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# QCD phase diagram from the lattice via effective Polyakov line actions relative weights and mean field

## arXiv:1708:08031

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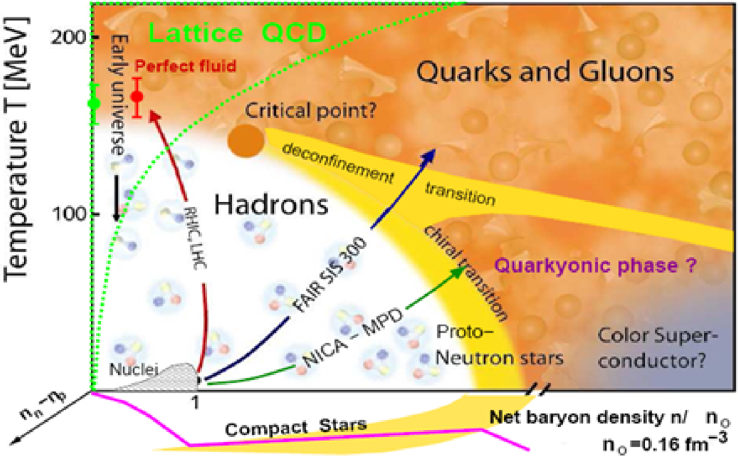
# Motivation - The QCD Phase Diagram



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- hard to compute $\exp[S_P(P_x)]$ directly, but action ratios are easily computed as expectation values $\rightarrow$ relative weights via derivatives of $S_P$ w.r.t. Fourier components $a_k$ of $P_x$

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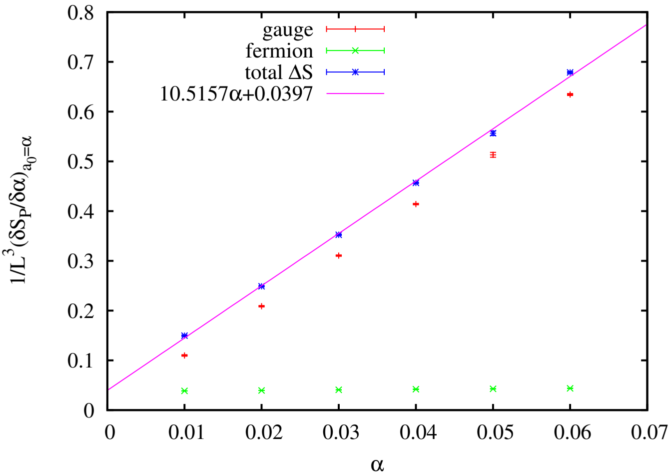
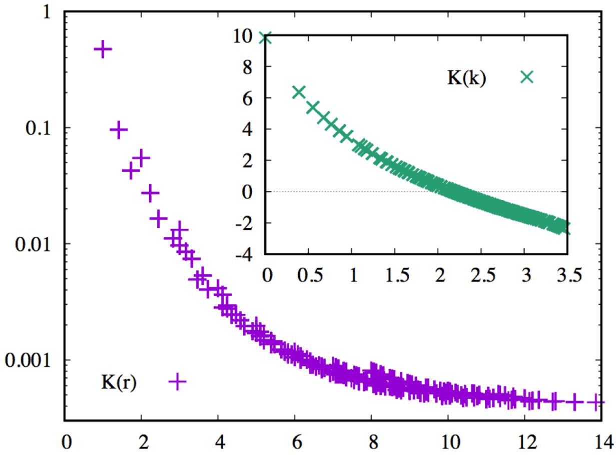
$$\frac{1}{L^3}\left(\frac{\partial S_P}{\partial a_k}\right)_{a_k=\alpha} = 2K(k)\alpha + \frac{p}{L^3} \sum_x (3he^{ikx} + 3h^2 e^{-ikx} + c.c.)$$

