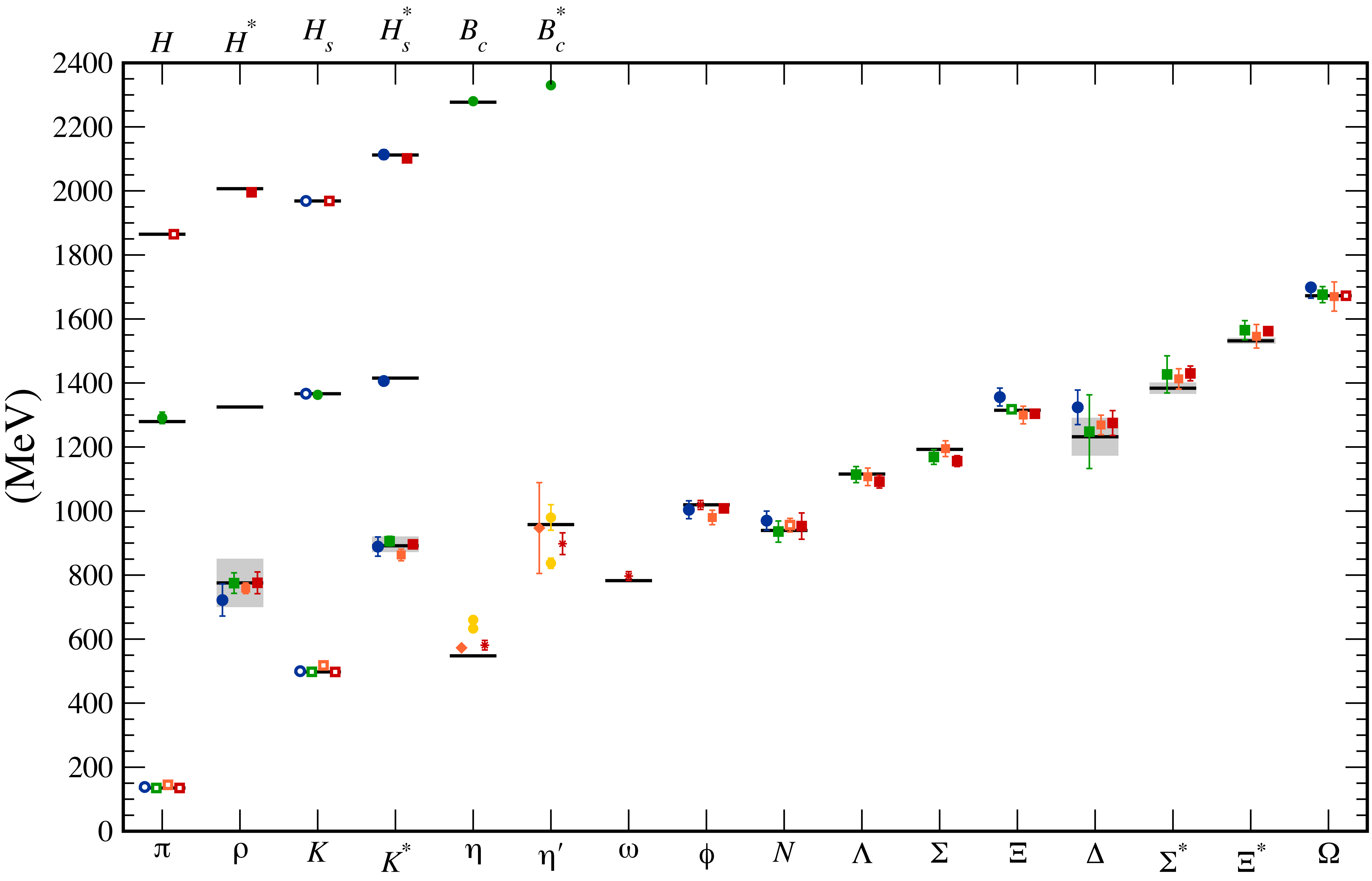


input from effective theory needed for B-physics

cross-validation between approaches crucial

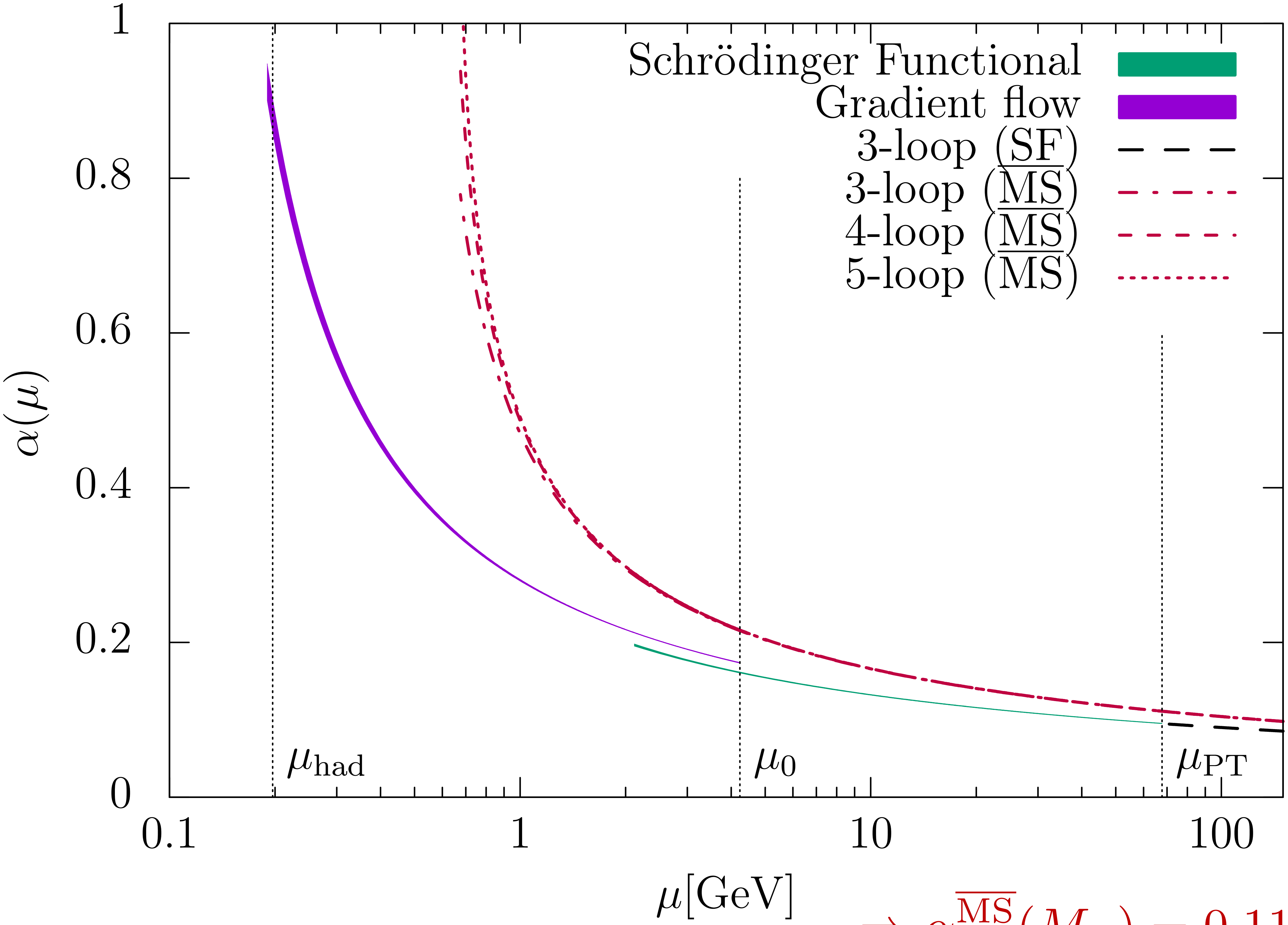
[Herdoíza summer 2015 + (partial) updates]

lattice QCD: validation



[Kronfeld, Annu. Rev. Nucl. Part. Sci. 62 (2012)]

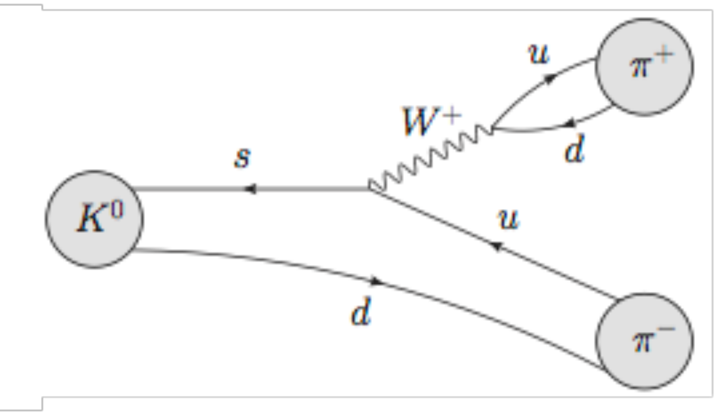
lattice QCD: state-of-the-art



$$\Rightarrow \alpha_{\mathrm{s}}^{\overline{\mathrm{MS}}}(M_Z) = 0.11852(84)$$

[ALPHA Collaboration, PRL 119 (2017) 102001]

lattice QCD: state-of-the-art



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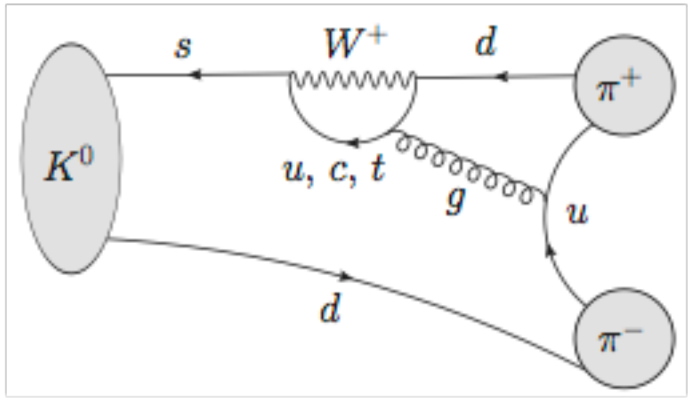

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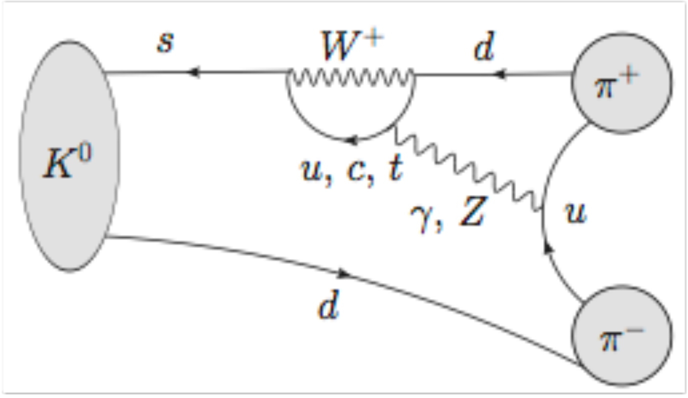

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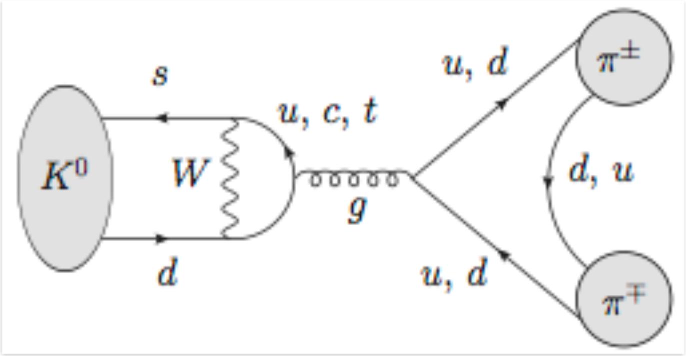

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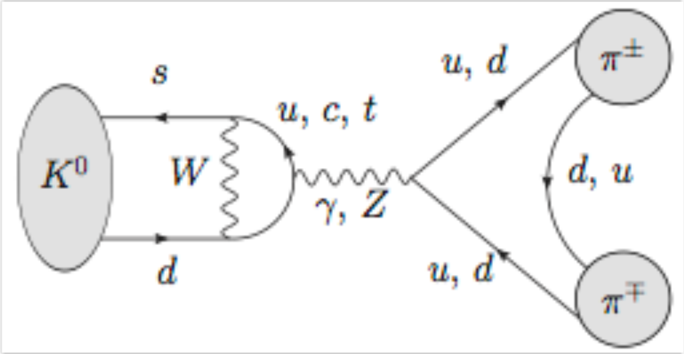

$+\cdots$

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{SM}} = 1.38(5.15)(4.43)\times 10^{-4}$$

cf. $$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{exp}} = 16.6(2.3)\times 10^{-4}$$

[RBC/UKQCD Collaboration, PRL 115 (2015) 212001]

lattice QCD for phenomenology: FLAG

Flavour Lattice Averaging Group: your one-stop repository of lattice results, world averages / estimates

3rd edition: results up to 30/11/2015 + updates from 2016

[Aoki et al., EPJC (2017) 77:112]

advisory board: S. Aoki, C. Bernard, H. Leutwyler, C. Sachrajda

editorial board: G. Colangelo, S. Hashimoto, A. Jüttner, S. Sharpe, A. Vladikas, U. Wenger

working groups:

quark masses	T. Blum, L. Lellouch, V. Lubicz
V_{ud}, V_{us}	P. Boyle, T. Kaneko, S. Simula
LECs	S. Dürr, H. Fukaya, U. Heller
kaon mixing	P. Dimopoulos, B. Mawhinney, H. Wittig
α_s	R. Horsley, T. Onogi, R. Sommer
heavy leptonic + mixing	Y. Aoki, M. Della Morte, D. Lin
heavy semileptonic	D. Bečirević, S. Gottlieb, E. Lunghi, CP

4th edition: early 2019, results up to Sep 2018

lattice QCD for phenomenology: FLAG

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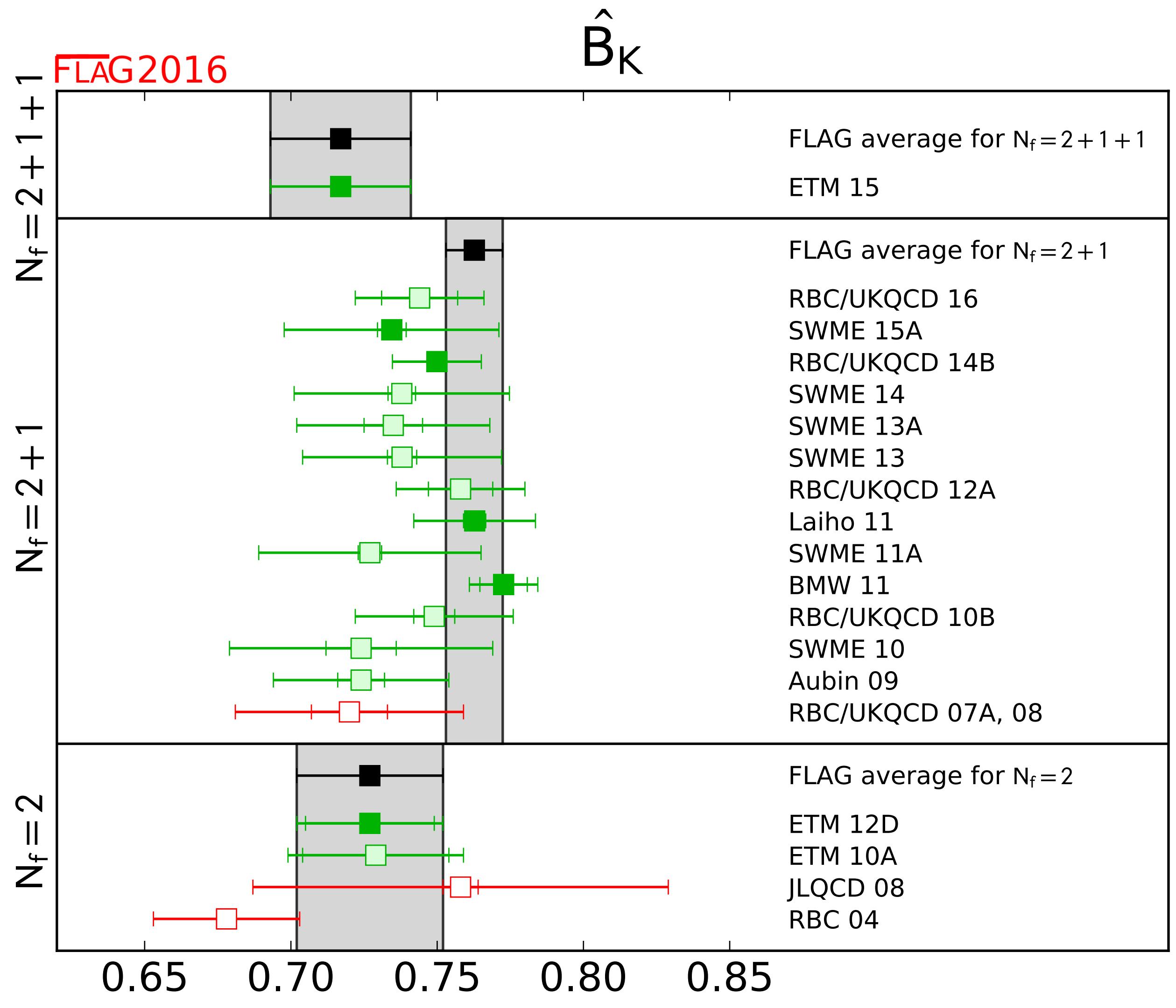

light quark physics: simple quantities at % precision level; start working hard on isospin+QED corrections etc.

lattice QCD for phenomenology: FLAG

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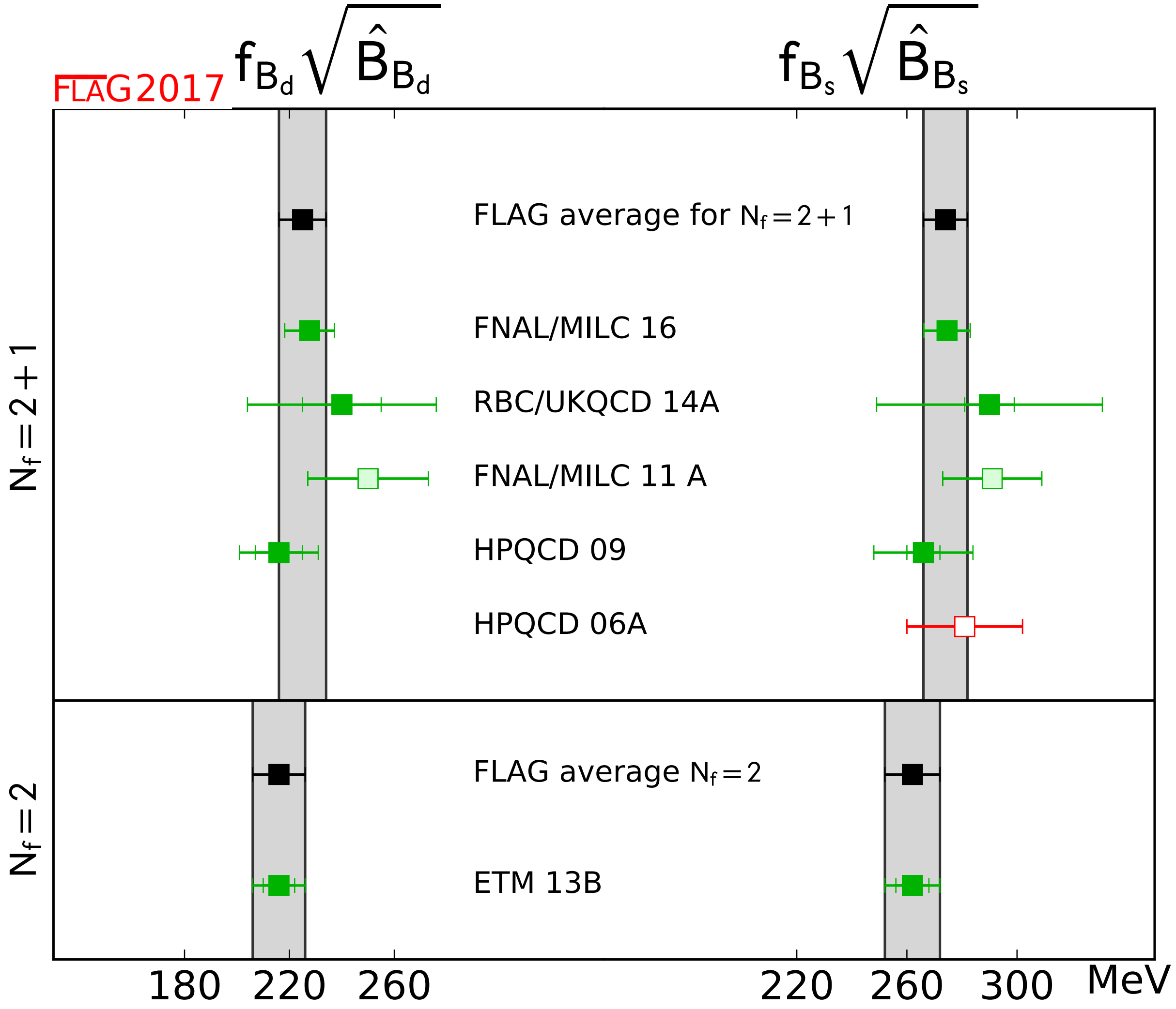

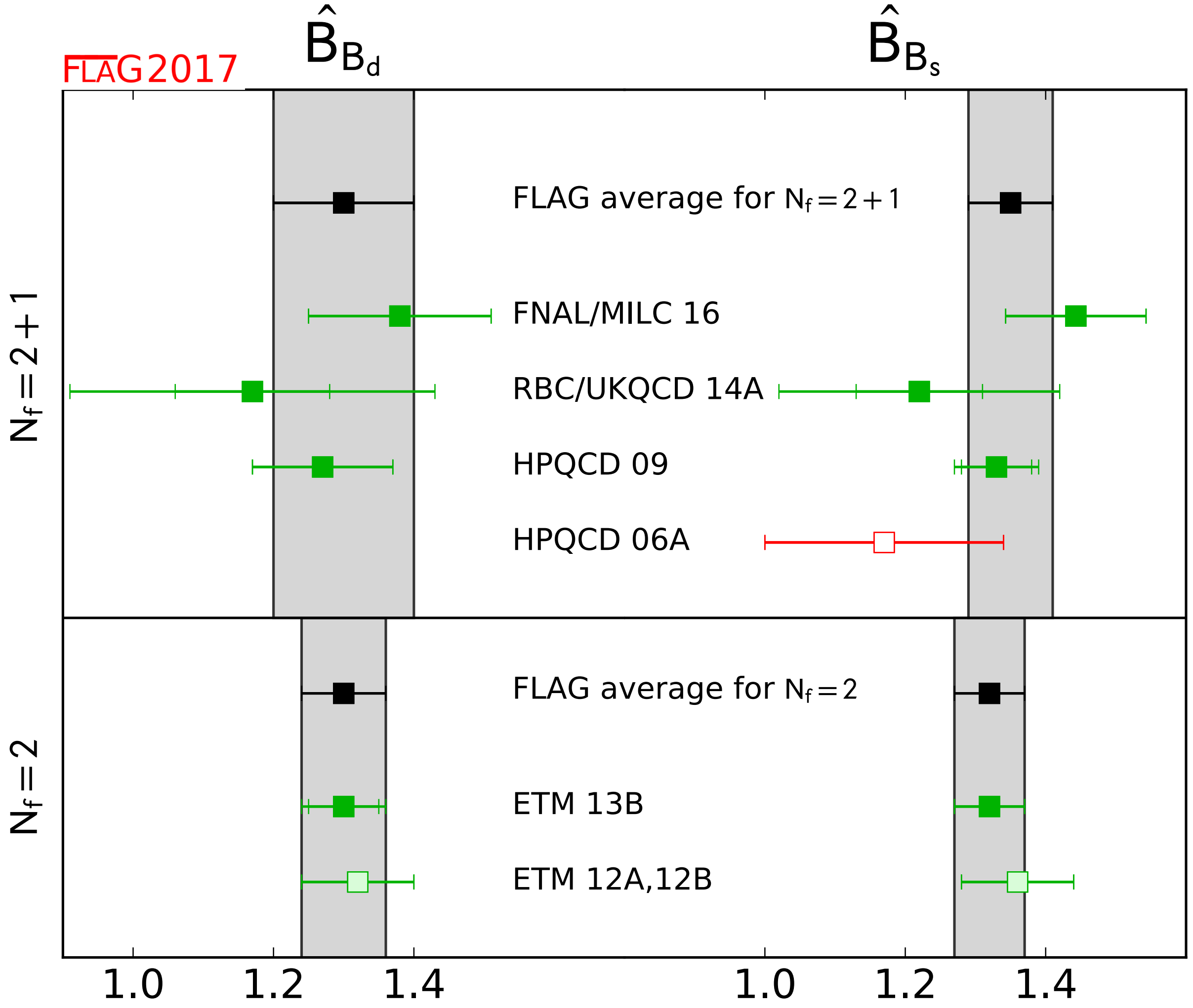

heavy quark physics: often significantly worse precision, still important to crosscheck approaches until significantly finer lattices are available

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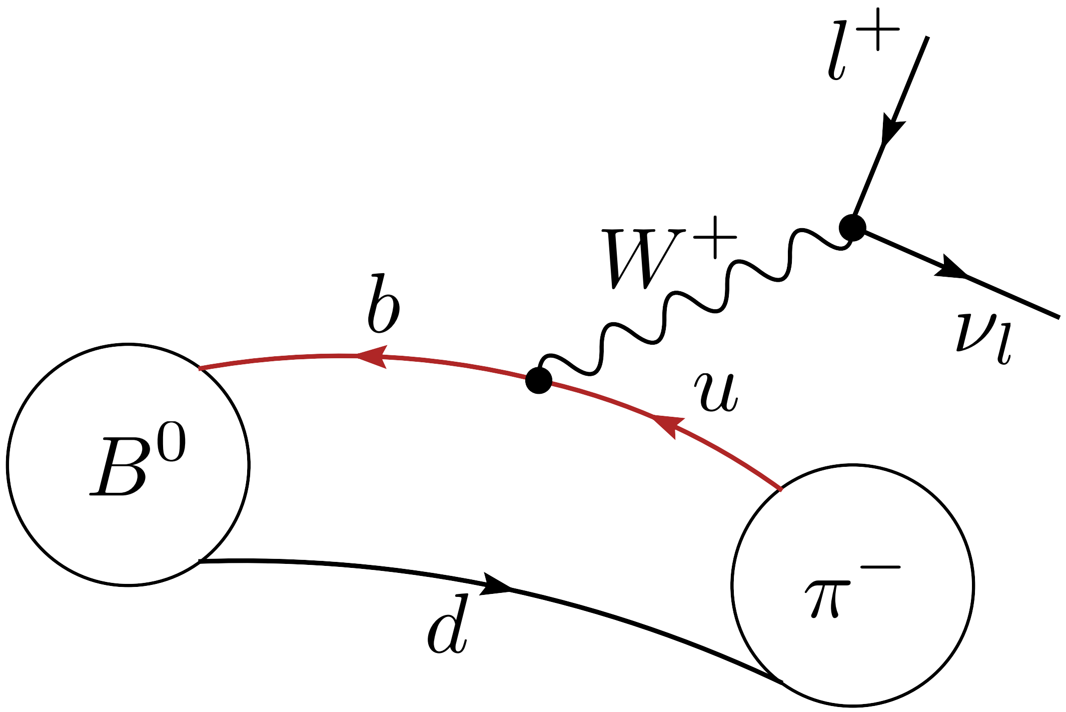
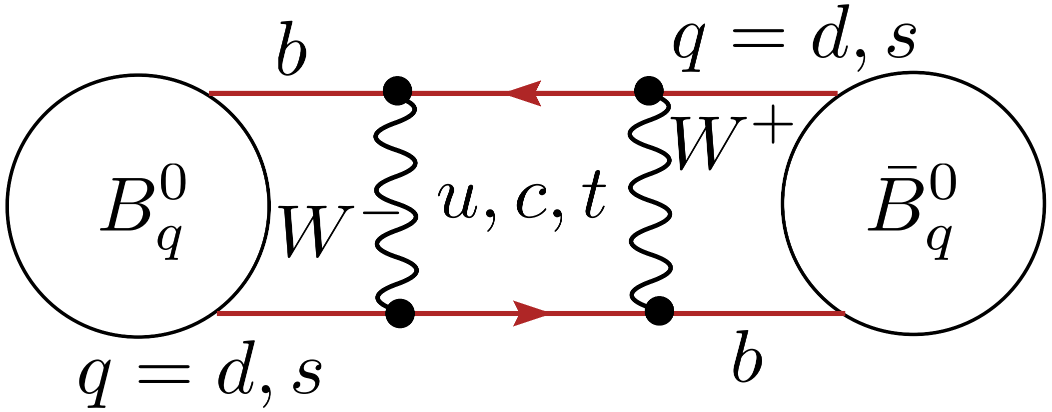
lattice QCD input for flavour physics



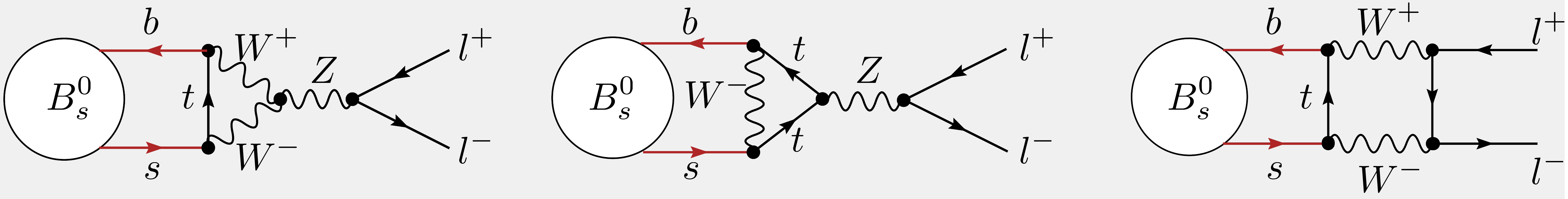
determination of CKM matrix elements from tree-level EW decays



rare decays and mixing: study (mostly loop-level) processes sensitive to new physics

B leptonic decay

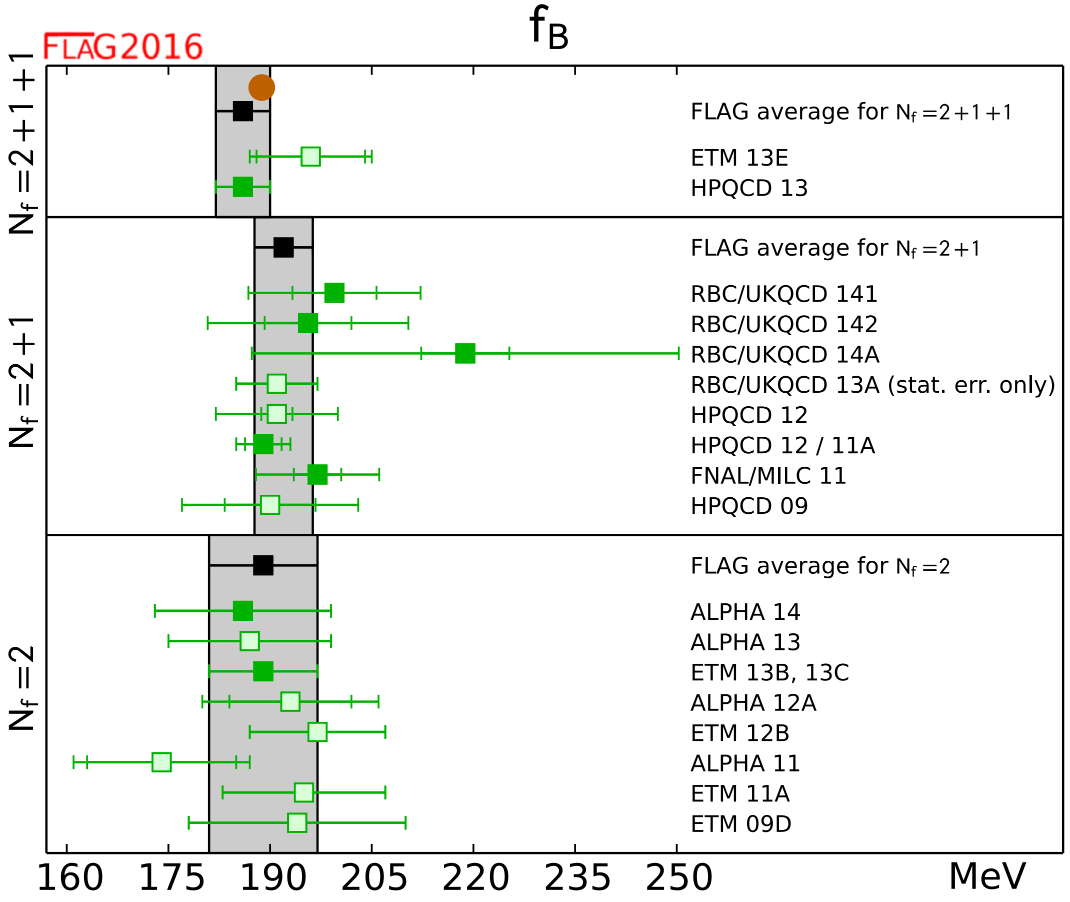
$$\frac{\mathcal{B}(B_{(c)} \to l\nu_l)}{\tau_{B_{(c)}}} = \frac{G_{\mathrm{F}}^2}{8\pi} m_l^2 m_{B_{(c)}} \left(1 - \frac{m_l^2}{m_{B_{(c)}}^2}\right)^2 |V_{qb}|^2 \, f_{B_{(c)}}^2 \; \text{[+ h.o. OPE]}$$

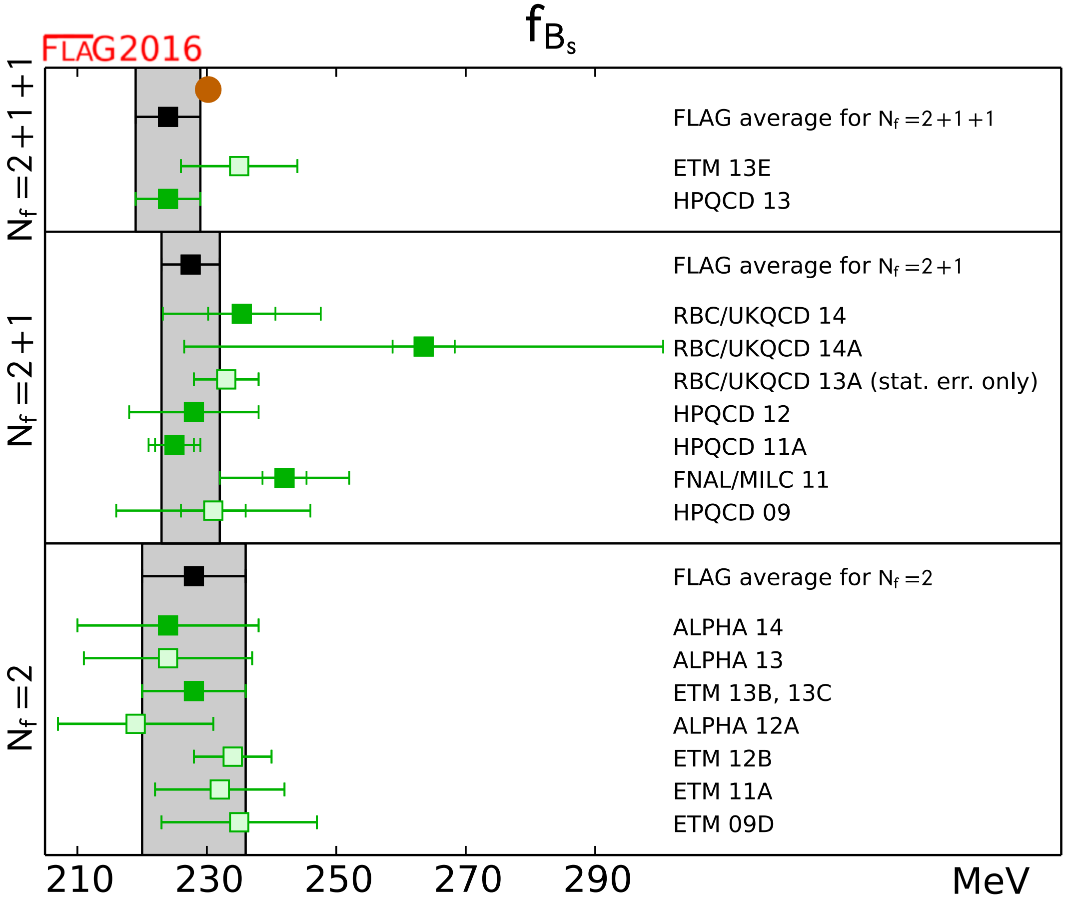


$$\frac{\mathcal{B}(B_q \to l^+ l^-)}{\tau_{B_q}} = \frac{G_{\mathrm{F}}^2}{\pi} Y \left(\frac{\alpha}{4\pi \sin^2 \theta_{\mathrm{W}}}\right)^2 m_{B_q} m_l^2 \sqrt{1 - 4\frac{m_l^2}{m_{B_q}^2}} \, |V_{tb}^* V_{tq}|^2 \, f_{B_q}^2$$

$$\langle 0 | \bar{b} \gamma^\mu \gamma^5 q | B_q(p) \rangle = f_{B_q} \, p^\mu$$

FLAG-3 — B decay constants





N_{f}	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
2	188(7)	227(7)	1.206(23)
2+1	192.0(4.3)	228.4(3.7)	1.201(16)
2+1+1	186(4)	224(5)	1.205(7)
	196(6)	236(7)	1.207(7)
	189.4(1.4)	230.7(1.2)	1.2180(49)

[HPQCD arXiv:1711.09981; FNAL/MILC arXiv:1712.09262]

FLAG-3 — B decay constants

- errors in the few- → sub-% ballpark: theory way ahead of experimental uncertainties
- estimates/averages dominated by few results: strong need of crosschecks from other HQ treatments to improve confidence in systematics
- electromagnetic corrections?

