


"Excited QCD 2018", Kopaonik, 11–15 March 2018

Scalar Mesons: Fifty Years of Challenging the Quark Model

George Rupp

CeFEMA, Instituto Superior Técnico, Lisbon

40-year collaborator.: Eef van Beveren (Coimbra))

I. Introduction: PDG and σ & κ pole positions over the years

II. Selection of models for the light scalar mesons

III. Very recent lattice results for σ

IV. Conclusions

$f_0(500)$ or σ
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

A REVIEW GOES HERE – Check our WWW List of Reviews

$f_0(500)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\ \text{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
(400–550)−*i*(200–350) OUR ESTIMATE				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$(512 \pm 15)-i(188 \pm 12)$	[1] ABLIKIM	17	BES3	$J/\psi \to \gamma 3\pi$
$(440 \pm 10)-i(238 \pm 10)$	[2] ALBALADEJO	12	RVUE	Compilation
$(445 \pm 25)-i(278^{+22}_{-18})$	[3,4] GARCIA-MAR..	11	RVUE	Compilation
$(457^{+14}_{-13})-i(279^{+11}_{-7})$	[3,5] GARCIA-MAR..	11	RVUE	Compilation
$(442^{+5}_{-8})-i(274^{+6}_{-5})$	[6] MOUSSALLAM	11	RVUE	Compilation
$(452 \pm 13)-i(259 \pm 16)$	[7] MENNESSIER	10	RVUE	Compilation
$(448 \pm 43)-i(266 \pm 43)$	[8] MENNESSIER	10	RVUE	Compilation
$(455 \pm 6^{+31}_{-13})-i(278 \pm 6^{+34}_{-43})$	[9] CAPRINI	08	RVUE	Compilation
$(463 \pm 6^{+31}_{-17})-i(259 \pm 6^{+33}_{-34})$	[10] CAPRINI	08	RVUE	Compilation
$(552^{+84}_{-106})-i(232^{+81}_{-72})$	[11] ABLIKIM	07A	BES2	$\psi(2S) \to \pi^+\pi^- J/\psi$
$(466 \pm 18)-i(223 \pm 28)$	[12] BONVICINI	07	CLEO	$D^+ \to \pi^-\pi^+\pi^+$
$(472 \pm 30)-i(271 \pm 30)$	[13] BUGG	07A	RVUE	Compilation
$(484 \pm 17)-i(255 \pm 10)$	GARCIA-MAR..	07	RVUE	Compilation
$(430)-i(325)$	[14] ANISOVICH	06	RVUE	Compilation
$(441^{+16}_{-8})-i(272^{+9}_{-12.5})$	[15] CAPRINI	06	RVUE	$\pi\pi \to \pi\pi$
$(470 \pm 50)-i(285 \pm 25)$	[16] ZHOU	05	RVUE	
$(541 \pm 39)-i(252 \pm 42)$	[17] ABLIKIM	04A	BES2	$J/\psi \to \omega\pi^+\pi^-$
$(528 \pm 32)-i(207 \pm 23)$	[18] GALLEGOS	04	RVUE	Compilation
$(440 \pm 8)-i(212 \pm 15)$	[19] PELAEZ	04A	RVUE	$\pi\pi \to \pi\pi$
$(533 \pm 25)-i(249 \pm 25)$	[20] BUGG	03	RVUE	
$517 - i240$	BLACK	01	RVUE	$\pi^0\pi^0 \to \pi^0\pi^0$
$(470 \pm 30)-i(295 \pm 20)$	[15] COLANGELO	01	RVUE	$\pi\pi \to \pi\pi$
$(535^{+48}_{-36})-i(155^{+76}_{-53})$	[21] ISHIDA	01		$\Upsilon(3S) \to \Upsilon\pi\pi$
$610 \pm 14 - i620 \pm 26$	[22] SUROVTSEV	01	RVUE	$\pi\pi \to \pi\pi, K\overline{K}$
$(540^{+36}_{-29})-i(193^{+32}_{-40})$	ISHIDA	00B		$p\overline{p} \to \pi^0\pi^0\pi^0$
$445 - i235$	HANNAH	99	RVUE	π scalar form factor
$(523 \pm 12)-i(259 \pm 7)$	KAMINSKI	99	RVUE	$\pi\pi \to \pi\pi, K\overline{K}, \sigma\sigma$
$442 - i\ 227$	OLLER	99	RVUE	$\pi\pi \to \pi\pi, K\overline{K}$
$469 - i203$	OLLER	99B	RVUE	$\pi\pi \to \pi\pi, K\overline{K}$
$445 - i221$	OLLER	99C	RVUE	$\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$
$(1530^{+90}_{-250})-i(560 \pm 40)$	ANISOVICH	98B	RVUE	Compilation
$420 - i\ 212$	LOCHER	98	RVUE	$\pi\pi \to \pi\pi\ , K\overline{K}$
$440 - i245$	[23] DOBADO	97	RVUE	Compilation
$(602 \pm 26)-i(196 \pm 27)$	[24] ISHIDA	97		$\pi\pi \to \pi\pi$
$(537 \pm 20)-i(250 \pm 17)$	[25] KAMINSKI	97B	RVUE	$\pi\pi \to \pi\pi, K\overline{K}, 4\pi$

$470 - i250$	[26,27]	TORNQVIST	96	RVUE	$\pi\pi \to \pi\pi, K\overline{K}, K\pi, \eta\pi$
$387 - i305$	[27,28]	JANSSEN	95	RVUE	$\pi\pi \to \pi\pi, K\overline{K}$
$420 - i370$	[29]	ACHASOV	94	RVUE	$\pi\pi \to \pi\pi$
$(506 \pm 10) - i(247 \pm 3)$		KAMINSKI	94	RVUE	$\pi\pi \to \pi\pi, K\overline{K}$
$370 - i356$	[30]	ZOU	94B	RVUE	$\pi\pi \to \pi\pi, K\overline{K}$
$408 - i342$	[27,30]	ZOU	93	RVUE	$\pi\pi \to \pi\pi, K\overline{K}$
$470 - i208$	[31]	VANBEVEREN	86	RVUE	$\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \ldots$
$(750 \pm 50) - i(450 \pm 50)$	[32]	ESTABROOKS	79	RVUE	$\pi\pi \to \pi\pi, K\overline{K}$
$(660 \pm 100) - i(320 \pm 70)$		PROTOPOP...	73	HBC	$\pi\pi \to \pi\pi, K\overline{K}$
$650 - i370$	[33]	BASDEVANT	72	RVUE	$\pi\pi \to \pi\pi$

[1] S-matrix pole; 8595 events.

[2] Applying the chiral unitary approach at NLO to the K_{e4} data of BATLEY 10 and $\pi N \to \pi\pi N$ data of HYAMS 73, GRAYER 74, and PROTOPOPESCU 73.

[3] Uses the K_{e4} data of BATLEY 10C and the $\pi N \to \pi\pi N$ data of HYAMS 73, GRAYER 74, and PROTOPOPESCU 73.

[4] Analytic continuation using Roy equations.

[5] Analytic continuation using GKPY equations.

[6] Using Roy equations.

[7] Average of three variants of the analytic K-matrix model. Uses the K_{e4} data of BATLEY 08A and the $\pi N \to \pi\pi N$ data of HYAMS 73 and GRAYER 74.

[8] Average of the analyses of three data sets in the K-matrix model. Uses the data of BATLEY 08A, HYAMS 73, and GRAYER 74, partially of COHEN 80 or ETKIN 82B.

[9] From the K_{e4} data of BATLEY 08A and $\pi N \to \pi\pi N$ data of HYAMS 73.

[10] From the K_{e4} data of BATLEY 08A and $\pi N \to \pi\pi N$ data of PROTOPOPESCU 73, GRAYER 74, and ESTABROOKS 74.

[11] From a mean of three different $f_0(500)$ parametrizations. Uses 40k events.

[12] From an isobar model using 2.6k events.

