$B_c \to J/\psi$ form factors
and lepton flavor universality violation in $R(J/\psi)$

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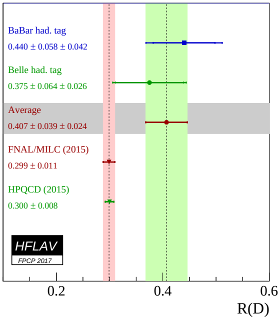
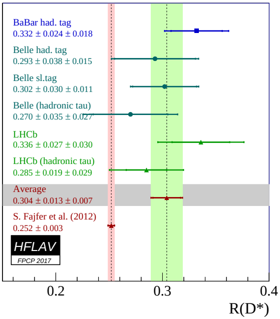
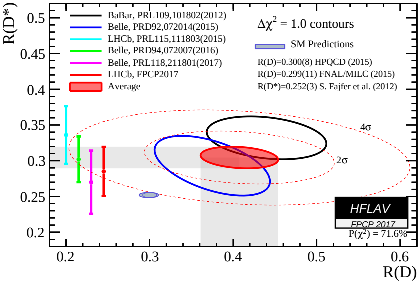
Possible cases of LFV in B decays

- reminder on experimental results:
- $R_{K^{(*)}}$
  - $b \to s$ loop induced FCNC in SM (penguin, box)
  - $R_K = \frac{\mathcal{B}(B^+ \to K^+ \mu\mu)}{\mathcal{B}(B^+ \to K^+ ee)} = 0.745 \pm {}^{0.090}_{0.074} \pm 0.036$ [1] ($q^2 \in [1, 6]$ GeV^2)
  - $R_{K^*} = \frac{\mathcal{B}(B \to K^* \mu\mu)}{\mathcal{B}(B \to K^* ee)} = 0.660 \pm {}^{0.110}_{0.070} \pm 0.024$ [2] ($q^2 \in [0.045, 1.1]$ GeV^2)
  - $R_{K^*} = \frac{\mathcal{B}(B \to K^* \mu\mu)}{\mathcal{B}(B \to K^* ee)} = 0.685 \pm {}^{0.113}_{0.069} \pm 0.047$ [2] ($q^2 \in [1.1, 6]$ GeV^2)
    - respectively $\sim 2.6\sigma$, $(2.2 - 2.4)\sigma$, combined $\sim 4\sigma$
- $R_{D^{(*)}}$
  - $b \to c$ charged current tree
  - $R_D = \frac{\mathcal{B}(B \to D\tau\bar{\nu}_\tau)}{\mathcal{B}(B \to Dl\bar{\nu}_l)} = 0.407 \pm 0.039 \pm 0.024$
  - $R_{D^*} = \frac{\mathcal{B}(B \to D^*\tau\bar{\nu}_\tau)}{\mathcal{B}(B \to D^*l\bar{\nu}_l)} = 0.304 \pm 0.013 \pm 0.007$
    - respectively $\sim 2.3\sigma$, 3.3σ, combined $\sim 4\sigma$

[1] LHCb (2014) 1406.6482
[2] LHCb (2017) 1705.05802


BaBar had. tag
0.440 ± 0.058 ± 0.042
Belle had. tag
0.375 ± 0.064 ± 0.026
Average
0.407 ± 0.039 ± 0.024
FNAL/MILC (2015)
0.299 ± 0.011
HPQCD (2015)
0.300 ± 0.008
HFLAV
FPCP 2017
0.2
0.4
0.6
R(D)


BaBar had. tag
0.332 ± 0.024 ± 0.018
Belle had. tag
0.293 ± 0.038 ± 0.015
Belle sl.tag
0.302 ± 0.030 ± 0.011
Belle (hadronic tau)
0.270 ± 0.035 ± 0.027
LHCb
0.336 ± 0.027 ± 0.030
LHCb (hadronic tau)
0.285 ± 0.019 ± 0.029
Average
0.304 ± 0.013 ± 0.007
S. Fajfer et al. (2012)
0.252 ± 0.003
HFLAV
FPCP 2017
0.2
0.3
0.4
R(D*)


