

Thermodynamic Characterizations of Exotic and Missing States

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Excited QCD 2018

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PRD94 (2016) 096010; arXiv:1711.09837.

Issues

- 1 Introduction
 - QCD Thermodynamics
 - Hadron Spectrum
- 2 Entropy Shift and Missing States
 - Entropy Shift below T_c
 - Entropy Shift above T_c
- 3 Fluctuations of Conserved Charges in a Thermal Medium
 - Fluctuations of Conserved Charges
 - Fluctuations in the HRG model
- 4 Correlations in a Thermal Medium
 - Free particles of spin 1/2
 - Free particles of any spin
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References

- **QCD at finite T:** Phys.Lett. B563 (2003) 173-178; Phys.Rev. D69 (2004). 116003.
- **Polyakov-Nambu–Jona-Lasinio:** hep-ph/0410053; AIP Conf.Proc. 756 (2005) 436-438; Phys.Rev. D74 (2006) 065005; Rom.Rep.Phys. 58 (2006) 081-086; PoS JHW2005 (2006) 025; AIP Conf.Proc. 892 (2007) 444-447; Eur.Phys.J. A31 (2007).
- **Dim-2 Condensates:** JHEP 0601 (2006) 073; Phys.Rev. D75 (2007) 105019; Nucl.Phys.Proc.Suppl. 186 (2009) 256-259; Phys.Rev. D81 (2010) 096009.
- **Hadron Resonance Gas for Polyakov loop:** Phys.Rev.Lett. 109 (2012) 151601; Nucl. Phys. Proc. Suppl. 234 (2013) 313-316; Acta Phys. Polon. B 45, 2407 (2014).
- **Polyakov loop Spectroscopy:** Phys.Rev. D89 (2014) 076006; Nucl. Part. Phys. Proc. 258-259 (2015) 109.
- **Heavy Quark Physics:** Nucl. Part. Phys. Proc. 270-272 (2016) 170-174; Phys.Rev. D94 (2016) 096010.
- **Fluctuations:** 1612.07091; 1711.09837.

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QCD Thermodynamics

- The **partition function of QCD**

$$Z_{\text{QCD}} = \text{Tr} e^{-H_{\text{QCD}}/T} = \sum_n e^{-E_n/T}, \quad H_{\text{QCD}}\psi_n = E_n\psi_n,$$

- Spectrum of QCD** \rightarrow Thermodynamics

Hadron Resonance Gas Model

- In the **confined phase**: Colour singlet states (hadrons + ... ???)
 \rightarrow Low temperature partonic expansion [EM, E. Ruiz Arriola, L.L. Salcedo, Acta Phys. Pol. B45 '14]:

$$Z = \underbrace{Z_0}_{\text{Vacuum}} \cdot \underbrace{Z_{q\bar{q}}}_{\text{Mesons}} \cdot \underbrace{Z_{qqq}}_{\text{Baryons}} \cdot \underbrace{Z_{\bar{q}\bar{q}\bar{q}}}_{\text{Hybrids}} \cdot \underbrace{Z_{q\bar{q}g}}_{\text{Tetraquarks}} \cdot \dots$$

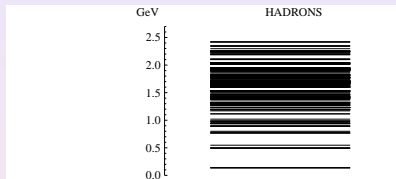
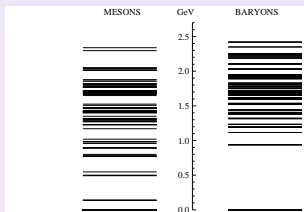
- In the **deconfined phase**: quarks and gluons \rightarrow quark-gluon plasma.
- Phase transition is a crossover** \rightarrow Do we see quark-gluon substructure BELOW the “*phase transition*”?