[13] Reanalysis of ABLIKIM 04A, PISLAK 01, and HYAMS 73 data.

[14] Using the N/D method.

[15] From the solution of the Roy equation (ROY 71) for the isoscalar S-wave and using a phase-shift analysis of HYAMS 73 and PROTOPOPESCU 73 data.

[16] Reanalysis of the data from PROTOPOPESCU 73, ESTABROOKS 74, GRAYER 74, ROSSELET 77, PISLAK 03, and AKHMETSHIN 04.

[17] From a mean of six different analyses and $f_0(500)$ parameterizations.

[18] Using data on $\psi(2S) \to J/\psi\pi\pi$ from BAI 00E and on $\Upsilon(nS) \to \Upsilon(mS)\pi\pi$ from BUTLER 94B and ALEXANDER 98.

[19] Reanalysis of data from PROTOPOPESCU 73, ESTABROOKS 74, GRAYER 74, and COHEN 80 in the unitarized ChPT model.

[20] From a combined analysis of HYAMS 73, AUGUSTIN 89, AITALA 01B, and PISLAK 01.

[21] A similar analysis (KOMADA 01) finds $(580^{+79}_{-30}) - i(190^{+107}_{-49})$ MeV.

[22] Coupled channel reanalysis of BATON 70, BENSINGER 71, BAILLON 72, HYAMS 73, HYAMS 75, ROSSELET 77, COHEN 80, and ETKIN 82B using the uniformizing variable.

[23] Using the inverse amplitude method and data of ESTABROOKS 73, GRAYER 74, and PROTOPOPESCU 73.

[24] Reanalysis of data from HYAMS 73, GRAYER 74, SRINIVASAN 75, and ROSSELET 77 using the interfering amplitude method.

[25] Average and spread of 4 variants ("up" and "down") of KAMINSKI 97B 3-channel model.

[26] Uses data from BEIER 72B, OCHS 73, HYAMS 73, GRAYER 74, ROSSELET 77, CASON 83, ASTON 88, and ARMSTRONG 91B. Coupled channel analysis with flavor symmetry and all light two-pseudoscalars systems.

[27] Demonstrates explicitly that $f_0(500)$ and $f_0(1370)$ are two different poles.

[28] Analysis of data from FALVARD 88.

[29] Analysis of data from OCHS 73, ESTABROOKS 75, ROSSELET 77, and MUKHIN 80.
[30] Analysis of data from OCHS 73, GRAYER 74, and ROSSELET 77.
[31] Coupled-channel analysis using data from PROTOPOPESCU 73, HYAMS 73, HYAMS 75, GRAYER 74, ESTABROOKS 74, ESTABROOKS 75, FROGGATT 77, CORDEN 79, BISWAS 81.
[32] Analysis of data from APEL 72C, GRAYER 74, CASON 76, PAWLICKI 77. Includes spread and errors of 4 solutions.
[33] Analysis of data from BATON 70, BENSINGER 71, COLTON 71, BAILLON 72,PROTOPOPESCU 73, and WALKER 67.

$f_0(500)$ BREIT-WIGNER MASS OR K-MATRIX POLE PARAMETERS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
(400–550) OUR ESTIMATE				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
513 ± 32	[34] MURAMATSU	02	CLEO	$e^+e^- \approx$ 10 GeV
$478^{+24}_{-23} \pm 17$	AITALA	01B	E791	$D^+ \to \pi^-\pi^+\pi^+$
563^{+58}_{-29}	[35] ISHIDA	01		$\Upsilon(3S) \to \Upsilon\pi\pi$
555	[36] ASNER	00	CLE2	$\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$
540 ± 36	ISHIDA	00B		$p\overline{p} \to \pi^0\pi^0\pi^0$
750 ± 4	ALEKSEEV	99	SPEC	$1.78\ \pi^- p_{polar} \to \pi^-\pi^+ n$
744 ± 5	ALEKSEEV	98	SPEC	$1.78\ \pi^- p_{polar} \to \pi^-\pi^+ n$
759 ± 5	[37] TROYAN	98		$5.2\ np \to np\pi^+\pi^-$
780 ± 30	ALDE	97	GAM2	$450\ pp \to pp\pi^0\pi^0$
585 ± 20	[38] ISHIDA	97		$\pi\pi \to \pi\pi$
761 ± 12	[39] SVEC	96	RVUE	$6–17\ \pi N_{polar} \to \pi^+\pi^- N$
$\sim$ 860	[40,41] TORNQVIST	96	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $K\pi$, $\eta\pi$
1165 ± 50	[42,43] ANISOVICH	95	RVUE	$\pi^- p \to \pi^0\pi^0 n$, $\overline{p}p \to \pi^0\pi^0\pi^0$, $\pi^0\pi^0\eta$, $\pi^0\eta\eta$
$\sim$ 1000	[44] ACHASOV	94	RVUE	$\pi\pi \to \pi\pi$
414 ± 20	[39] AUGUSTIN	89	DM2	

[34] Statistical uncertainty only.
[35] A similar analysis (KOMADA 01) finds 526^{+48}_{-37} MeV.
[36] From the best fit of the Dalitz plot.
[37] 6σ effect, no PWA.
[38] Reanalysis of data from HYAMS 73, GRAYER 74, SRINIVASAN 75, and ROSSELET 77 using the interfering amplitude method.
[39] Breit-Wigner fit to S-wave intensity measured in $\pi N \to \pi^-\pi^+ N$ on polarized targets. The fit does not include $f_0(980)$.
[40] Uses data from ASTON 88, OCHS 73, HYAMS 73, ARMSTRONG 91B, GRAYER 74, CASON 83, ROSSELET 77, and BEIER 72B. Coupled channel analysis with flavor symmetry and all light two-pseudoscalars systems.
[41] Also observed by ASNER 00 in $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$ decays.
[42] Uses $\pi^0\pi^0$ data from ANISOVICH 94, AMSLER 94D, and ALDE 95B, $\pi^+\pi^-$ data from OCHS 73, GRAYER 74 and ROSSELET 77, and $\eta\eta$ data from ANISOVICH 94.
[43] The pole is on Sheet III. Demonstrates explicitly that $f_0(500)$ and $f_0(1370)$ are two different poles.
[44] Analysis of data from OCHS 73, ESTABROOKS 75, ROSSELET 77, and MUKHIN 80.

$K_0^*(800)$
or κ

$I(J^P) = \frac{1}{2}(0^+)$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$K_0^*(800)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
682 ±29 OUR AVERAGE		Error includes scale factor of 2.4. See the ideogram below.			
826 ±49 $^{+49}_{-34}$	1338	[1] ABLIKIM	11B	BES2	$J/\psi \to K_S^0 K_S^0 \pi^+ \pi^-$
849 ±77 $^{+18}_{-14}$	1421	[2,3] ABLIKIM	10E	BES2	$J/\psi \to K^\pm K_S^0 \pi^\mp \pi^0$
841 ±30 $^{+81}_{-73}$	25k	[4,5] ABLIKIM	06C	BES2	$J/\psi \to \overline{K}^*(892)^0 K^+ \pi^-$
658 ±13		[6] DESCOTES-G..	06	RVUE	$\pi K \to \pi K$
797 ±19 ±43	15k	[7,8] AITALA	02	E791	$D^+ \to K^- \pi^+ \pi^+$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
663 ± 8 ±34		[9] BUGG	10	RVUE	S-matrix pole
706.0± 1.8±22.8	141k	[10] BONVICINI	08A	CLEO	$D^+ \to K^- \pi^+ \pi^+$
856 ±17 ±13	54k	[11] LINK	07B	FOCS	$D^+ \to K^- \pi^+ \pi^+$
750 $^{+30}_{-55}$		[12] BUGG	06	RVUE	
855 ±15	0.6k	[13] CAWLFIELD	06A	CLEO	$D^0 \to K^+ K^- \pi^0$
694 ±53		[3,14] ZHOU	06	RVUE	$K p \to K^- \pi^+ n$
753 ±52		[15] PELAEZ	04A	RVUE	$K\pi \to K\pi$
594 ±79		[14] ZHENG	04	RVUE	$K^- p \to K^- \pi^+ n$
722 ±60		[16] BUGG	03	RVUE	11 $K^- p \to K^- \pi^+ n$
905 $^{+65}_{-30}$		[17] ISHIDA	97B	RVUE	11 $K^- p \to K^- \pi^+ n$

[1] The Breit-Wigner parameters from a fit with seven intermediate resonances. The S-matrix pole position is $(764 \pm 63^{+71}_{-54}) - i\,(306 \pm 149^{+143}_{-85})$ MeV.

