Radiative corrections and strong interactions

Marc Knecht

Centre de Physique Théorique UMR7332,
CNRS Luminy Case 907, 13288 Marseille cedex 09 - France
knecht@cpt.univ-mrs.fr

Getting to grips with QCD – Campus des Cordeliers, Paris, 4 - 6 April, 2018

OUTLINE

- Introduction

- Radiative corrections in pion and kaon decays: the low-energy EFT point of view

- A case study: K_{e4}^{00} or radiative corrections in real life

- Radiative corrections to semileptonic decays in the SM

- Conclusion

(For the lattice point of view $\longrightarrow$ cf. talk by F. Sanfilippo)

Introduction

Precision measurements of (semileptonic) decays of K, D, B mesons, or hadronic decay modes of the τ lepton, allow to put constraints on physics beyond the standard model (tests of lepton flavour universality or of CKM unitarity, CP violation, admixture of right-handed currents,...)...

... but also provide information on low-energy strong interactions (e.g. decay constants, structure of form factors, $\pi\pi$ scattering lengths,...), that allow to test predictions or to determine non-perturbative parameters (low-energy constants in the case of kaons) that occur also in other processes

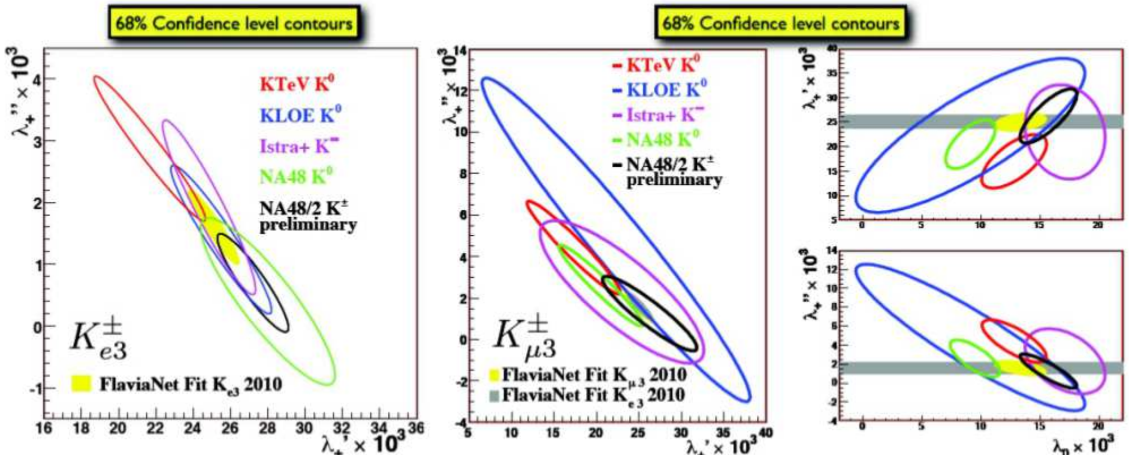
Theoretical predictions are often made in a world where $\alpha = 0$ ($M_\pi = M_{\pi^0}$), and even $m_u = m_d$. One has to connect this "theoretician's paradise" (J. Gasser) to the real world, where $\alpha \neq 0$ (which means in particular $M_\pi \neq M_{\pi^0}$)

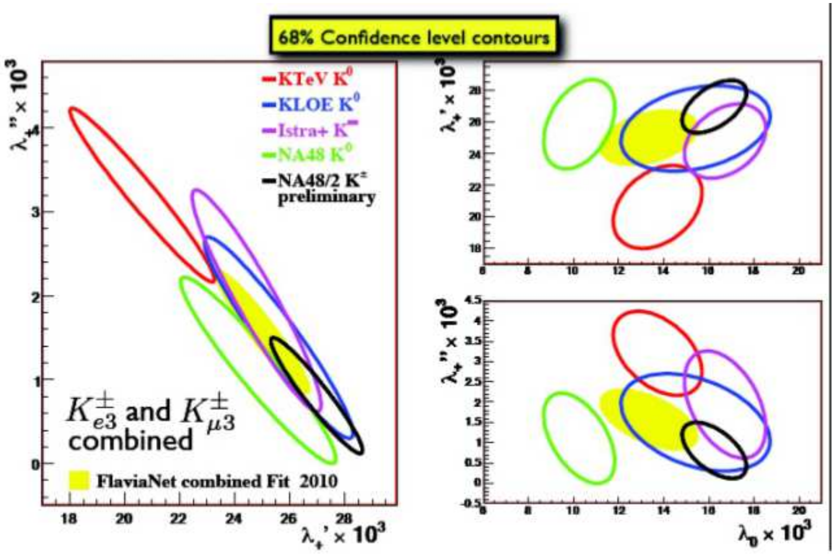
$\longrightarrow$ quantitative control over radiative, or more generally, isospion-breaking (IB) corrections has become mandatory

A lot of progress on the experimental side during the last decade or so on "traditional" (i.e. non rare) kaon decay modes (ISTRA+ @ IHEP, KTeV @ FNAL, KLOE and KLOE2 @ DAΦNE, NA48 and NA48/2 @ SPS), and more is to come, e.g. NA62

Illustration with structure of $K_{\ell 3}$ form factors...

M. Antonelli et al., Eur Phys. J. C 69, 399 (2010)





D. Madigozhin [NA48/2 Coll.], PoS DIS2013, 135 (2013)

... or with $K^{\pm} \to \pi^+\pi^- e^{\pm}\nu$

- Geneva-Saclay high-statistics experiment: $3 \cdot 10^4$ events, a_0 at 20%

L. Rosselet et al., Phys. Rev. D 15, 574 (1977)

- BNL-E865: $4 \cdot 10^5$ events

S. Pislak et al., Phys. Rev. 67, 072004 (2003) [Phys. Rev. 81, 119903 (2010)] [hep-ex/0301040]

- NA48/2: $1.1 \cdot 10^6$ events, a_0 at 6%

J. R. Batley et al., Eur. Phys. J. C 70, 635 (2010)

The experimental values of the two S-wave scattering lengths

$$a_0 = 0.222(14) \qquad a_2 = -0.0432(97)$$

compare quite well with the prediction from two-loop chiral perturbation theory

$$a_0 = 0.220(5) \qquad a_2 = -0.0444(10)$$

G. Colangelo, J. Gasser, H. Leutwyler, Nucl. Phys. B 603, 125 (2001)

But taking isospin corrections ($m_u \neq m_d$ and $M_\pi \neq M_{\pi^0}$) into account turns out to be crucial in order to reach this agreement

J. Gasser, PoS KAON , 033 (2008), arXiv:0710.3048 [hep-ph]

Although it will not always be mentioned explicitly, only infrared finite radiatively-corrected observables will be considered [in particular, amplitudes include emission of one (soft) photon]

Radiative corrections to total decay rates are typically at the level of a few %

$$\Gamma = \Gamma_0 \left[1 + \alpha \frac{\Delta\Gamma}{\Gamma_0} \right] \qquad \alpha \frac{\Delta\Gamma}{\Gamma_0} \sim \pm(1-3)\%$$

Radiative corrections to differential decay rates can, locally, be more important, e.g. $\sim \pm 10\%$

$$\frac{d^2\Gamma}{dxdy} = \frac{d^2\Gamma_0}{dxdy} \left[1 + \alpha\delta(x,y) \right] \qquad \alpha\delta(x,y) \sim \pm(1-10)\%$$

[cf. also situations where there are experimental cuts...]