# Fitting to lattice data

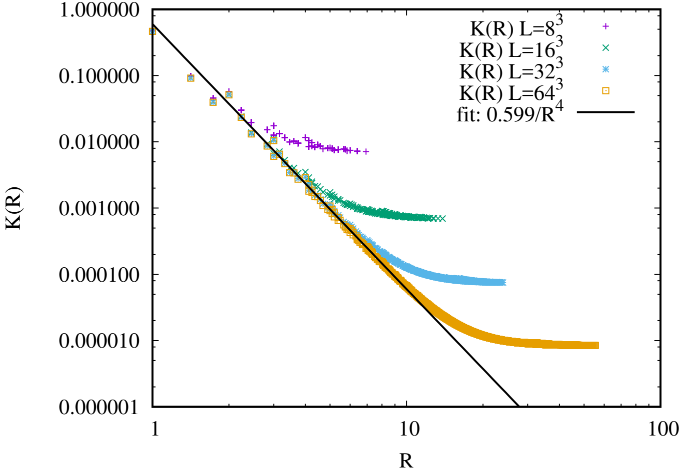


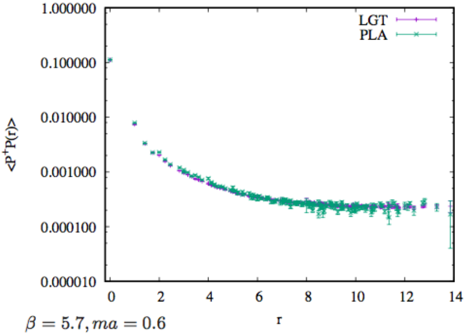

k=0

# Fourier transform $K(k)$ to $K(r)$

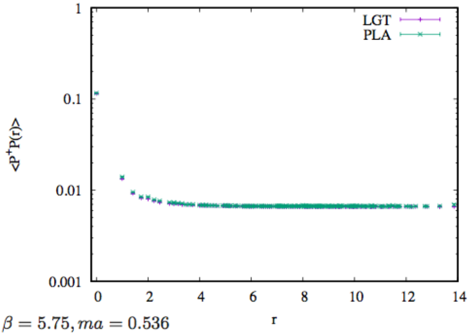


# Finite size cutoff $R_{cut}$ for $K(r)$






$\beta = 5.7, ma = 0.6$

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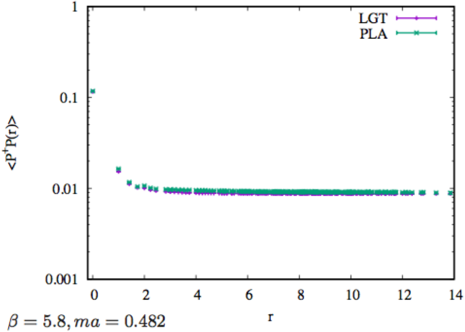


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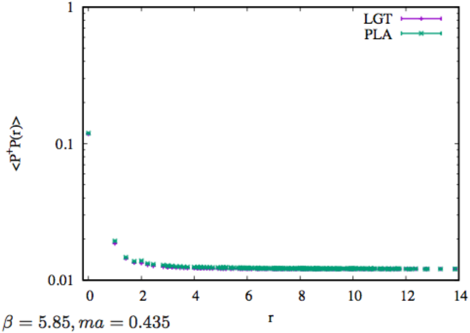


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$$S_P^0 = \frac{1}{9}[\sum_{x,y\neq x} \mathrm{Tr}U_x \mathrm{Tr}U_y^\dagger K(x-y) + \sum_x \mathrm{Tr}U_x \mathrm{Tr}U_x^\dagger K(0)]$$

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$$\mathrm{Tr}U_x = (\mathrm{Tr}U_x - u) + u \ , \ \mathrm{Tr}U_x^\dagger = (\mathrm{Tr}U_x^\dagger - v) + v$$

$$S_P^0 = \frac{1}{9}\sum_{x\neq 0} K(x)[\sum_x (v\mathrm{Tr}U_x + u\mathrm{Tr}U_x^\dagger) - uvL^3]$$