[2] From a fit including ten additional resonances and energy-independent Breit-Wigner width.

[3] S-matrix pole.

[4] S-matrix pole. GUO 06 in a chiral unitary approach report a mass of 757 ± 33 MeV and a width of 558 ± 82 MeV.

[5] A fit in the $K_0^*(800) + K^*(892) + K^*(1410)$ model with mass and width of the $K_0^*(800)$ from ABLIKIM 06C well describes the left slope of the $K_S^0 \pi^-$ invariant mass spectrum in $\tau^- \to K_S^0 \pi^- \nu_\tau$ decay studied by EPIFANOV 07.

[6] S-matrix pole. Using Roy-Steiner equations (ROY 71) as well as unitarity, analyticity and crossing symmetry constraints.

[7] Not seen by KOPP 01 using 7070 events of $D^0 \to K^- \pi^+ \pi^0$. LINK 02E and LINK 05I show clear evidence for a constant non-resonant scalar amplitude rather than $K_0^*(800)$ in their high statistics analysis of $D^+ \to K^- \pi^+ \mu^+ \nu_\mu$.

$f_0(980)$

$I^G(J^{PC}) = 0^+(0^{++})$

See also the minireview on scalar mesons under $f_0(500)$. (See the index for the page number.)

$f_0(980)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
990 ±20 OUR ESTIMATE					
• • • We do not use the following data for averages, fits, limits, etc. • • •					
989.4 ± 1.3	424	ABLIKIM	15P	BES3	$J/\psi \to K^+K^-3\pi$
989.9 ± 0.4	706	ABLIKIM	12E	BES3	$J/\psi \to \gamma 3\pi$
$1003\ ^{+5}_{-27}$		[1,2] GARCIA-MAR..11		RVUE	Compilation
996 ± 7		[1,3] GARCIA-MAR..11		RVUE	Compilation
$996\ ^{+4}_{-14}$		[4] MOUSSALLAM	11	RVUE	Compilation
981 ±43		[5] MENNESSIER	10	RVUE	Compilation
$1030\ ^{+30}_{-10}$		[6] ANISOVICH	09	RVUE	0.0 $\overline{p}p$, πN
$977\ ^{+11}_{-9} \pm 1$	44	[7] ECKLUND	09	CLEO	4.17 $e^+e^- \to D_s^- D_s^{*+}$ + c.c.
$982.2 \pm 1.0\ ^{+8.1}_{-8.0}$		[8] UEHARA	08A	BELL	10.6 $e^+e^- \to e^+e^-\pi^0\pi^0$
$976.8 \pm 0.3\ ^{+10.1}_{-0.6}$	64k	[9] AMBROSINO	07	KLOE	1.02 $e^+e^- \to \pi^0\pi^0\gamma$
$984.7 \pm 0.4\ ^{+2.4}_{-3.7}$	64k	[10] AMBROSINO	07	KLOE	1.02 $e^+e^- \to \pi^0\pi^0\gamma$
973 ± 3	262 ± 30	[11] AUBERT	07AK	BABR	10.6 $e^+e^- \to \phi\pi^+\pi^-\gamma$
970 ± 7	54 ± 9	[11] AUBERT	07AK	BABR	10.6 $e^+e^- \to \phi\pi^0\pi^0\gamma$
953 ±20	2.6k	[12] BONVICINI	07	CLEO	$D^+ \to \pi^-\pi^+\pi^+$
$985.6\ ^{+1.2}_{-1.5}\ ^{+1.1}_{-1.6}$		[13] MORI	07	BELL	10.6 $e^+e^- \to e^+e^-\pi^+\pi^-$
$983.0 \pm 0.6\ ^{+4.0}_{-3.0}$		[14] AMBROSINO	06B	KLOE	1.02 $e^+e^- \to \pi^+\pi^-\gamma$
$977.3 \pm 0.9\ ^{+3.7}_{-4.3}$		[15] AMBROSINO	06B	KLOE	1.02 $e^+e^- \to \pi^+\pi^-\gamma$
950 ± 9	4286	[16] GARMASH	06	BELL	$B^+ \to K^+\pi^+\pi^-$
965 ±10		[17] ABLIKIM	05	BES2	$J/\psi \to \phi\pi^+\pi^-$, ϕK^+K^-
1031 ± 8		[18] ANISOVICH	03	RVUE	
1037 ±31		TIKHOMIROV	03	SPEC	40.0 $\pi^- C \to K_S^0 K_S^0 K_L^0 X$
973 ± 1	2438	[19] ALOISIO	02D	KLOE	$e^+e^- \to \pi^0\pi^0\gamma$
977 ± 3 ± 2	848	[20] AITALA	01A	E791	$D_s^+ \to \pi^-\pi^+\pi^+$
969.8 ± 4.5	419	[21] ACHASOV	00H	SND	$e^+e^- \to \pi^0\pi^0\gamma$
$985\ ^{+16}_{-12}$	419	[22,23] ACHASOV	00H	SND	$e^+e^- \to \pi^0\pi^0\gamma$
976 ± 5 ± 6		[24] AKHMETSHIN	99B	CMD2	$e^+e^- \to \pi^+\pi^-\gamma$