BaBar, PRL109,101802(2012)
Belle, PRD92,072014(2015)
LHCb, PRL115,111803(2015)
Belle, PRD94,072007(2016)
Belle, PRL118,211801(2017)
LHCb, FPCP2017
Average
Δχ² = 1.0 contours
SM Predictions
R(D)=0.300(8) HPQCD (2015)
R(D)=0.299(11) FNAL/MILC (2015)
R(D*)=0.252(3) S. Fajfer et al. (2012)
4σ
2σ
HFLAV
FPCP 2017
P(χ²) = 71.6%
R(D*)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.2
0.3
0.4
0.5
0.6
R(D)

New observable: $R_{J/\psi}$

- new measurement of $R_{J/\psi} = \frac{\mathcal{B}(B_c^+ \to J/\psi \tau^+ \bar{\nu}_\tau)}{\mathcal{B}(B_c^+ \to J/\psi \mu^+ \bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18$ [3]
- $\sim 2\sigma$ from SM ($R^{\rm SM}_{J/\psi} = 0.22 - 0.28$)
- even though the value is much larger, errors are still huge

[3] LHCb (2017/8) 1711.05623

The QCD Sum Rule approach

- assume Quark-Hadron Duality
- cf. sum rule (hadronic side ≈ parton side) → perturbative + "nonperturbative" part (modeled by OPE)
- here, approximate mesons by currents

$$j_{B_c} \sim \bar{b} \mathrm{i} \gamma_5 c$$
$$j^{\mu}_{J/\psi} \sim \bar{c} \gamma^{\mu} c$$

- through dispersion relations obtain spectral densities by using **Cutkosky cut rules**

General strategy (2 pt. QCDSR)

- definitions of decay constants

$$\langle 0|\, \bar{c}\gamma_\nu c\, |J/\psi(p_2)\rangle = f_{J/\psi} m_{J/\psi} \epsilon_\nu$$

$$\langle B_c(p_1)|\, \bar{b} \mathrm{i}\gamma_5 c\, |0\rangle = -\frac{f_{B_c} m_{B_c}^2}{m_b + m_c}$$

- f_{B_c} known from lattice [4], $f_{J/\psi}$ both from lattice and experiment [5]
- calculate current correlators $\langle 0|\mathcal{T}\{j_{B_c} j_{B_c}^\dagger\}|0\rangle$ and $\langle 0|\mathcal{T}\{j_{J/\psi}^\mu j_{J/\psi}^{\nu\,\dagger}\}|0\rangle$ in mom. space perturbatively
- use the fact that

$$\Pi(q^2)_i \approx \frac{1}{\pi} \int_{(m_{q_1}+m_{q_2})^2}^{s_0^{\mathrm{eff.}}} \mathrm{d}s \frac{\mathrm{Im}[\Pi_i(s)]}{s - q^2} \tag{1}$$

[4] McNeile, Davies, Follana, Hornbostel, Lepage 1207.0994

[5] Bečirević, Duplančić, Klajn, Melić, Sanfilippo 1312.2858; Bailas, Blossier, Morénas 1803.09673

- obtain imaginary part by "cutting" the diagrams
- to improve convergence, Borel transform by

$$\hat{B}_{p^2}(M^2)\frac{1}{(p^2-m^2)^n} = \frac{1}{(n-1)!}(-1)^n \frac{1}{M^{2n}} \mathrm{e}^{-m^2/M^2} \quad (2)$$

- fix thresholds so that they would reproduce values from lattice/experiment
- work where constants are stable in Borel mass parameter ($M^2_{B_c} = 60 - 80$ GeV^2, $M^2_{J/\psi} = 20 - 25$ GeV^2)
- we get

$s_{J/\psi}$ [GeV^2]	$f_{J/\psi}$ [GeV]
15.5	0.385
16	0.402
16.5	0.407

s_{B_c} [GeV^2]	f_{B_c} [GeV]
52	0.406
53	0.424
54	0.439

- working with pole mass, and $m_c = z\, m_b$, where $z = 0.28 - 0.32$, and $m_b = 4.6$ GeV

General strategy (3 pt. QCDSR)