N_f	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
2	188(7)	227(7)	1.206(23)
2+1	192.0(4.3)	228.4(3.7)	1.201(16)
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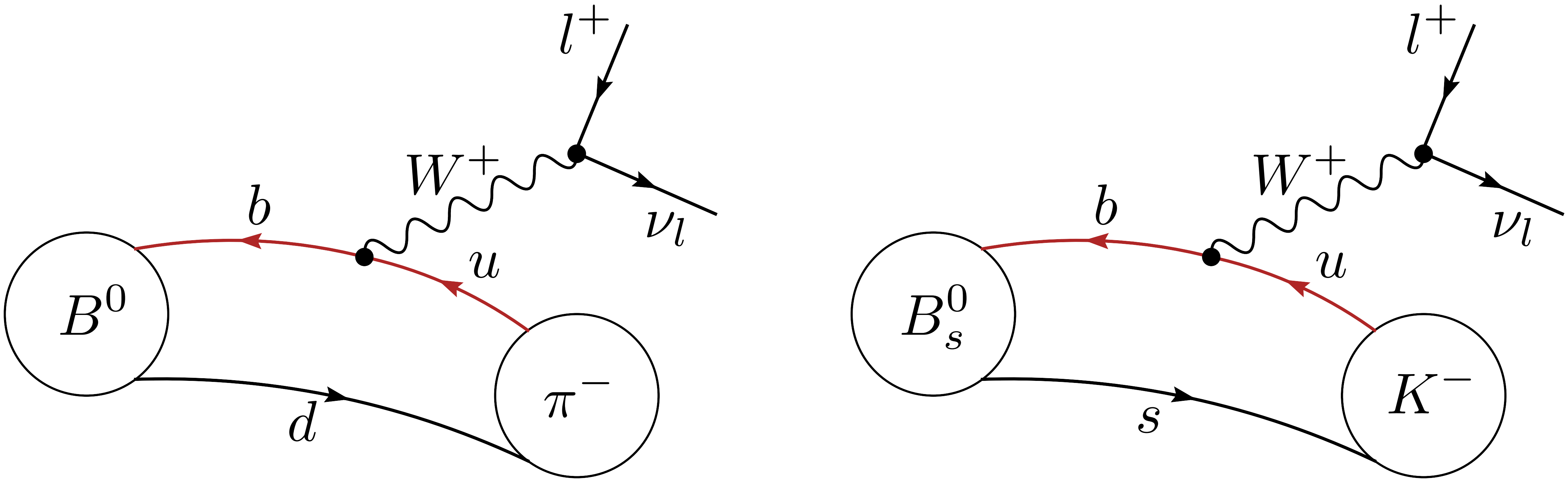
`[HPQCD arXiv:1711.09981; FNAL/MILC arXiv:1712.09262]`

$b \to u$ semileptonic

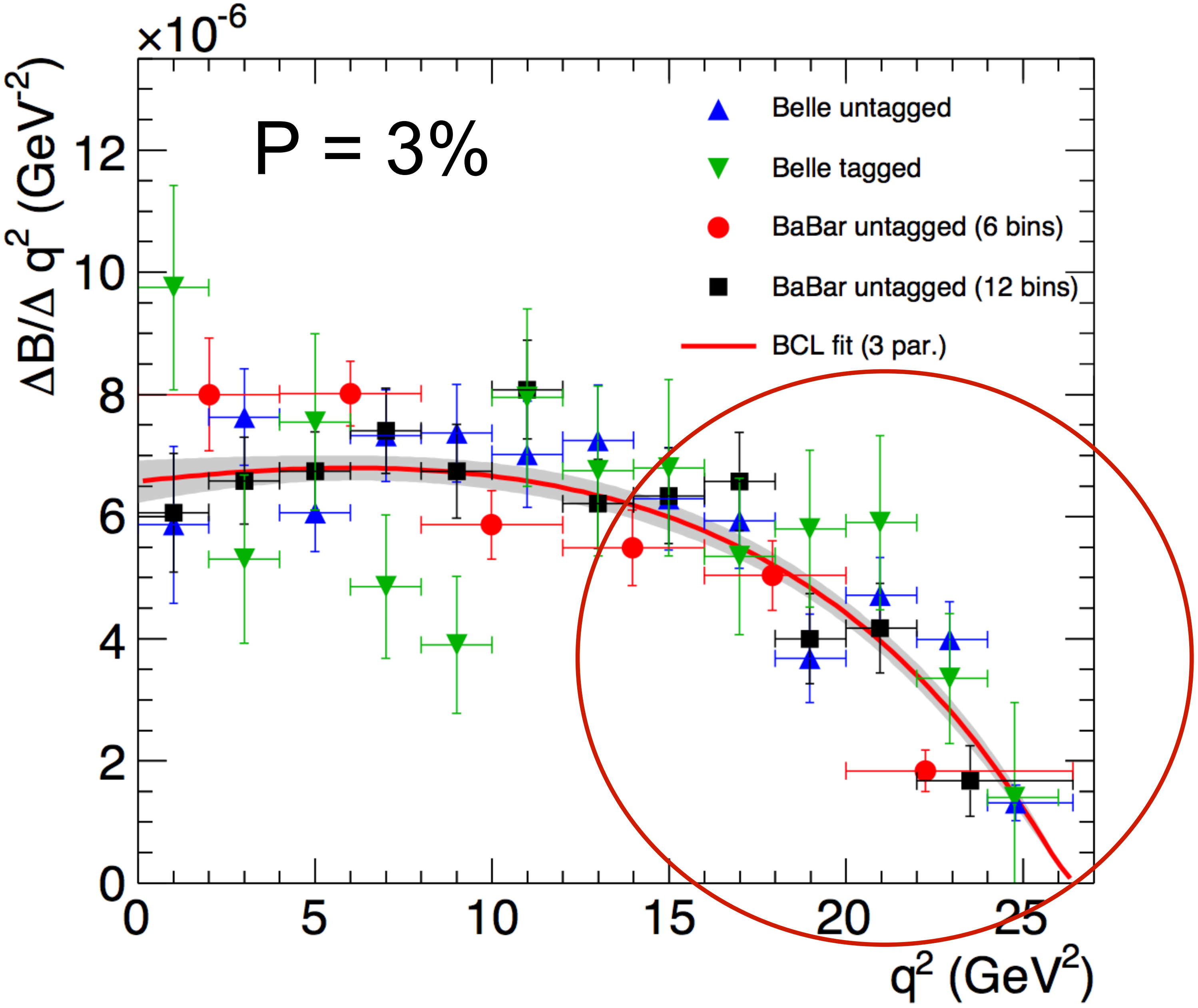


$$\frac{\mathrm{d}\Gamma(B_{(s)} \to Pl\nu)}{\mathrm{d}q^2} = \frac{G_\mathrm{F}^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 \right.$$
$$\left. + \frac{3m_l^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

$$\langle P(p') | \bar{b} \gamma_\mu q | B_q(p) \rangle = f_+(q^2) \left(p_\mu + p'_\mu - \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu \right) + f_0(q^2) \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu \,, \qquad q = p - p'$$

$B \to \pi l \nu$

large phase space ⇒ accurate description of q^2 dependence over a significant region crucial for a precise CKM determination



[HFAG]

easily accessible kinematics on the lattice (not-too-fast pions)

$B \to \pi l \nu$

large phase space ⇒ accurate description of q^2 dependence over a significant region crucial for a precise CKM determination

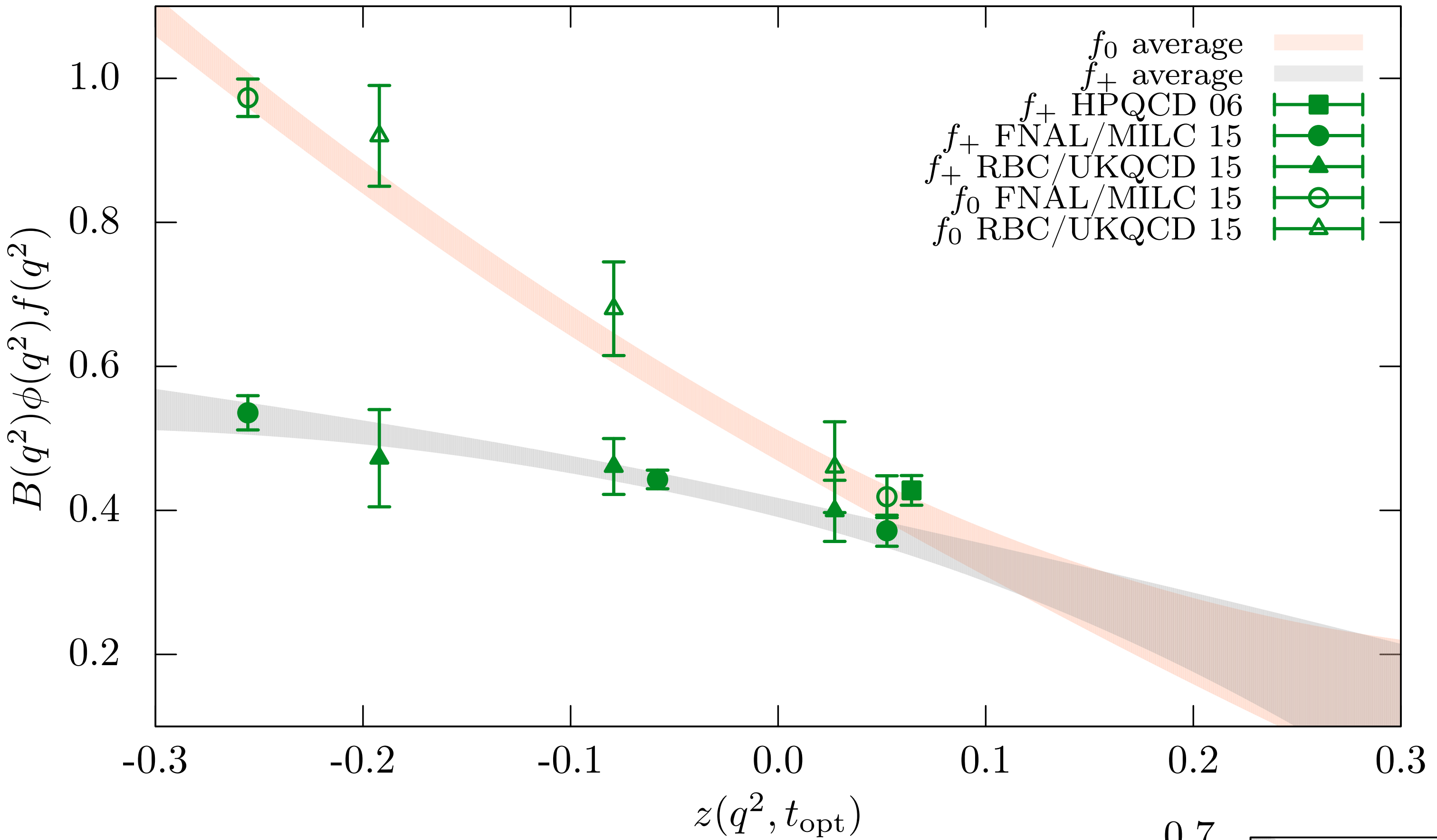
z-parameterisations (specially BCL) becoming de facto standard in LQCD, driven by their success in the $B \to \pi$ vector channel

[Okubo PRD 3 (1971) 2807, 4 (1971) 725]
[Bourrely, Machet, de Rafael NPB 189 (1981) 157]
[Boyd, Grinstein, Lebed PRL 74 (1995) 4603]
[Lellouch NPB 479 (1996) 353]
[Bourrely, Caprini, Micu EJPC 27 (2003) 439]
[Arnesen, Grinstein, Rothstein, Stewart PRL 95 (2005) 071802]
[Becher, Hill PLB 633 (2006) 61]
[Flynn, Nieves PRD 75 (2007) 013008]
[Bourrely, Caprini, Lellouch PRD 79 (2009) 013008]

however, for basically every other process/channel there may be not enough information to control systematics via unitarity constraints ⇒ inspecting systematics still crucial!

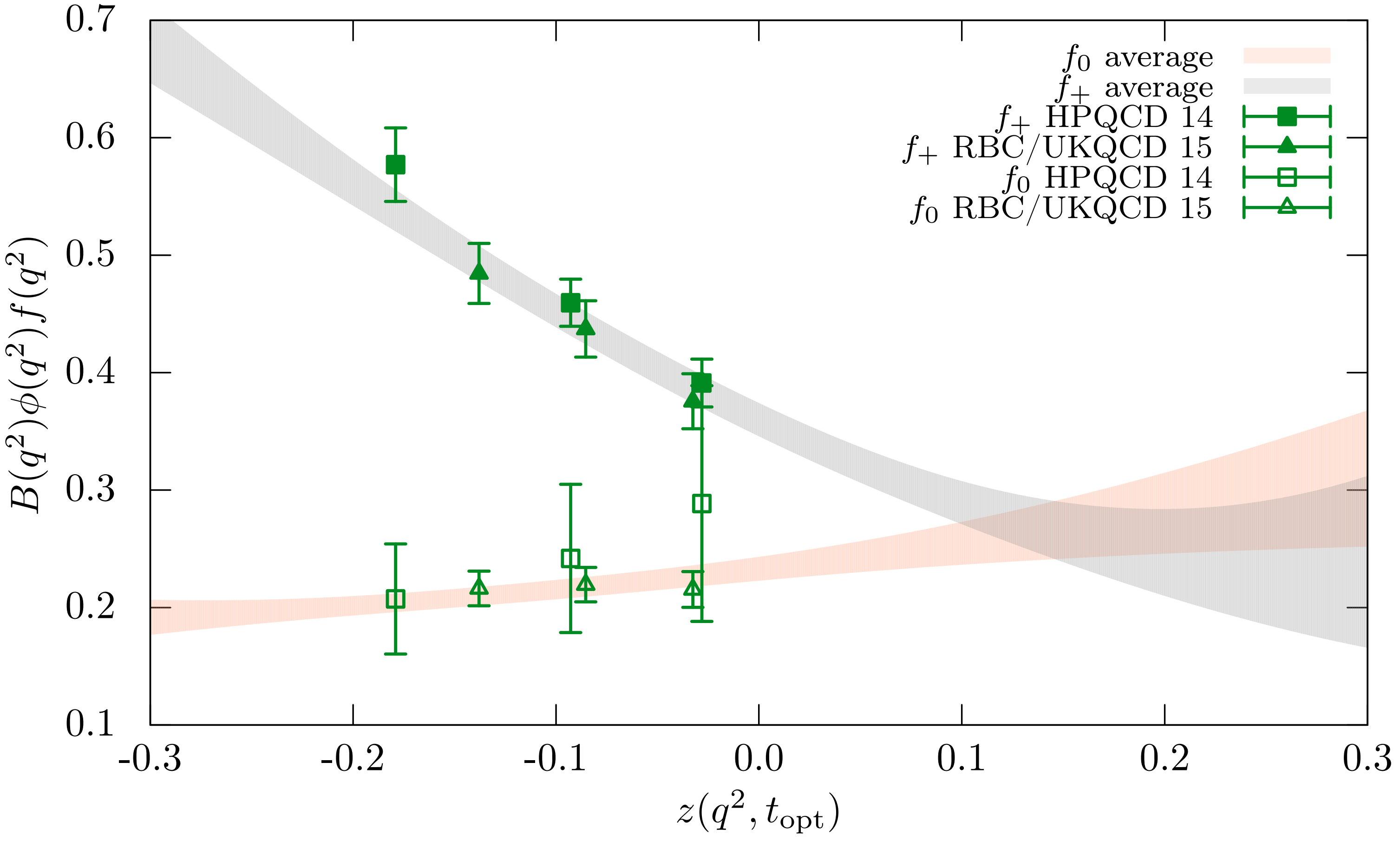
[Gambino, Moriond EW 2018]

FLAG-3 — $B\to\pi l\nu$ and $B_s\to K l\nu$

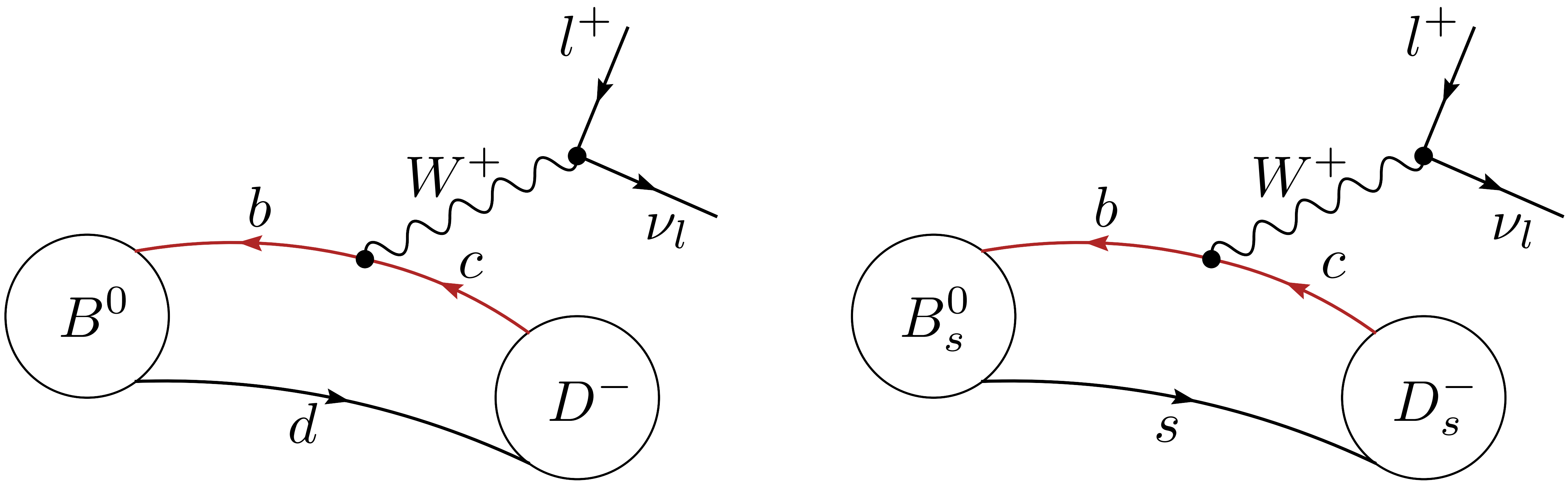


$B\to\pi$

$B_s\to K$



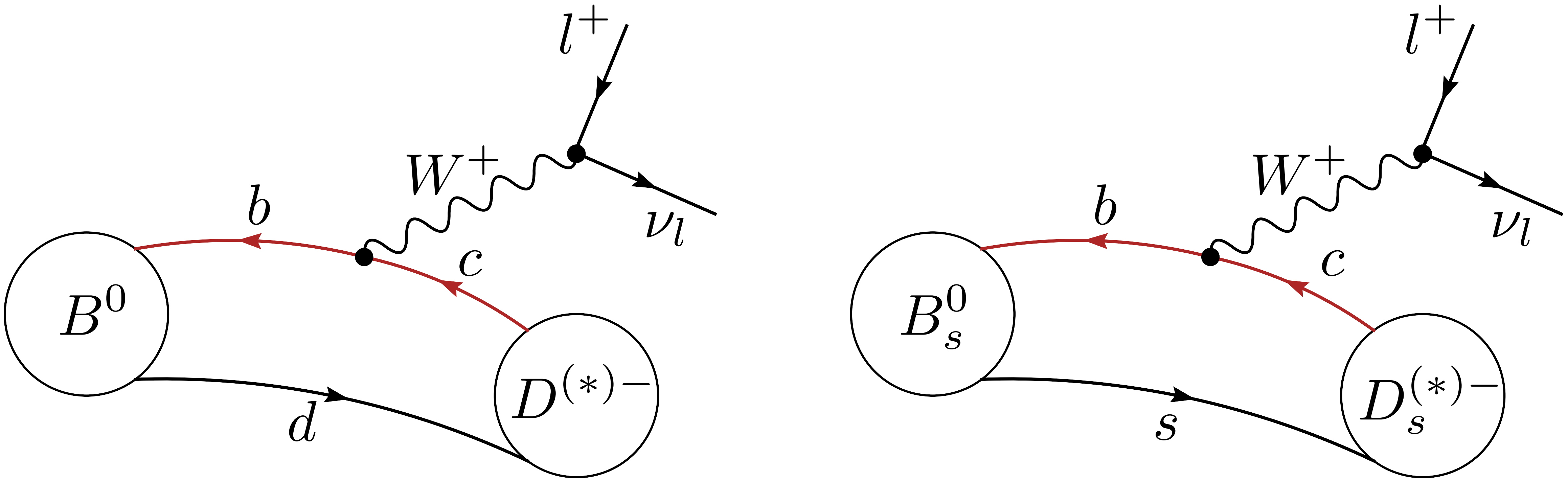
$b \to c$ semileptonic



$$\frac{\mathrm{d}\Gamma(B_{(s)} \to Pl\nu)}{\mathrm{d}q^2} = \frac{G_{\mathrm{F}}^2 \, |V_{cb}|^2}{24\pi^3} \, \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_l^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

$$\langle P(p') | \bar{b} \gamma_\mu q | B_q(p) \rangle = f_+(q^2) \left(p_\mu + p'_\mu - \frac{m_{B_q}^2 - m_P^2}{q^2} \, q_\mu \right) + f_0(q^2) \, \frac{m_{B_q}^2 - m_P^2}{q^2} \, q_\mu \, , \qquad q = p - p'$$

$b\to c$ semileptonic



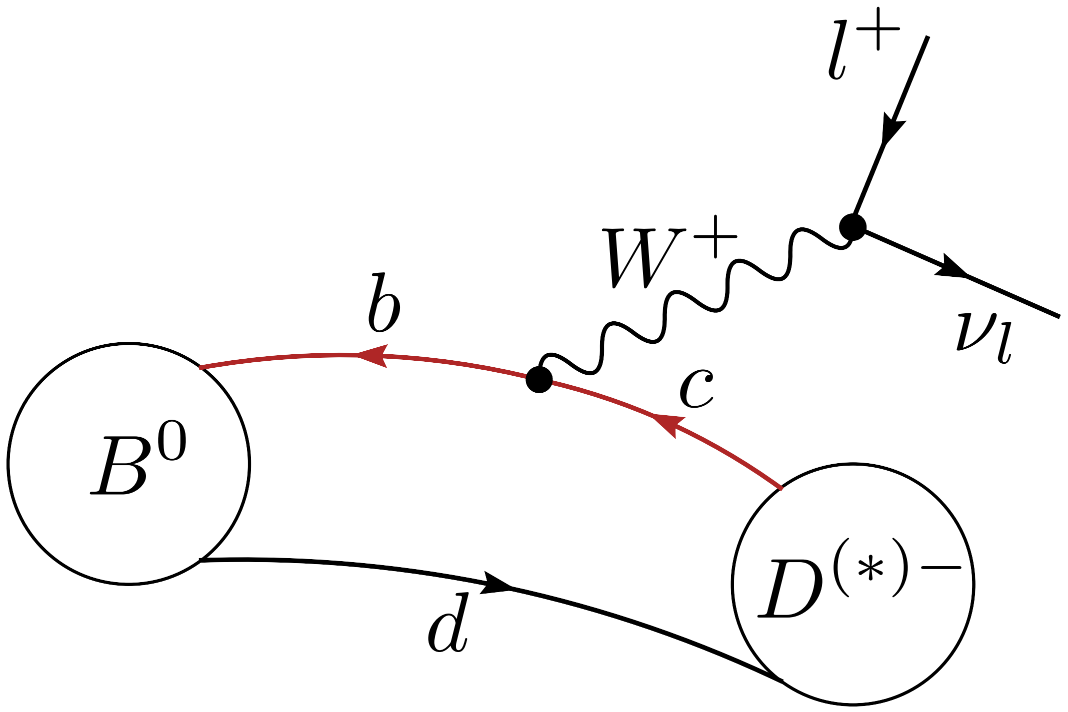
$$\frac{\mathrm{d}\Gamma(B\to Dl\nu_l)}{\mathrm{d}w} = \frac{G_{\mathrm{F}}^2}{48\pi^3}(m_B+m_D)^2(w^2-1)^{3/2}|\eta_{\mathrm{EW}}|^2\,|V_{cb}|^2\,|\mathcal{G}(w)|^2 \;+\; \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

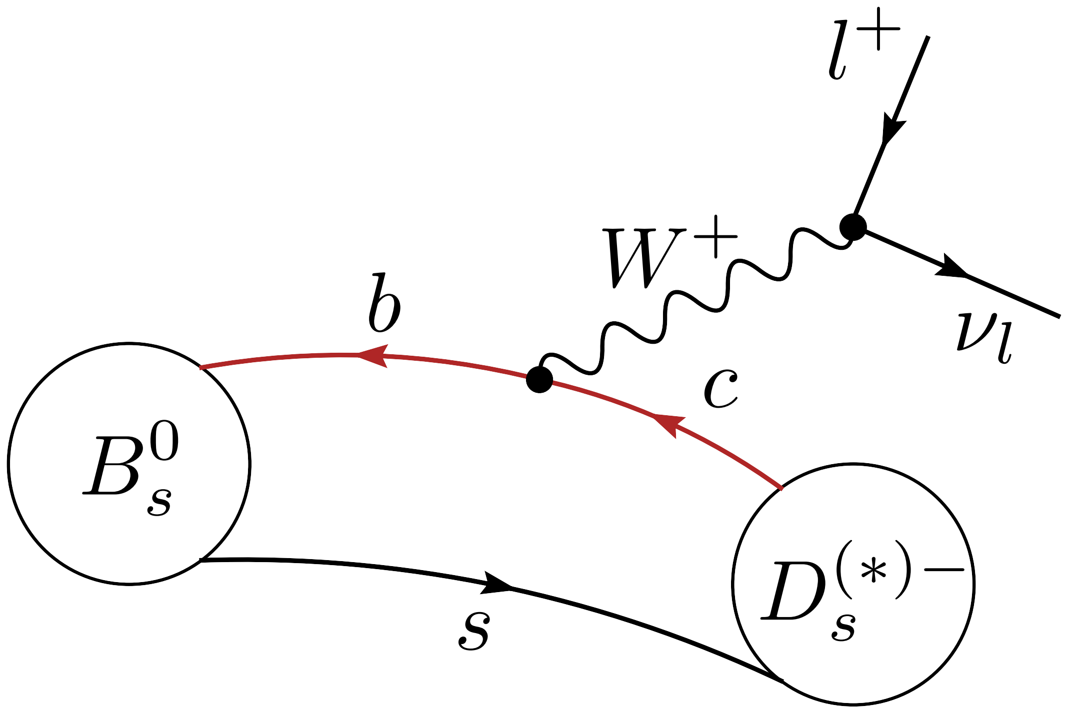
$$\frac{\mathrm{d}\Gamma(B\to D^*l\nu_l)}{\mathrm{d}w} = \frac{G_{\mathrm{F}}^2}{4\pi^3}(m_B-m_{D^*})^2(w^2-1)^{1/2}|\eta_{\mathrm{EW}}|^2\chi(w)\,|V_{cb}|^2\,|\mathcal{F}(w)|^2 \;+\; \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

low recoil region accessible to lattice computations ⇒ CKM can be determined by computing form factors at w=1

shape information relevant as precision increases + LFU

$b \rightarrow c$ semileptonic



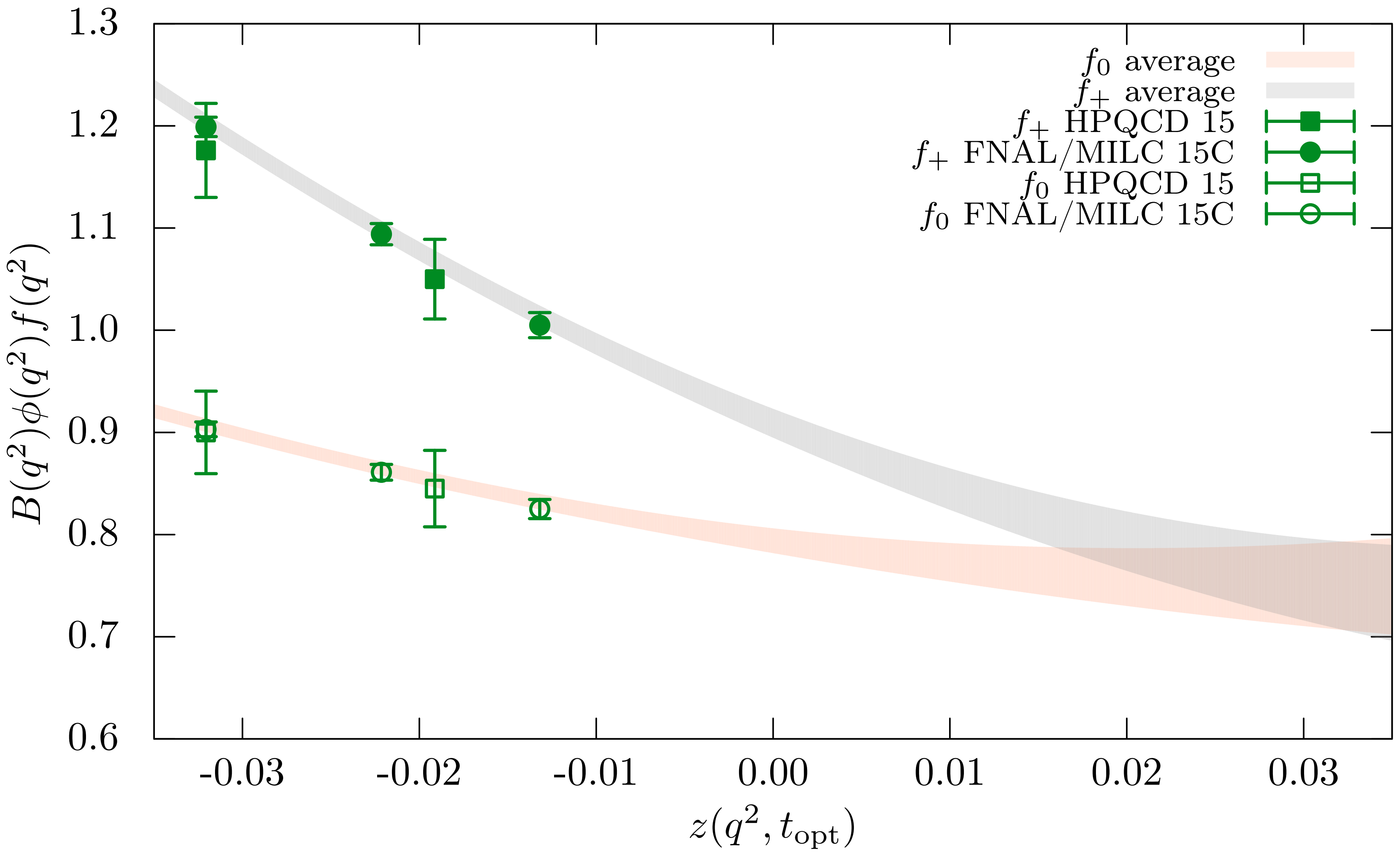


$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}$$

only zero recoil for D^* so far (including recent HPQCD update), but ongoing FNAL/MILC work should provide full results at $w \neq 1$ soon

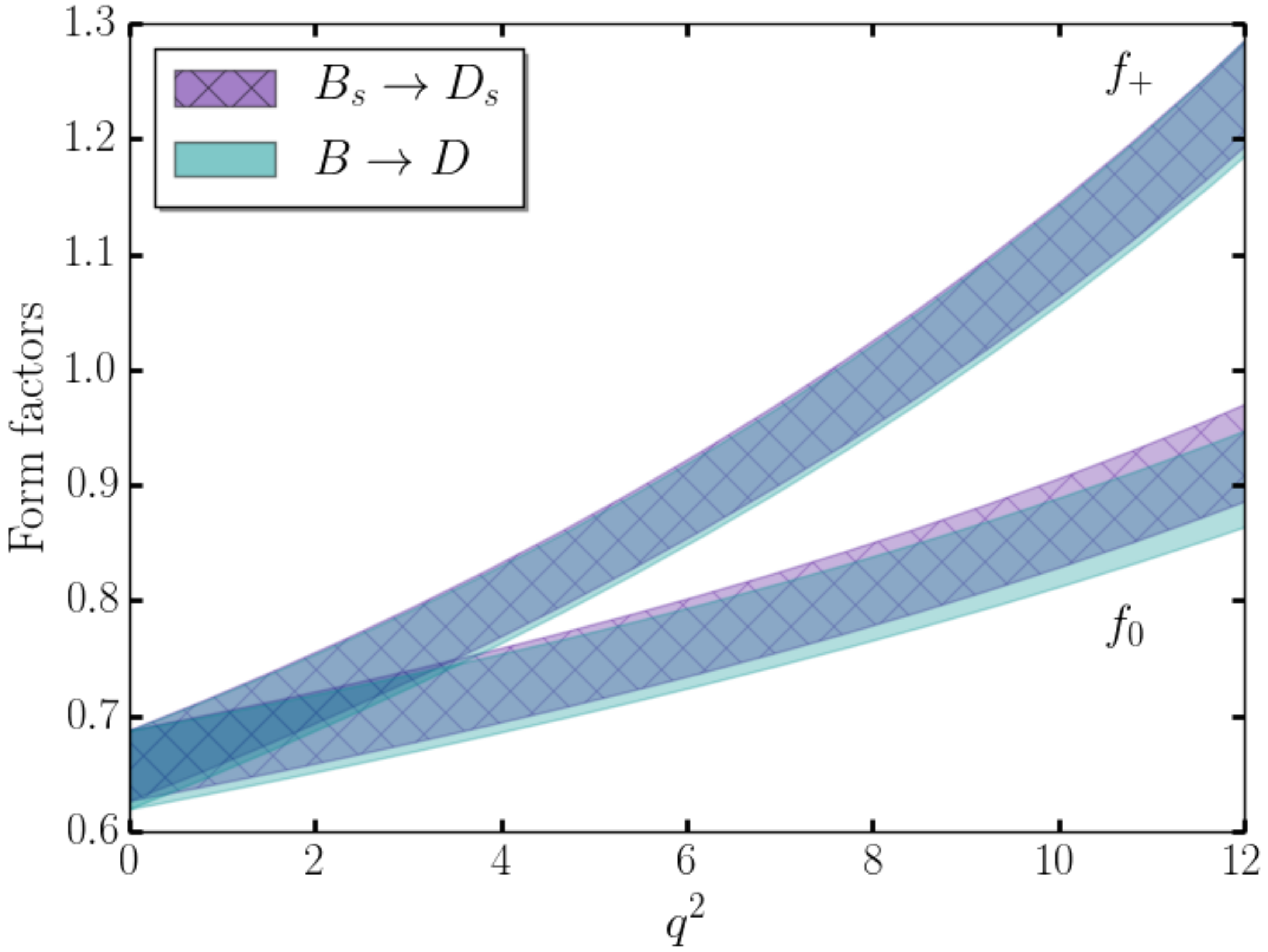
`[HPQCD arXiv:1711.11013]`
`[FNAL/MILC arXiv:1710.09817]`

FLAG-3 — $B\to D l\nu$



newer $B_{(s)}\to D_{(s)}$ computations include results $w>1$ ⇒ $R(D)=0.300(8)$

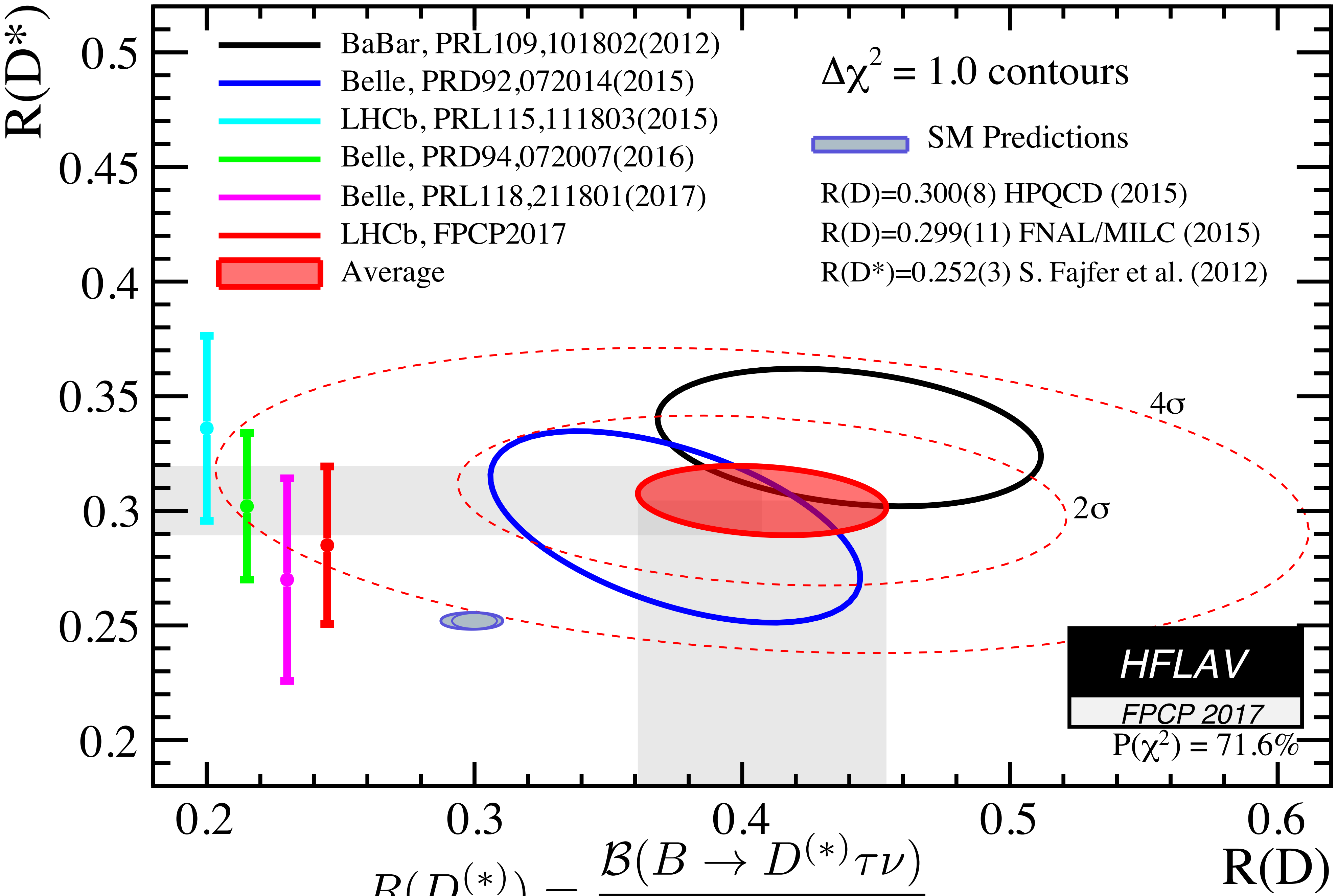
$B_s \rightarrow D_s l\nu$ new results



newer $B_{(s)} \rightarrow D_{(s)}$ computations include results $w > 1$ ⇒ $R(D_s) = 0.301(6)$