977 ± 3 ± 6	268	[24] AKHMETSHIN	99C	CMD2	$e^+e^- \rightarrow \pi^0\pi^0\gamma$
975 ± 4 ± 6		[25] AKHMETSHIN	99C	CMD2	$e^+e^- \rightarrow \pi^0\pi^0\gamma$
975 ± 4 ± 6		[26] AKHMETSHIN	99C	CMD2	$e^+e^- \rightarrow \pi^+\pi^-\gamma$, $\pi^0\pi^0\gamma$
985 ±10		BARBERIS	99	OMEG	450 $pp \rightarrow p_s p_f K^+K^-$
982 ± 3		BARBERIS	99B	OMEG	450 $pp \rightarrow p_s p_f \pi^+\pi^-$
982 ± 3		BARBERIS	99C	OMEG	450 $pp \rightarrow p_s p_f \pi^0\pi^0$
987 ± 6 ± 6		[27] BARBERIS	99D	OMEG	450 $pp \rightarrow K^+K^-$, $\pi^+\pi^-$
989 ±15		BELLAZZINI	99	GAM4	450 $pp \rightarrow pp\pi^0\pi^0$
991 ± 3		[28] KAMINSKI	99	RVUE	$\pi\pi \rightarrow \pi\pi$, $K\overline{K}$, $\sigma\sigma$
∼ 980		[28] OLLER	99	RVUE	$\pi\pi \rightarrow \pi\pi$, $K\overline{K}$
∼ 993.5		OLLER	99B	RVUE	$\pi\pi \rightarrow \pi\pi$, $K\overline{K}$
∼ 987		[28] OLLER	99C	RVUE	$\pi\pi \rightarrow \pi\pi$, $K\overline{K}$, $\eta\eta$
957 ± 6		[29] ACKERSTAFF	98Q	OPAL	$Z \rightarrow f_0$ X
960 ±10		ALDE	98	GAM4	
1015 ±15		[28] ANISOVICH	98B	RVUE	Compilation
1008		[30] LOCHER	98	RVUE	$\pi\pi \rightarrow \pi\pi$, $K\overline{K}$
955 ±10		[29] ALDE	97	GAM2	450 $pp \rightarrow pp\pi^0\pi^0$
994 ± 9		[31] BERTIN	97C	OBLX	0.0 $\overline{p}p \rightarrow \pi^+\pi^-\pi^0$
993.2 ± 6.5 ± 6.9		[32] ISHIDA	96	RVUE	$\pi\pi \rightarrow \pi\pi$, $K\overline{K}$
1006		TORNQVIST	96	RVUE	$\pi\pi \rightarrow \pi\pi$, $K\overline{K}$, $K\pi$, $\eta\pi$
997 ± 5	3k	[33] ALDE	95B	GAM2	38 $\pi^- p \rightarrow \pi^0\pi^0 n$
960 ±10	10k	[34] ALDE	95B	GAM2	38 $\pi^- p \rightarrow \pi^0\pi^0 n$
994 ± 5		AMSLER	95B	CBAR	0.0 $\overline{p}p \rightarrow 3\pi^0$
∼ 996		[35] AMSLER	95D	CBAR	0.0 $\overline{p}p \rightarrow \pi^0\pi^0\pi^0$, $\pi^0\eta\eta$, $\pi^0\pi^0\eta$
987 ± 6		[36] ANISOVICH	95	RVUE	
1015		JANSSEN	95	RVUE	$\pi\pi \rightarrow \pi\pi$, $K\overline{K}$
983		[37] BUGG	94	RVUE	$\overline{p}p \rightarrow \eta 2\pi^0$
973 ± 2		[38] KAMINSKI	94	RVUE	$\pi\pi \rightarrow \pi\pi$, $K\overline{K}$
988		[39] ZOU	94B	RVUE	
988 ±10		[40] MORGAN	93	RVUE	$\pi\pi(K\overline{K}) \rightarrow \pi\pi(K\overline{K})$, $J/\psi \rightarrow \phi\pi\pi(K\overline{K})$, $D_s \rightarrow \pi(\pi\pi)$
971.1 ± 4.0		[29] AGUILAR-...	91	EHS	400 pp
979 ± 4		[41] ARMSTRONG	91	OMEG	300 $pp \rightarrow pp\pi\pi$, $ppK\overline{K}$
956 ±12		BREAKSTONE	90	SFM	$pp \rightarrow pp\pi^+\pi^-$
959.4 ± 6.5		[29] AUGUSTIN	89	DM2	$J/\psi \rightarrow \omega\pi^+\pi^-$
978 ± 9		[29] ABACHI	86B	HRS	$e^+e^- \rightarrow \pi^+\pi^-$X
$985.0^{+9.0}_{-39.0}$		ETKIN	82B	MPS	23 $\pi^- p \rightarrow n2K^0_S$
974 ± 4		[41] GIDAL	81	MRK2	$J/\psi \rightarrow \pi^+\pi^-$X
975		[42] ACHASOV	80	RVUE	
986 ±10		[41] AGUILAR-...	78	HBC	0.7 $\overline{p}p \rightarrow K^0_S K^0_S$
969 ± 5		[41] LEEPER	77	ASPK	2–2.4 $\pi^- p \rightarrow \pi^+\pi^- n$, $K^+K^- n$
987 ± 7		[41] BINNIE	73	CNTR	$\pi^- p \rightarrow n$MM

1012 ± 6	[43] GRAYER	73	ASPK	17	$\pi^- p \to \pi^+\pi^- n$
1007 ±20	[43] HYAMS	73	ASPK	17	$\pi^- p \to \pi^+\pi^- n$
997 ± 6	[43] PROTOPOP...	73	HBC	7	$\pi^+ p \to \pi^+ p \pi^+\pi^-$

[1] Quoted number refers to real part of pole position.
[2] Analytic continuation using Roy equations. Uses the K_{e4} data of BATLEY 10C and the $\pi N \to \pi\pi N$ data of HYAMS 73, GRAYER 74, and PROTOPOPESCU 73.
[3] Analytic continuation using GKPY equations. Uses the K_{e4} data of BATLEY 10C and the $\pi N \to \pi\pi N$ data of HYAMS 73, GRAYER 74, and PROTOPOPESCU 73.
[4] Pole position. Used Roy equations.
[5] Average of the analyses of three data sets in the K-matrix model. Uses the data of BATLEY 08A, HYAMS 73, and GRAYER 74, partially of COHEN 80 or ETKIN 82B.
[6] On sheet II in a 2-pole solution. The other pole is found on sheet III at $(850-100i)$ MeV
[7] Using a relativistic Breit-Wigner function and taking into account the finite D_s mass.
[8] Breit-Wigner mass. Using finite width corrections according to FLATTE 76 and ACHASOV 05, and the ratio $g_{f_0 KK}/g_{f_0\pi\pi} = 0$.
[9] In the kaon-loop fit.
[10] In the no-structure fit.
[11] Systematic errors not estimated.
[12] FLATTE 76 parameterization. $g_{f_0\pi\pi} = 329 \pm 96$ MeV/c^2 assuming $g_{f_0 K\overline{K}}/g_{f_0\pi\pi}$=2.
[13] Breit-Wigner mass. Using finite width corrections according to FLATTE 76 and ACHASOV 05, and the ratio $g_{f_0 KK}/g_{f_0\pi\pi} = 4.21 \pm 0.25 \pm 0.21$ from ABLIKIM 05.
[14] In the kaon-loop fit following formalism of ACHASOV 89.
[15] In the no-structure fit assuming a direct coupling of ϕ to $f_0\gamma$.
[16] FLATTE 76 parameterization. Supersedes GARMASH 05.
[17] FLATTE 76 parameterization, $g_{f_0 K\overline{K}}/g_{f_0\pi\pi} = 4.21 \pm 0.25 \pm 0.21$.
[18] K-matrix pole from combined analysis of $\pi^- p \to \pi^0\pi^0 n$, $\pi^- p \to K\overline{K} n$, $\pi^+\pi^- \to \pi^+\pi^-$, $\overline{p}p \to \pi^0\pi^0\pi^0$, $\pi^0\eta\eta$, $\pi^0\pi^0\eta$, $\pi^+\pi^-\pi^0$, $K^+K^-\pi^0$, $K^0_S K^0_S\pi^0$, $K^+K^0_S\pi^-$ at rest, $\overline{p}n \to \pi^-\pi^-\pi^+$, $K^0_S K^-\pi^0$, $K^0_S K^0_S\pi^-$ at rest.
[19] From the negative interference with the $f_0(500)$ meson of AITALA 01B using the ACHASOV 89 parameterization for the $f_0(980)$, a Breit-Wigner for the $f_0(500)$, and ACHASOV 01F for the $\rho\pi$ contribution.
[20] Coupled-channel Breit-Wigner, couplings g_π=0.09±0.01±0.01, g_K=0.02±0.04±0.03.
[21] Supersedes ACHASOV 98I. Using the model of ACHASOV 89.
[22] Supersedes ACHASOV 98I.
[23] In the "narrow resonance" approximation.
[24] Assuming $\Gamma(f_0)$= 40 MeV.
[25] From a narrow pole fit taking into account $f_0(980)$ and $f_0(1200)$ intermediate mechanisms.
[26] From the combined fit of the photon spectra in the reactions $e^+e^- \to \pi^+\pi^-\gamma$, $\pi^0\pi^0\gamma$.
[27] Supersedes BARBERIS 99 and BARBERIS 99B
[28] T-matrix pole.
[29] From invariant mass fit.
[30] On sheet II in a 2 pole solution. The other pole is found on sheet III at $(1039-93i)$ MeV.
[31] On sheet II in a 2 pole solution. The other pole is found on sheet III at (963-29i) MeV.
[32] Reanalysis of data from HYAMS 73, GRAYER 74, SRINIVASAN 75, and ROSSELET 77 using the interfering amplitude method.
[33] At high $|t|$.
[34] At low $|t|$.

$a_0(980)$

$I^G(J^{PC}) = 1^-(0^{++})$

See our minireview on scalar mesons under $f_0(500)$. (See the index for the page number.)