Emission of soft photons can sometimes lift the helicity suppression:
for instance in $B \to \mu\nu_\mu$

$$\left(\frac{M_B}{m_\mu}\right)^2 \times \alpha \quad \text{is not small...}$$

D. Bećirević, B. Haas, E. Kou, Phys. Lett. B 681, 257 (2009)

Radiative corrections are often computed within a framework (ChPT, QED with point-like mesons,...) where the quantities one wishes to measure (slopes of form factors, scattering lengths,...) are actually fixed to their LO or NLO values

$\longrightarrow$ can produce biases...

$\longrightarrow$ one should provide a model-independent or input-free framework

Most of the time, radiative corrections are small, knowing them at 10% or even 20% precision is usually sufficient

Radiative corrections in pion and kaon decays: the low-energy EFT point of view

Several theoretical tools available

Tool 1 : chiral lagrangian

At energies well below the electroweak scale, the weak interactions are described by effective lagrangians involving four-fermion operators

- For the $\Delta S = 1$ non-leptonic transitions:

$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_{\rm F}}{\sqrt{2}} V_{ud} V_{us}^{*} \sum_i C_i(\mu) Q_i(\mu)$$

- $C_i(\mu)$ $\longrightarrow$ perturbative QCD corrections from M_W down to $\mu \lesssim m_c$

- For the semi-leptonic transitions:

$$\mathcal{L}_{\rm eff}^{SL} = -\frac{G_{\rm F}}{\sqrt{2}} \left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\right] \left\{V_{ud}\left[\bar{u}\gamma^\mu(1-\gamma_5)d\right] + V_{us}\left[\bar{u}\gamma^\mu(1-\gamma_5)s\right]\right\} + {\rm h.\,c.}$$

- No QCD corrections in $\mathcal{L}_{\rm eff}^{SL}$ $\longrightarrow$ factorized form, the description of semi-leptonic decays amounts to the evaluation of the relevant form factors

For $\mu << \Lambda_{\rm had} \sim 1{\rm GeV}$ (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks $m_{u,d,s} \to 0$

$\longrightarrow$ construct an effective lagrangian that describes the interactions among these pseudoscalar mesons in a systematic low-energy expansion

S. Weinberg, Physica A 96, 327 (1979)

J. Gasser, H. Leutwyler, Annals Phys. 158, 142 (1984); Nucl. Phys. B 250, 465 (1985)

- strong interactions among mesons at low-energies

$$\mathcal{L}^{\rm str} = \mathcal{L}_2^{\rm str}(2) + \mathcal{L}_4^{\rm str}(10+0) + \mathcal{L}_6^{\rm str}(90+23) + \cdots$$

- $\Delta S = 1$ transitions

$$\mathcal{L}_{\rm eff}^{\Delta S=1} \longrightarrow \mathcal{L}_2^{\Delta S=1}(1+1) + \mathcal{L}_4^{\Delta S=1}(22+28) + \cdots$$

J. A. Cronin, Phys. Rev. 161, 1483 (1967)

J. Kambor, J. H. Missimer, D. Wyler, Nucl. Phys. B 346, 17 (1990)

G. Esposito-Farese, Z. Phys. C 50, 255 (1991)

G. Ecker, J. Kambor, D. Wyler, Nucl. Phys. B 394, 101 (1993)

Adding electromagnetic interactions requires to include the photon as a low-energy degree of freedoms (loops involving virtual photons will produce their own divergences, which require additional low-energy constants)...

$$\mathcal{L}^{\rm str;EM} = \mathcal{L}_2^{\rm str;EM}(1) + \mathcal{L}_4^{\rm str;EM}(13+0) + \cdots$$

$$\mathcal{L}_2^{\rm str;EM} = e^2 C \langle Q U^\dagger Q U \rangle \qquad \mathcal{L}_4^{\rm str;EM}(13+0) = \sum_{i=1}^{13} K_i \mathcal{O}_i^{\rm str;EM}$$

G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321, 311 (1989)

R. Urech, Nucl. Phys. B 433, 234 (1995)

H. Neufeld, H. Rupertsberger, Z. Phys. C 71, 131 (1996)

$$\mathcal{L}^{\Delta S=1;EM} = \mathcal{L}_2^{\Delta S=1;EM}(1) + \mathcal{L}_4^{\Delta S=1;EM}(14+?) + \cdots$$

$$\mathcal{L}_2^{\Delta S=1;EM} = e^2 G_8 F_0^6 g_{\rm weak} \langle \lambda_{23} U^\dagger Q U \rangle \qquad \mathcal{L}_4^{\Delta S=1;EM} = e^2 G_8 F_0^4 \sum_{i=1}^{14} Z_i \mathcal{O}_i^{\Delta S=1;EM}$$

J. Bijnens, M. B. Wise, Phys. Lett. B 137, 245 (1984)

G. Ecker, G. Isidori, Müller, H. Neufeld, A. Pich, Nucl. Phys. 591, 1419 (2000)

... as well as the light leptons (for the description of radiative corrections to semi-leptonic decays)

$$\mathcal{L}^{\rm lept} = \mathcal{L}_2^{\rm lept}(0) + \mathcal{L}_4^{\rm lept}(5) + \cdots \qquad \mathcal{L}_4^{\rm lept} = \sum_{i=1}^{5} X_i \mathcal{O}_i^{\rm lept}$$

M. K., H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)

Crucial issue: determination of low-energy constants

- K_i

- identify the corresponding QCD correlators (two-, three- and four-point functions), convoluted with the free photon propagator