[HPQCD PRD 95 (2017) 114506]

$R(D^{(*)})$

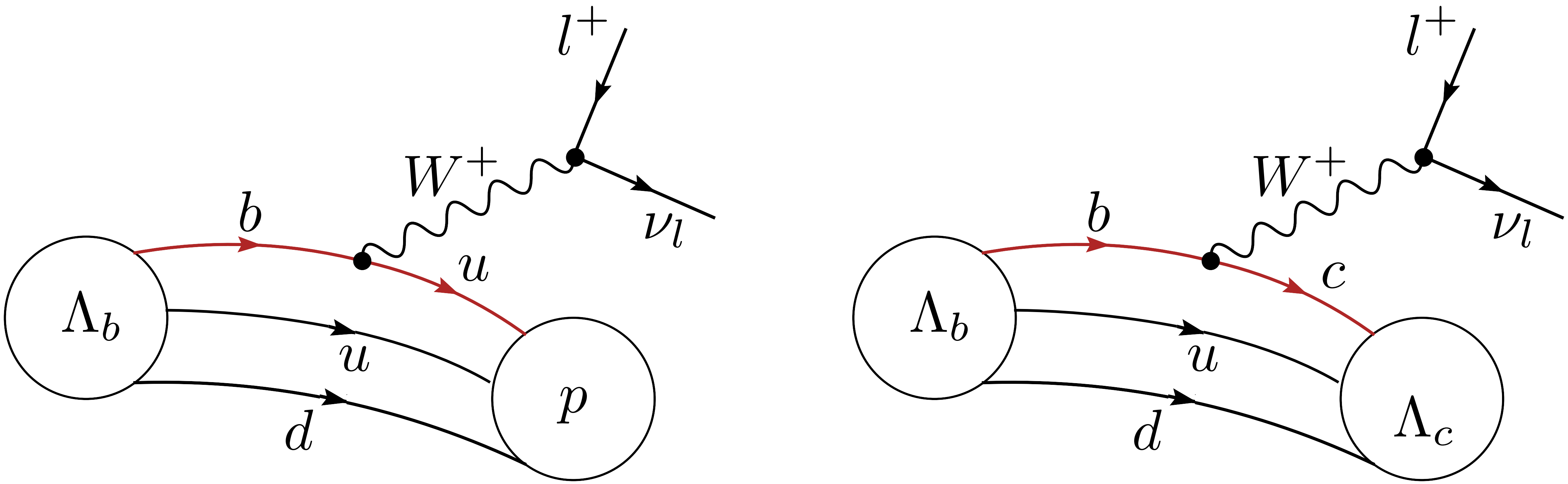


$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\mu\nu)}$$

ongoing work on lattice determination of $R(D^*)$

[FNAL/MILC EPJ Web Conf. 175 (2018) 13003]

baryonic decays

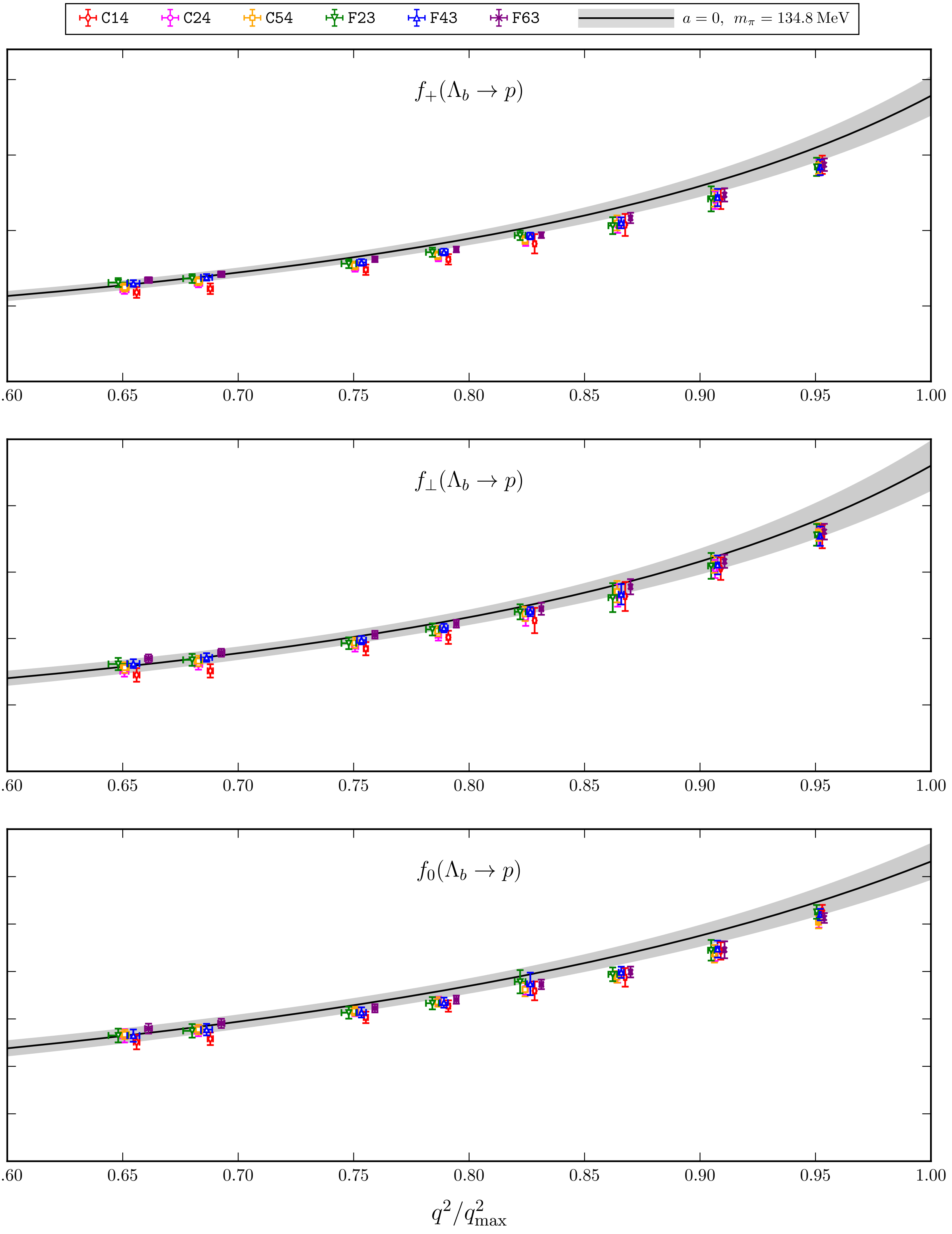


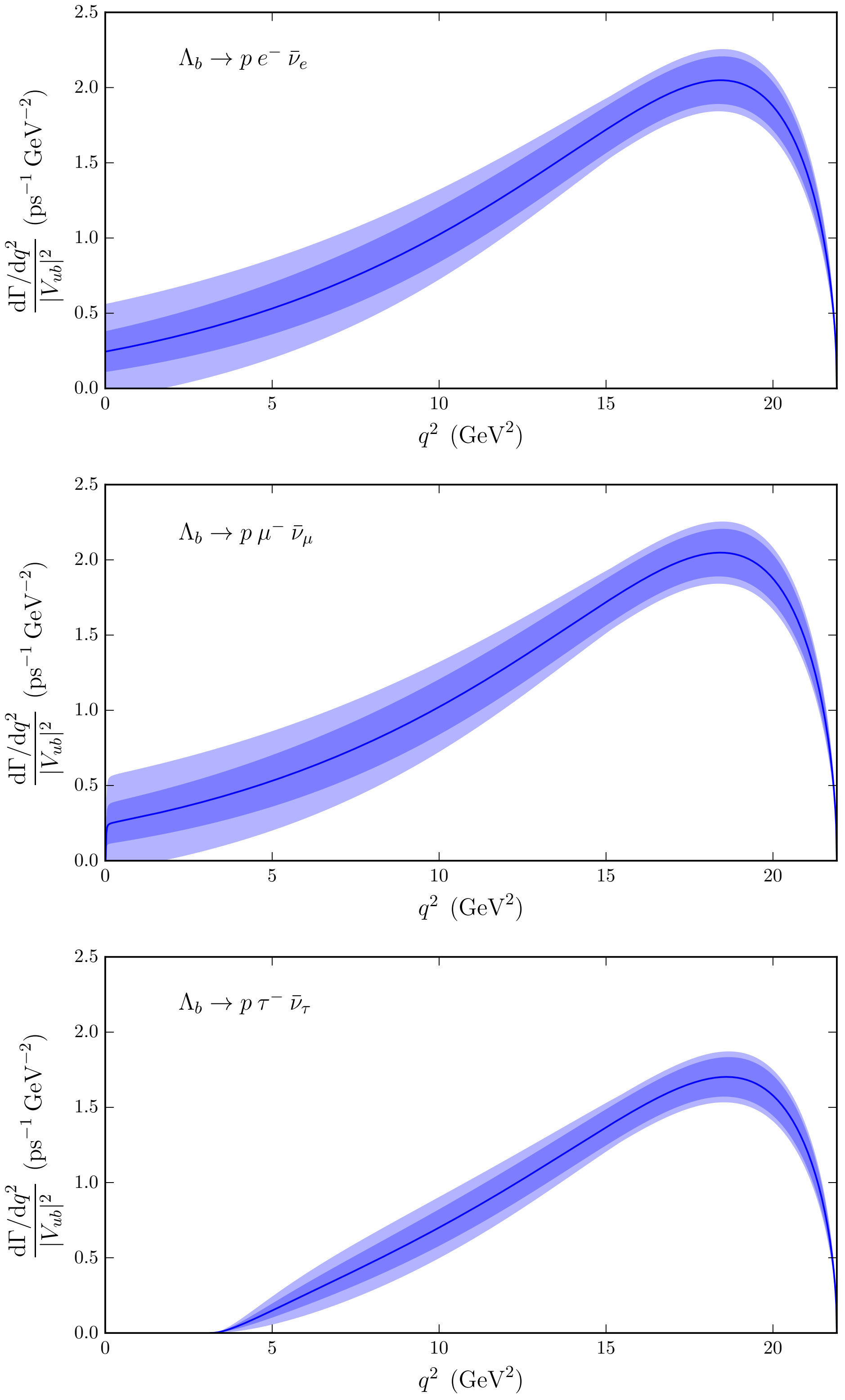
new exclusive determination of $|V_{cb}|/|V_{ub}|$ from LHCb measurement + LQCD computation of form factors

baryonic decays

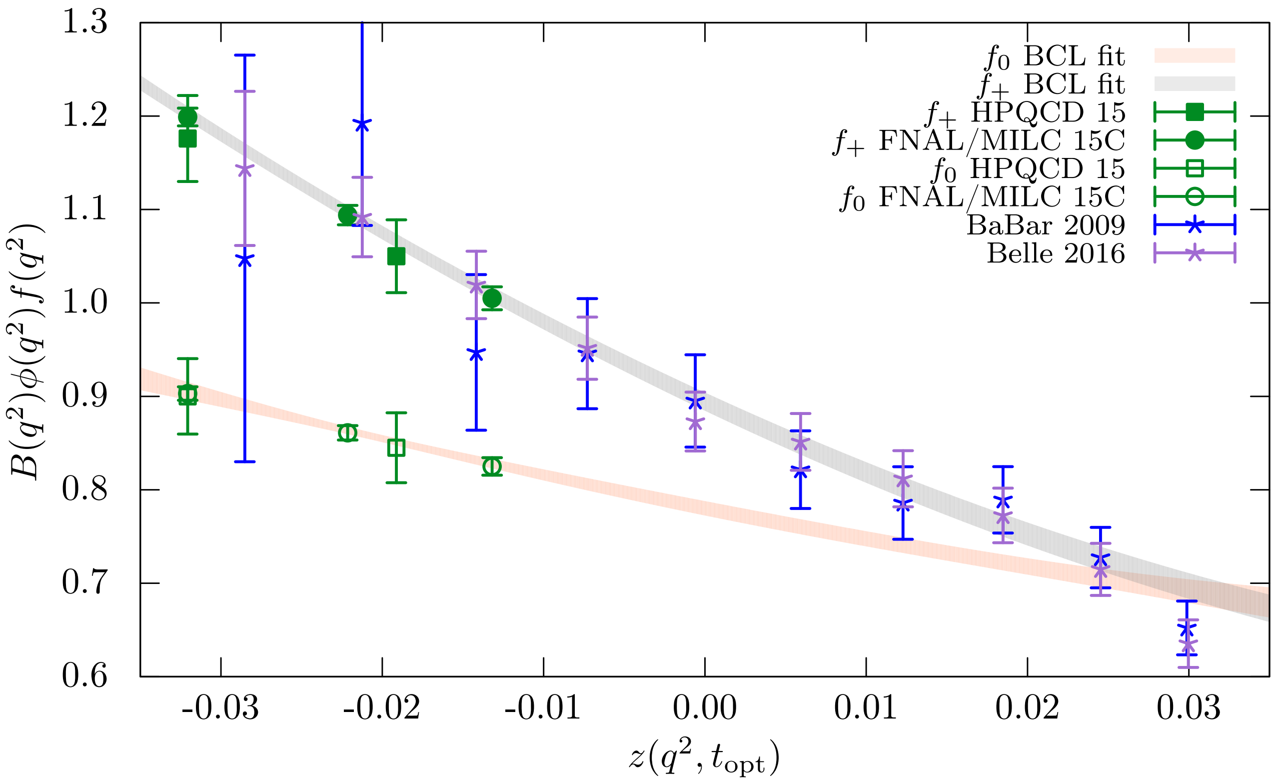
[Detmold, Lehner, Meitnel PRD 92 (2015) 034503]

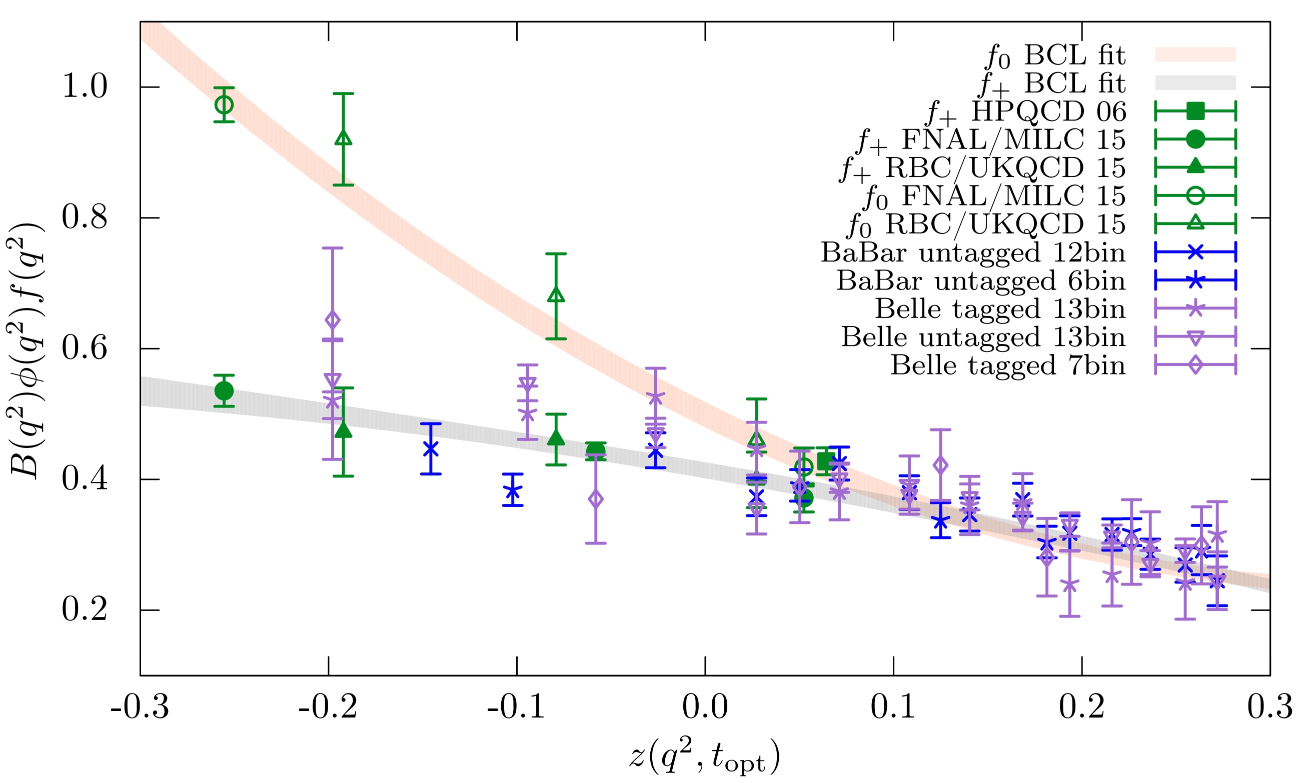
[cf. Detmold, Lin, Meitnel, Wingate PRD 88 (2013) 014512]

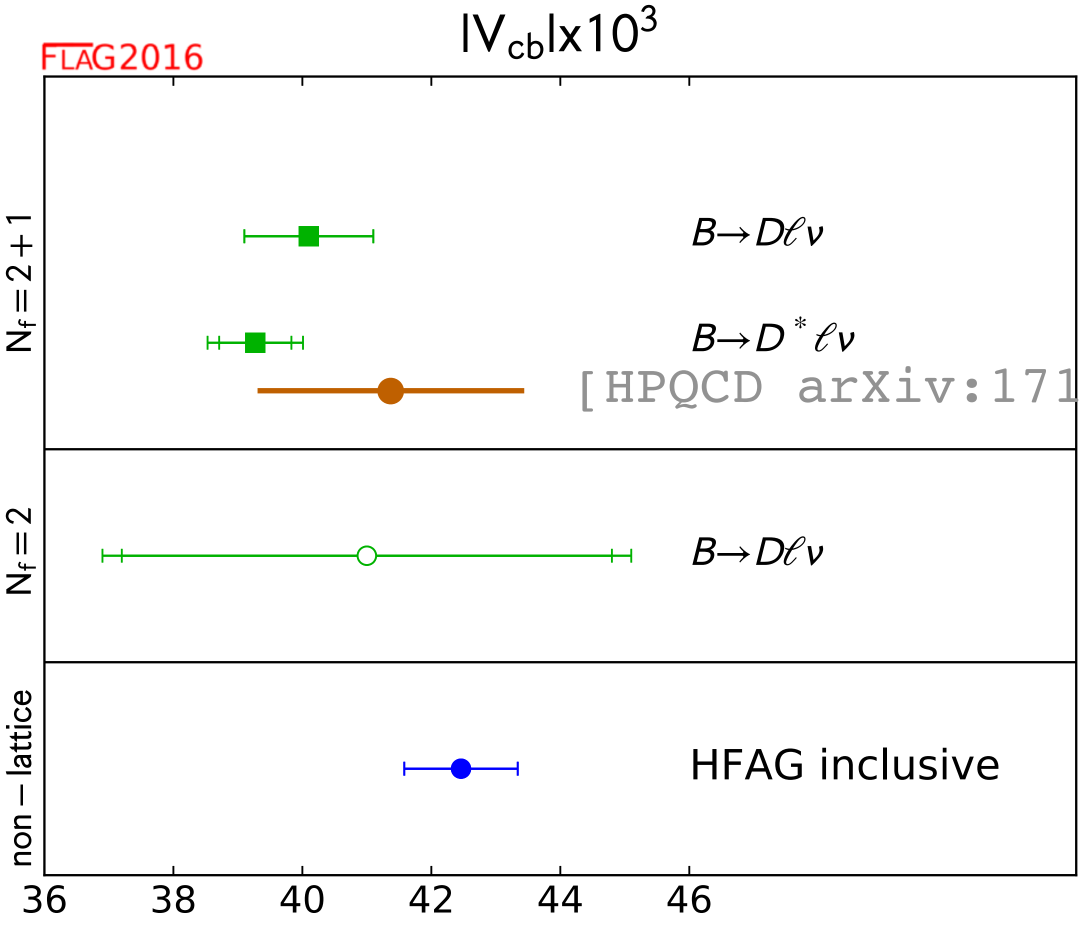




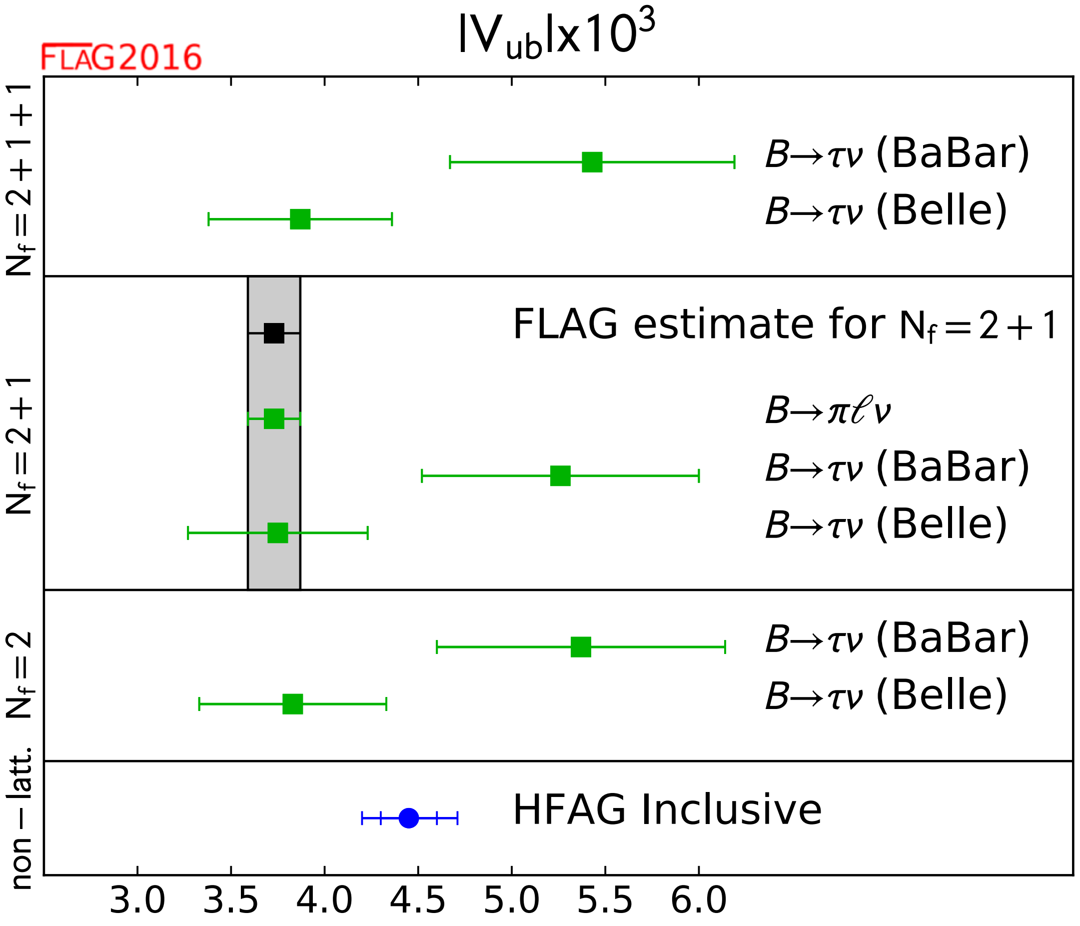
FLAG-3 — 3rd row CKM



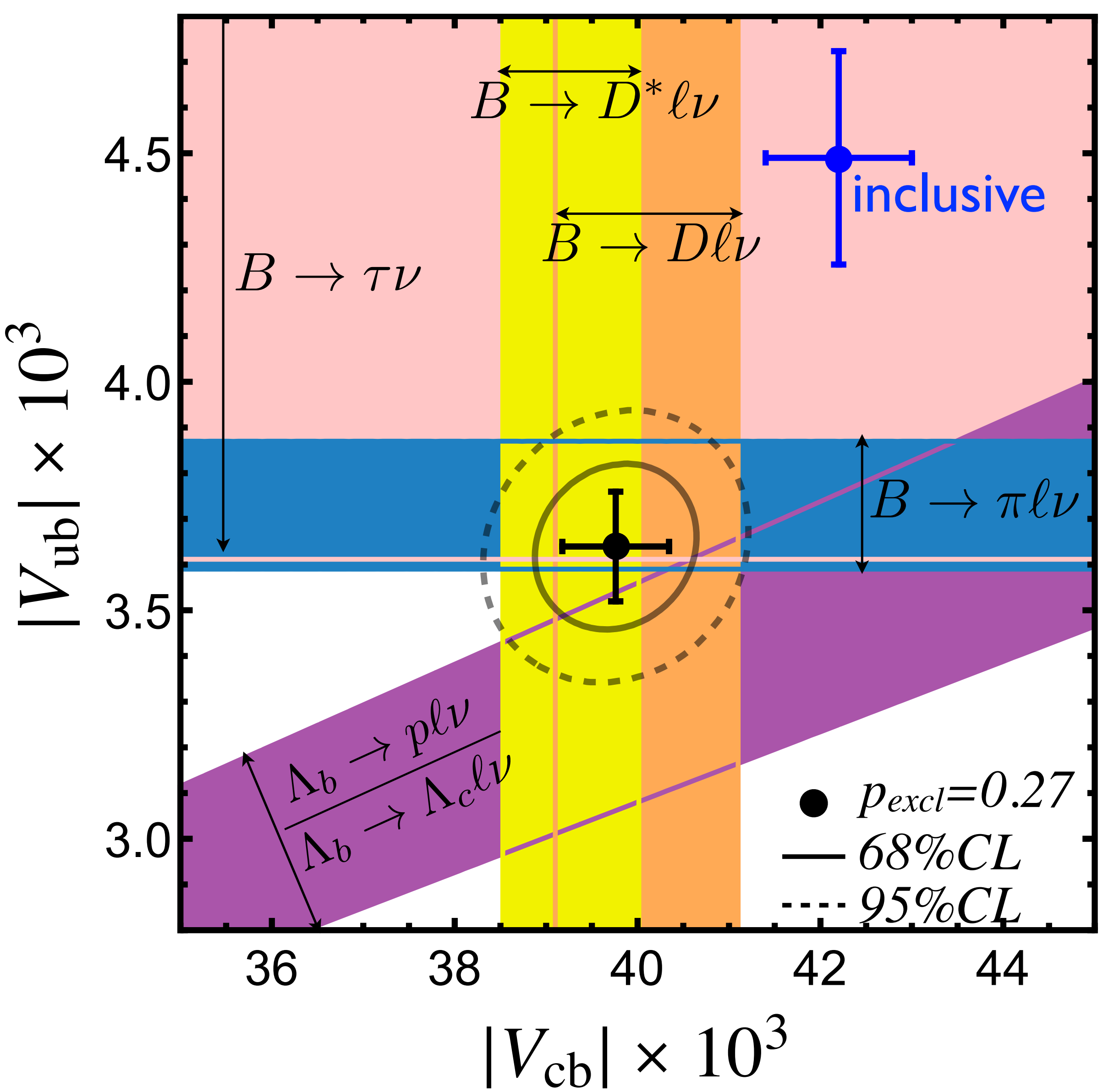




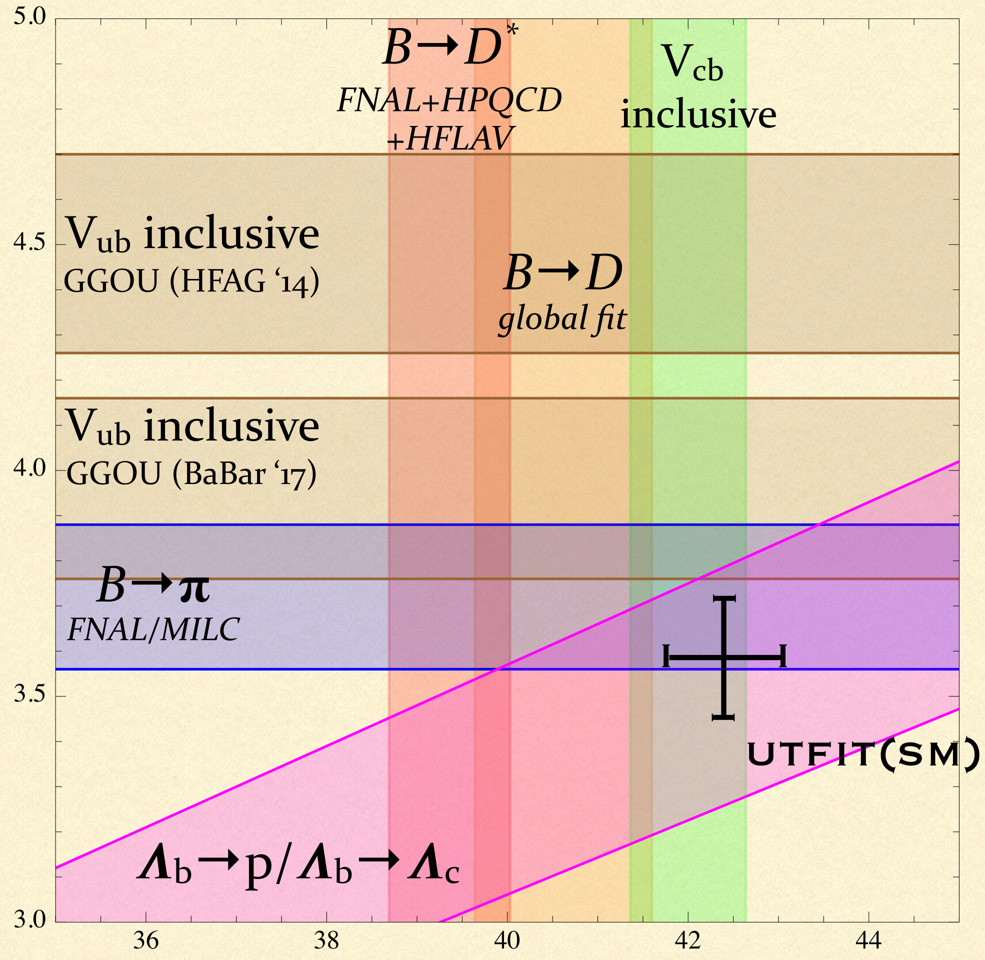
[HPQCD arXiv:1711.11013]



FLAG-3 — 3rd row CKM



[Lunghi, Moriond QCD 2017]



New V_{ub} incl
by Babar
in agreement
with exclusive
PRD 95 (2017) 7, 072001

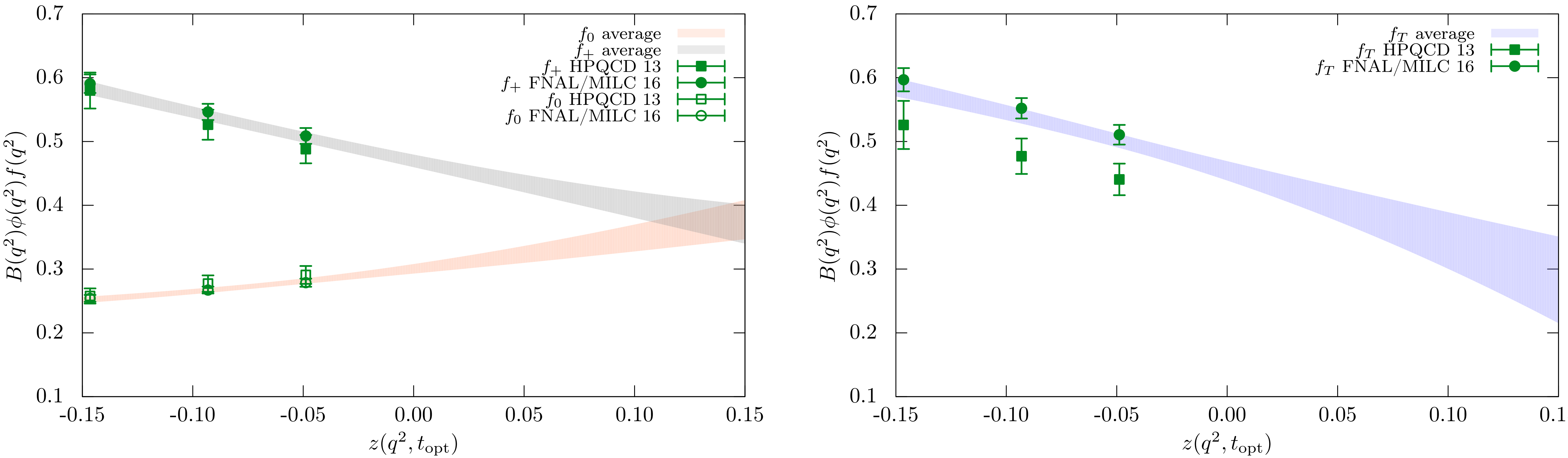
New HPQCD
$B \to D^*$ result
at zero recoil
arXiv:1711.11013

New Belle $B \to D^*$
result: with FNAL
V_{cb}=37.4(1.3) 10^{-3}
arXiv:1702.01521

[Gambino, Moriond EW 2018]

rare decays: form factors for $B\to K$

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization	Heavy-quark treatment
HPQCD 13E	[515]	$2+1$	A	○	○	○	○	✓
FNAL/MILC 15D	[516]	$2+1$	A	★	○	★	○	✓



[FLAG-3, Aoki et al., EPJC (2017) 77:112]

⇒ O_7, O_9, O_{10} (similar results for $B\to\pi$ [FNAL/MILC PRL 115(2015)152002])

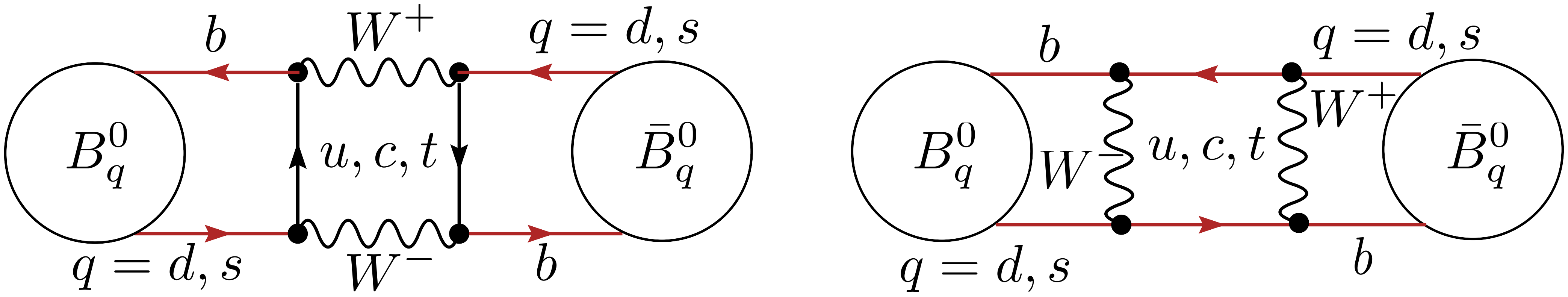
rare decays: form factors for $B \to K$

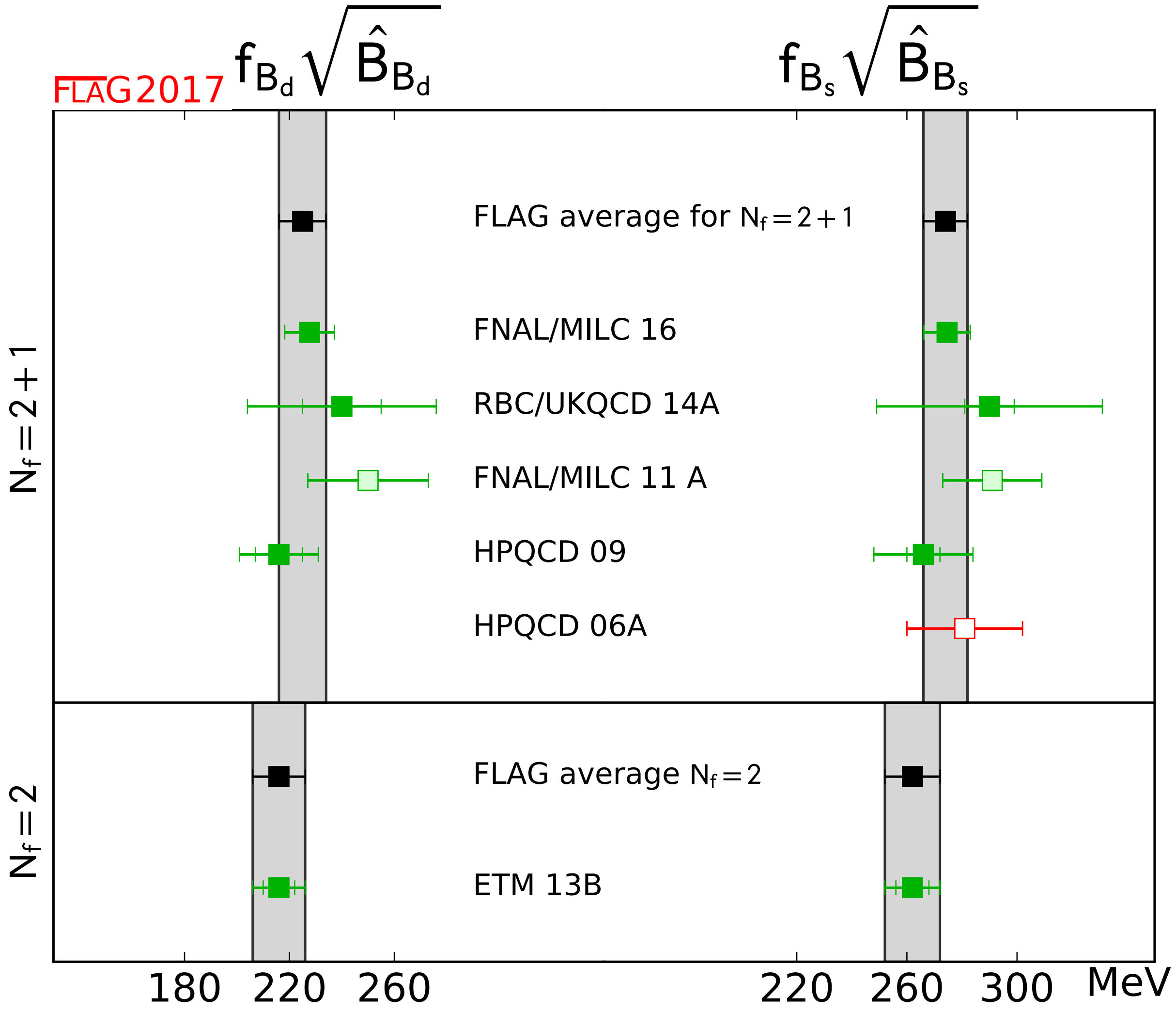
- lattice results at similar level of maturity as for SM tree-level decays

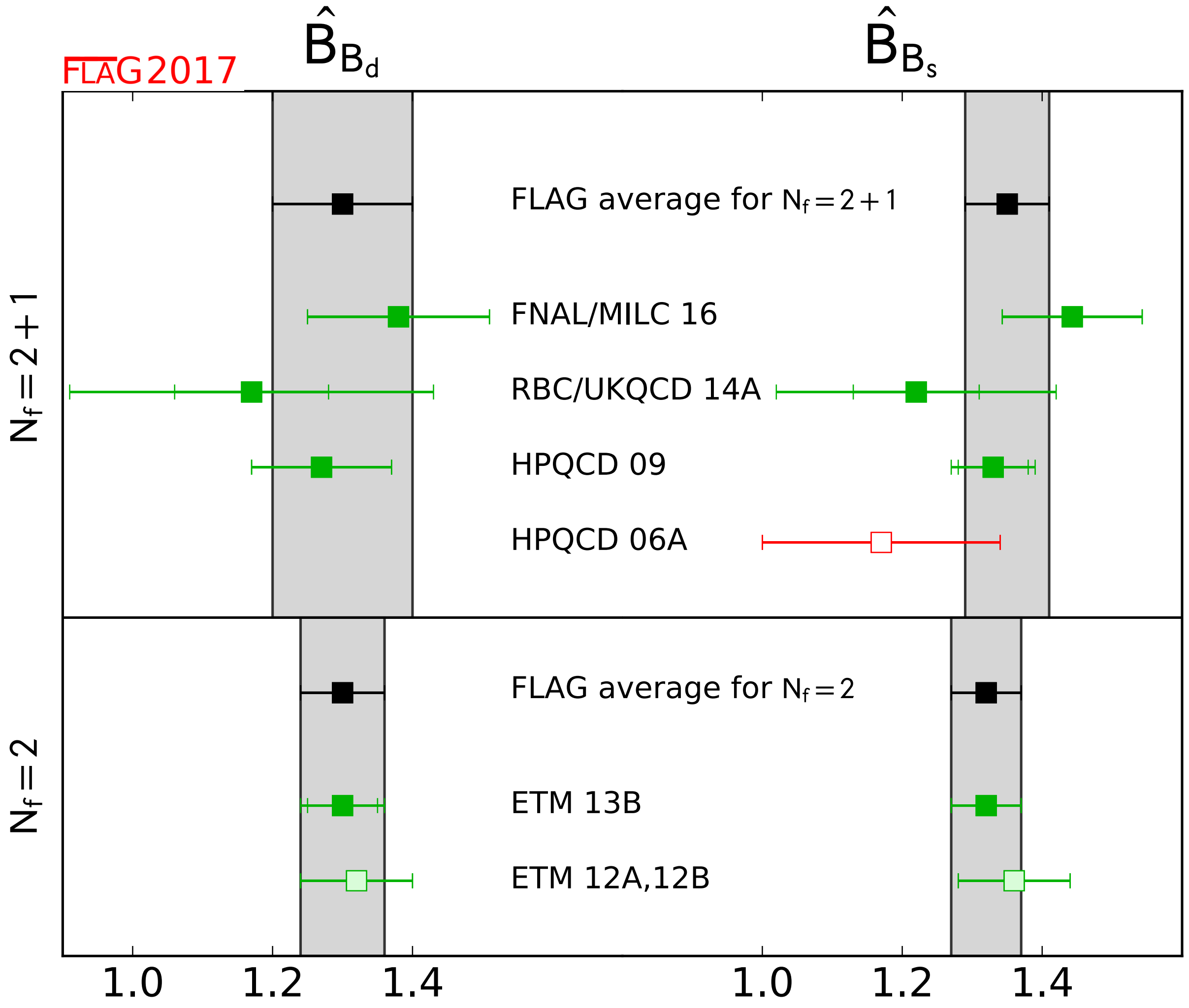
- channels with vectors in final state (e.g. K^*) much more complicated: treatment of resonances in Euclidean amplitudes quite non-trivial

- matrix elements of charmed penguins in effective Hamiltonian involve similar difficulties as their relatives in non-leptonic K and B decay — a notoriously difficult nut to crack. (bounds?)