$a_0(980)$ MASS

VALUE (MeV)	*DOCUMENT ID*
980±20 OUR ESTIMATE	Mass determination very model dependent

$\eta\pi$ FINAL STATE ONLY

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$982.5 \pm 1.6 \pm 1.1$	16.9k	[1] AMBROSINO	09F	KLOE		$1.02\ e^+e^- \to \eta\pi^0\gamma$
986 ± 4		ANISOVICH	09	RVUE		$0.0\ \overline{p}p,\ \pi N$
$982.3\ ^{+0.6}_{-0.7}\ ^{+3.1}_{-4.7}$		[2] UEHARA	09A	BELL		$\gamma\gamma \to \pi^0\eta$
$987.4 \pm 1.0 \pm 3.0$		[3,4] BUGG	08A	RVUE	0	$\overline{p}p \to \pi^0\pi^0\eta$
$989.1 \pm 1.0 \pm 3.0$		[4,5] BUGG	08A	RVUE	0	$\overline{p}p \to \pi^0\pi^0\eta$
$985 \pm 4 \pm 6$	318	ACHARD	02B	L3		$183–209\ e^+e^- \to e^+e^-\eta\pi^+\pi^-$
$995\ ^{+52}_{-10}$	36	[6] ACHASOV	00F	SND		$e^+e^- \to \eta\pi^0\gamma$
$994\ ^{+33}_{-8}$	36	[7] ACHASOV	00F	SND		$e^+e^- \to \eta\pi^0\gamma$
975 ± 7		BARBERIS	00H			$450\ pp \to p_f\eta\pi^0 p_s$
988 ± 8		BARBERIS	00H			$450\ pp \to \Delta_f^{++}\eta\pi^- p_s$
~ 1055		[8] OLLER	99	RVUE		$\eta\pi,\ K\overline{K}$
~ 1009.2		[8] OLLER	99B	RVUE		$\pi\pi \to \pi\pi,\ K\overline{K}$
993.1 ± 2.1		[9] TEIGE	99	B852		$18.3\ \pi^-p \to \eta\pi^+\pi^-n$
988 ± 6		[8] ANISOVICH	98B	RVUE		Compilation
987		TORNQVIST	96	RVUE		$\pi\pi \to \pi\pi,\ K\overline{K},\ K\pi,\ \eta\pi$
991		JANSSEN	95	RVUE		$\eta\pi \to \eta\pi,\ K\overline{K},\ K\pi,\ \eta\pi$
$984.45 \pm 1.23 \pm 0.34$		AMSLER	94C	CBAR		$0.0\ \overline{p}p \to \omega\eta\pi^0$
982 ± 2		[10] AMSLER	92	CBAR		$0.0\ \overline{p}p \to \eta\eta\pi^0$
984 ± 4	1040	[10] ARMSTRONG	91B	OMEG	±	$300\ pp \to pp\eta\pi^+\pi^-$
976 ± 6		ATKINSON	84E	OMEG	±	$25–55\ \gamma p \to \eta\pi n$
986 ± 3	500	[11] EVANGELIS...	81	OMEG	±	$12\ \pi^-p \to \eta\pi^+\pi^-\pi^-p$
990 ± 7	145	[11] GURTU	79	HBC	±	$4.2\ K^-p \to \Lambda\eta 2\pi$
980 ± 11	47	CONFORTO	78	OSPK	−	$4.5\ \pi^-p \to pX^-$
978 ± 16	50	CORDEN	78	OMEG	±	$12–15\ \pi^-p \to n\eta 2\pi$
977 ± 7		GRASSLER	77	HBC	−	$16\ \pi^\mp p \to p\eta 3\pi$
989 ± 4	70	WELLS	75	HBC	−	$3.1–6\ K^-p \to \Lambda\eta 2\pi$

972	±10	150	DEFOIX	72	HBC	±	0.7 $\overline{p}p \to 7\pi$
970	±15	20	BARNES	69C	HBC	−	4–5 $K^- p \to \Lambda\eta 2\pi$
980	±10		CAMPBELL	69	DBC	±	2.7 $\pi^+ d$
980	±10	15	MILLER	69B	HBC	−	4.5 $K^- N \to \eta\pi\Lambda$
980	±10	30	AMMAR	68	HBC	±	5.5 $K^- p \to \Lambda\eta 2\pi$

[1] Using the model of ACHASOV 89 and ACHASOV 03B.
[2] From a fit with the S-wave amplitude including two interfering Breit-Wigners plus a background term.
[3] Parameterizes couplings to $\overline{K}K$, $\pi\eta$, and $\pi\eta'$.
[4] Using AMSLER 94D and ABELE 98.
[5] From the T-matrix pole on sheet II.
[6] Using the model of ACHASOV 89. Supersedes ACHASOV 98B.
[7] Using the model of JAFFE 77. Supersedes ACHASOV 98B.
[8] T-matrix pole.
[9] Breit-Wigner fit, average between $a_0^{\pm}$ and a_0^0. The fit favors a slightly heavier $a_0^{\pm}$.
[10] From a single Breit-Wigner fit.
[11] From $f_1(1285)$ decay.

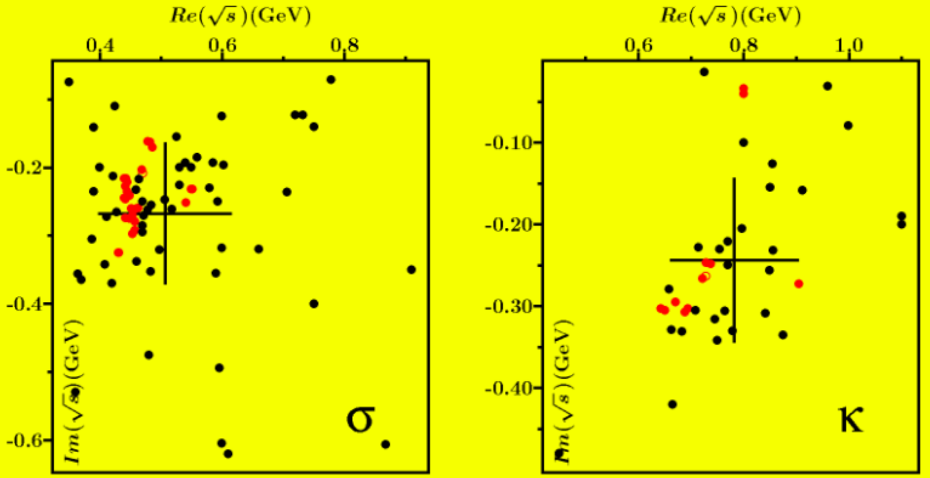
$K\overline{K}$ ONLY

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
925 ± 5 ±8	190k	[1] AAIJ	16N	LHCB		$D^0 \to K_S^0 K^{\pm}\pi^{\mp}$
• • • We do not use the following data for averages, fits, limits, etc. • • •						
∼ 1053		[2] OLLER	99C	RVUE		$\pi\pi \to \pi\pi$, $K\overline{K}$
982 ± 3		[3] ABELE	98	CBAR		0.0 $\overline{p}p \to K_L^0 K^{\pm}\pi^{\mp}$
975 ±15		BERTIN	98B	OBLX	±	0.0 $\overline{p}p \to K^{\pm}K_S\pi^{\mp}$
976 ± 6	316	DEBILLY	80	HBC	±	1.2–2 $\overline{p}p \to f_1(1285)\omega$
1016 ±10	100	[4] ASTIER	67	HBC	±	0.0 $\overline{p}p$
1003.3± 7.0	143	[5] ROSENFELD	65	RVUE	±	

[1] Using a two-channel resonance parametrization with couplings fixed to ABELE 98.
[2] T-matrix pole.
[3] T-matrix pole on sheet II, the pole on sheet III is at 1006-i49 MeV.
[4] ASTIER 67 includes data of BARLOW 67, CONFORTO 67, ARMENTEROS 65.
[5] Plus systematic errors.