Issues

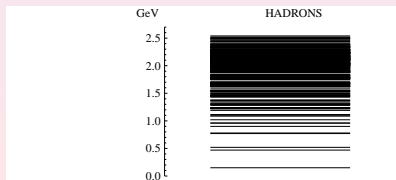
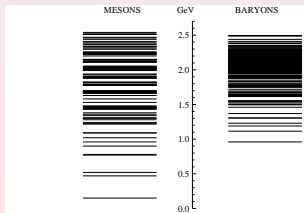
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Hadron Spectrum (u,d,s)

- Particle Data Group (PDG) compilation 2016



- Relativized Quark Model (RQM), Isgur, Godfrey, Capstick 1985



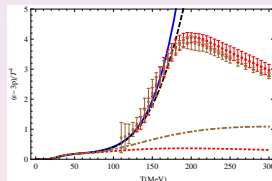
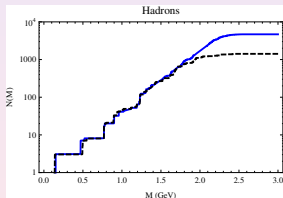
Cumulative number of states

- Cumulative number \equiv number of bound states below M .

$$N(M) = \sum_n \Theta(M - M_n),$$

- Which states count?

$$N(M) = N_{q\bar{q}}(M) + N_{qqq}(M) + \dots,$$



$$N_{q\bar{q}} \sim M^6, \quad N_{qqq} \sim M^{12}, \quad N_{qq\bar{q}\bar{q}} \sim M^{18} \quad \text{and} \quad N_{\text{hadrons}} \sim e^{M/T_H}$$

$$Z = \text{Tr} e^{-H/T} \xrightarrow{T \rightarrow T_H^-} \infty, \quad T_H \sim 150 \text{ MeV} \equiv \text{Hagedorn temperature}$$

- Non-interacting Hadron-Resonance Gas works for $T \lesssim 0.8T_c$.

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Entropy Shift and Polyakov loop

- **Physical situation**: Add to the QCD thermal vacuum one extra **heavy charge** belonging to representation **R**.
- Charge is screened by dynamical constituents to form color neutral states: $[Q\bar{q}]$, $[Qqq]$, $[Q\bar{q}g]$, $[Q\bar{q}q\bar{q}]$, $[Q\bar{q}qqq]$, \dots
- Energy of the states changes under the presence of the charge

$$E_n \rightarrow E_n^{\mathbf{R}} \rightarrow \Delta_n^{\mathbf{R}} + m_{\mathbf{R}} + \dots$$

$\Delta_n^{\mathbf{R}}$ remains finite in the limit $m_{\mathbf{R}} \rightarrow \infty$.

- In the static gauge $\partial_0 A_0 = 0$ the **Polyakov loop** operator reads

$$\text{tr}_{\mathbf{R}} \Omega(\vec{r}) = \text{tr}_{\mathbf{R}} e^{iA_0(\vec{r})/T}.$$

- Ratio of partition functions \rightarrow **Free energy shift** $\equiv \Delta F_{\mathbf{R}}$

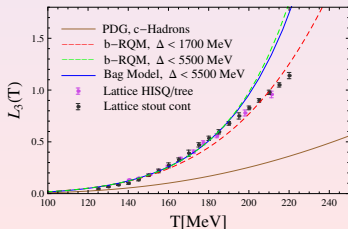
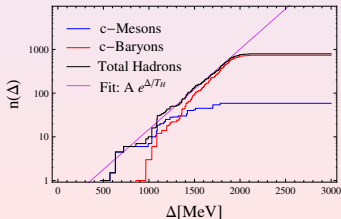
$$L_{\mathbf{R}} \equiv \langle \text{tr}_{\mathbf{R}} \Omega(0) \rangle = \frac{Z_{\mathbf{R}}}{Z_0} = e^{-\Delta F_{\mathbf{R}}/T} = \frac{\frac{1}{2} \sum_n e^{-\Delta_n^{\mathbf{R}}/T}}{1 + \dots}$$

Polyakov loop and Hadron Resonance Gas model

- Counting states \rightarrow Mesons and Baryons with 1 heavy quark ($\mathbf{R} = \mathbf{3}$) \rightarrow HRG model for the Polyakov loop in the fund. rep. [EM, E.Ruiz Arriola, L.L.Salcedo, PRL 109 (2012)]

$$n(\Delta) = \sum_n \Theta(\Delta - \Delta_n) \sim e^{\Delta/T_{H,L}} \rightarrow L_3(T) = \frac{1}{2} \int d\Delta \frac{\partial n(\Delta)}{\partial \Delta} e^{-\Delta/T}$$