FLAG-3 — *B* mixing







ballpark ≲ 10% accuracy, few computations; still hard work to do to constrain NP strongly

conclusions and outlook

- B-physics on the lattice making remarkable progress, most notably in semileptonic decays

- N.B. predictions for B_c leptonic and semileptonic rates

  [HPQCD arXiv:1503.05762/1605.05645/1611.01987]

- still way to go to meet the new era precision requirements
  - crosscheck HQ approaches as much as possible
  - full incorporation of available ensembles to HQ physics
  - many systematics to be improved: use of perturbation theory, momentum dependence of FFs, incorporation of QED effects, resonances ...

    [talk by F Sanfilippo]

- smart ways to improve our understanding of rare decays?

- decrease the lattice spacing and get direct access to the b region

conclusions and outlook

- much other interesting stuff going on:
  - precision of strong coupling, masses consistently below 1%
  - permille kaon physics, few % charm physics [G Salerno, D Mohler]
  - crack $K \to \pi\pi$; extend to heavy sector?
  - $K \to \pi\nu\bar{\nu}$ within reach (ongoing, RBC/UKQCD)

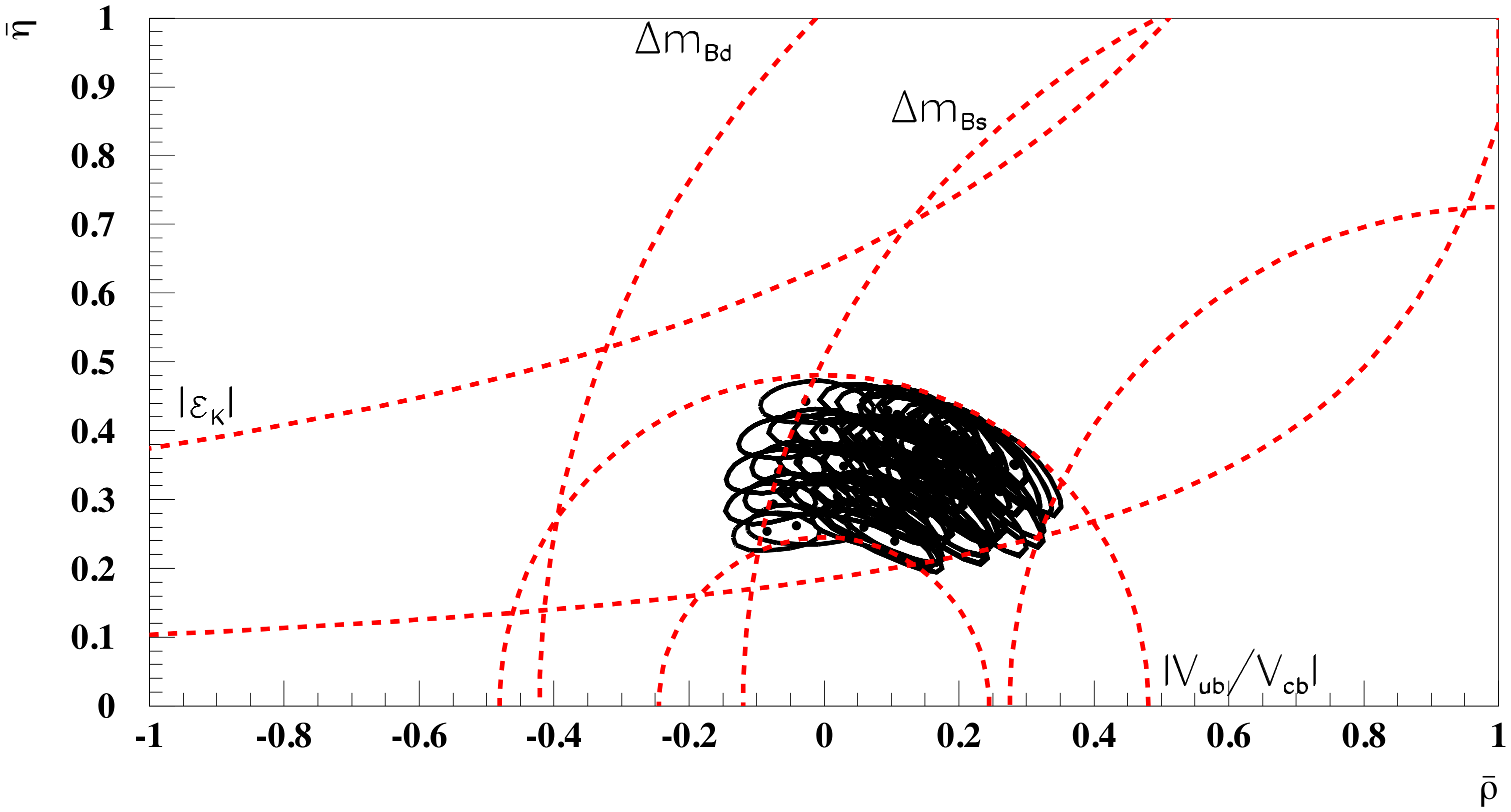
- non-flavour (SM parameters, $(g\text{-}2)_\mu$, PDFs, ...)

  [talks by G Colangelo and A Gérardin]

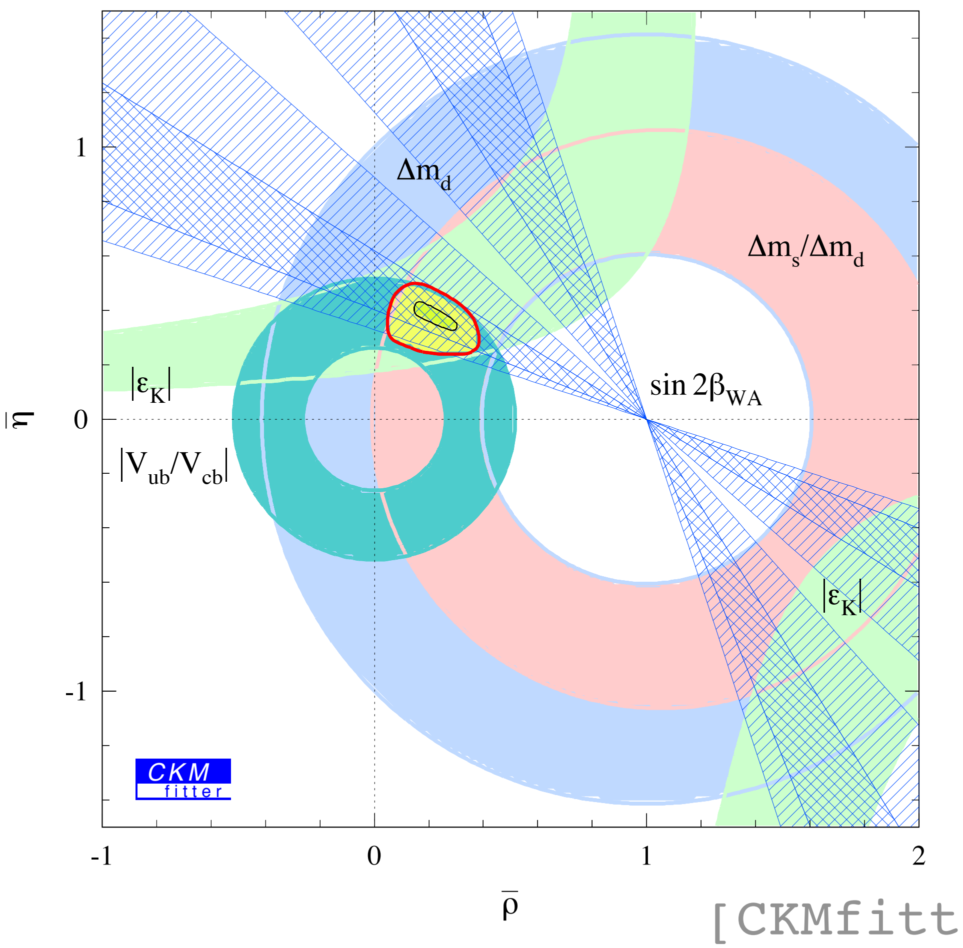
- **large programme aimed at keeping experimental pace**

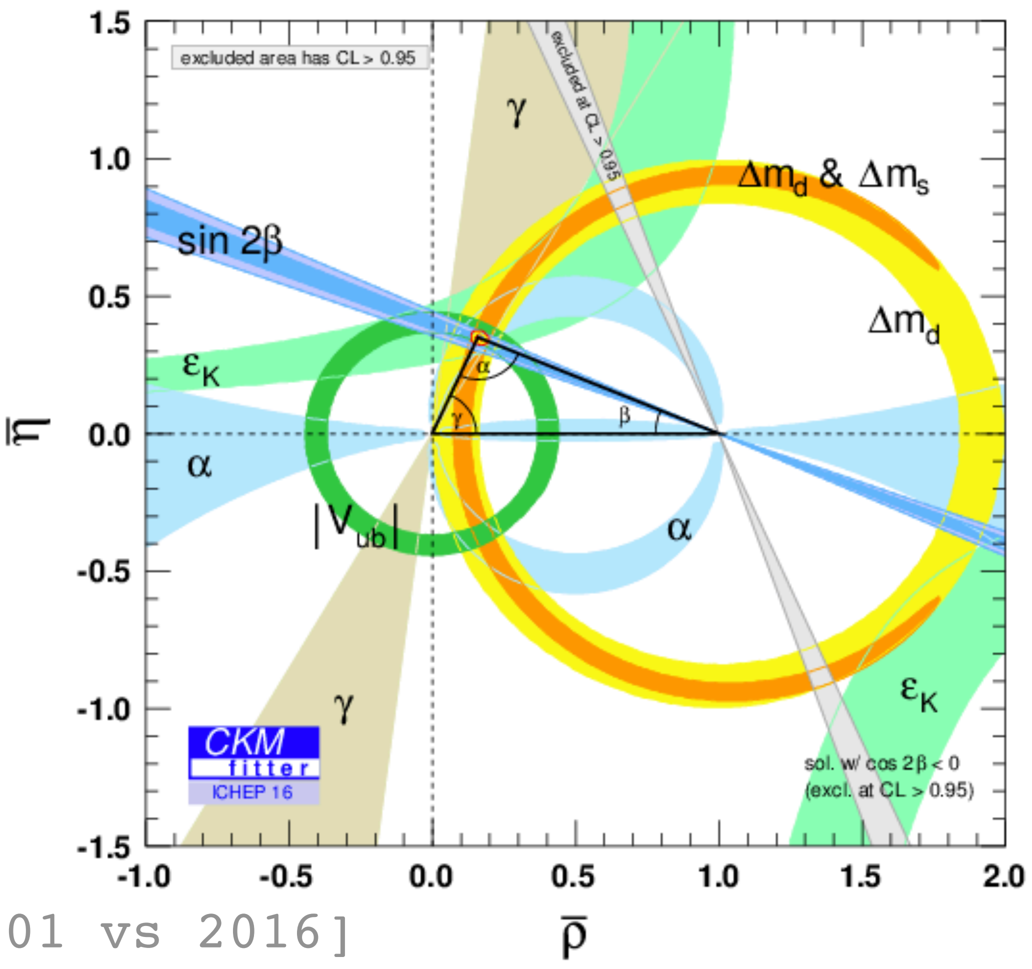
extra slides

why we care



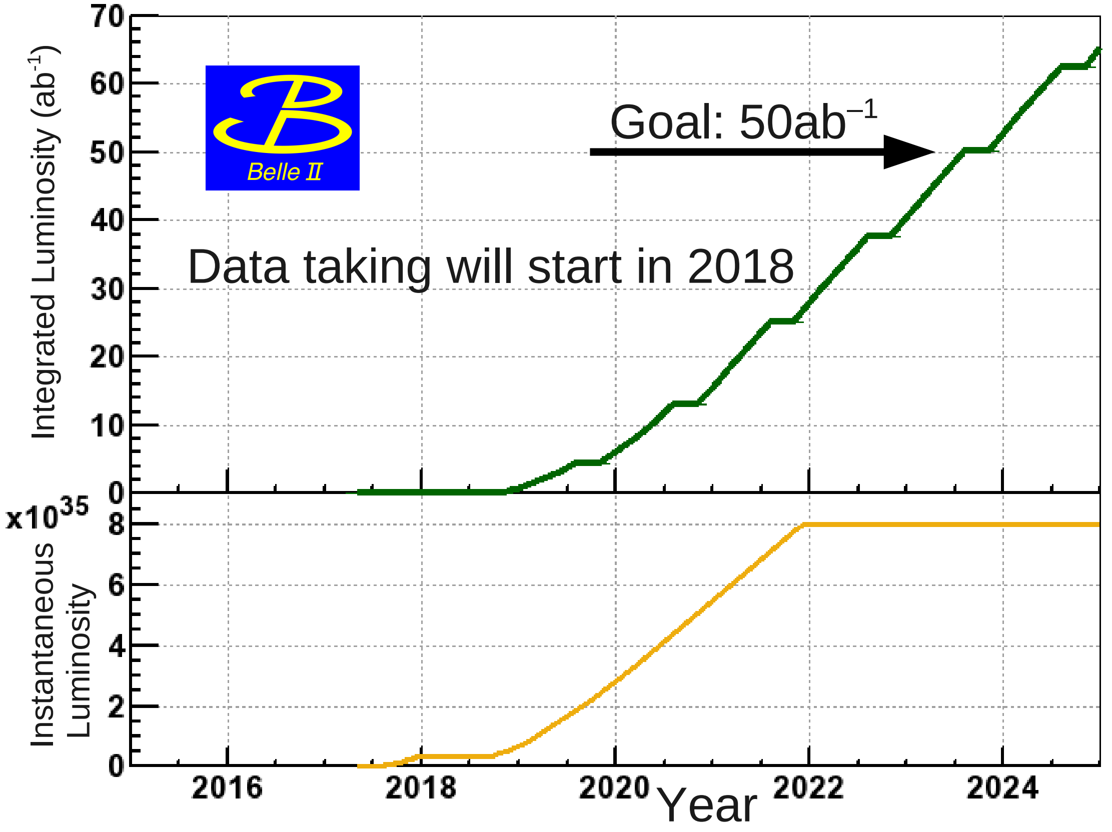
[BaBar Physics Book, 1999]

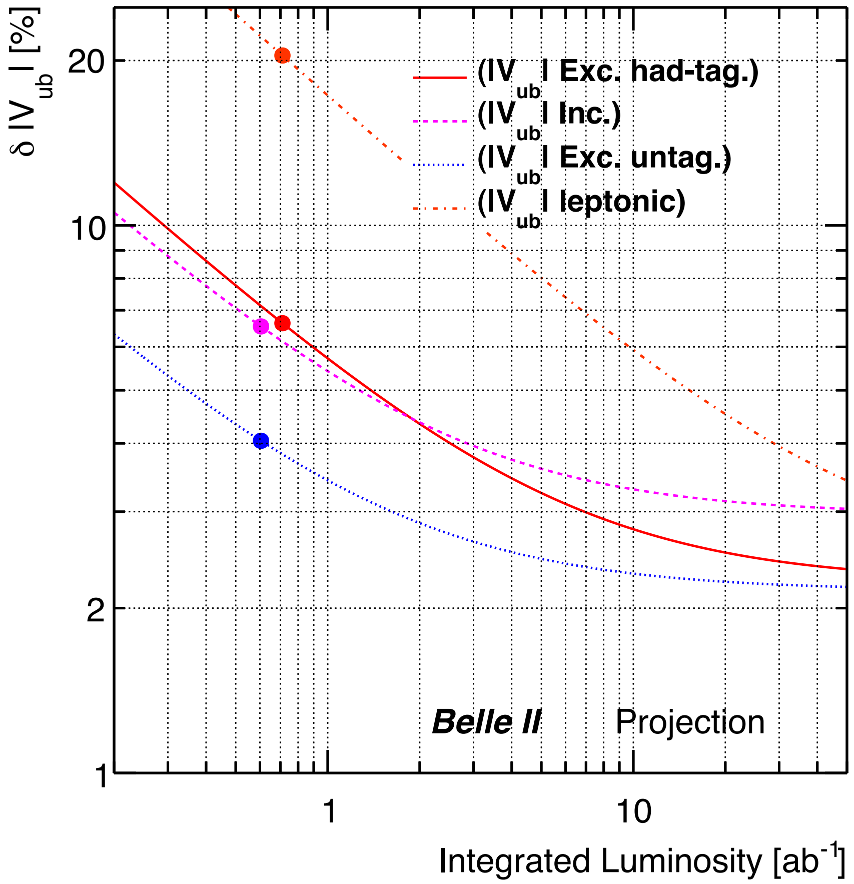




[CKMfitter 2001 vs 2016]

Belle II projections

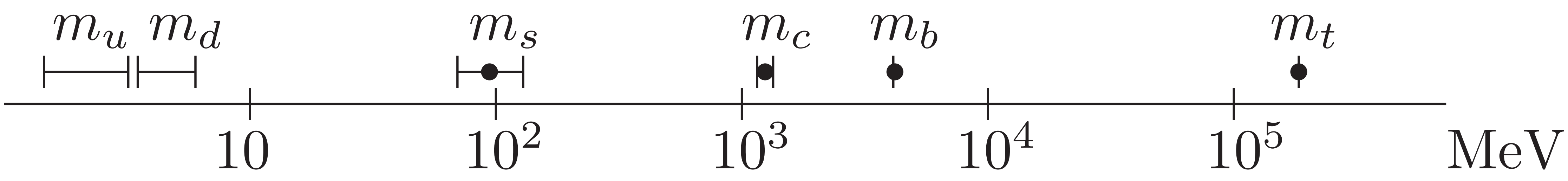




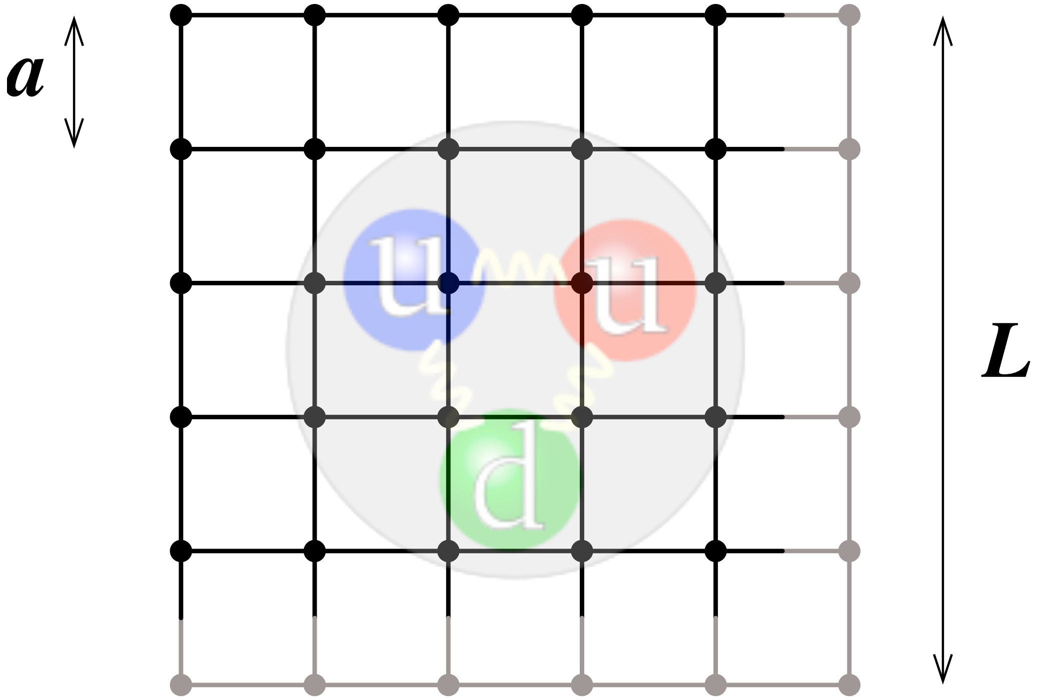
Lattice Quantity	CKM element	WA Expt. Error	Lattice error		
			2013 (Present)	2014	2018
$F(1)$ $(B \to D^* \ell \nu)$	$\|V_{cb}\|$	1.3	1.8	1.5	<1
$G(1)$ $(B \to D \ell \nu)$	$\|V_{cb}\|$	1.3	1.8	1.5	<1
$G_s(1)$ $(B_s \to D_s^* \ell \nu)$	$\|V_{cb}\|$	–	4.6	–	–
$\zeta(B \to \pi \ell \nu)$	$\|V_{ub}\|$	4.1	8.7	4	2
f_B $(B \to \tau \nu, \mu \nu)$	$\|V_{ub}\|$	9.0	2.5	1.5	<1
$R(D)(B \to D \tau \nu)$	–	13	4.3	4	< 2
Mixing $\zeta(\Delta m_d / \Delta m_s)$	$\|V_{td}\|/\|V_{ts}\|$	0.4	4.0	–	< 1

[BELLE2-NOTE-PH-2015-002, retrieved from B2TiP]

lattice QCD



first-principles approach = control all systematic uncertainties



- cover all relevant scales: $L^{-1} \ll \mu \ll a^{-1}$
- control scaling (exploit universality!) → renormalisation
- reconstruct Minkowskian amplitudes (non-trivial for multihadron final states)

complement with other first-principles/systematic approaches: dispersion relations, effective theories, ...

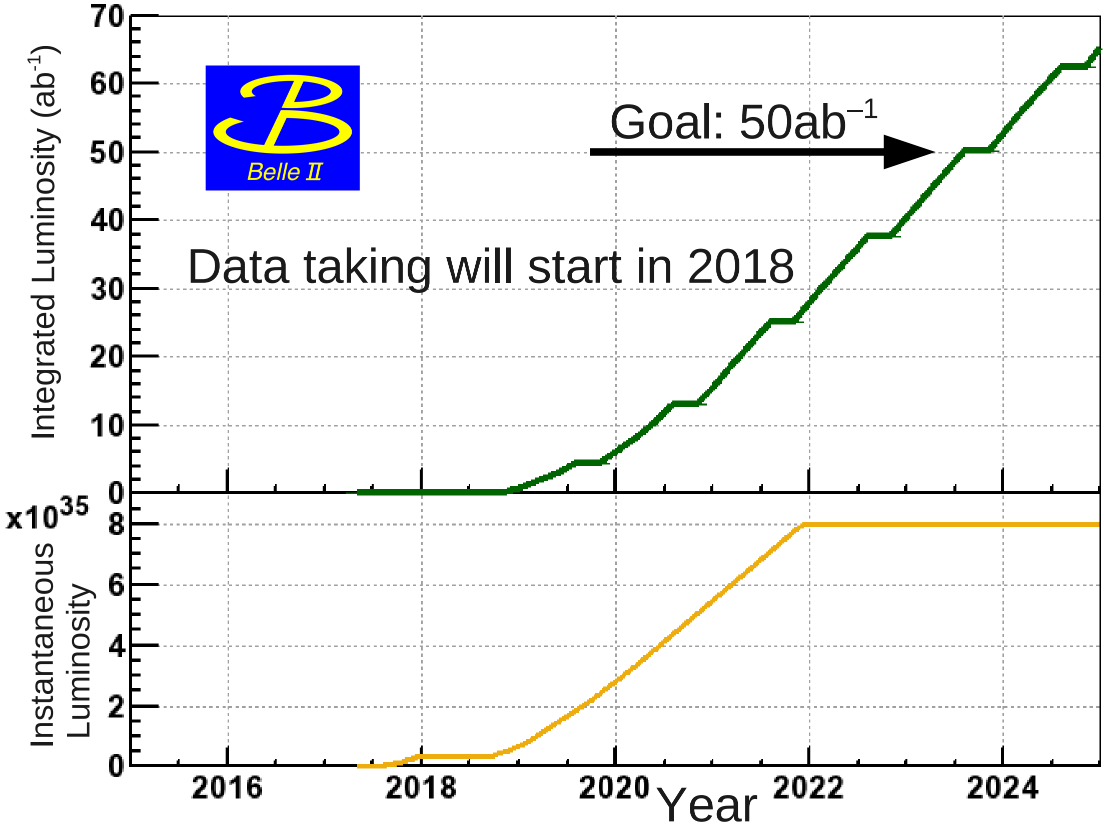
Belle II projections

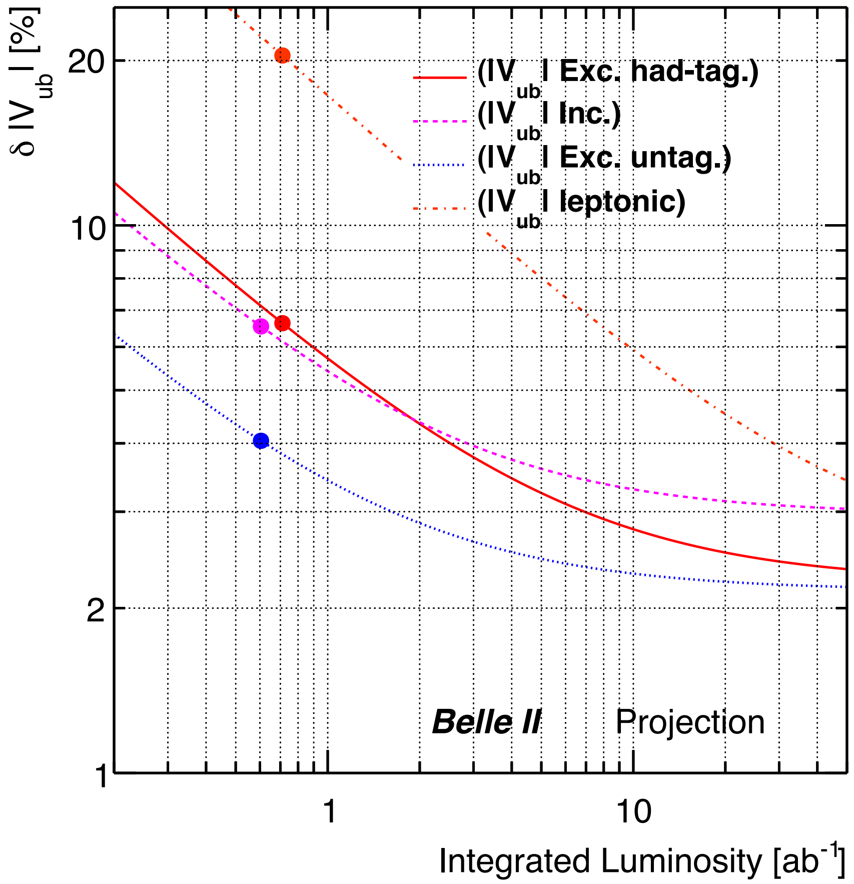
Observables	Belle (2014)	Belle II 5 ab^{-1}	 50 ab^{-1}	$\mathcal{L}_s$ [ab^{-1}]
$\sin 2\beta$	$0.667 \pm 0.023 \pm 0.012$	± 0.012	± 0.008	6
α		$\pm 2^\circ$	$\pm 1^\circ$	
γ	$\pm 14^\circ$	$\pm 6^\circ$	$\pm 1.5^\circ$	
$S(B \to \phi K^0)$	$0.90^{+0.09}_{-0.19}$	± 0.053	± 0.018	>50
$S(B \to \eta' K^0)$	$0.68 \pm 0.07 \pm 0.03$	± 0.028	± 0.011	>50
$S(B \to K^0_S K^0_S K^0_S)$	$0.30 \pm 0.32 \pm 0.08$	± 0.100	± 0.033	44
$\lvert V_{cb}\rvert$ incl.	$\pm 2.4\%$	$\pm 1.0\%$		< 1
$\lvert V_{cb}\rvert$ excl.	$\pm 3.6\%$	$\pm 1.8\%$	$\pm 1.4\%$	< 1
$\lvert V_{ub}\rvert$ incl.	$\pm 6.5\%$	$\pm 3.4\%$	$\pm 3.0\%$	2
$\lvert V_{ub}\rvert$ excl. (had. tag.)	$\pm 10.8\%$	$\pm 4.7\%$	$\pm 2.4\%$	20
$\lvert V_{ub}\rvert$ excl. (untag.)	$\pm 9.4\%$	$\pm 4.2\%$	$\pm 2.2\%$	3
$\mathcal{B}(B \to \tau\nu)$ [10^{-6}]	96 ± 26	$\pm 10\%$	$\pm 5\%$	46
$\mathcal{B}(B \to \mu\nu)$ [10^{-6}]	< 1.7	5σ	$>> 5\sigma$	>50
$R(B \to D\tau\nu)$	$\pm 16.5\%$	$\pm 5.6\%$	$\pm 3.4\%$	4
$R(B \to D^*\tau\nu)$	$\pm 9.0\%$	$\pm 3.2\%$	$\pm 2.1\%$	3
$\mathcal{B}(B \to K^{*+}\nu\overline{\nu})$ [10^{-6}]	< 40		$\pm 30\%$	>50
$\mathcal{B}(B \to K^{+}\nu\overline{\nu})$ [10^{-6}]	< 55		$\pm 30\%$	>50
$\mathcal{B}(B \to X_s\gamma)$ [10^{-6}]	$\pm 13\%$	$\pm 7\%$	$\pm 6\%$	< 1
$A_{CP}(B \to X_s\gamma)$		± 0.01	± 0.005	8
$S(B \to K^0_S\pi^0\gamma)$	$-0.10 \pm 0.31 \pm 0.07$	± 0.11	± 0.035	> 50
$S(B \to \rho\gamma)$	$-0.83 \pm 0.65 \pm 0.18$	± 0.23	± 0.07	> 50
C_7/C_9 $(B \to X_s\ell\ell)$	$\sim 20\%$	10%	5%	
$\mathcal{B}(B_s \to \gamma\gamma)$ [10^{-6}]	< 8.7	± 0.3		
$\mathcal{B}(B_s \to \tau^+\tau^-)$ [10^{-3}]		< 2		

Observables	Belle (2014)	Belle II 5 ab^{-1}	 50 ab^{-1}	$\mathcal{L}_s$ [ab^{-1}]
$\mathcal{B}(D_s \to \mu\nu)$	$5.31 \times 10^{-3}(1 \pm 0.053 \pm 0.038)$	$\pm 2.9\%$	$\pm(0.9\%$-$1.3\%)$	> 50
$\mathcal{B}(D_s \to \tau\nu)$	$5.70 \times 10^{-3}(1 \pm 0.037 \pm 0.054)$	$\pm(3.5\%$-$4.3\%)$	$\pm(2.3\%$-$3.6\%)$	3-5
y_{CP} [10^{-2}]	$1.11 \pm 0.22 \pm 0.11$	$\pm(0.11$-$0.13)$	$\pm(0.05$-$0.08)$	5-8
A_Γ [10^{-2}]	$-0.03 \pm 0.20 \pm 0.08$	± 0.10	$\pm(0.03$-$0.05)$	7 - 9
$A_{CP}^{K^+K^-}$ [10^{-2}]	$-0.32 \pm 0.21 \pm 0.09$	± 0.11	± 0.06	15
$A_{CP}^{\pi^+\pi^-}$ [10^{-2}]	$0.55 \pm 0.36 \pm 0.09$	± 0.17	$\pm$ 0.06	> 50
$A_{CP}^{\phi\gamma}$ [10^{-2}]	$\pm$ 5.6	± 2.5	± 0.8	> 50
$x^{K_S\pi^+\pi^-}$ [10^{-2}]	$0.56 \pm 0.19 \pm {}^{0.07}_{0.13}$	± 0.14	± 0.11	3
$y^{K_S\pi^+\pi^-}$ [10^{-2}]	$0.30 \pm 0.15 \pm {}^{0.05}_{0.08}$	± 0.08	± 0.05	15
$\lvert q/p\rvert^{K_S\pi^+\pi^-}$	$0.90 \pm {}^{0.16}_{0.15} \pm {}^{0.08}_{0.06}$	± 0.10	± 0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ [$^\circ$]	$-6 \pm 11 \pm {}^{4}_{5}$	± 6	± 4	10
$A_{CP}^{\pi^0\pi^0}$ [10^{-2}]	$-0.03 \pm 0.64 \pm 0.10$	± 0.29	± 0.09	> 50
$A_{CP}^{K^0_S\pi^0}$ [10^{-2}]	$-0.10 \pm 0.16 \pm 0.09$	± 0.08	± 0.03	> 50
$Br(D^0 \to \gamma\gamma)$ [10^{-6}]	< 1.5	$\pm 30\%$	$\pm 25\%$	2
	$\tau \to \mu\gamma$ [10^{-9}]	< 45	< 14.7	< 4.7
	$\tau \to e\gamma$ [10^{-9}]	< 120	< 39	< 12
	$\tau \to \mu\mu\mu$ [10^{-9}]	< 21.0	< 3.0	< 0.3

[BELLE2-NOTE-PH-2015-002, retrieved from B2TiP]

Belle II projections





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[BELLE2-NOTE-PH-2015-002, retrieved from B2TiP]

LHCb Run 2 + upgrade projections

Type	Observable	LHC Run 1	LHCb 2018	**LHCb upgrade**	Theory
B_s^0 mixing	$\phi_s(B_s^0 \to J/\psi\,\phi)$ (rad)	0.049	0.025	**0.009**	~ 0.003
	$\phi_s(B_s^0 \to J/\psi\, f_0(980))$ (rad)	0.068	0.035	**0.012**	~ 0.01
	$A_{\rm sl}(B_s^0)$ (10^{-3})	2.8	1.4	**0.5**	0.03
Gluonic penguin	$\phi_s^{\rm eff}(B_s^0 \to \phi\phi)$ (rad)	0.15	0.10	**0.018**	0.02
	$\phi_s^{\rm eff}(B_s^0 \to K^{*0}\bar{K}^{*0})$ (rad)	0.19	0.13	**0.023**	< 0.02
	$2\beta^{\rm eff}(B^0 \to \phi K_{\rm S}^0)$ (rad)	0.30	0.20	**0.036**	0.02
Right-handed currents	$\phi_s^{\rm eff}(B_s^0 \to \phi\gamma)$ (rad)	0.20	0.13	**0.025**	< 0.01
	$\tau^{\rm eff}(B_s^0 \to \phi\gamma)/\tau_{B_s^0}$	5%	3.2%	**0.6%**	0.2 %
Electroweak penguin	$S_3(B^0 \to K^{*0}\mu^+\mu^-; 1 < q^2 < 6\,{\rm GeV}^2/c^4)$	0.04	0.020	**0.007**	0.02
	$q_0^2\, A_{\rm FB}(B^0 \to K^{*0}\mu^+\mu^-)$	10%	5%	**1.9%**	$\sim 7\%$
	$A_{\rm I}(K\mu^+\mu^-; 1 < q^2 < 6\,{\rm GeV}^2/c^4)$	0.09	0.05	**0.017**	~ 0.02
	$\mathcal{B}(B^+ \to \pi^+\mu^+\mu^-)/\mathcal{B}(B^+ \to K^+\mu^+\mu^-)$	14%	7%	**2.4%**	$\sim 10\%$
Higgs penguin	$\mathcal{B}(B_s^0 \to \mu^+\mu^-)$ (10^{-9})	1.0	0.5	**0.19**	0.3
	$\mathcal{B}(B^0 \to \mu^+\mu^-)/\mathcal{B}(B_s^0 \to \mu^+\mu^-)$	220%	110%	**40%**	$\sim 5\,\%$
Unitarity triangle angles	$\gamma(B \to D^{(*)}K^{(*)})$	7°	4°	**0.9°**	negligible
	$\gamma(B_s^0 \to D_s^{\mp}K^{\pm})$	17°	11°	**2.0°**	negligible
	$\beta(B^0 \to J/\psi\, K_{\rm S}^0)$	1.7°	0.8°	**0.31°**	negligible
Charm *CP* violation	$A_\Gamma(D^0 \to K^+K^-)$ (10^{-4})	3.4	2.2	**0.4**	–
	ΔA_{CP} (10^{-3})	0.8	0.5	**0.1**	–

[LHCb-PUB-2014-040]

n.b.: LHCb making key contributions (B_s leptonic, Λ_b, ...)

QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2g^2}\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \sum_{q=1}^{N_{\mathrm{f}}} \bar{\psi}_q \left[i\not{D} - m_q\right] \psi_q \underbrace{+ \frac{i\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left[F_{\mu\nu}F_{\rho\sigma}\right]}_{\not{\mathrm{C}}\not{\mathrm{P}}}$$

$1 + N_{\mathrm{f}} + 1$ free parameters:

α_{s}, m_q ⟶ fixed by hadron masses/decay constants

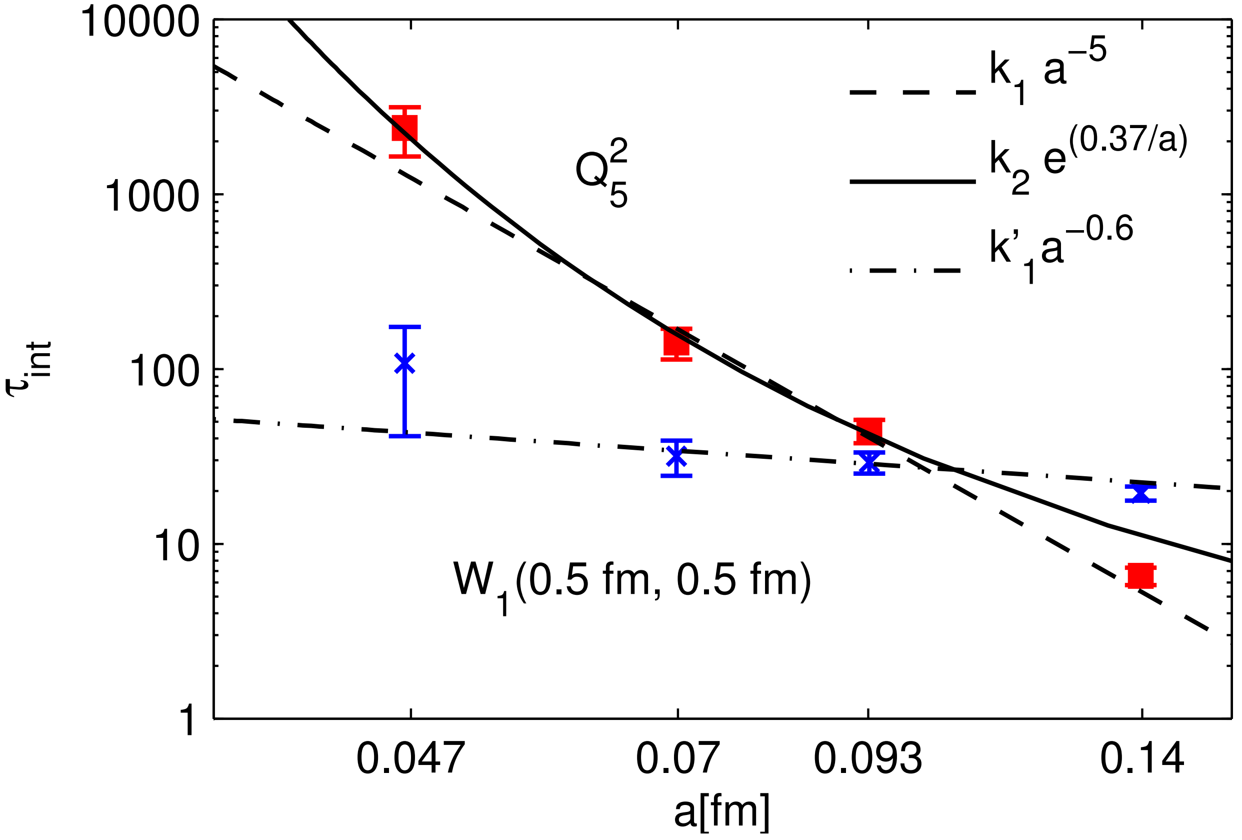
$\theta = 0$ ⟶ fixed by neutron EDM

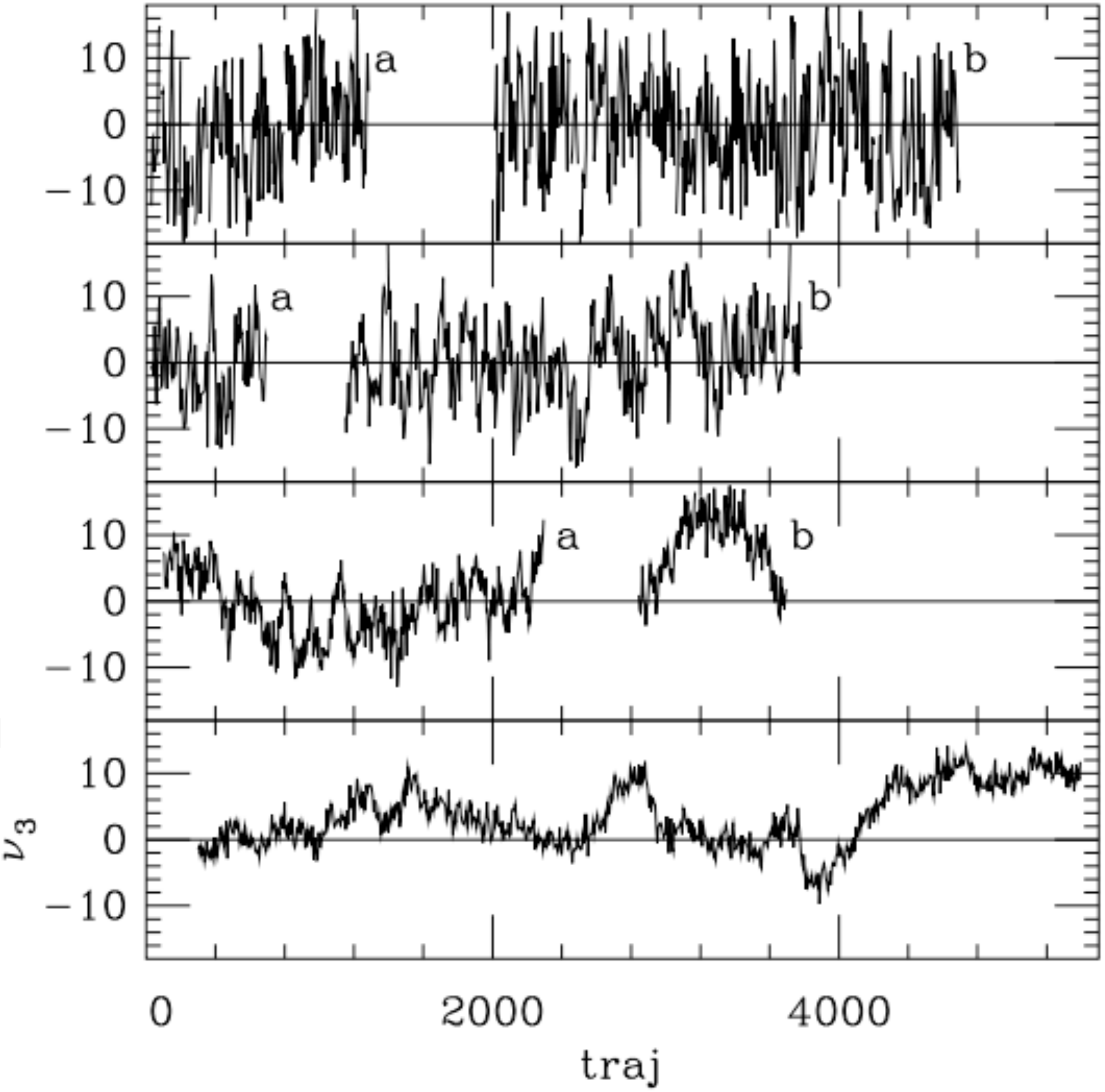
once the Lagrangian parameters are fixed, everything else is a prediction

calculations in the hadronic regime are challenging: low-energy QCD is strongly non-perturbative

towards a fully relativistic b

crucial issue: strong lattice space dependence of autocorrelations





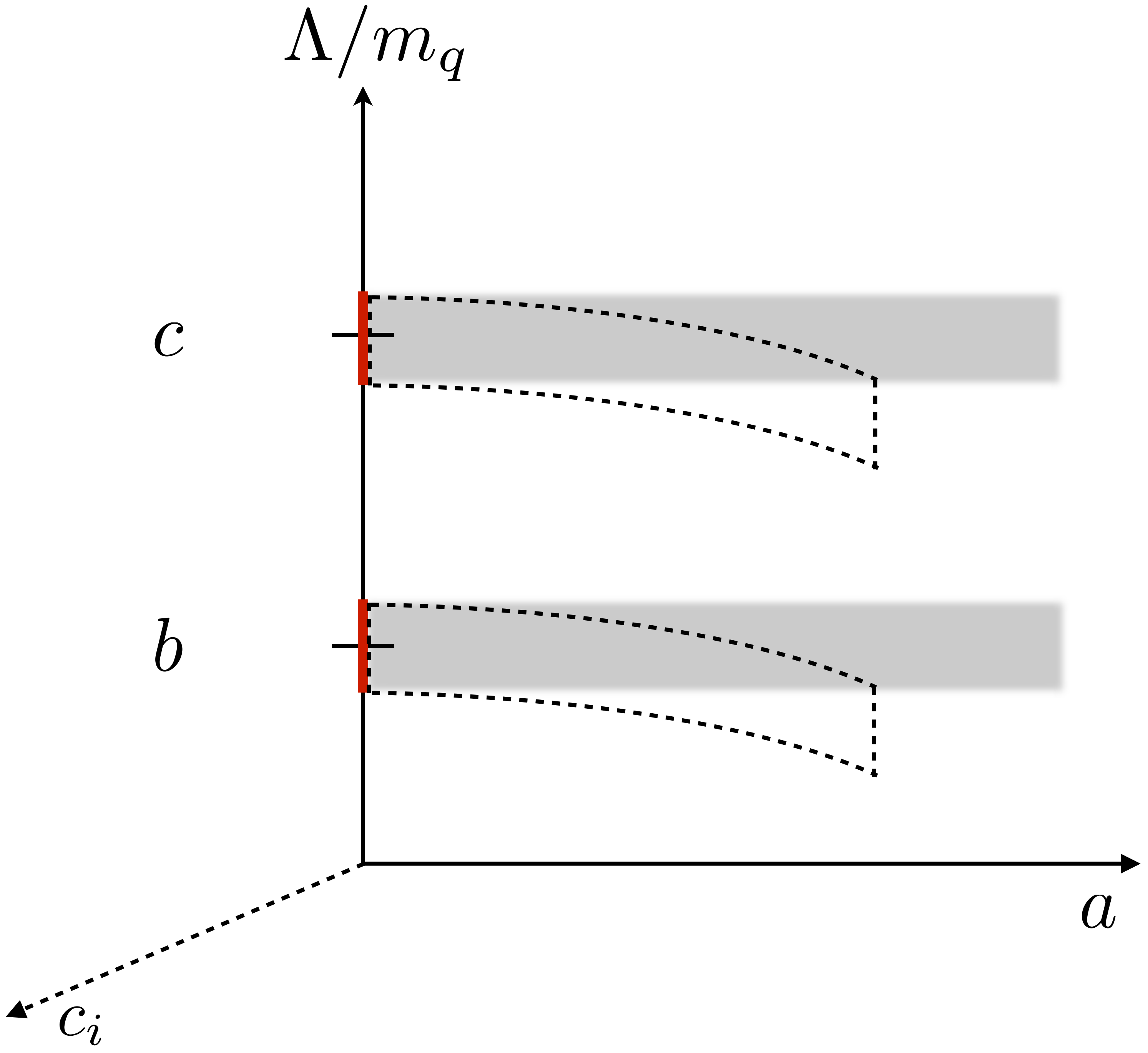
[Del Debbio, Panagopoulos, Vicari 2002]
[Schaefer, Sommer, Virotta 2010]

[Lüscher, Schaefer 2011; CLS N_f=2+1 obc programme]
[Mages et al. 2015; Laio et al. 2015; Brower et al. 2015; Detmold, Endres 2016]

[MILC N_f=2+1 ensembles]

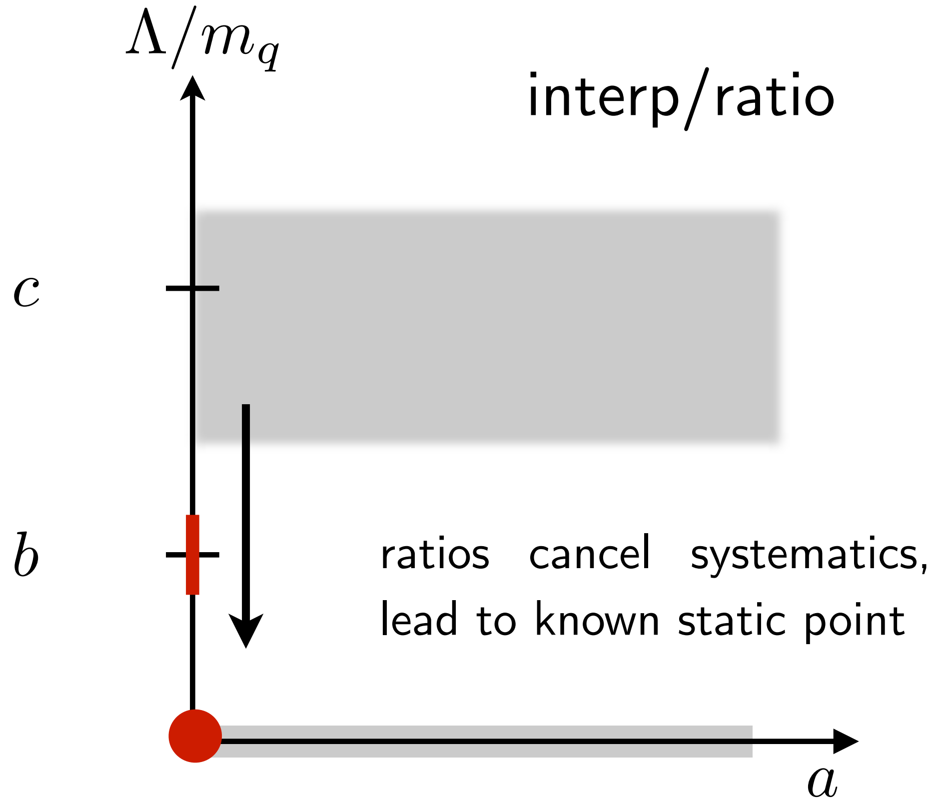
approaches to B physics

what one would like to do



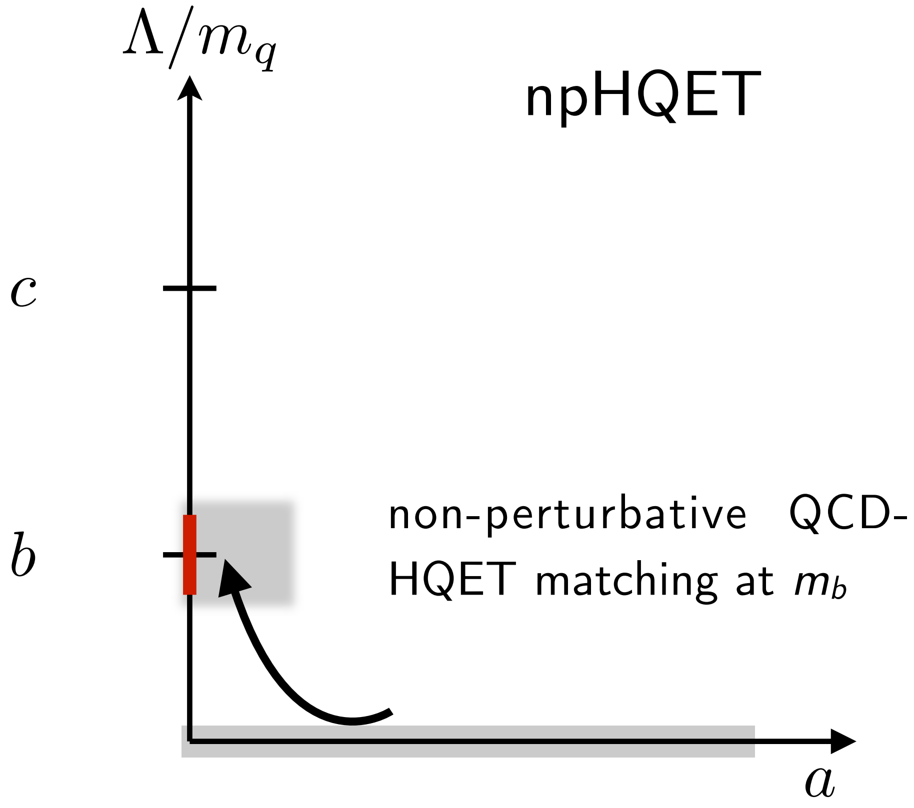
approaches to B physics

effective theory used differently, different pros/cons balance: **crosschecks crucial**



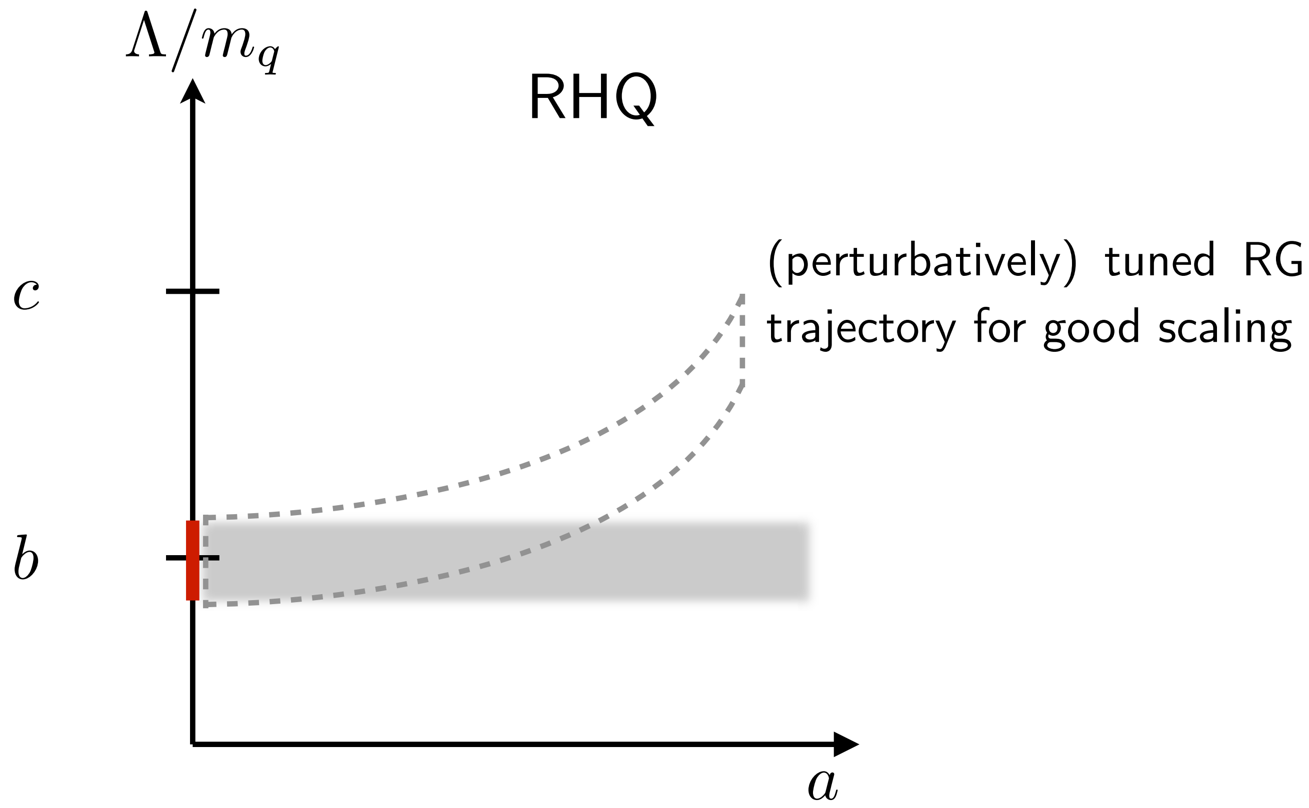
interp/ratio

ratios cancel systematics, lead to known static point



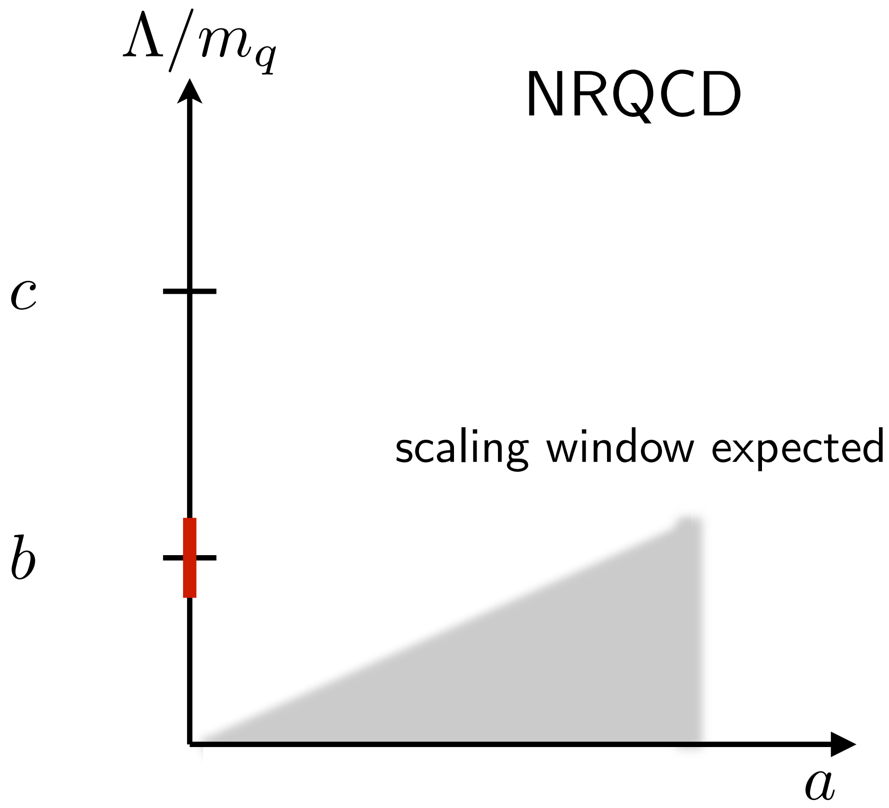
npHQET

non-perturbative QCD-HQET matching at m_b



RHQ

(perturbatively) tuned RG trajectory for good scaling



NRQCD

scaling window expected

FLAG

Flavour Lattice Averaging Group: your one-stop repository of lattice results, world averages / estimates

advisory board: S. Aoki, C. Bernard, H. Leutwyler, C. Sachrajda

editorial board: G. Colangelo, S. Hashimoto, A. Jüttner, S. Sharpe, A. Vladikas, U. Wenger

working groups:

quark masses	T. Blum, L. Lellouch, V. Lubicz
V_{ud}, V_{us}	P. Boyle, T. Kaneko, S. Simula
LECs	S. Dürr, H. Fukaya, U. Heller
kaon mixing	P. Dimopoulos, B. Mawhinney, H. Wittig
α_s	R. Horsley, T. Onogi, R. Sommer
heavy leptonic + mixing	Y. Aoki, M. Della Morte, D. Lin
heavy semileptonic	D. Bečirević, S. Gottlieb, E. Lunghi, CP

3rd edition: results up to 30/11/2015 + updates from 2016

[Aoki et al., EPJC (2017) 77:112]

FLAG

Flavour Lattice Averaging Group: your one-stop repository of lattice results, world averages / estimates

advisory board: S Aoki, M Golterman, R Van De Water, A Vladikas

editorial board: G Colangelo, S Hashimoto, A Jüttner, S Sharpe, U Wenger

working groups:

quark masses	T Blum, A Portelli, A Ramos
V_{us}	T Kaneko, J Simone, S Simula
LECs	S Dürr, H Fukaya, U Heller
kaon mixing	P Dimopoulos, G Herdoíza, B Mawhinney
α_s	R. Horsley, T. Onogi, R. Sommer
heavy leptonic + mixing	Y. Aoki, M. Della Morte, D. Lin
heavy semileptonic	D. Bečirević, S. Gottlieb, E. Lunghi, CP
NME	S Collins, R Gupta, A Nicholson, H Wittig

lineup for 4th edition (due early 2019)

FLAG

what FLAG provides for each quantity:

- complete list of references
- summary of relevant formulae and notation
- quick-look summary tables
- quality assessment of computation setup: colour-coded tables
- averages/estimates (if sensible)
- a “lattice dictionary” for non-experts
- thorough appendix tables with details of all computations for experts
- between-editions updates at http://itpwiki.unibe.ch/flag

cite the original works!

FLAG

tables:

★/✓ allows for satisfactory control of systematics

○ allows for reasonable (but improvable) estimate of systematics

■ unlikely to allow for reasonable control of systematics

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization/ matching	Heavy-quark treatment	f_{B_s}/f_{B^+}	f_{B_s}/f_{B^0}	f_{B_s}/f_B
ETM 13E	[456]	$2+1+1$	C	★	○	○	○	✓	–	–	1.201(25)
HPQCD 13	[52]	$2+1+1$	A	★	★	★	○	✓	1.217(8)	1.194(7)	1.205(7)
RBC/UKQCD 14	[53]	$2+1$	A	○	○	○	○	✓	1.223(71)	1.197(50)	–
RBC/UKQCD 14A	[54]	$2+1$	A	○	○	○	○	✓	–	–	1.193(48)
RBC/UKQCD 13A	[457]	$2+1$	C	○	○	○	○	✓	–	–	1.20(2)$_{\text{stat}}$[a]
HPQCD 12	[55]	$2+1$	A	○	○	○	○	✓	–	–	1.188(18)
FNAL/MILC 11	[48]	$2+1$	A	○	○	★	○	✓	1.229(26)	–	–
RBC/UKQCD 10C	[464]	$2+1$	A	■	■	■	○	✓	–	–	1.15(12)
HPQCD 09	[59]	$2+1$	A	○	○	○	○	✓	–	–	1.226(26)
ALPHA 14	[57]	2	A	★	★	★	★	✓	–	–	1.203(65)
ALPHA 13	[458]	2	C	★	★	★	★	✓	–	–	1.195(61)(20)
ETM 13B, 13C[b]	[20,58]	2	A	★	○	★	○	✓	–	–	1.206(24)
ALPHA 12A	[459]	2	C	★	★	★	★	✓	–	–	1.13(6)
ETM 12B	[460]	2	C	★	○	★	○	✓	–	–	1.19(5)
ETM 11A	[182]	2	A	○	○	★	○	✓	–	–	1.19(5)

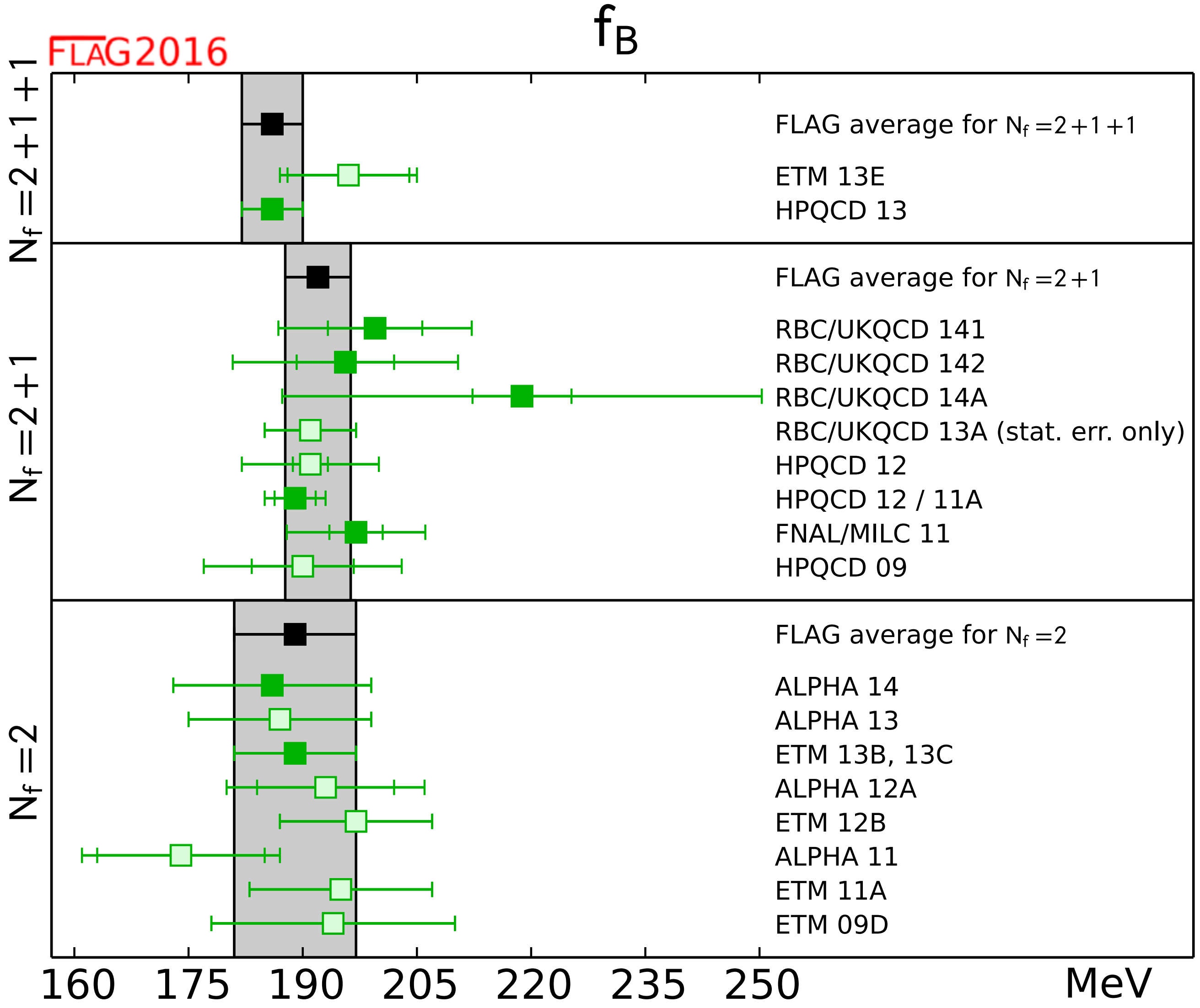
[a] Statistical errors only

[b] Update of ETM 11A and 12B

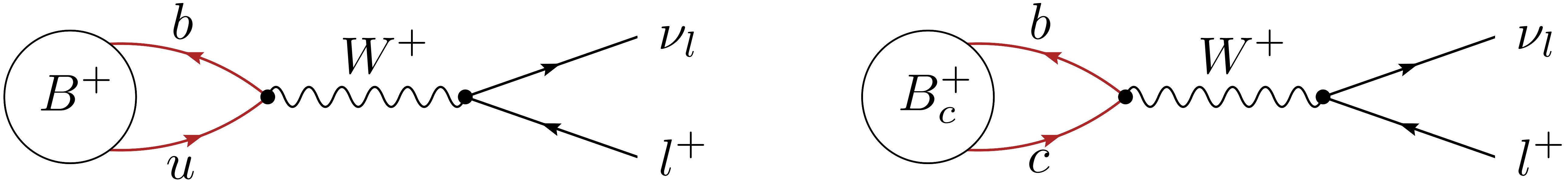
FLAG

plots:

- 🟩 result included in average or estimate
- □ result OK but not included (e.g. superseded, unpublished, ...)
- □ all other results



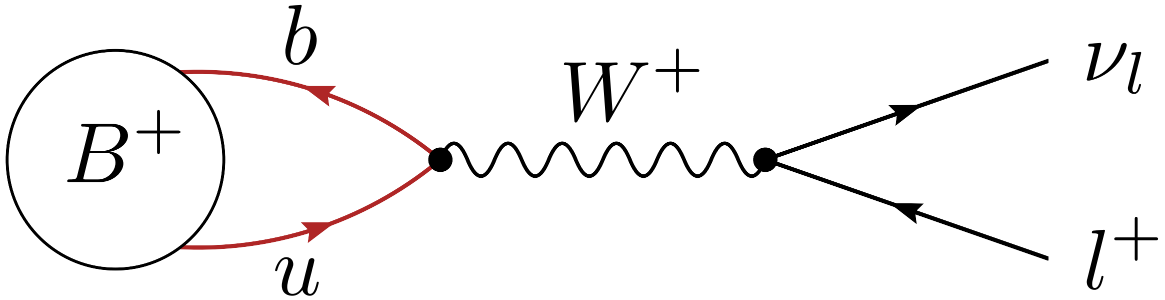
B leptonic decay

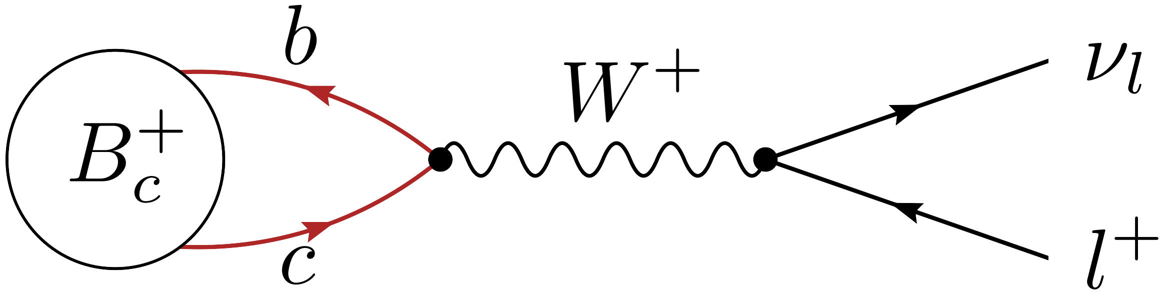


$$\frac{\mathcal{B}(B_{(c)} \to l\nu_l)}{\tau_{B_{(c)}}} = \frac{G_{\mathrm{F}}^2}{8\pi} m_l^2 m_{B_{(c)}} \left(1 - \frac{m_l^2}{m_{B^2_{(c)}}}\right)^2 |V_{qb}|^2 \, f^2_{B_{(c)}} \quad \text{[+ h.o. OPE]}$$

$$\langle 0|\bar{b}\gamma^\mu\gamma^5 q|B_q(p)\rangle = f_{B_q}\, p^\mu$$

B leptonic decay





$$\frac{\mathcal{B}(B \to l\nu_l)}{\underline{\tau_B}} = \underline{\frac{G_F^2}{8\pi} m_l^2 m_B \left(1 - \frac{m_l^2}{m_B^2}\right)^2} |V_{ub}|^2 f_B^2 \quad \underline{\text{[+ h.o. OPE]}}$$

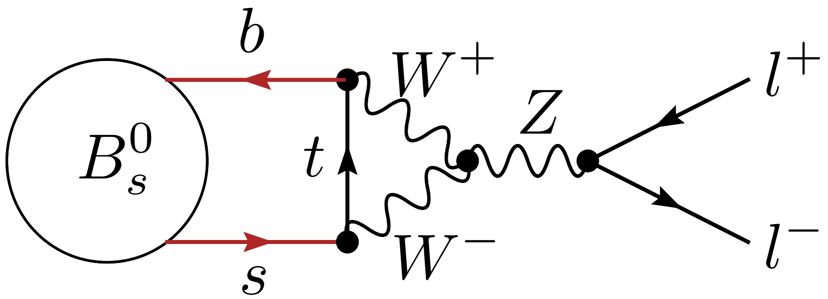
(negligible)

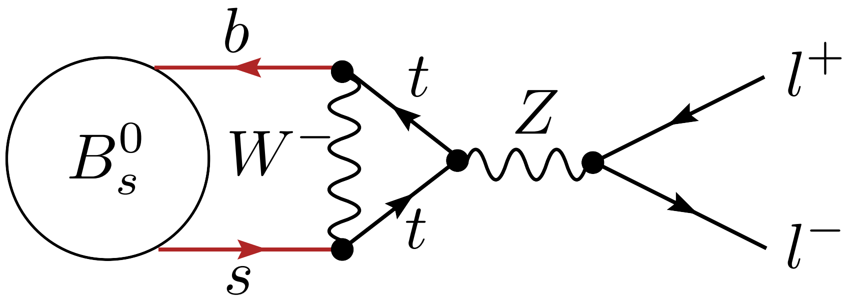
$\Delta = 0.2\%$

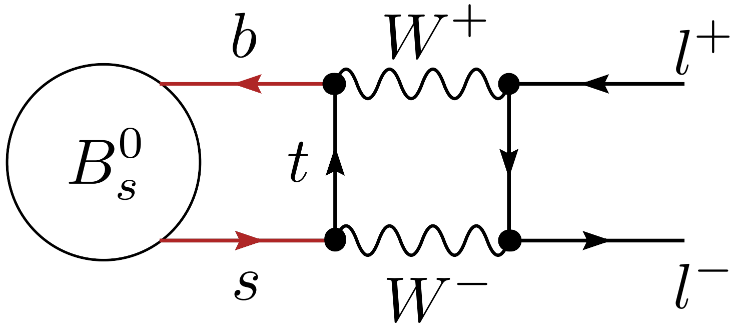
$\Delta \sim \mathcal{O}\left(\frac{m_B^2}{M_W^2}\right) \sim 0.4\%$

(+ e.m. corrections!)