$a_0(980)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
50 to 100 OUR ESTIMATE Width determination very model dependent. Peak width in $\eta\pi$ is about 60 MeV, but decay width can be much larger.						
• • • We do not use the following data for averages, fits, limits, etc. • • •						
75.6 ± 1.6 $^{+17.4}_{-10.0}$		[1] UEHARA	09A	BELL		$\gamma\gamma \to \pi^0\eta$
80.2 ± 3.8 ± 5.4		[2] BUGG	08A	RVUE	0	$\overline{p}p \to \pi^0\pi^0\eta$
50 ±13 ± 4	318	ACHARD	02B	L3		183–209 $e^+e^- \to e^+e^-\eta\pi^+\pi^-$
72 ±16		BARBERIS	00H			450 $pp \to p_f\eta\pi^0 p_s$
61 ±19		BARBERIS	00H			450 $pp \to \Delta_f^{++}\eta\pi^- p_s$
∼ 42		[3] OLLER	99	RVUE		$\eta\pi$, $K\overline{K}$
∼ 112		[3] OLLER	99B	RVUE		$\pi\pi \to \eta\pi$, $K\overline{K}$



Average σ pole position

(507 ± 110) -i (267 ± 104) MeV

Average κ pole position

(782 ± 123) -i (244 ± 101) MeV

reference	σ (MeV)	nearby poles	κ (MeV)	nearby poles	<<<
1968			725 - i 14		return
1968			1100 - i 200		return
1969			1100 - i 190		return
1970	425 - i 110				return
1971	428 - i 265				return
1972	480 - i 475				return
1973	660 - i 320				return
1973	460 - i 338		665 - i 420		return
1974			800 - i 100		return
1979	750 - i 400				return
1982	750 - i 140		800 - i 40	2003	return
1986	470 - i 208	1999	727 - i 263	2009	return
1987	910 - i 350				return
1988			905 - i 273	2000 2004	return
1989	482 - i 163	2001 2010			return
1993	408 - i 342				return
1994	420 - i 370				return
1994	370 - i 365				return
1994	506 - i 247				return
1995	387 - i 305				return
1996	470 - i 250				return
1996	559 - i 185				return

D. Iagolnitzer, J. Zinn-Justin, J. B. Zuber, Nucl. Phys. B **60** (1973) 233

- Lagrangian model of pseudoscalar-meson scattering, with exchanges of the vector mesons ρ, $K^\star$, ϕ.
- Model is not renormalisable, so subtraction constants are used.
- Born plus 1-loop diagrams are unitarised with Padé method.

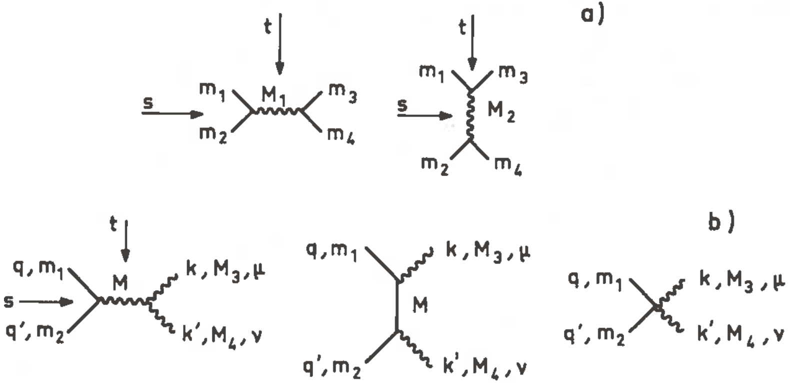


Fig. 15. First order diagrams contributing to: (a) $0^- + 0^- \to 0^- + 0^-$ amplitudes; (b) $0^- + 0^- \to 1^- + 1^-$ amplitudes.

Predicted resonances: due to present uncertainties, the values of experimental masses and widths which are reported here are to be taken as indicative as far as the 0^+ nonet and some of the 2^+ mesons are concerned.

Name	J^P	I^G	Mass (MeV)		Width (MeV)	
			Experiment	Our model	Experiment	Our model
ϵ	0^+	0^+	500 to 750	460	$\geqslant 100$	675
π_N	0^+	1^-	975	775	58	610 [b]
S^*	0^+	0^+	~ 1000	990	20–50	40 [b]
κ	0^+	$\frac{1}{2}$	–	665	–	840
ρ	1^-	1^+	765	764	125 ± 20	83, (130) [a]
K^*	1^-	$\frac{1}{2}$	892	845	50	52, (38) [a]
φ	1^-	0^-	1020	1022	1.8 (partial width $\varphi \to K\bar{K}$)	0, (4.5) [a]
f_0	2^+	0^+	1270	1365	155 ± 25	165
f'_0	2^+	0^+	1515	1536	73 ± 23	8
A_2	2^+	1^-	1310	1332	85	143
K^*	2^+	$\frac{1}{2}$	1410	1539	107	5

R. L. Jaffe, Phys. Rev. D **15** (1977) 267

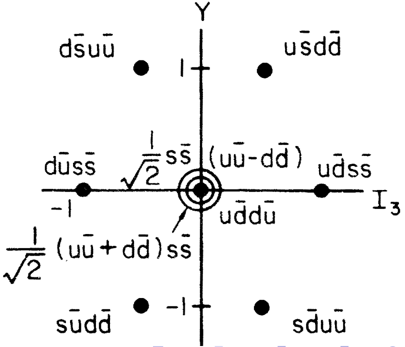
- Light scalar mesons described as $q^2\bar{q}^2$ states in the MIT Bag Model.
- Pure bound-state calculation, without dynamical effects of decay.
- Very large attractive colour-hyperfine mass shifts for $q^2\bar{q}^2$ nonet.
- Since here δ ("$a_0(980)$") and $S^\star$ ("$f_0(980)$") have the same quark content, they come out degenerate in mass (1100 MeV).
- Masses of ϵ ("$f_0(500)$"), κ ("$K_0^\star(800)$"): 650 MeV, 900 MeV.

$$\epsilon(700) = C^0(\underline{9}, 0^+) = u\bar{u}d\bar{d}\ ,$$

$$S^*(993) = C^s(\underline{9}, 0^+) = \frac{1}{\sqrt{2}}\ s\bar{s}(u\bar{u} + d\bar{d})$$

$$\delta(976) = C^s_\pi(\underline{9}, 0^+) = u\bar{d}s\bar{s}, \text{etc.}\ ,$$

$$\kappa(?) = C_K(\underline{9}, 0^+) = u\bar{s}d\bar{d}\ , \text{etc}\ .$$



R. Delbourgo, M. D. Scadron, Phys. Rev. Lett. **48** (1982) 379

- In the quark-level linear σ model, the quark loop for the pion decay constant is given by

$$if_\pi = g_{\pi qq} \frac{N_c \times 4}{(2\pi)^4} \int d^4 p \, \frac{m_{\rm dyn}(p^2)}{[p^2 - m_{\rm dyn}^2(p^2)]^2} \, .$$

- Self-consistency via the quark-level Goldberger-Treiman relation then leads to $m_{\rm dyn} = f_\pi \, 2\pi/\sqrt{N_c} \approx 315$ MeV in the chiral limit.
- In the quark-level linear σ model, this gives just like in NJL models $m_\sigma = 2m_{\rm dyn} \approx 630$ MeV in the chiral limit.
- Later, Delbourgo and Scadron formulated the quark-level linear σ model fully self-consistently via bootstrap at loop order. They also generalised the model to flavour $SU(3)$.