Hadron spectrum with one c-quark, and Polyakov loop: $\Delta_n = M_n - m_Q$



Entropy Shift and Missing States

[EM, E.Ruiz Arriola, L.L.Salcedo, PRD94 (2016)]

- Polyakov loop renormalization ambiguity [S.Gupta, K.Hübner, O.Kaczmarek, PRD77 (2008)]:

$$L_R = e^{c/T} \cdot L'_R.$$

- The ambiguity can be removed by computing the entropy shift

$$L_R = \langle \text{tr}_R \Omega(0) \rangle_T = e^{-\Delta F_R(T)/T} \rightarrow \Delta S_R(T) = -\frac{\partial}{\partial T} \Delta F_R(T).$$

- Third principle of thermodynamics for degenerate states

$$\Delta S_Q(0) = \log(2N_f), \quad \Delta S_Q(\infty) = \log N_c.$$

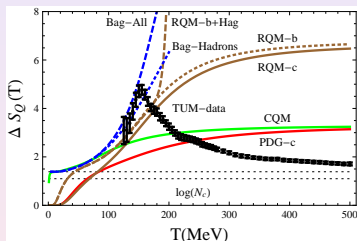
- Renormalization Group Equation:

$$0 = \mu \frac{d\Delta S_Q}{d\mu} = \beta(g) \frac{\partial \Delta S_Q}{\partial g} - \sum_q m_q (1 + \gamma_q) \frac{\partial \Delta S_Q}{\partial m_q} - T \frac{\partial \Delta S_Q}{\partial T}.$$

Entropy Shift and Missing States

Lattice data for the Entropy Shift ($N_c = 3, N_f = 2 + 1$)

[A.Bazavov et al., PRD93 (2016) 114502].



- Entropy shift as a function of the temperature:
 - PDG not enough to describe lattice data.
 - Relativistic Quark Model (Mesons + Baryons) [S.Godfrey, N.Isgur, PRD32 '85].
 - MIT Bag Model [A. Chodos et al., PRD9 '74] including All = ($[Q\bar{q}]$, $[Qqq]$, $[Q\bar{q}g]$ and $[Qqqg]$)

Issues

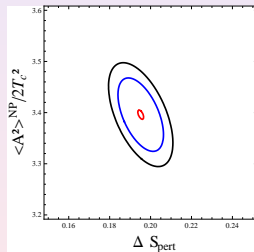
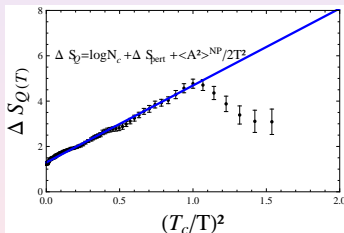
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Power corrections above T_C

- Polyakov loop at high temperatures:

$$\langle \text{tr}(e^{igA_0/T}) \rangle \sim N_c e^{-g^2 \langle \text{tr} A_0^2 \rangle / 2N_c T^2} + \dots$$

- $\rightarrow \Delta S_Q(T) = \log(N_c) + \Delta S_{\text{pert}}(T) + \langle A^2 \rangle^{\text{NP}} / 2T^2$



- *Left panel:* Lattice data [A.Bazavov et al, PRD93 '16]. The straight line is the fit using the dim-2 condensate.
- *Right panel:* Correlation plot between the dim-2 condensate and the perturbative entropy.

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Fluctuations of Conserved Charges

- Conserved charges $\rightarrow [Q_a, H] = 0$.
- In the uds sector the only conserved charges are:

$Q \equiv$ Electric charge, $B \equiv$ Baryon number, $S \equiv$ Strangeness.