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- equivalent to the stationarity of the mean field free energy with respect to variations in $u$ and $v$ $\rightarrow$ solve numerically

$$u - \frac{1}{G}\frac{\partial G}{\partial A} = 0 \quad \text{and} \quad v - \frac{1}{G}\frac{\partial G}{\partial B} = 0,$$

with $A = J_0 v$, $B = J_0 u$, $J_0 = \sum_{x\neq 0} K(x)/9$ and

$$G(A,B) = \mathcal{D}\left(\mu, \frac{\partial}{\partial A}, \frac{\partial}{\partial B}\right) \sum_{s=-\infty}^{\infty} \det\left[D_{ij}^{-s} I_0[2\sqrt{AB}]\right],$$

where $I_0$ is a Bessel function and $D_{ij}^{-s}$ is the $i,j$-th component of a matrix of differential operators

$$D_{ij}^{s} = \begin{cases} D_{i,j+s} & s \geq 0 \\ D_{i+|s|,j} & s < 0 \end{cases},$$

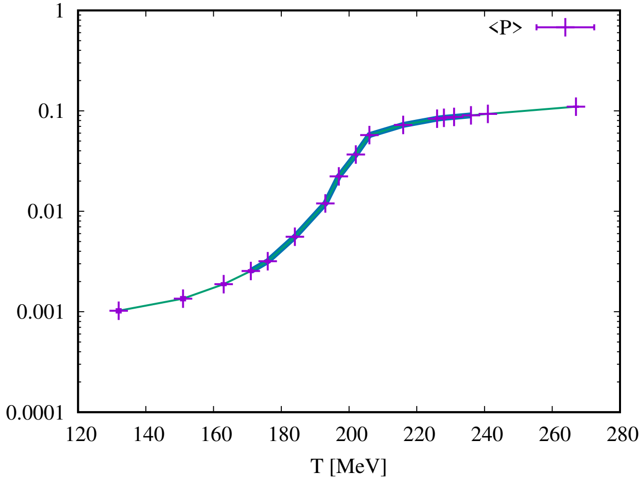
$$D_{ij} = \begin{cases} \left(\frac{\partial}{\partial B}\right)^{i-j} & i \geq j \\ \left(\frac{\partial}{\partial A}\right)^{j-i} & i < j \end{cases},$$

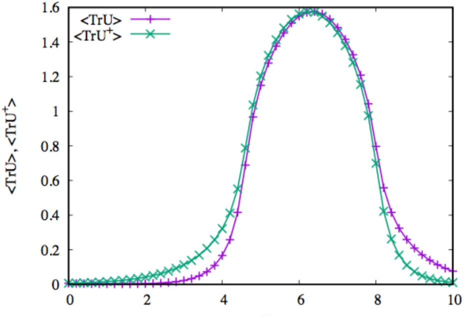
# Simulation parameters and mean field results

- for effective Polyakov line actions derived from LGT
- on $16^3$x6 lattices with Wilson gauge action and
- dynamical staggered fermions with $m_q = 695$MeV
- scale setting via $a$ from Necco-Sommer expression
- we keep $N_t = 6$ and $m_q = 695$MeV fixed and vary $T$ via $\beta$
- $a_0 = K(x=0)/9$, $J_0 = \sum_{x\neq 0} K(x)/9$, note small $h$!!!

