QCD phase diagram from the lattice via effective Polyakov line actions relative weights and mean field

arXiv:1708:08031

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Motivation - The QCD Phase Diagram
Effective Polyakov Line Action

- map LGT to Polyakov line action (SU(3) spin) model
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- fix Polyakov line holonomies $U_0(\vec{x}, 0) = P_x$ (temporal gauge) and integrate out all other d.o.f.
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$$e^{S_P(P_x)} = \int DU_0(\vec{x}, 0) DU_k D\psi \prod_x \delta[P_x - U_0(\vec{x}, 0)] e^{S_L}$$
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- derive \( S_P \) at \( \mu = 0 \), for \( \mu > 0 \) we have (true to all orders of strong coupling/hopping parameter expansion)
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$$S_{P}^{\mu}(P_x, P_x^\dagger) = S_{P}^{\mu=0}[e^{N_t \mu} P_x, e^{-N_t \mu} P_x^\dagger]$$
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- derive \( S_P \) at \( \mu = 0 \), for \( \mu > 0 \) we have (true to all orders of strong coupling/hopping parameter expansion)

\[
S^\mu_K(P_x, P_x^\dagger) = S^\mu_K=0[e^{N_t \mu} P_x, e^{-N_t \mu} P_x^\dagger]
\]

- hard to compute \( \exp[S_P(P_x)] \) directly, but action ratios are easily computed as expectation values \( \rightarrow \) relative weights via derivatives of \( S_P \) w.r.t. Fourier components \( a_k \) of \( P_x \)
setting a particular $a_k = 0$, we construct from the resulting configuration
\[ \tilde{P}_x = P_x - e^{ikx} \sum_y P_y e^{-iky} / L^3 \]
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$$S_{\text{eff}} [P_x] = \sum_{x,y} P_x K(x - y) P_y$$

$$+ p \sum_x \log(1 + he^{\mu / T} Tr[P_x] + h^2 e^{2\mu / T} Tr[P_x^\dagger] + h^3 e^{3\mu / T})$$

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determine $K(x - y)$ and $h$ from fitting to lattice data
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S_{\text{eff}}[P_x] = \sum_{x,y} P_x K(x - y) P_y + p \sum_x \log(1 + he^{\mu/T} Tr[P_x] + h^2 e^{2\mu/T} Tr[P_x^\dagger] + h^3 e^{3\mu/T}) \log(1 + he^{-\mu/T} Tr[P_x] + h^2 e^{-2\mu/T} Tr[P_x^\dagger] + h^3 e^{-3\mu/T})
\]

determine \( K(x - y) \) and \( h \) from fitting to lattice data

\[
\frac{1}{L^3} \left( \frac{\partial S_P}{\partial a_k} \right)_{a_k = \alpha} = 2K(k)\alpha + \frac{p}{L^3} \sum_x (3he^{ikx} + 3h^2 e^{-ikx} + \text{c.c.})
\]
Fitting to lattice data

\[ \frac{1}{L^3} \left( \frac{\delta S_p}{\delta \alpha} \right)_{\alpha=\alpha_0} = \alpha \]

- Gauge
- Fermion
- Total $\Delta S$

10.5157$\alpha + 0.0397$

$k=0$
Fourier transform $K(k)$ to $K(r)$
Finite size cutoff $R_{cut}$ for $K(r)$

![Graph showing the finite size cutoff $R_{cut}$ for $K(r)$](image)

- $K(R) L=8^3$
- $K(R) L=16^3$
- $K(R) L=32^3$
- $K(R) L=64^3$

Fit: $0.599/R^4$
Solve sign problem for the effective action

remaining sign problem can be solved by mean field theory
(see also Splittorff and Greensite, 2012)
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$$S^0_P = \frac{1}{9} [\sum_{x,y \neq x} \text{Tr} U_x \text{Tr} U_y^\dagger K(x - y) + \sum_x \text{Tr} U_x \text{Tr} U_x^\dagger K(0)]$$
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$$\text{Tr}U_x = (\text{Tr}U_x - u) + u, \text{ Tr}U_x^\dagger = (\text{Tr}U_x^\dagger - v) + v$$
\[ S_P^0 = \frac{1}{9} \sum_{x \neq 0} K(x) \left[ \sum_x (v \text{Tr} U_x + u \text{Tr} U_x^\dagger) - uvL^3 \right] \]

\[ + \frac{1}{9} \sum_x \text{Tr}[U_x] \text{Tr} U_x^\dagger K(0) + E_0 \]

with \( E_0 = \sum_{x,y \neq x} (\text{Tr} U_x - u)(\text{Tr} U_y^\dagger - v)K(x - y) \)
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- equivalent to the stationarity of the mean field free energy with respect to variations in \( u \) and \( v \) → solve numerically
QCD phase diagram from the lattice

\[ u - \frac{1}{G} \frac{\partial G}{\partial A} = 0 \quad \text{and} \quad v - \frac{1}{G} \frac{\partial G}{\partial B} = 0, \]

with \( A = J_0 v, \ B = J_0 u, \ J_0 = \sum_{x \neq 0} K(x)/9 \) and

\[ G(A, B) = \mathcal{D} \left( \mu, \frac{\partial}{\partial A}, \frac{\partial}{\partial B} \right) \sum_{s=-\infty}^{\infty} \det \left[ D_{ij}^{-s} I_0 [2\sqrt{AB}] \right], \]

where \( I_0 \) is a Bessel function and \( D_{ij}^{-s} \) is the \( i, j \)-th component of a matrix of differential operators

\[
D_{ij}^s = \begin{cases} 
D_{i,j+s} & s \geq 0 \\
D_{i+|s|,j} & s < 0 
\end{cases}, \\
D_{ij} = \begin{cases} 
\left( \frac{\partial}{\partial B} \right)^{i-j} & i \geq j \\
\left( \frac{\partial}{\partial A} \right)^{j-i} & i < j 
\end{cases}. 
\]
Simulation parameters and mean field results