- In the hot vacuum (no chemical potential):

$$\langle Q \rangle_T = 0, \quad \langle B \rangle_T = 0, \quad \langle S \rangle_T = 0.$$

- Fluctuations \rightarrow Susceptibilities:

$$\chi_{ab}(T) \equiv \frac{1}{VT^3} \langle \Delta Q_a \Delta Q_b \rangle_T, \quad \Delta Q_a = Q_a - \langle Q_a \rangle_T.$$

At high temperature \rightarrow
$$\begin{cases} \chi_{BB}(T) \propto \langle B^2 \rangle_T \rightarrow 1/N_c \\ \chi_{QQ}(T) \propto \langle Q^2 \rangle_T \rightarrow \sum_{i=1}^{N_f} q_i^2 \\ \chi_{SS}(T) \propto \langle S^2 \rangle_T \rightarrow 1 \end{cases}.$$

Fluctuations of Conserved Charges

[M.Asakawa, M.Kitazawa, Prog. Part. Nucl. Phys. 90 (2016)].

- Fluctuations of conserved charges can be computed from the grand-canonical partition function:

$$Z = \text{Tr} \exp \left[- \left(H - \sum_a \mu_a Q_a \right) / T \right], \quad \Omega = -T \log Z,$$

by differentiation

$$-\left. \frac{\partial \Omega}{\partial \mu_a} \right|_{\mu_a=0} = \langle Q_a \rangle_T, \quad -T \left. \frac{\partial^2 \Omega}{\partial \mu_a \partial \mu_b} \right|_{\mu_a=0=\mu_b} = \langle \Delta Q_a \Delta Q_b \rangle_T \equiv VT^3 \chi_{ab}(T)$$

- $Q_a \in \{Q, B, S\}$, or, in the quark-flavor basis, $Q_a \in \{u, d, s\}$ where

$$B = \frac{1}{3}(u + d + s), \quad Q = \frac{1}{3}(2u - d - s), \quad S = -s.$$

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Fluctuations in the HRG model

- Fluctuations in the HRG model:

$$Q_a = \sum_{i \in \text{Hadrons}} q_a^i N_i, \quad \chi_{ab}(T) = \sum_{i,j \in \text{Hadrons}} q_a^i q_b^j \langle \Delta N_i \Delta N_j \rangle_T, \quad a, b \in \{Q, B, S\}$$

where $q_a^i \in \{Q_i, B_i, S_i\} \equiv$ charge of the i th-hadron corresponding to symmetry a .

- Averaged number of hadrons of type i is

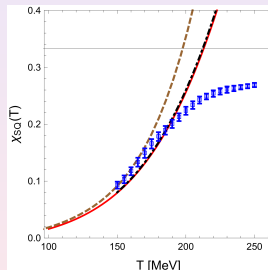
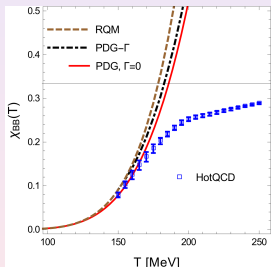
$$\langle N_i \rangle_T = V \int \frac{d^3k}{(2\pi)^3} \frac{g_i}{e^{E_{k,i}/T} - \xi_i},$$

with $E_{k,i} = \sqrt{k^2 + M_i^2}$, and $\xi = \pm 1$ for bosons/fermions.

Fluctuations and Missing States

- **Fluctuations of Conserved Charges** → Good description of lattice data for $T \lesssim 160$ MeV.

Lattice data of Fluctuations [A.Bazavov et al., PRD86 (2012)]

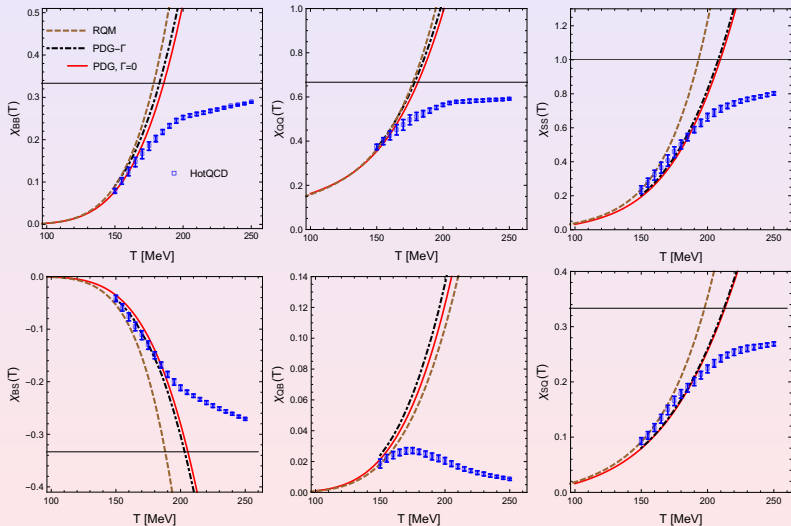


→ *Fluctuations as a diagnostic tool to study missing states.*

Example: **RQM** seems to have too many baryonic states, but not too many charged states.