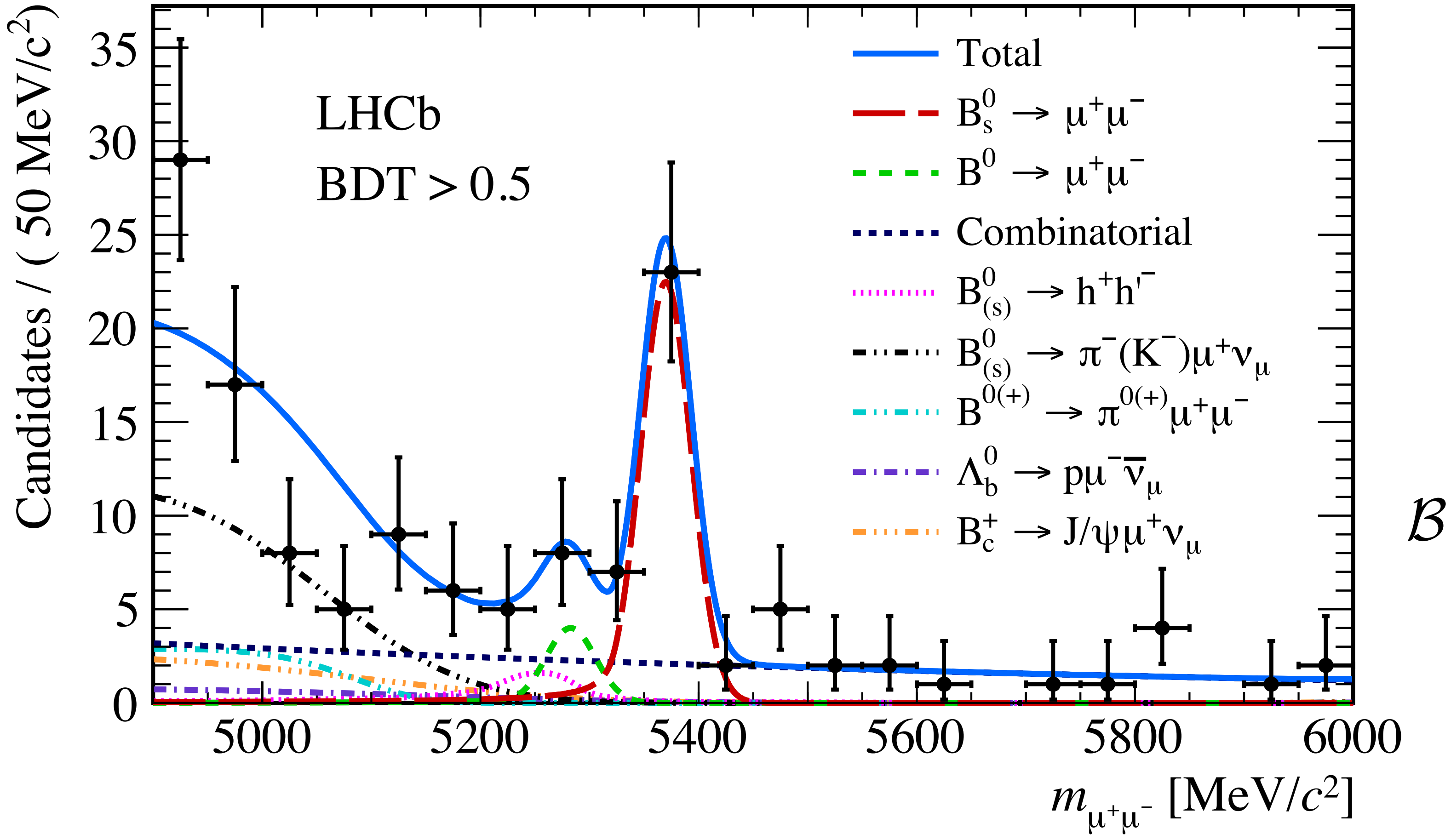
B leptonic decay







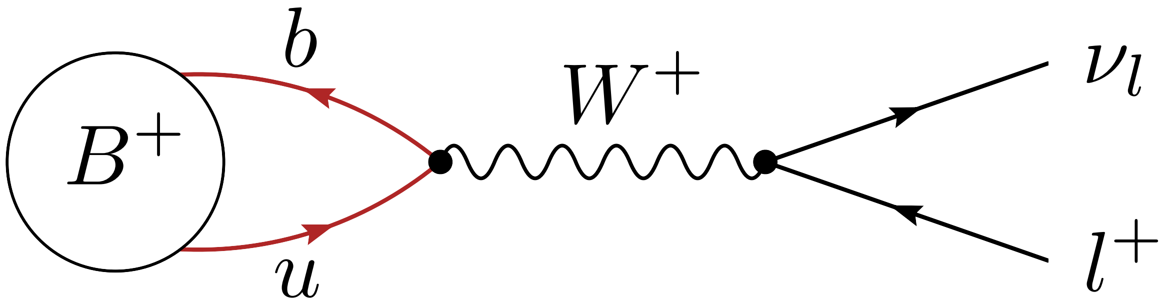
$$\frac{\mathcal{B}(B_q \to l^+ l^-)}{\tau_{B_q}} = \frac{G_F^2}{\pi}\, Y\left(\frac{\alpha}{4\pi\sin^2\theta_W}\right)^2 m_{B_q} m_l^2 \sqrt{1-4\frac{m_l^2}{m_{B_q}^2}}\, |V_{tb}^* V_{tq}|^2\, f_{B_q}^2$$

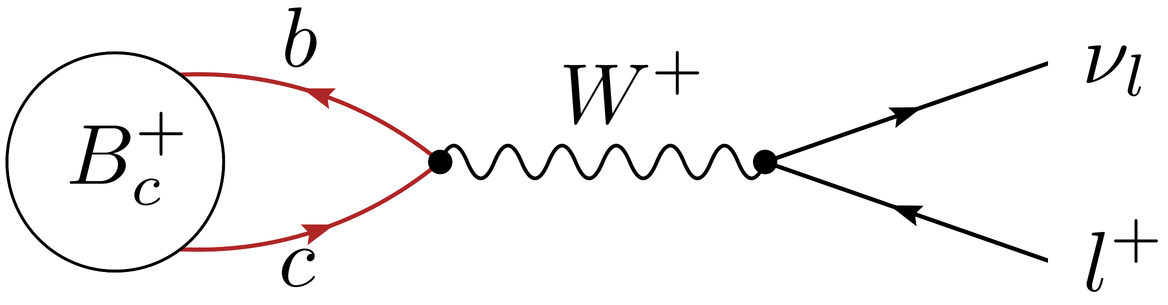


$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$$

[LHCb Nature 522 (2015) 68]
[LHCb PRL 118 (2017) 191801]

B leptonic decay





$$\frac{\mathcal{B}(B_c \to l\nu_l)}{\underline{\tau_{B_c}}} = \underline{\frac{G_{\mathrm{F}}^2}{8\pi} m_l^2 m_{B_c} \left(1 - \frac{m_l^2}{m_{B_c^2}}\right)^2} \; |V_{qb}|^2 \, f_{B_c}^2 \quad \underline{[+ \text{ h.o. OPE}]}$$

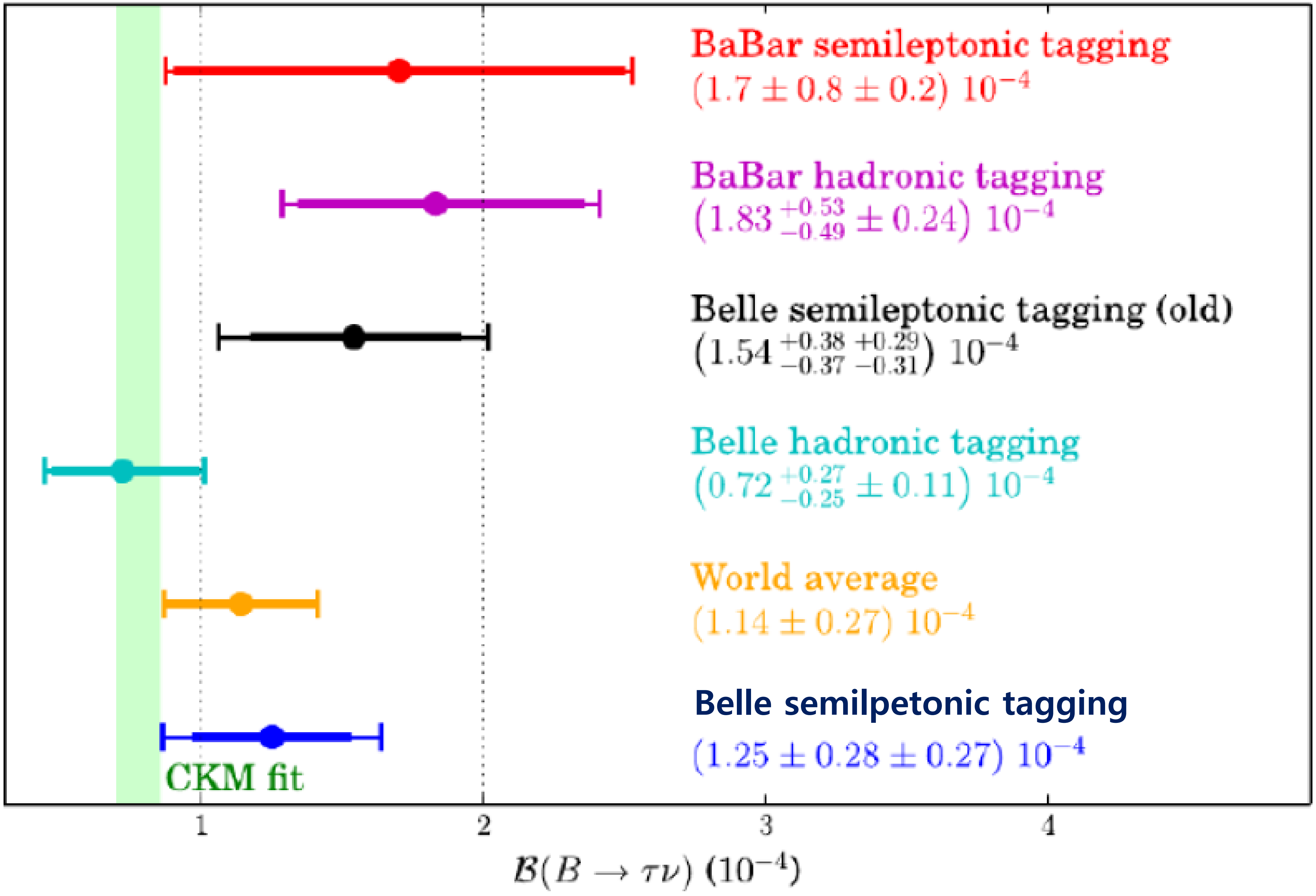
(negligible)

$$\Delta = 7.3\%$$

$$\Delta \sim \mathcal{O}\left(\frac{m_{B_c}^2}{M_W^2}\right) \sim 0.6\%$$

(+ e.m. corrections!)

B leptonic decay



[from C Park's talk at FPCP 2015]

Belle II projections

Observables	Belle	Belle II		$\mathcal{L}_s$
	(2014)	5 ab^{-1}	50 ab^{-1}	[ab^{-1}]
$\sin 2\beta$	$0.667 \pm 0.023 \pm 0.012$	± 0.012	± 0.008	6
α		$\pm 2^\circ$	$\pm 1^\circ$	
γ	$\pm 14^\circ$	$\pm 6^\circ$	$\pm 1.5^\circ$	
$S(B \to \phi K^0)$	$0.90^{+0.09}_{-0.19}$	± 0.053	± 0.018	>50
$S(B \to \eta' K^0)$	$0.68 \pm 0.07 \pm 0.03$	± 0.028	± 0.011	>50
$S(B \to K^0_S K^0_S K^0_S)$	$0.30 \pm 0.32 \pm 0.08$	± 0.100	± 0.033	44
$\vert V_{cb}\vert$ incl.	$\pm 2.4\%$	$\pm 1.0\%$		< 1
$\vert V_{cb}\vert$ excl.	$\pm 3.6\%$	$\pm 1.8\%$	$\pm 1.4\%$	< 1
$\vert V_{ub}\vert$ incl.	$\pm 6.5\%$	$\pm 3.4\%$	$\pm 3.0\%$	2
$\vert V_{ub}\vert$ excl. (had. tag.)	$\pm 10.8\%$	$\pm 4.7\%$	$\pm 2.4\%$	20
$\vert V_{ub}\vert$ excl. (untag.)	$\pm 9.4\%$	$\pm 4.2\%$	$\pm 2.2\%$	3
$\mathcal{B}(B \to \tau\nu)$ [10^{-6}]	96 ± 26	$\pm 10\%$	$\pm 5\%$	46
$\mathcal{B}(B \to \mu\nu)$ [10^{-6}]	< 1.7	5σ	$>> 5\sigma$	>50
$R(B \to D\tau\nu)$	$\pm 16.5\%$	$\pm 5.6\%$	$\pm 3.4\%$	4
$R(B \to D^*\tau\nu)$	$\pm 9.0\%$	$\pm 3.2\%$	$\pm 2.1\%$	3
$\mathcal{B}(B \to K^{*+}\nu\overline{\nu})$ [10^{-6}]	< 40		$\pm 30\%$	>50
$\mathcal{B}(B \to K^{+}\nu\overline{\nu})$ [10^{-6}]	< 55		$\pm 30\%$	>50
$\mathcal{B}(B \to X_s\gamma)$ [10^{-6}]	$\pm 13\%$	$\pm 7\%$	$\pm 6\%$	< 1
$A_{CP}(B \to X_s\gamma)$		± 0.01	± 0.005	8
$S(B \to K^0_S\pi^0\gamma)$	$-0.10 \pm 0.31 \pm 0.07$	± 0.11	± 0.035	> 50
$S(B \to \rho\gamma)$	$-0.83 \pm 0.65 \pm 0.18$	± 0.23	± 0.07	> 50
C_7/C_9 $(B \to X_s\ell\ell)$	$\sim$20%	10%	5%	
$\mathcal{B}(B_s \to \gamma\gamma)$ [10^{-6}]	< 8.7	± 0.3		
$\mathcal{B}(B_s \to \tau^+\tau^-)$ [10^{-3}]		< 2		

Observables	Belle	Belle II		$\mathcal{L}_s$
	(2014)	5 ab^{-1}	50 ab^{-1}	[ab^{-1}]
$\mathcal{B}(D_s \to \mu\nu)$	$5.31 \times 10^{-3}(1 \pm 0.053 \pm 0.038)$	$\pm 2.9\%$	$\pm$(0.9%-1.3%)	> 50
$\mathcal{B}(D_s \to \tau\nu)$	$5.70 \times 10^{-3}(1 \pm 0.037 \pm 0.054)$	$\pm$(3.5%-4.3%)	$\pm$(2.3%-3.6%)	3-5
y_{CP} [10^{-2}]	$1.11 \pm 0.22 \pm 0.11$	$\pm$(0.11-0.13)	$\pm$(0.05-0.08)	5-8
A_Γ [10^{-2}]	$-0.03 \pm 0.20 \pm 0.08$	± 0.10	$\pm$(0.03-0.05)	7 - 9
$A_{CP}^{K^+K^-}$ [10^{-2}]	$-0.32 \pm 0.21 \pm 0.09$	± 0.11	± 0.06	15
$A_{CP}^{\pi^+\pi^-}$ [10^{-2}]	$0.55 \pm 0.36 \pm 0.09$	± 0.17	$\pm$ 0.06	> 50
$A_{CP}^{\phi\gamma}$ [10^{-2}]	$\pm$ 5.6	± 2.5	± 0.8	> 50
$x^{K_S\pi^+\pi^-}$ [10^{-2}]	$0.56 \pm 0.19 \pm {}^{0.07}_{0.13}$	± 0.14	± 0.11	3
$y^{K_S\pi^+\pi^-}$ [10^{-2}]	$0.30 \pm 0.15 \pm {}^{0.05}_{0.08}$	± 0.08	± 0.05	15
$\vert q/p\vert^{K_S\pi^+\pi^-}$	$0.90 \pm {}^{0.16}_{0.15} \pm {}^{0.08}_{0.06}$	± 0.10	± 0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ [$^\circ$]	$-6 \pm 11 \pm {}^{4}_{5}$	± 6	± 4	10
$A_{CP}^{\pi^0\pi^0}$ [10^{-2}]	$-0.03 \pm 0.64 \pm 0.10$	± 0.29	± 0.09	> 50
$A_{CP}^{K^0_S\pi^0}$ [10^{-2}]	$-0.10 \pm 0.16 \pm 0.09$	± 0.08	± 0.03	> 50
$Br(D^0 \to \gamma\gamma)$ [10^{-6}]	< 1.5	$\pm 30\%$	$\pm 25\%$	2
	$\tau \to \mu\gamma$ [10^{-9}]	< 45	< 14.7	< 4.7
	$\tau \to e\gamma$ [10^{-9}]	< 120	< 39	< 12
	$\tau \to \mu\mu\mu$ [10^{-9}]	< 21.0	< 3.0	< 0.3

[BELLE2-NOTE-PH-2015-002, retrieved from B2TiP]

FLAG-3 — B decay constants

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization/ matching	Heavy-quark treatment
ETM 13E	[456]	$2+1+1$	C	★	○	○	○	✓
HPQCD 13	[52]	$2+1+1$	A	★	★	★	○	✓
RBC/UKQCD 14	[53]	$2+1$	A	○	○	○	○	✓
RBC/UKQCD 14A	[54]	$2+1$	A	○	○	○	○	✓
RBC/UKQCD 13A	[457]	$2+1$	C	○	○	○	○	✓
HPQCD 12	[55]	$2+1$	A	○	○	○	○	✓
FNAL/MILC 11	[48]	$2+1$	A	○	○	★	○	✓
RBC/UKQCD 10C	[464]	$2+1$	A	■	■	■	○	✓
HPQCD 09	[59]	$2+1$	A	○	○	○	○	✓
ALPHA 14	[57]	2	A	★	★	★	★	✓
ALPHA 13	[458]	2	C	★	★	★	★	✓
ETM 13B, 13C[b]	[20,58]	2	A	★	○	★	○	✓
ALPHA 12A	[459]	2	C	★	★	★	★	✓
ETM 12B	[460]	2	C	★	○	★	○	✓
ETM 11A	[182]	2	A	○	○	★	○	✓

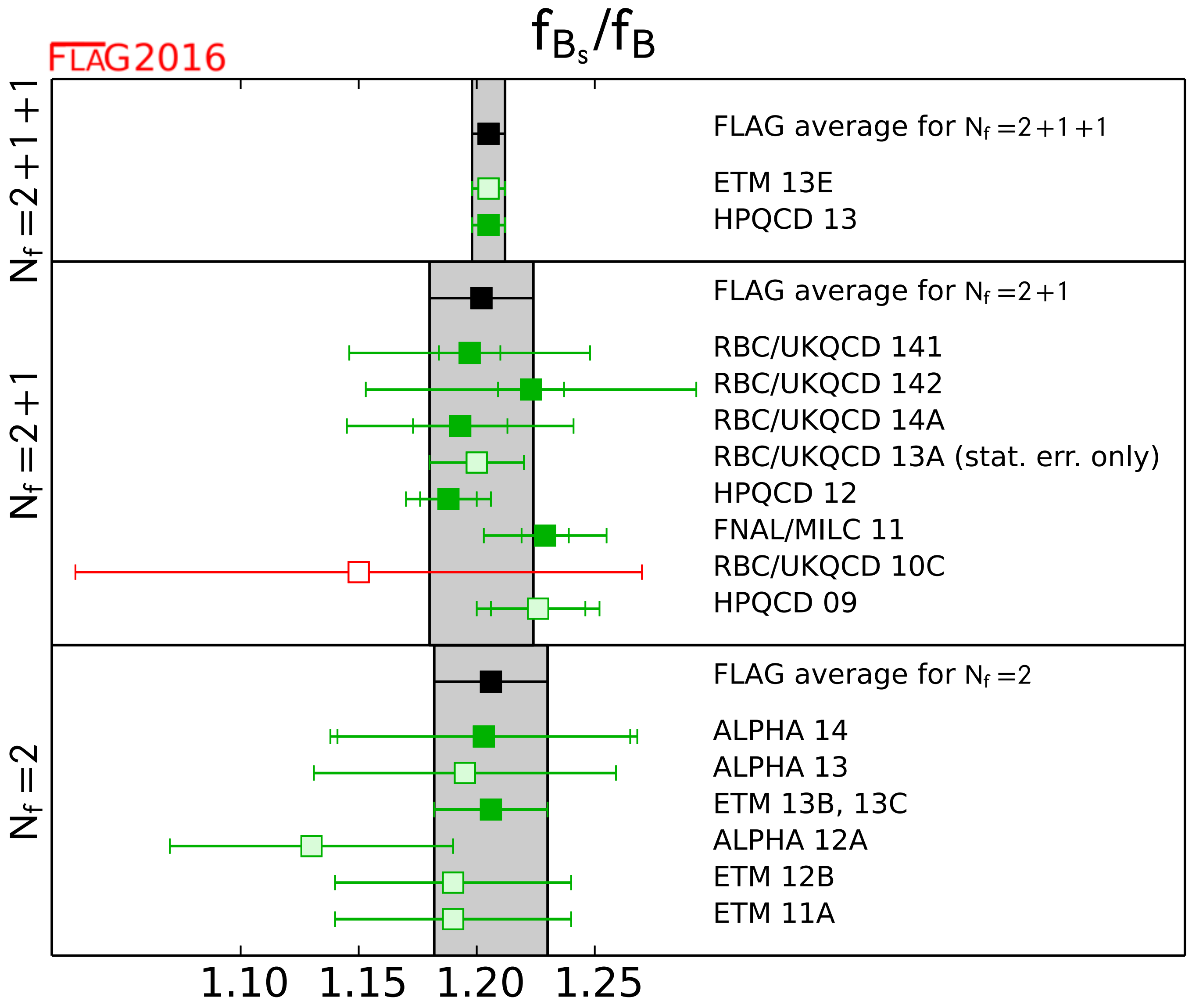
---

[a] Statistical errors only

[b] Update of ETM 11A and 12B

(+ HPQCD result for f_{B_c}) `[PRD 86 (2012) 074503]`

FLAG-3 — B decay constants



N_f	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
2	188(7)	227(7)	1.206(23)
2+1	192.0(4.3)	228.4(3.7)	1.201(16)
2+1+1	186(4)	224(5)	1.205(7)

Belle II projections

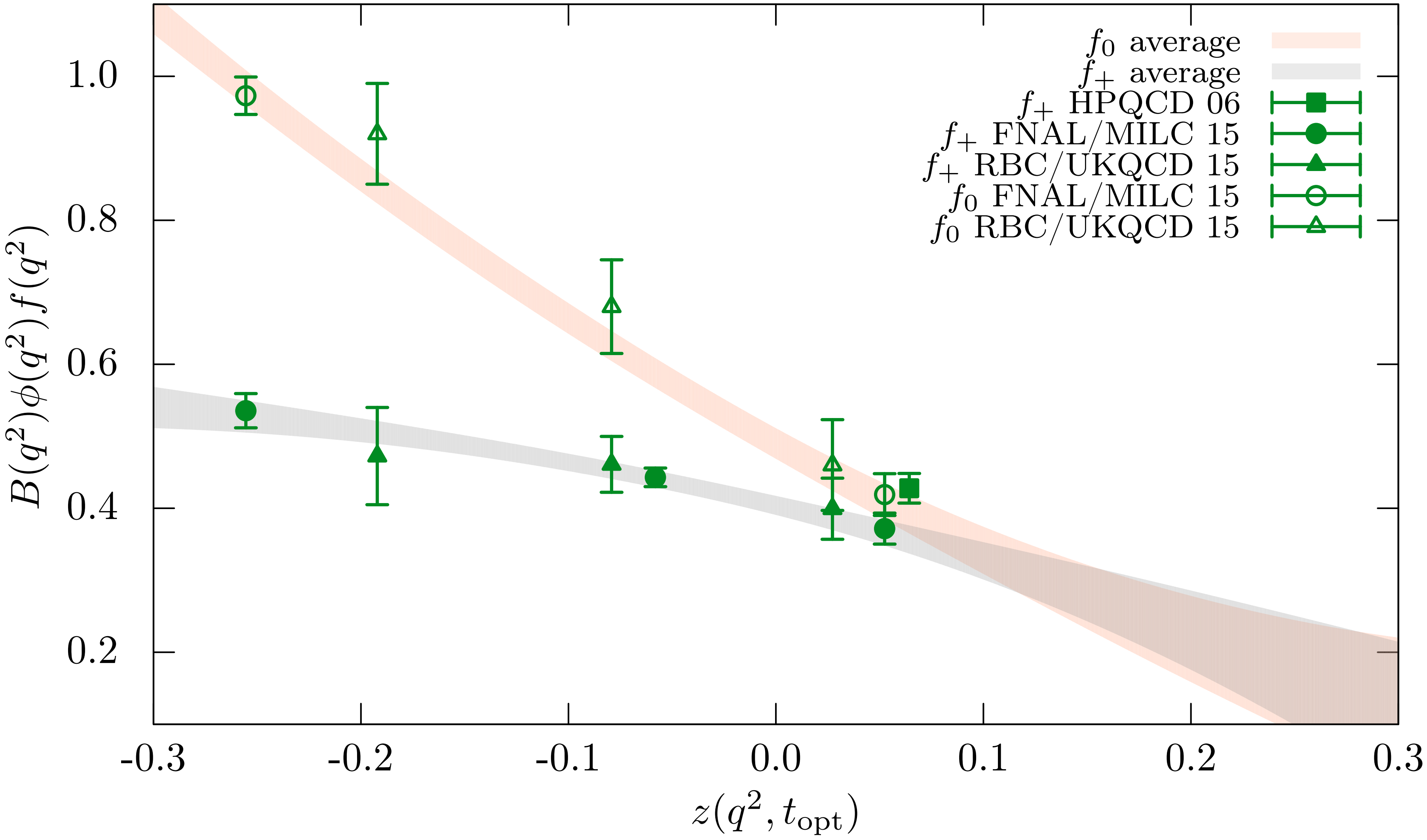
Observables	Belle	Belle II		$\mathcal{L}_s$
	(2014)	5 ab^{-1}	50 ab^{-1}	$[\text{ab}^{-1}]$
$\sin 2\beta$	$0.667 \pm 0.023 \pm 0.012$	±0.012	±0.008	6
α		$\pm2^\circ$	$\pm1^\circ$	
γ	$\pm14^\circ$	$\pm6^\circ$	$\pm1.5^\circ$	
$S(B \to \phi K^0)$	$0.90^{+0.09}_{-0.19}$	±0.053	±0.018	>50
$S(B \to \eta' K^0)$	$0.68 \pm 0.07 \pm 0.03$	±0.028	±0.011	>50
$S(B \to K^0_S K^0_S K^0_S)$	$0.30 \pm 0.32 \pm 0.08$	±0.100	±0.033	44
$\|V_{cb}\|$ incl.	$\pm2.4\%$	$\pm1.0\%$		< 1
$\|V_{cb}\|$ excl.	$\pm3.6\%$	$\pm1.8\%$	$\pm1.4\%$	< 1
$\|V_{ub}\|$ incl.	$\pm6.5\%$	$\pm3.4\%$	$\pm3.0\%$	2
$\|V_{ub}\|$ excl. (had. tag.)	$\pm10.8\%$	$\pm4.7\%$	$\pm2.4\%$	20
$\|V_{ub}\|$ excl. (untag.)	$\pm9.4\%$	$\pm4.2\%$	$\pm2.2\%$	3
$\mathcal{B}(B \to \tau\nu)$ $[10^{-6}]$	96 ± 26	$\pm10\%$	$\pm5\%$	46
$\mathcal{B}(B \to \mu\nu)$ $[10^{-6}]$	< 1.7	5σ	$>> 5\sigma$	>50
$R(B \to D\tau\nu)$	$\pm16.5\%$	$\pm5.6\%$	$\pm3.4\%$	4
$R(B \to D^*\tau\nu)$	$\pm9.0\%$	$\pm3.2\%$	$\pm2.1\%$	3
$\mathcal{B}(B \to K^{*+}\nu\overline{\nu})$ $[10^{-6}]$	< 40		$\pm30\%$	>50
$\mathcal{B}(B \to K^{+}\nu\overline{\nu})$ $[10^{-6}]$	< 55		$\pm30\%$	>50
$\mathcal{B}(B \to X_s\gamma)$ $[10^{-6}]$	$\pm13\%$	$\pm7\%$	$\pm6\%$	< 1
$A_{CP}(B \to X_s\gamma)$		±0.01	±0.005	8
$S(B \to K^0_S\pi^0\gamma)$	$-0.10 \pm 0.31 \pm 0.07$	±0.11	±0.035	> 50
$S(B \to \rho\gamma)$	$-0.83 \pm 0.65 \pm 0.18$	±0.23	±0.07	> 50
C_7/C_9 $(B \to X_s\ell\ell)$	$\sim$20%	10%	5%	
$\mathcal{B}(B_s \to \gamma\gamma)$ $[10^{-6}]$	< 8.7	±0.3		
$\mathcal{B}(B_s \to \tau^+\tau^-)$ $[10^{-3}]$		< 2		

Observables	Belle	Belle II		$\mathcal{L}_s$
	(2014)	5 ab^{-1}	50 ab^{-1}	$[\text{ab}^{-1}]$
$\mathcal{B}(D_s \to \mu\nu)$	$5.31 \times 10^{-3}(1 \pm 0.053 \pm 0.038)$	$\pm2.9\%$	$\pm$(0.9%-1.3%)	> 50
$\mathcal{B}(D_s \to \tau\nu)$	$5.70 \times 10^{-3}(1 \pm 0.037 \pm 0.054)$	$\pm$(3.5%-4.3%)	$\pm$(2.3%-3.6%)	3-5
y_{CP} $[10^{-2}]$	$1.11 \pm 0.22 \pm 0.11$	$\pm$(0.11-0.13)	$\pm$(0.05-0.08)	5-8
A_Γ $[10^{-2}]$	$-0.03 \pm 0.20 \pm 0.08$	±0.10	$\pm$(0.03-0.05)	7 - 9
$A_{CP}^{K^+K^-}$ $[10^{-2}]$	$-0.32 \pm 0.21 \pm 0.09$	±0.11	±0.06	15
$A_{CP}^{\pi^+\pi^-}$ $[10^{-2}]$	$0.55 \pm 0.36 \pm 0.09$	±0.17	$\pm$ 0.06	> 50
$A_{CP}^{\phi\gamma}$ $[10^{-2}]$	$\pm$ 5.6	±2.5	±0.8	> 50
$x^{K_S\pi^+\pi^-}$ $[10^{-2}]$	$0.56 \pm 0.19 \pm {}^{0.07}_{0.13}$	±0.14	±0.11	3
$y^{K_S\pi^+\pi^-}$ $[10^{-2}]$	$0.30 \pm 0.15 \pm {}^{0.05}_{0.08}$	±0.08	±0.05	15
$\|q/p\|^{K_S\pi^+\pi^-}$	$0.90 \pm {}^{0.16}_{0.15} \pm {}^{0.08}_{0.06}$	±0.10	±0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ $[^\circ]$	$-6 \pm 11 \pm {}^{4}_{5}$	±6	±4	10
$A_{CP}^{\pi^0\pi^0}$ $[10^{-2}]$	$-0.03 \pm 0.64 \pm 0.10$	±0.29	±0.09	> 50
$A_{CP}^{K^0_S\pi^0}$ $[10^{-2}]$	$-0.10 \pm 0.16 \pm 0.09$	±0.08	±0.03	> 50
$Br(D^0 \to \gamma\gamma)$ $[10^{-6}]$	< 1.5	$\pm30\%$	$\pm25\%$	2
	$\tau \to \mu\gamma$ $[10^{-9}]$	< 45	< 14.7	< 4.7
	$\tau \to e\gamma$ $[10^{-9}]$	< 120	< 39	< 12
	$\tau \to \mu\mu\mu$ $[10^{-9}]$	< 21.0	< 3.0	< 0.3

[BELLE2-NOTE-PH-2015-002, retrieved from B2TiP]

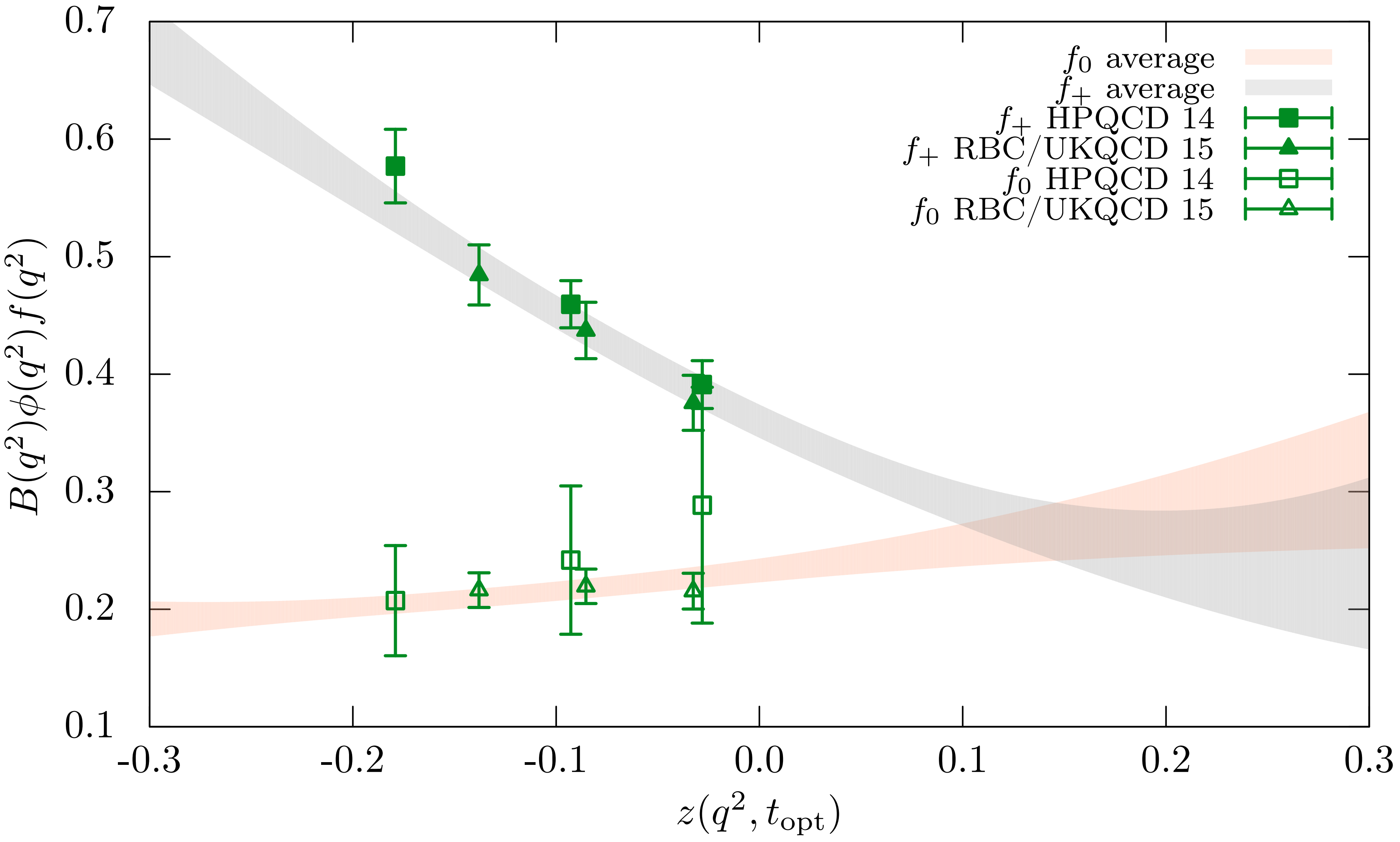
FLAG-3 — $B \to \pi l \nu$

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization	Heavy-quark treatment
FNAL/MILC 15	[504]	$2+1$	A	★	○	★	○	✓
RBC/UKQCD 15	[505]	$2+1$	A	○	○	○	○	✓
HPQCD 06	[503]	$2+1$	A	○	○	○	○	✓