- study their short-distance behaviour

- write spectral sum rules

- saturate with lowest-lying narrow-width resonances

B. Moussallam, Nucl. Phys. B 504, 391 (1997) [hep-ph/9701400]

B. Ananthanarayan, B. Moussallam, JHEP06, 047 (2004) [hep-ph/0405206]

Analogous to the DGMLY sum-rule for C

$$C = -\frac{1}{16\pi^2}\frac{3}{2\pi}\int_0^{\infty} ds\, s\, \ln\frac{s}{\mu^2}\,[\rho_{VV}(s) - \rho_{AA}(s)]$$

T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low and J. E. Young, Phys. Rev. Lett. 18, 759 (1967)

B. Moussallam, Eur. Phys. J. C 6, 681 (1999) [hep-ph/9804271]

Crucial issue: determination of low-energy constants

• X_i

- two-step matching procedure:

i) compute radiative corrections to $\bar{q}q' \to \ell\nu$ in the SM and in the four-fermion theory

ii) match the radiatively corrected four-fermion theory to the chiral lagrangian, by identifying the QCD correlators (convoluted with the free photon propagator) that describe the X_i's
Saturate the resulting spectral sum rules with lowest-lying resonance states

S. Descotes-Genon, B. Moussallam, Eur. Phys. J. C 42, 403 (2005) [hep-ph/0505077]

• $\mathrm{g}_{\mathrm{weak}}$ and Z_i

Have been estimated in the large-N_c limit

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Eur. Phys. J. C 33, 269 (2004)

For instance

$$(g_8 e^2 \mathrm{g}_{\mathrm{weak}})^{\infty} = -\left(\frac{\langle\bar{\psi}\psi\rangle}{F_0^3}\right)^2 \left[3C_8(\mu) + \frac{16}{3}e^2 C_6(\mu)(K_9 - 2K_{10})\right]$$

Crucial issue: determination of low-energy constants

The dependence on the short-distance scale vanishes at leading-order in the large-N_c limit. A scale dependence remains at subleading order in $1/N_c$.
The (subleading order) contribution of Q_7 can also be computed,

$$(g_8 e^2 \mathrm{g}_{\mathrm{weak}})^{1/N_c;Q_7} = -\frac{9}{8\pi^2} C_7(\mu) \frac{M_\rho^2}{F_0^2} \left[\ln \frac{\mu^2}{M_\rho^2} + \frac{1}{3} - 2 \ln 2 \right]$$

M. K., S. Peris, E. de Rafael, Phys. Lett. B 457, 227 (1999)

but this does not completely remove the residual scale dependence

Applications to many examples (non-exhaustive list)

- $\pi \to \ell\nu_\ell(\gamma)$ and $K \to \ell\nu_\ell(\gamma)$
  M. K., H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)
  V. Cirigliano, I. Rosell, JHEP 0710, 005 (2007)
  J. Gasser, G. R. S. Zarnauskas, Phys. Lett. B 693, 122 (2010)
  V. Cirigliano, H. Neufeld, Phys. Lett. B 700, 7 (2011)

- $K \to \pi\ell\nu_\ell(\gamma)$
  V. Cirigliano, M. K., H. Neufeld, H. Rupertsberger and P. Talavera, Eur. Phys. J. C 23, 121 (2002)
  A. Kastner, H. Neufeld, Eur. Phys. J. C 57, 541 (2008)
  V. Cirigliano, M. Giannotti, H. Neufeld, JHEP 0811, 006 (2008)
  J. Gasser, B. Kubis, N. Paver, M. Verbeni, Eur. Phys. J. C 40, 205 (2005)

- $\pi^+ \to \pi^0 e\nu_e$
  V. Cirigliano, M. K., H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003)

- $K^+ \to \pi^+\pi^-\ell\nu_\ell$
  V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274
  A. Nehme, Nucl. Phys. B 682, 289 (2004)
  P. Stoffer, Eur. Phys. J. C 74, 2749 (2014)

- $K \to \pi\pi$
  V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Phys. Rev. Lett. 91, 162001 (2003)
  V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Eur. Phys. J. C 33, 269 (2004)
  V. Cirigliano, G. Ecker, A. Pich, Phys. Lett. B 679, 445 (2009)

- $K \to \pi\pi\pi$
  J. Bijnens, F. Borg, Nucl Phys. B 697, 319 (2004); Eur. Phys. J. C 39, 347 (2005); C 40, 383 (2005)

- $\ldots$
  V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84, 399 (2012)

Tool 2: non-relativistic effective field theory

$K \to \pi\pi^0\pi^0$: Important experimental feature: cusp at $M_{00} = 2M_\pi$ in the invariant mass distribution of the two neutral pions

First observed by NA48/2 in a sample of $2.3 \cdot 10^7\ K^\pm \to \pi^\pm\pi^0\pi^0$

J. R. Batley et al., Phys. Lett. B 633, 176 (2006)

Correctly interpreted as a rescattering effect $\pi^+\pi^- \to \pi^0\pi^0$ $(M_\pi \neq M_{\pi^0})$, corresponding to the combination $a_0 - a_2$ of S-wave scattering lengths

N. Cabibbo, Phys. Rev. Lett. 93, 121801 (2004)

But simple phenomenological parametrizations

N. Cabibbo and G. Isidori, JHEP0503, 021 (2005)

E. Gamiz, J. Prades and I. Scimemi, Eur. Phys. J. C 50, 405 (2007)

or one-loop ChPT calculations including isospin breaking

J. Bijnens, F. Borg, Nucl Phys. B 697, 319 (2004); Eur. Phys. J. C 39, 347 (2005); Eur. Phys. J. C 40, 383 (2005)

either do not give the correct analyticity properties or do not give a sufficiently accurate description of the cusp

Tool 2: non-relativistic effective field theory

Better description obtained by combining a non relativistic EFT framework

$$|\mathbf{p}|/M_\pi \sim \mathcal{O}(\epsilon)$$

and a *systematic* expansion in powers of the *scattering lengths* (treated as free parameters), including orders ϵ^2, $a\epsilon^3$, $a^2\epsilon^2$

G. Colangelo, J. Gasser, B. Kubis and A. Rusetsky, Phys. Lett. B 638, 187 (2006)

J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B 850, 96 (2011)

Radiative corrections were also included

M. Bissegger, A. Fuhrer, J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B 806, 178 (2009)

$$\longrightarrow a_0 - a_2 = 0.2571 \pm 0.0056$$

J. R. Batley et al, Eur. Phys. J. C 64, 589 (2009)

Later also observed by KTeV in a sample of $6.8 \cdot 10^7$ $K_L \to \pi^0\pi^0\pi^0$ events but the rescattering effect is quite smaller

$$a_0 - a_2 = 0.215 \pm 0.031$$

E. Abouzaid et al., Phys. Rev. D 78, 032009 (2008)

Tool 3: Dispersive constructions of amplitudes and form factors

Illustration: $M_\pi \neq M_{\pi^0}$ effects in the phases of $K_{\ell 4}$ form factors

Standard angular analysis of the K_{e4}^{+-} form factors provides information on low-energy $\pi\pi$ scattering (Watson's theorem) through the phase difference

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}}$$

N. Cabibbo, A. Maksymowicz, Phys. Rev. B 137, 438 (1965); Erratum-ibid 168, 1926 (1968)

F.A. Berends, A. Donnachie, G.C. Oades, Phys. Rev. 171, 1457(1968)

measurable in the interference of the F^{+-} and G^{+-} form factors.