- wanted matrix element is parametrized using form factors

$$
\begin{aligned}
-\mathrm{i}\,\langle J/\psi(p_2)|\,\gamma_\mu(1-\gamma_5)\,|B_c(p_1)\rangle =& \frac{2V(q^2)}{m_{B_c}+m_{J/\psi}}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p_2^\alpha p_1^\beta \\
&+\mathrm{i}(m_{B_c}+m_{J/\psi})A_1(q^2)\epsilon^*_\mu \\
&-\mathrm{i}\frac{A_2(q^2)}{m_{B_c}+m_{J/\psi}}(\epsilon^*\cdot q)(p_1+p_2)_\mu \\
&-\mathrm{i}\frac{2m_{J/\psi}}{q^2}(A_3(q^2)-A_0(q^2))(\epsilon^*\cdot q)q_\mu
\end{aligned}
$$

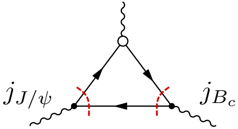
$$
\begin{aligned}
-\mathrm{i}\,\langle J/\psi(p_2)|\,\sigma_{\mu\nu}\gamma_5\,|B_c(p_1)\rangle = -\mathrm{i}A(q^2)\{\varepsilon^*_\mu(p_1+p_2)_\nu-(p_1+p_2)_\mu\varepsilon^*_\nu\} \\
+\mathrm{i}B(q^2)\{\varepsilon^*_\mu q_\nu - q_\mu\varepsilon^*_\nu\} + 2\mathrm{i}C(q^2)\frac{\varepsilon^* q}{m^2_{B_c}-m^2_{J/\psi}}\{p_{2\mu}q_\nu - q_\mu p_{2\nu}\}
\end{aligned}
$$

- for the tensor form factors, we prefer different form

$$T_1(q^2) = A(q^2) \qquad T_2(q^2) = A(q^2) - \frac{q^2}{m_{B_c}^2 - m_{J/\psi}^2} B(q^2)$$

$$T_3(q^2) = B(q^2) + C(q^2) \qquad \tilde{T}_3(q^2) = A(q^2) + \frac{q^2}{m_{B_c}^2 - m_{J/\psi}^2} C(q^2)$$

- now perturbatively calculate correlator of three currents by "cutting" in two channels [6]



- sequential cuts in two channels

$$\mathcal{D}_{s_1,s_2}[\Pi_{\mu\nu}] = \mathcal{D}_{s_2}\{\mathcal{D}_{s_1}[\Pi_{\mu\nu}]\} = (2\mathrm{i})\mathrm{Im}_{s_2}\{(2\mathrm{i})\mathrm{Im}_{s_1}[\Pi_{\mu\nu}]\} = (-4)\mathrm{Im}_{s_1,s_2}[\Pi_{\mu\nu}]$$

[6] Britto 1401.3546

- decomposition

$$\Pi_{\mu\nu} = \sum_i \Pi^i(p_1^2, p_2^2, q^2)\Gamma^i_{\mu\nu} \tag{3}$$

has double dispersion relations

$$\Pi_i^{\rm ph}(p_1^2, p_2^2, q^2) = -\frac{1}{(2\pi)^2}\iint \frac{\rho_i^{\rm ph}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} {\rm d}s_1 {\rm d}s_2 \tag{4}$$

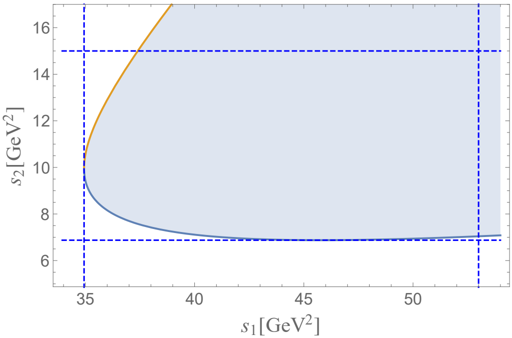
- spectral density: resonances + continuum $\rightarrow$

$$\int\limits_{s^0_{\rm ph1}} \int\limits_{s^0_{\rm ph2}} \rho_i^{\rm cont}(s_1, s_2, q^2) {\rm e}^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} {\rm d}s_1 {\rm d}s_2$$

$$\approx \int\limits_{s^0_{\rm eff1}} \int\limits_{s^0_{\rm eff2}} \rho_i^{\rm pert}(s_1, s_2, q^2) {\rm e}^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} {\rm d}s_1 {\rm d}s_2$$

- now use the thresholds used in 2pt. SR to obtain decay constants!