- for effective Polyakov line actions derived from LGT
- on $16^3 \times 6$ lattices with Wilson gauge action and
dynamical staggered fermions with $m_q = 695$ MeV
- scale setting via $a$ from Necco-Sommer expression
- we keep $N_t = 6$ and $m_q = 695$ MeV fixed and vary $T$ via $\beta$
- $a_0 = K(x = 0)/9$, $J_0 = \sum_{x \neq 0} K(x)/9$, note small $h$!!!
Finite temperature transition at $\mu = 0$
QCD phase diagram from the lattice

\[ \beta = 5.63, m_\Lambda = 0.711 \]

\[ \beta = 5.65, m_\Lambda = 0.677 \]

\[ \beta = 5.66, m_\Lambda = 0.66 \]

\[ \beta = 5.68, m_\Lambda = 0.63 \]
QCD phase diagram from the lattice

\[ \beta = 5.7, m_a = 0.6 \]

\[ \beta = 5.73, m_a = 0.561 \]

\[ \beta = 5.75, m_a = 0.536 \]

\[ \beta = 5.77, m_a = 0.513 \]
Free energy \( f_{mf}/T = J_0 \mu v - \log G(A, B) \)
Number density \( n = \frac{(\partial G/\partial \mu)}{G} \)
Preliminary Phase Diagram

![Phase Diagram Image]

- Phase transition line
- Critical endpoints

**Axes:**
- T [MeV] on the y-axis
- \(\mu\) [MeV] on the x-axis

**Legend:**
- Phase transition line
- Critical endpoints

**Graph Details:**
- Data points indicating key transitions and endpoints in the QCD phase diagram from the lattice.
Comparison to other methods

- tricky because of different lattice fermions, number of flavors and quark masses
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- analytical continuation from imaginary $\mu$ (M. d’Elia and M.-P. Lombardo, 2003)
  - four flavors of staggered quarks, $ma = 0.05$
  - $T(\mu) = T_c(1 - 0.021 \frac{\mu^2}{2T_c^2})$, fit $T_c \approx 220\text{MeV}$
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- heavy-dense complex Langevin (G. Aarts, F. Attanasio, B. Jäger, and D. Sexty, 2016)
  - two flavors of Wilson fermions, $\kappa = 0.04$ (heavy!)
  - $T(\mu) = 481(1 - \frac{\mu^2}{\mu_0^2}) - 279.3(1 - \frac{\mu^2}{\mu_0^2})^2$
  - $\mu_0 = -\log(2\kappa)$ motivated by hopping parameter expansion
  - take a $\mu_0$ to give the closest fit to our data
Analytical Continuation from imaginary $\mu$

![Graph showing relative weights and analytic continuation]
Heavy-dense Complex Langevin

QCD phase diagram from the lattice

relative weights
heavy dense CL

T
µ

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Conclusions

- determined effective Polyakov line action for staggered fermions with $m_q = 695\text{MeV}$ with standard Wilson gauge action for a range of gauge couplings on $16^3 \times 6$ lattices

good agreement for the Polyakov line correlators computed in the effective theory and underlying lattice gauge theory solved sign problem for the effective theory by mean field and find a phase transition line and correct density limit good agreement with analytical continuation from imaginary $\mu$ (d'Elia and Lombardo, 2003) comparison to heavy dense complex Langevin and other methods tricky because of different lattice fermions, number of flavors and quark masses

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- second critical endpoint suggests heavy quark regime or maybe quark-hadron continuity (smooth superfluid nuclear to superconducting quark matter transition)

- simple PLA ansatz may not hold on finer lattices (diverging interaction range) and for lighter quarks?

- Polyakov lines in higher representations, trilinear couplings, etc., maybe required at higher densities?

- higher order (multi-body interactions) and chiral/center symmetry breaking terms suppressed by small $h$?

- can the mean-field PLA still locate transition lines and determine critical properties reliably?

- supplement RW and MF approach with other methods, e.g. inverse Monte-Carlo (Wozar et al., Bahrampour et al.), strong coupling effective PLA (G. Bergner, J. Langelage, O. Philipsen)
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Questions?

arXiv:1708.08031

Thank You &

Tareq Alhalholy, Derar Altarawneh, Michael Engelhardt, Manfried Faber, Martin Gal, Jeff Greensite, Urs M. Heller, James Hettrick, Andrei Ivanov, Francesco Knechtli, Tomasz Korzec, Thomas Layer, Štefan Olejnik, Luis Oxman, Mario Pitschmann, Jesus Saenz, Thomas Schweigler, Wolfgang Söldner, David Vercauteren, Markus Wellenzohn

13.3.2018 Excited QCD 2018, Kopaonik, Serbia
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