Comparison with solutions of the Roy equations

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2)$$

allows to extract the values of the $\pi\pi$ S-wave scattering lengths in the isospin channels $I = 0, 2$

$f_{\text{Roy}}(s; a_0^2, a_0^2)$ follows from:

- dispersion relations (analyticity, unitarity, crossing, Froissard bound)
- $\pi\pi$ data at energies $\sqrt{s} \geq 1$ GeV
- isospin symmetry

S.M. Roy, Phys. Lett. B 36, 353 (1971)

Solutions can be constructed for $(a_0^0, a_0^2) \in$ Universal Band

B. Ananthanarayan, G. Colangelo, J. Gasser, H. Leutwyler, Phys. Rep. 353, 207 (2001)

Tool 3: Dispersive constructions of amplitudes and form factors

Once standard radiative corrections have been taken care of (more below), it is still important to take isospin-breaking corrections due to $M_\pi \neq M_{\pi^0}$ [also an $\mathcal{O}(\alpha)$ effect!] into account before analyzing data

J. Gasser, PoS KAON, 033 (2008)

Evaluation of IB corrections in ChPT

G. Colangelo, J. Gasser, A. Rusetsky, Eur. Phys. J. C 59, 777 (2009)

$$\longrightarrow a_0^0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}\,(37)_{\text{th}} \qquad a_0^2 = -0.0432(86)_{\text{stat}}(34)_{\text{syst}}(28)_{\text{th}}$$

However, IB corrections were evaluated at fixed values of the scattering lengths

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2) + \delta f_{\text{IB}}(s; (a_0^0)^{\text{LO}}_{\text{ChPT}}, (a_0^2)^{\text{LO}}_{\text{ChPT}})$$

Drawback shared by other studies devoted to isospin breaking in ChPT (QCD+QED)

V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274

A. Nehme, Nucl. Phys. B 682, 289 (2004)

P. Stoffer, Eur. Phys. J. C 74, 2749 (2004)

Is it possible to obtain

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2) + \delta f_{\text{IB}}(s; a_0^0, a_0^2) \quad ?$$

What is the quantitative effect in the determination of the scattering lengths?

Tool 3: Dispersive constructions of amplitudes and form factors

Adapt the approach ("reconstruction theorem") described in

J. Stern, H. Sazdjian, N. H. Fuchs, Phys. Rev. D 47, 3814 (1993)

for the $\pi\pi$ scattering amplitude, and implemented in

M. Knecht, B. Moussallam, J. Stern, N.H. Fuchs, Nucl. Phys. B 457, 513 (1995)

Rests on very general principle

a) Relativistic invariance

b) Analyticity, unitarity, crossing

c) Chiral counting

Note: isospin symmetry not required

$\longrightarrow \delta f_{\text{IB}}(s; a_0^0, a_0^2)$ worked out at NLO

S. Descotes-Genon, M. K., Eur. Phys. J. C 72, 1962 (2012)

V. Bernard, S. Descotes-Genon, M. K., Eur. Phys. J. C 73, 2478 (2013)

Re-analysis of NA48/2 data

$$a_0^0 = 0.221 \pm 0.018 \qquad a_0^2 = -0.0453 \pm 0.0106$$

to be compared to

$$a_0^0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}\,(37)_{\text{th}} \qquad a_0^2 = -0.0432(86)_{\text{stat}}(34)_{\text{syst}}(28)_{\text{th}}$$

A case study:

K_{e4}^{00} or radiative corrections in real life

NA48/2 has measured the two $K_{e4}^{\pm}$ channels:

K_{e4}^{+-} [i.e. $K^{\pm} \to \pi^+\pi^- e^{\pm}\nu_e$], about 10^6 events

J. R. Batley et al. [NA48/2 Coll.], Phys. Lett. B 715, 105 (2012)

K_{e4}^{00} [i.e. $K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu_e$], about $6.5 \cdot 10^4$ events (unitarity cusp in $M_{\pi^0\pi^0}$ seen)

J. R. Batley et al. [NA48/2 Coll.], JHEP 1408, 159 (2014)

The two matrix elements have a form factor ($F^{+-} = F^{00}$) in common in the isospin limit

$$(1+\delta_{EM})\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010$$

Can one understand this 6.5% effect in terms of isospin breaking?

About two thirds of the effect can be ascribed to isospin breaking in the quark masses

$$\left.\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]}\right|_{\rm LO} = 1.039 \pm 0.002 \qquad \left[R = \frac{m_s - m_{ud}}{m_d - m_u} = 38.2(1.1)(0.8)(1.4)\right]$$

V. Cuplov, PhD Thesis (2004); A. Nehme, Nucl. Phys. B 682, 289 (2004)

Z. Fodor et al. [BMW Coll.], Phys. Rev. Lett. 117, 082001 (2016)

Radiative corrections? $\delta_{EM} \sim 2.5\%$?

$\longrightarrow$ need to understand how radiative corrections were treated in the K_{e4}^{+-} mode...

Treatment of radiative corrections in the data analyses:

K^{00}_{e4}: no radiative corrections whatsoever applied (hence the factor $\delta_{\rm EM}$!)