- from delta functions → phase space restrictions



- after integrating numerically over densities → form factors
- estimate errors by varying:
  - thresholds and decay constants simultaneously
  - Borel mass parameter
  - ratio of masses z

Leading correction - $\langle GG \rangle$

- there are no quark condensate contributions
- main correction comes from $\langle GG \rangle$
- extracted in "fixed-point" (Schwinger-Fock) gauge defined by

$$(x - x_0)_\mu A^\mu(x) = 0$$

- where

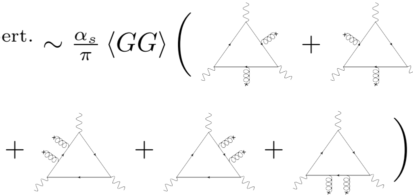
$$A^\mu(x) = -\mathrm{i}\frac{(2\pi)^4}{2} G_{\alpha\mu}(0) \frac{\partial}{\partial k_\alpha} \delta^{(4)}(k) + \dots$$

$\rightarrow$ allows for parametrizing with $\langle GG \rangle$

Leading correction - $\langle GG \rangle$

- the contributions to the spectral density look like

$$\Pi_{\mu\nu}^{\text{nonpert.}} \sim \frac{\alpha_s}{\pi} \langle GG \rangle \Big(\cdots \Big)$$



- tensor reduction to $D-$dim. scalar integrals using ROLI [7]

$$I_0^D(a,b,c) = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{1}{\left[k^2 - m_c^2\right]^a \left[(k+p_1)^2 - m_b^2\right]^b \left[(k+p_2)^2 - m_c^2\right]^c}$$

  - easily "Borelised"

- amount only up to couple % (depending on value of $\langle GG \rangle$)

[7]Duplančić, Nižić 03031804

FF fit

- QCDSR is reliable up to low values of q^2
- for higher q^2, need to extrapolate
- use the "z parametrization":

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

where $t_0 = 0$ in our case, and $t_+ = (m_{B_c} + m_{J/\psi})^2$

- form factors are fit to a function [8]

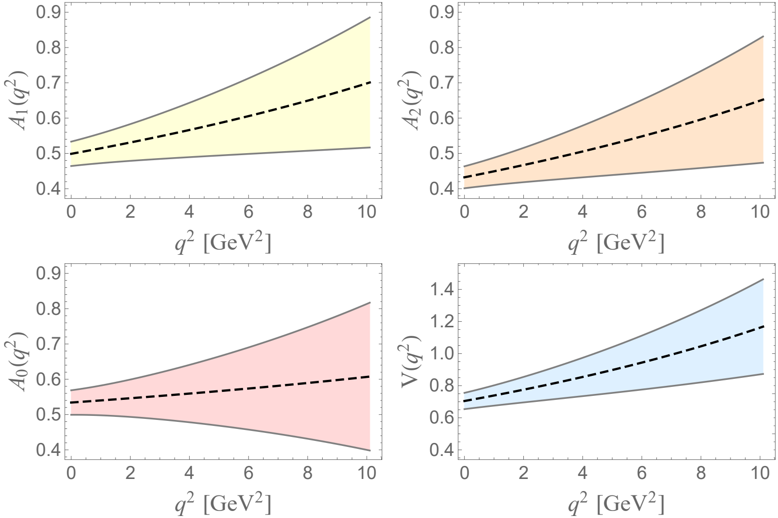
$$F_i(q^2) = \frac{1}{1 - q^2/m_{B_c}^{*2}} \sum_{k=0}^{K} \frac{a_k}{k!} z^k$$