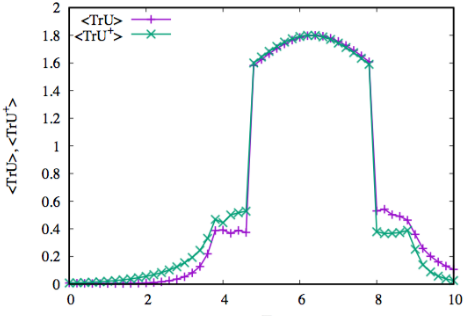
| $\beta$ | $T$[MeV] | $a$[fm] | $ma$ | $\langle P\rangle$ | $K(0)$ | $a_0$ | $J_0$ | $h$(mfd) | $\mu/T$ | $\mu$[MeV] |
|---|---|---|---|---|---|---|---|---|---|---|
| 5.63 | 163 | 0.201 | 0.711 | 0.00188 | 7.648 | -0.0079 | 0.6803 | 0.00183 | - | - |
| 5.66 | 176 | 0.187 | 0.660 | 0.00318 | 8.764 | -0.0201 | 0.8224 | 0.00184 | 5.425 | 955 |
| 5.68 | 184 | 0.178 | 0.630 | 0.00558 | 9.069 | -0.0192 | 0.8381 | 0.00288 | 5.075 | 934 |
| 5.70 | 193 | 0.170 | 0.601 | 0.01198 | 9.382 | -0.0256 | 0.8646 | 0.00513 | 4.625 | 893 |
| 5.73 | 206 | 0.159 | 0.561 | 0.05734 | 10.221 | -0.0360 | 0.8709 | 0.01527 | 3.525 | 726 |
| 5.75 | 216 | 0.152 | 0.536 | 0.07235 | 9.851 | -0.0334 | 0.8608 | 0.02971 | 2.825 | 610 |
| 5.77 | 226 | 0.145 | 0.513 | 0.08354 | 9.760 | -0.0380 | 0.7753 | 0.03940 | 1.125 | 254 |
| 5.775 | 229 | 0.144 | 0.508 | 0.08522 | 9.719 | -0.0364 | 0.7920 | 0.03530 | 1.825 | 418 |
| 5.78 | 231 | 0.142 | 0.502 | 0.08703 | 9.834 | -0.0454 | 0.7622 | 0.04515 | 0.775 | 179 |
| 5.80 | 241 | 0.136 | 0.482 | 0.09332 | 10.039 | -0.0438 | 0.7623 | 0.04639 | 0.675 | 163 |
| 5.85 | 267 | 0.123 | 0.435 | 0.10992 | 10.151 | -0.0540 | 0.6850 | 0.07716 | - | - |

# Finite temperature transition at $\mu = 0$

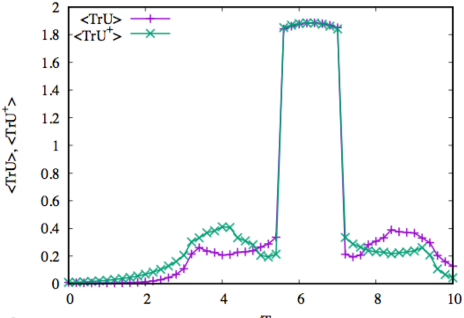




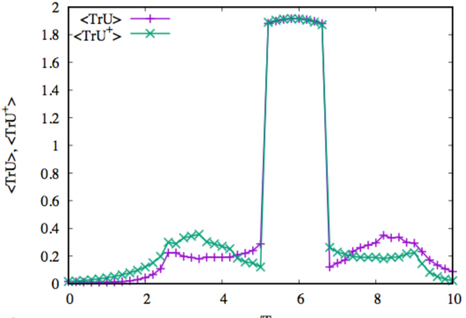

$\beta = 5.63, ma = 0.711$



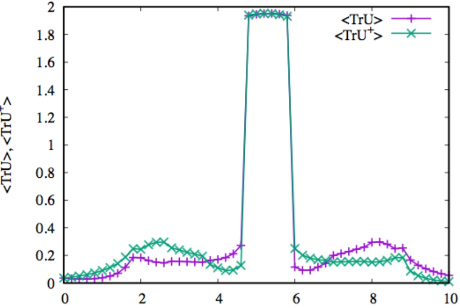

$\beta = 5.65, ma = 0.677$




$\beta = 5.66, ma = 0.66$




$\beta = 5.68, ma = 0.63$




μ/T

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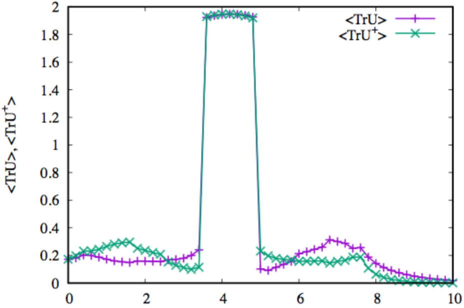
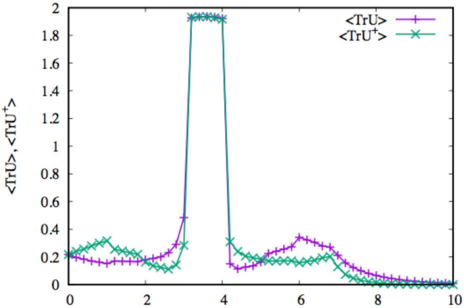