K^{+-}_{e4}:

– Sommerfeld-Gamow-Sakharov factors applied to each pair of charged legs

– Corrections induced by emission of real photons treated with PHOTOS

Z. Was et al., Comp. Phys. Comm. 79, 291 (1994); Eur. Phys. J. C 45, 97 (2006); C 51, 569 (2007); Chin. Phys. C 34, 889 (2010)

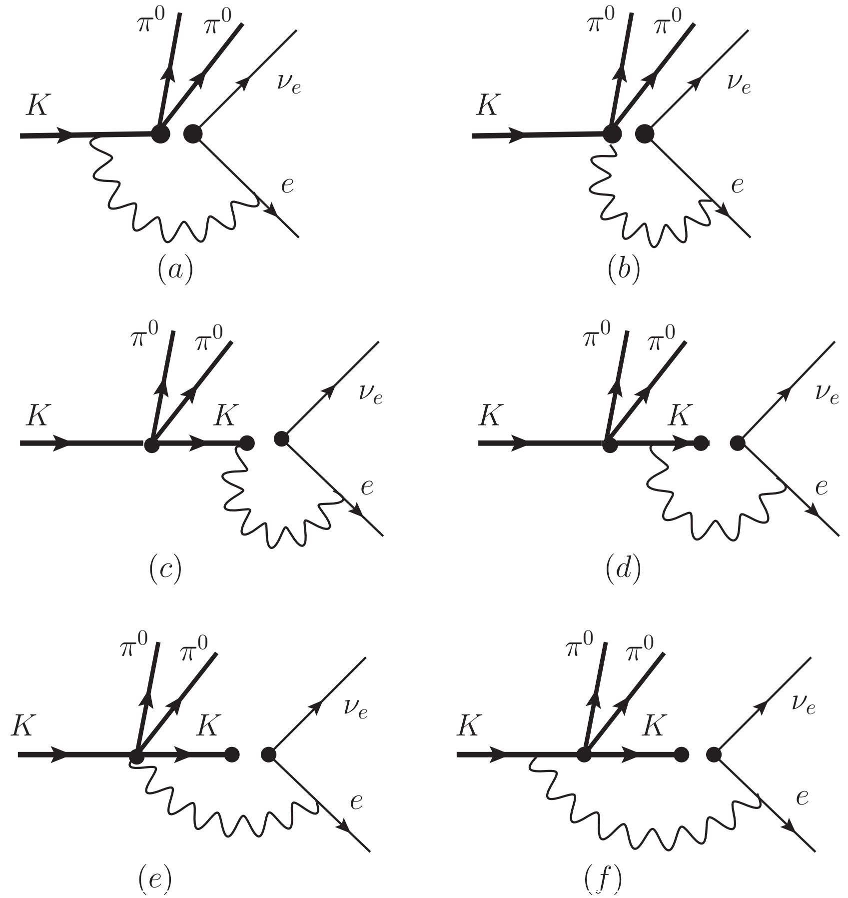
– PHOTOS also implements (1 loop QED) w.f.r. on the external charged legs [$\longrightarrow$ no IR divergences], based on

Y. M. Bystritskiy, S. R. Gevorkian, E. A. Kuraev, Eur. Phys. J. C 67, 47 (2009)

– All structure-dependent corrections are discarded (gauge invariant truncation)

– R^{+-} form factor neglected (appears multiplied by m_e^2 in the differential decay rate), but there is a contribution to ΔF^{+-} of the type $\mathcal{O}(\alpha) \times R^{+-}$ (i.e. $m_e = 0$ is not equivalent to $R^{+-} = 0$ in the presence of radiative corrections)

– UV divergences not treated



Non factorizable radiative corrections

Besides w.f. factors of QED, only diagram (a) is considered in a PHOTOS-like treatment of radiative corrections [diagrams (b), (c), and (d) vanish for $m_e \to 0$]

Adding the diagrams for the emission of a soft photon, one obtains

$$\Gamma^{\text{tot}} = \Gamma(K_{e4}^{00}) + \bar{\Gamma}^{\text{soft}}(K_{e4\gamma}^{00}) = \Gamma_0(K_{e4}^{00}) \times (1 + 2\delta_{EM})$$

with $\delta_{EM} = 0.018 \;\longrightarrow\; \frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010 - 0.018 \sim \left(1 + \frac{3}{2R}\right)$

V. Bernard, S. Descotes-Genon, M. K., Eur. Phys. J. C 75, 145 (2015)

Radiative corrections to semileptonic decays in the SM

MOTIVATION

• Increasing precision on hadronic decays of the tau meson, and on semileptonic decays of heavy-light mesons $\longrightarrow$ cf. talk by C. Pena

- Low-energy effective theory is no longer the appropriate framework

- SM is the appropriate framework

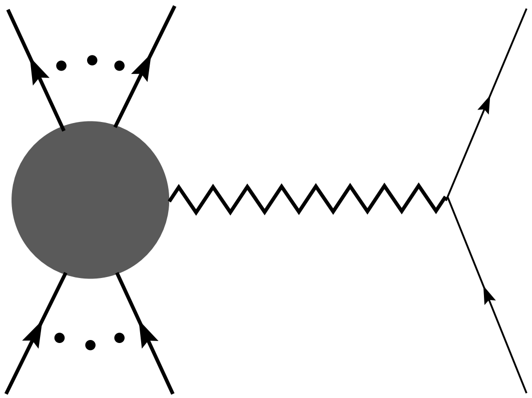
A. Sirlin, Rev. Mod. Phys. 50 (1978)

S. Weinberg, Phys. Rev. D 8 (1973)

G. Preparata, W. I. Weisberger, Phys Rev. 175 (1968)

Abers et al., Phys. Rev. 167 (1968)

Tree level



$$\mathcal{A}^{(0)} = \mathcal{A}^{(0)}_{(W)} + \mathcal{A}^{(0)}_{(\phi^\pm)}$$

$$\mathcal{A}^{(0)}_{(W)} = \left(\frac{-g}{\sqrt{2}}\right)^2 \times i \langle F \,|\, J^{\mu}_{(W)}(0) \,|\, I \rangle_{QCD} \times \frac{(-i)}{p^2 - M_W^2} \times i\, L^\dagger_\mu$$

$$\mathcal{A}^{(0)}_{(\phi^\pm)} = \left(\frac{-g}{\sqrt{2} M_W}\right)^2 \times i \langle F \,|\, \partial \cdot J_{(W)}(0) \,|\, I \rangle_{QCD} \times \frac{i}{p^2 - M_W^2} \times i(p_\ell - p_{\nu_\ell}) \cdot L^\dagger$$

$$L^\dagger_\mu = \frac{1}{2} \bar{u}_{\nu_\ell}(p_{\nu_\ell}) \gamma_\mu (1 - \gamma_5) v_\ell(p_\ell), \quad p = P_I - P_F = p_\ell + p_{\nu_\ell}$$

- of order $\mathcal{O}(G_F)$, $p^2 \ll M_W^2$ (Fermi theory)