it is found that it is enough to use $K = 1$

[8] Bourrely, Caprini, Lellouch 0807.2722

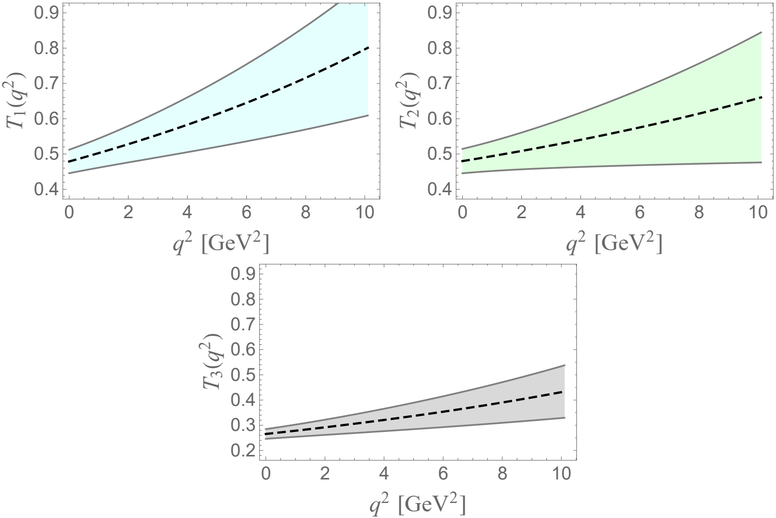
Form factor fits

- vector form factors are then



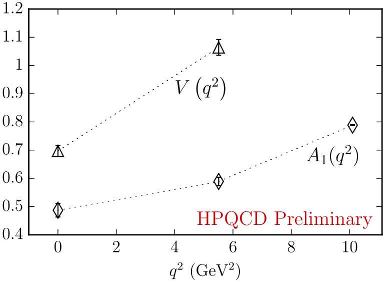
Form factor fits

- and, remaining, tensor form factors



Comparison with HPQCD

- there is some preliminary lattice data (for A_1 and V) available [9], with which we nicely agree



- $V^{\text{lat.}}(0) = 0.70 \pm 0.02$, while $V^{\text{us}}(0) = 0.70 \pm 0.07$
- $A_1^{\text{lat.}}(0) = 0.49 \pm 0.03$, while $A_1^{\text{us}}(0) = 0.50 \pm 0.05$

[9] HPQCD 1611.01987

$R_{J/\psi}$

- and finally our

$$R_{J/\Psi} = \frac{\mathcal{B}(B_c^+ \to J/\psi \tau^+ \bar{\nu}_\tau)}{\mathcal{B}(B_c^+ \to J/\psi \mu^+ \bar{\nu}_\mu)} = 0.23(1)$$

  in Standard Model

- other known calculations are

$R_{J/\psi}$	author	type
0.228(8)	Ivanov *et al.* 0007169	QM
0.216(6)	Kiselev 0211021	QCDSR
0.278(9)	Ebert *et al.* 0306306	QM
0.264(7)	Hernandez *et al.* 0607150	QM
0.179(1)	Watanabe 1709.08644	pQCD

- so relatively consistent with previous calculations

Testing NP in $R_D - R_{J/\psi}$

- general effective Lagrangian involving four-Fermi operators (with the assumption of coupling only to τ)

$$-\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[(1 + g_{V_1}) \mathcal{O}_{V_L} + g_{V_R} \mathcal{O}_{V_R} + g_{S_L} \mathcal{O}_{S_L} + g_{S_R} \mathcal{O}_{S_R} + g_T \mathcal{O}_T \right] + \text{h.c.}$$

