KU LEUVEN

A fresh look at the (non-)Abelian Landau-Khalatnikov-Fradkin transformations

Conference Presentation Excited QCD
ArXiv: 1801.01703

Tim De Meerleer - tim.demeerleer@kuleuven.be

KU Leuven

March 2018

0 Outline

KU LEUVEN

1 Introduction (I)

The Landau-Fradkin-Khalatnikov transformations (LKFT's) show us how n-point correlators relate between different gauges.
Motivations

- Gauge fixing is required. Most calculations are limited to Landau gauge, but other gauge choices are emerging. But how are these different computations related?
- In practice one often needs an Ansätz, for instance photon-fermion vertex in QED. These Ansätze are gauge specific, but the LKFT can show how this vertext looks in a different gauge.
- Finally, all physics should be gauge independent. The question is, are these Anzätze consistent with gauge convariance?

1 Introduction (II)

Our final goal: LKFT for the gluon and quark n-point correlator function

$$\langle A_\mu A_\nu \rangle_\alpha = \langle A_\mu A_\nu \rangle_{\alpha'} + something$$

We start by recovering the (known) Abelian LKFT for the photon, as a first proof of concept.
Next, we show how to expand this to non-Abelian theories, retrieving the LKFT for the gluon propagator.

2 The QCD action

$$S = S_{FP} + S_f + S_h{}^{[1]}$$

$$S_{FP} = \int \mathrm{d}^4x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

$$S_f = \int \mathrm{d}^4x \left(\bar{\psi}(i\not{D} + m_f)\psi \right)$$

$$S_h = \int \mathrm{d}^4x \left(\tau^a \partial_\mu A^{h,a}_\mu + \frac{m^2}{2} A^{h,a}_\mu A^{h,a}_\mu + \bar{\eta}^a \partial_\mu D^{ab}_\mu (A^h) \eta^b \right)$$

Where A^h was introduced to minimise $\int \mathrm{d}^4x\, A^u_\mu A^u_\mu$, related to the problem of Gribov copies.

---

[1] M. A. L. Capri, D. Fiorentini, M. S. Guimaraes, B. W. Mintz, L. F. Palhares and S. P. Sorella, (2016) - ArXiv:1606.06601v3

2 The additional "field" A^h

The transversal "field" A^h

$$A_\mu^h = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h$$
$$h = e^{ig\phi^a T^a}$$

Minimalisation of A^h leads to

$$A_\mu^h = A_\mu - \frac{\partial_\mu}{\partial^2}\partial A + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\mu \frac{1}{\partial^2}\partial A\right] + ig\frac{\partial_\mu}{\partial^2}\left[\frac{\partial_\nu}{\partial^2}\partial A, A_\nu\right] + i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + \mathcal{O}\left(A^3\right) \tag{1}$$

KU LEUVEN

2 Properties of A^h

- Landau gauge ($\partial A = 0$): $A^h_\mu = A_\mu$
- BRST invariant
- Renormalisability of A^h has been proven[2].
- Looking at transformations of A_μ and h, A^h is gauge invariant by construction

$$\left\langle A^h_\mu A^h_\nu \right\rangle_\alpha = \left\langle A^h_\mu A^h_\nu \right\rangle_{\alpha'} \qquad (2)$$

[2] M. A. L. Capri, D. Fiorentini, M. S. Guimaraes, B. W. Mintz, L. F. Palhares and S. P. Sorella, (2016) - ArXiv: 1606.06601v3

2 Properties of A^h

- Landau gauge ($\partial A = 0$): $A^h_\mu = A_\mu$
- BRST invariant
- Renormalisability of A^h has been proven[2].
- Looking at transformations of A_μ and h, A^h is gauge invariant by construction

$$\left\langle A^h_\mu A^h_\nu \right\rangle_\alpha = \left\langle A^h_\mu A^h_\nu \right\rangle_{\alpha'=0} \tag{2}$$

[2] M. A. L. Capri, D. Fiorentini, M. S. Guimaraes, B. W. Mintz, L. F. Palhares and S. P. Sorella, (2016) - ArXiv: 1606.06601v3