[E.Ruiz Arriola, W.Broniowski, EM, L.L.Salcedo, 1612.07091].

Fluctuations in the HRG model



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Current correlators of free particles of spin 1/2

- Lagrangian density for Dirac fermions

$$\mathcal{L}(x) = \bar{\Psi}(x)(\not{D} + m)\Psi(x)$$

where m is the mass and $\not{D} = \gamma^\mu D_\mu$.

- Partition function of the system

$$Z = \int \mathcal{D}\Psi(x)\mathcal{D}\bar{\Psi}(x) e^{-\int_0^\beta dx_0 \int d^3x \mathcal{L}(x)}$$

- Fermions are antiperiodic: $\Psi(x_0 + \beta, \vec{x}) = -\Psi(x_0, \vec{x})$ with $\beta = 1/T$ the inverse of temperature.
- Vector currents

$$j^\mu(x) = \bar{\Psi}(x)\gamma^\mu\Psi(x).$$

- Zero components are the conserved charges: $\rho(x) \equiv j^0(x)$.
- Retarded correlator

$$C_{1/2}^{\mu\nu}(x) \equiv \langle j^\mu(x)j^\nu(0) \rangle$$

Current correlators of free particles of spin 1/2

- Propagator of fermions of spin 1/2 in position space

$$S_{1/2}(x) = - \int \frac{d^4 k}{(2\pi)^4} \frac{i\not{k} + m}{k^2 + m^2} e^{-ikx}.$$

- The correlator writes

$$\langle j^\mu(x) j^\nu(0) \rangle = \langle S_{1/2}(x) \gamma^\mu S_{1/2}(-x) \gamma^\nu \rangle$$

- Correlator at zero temperature:**

$$\langle j^\mu(x) j^\nu(0) \rangle = 4 [2(\partial^\mu \Delta(x))(\partial^\nu \Delta(x)) - ((\partial_\alpha \Delta(x))^2 + m^2 \Delta(x)^2) \eta^{\mu\nu}]$$

where

$$\Delta(x) = \int \frac{d^4 k}{(2\pi)^4} \frac{e^{-ik_\mu x^\mu}}{k^2 + m^2} = \frac{m}{4\pi^2} \frac{K_1(m|x|)}{|x|}, \quad |x| = \sqrt{x_0^2 + \vec{x}^2}.$$

$K_1 \equiv$ Bessel function of the second kind.

Free particles of spin 1/2: $T = 0$

- Explicit result for the correlator at $T = 0$:

$$\langle j^\mu(x) j^\nu(0) \rangle = \frac{4m^4}{(4\pi^2)^2} \left[\left(\frac{K_2(m|x|)}{|x|^2} \right)^2 [2x^\mu x^\nu - \eta^{\mu\nu} x^2] - \left(\frac{K_1(m|x|)}{|x|} \right)^2 \eta^{\mu\nu} \right]$$

- Conservation of the current

$$\partial_\mu \langle j^\mu(x) j^\nu(0) \rangle = 0.$$

- Behavior at small distances

$$\langle j^0(\vec{x}) j^0(0) \rangle \simeq -\frac{1}{\pi^4 r^6} + \frac{m^2}{4\pi^4 r^4} + \mathcal{O}(r^{-2}), \quad \text{with } r = |\vec{x}|.$$

Free particles of spin 1/2: $T \neq 0$

- **Correlator at finite temperature.** Using Poisson's formula

$$\int \frac{dk_0}{2\pi} F(k_0, \vec{k}) \rightarrow i \sum_{n=-\infty}^{\infty} \xi^n \int \frac{dk_4}{2\pi} F(ik_4, \vec{k}) e^{ink_4/T},$$

where $\xi = \pm 1$ for bosons (fermions).