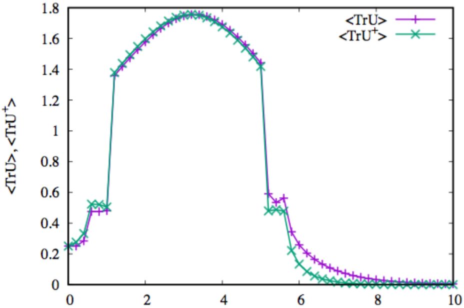

μ/T

$\beta = 5.73, ma = 0.561$



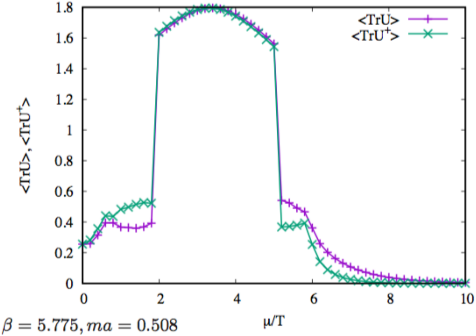

μ/T

$\beta = 5.75, ma = 0.536$

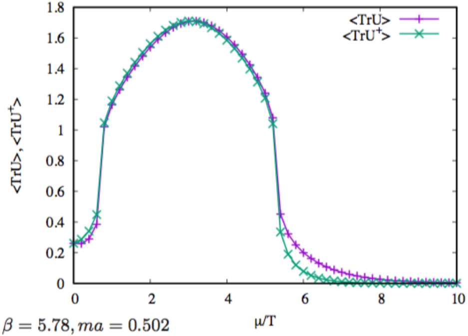



μ/T

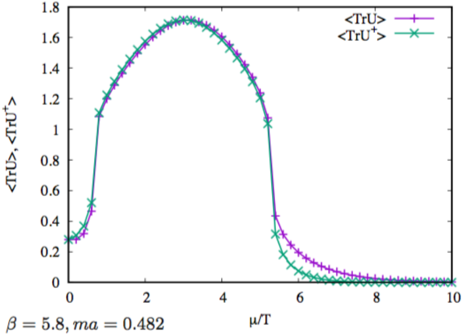
$\beta = 5.77, ma = 0.513$




$\beta = 5.775, ma = 0.508$




$\beta = 5.78, ma = 0.502$

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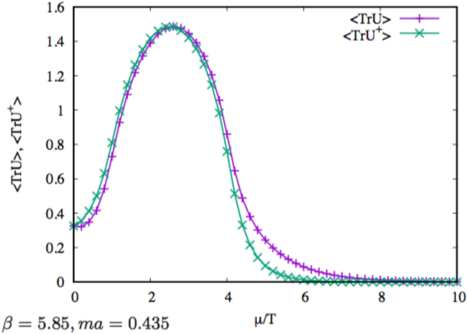