2 Properties of A^h

- Landau gauge ($\partial A = 0$): $A^h_\mu = A_\mu$
- BRST invariant
- Renormalisability of A^h has been proven[2].
- Looking at transformations of A_μ and h, A^h is gauge invariant by construction

$$\left\langle A^h_\mu A^h_\nu \right\rangle_\alpha = \left\langle A^h_\mu A^h_\nu \right\rangle_{\alpha'=0} = \langle A_\mu A_\nu \rangle_{\alpha'=0} \tag{2}$$

[2] M. A. L. Capri, D. Fiorentini, M. S. Guimaraes, B. W. Mintz, L. F. Palhares and S. P. Sorella, (2016) - ArXiv: 1606.06601v3

2 Evaluating propagators (I)

Photon propagator: $A^h_\mu = A_\mu - \partial_\mu \phi$

$$\left\langle A^a_\mu(p) A^b_\nu(-p) \right\rangle_\alpha = \left\langle A^a_\mu(p) A^b_\nu(-p) \right\rangle_{\alpha=0} + \alpha \frac{p_\mu p_\nu}{p^4} \delta^{ab}$$

Which retrieves the known LKFT for the photon.

2 Evaluating propagators (II)

Gluon propagator

Inserting full order expansion (1) yields gluon propagator.
(Work in progress)

General LKFT

Invariant fermion field $\psi^h = h^\dagger \psi$

$$\left\langle A_\mu^h \dots \psi^h \dots \bar{\psi}^h \dots \right\rangle_\alpha = \left\langle A_\mu^h \dots \psi^h \dots \bar{\psi}^h \dots \right\rangle_{\alpha'}$$

$$
\begin{aligned}
&\Big\langle A^{h,a}_\mu(p) A^{h,b}_\nu(-p) \Big\rangle \qquad (3) \\
&= \Big\langle A^a_\mu(p) A^b_\nu(-p) \Big\rangle + \Big\langle A^a_\mu(p) \partial_\nu \phi^b(-p) \Big\rangle + \Big\langle \partial_\mu \phi^a(p) A^b_\nu(-p) \Big\rangle + \Big\langle \partial_\mu \phi^a(p) \partial_\nu \phi^b(-p) \Big\rangle \\
&+ g f^{bcd} \Big[- \Big\langle A^a_\mu(p) A^c_\nu(-p) \phi^d(-p) \Big\rangle + \Big\langle \partial_\mu \phi^a(p) A^c_\nu(-p) \phi^d(-p) \Big\rangle \\
&\quad - \frac{1}{2} \Big\langle A_\mu(p)^a \phi^c(-p) \partial_\nu \phi^d(-p) \Big\rangle + \frac{1}{2} \Big\langle \partial_\mu \phi^a(p) \phi^c(-p) \partial_\nu \phi^d(-p) \Big\rangle \Big] \\
&+ g f^{acd} \Big[- \Big\langle A^c_\mu(p) \phi^d(p) A^b_\nu(-p) \Big\rangle + \Big\langle A^c_\mu(p) \phi^d(p) \partial_\nu \phi^b(-p) \Big\rangle \\
&\quad - \frac{1}{2} \Big\langle \phi^c(p) \partial_\mu \phi^d(p) A^b_\nu(-p) \Big\rangle + \frac{1}{2} \Big\langle \phi^c(p) \partial_\mu \phi^d(p) \partial_\nu \phi^b(-p) \Big\rangle \Big] \\
&+ \frac{g^2}{3!} D^{bcde} \Big[\Big\langle A^a_\mu(p) \partial_\nu \phi^c(-p) \phi^d(-p) \phi^e(-p) \Big\rangle - 2 \Big\langle A^a_\mu(p) \phi^c(-p) \partial_\nu \phi^d(-p) \phi^e(-p) \Big\rangle \\
&\quad + \Big\langle A^a_\mu(p) \phi^c(-p) \phi^d(-p) \partial_\nu \phi^e(-p) \Big\rangle - \Big\langle \partial_\mu \phi^a(p) \partial_\nu \phi^c(-p) \phi^d(-p) \phi^e(-p) \Big\rangle \\
&\quad + 2 \Big\langle \partial_\mu \phi^a(p) \phi^c(-p) \partial_\nu \phi^d(-p) \phi^e(-p) \Big\rangle - \Big\langle \partial_\mu \phi^a(p) \phi^c(-p) \phi^d(-p) \partial_\nu \phi^e(-p) \Big\rangle \Big] \\
&+ \frac{g^2}{3!} D^{acde} \Big[\Big\langle \partial_\mu \phi^c(p) \phi^d(p) \phi^e(p) A^b_\nu(-p) \Big\rangle - 2 \Big\langle \phi^c(p) \partial_\mu \phi^d(p) \phi^e(p) A^b_\nu(-p) \Big\rangle \\
&\quad + \Big\langle \phi^c(p) \phi^d(p) \partial_\mu \phi^e(p) A^b_\nu(-p) \Big\rangle - \Big\langle \partial_\mu \phi^c(p) \phi^d(p) \phi^e(p) \partial_\nu \phi^b(-p) \Big\rangle \\
&\quad + 2 \Big\langle \phi^c(p) \partial_\mu \phi^d(p) \phi^e(p) \partial_\nu \phi^b(-p) \Big\rangle - \Big\langle \phi^c(p) \phi^d(p) \partial_\mu \phi^e(p) \partial_\nu \phi^b(-p) \Big\rangle \Big] \\
&- \frac{g^2}{2} D^{bcde} \Big[\Big\langle A^a_\mu(p) A^c_\nu(-p) \phi^d(-p) \phi^e(-p) \Big\rangle - 2 \Big\langle A^a_\mu(p) \phi^c(-p) A^d_\nu(-p) \phi^e(-p) \Big\rangle \\
&\quad + \Big\langle A^a_\mu(p) \phi^c(-p) \phi^d(-p) A^e_\nu(-p) \Big\rangle - \Big\langle \partial_\mu \phi^a(p) A^c_\nu(-p) \phi^d(-p) \phi^e(-p) \Big\rangle \\
&\quad + 2 \Big\langle \partial_\mu \phi^a(p) \phi^c(-p) A^d_\nu(-p) \phi^e(-p) \Big\rangle - \Big\langle \partial_\mu \phi^a(p) \phi^c(-p) \phi^d(-p) A^e_\nu(-p) \Big\rangle \Big]
\end{aligned}
$$