E. van Beveren, T. A. Rijken, K. Metzger, C. Dullemond, G. Rupp, J. E. Ribeiro, Z. Phys. C **30** (1986) 615 [arXiv:0710.4067 [hep-ph]]

Abstract

A unitarized nonrelativistic meson model which is successful for the description of the heavy and light vector and pseudoscalar mesons yields, in its extension to the scalar mesons but for the same model parameters, a complete nonet below 1 GeV. In the unitarization scheme, real and virtual meson-meson decay channels are coupled to the quark-antiquark confinement channels. The flavor-dependent harmonic-oscillator confining potential itself has bound states $\epsilon(1.3$ GeV$)$, $S(1.5$ GeV$)$, $\delta(1.3$ GeV$)$, $\kappa(1.4$ GeV$)$, similar to the results of other bound-state $q\bar{q}$ models. However, the full coupled-channel equations show poles at $\epsilon(0.5$ GeV$)$, $S(0.99$ GeV$)$, $\delta(0.97$ GeV$)$, $\kappa(0.73$ GeV$)$. Not only can these pole positions be calculated in our model, but also cross sections and phase shifts in the meson-scattering channels, which are in reasonable agreement with the available data for $\pi\pi$, $\eta\pi$ and $K\pi$ in S-wave scattering.

- Unitarised quark-meson model, with all parameters fixed from previous work.
- All decay channels with pseudoscalar and vector mesons included.
- Poles of light scalar mesons found at:
  $f_0(470-i208)$, $K_0^*(727-i263)$, $a_0(968-i28)$, $f_0(994-i20)$.
- Additional poles found for $f_0(1370)$, $K_0^*(1430)$, $a_0(1450)$, $f_0(1500)$, at reasonable values.
- Moreover, S-wave scattering data were reasonably reproduced.

R. Kaminski, L. Lesniak, J. P. Maillet, Phys. Rev. D **50** (1994) 3145

- Purely mesonic model with two coupled channels ($\pi\pi$ and $K\bar{K}$).
- Phenomenological separable potentials of rank 2 ($\pi\pi$) and 1 ($K\bar{K}$).
- Lippmann-Schwinger equation used with relativistic propagators.
- The 8 parameters were fitted to data on $\delta_{\pi\pi}$, $\delta_{K\bar{K}}$, and η.

Table V: Masses and widths of resonances obtained in fits to the data sets 1 and 2 compared with values of the Particle Data Group [34] and Ref. [39] for $f_0(500)$.

pole	set 1		set 2		Particle Data Group	
	M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)
$f_0(500)$	506±10	494±5	505±10	497±5	≤ 700	≥ 600
$f_0(975)$	973±2	29±2	974±2	30±1	974.1±2.5	47±9
$f_0(1400)$	1430±5	145±25	1428^{+13}_{-7}	157^{+43}_{-29}	~ 1400	$150 \div 400$

G. Janssen, B. C. Pearce, K. Holinde, J. Speth, Phys. Rev. D **52** (1995) 2690 [arXiv:nucl-th/9411021]

- Pseudoscalar-meson scattering with Blankenblecler-Sugar equation.
- Exchanges of ρ, $K^\star$, ω, ϕ (t-channel); ϵ (f_0), ρ, f_2 (s-channel).
- Model needs several cutoffs, apart from other parameters.
- $I = 1/2$ case ($\kappa = K_0^\star(800)$) is not treated.

TABLE V. A summary of all poles found in the $\pi\pi - K\overline{K} - \pi\eta$ system.

I	J	Sheet	Pole position [MeV]	Comment
0	0	[bt] (II)	$(387, \pm 305)$	$\sigma(400)$
0	0	[bb] (III)	$(314, \pm 428)$	$\sigma(400)$ shadow pole
0	0	[bt] (II)	$(1015, \pm 15)$	$f_0(980)$
0	0	[bb] (III)	$(1346, \pm 249)$	effective $f_0(1400) - f_0(1590)$
1	1	[bt] (II)	$(775, \pm 82)$	ρ
1	0	[bt] (II)	$(991, \pm 101)$	$a_0(980)$

N. A. Törnqvist, M. Roos, Phys. Rev. Lett. **76** (1996) 1575

- Unitarised relativistic quark-meson model for the light scalar mesons.
- For each scalar, one bare $q\bar{q}$ state is coupled to channels of two pseudoscalar mesons.
- Six parameters, including an *ad hoc* negative Adler zero for $K\pi$, were fitted to the data.
- Note: no $K_0^\star(800)$ (κ) was found, probably due to the unrealistic Adler zero (see AIP Conf. Proc. 756 (2005) 360 [hep-ph/0412078].

TABLE I. The $^3P_0$ resonance parameters in units of MeV. The first resonance is the σ. The two following are both manifestations of the same $s\bar{s}$ state. The $f_0(980)$ and $a_0(980)$ have no Breit-Wigner-like description, and the Γ_{BW} for the latter is rather the peak width. The last entry is an image pole to the $a_0(980)$, which in an improved fit could represent the $a_0(1450)$. The $f_0(1300)$ and $K_0^*(1430)$ poles appear simultaneously on two sheets since the $\eta\eta$ and the $K\eta$ couplings, respectively, nearly vanish. The mixing angle δ_S for the σ is with respect to $u\bar{u} + d\bar{d}$, while for the two heavier f_0's it is with respect to $s\bar{s}$. Pure SU3$_f$ states have $\delta_S = -35.3°$.

Resonance	m_{BW}	Γ_{BW}	$\delta_{S,BW}$	$[\mathrm{Re}s_{pole}]^{1/2}$	$\frac{-\mathrm{Im}s_{pole}}{m_{pole}}$	$s_{pole}^{1/2}$	$\delta_{S,pole}$	Sheet	Comment
$f_0(400-900)$	860	880	$(-9+i8.5)°$	397	590	$470-i250$	$(-3.4+i1.5)°$	II	The σ meson; near $u\bar{u}+d\bar{d}$ state
$f_0(980)$	⋯	⋯	⋯	1006	34	$1006-i17$	$(0.4+i39)°$	II	First near $s\bar{s}$ state
$f_0(1300)$	1186	350	$(-32+i1)°$	1202	338	$1214-i168$	$(-36+i2)°$	III,V	Second near $s\bar{s}$ state
$K_0^*(1430)$	1349	498	⋯	1441	320	$1450-i160$	⋯	II,III	The $s\bar{d}$ state
$a_0(980)$	987	≈100	⋯	1084	270	$1094-i145$	⋯	II	First $u\bar{d}$ state
$a_0(1450)$	⋯	⋯	⋯	1566	578	$1592-i284$	⋯	III	Second $u\bar{d}$ state?

M. Harada, F. Sannino, J. Schechter, Phys. Rev. D **54** (1996) 1991

- Amplitudes constructed from an effective nonlocal chiral Lagrangian.
- Local unitarity by assuming the amplitudes to be the sum of a relativistic Breit-Wigner form plus a non-resonant background.
- Fits of the few parameters are done to $\pi\pi$ data up to 1.2 GeV, in different scenarios.

$I^G(J^{PC})$	M(MeV)	Γ_{tot}(MeV)	$B(2\pi)$%
$0^+(0^{++})$	559	370	–
$1^+(1^{--})$	769.9	151.2	100
$0^+(0^{++})$	980	40–400	78.1
$0^+(2^{++})$	1275	185	84.9
$0^+(0^{++})$	1000–1500	150–400	93.6
$1^+(1^{--})$	1465	310	Seen

S. Ishida, M. Ishida, H. Takahashi, T. Ishida, K. Takamatsu, T. Tsuru, Prog. Theor. Phys. **95** (1996) 745 [hep-ph/9610325]

- Unitary "interfering-amplitude" method used for the channels $\pi\pi$ and $K\bar{K}$ to describe the light isoscalar mesons.
- An *ad hoc* negative background phase is introduced for$\pi\pi$ instead of the usual Adler zero.
- The σ is supposed not to couple to $K\bar{K}$.