- factorization between the leptonic and the hadronic part (form factors)

Including radiative corrections :

- factorization no longer holds

- all scales of the SM involved

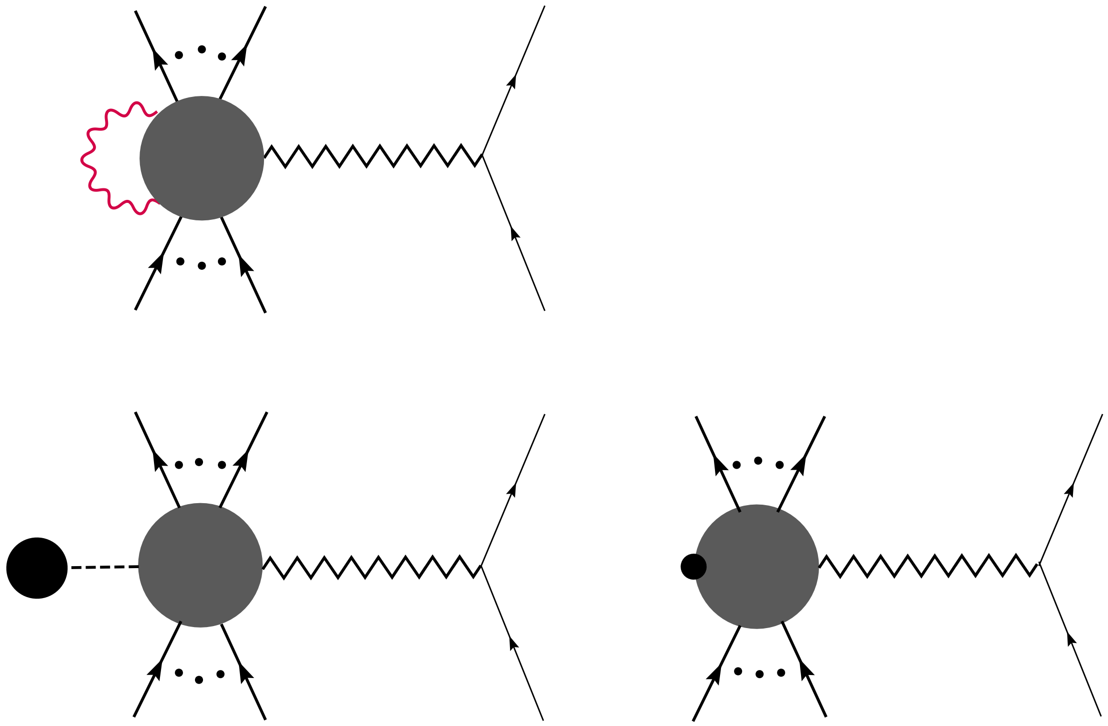
$$m_e, m_\mu, m_u, m_d \ll m_s \ll \Lambda_H \sim 1\,\text{GeV} < m_c, m_\tau < m_b \ll M_W, M_Z, M_H, m_t$$

The NLO amplitude receives contributions $\mathcal{O}(\alpha G_F)$, but also

$$\mathcal{O}(\alpha G_F \times \frac{m_\ell^2}{M_{W,Z}^2}) \qquad \mathcal{O}(\alpha G_F \times \frac{m_q m_{q'}}{M_{W,Z}^2}) \qquad \mathcal{O}(\alpha G_F \times \frac{\Lambda_H^2}{M_{W,Z}^2})$$

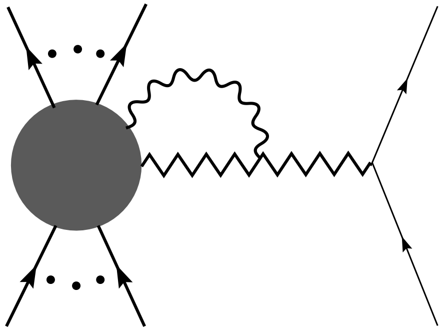
which are neglected, being $\sim \mathcal{O}(G_F^2)$

Corrections to the hadronic matrix element



- loops of gauge bosons and scalars
- tadpoles and tadpole counterterms
- counterterms

The Z boson and photon exchange contributions



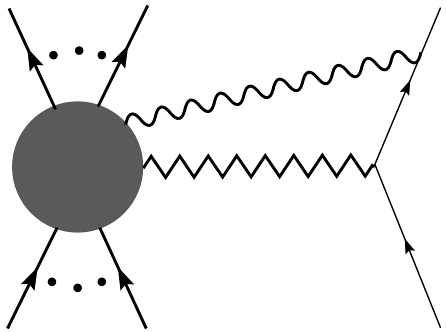
$$\mathcal{U}_{(Z)}(p) \;=\; \left(-i\,\frac{g}{\sqrt{2}}\right)^2 \times \frac{(-i)}{p^2-M_W^2} \times L^{\mu\dagger} \times \int \frac{d^4q}{(2\pi)^4}\,\frac{(-i)}{q^2-M_Z^2}\,\frac{(-i)}{(q-p)^2-M_W^2}$$
$$\times\,(ig\cos\theta_w)\,V_{\mu\nu\rho}(q,p) \times\,(-i)\,\frac{\sqrt{g^2+g'^2}}{2}\,\mathcal{T}^{\nu\rho}_{(Z)}(q,p)$$

$$\mathcal{U}_{(\gamma)}(p) \;=\; \left(-i\,\frac{g}{\sqrt{2}}\right)^2 \times \frac{(-i)}{p^2-M_W^2} \times L^{\mu\dagger} \times \int \frac{d^4q}{(2\pi)^4}\,\frac{(-i)}{q^2}\,\frac{(-i)}{(q-p)^2-M_W^2}$$
$$\times\,(ig\sin\theta_w)\,V_{\mu\nu\rho}(q,p) \times\,(-i)\,g\sin\theta_w\,\mathcal{T}^{\nu\rho}_{(\gamma)}(q,p)$$

$$V_{\mu\nu\rho}(q,p) \;=\; (2q-p)_\mu\eta_{\nu\rho} \;+\; (2p-q)_\nu\eta_{\mu\rho} \;-\; (p+q)_\rho\eta_{\mu\nu}$$

$$\mathcal{T}^{\nu\rho}_{(Z,\gamma)}(q,p) \sim \int d^4x e^{-q\cdot x}\langle\, F\,|\,T\{J_{(Z,\gamma)}J^{\mu}_{(W)}(0)\,|\,I\,\rangle_{QCD}$$