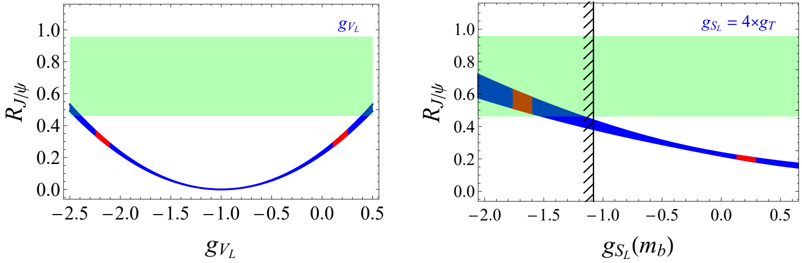
where

$$\begin{aligned}
\mathcal{O}_{V_L} &= (\bar{c}_L \gamma^\mu b_L)(\bar{\tau}_L \gamma_\mu \nu_L) \\
\mathcal{O}_{V_R} &= (\bar{c}_R \gamma^\mu b_R)(\bar{\tau}_L \gamma_\mu \nu_L) \\
\mathcal{O}_{S_L} &= (\bar{c}_L b_R)(\bar{\tau}_R \gamma_\mu \nu_L) \\
\mathcal{O}_{S_R} &= (\bar{c}_R b_L)(\bar{\tau}_R \gamma_\mu \nu_L) \\
\mathcal{O}_T &= (\bar{c}_R \sigma^{\mu\nu} b_L)(\bar{\tau}_R \sigma_{\mu\nu} \nu_L)
\end{aligned}$$

- try to accomodate for both large R_D and $R_{J/\psi}$ (if remains large after more data - RN signal strength 1400 ± 300)

Testing NP in $R_D - R_{J/\psi}$

- plotting just the 1σ bounds

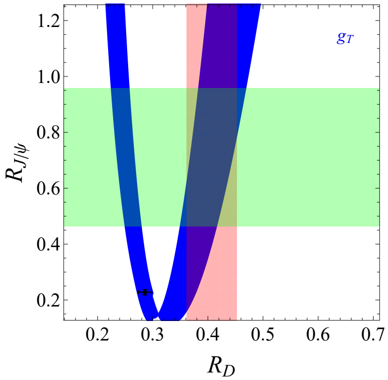


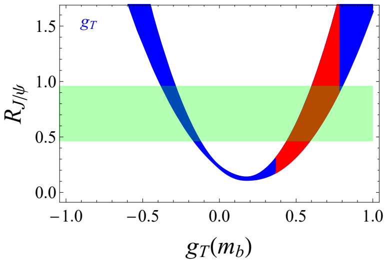
- lower bound [10] comes from demand that contribution of $B_c \to \tau\bar{\nu}$ doesn't exceed fraction of total width allowed by B_c lifetime in SM (would lift the slightly chirally suppressed $B_c \to \tau\bar{\nu}$)

[10] Alonso, Grinstein, Camalich 1611.06676

Testing NP in $R_D - R_{J/\psi}$

- if just $\mathcal{O}_T$ is turned on



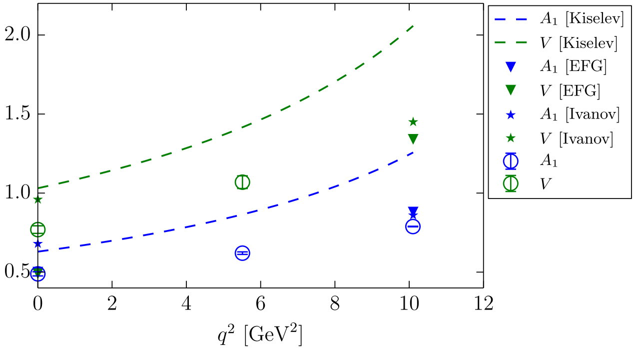


- one the more common ways of explaining $R_{D^{(*)}}$ are various leptoquark models
- in these terms LQs $S_1(3,1,-1/3) + R_2(3,2,7/6)$ would be plausible

Thank you!

BACKUP

- comparison of prev. results with lattice



BACKUP

- error budget from experiment

Source of uncertainty	**Size ($\times 10^{-2}$)**
Limited size of simulation samples	8.0
$B_c^+ \to J/\psi$ form factors	12.1
$B_c^+ \to \psi(2S)$ form factors	3.2
Fit bias correction	5.4
Z binning strategy	5.6
Misidentification background strategy	5.6
Combinatorial background cocktail	4.5
Combinatorial J/ψ sideband scaling	0.9
$B_c^+ \to J/\psi H_c X$ contribution	3.6
Semitauonic $\psi(2S)$ and χ_c feed-down	0.9
Weighting of simulation samples	1.6
Efficiency ratio	0.6
$\mathcal{B}(\tau^+ \to \mu^+ \nu_\mu \overline{\nu}_\tau)$	0.2
Total systematic uncertainty	**17.7**
Statistical uncertainty	**17.3**