Table I. Parameters in the best fit below the $K\overline{K}$ threshold with 2 resonances and the negative background phase. Properties of $f_0(980)$ in this table may not be definitive, due to omitting data over the $K\overline{K}$ threshold. For the property of $f_0(980)$, see §3.2 and Table II. The value of $g_{K\overline{K}}$ is quoted from the result of fitting data including those over the $K\overline{K}$ threshold. The decay width Γ and the "peak width" $\Gamma^{(p)}$ are defined, respectively, as $\Gamma_i = \int_0^\infty ds\Gamma_i(s)\sqrt{s}\Gamma_i^{\rm tot}(s) /(\pi[(s-M_i^2)^2 + s\Gamma_i^{\rm tot}(s)^2])$ and $\Gamma_i^{(p)} = \Gamma_i(s = M_i^2)$. The relation of g and $\Gamma(s)$ is given in our BW formula Eq.(2·14). If we use another form of BW formula, instead of (2·14), as $-M\Gamma(M^2) /(s - M^2 + iM\Gamma(M^2))$, the pole position is simply given as $s_{\rm pole}/M = M - i\Gamma(M^2)$. See the discussions in §4(A).

	mass(MeV)	$g_{\pi\pi}$(MeV)	$g_{K\overline{K}}/g_{\pi\pi}$	$\Gamma_{\rm tot}$(MeV)=$\Gamma_{\pi\pi}$	$\Gamma^{(p)}$(MeV)
σ	553.3±0.5	3336±12	0.	242.6±1.2	349.3±2.5
($f_0(980)$)	(970.7±2.2)	(1768±24)			

	2 BW(GeV^{-1})
r_c	3.46±0.01

J. A. Oller, E. Oset, J. R. Pelaez, Phys. Rev. D **59** (1999) 074001 [Errata-ibid **60** (1999) 099906, **75** (2007) 099903]

- Amplitudes from $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ chiral Langrangians are uniterised with the Inverse Amplitude Method.
- The seven free parameters of the $\mathcal{O}(p^2)$ Lagrangian are fitted to the data.
- Pole positions are found for the light scalar and vector mesons.

TABLE III. Masses and partial widths in MeV.

Channel (I,J)	Resource	Mass from pole	Width from pole	Mass effective	Width effective	Partial widths
(0,0)	σ	442	454	$\approx$600	very large	$\pi\pi-100\%$
						$\pi\pi-65\%$
(0,0)	$f_0(980)$	994	28 $\approx$980	$\approx$30		
						$K\bar{K}-35\%$
(0,1)	$\phi(1020)$	980	0	980	0	
(1/2,0)	κ	770	500 $\approx$850	very large	$K\pi-100\%$	
(1/2,1)	$k_*(890)$	892	42 *895	42	$K\pi-100\%$	
						$\pi\eta-50\%$
(1,0)	$a_0(980)$	1055	42	980	40	
						$K\bar{K}-50\%$
(1,1)	$\rho(770)$	759	141	771	147	$\pi\pi=100\%$

D. V. Bugg, Phys. Lett. B **572** (2003) 1 [Erratum-ibid **595** (2004) 556]

Abstract

Evidence for the σ pole has been reported in production processes such as $D^+ \to \pi^+\pi^-\pi^+$; likewise evidence for the κ pole appears in $D^+ \to K^-\pi^+\pi^+$. Their effects in $\pi\pi$ and $K\pi$ elastic scattering are much less conspicuous. However, consistent fits to both production data and elastic scattering may be obtained by including the Adler zero into an s-dependent width for each resonance. These zeros suppress strongly the effects of the σ and κ poles in elastic scattering; the zeros are absent from amplitudes for production data. With this prescription, data from $\pi\pi \to \pi\pi$, K_{e4} decays and CP violation in K^0 decays give a σ pole position of $(525 \pm 40) - i(247 \pm 25)$ MeV. A combined analysis with production data gives a better determination of $(533 \pm 25) - i(249 \pm 25)$ MeV. The analysis of LASS data for $K\pi$ elastic scattering, including the Adler zero, determines a κ pole at $(722 \pm 60) - i(386 \pm 50)$ MeV.

The Fourier transform of the matrix element for $\sigma \to \pi\pi$ reveals a compact interaction region with RMS radius ~ 0.4 fm.

- Relativistic Breit-Wigner forms used for σ in $\pi\pi$ and κ in $K\pi$.
- Theoretical Adler zeros introduced via s-dependent widths.
- Combined fits were done to both elastic-scattering and production data.
- The Adler zeros were argued to prevent the formation of bound states, pushing the σ and κ poles away into the complex plane.

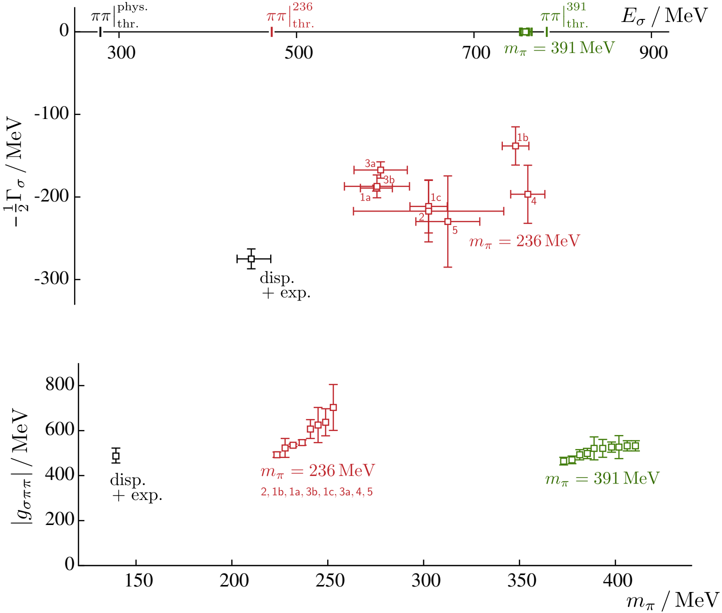
I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett. **96** (2006) 132001 [arXiv:hep-ph/0512364]

- Employs Roy equation for the S-wave $\pi\pi$ scattering amplitude, which is a twice-subtracted dispersion relation:

$$t_0^0(s) = a + (s - 4M_\pi^2)b + \int_{4M_\pi^2}^{\Lambda^2} ds'\{K_0(s,s')\,\mathrm{Im}t_0^0(s') + K_1(s,s')\,\mathrm{Im}t_1^1(s') + K_2(s,s')\,\mathrm{Im}t_0^2(s')\} + d_0^0(s)$$

- where K_0, K_1, K_2 are S-, P-, and D-wave kernels, respectively, with contributions from the right- and left-hand cuts, $\mathrm{Im}\,t_0^0$, $\mathrm{Im}\,t_1^1$, and $\mathrm{Im}\,t_0^2$ (exotic S-wave).
- Also, $d_0^0(s)$ is an estimate of contributions from higher partial waves plus the remainder of the integral over S- and P-waves above the cutoff $\Lambda \geq 1.4$ GeV.
- The subtraction constants a, b can be expressed in terms of the S-wave scattering lengths a_0^0, a_0^2, from chiral perturbation theory.
- **Resulting σ ($f_0(500)$) pole:** $(441 - i272)$ **MeV.**

R. A. Briceno, J. J. Dudek, R. G. Edwards, D. J. Wilson, Phys. Rev. Lett. **118** (2017) 022002



IV. Conclusions

- The light scalar mesons are awkward in the traditional static quark model, due to their low masses and the unusual mass pattern.
- An ingenious way out were the tetraquarks proposed by Jaffe in the context of the MIT bag model, due to very large negative mass shifts for the lowest nonet of scalars from the colour-spin "hyperfine" interaction.
- However, such tetraquark approaches usually neglect the probably large dynamical effects of decay to S-wave two-meson channels.
- The Helsinki and Nijmegen unitarised models showed long ago that unitarisation effects can be large enough to explain the light scalars, via the generation of additional, dynamical states.
- In particular, in 1986 the Nijmegen model predicted a complete light scalar nonet with masses and widths still compatible with present-day PDG values, using parameters fixed in previous work.
- Very recent lattice results by the JLab and Graz groups lend support to this picture of scalar mesons as $q\bar{q}$ states with large meson-meson admixtures.

Хвала на пажњи!