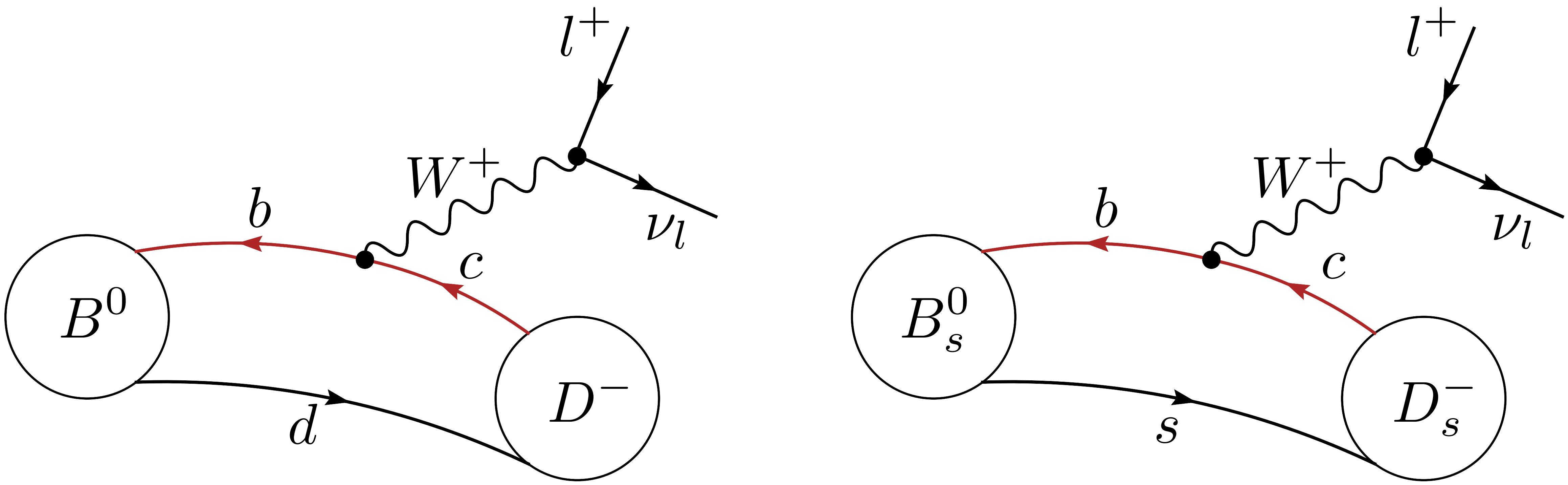


FLAG-3 — $B_s \to K l \nu$

Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization	Heavy-quark treatment
RBC/UKQCD 15	[505]	$2+1$	A	○	○	○	○	✓
HPQCD 14	[511]	$2+1$	A	○	○	○	○	✓



$b \to c$ semileptonic

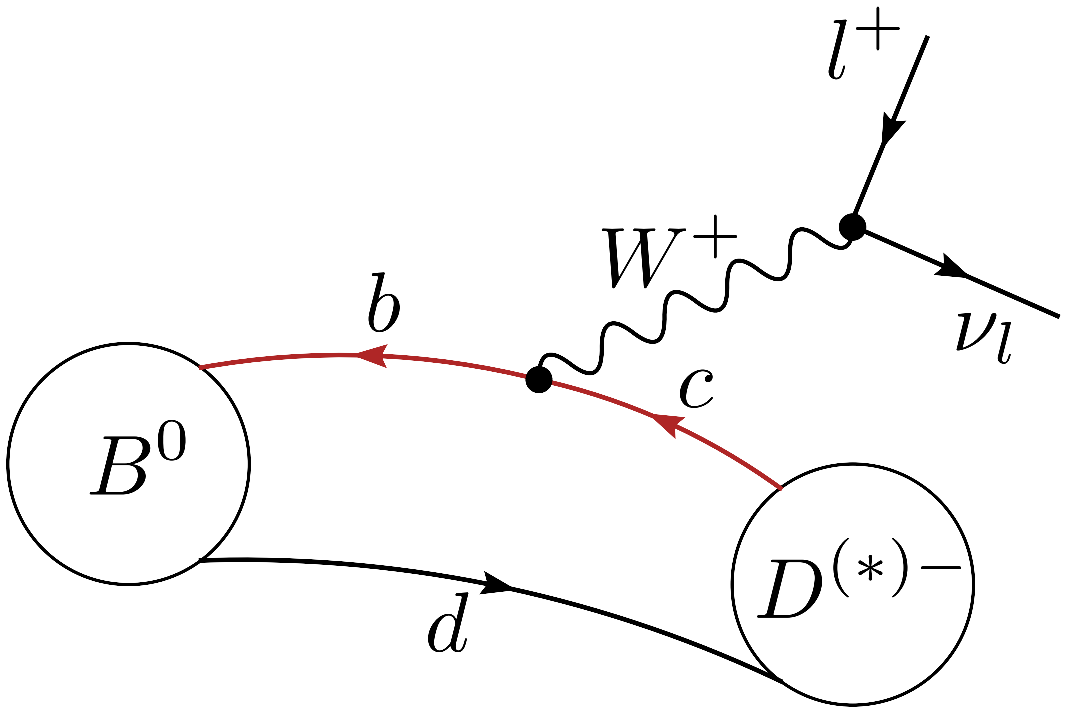


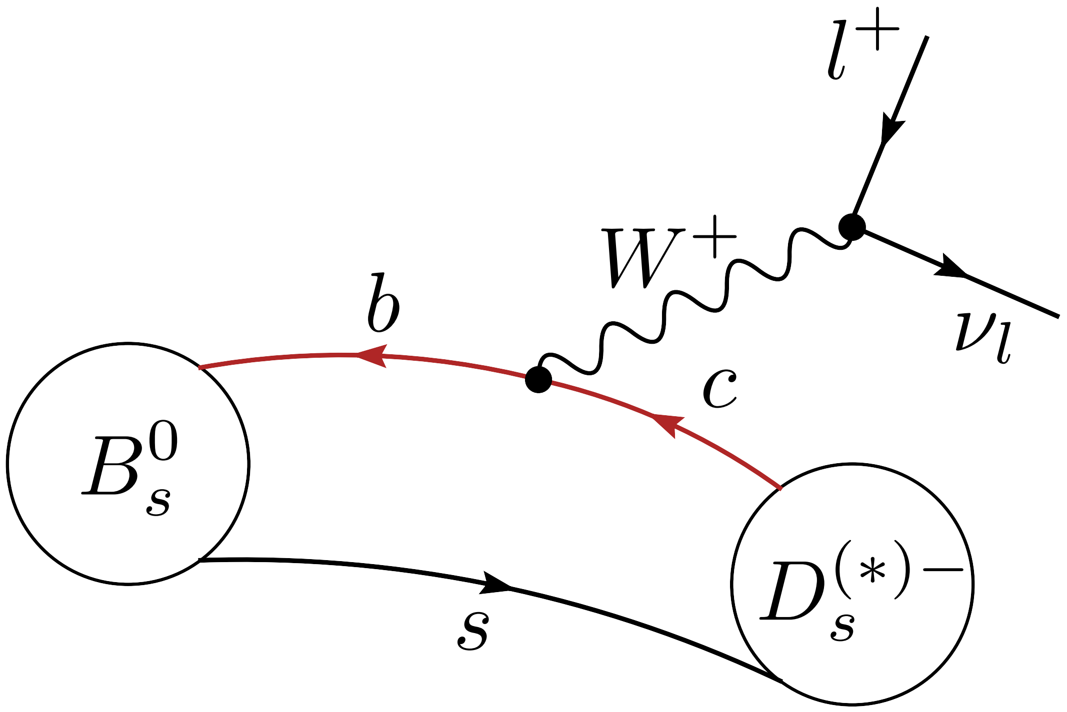
$$\frac{\mathrm{d}\Gamma(B_{(s)} \to P l \nu)}{\mathrm{d}q^2} = \frac{G_F^2 \, |V_{cb}|^2}{24\pi^3} \, \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \boxed{\frac{3m_l^2}{8q^2}} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

e,μ suppressed

uncertainties from kinematical factors / neglected h.o. OPE at the permille level

$b \to c$ semileptonic





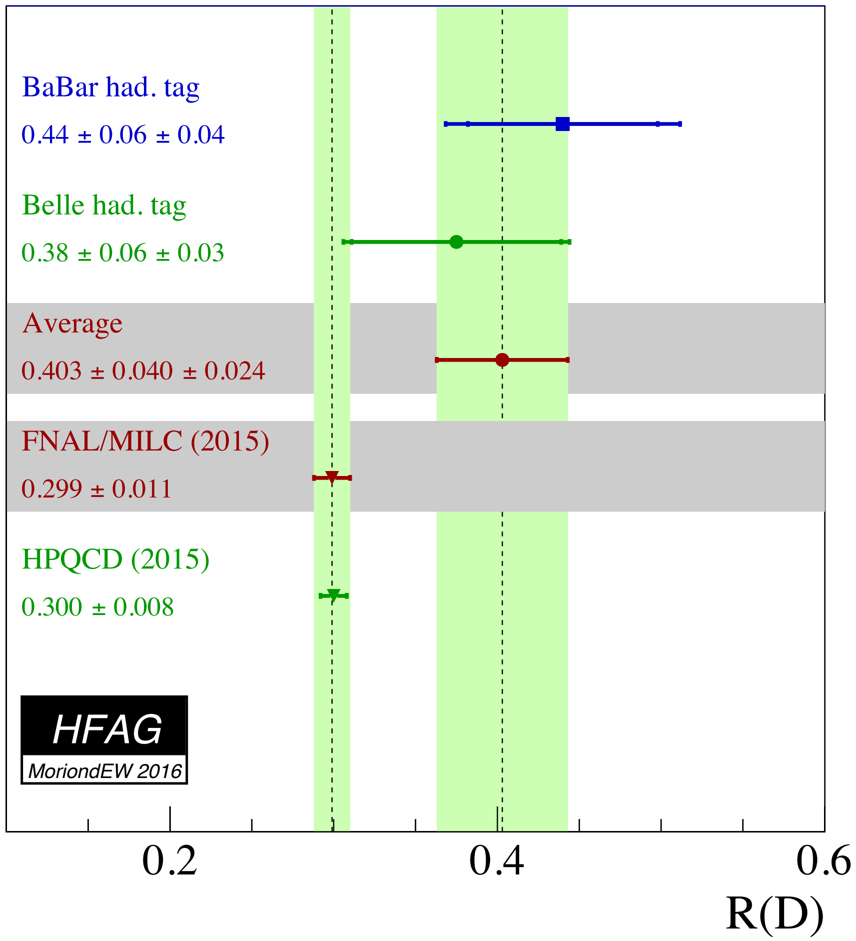
$$\frac{\mathrm{d}\Gamma(B \to D l \nu_l)}{\mathrm{d}w} = \frac{G_{\mathrm{F}}^2}{48\pi^3}(m_B + m_D)^2 (w^2 - 1)^{3/2} |\eta_{\mathrm{EW}}|^2 \, |V_{cb}|^2 \, |\mathcal{G}(w)|^2 \; + \; \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

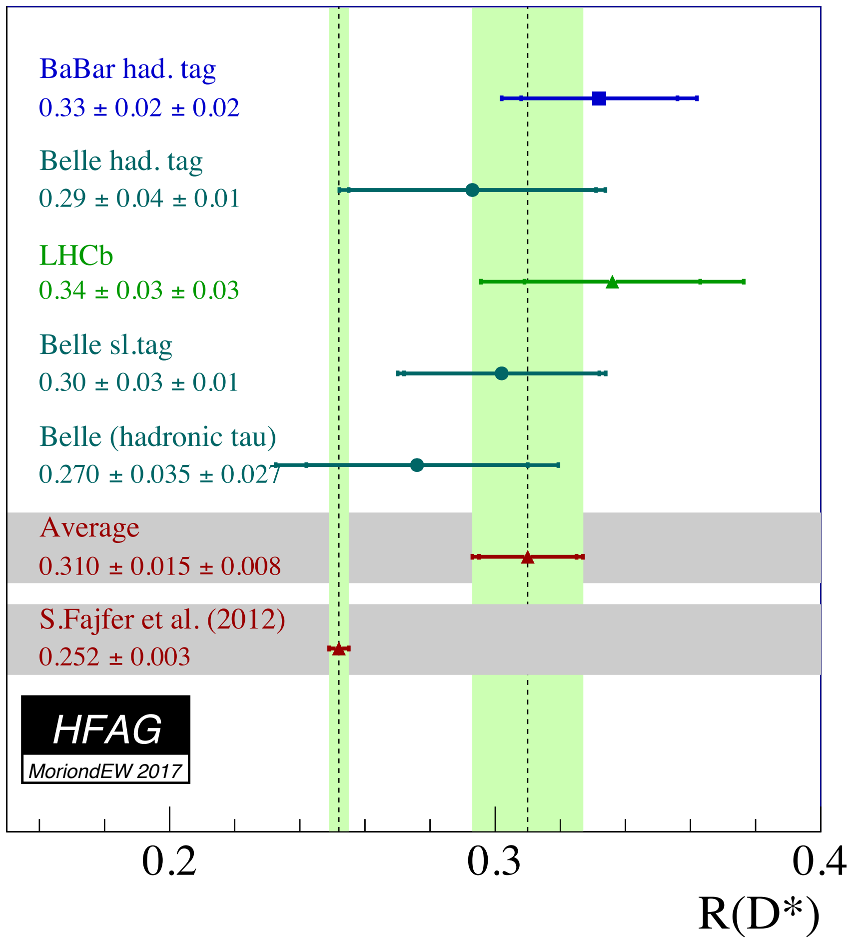
$$\frac{\mathrm{d}\Gamma(B \to D^* l \nu_l)}{\mathrm{d}w} = \frac{G_{\mathrm{F}}^2}{4\pi^3}(m_B - m_{D^*})^2 (w^2 - 1)^{1/2} |\eta_{\mathrm{EW}}|^2 \chi(w) \, |V_{cb}|^2 \, |\mathcal{F}(w)|^2 \; + \; \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

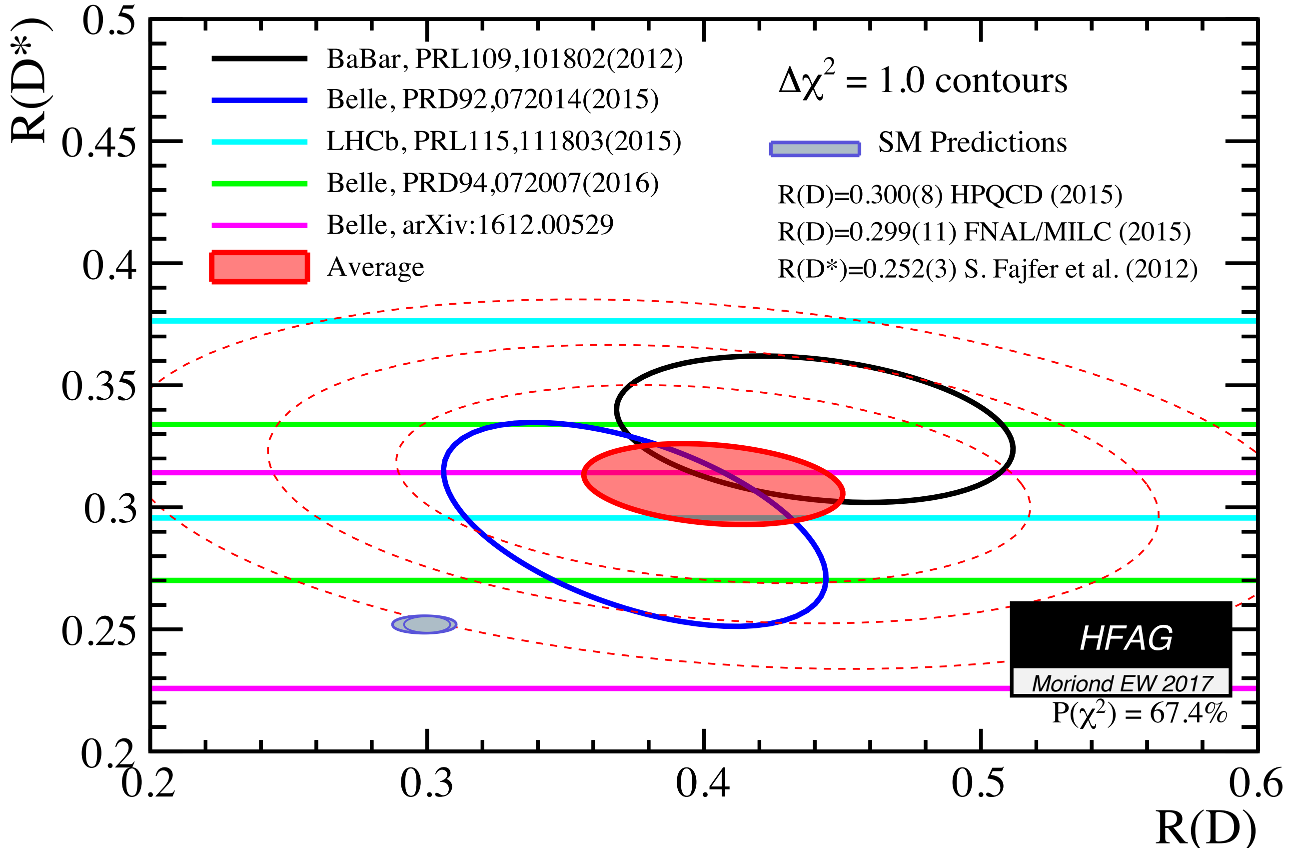
$$w = \frac{p_B \cdot p_{D^{(*)}}}{m_B m_{D^{(*)}}} \qquad\qquad \mathcal{G}(w) = \frac{4\,\frac{m_D}{m_B}}{1 + \frac{m_D}{m_B}} \, f_+(q^2)$$

etc

$b \to c$ semileptonic







Belle II projections

Observables	Belle	Belle II		$\mathcal{L}_s$
	(2014)	5 ab^{-1}	50 ab^{-1}	[ab^{-1}]
$\sin 2\beta$	$0.667 \pm 0.023 \pm 0.012$	± 0.012	± 0.008	6
α		$\pm 2^\circ$	$\pm 1^\circ$	
γ	$\pm 14^\circ$	$\pm 6^\circ$	$\pm 1.5^\circ$	
$S(B \to \phi K^0)$	$0.90^{+0.09}_{-0.19}$	± 0.053	± 0.018	>50
$S(B \to \eta' K^0)$	$0.68 \pm 0.07 \pm 0.03$	± 0.028	± 0.011	>50
$S(B \to K^0_S K^0_S K^0_S)$	$0.30 \pm 0.32 \pm 0.08$	± 0.100	± 0.033	44
$\lvert V_{cb} \rvert$ incl.	$\pm 2.4\%$	$\pm 1.0\%$		< 1
$\lvert V_{cb} \rvert$ excl.	$\pm 3.6\%$	$\pm 1.8\%$	$\pm 1.4\%$	< 1
$\lvert V_{ub} \rvert$ incl.	$\pm 6.5\%$	$\pm 3.4\%$	$\pm 3.0\%$	2
$\lvert V_{ub} \rvert$ excl. (had. tag.)	$\pm 10.8\%$	$\pm 4.7\%$	$\pm 2.4\%$	20
$\lvert V_{ub} \rvert$ excl. (untag.)	$\pm 9.4\%$	$\pm 4.2\%$	$\pm 2.2\%$	3
$\mathcal{B}(B \to \tau\nu)$ [10^{-6}]	96 ± 26	$\pm 10\%$	$\pm 5\%$	46
$\mathcal{B}(B \to \mu\nu)$ [10^{-6}]	< 1.7	5σ	$>> 5\sigma$	>50
$R(B \to D\tau\nu)$	$\pm 16.5\%$	$\pm 5.6\%$	$\pm 3.4\%$	4
$R(B \to D^*\tau\nu)$	$\pm 9.0\%$	$\pm 3.2\%$	$\pm 2.1\%$	3
$\mathcal{B}(B \to K^{*+}\nu\overline{\nu})$ [10^{-6}]	< 40		$\pm 30\%$	>50
$\mathcal{B}(B \to K^{+}\nu\overline{\nu})$ [10^{-6}]	< 55		$\pm 30\%$	>50
$\mathcal{B}(B \to X_s\gamma)$ [10^{-6}]	$\pm 13\%$	$\pm 7\%$	$\pm 6\%$	< 1
$A_{CP}(B \to X_s\gamma)$		± 0.01	± 0.005	8
$S(B \to K^0_S \pi^0 \gamma)$	$-0.10 \pm 0.31 \pm 0.07$	± 0.11	± 0.035	> 50
$S(B \to \rho\gamma)$	$-0.83 \pm 0.65 \pm 0.18$	± 0.23	± 0.07	> 50
C_7/C_9 $(B \to X_s \ell\ell)$	$\sim$20%	10%	5%	
$\mathcal{B}(B_s \to \gamma\gamma)$ [10^{-6}]	< 8.7	± 0.3		
$\mathcal{B}(B_s \to \tau^+\tau^-)$ [10^{-3}]		< 2		

Observables	Belle	Belle II		$\mathcal{L}_s$
	(2014)	5 ab^{-1}	50 ab^{-1}	[ab^{-1}]
$\mathcal{B}(D_s \to \mu\nu)$	$5.31 \times 10^{-3}(1 \pm 0.053 \pm 0.038)$	$\pm 2.9\%$	$\pm$(0.9%-1.3%)	> 50
$\mathcal{B}(D_s \to \tau\nu)$	$5.70 \times 10^{-3}(1 \pm 0.037 \pm 0.054)$	$\pm$(3.5%-4.3%)	$\pm$(2.3%-3.6%)	3-5
y_{CP} [10^{-2}]	$1.11 \pm 0.22 \pm 0.11$	$\pm$(0.11-0.13)	$\pm$(0.05-0.08)	5-8
A_Γ [10^{-2}]	$-0.03 \pm 0.20 \pm 0.08$	± 0.10	$\pm$(0.03-0.05)	7 - 9
$A_{CP}^{K^+K^-}$ [10^{-2}]	$-0.32 \pm 0.21 \pm 0.09$	± 0.11	± 0.06	15
$A_{CP}^{\pi^+\pi^-}$ [10^{-2}]	$0.55 \pm 0.36 \pm 0.09$	± 0.17	$\pm$ 0.06	> 50
$A_{CP}^{\phi\gamma}$ [10^{-2}]	$\pm$ 5.6	± 2.5	± 0.8	> 50
$x^{K_S\pi^+\pi^-}$ [10^{-2}]	$0.56 \pm 0.19 \pm {}^{0.07}_{0.13}$	± 0.14	± 0.11	3
$y^{K_S\pi^+\pi^-}$ [10^{-2}]	$0.30 \pm 0.15 \pm {}^{0.05}_{0.08}$	± 0.08	± 0.05	15
$\lvert q/p \rvert^{K_S\pi^+\pi^-}$	$0.90 \pm {}^{0.16}_{0.15} \pm {}^{0.08}_{0.06}$	± 0.10	± 0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ [°]	$-6 \pm 11 \pm {}^{4}_{5}$	± 6	± 4	10
$A_{CP}^{\pi^0\pi^0}$ [10^{-2}]	$-0.03 \pm 0.64 \pm 0.10$	± 0.29	± 0.09	> 50
$A_{CP}^{K^0_S\pi^0}$ [10^{-2}]	$-0.10 \pm 0.16 \pm 0.09$	± 0.08	± 0.03	> 50
$Br(D^0 \to \gamma\gamma)$ [10^{-6}]	< 1.5	$\pm 30\%$	$\pm 25\%$	2
	$\tau \to \mu\gamma$ [10^{-9}]	< 45	< 14.7	< 4.7
	$\tau \to e\gamma$ [10^{-9}]	< 120	< 39	< 12
	$\tau \to \mu\mu\mu$ [10^{-9}]	< 21.0	< 3.0	< 0.3

[BELLE2-NOTE-PH-2015-002, retrieved from B2TiP]

FLAG-3 — $b \to c$ semileptonic

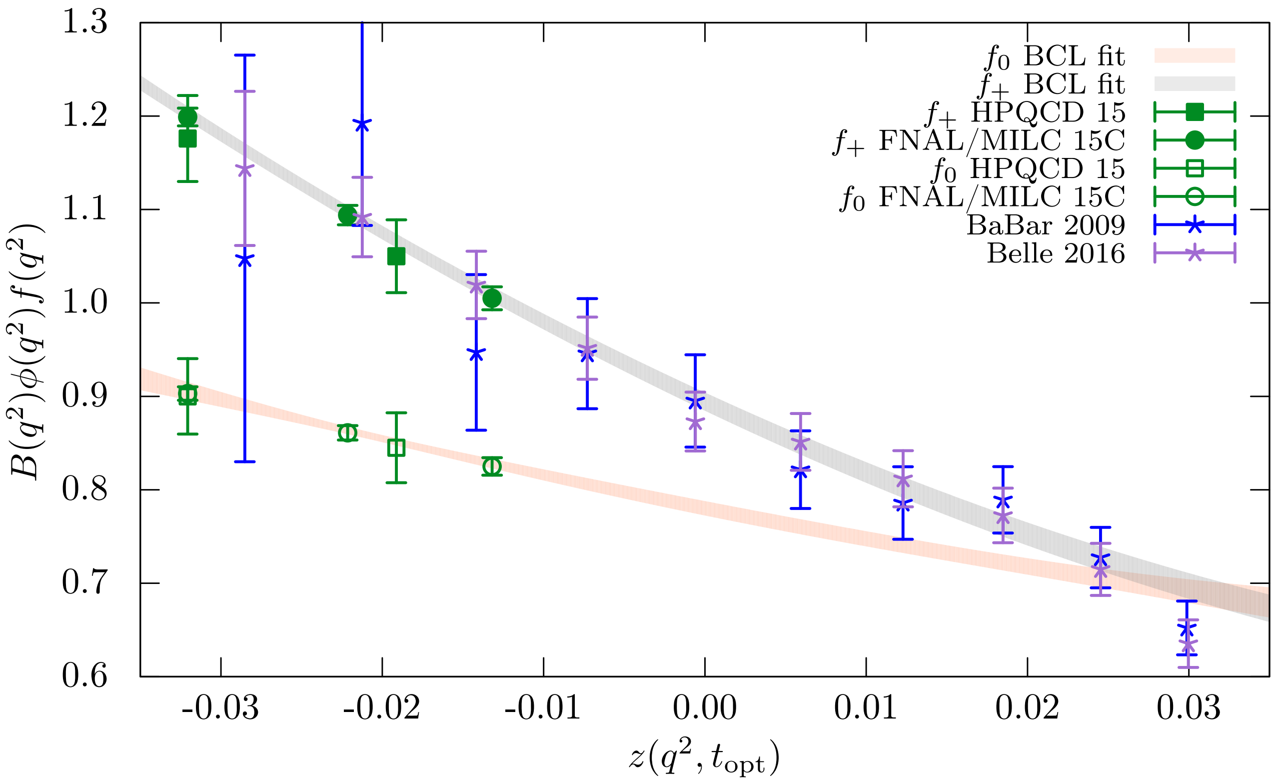
Table 39 Lattice results for the $B \to D^* \ell \nu$, $B \to D \ell \nu$, and $B_s \to D_s \ell \nu$ semileptonic form factors and $R(D)$

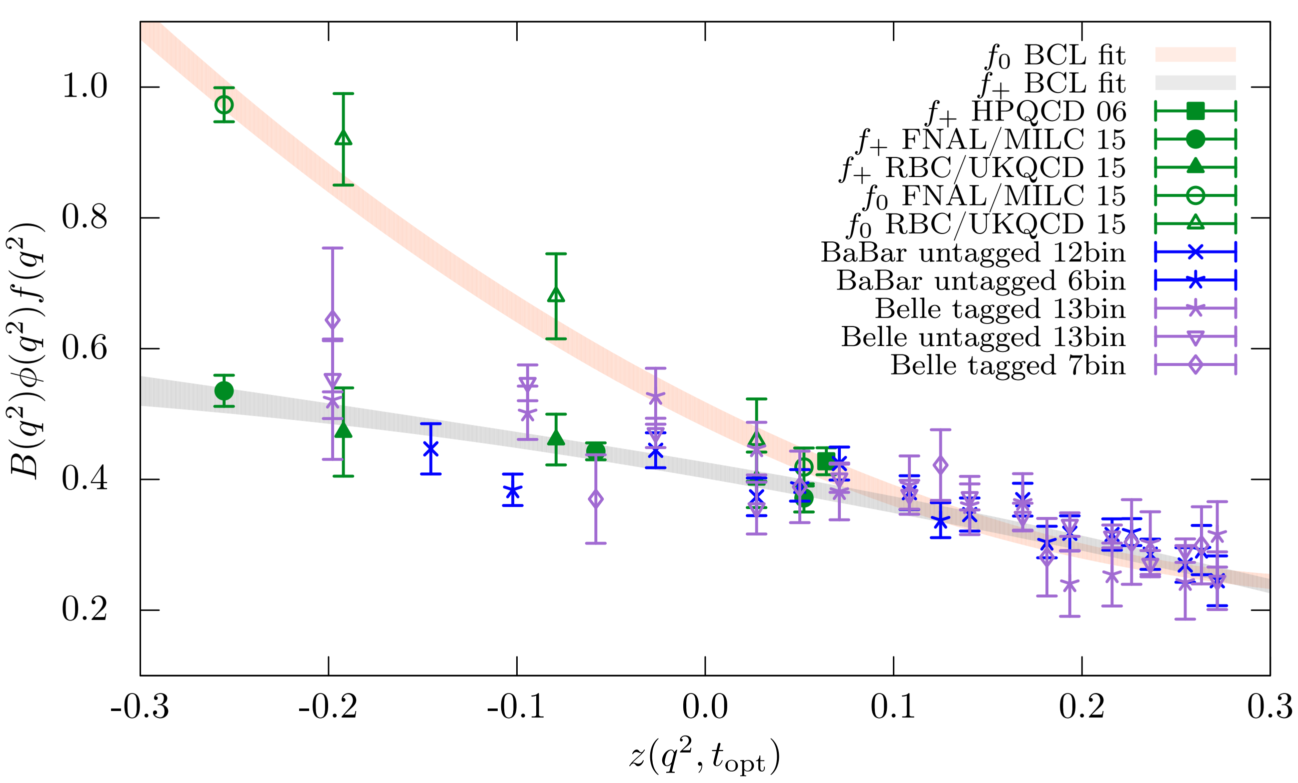
Collaboration	Refs.	N_f	Publication status	Continuum extrapolation	Chiral extrapolation	Finite volume	Renormalization	Heavy-quark treatment	$w = 1$ form factor/ratio	
FNAL/MILC 14	[539]	$2+1$	A	★	○	★	○	✓	$\mathcal{F}^{B \to D^*}(1)$	0.906(4)(12)
HPQCD 15	[541]	$2+1$	A	○	○	○	○	✓	$\mathcal{G}^{B \to D}(1)$	1.035(40)
FNAL/MILC 15C	[540]	$2+1$	A	★	○	★	○	✓	$\mathcal{G}^{B \to D}(1)$	1.054(4)(8)
HPQCD 15	[541]	$2+1$	A	○	○	○	○	✓	$R(D)$	0.300(8)
FNAL/MILC 15C	[540]	$2+1$	A	★	○	★	○	✓	$R(D)$	0.299(11)
Atoui 13	[537]	2	A	★	○	★	–	✓	$\mathcal{G}^{B \to D}(1)$	1.033(95)
Atoui 13	[537]	2	A	★	○	★	–	✓	$\mathcal{G}^{B_s \to D_s}(1)$	1.052(46)

newer $B \to D$ computations include results $w > 1$

[FLAG-3, Aoki et al., EPJC (2017) 77:112]

FLAG-3 — 3rd row CKM





	From	$\lvert V_{cb}\rvert \times 10^3$
Our average for $N_f = 2+1$	$B \to D^* \ell \nu$	39.27(56)(49)
Our average for $N_f = 2+1$	$B \to D \ell \nu$	40.1(1.0)
Our average for $N_f = 2$	$B \to D \ell \nu$	41.0(3.8)(1.5)
HFAG inclusive average	$B \to X_c \ell \nu$	42.46(88)

$B \to \pi \ell \nu$ $(N_f = 2+1)$	
	Central values
$V_{ub} \times 10^3$	3.73 (14)

$N_f = 2$ Belle $B \to \tau \nu_\tau$: $|V_{ub}| = 3.83(48)(15) \times 10^{-3}$,

$N_f = 2+1$ Belle $B \to \tau \nu_\tau$: $|V_{ub}| = 3.75(47)(9) \times 10^{-3}$,

$N_f = 2+1+1$ Belle $B \to \tau \nu_\tau$: $|V_{ub}| = 3.87(48)(9) \times 10^{-3}$;

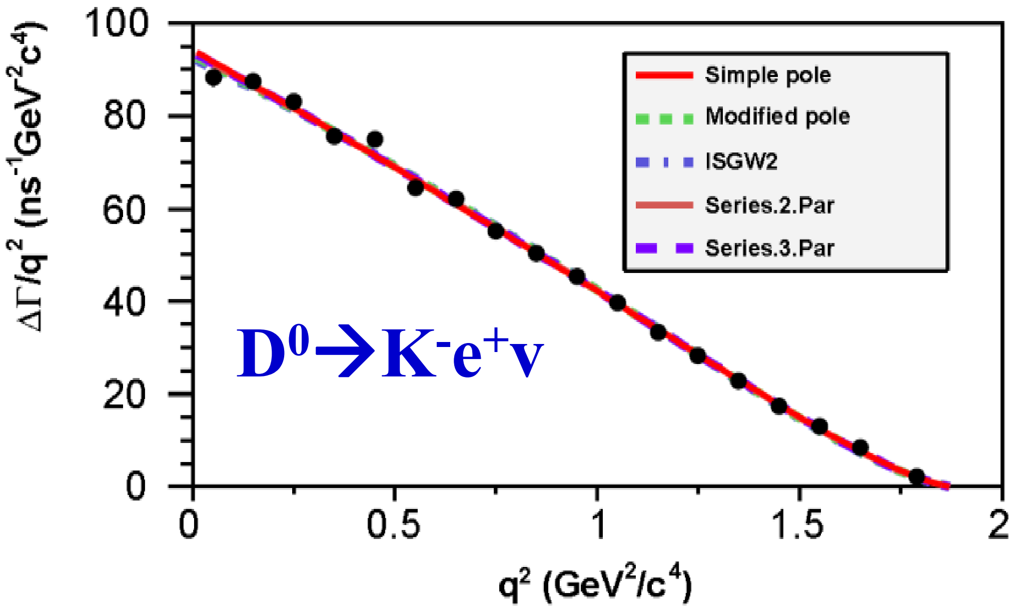
$N_f = 2$ Babar $B \to \tau \nu_\tau$: $|V_{ub}| = 5.37(74)(21) \times 10^{-3}$,

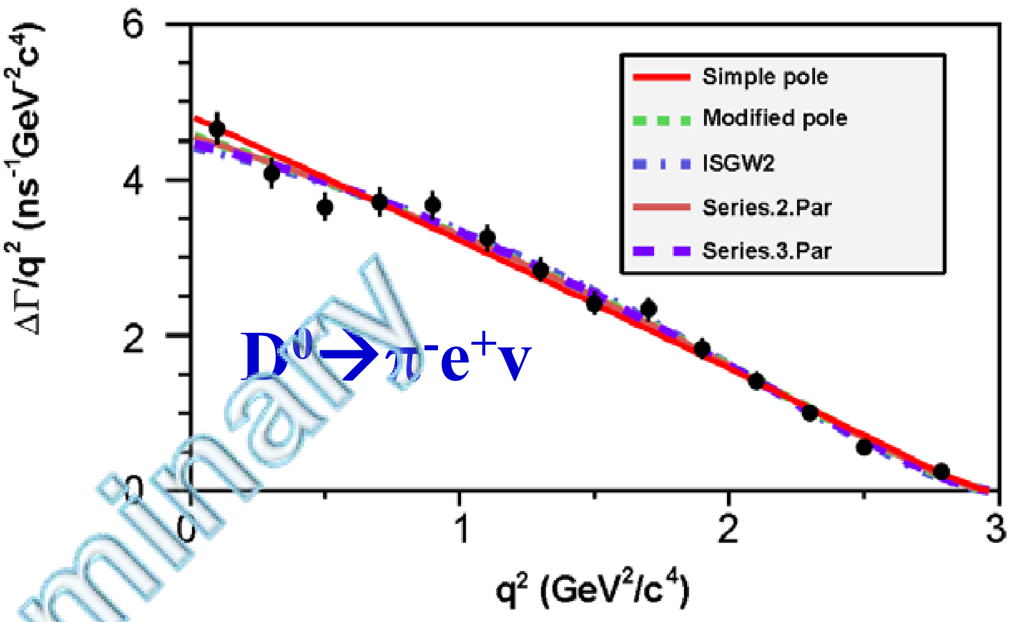
$N_f = 2+1$ Babar $B \to \tau \nu_\tau$: $|V_{ub}| = 5.26(73)(12) \times 10^{-3}$,

$N_f = 2+1+1$ Babar $B \to \tau \nu_\tau$: $|V_{ub}| = 5.43(75)(12) \times 10^{-3}$.