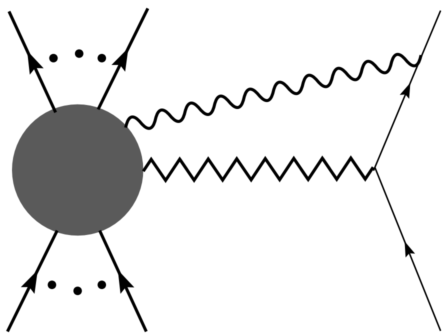
The Z-boson box contribution



$$\mathcal{A}_{(Z)}^{\ell\text{-box}}(p) \;=\; \left(-\frac{ig}{\sqrt{2}}\right)^2 \times \left(-\frac{i\sqrt{g^2+g'^2}}{2}\right)^2 \times \int \frac{d^4q}{(2\pi)^4}\,\frac{(-i)}{q^2-M_Z^2}\,\frac{(-i)}{(q-p)^2-M_W^2}$$
$$\times\,\bar{u}_{\nu_\ell}(p_{\nu_\ell})\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)\frac{i}{\not{q}-\not{p}_\ell-m_\ell}\gamma_\nu\,\left[T_\ell^3(1-\gamma_5)\;-\;2Q_\ell\sin^2\theta_w\right]v_\ell(p_\ell)\,\mathcal{T}_{(Z)}^{\nu\rho}(q,p)$$

$$\mathcal{A}_{(Z)}^{\nu_\ell\text{-box}}(p) \;=\; \left(-\frac{ig}{\sqrt{2}}\right)^2 \times \left(-\frac{i\sqrt{g^2+g'^2}}{2}\right)^2 \times \int \frac{d^4q}{(2\pi)^4}\,\frac{(-i)}{q^2-M_Z^2}\,\frac{(-i)}{(q-p)^2-M_W^2}$$
$$\times\,\bar{u}_{\nu_\ell}(p_{\nu_\ell})\gamma_\nu\,T_{\nu_\ell}^3(1-\gamma_5)\,\frac{i}{\not{p}_{\nu_\ell}-\not{q}}\gamma_\rho\left(\frac{1-\gamma_5}{2}\right)v_\ell(p_\ell)\,\mathcal{T}_{(Z)}^{\nu\rho}(q,p)$$

The photon box contribution



$$\mathcal{A}^{\text{box}}_{(\gamma)}(p) \quad = \quad \left(-\frac{ig}{\sqrt{2}}\right)^2 \times (-ie) \times (-ieQ_\ell) \times \int \frac{d^4q}{(2\pi)^4} \frac{(-i)}{q^2} \frac{(-i)}{(q-p)^2 - M_W^2}$$

$$\times\, \bar{u}_{\nu_\ell}(p_{\nu_\ell})\gamma_\rho \left(\frac{1-\gamma_5}{2}\right) \frac{i}{\not{q} - \not{p}_\ell - m_\ell} \gamma_\nu\, v_\ell(p_\ell)\, \mathcal{T}^{\nu\rho}_{(\gamma)}(q,p)$$

Other corrections

- W-propagator self-energy corrections
- Corrections from the Higgs sector
- Leptonic wave-function and vertex corrections
- Real photon emission

Putting it all together

$$\Gamma(P^+ \to \ell^+\nu_\ell(\gamma)) = \Gamma^{(0)}(P^+ \to \ell^+\nu_\ell) \times \Bigg\{1 + \left(\frac{\alpha}{2\pi}\right)\Bigg[\delta_{\text{IB}} + \delta_{\text{SD}} + \delta_{\text{INT}} + \delta_{(\gamma<)} + \delta^{\text{res. 3-pt.}}_{(\gamma<)} + \delta^{\text{box}}_{(\gamma)} + \delta^{\text{w.f.}}_{(\gamma<)} + \delta^{\text{div}}_{(\gamma)} + \delta^{\text{fin}}_{(\gamma)} + \delta^{\text{div}}_{(Z)} + \delta^{\text{fin}}_{(Z)} + \delta^{\text{box}}_{(Z)} + \delta^{\text{self}} + \delta^{\text{vertex}} + \delta^{\text{w.f. res.}}\Bigg]\Bigg\}.$$

A similar formula can be established for the muon decay

$$\Gamma(\mu^+ \to e^+\nu_\ell\bar{\nu}_\mu(\gamma)) = \Gamma^{(0)}(\mu^+ \to e^+\nu_\ell\bar{\nu}_\mu) \times \Bigg\{1 + \left(\frac{\alpha}{2\pi}\right)\Bigg[\widetilde{\delta}_{\text{IB}} + \widetilde{\delta}_{(\gamma<)} + \widetilde{\delta}^{\text{res. 3-pt.}}_{(\gamma<)} + \widetilde{\delta}^{\text{box}}_{(\gamma)} + \widetilde{\delta}^{\text{w.f.}}_{(\gamma<)} + \widetilde{\delta}^{\text{div}}_{(\gamma)} + \widetilde{\delta}^{\text{fin}}_{(\gamma)} + \widetilde{\delta}^{\text{div}}_{(Z)} + \widetilde{\delta}^{\text{fin}}_{(Z)} + \widetilde{\delta}^{\text{box}}_{(Z)} + \widetilde{\delta}^{\text{self}} + \widetilde{\delta}^{\text{vertex}} + \widetilde{\delta}^{\text{w.f. res.}}\Bigg]\Bigg\}.$$

The universality structure of the weak interactions induces a certain number of relations among the corrections occuring in the two decays

$$\begin{aligned}
\delta^{\text{div}}_{(\gamma)} &= \widetilde{\delta}^{\text{div}}_{(\gamma)} \\
\delta^{\text{div}}_{(Z)} &= \widetilde{\delta}^{\text{div}}_{(Z)} \\
\delta^{\text{self}} &= \widetilde{\delta}^{\text{self}} \\
\delta^{\text{vertex}} &= \widetilde{\delta}^{\text{vertex}} \\
\delta^{\text{w.f. res.}} &= \widetilde{\delta}^{\text{w.f. res.}}
\end{aligned}$$

The fact that the divergent pieces are identical allows to define the same renormalized weak coupling constant from both processes, and reflects the renormalizability of the standard model. The presence of the strong interactions induces α_s dependent violations of universality in the finite pieces

$$\begin{aligned}
\delta^{\text{fin}}_{(Z)} &= \widetilde{\delta}^{\text{fin}}_{(Z)} + \frac{1}{2}\cot^2\theta_w \times \Delta_{(Z)}(\alpha_s) \\
\delta^{\text{fin}}_{(\gamma)} &= \widetilde{\delta}^{\text{fin}}_{(\gamma)} + \frac{1}{2} \times \Delta_{(\gamma)}(\alpha_s)
\end{aligned}$$