- $n \equiv$ number of thermal loops:
 $n = 0 \rightarrow T = 0$ contribution, $n \neq 0 \rightarrow$ finite T corrections.
- **Correlator at finite temperature:**

$$\langle j^\mu(x) j^\nu(0) \rangle_T = 4 \left[2(\partial^\mu \Delta_T(x))(\partial^\nu \Delta_T(x)) - ((\partial_\alpha \Delta_T(x))^2 + m^2 \Delta_T(x)^2) \eta^{\mu\nu} \right]$$

$$\Delta_T(x) = \frac{m}{4\pi^2} \sum_{n=-\infty}^{+\infty} \xi^n \frac{K_1(m|x|)}{|x|}, \quad |x| = \sqrt{\left(x_0 - \frac{n}{T}\right)^2 + \vec{x}^2}.$$

- At small distances \rightarrow finite T correction starting at $\mathcal{O}(r^{-2})$

$$\langle j^0(\vec{x}) j^0(0) \rangle_T - \langle j^0(\vec{x}) j^0(0) \rangle_{T=0} = \frac{1}{r^2} \frac{m^2 T}{\pi^2} \left(m K_1\left(\frac{m}{T}\right) + 2TK_2\left(\frac{m}{T}\right) \right) + \dots$$

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Current correlators of free particles of any spin

- By using the formalism of [V.Bargmann, E.Wigner, Proc. Nat. Acad. Sci 34 (1948)], in Euclidean space

$$C_{\mu\nu}^J(x) \equiv \langle j_\mu(x) j_\nu(0) \rangle = (2m)^2 \frac{(-1)^n}{n^2} \left(a_n L_1^{n-1} L_{\mu\nu} + a_{n-1} L_1^{n-2} L_\mu L_\nu \right) \Delta^2(x)$$

for $J \geq 1/2$, where $a_n = 2^{1-n} \binom{n+2}{3}$, ($a_0 = 0$), $n = 2J$, where J is the spin,

$$C_{\mu\nu}^0(x) = m^2 L_\mu L_\nu \Delta^2(x),$$

and L are differential operators

$$L_1 = 1 - \frac{1}{m^2} \partial_\alpha^1 \partial_\alpha^2, \quad L_\mu = \frac{1}{m} \left(\partial_\mu^1 - \partial_\mu^2 \right),$$

$$L_{\mu\nu} = \delta_{\mu\nu} \left(1 + \frac{1}{m^2} \partial_\alpha^1 \partial_\alpha^2 \right) - \frac{1}{m^2} \left(\partial_\mu^1 \partial_\nu^2 + \partial_\nu^1 \partial_\mu^2 \right).$$

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Current correlators in the HRG model

- Within the HRG model the correlator writes

$$C_{\mu\nu}^{ab}(x) \equiv \langle j_{\mu}^a(x) j_{\nu}^b(0) \rangle = \sum_i \frac{1}{2} q_i^a q_i^b C_{\mu\nu}^{J_i}(x), \quad q_i^a \in \{Q_i, B_i, S_i\}.$$

- i stands for any hadron, distinguishing between **spin** J_i , **isospin** and **particle-antiparticle**.
- Lowest lying states in the meson and hadron spectrum corresponding to pions and protons/neutrons:

$$i \in \{\pi^+, \pi^0, \pi^-, p \uparrow, p \downarrow, \bar{p} \uparrow, \bar{p} \downarrow, n \uparrow, n \downarrow, \bar{n} \uparrow, \bar{n} \downarrow\}.$$

- Small distance behavior: $C_{00}^J(r) \underset{r \rightarrow 0}{\sim} \frac{m^2}{r^4} \frac{1}{(mr)^{4J}}$.
- After summation over hadrons of higher and higher spin

$$C_{00}^{HRG}(r) = \sum_J C_{00}^J(r) \xrightarrow{r \rightarrow r_H^+} \infty$$

$r_H \equiv$ Hagedorn distance \longleftrightarrow Analogous to T_H at finite T .