3 Path integral formalism

Starting from the Abelian limit, the QED action

$$S = \int \mathrm{d}^4x \left(\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \bar{\psi}\not{D}\psi + ib\partial_\mu A_\mu + \frac{\alpha}{2}b^2 + \bar{c}\partial^2 c + \bar{J}\psi + \bar{\psi}J + J_\mu A_\mu\right)$$

and corresponding path integral

$$Z(J) = \int [\mathrm{d}\mu]\ e^{-S}$$

3 Gauge transformations (I)

Under a general gauge transformation

$$
\begin{aligned}
U &= e^{-ie\phi} \\
A_\mu \to A'_\mu &= A_\mu - \partial_\mu \phi \\
\psi \to \psi' &= U\psi
\end{aligned}
$$

3 Gauge transformations (I)

Under a general gauge transformation

$$
\begin{aligned}
U &= e^{-ie\phi} \\
A_\mu \to A'_\mu &= A_\mu - \partial_\mu \phi \\
\psi \to \psi' &= U\psi
\end{aligned}
$$

We still have the freedom to chose the 'angle' of rotation ϕ

$$\phi = -X \frac{1}{\partial^2} \partial_\mu A_\mu \tag{4}$$

3 Gauge transformations (II)

Under this transformation

$$\partial_\mu A'_\mu = (1 + X)\partial_\mu A_\mu$$

To keep the action (up to source terms) invariant

$$b' = \frac{1}{1+X} b$$
$$\alpha' = (1+X)^2 \alpha$$

Rescaling of the gauge parameter!

3 The photon propagator revisited

$$\langle A_\mu(x)A_\nu(y)\rangle_\alpha = \int [\mathrm{d}\mu] A_\mu(x)A_\nu(y)e^{-S}$$
$$= \frac{\delta^2 Z_\alpha}{\delta J_\nu(y)\delta J_\mu(x)} = \frac{\delta^2 Z'_\alpha}{\delta J_\nu(y)\delta J_\mu(x)}$$

Note, the action remained invariant, up to the source terms!