An additional difference in the Z-box contributions arises as a consequence of the fact that the average charges in the quark and lepton multiplets do not coincide

$$\delta^{\text{box}}_{(Z)} = \frac{M_W^2}{M_Z^2 - M_W^2} \ln \frac{M_Z^2}{M_W^2} \left[\left(2 + \frac{1}{2}\right) \cot^2\theta_w + 3\overline{Q_q} \tan^2\theta_w + \Delta^{\text{box}}_{(Z)}(\alpha_s) \right], \quad \overline{Q_q} = 1/6$$

$$\widetilde{\delta}^{\text{box}}_{(Z)} = \frac{M_W^2}{M_Z^2 - M_W^2} \ln \frac{M_Z^2}{M_W^2} \left[\left(2 + \frac{1}{2}\right) \cot^2\theta_w + 3\overline{Q_\ell} \tan^2\theta_w \right], \quad \overline{Q_\ell} = -1/2$$

As far as the remaining contributions are concerned, one has ($r_\ell = m_\ell^2/M_{P+}^2$

$$\delta_{\text{IB}} + \delta^{\text{pt}}_{(\gamma<)} + \delta^{\text{res. 3-pt.}}_{(\gamma<);\,\text{pt}} + \delta^{\text{box}}_{(\gamma);\,\text{pt}} + \delta^{\text{w.f.}}_{(\gamma<)} = \frac{1}{2} H(r_\ell) + \frac{7}{2} - \frac{3}{2} \ln \frac{M_{P+}^2}{M_W^2}$$

$$\begin{aligned} H(z) = & \frac{23}{2} - \frac{3}{1-z} + 11 \ln z - \frac{2 \ln z}{1-z} - \frac{3 \ln z}{(1-z)^2} \\ & - 8 \ln(1-z) - \frac{4(1+z)}{1-z} \ln z \ln(1-z) - \frac{8(1+z)}{1-z} \text{Li}_2(1-z). \end{aligned}$$

while in the muon case, the analogous combination

$$\widetilde{\delta}_{\text{IB}} + \widetilde{\delta}_{(\gamma<)} + \widetilde{\delta}^{\text{res. 3-pt.}}_{(\gamma<)} + \widetilde{\delta}^{\text{box}}_{(\gamma)} + \widetilde{\delta}^{\text{w.f.}}_{(\gamma<)} = \frac{25}{4} - \pi^2$$

reproduces the finite result of the local Fermi theory

Introducing a new coupling constant, defined as

$$G_\mu = \frac{g^2}{4\sqrt{2}M_W^2}\left\{1 + \left(\frac{\alpha}{4\pi}\right)\left[\widetilde{\delta}^{\text{div}}_{(\gamma)} + \widetilde{\delta}^{\text{fin}}_{(\gamma)} + \widetilde{\delta}^{\text{div}}_{(Z)} + \widetilde{\delta}^{\text{fin}}_{(Z)} + \widetilde{\delta}^{\text{box}}_{(Z)} + \widetilde{\delta}^{\text{self}} + \widetilde{\delta}^{\text{vertex}} + \widetilde{\delta}^{\text{w.f. res.}}\right]\right\}$$

gives

$$\begin{aligned}
\Gamma(P^+ \to \ell^+\nu_\ell(\gamma)) \quad = \quad & \frac{G_\mu^2|V^*_{\text{CKM}}|^2}{4\pi}\, M^3_{P^+} F^2_{P^+} r_\ell (1-r_\ell)^2 \Bigg\{1 + \left(\frac{\alpha}{2\pi}\right)\Bigg[\frac{1}{2}H(r_\ell) + \frac{7}{2} - \frac{3}{2}\ln\frac{M^2_{P^+}}{M^2_W} \\
& + \delta_{\text{SD}} + \delta_{\text{INT}} + \delta^{\text{res}}_{(\gamma<)} + \delta^{\text{res. 3-pt.}}_{(\gamma<);\,\text{res}} + \delta^{\text{box}}_{(\gamma);\,\text{res}} \\
& + \frac{M_W^2}{M_Z^2 - M_W^2}\ln\frac{M_Z^2}{M_W^2}\left[3(\overline{Q_q} - \overline{Q_\ell})\tan^2\theta_w + \Delta^{\text{box}}_{(Z)}(\alpha_s)\right] \\
& + \frac{1}{2}\cot^2\theta_w\,\Delta_{(Z)}(\alpha_s) + \frac{1}{2}\Delta_{(\gamma)}(\alpha_s)\Bigg]\Bigg\}
\end{aligned}$$

Conclusions

High precision reached by the data concerning non-leptonic and semi-leptonic decay modes of the kaons has made the treatment of isospin-breaking effects ($m_u \neq m_d$ and $\alpha \neq 0$) unavoidable

A lot of activity has been going on, extending the scope of the low-energy EFT in order to meet this necessity (inclusion of photons, leptons). Only a fraction of the many applications has been mentioned here

The issue of additional low-energy constants has been dealt with in a rather satisfactory manner (progress on estimates of the Z_i's would be welcome, though)

The effects due to $M_\pi \neq M_{\pi^0}$ are important (especially for $K \to \pi\pi\pi$ and for K_{e4}).
ChPT at NLO is not always sufficient.
$\longrightarrow$ This issue can be dealt with through more elaborate/adapted approaches, like NREFT, dispersive representations,...

Watch out for possible biases if the radiative corrections to form factors and/or decay distributions are given for fixed values of the parameters one actually wants to extract from data
$\longrightarrow$ Not the case for $\delta_0(s) - \delta_1(s)$ extracted from K_{e4}

Treatment of radiative corrections in K_{e4} rather rudimentary, does not match the quality of the data
$\longrightarrow$ Improvements should be possible

SM provides a framework to compute radiative corrections to semileptonic decays of mesons in situations where low-energy effective theory does not apply (hadronic tau decays, semileptonic decays of B and D mesons)

Genuine $\mathcal{O}(\alpha G_F)$ effects can be disantagled from $\mathcal{O}(G_F^2)$ contributions in a systematic (and gauge invariant) manner)

Result is finite and involves three-current and two-current correlation functions of QCD, whose evaluation requires nonperturbative approaches (lattice, large-N_C)

In the case of light pseudoscalar mesons, alternative identification of the low-energy constants in terms of these QCD correlators

Thanks for your attention!