Current correlators in the HRG model

- An equivalent expression:

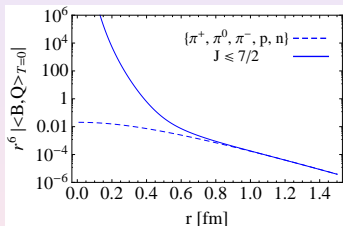
$$C_{\mu\nu}^{ab}(x) \equiv \langle j_{\mu}^a(x) j_{\nu}^b(0) \rangle = \sum_{M \in \text{Mesons}} \frac{1}{2} (2J_M + 1) q_M^a q_M^b C_{\mu\nu}^{J_M}(x) + \sum_{B \in \text{Baryons} > 0} (2J_B + 1) q_B^a q_B^b C_{\mu\nu}^{J_B}(x)$$

- M and B run now over the spin multiplets of mesons and baryons, each of them with degeneracy $(2J_M + 1)$ and $(2J_B + 1)$.
- Lowest lying states: $M \in \{\pi^+, \pi^-, \pi^0\}$ and $B \in \{p, n\}$.
- These correlators (in the static limit $x^0 = 0$) fulfill

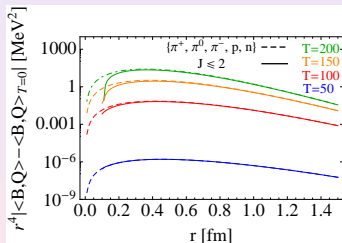
$$\chi_{ab}(T) = \int d^3x C_{00}^{ab}(0, \vec{x}).$$

Correlations in the confined phase of QCD

Static C_{00} correlator at $T = 0$



Static C_{00} correlator at $T \neq 0$

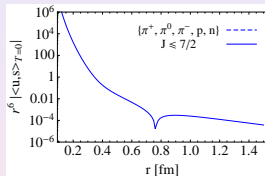
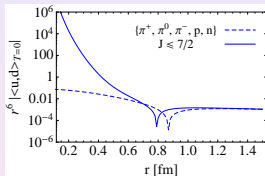
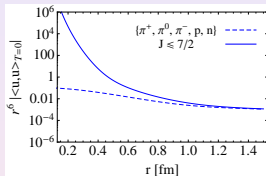


- Analogy: “correlators at $T=0$ ” \Leftrightarrow “susceptibilities $T \neq 0$ ”

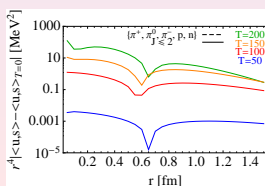
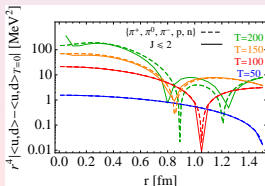
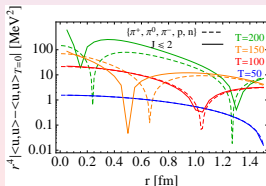
$$C_{00}^{ab}(0, \vec{x}) \underset{r \rightarrow \infty}{\sim} e^{-Mr} \quad \text{and} \quad \chi_{ab}(T) \underset{T \rightarrow 0}{\sim} e^{-M/T}$$

Correlations in the confined phase of QCD

• Correlations $T = 0$



• Correlations $T \neq 0$



Conclusions

- At low temperatures **hadrons** can be considered as a **complete basis of states in terms of a Hadron Resonance Gas (HRG) model**. The HRG works at $T \lesssim 0.8T_c$.
- Close T_c many hadrons are needed to saturate the sum rule \implies What states are needed when approaching T_c from below?
- **Polyakov loop** and **Entropy shift** due to a heavy quark suggests that there are in the QCD spectrum: i) conventional missing states ($Q\bar{q}$ and Qqq), and ii) hybrid states ($Q\bar{q}g$ and $Qqqg$).
- This establishes a **new tool** for **Polyakov loop spectroscopy** of the QCD spectrum including exotic states.
- **Fluctuations of conserved charges** in the confined phase of QCD allow to study missing states in three different sectors:
 - i) electric charge, ii) baryon number, and iii) strangeness.
- We have obtained results for the **correlations of conserved charges** at zero and finite temperature \rightarrow **Confronting these with future results on the lattice will help in the study of missing states!!!**

Thank You!