3 The photon propagator revisited

$$\langle A_\mu(x) A_\nu(y) \rangle_\alpha = \int [\mathrm{d}\mu] A_\mu(x) A_\nu(y) e^{-S}$$
$$= \frac{\delta^2 Z_\alpha}{\delta J_\nu(y) \delta J_\mu(x)} = \frac{\delta^2 Z'_\alpha}{\delta J_\nu(y) \delta J_\mu(x)}$$

Note, the action remained invariant, up to the source terms!

$$\langle A_\mu(p) A_\nu(-p) \rangle_\alpha = \langle A_\mu(p) A_\nu(-p) \rangle_{\alpha'} - (\alpha' - \alpha) \frac{p_\mu p_\nu}{p^4} \tag{5}$$

Again exactly the LKFT for the photon.

3 Fermion propagator

Fermions, and the corresponding source terms, transform as well. In this sector one finds (with $U = e^{ie\frac{X}{1+X}\frac{1}{\partial^2}\partial A'}$)

$$\left\langle \bar{\psi}(x)\psi(y) \right\rangle_\alpha = \left\langle \bar{\psi}'(x) e^{\frac{-ieX}{1+X}\frac{1}{\partial^2}\partial_\mu A'_\mu(x)} e^{\frac{ieX}{1+X}\frac{1}{\partial^2}\partial_\nu A'_\nu(y)} \psi'(y) \right\rangle_{\alpha'}$$

3 Fermion propagator

Fermions, and the corresponding source terms, transform as well. In this sector one finds (with $U = e^{ie\frac{X}{1+X}\frac{1}{\partial^2}\partial A'}$)

$$\left\langle \bar{\psi}(x)\psi(y) \right\rangle_{\alpha} = \left\langle \bar{\psi}'(x) e^{\frac{-ieX}{1+X}\frac{1}{\partial^2}\partial_\mu A'_\mu(x)} e^{\frac{ieX}{1+X}\frac{1}{\partial^2}\partial_\nu A'_\nu(y)} \psi'(y) \right\rangle_{\alpha'}$$

explicitly contracting these fields in the Landau gauge ($\alpha' = 0$)

$$\left\langle \bar{\psi}(x)\psi(y) \right\rangle_{\alpha} = \left\langle \bar{\psi}'(x)\psi'(y) \right\rangle_{\alpha'=0} \left\langle U^\dagger(x) U(y) \right\rangle_{\alpha'=0}$$

3 Extension to QCD

The 'full' transformation

$$A_\mu \to A'_\mu = U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger$$
$$U = e^{ig\phi} = 1 + ig\phi - \frac{g^2}{2}\phi^2 + \mathcal{O}\left(\phi^3\right)$$

requiring $\partial_\mu A'_\mu = (1+X)\partial_\mu A_\mu$ can be solved for all orders of ϕ

$$\phi = X\frac{1}{\partial^2}\partial_\mu A_\mu - igX\frac{1}{\partial^2}\left[\frac{1}{\partial^2}\partial_\mu\partial_\nu A_\nu, A_\mu\right]$$
$$- igX\frac{1}{\partial^2}\left[\frac{1}{\partial^2}\partial_\mu A_\mu, \partial_\nu A_\nu\right] - \frac{igX^2}{2}\frac{1}{\partial^2}\left[\frac{1}{\partial^2}\partial_\mu A_\mu, \partial_\nu A_\nu\right] + \mathcal{O}(A^3)$$

4 Conclusion & Outlook

- Investigating gluon propagator using FeynCalc (contracting the correspnding diagrams), due to the high number of diagrams (order of 100).
- Using this result to check the non-Abelian LKFT with the expected results (Lattice QCD).
- Linking this viewpoint of LKFT's to the theory of Nielsen identities.

4 Conclusion & Outlook

- Investigating gluon propagator using FeynCalc (contracting the correspnding diagrams), due to the high number of diagrams (order of 100).
- Using this result to check the non-Abelian LKFT with the expected results (Lattice QCD).
- Linking this viewpoint of LKFT's to the theory of Nielsen identities.

Any questions?