[FLAG-3, Aoki et al., EPJC (2017) 77:112]

q^2 dependence of form factors





		$D^0 \rightarrow K^- e^+ \nu$		$D^0 \rightarrow \pi^- e^+ \nu$
Simple Pole	$f_K^+(0)\lvert V_{cs}\rvert$	$0.7209 \pm 0.0022 \pm 0.0033$	$f_\pi^+(0)\lvert V_{cd}\rvert$	$0.1475 \pm 0.0014 \pm 0.0005$
	M_{pole}	$1.9207 \pm 0.0103 \pm 0.0069$	M_{pole}	$1.9114 \pm 0.0118 \pm 0.0038$
Mod. Pole	$f_K^+(0)\lvert V_{cs}\rvert$	$0.7163 \pm 0.0024 \pm 0.0034$	$f_\pi^+(0)\lvert V_{cd}\rvert$	$0.1437 \pm 0.0017 \pm 0.0008$
	α	$0.3088 \pm 0.0195 \pm 0.0129$	α	$0.2794 \pm 0.0345 \pm 0.0113$
ISGW2	$f_K^+(0)\lvert V_{cs}\rvert$	$0.7139 \pm 0.0023 \pm 0.0034$	$f_\pi^+(0)\lvert V_{cd}\rvert$	$0.1415 \pm 0.0016 \pm 0.0006$
	r_{ISGW2}	$1.6000 \pm 0.0141 \pm 0.0091$	r_{ISGW2}	$2.0688 \pm 0.0394 \pm 0.0124$
Series.2.Par	$f_K^+(0)\lvert V_{cs}\rvert$	$0.7172 \pm 0.0025 \pm 0.0035$	$f_\pi^+(0)\lvert V_{cd}\rvert$	$0.1435 \pm 0.0018 \pm 0.0009$
	r_1	$-2.2278 \pm 0.0864 \pm 0.0575$	r_1	$-2.0365 \pm 0.0807 \pm 0.0260$
Series.3.Par	$f_K^+(0)\lvert V_{cs}\rvert$	$0.7196 \pm 0.0035 \pm 0.0041$	$f_\pi^+(0)\lvert V_{cd}\rvert$	$0.1420 \pm 0.0024 \pm 0.0010$
	r_1	$-2.3331 \pm 0.1587 \pm 0.0804$	r_1	$-1.8434 \pm 0.2212 \pm 0.0690$
	r_2	$3.4223 \pm 3.9090 \pm 2.4092$	r_2	$-1.3871 \pm 1.4615 \pm 0.4677$

[from H Ma's talk on behalf of BESIII at CHARM 2015]

a benchmark case: $f_+(B \to \pi l \nu)$

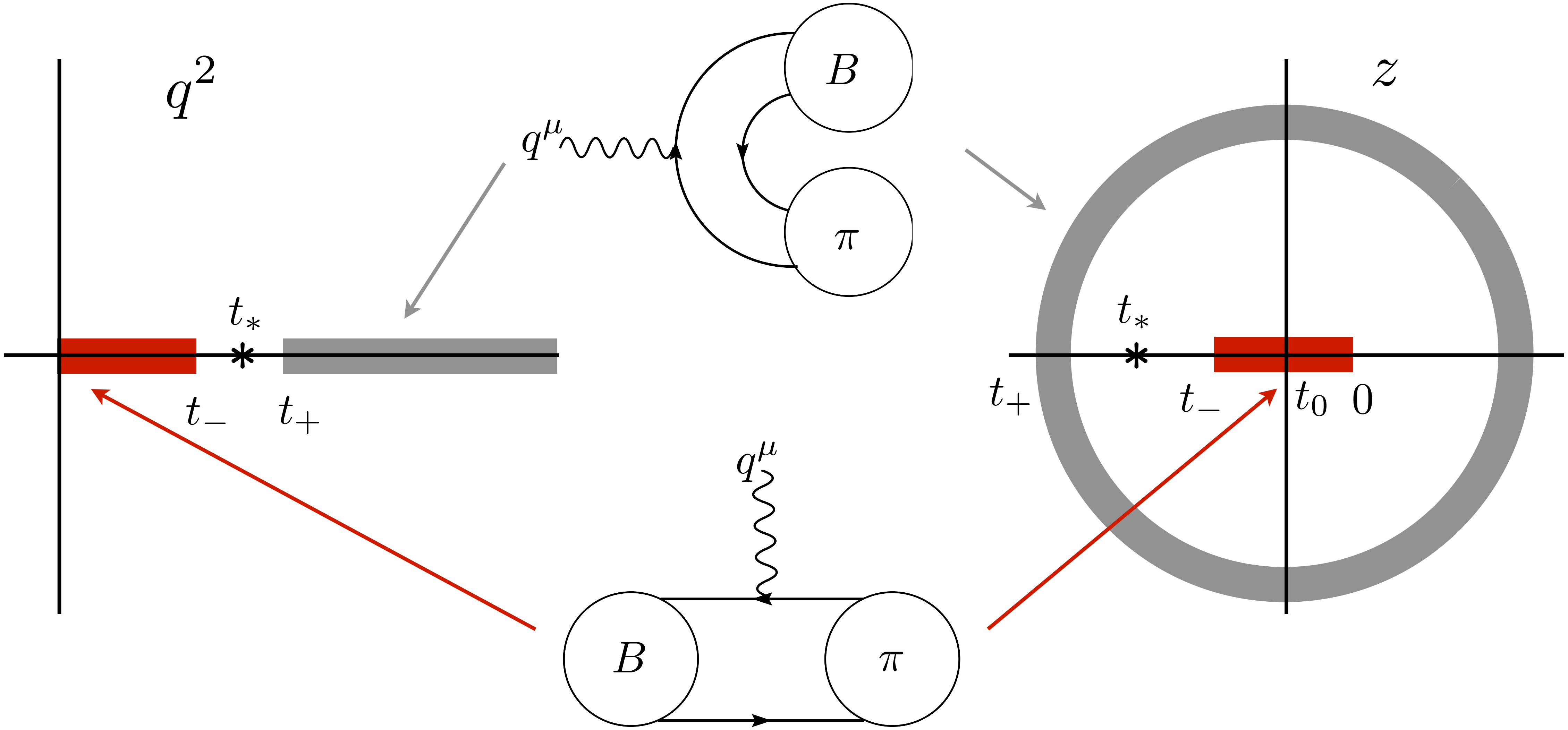
various parametrisations based on pole dominance: Bećirević-Kaidalov, Ball-Zwicky, Hill, ... difficult to systematically improve precision

[Bečirević, Kaidalov PLB 478 (2000) 417]
[Ball, Zwicky PRD 71 (2005) 014015]
[Hill PRD 73 (2006) 014012]

z-parametrisations proposed to solve this issue (almost) rigourously by exploiting unitarity and crossing symmetry

[Okubo PRD 3 (1971) 2807, 4 (1971) 725]
[Bourrely, Machet, de Rafael NPB 189 (1981) 157]
[Boyd, Grinstein, Lebed PRL 74 (1995) 4603]
[Lellouch NPB 479 (1996) 353]
[Bourrely, Caprini, Micu EJPC 27 (2003) 439]
[Arnesen, Grinstein, Rothstein, Stewart PRL 95 (2005) 071802]
[Becher, Hill PLB 633 (2006) 61]
[Flynn, Nieves PRD 75 (2007) 013008]
[Bourrely, Caprini, Lellouch PRD 79 (2009) 013008]

a benchmark case: $f_+(B \to \pi l \nu)$



$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$\Rightarrow \quad f_+(q^2) = \frac{1}{B(q^2)\phi(q^2, t_0)} \sum_{n \geq 0} a_n z(q^2, t_0)^n$$

$$t_+ = (m_B + m_\pi)^2 \,, \qquad t_0 < t_+$$

unitarity bound: $\sum_{m,n} B_{mn}^{(\phi)} a_m a_n \leq 1$

a benchmark case: $f_+(B \to \pi l \nu)$

$$f_+(q^2) = \frac{1}{B(q^2)\phi(q^2,t_0)} \sum_{n\geq 0} a_n z(q^2,t_0)^n \qquad B(q^2) = z(q^2, m^2_{B^*})$$

BGL: complicated outer function $\phi \quad \longrightarrow \quad \sum_{n\geq 0} |a_n|^2 \lesssim 1$

[Boyd, Grinstein, Lebed PRL 74 (1995) 4603]

BCL: $f_+(q^2) = \frac{1}{1-q^2/m^2_{B^*}} \sum_{n\geq 0} a_n z^n \quad \longrightarrow \quad \sum_{m,n\geq 0} B_{mn} a_m a_n \lesssim 1$

(recommended by FLAG)

[Bourrely, Caprini, Lellouch PRD 79 (2009) 013008]

crucial for optimal use:

- *all* sub-threshold poles included in Blaschke factor
- fixed kinematics (coefficients implicitly depend on quark masses)

does the unitarity bound apply?

- using a z-parametrisation as part of a global fit including a, m_q, ... (modified z-expansion) tricky
  - poles can cross threshold as quark masses change
  - complicated entanglement of (m_q,a) dependence (complete form factor vs. z-parametrisation coefficient)

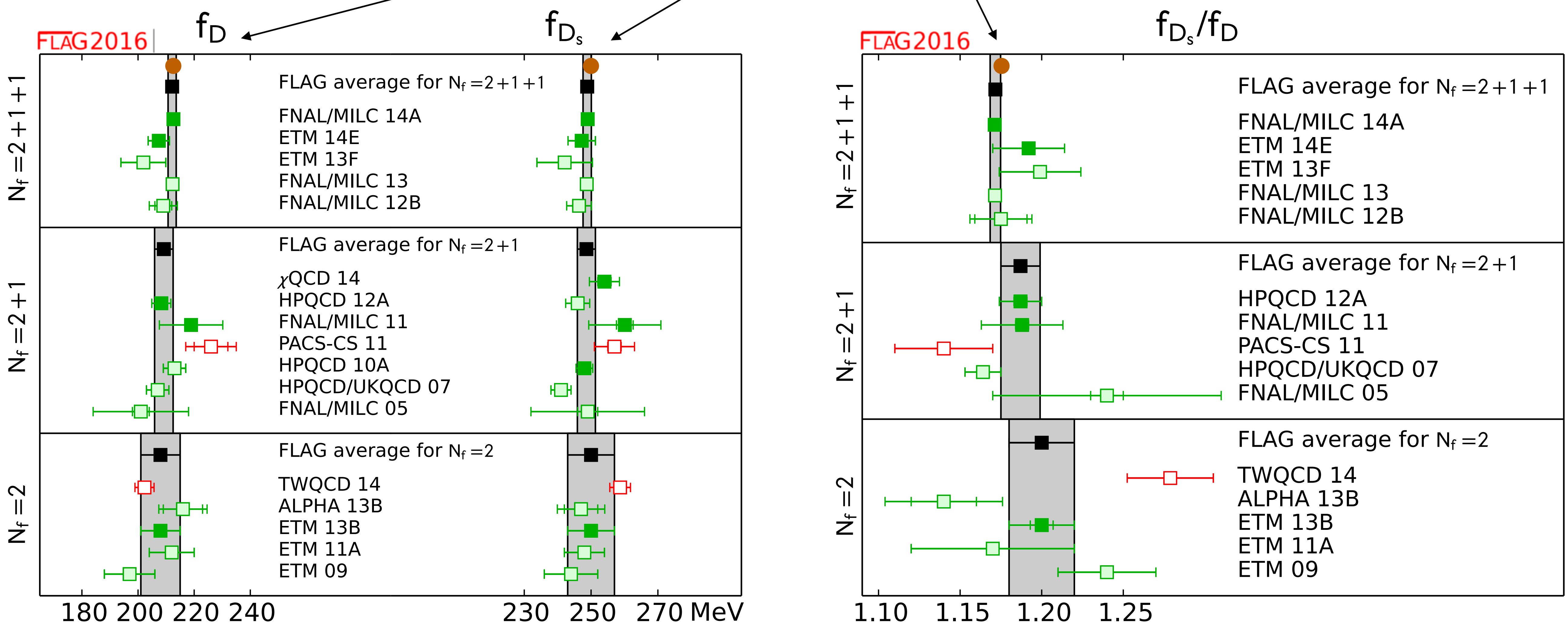
- pole structure not always well-known (scalar channels, D decay), or complicated (Λ_b decay)

- missing sub-threshold poles may imply convergence breakdown (proton charge radius analysis by Hill, Paz et al, D semileptonic decay data by Bećirević et al)

[Hill, Paz PRD 82 (2010) 113005]
[Bhattacharya, Hill, Paz PRD 84 (2011) 073006]
[Epstein, Paz, Roy PRD 90 (2014) 074027]
[Bećirević et al arXiv:1407.1019]

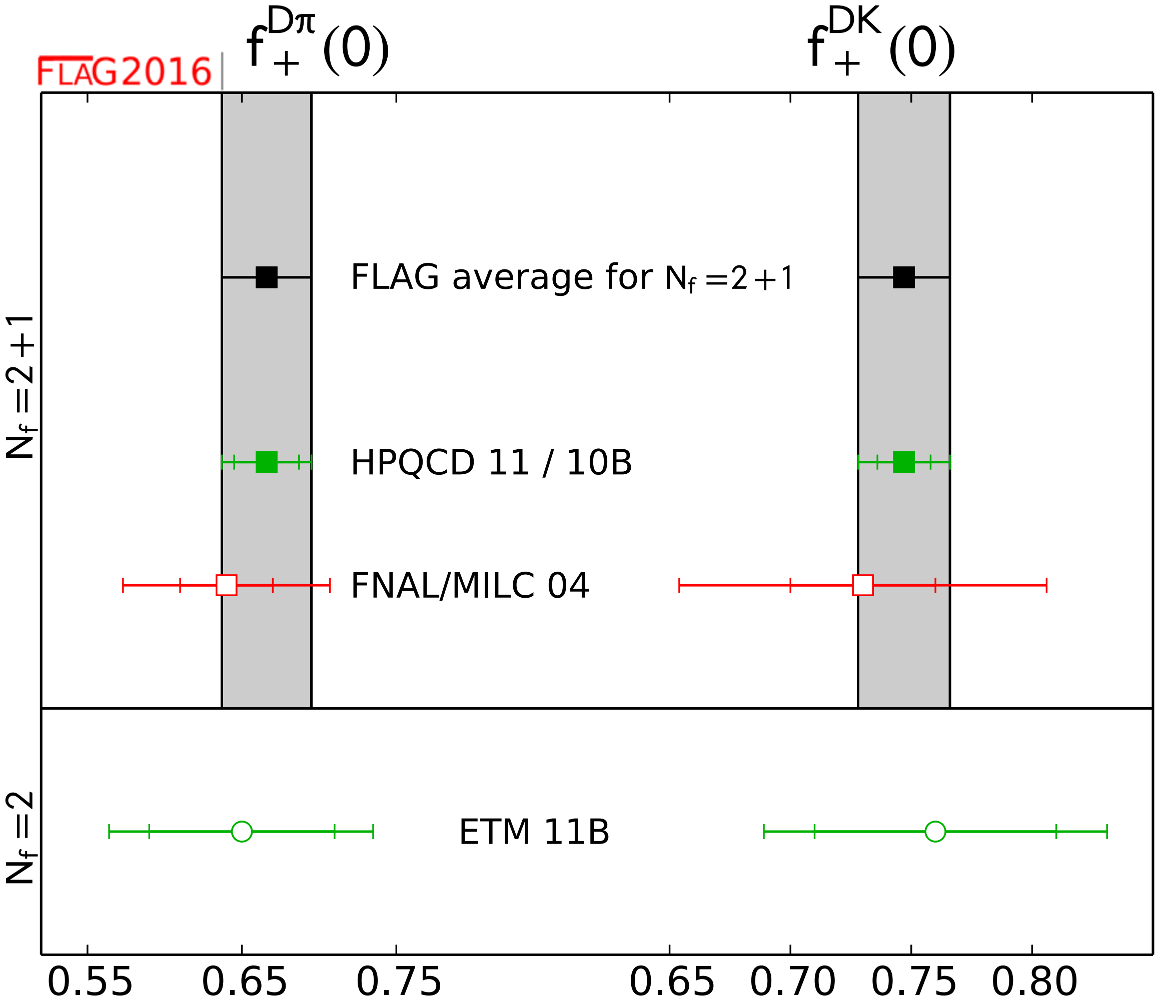
leptonic charm decay

[FNAL/MILC arXiv:1712.09262]

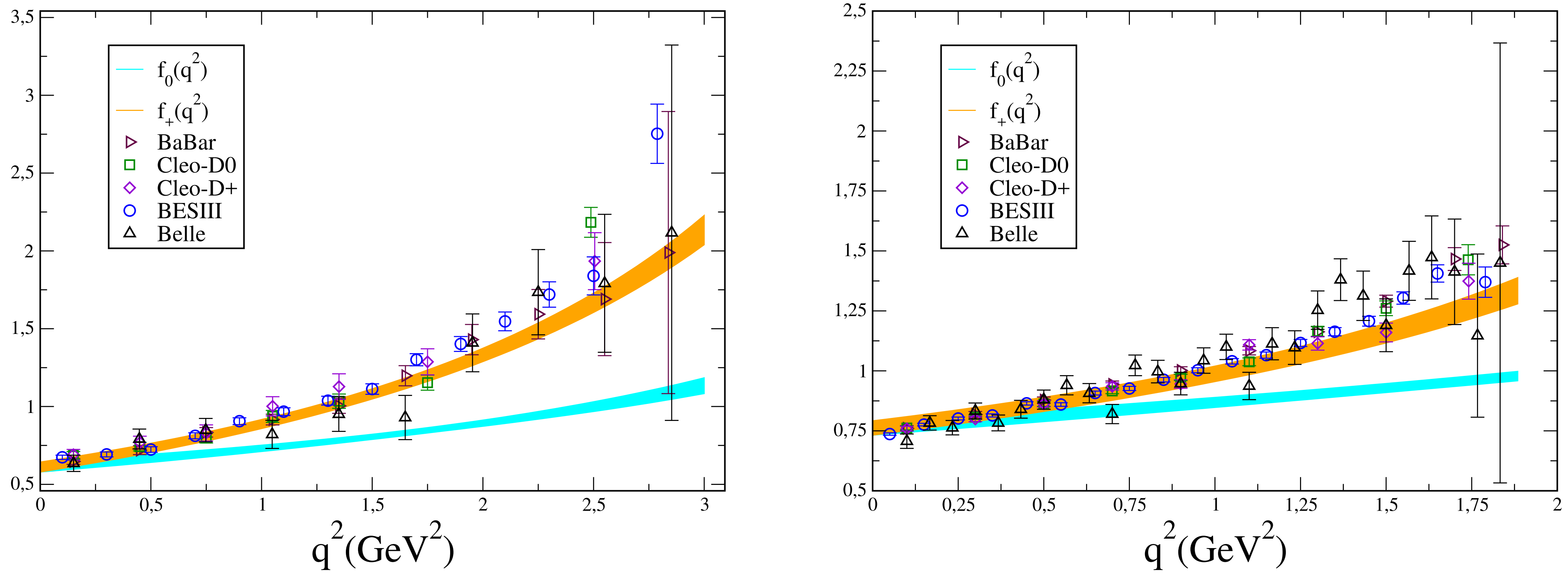


semileptonic charm decay

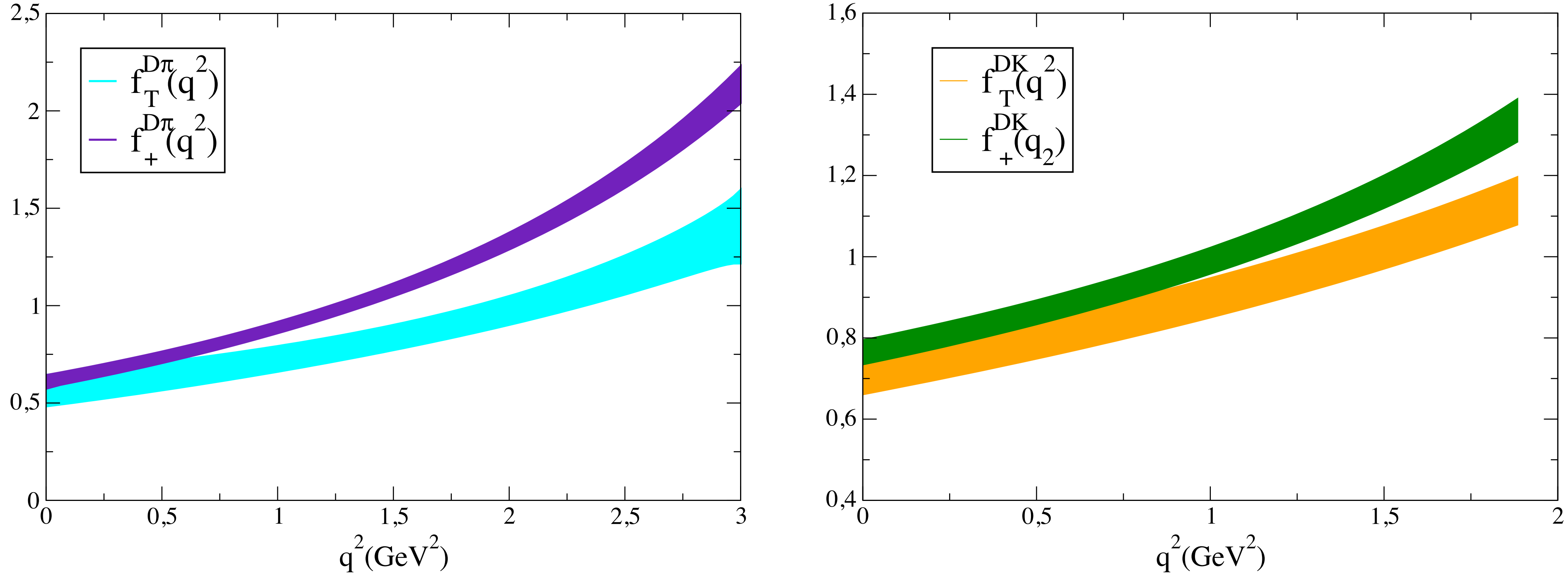
before 2017: very few computations, no q^2 dependence



semileptonic charm decay

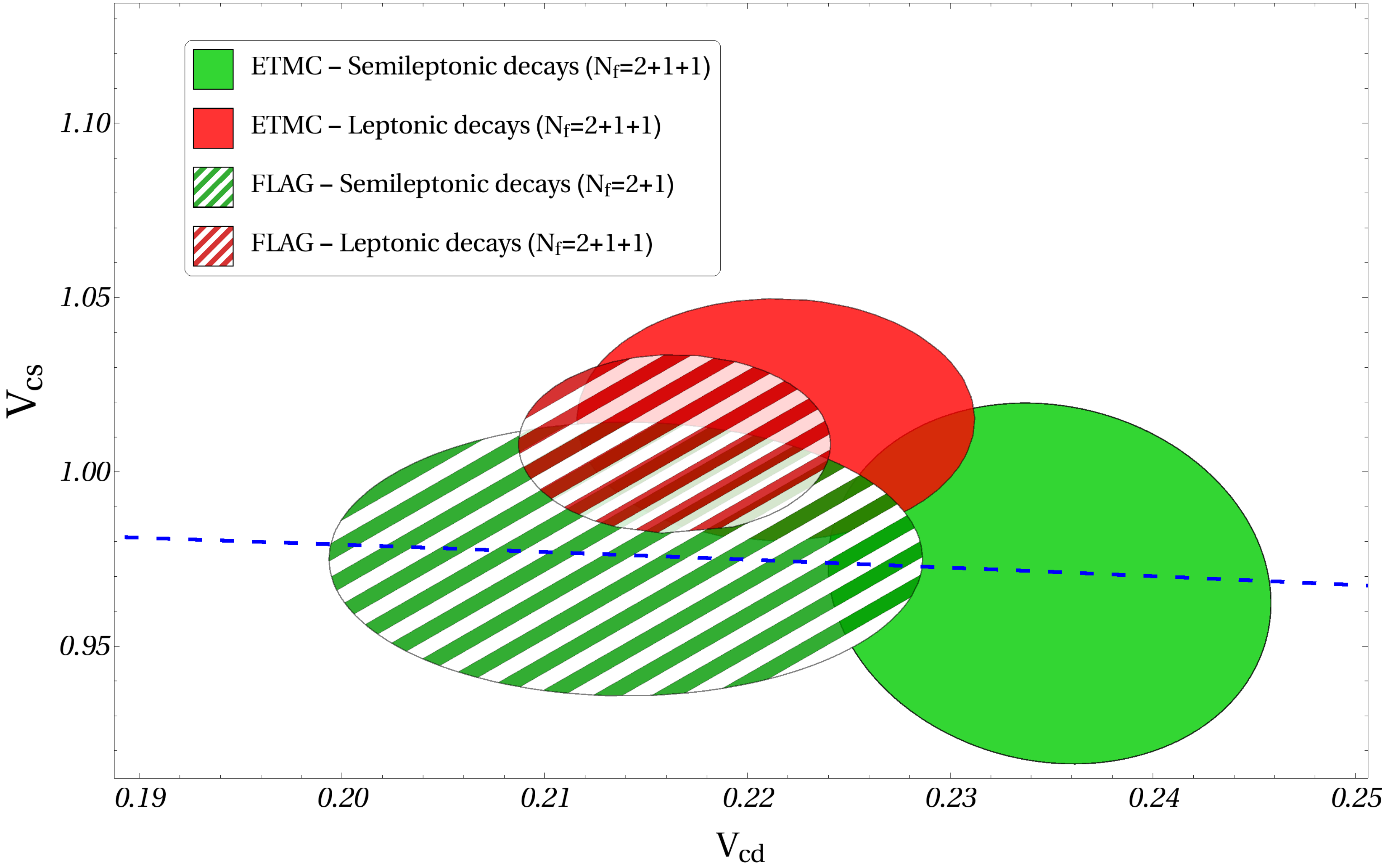


[ETMC PRD 96 (2017) 054514]



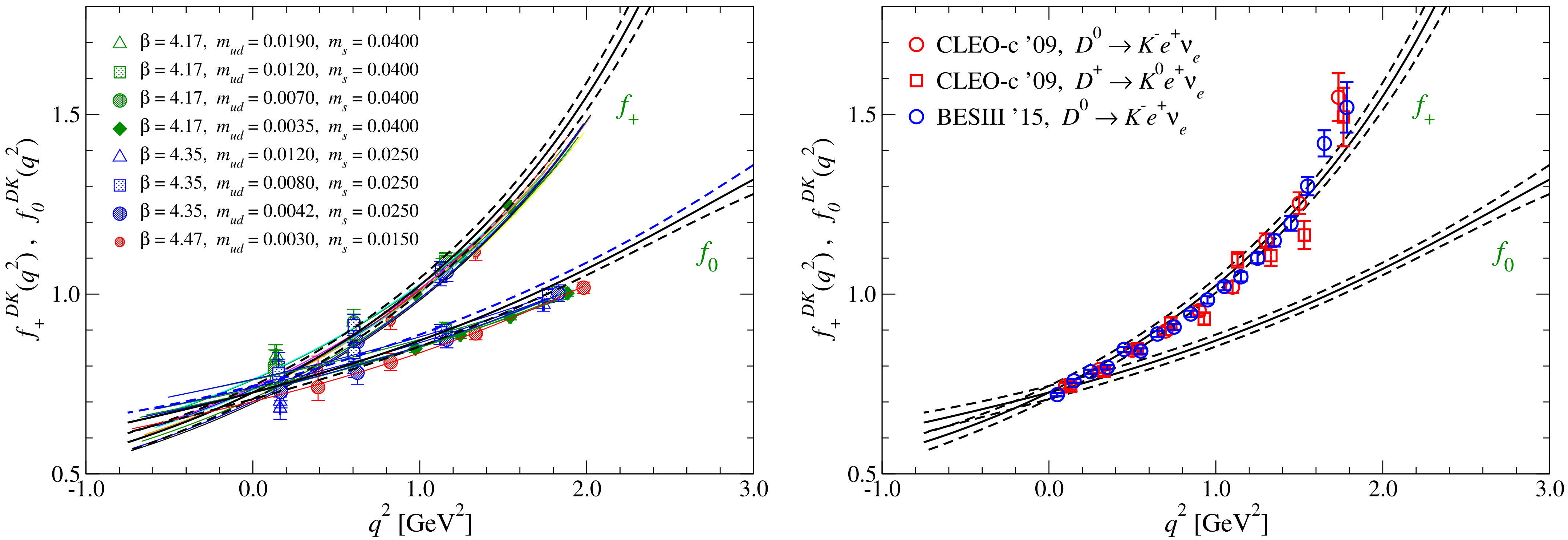
[ETMC arXiv:1710.07121]

semileptonic charm decay

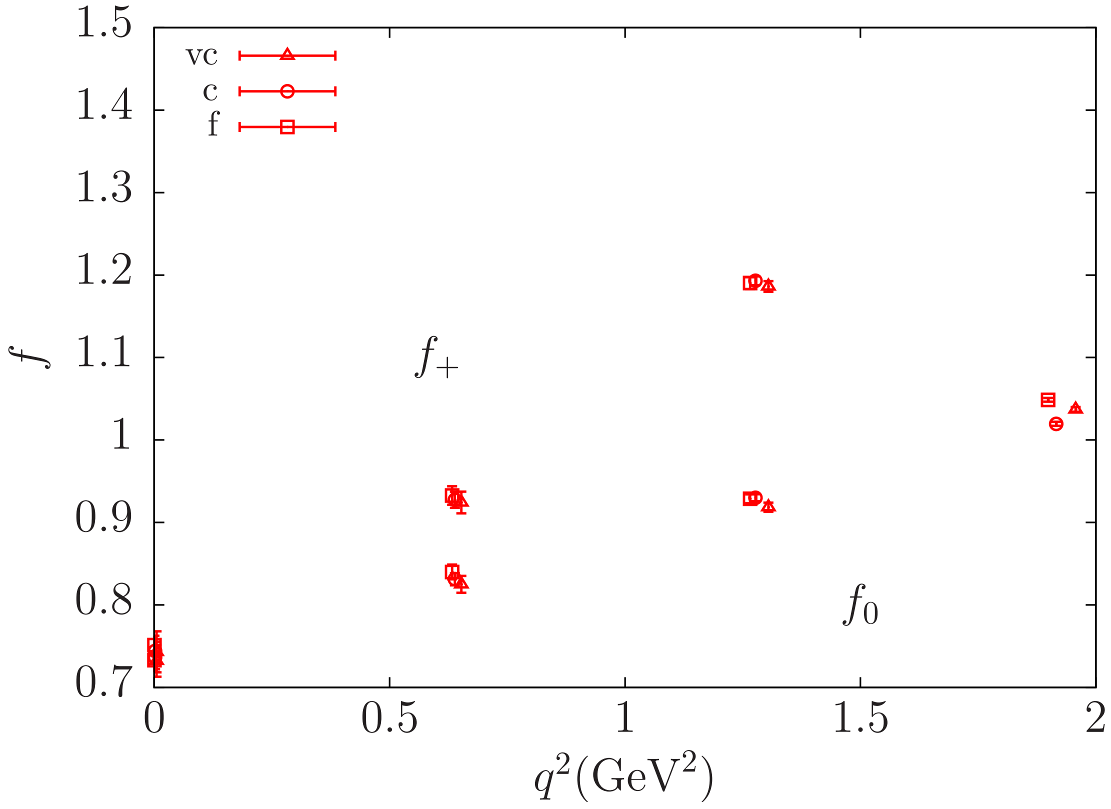


[ETMC arXiv:1706.03657]

semileptonic charm decay



[JLQCD arXiv:1711.11235]

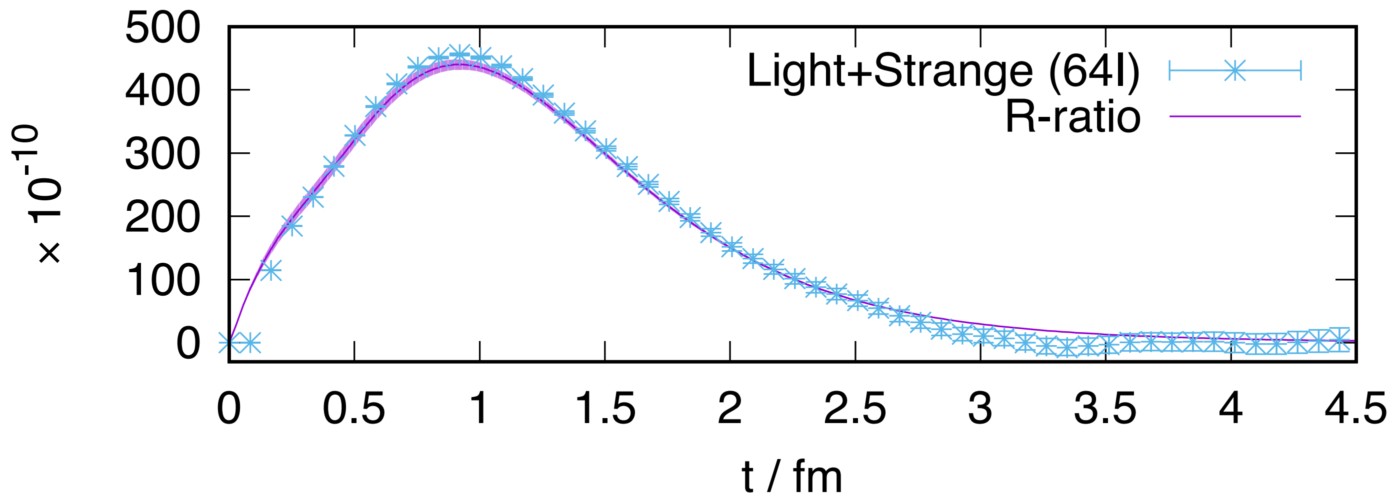


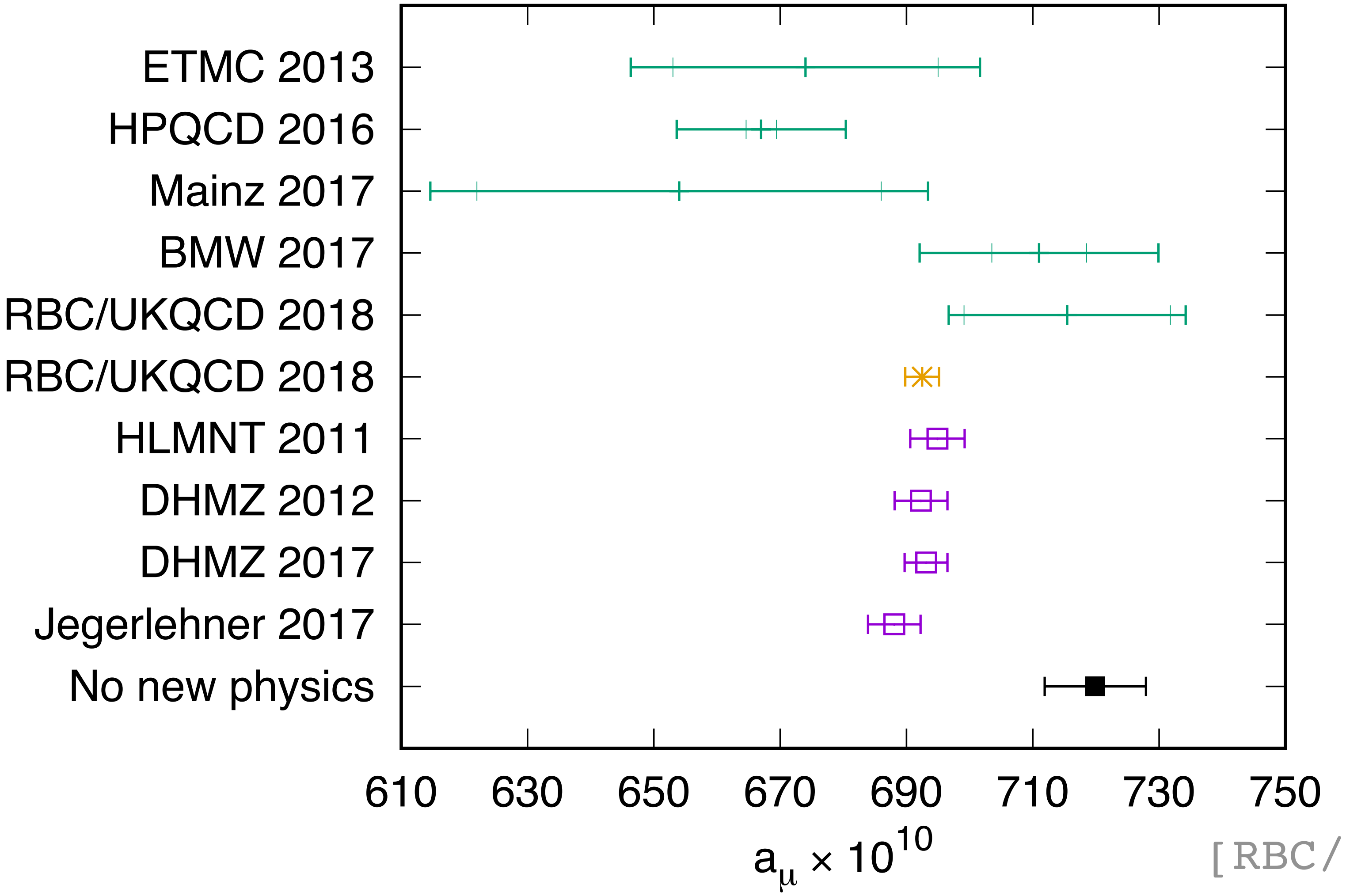
[HPQCD arXiv:1710.07334; ongoing]

$(g\text{-}2)_\mu$

$$\Pi(q^2) - \Pi(q^2=0) = \sum_t \left(\frac{\cos(qt)-1}{q^2} + \frac{1}{2}t^2 \right) C(t)$$

state-of-the-art: RBC/UKQCD QCD+QED determination combining [lattice for intermediate distances]+[exp $e^+e^- \rightarrow$hadrons for short/long distances]





[RBC/UKQCD arXiv:1801.07224]

2. PDFs and Quasi-PDFs

[slides from L Del Debbio's review, Lattice 2017]

$$\boxed{\mathcal{M}_i(\zeta, P) = \langle P|\bar{\psi}(\zeta)\Gamma_i \, \mathrm{P}\exp\left(-ig\int_0^{\zeta} d\eta\, A(\eta)\right)\psi(0)|P\rangle}$$

light-cone PDF – $\zeta = (0, y^-, \vec{0}_\perp)$:

$$f(x,\mu) = \int \frac{dy^-}{4\pi}\, e^{-i(xP^+)y^-}\, \mathcal{M}^+(y^-, P^+)$$

quasi-PDF, time-independent quantity – $\zeta = (0,0,0,z)$:

$$q(x,\mu,M_N,P_z) = \int \frac{dz}{4\pi}\, e^{-i(xP_z)z}\, \mathcal{M}^z(z,P_z) \xrightarrow{P_z\to\infty} f(x,\mu)$$

From quasi-PDFs to PDFs

`[slides from L Del Debbio’s review, Lattice 2017]`

Extracting PDFs from lattice simulations:

- renormalization of the lattice operator
    - ◇ RI/MOM prescription
    - ◇ matching to $\overline{\mathrm{MS}}$
    - ◇ trace operators and power divergencies
- Euclidean to Minkowski space
- factorization theorem for the renormalized quasi-PDF
- gradient flow