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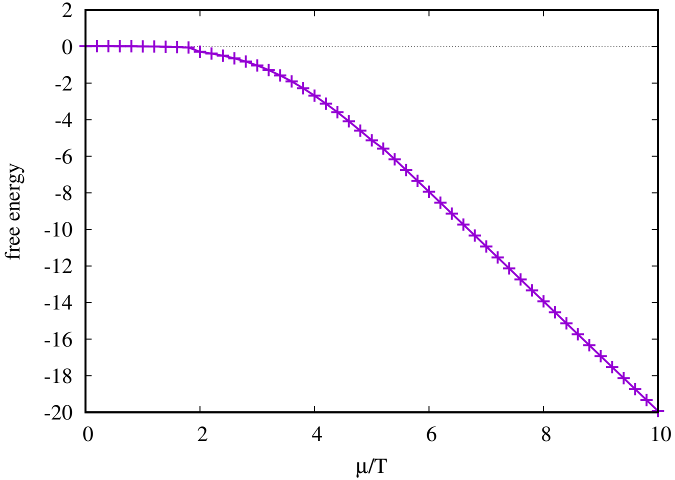
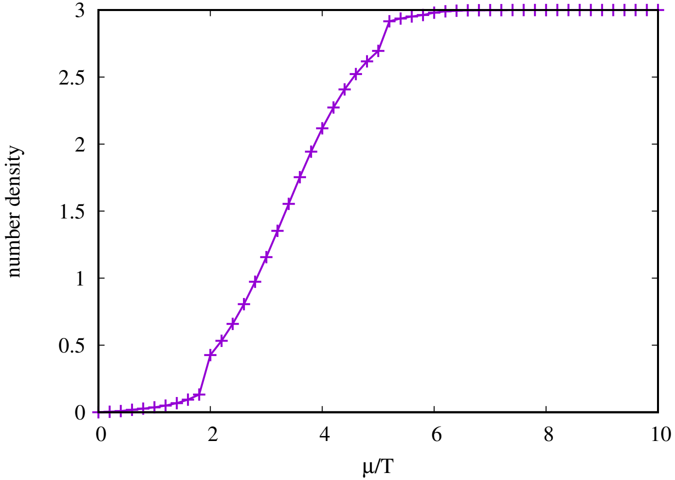
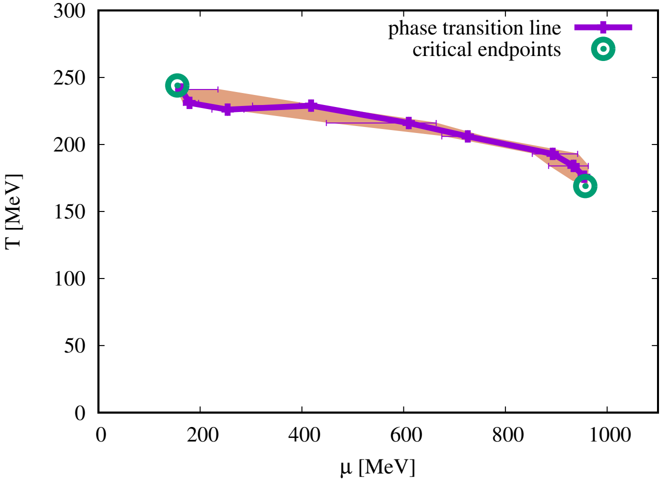
# Free energy $f_{mf}/T = J_0 uv - \log G(A, B)$



# Number density $n = (\partial G/\partial\mu)/G$



# Preliminary Phase Diagram



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  - four flavors of staggered quarks, $ma = 0.05$
  - $T(\mu) = T_c(1 - 0.021\frac{\mu^2}{2T_c^2})$, fit $T_c \approx 220\text{MeV}$

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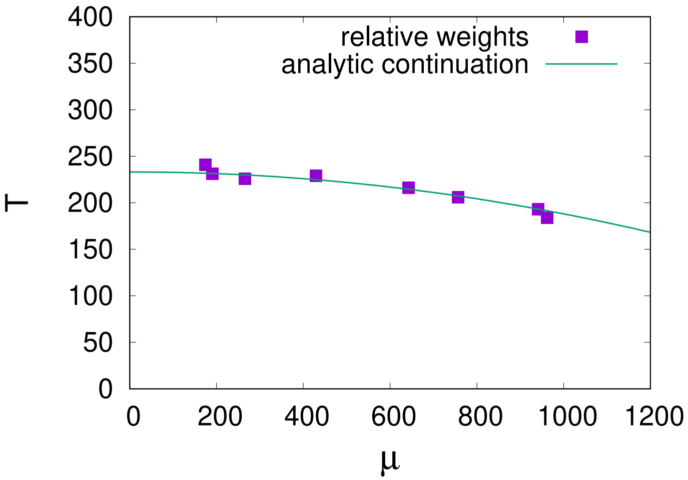
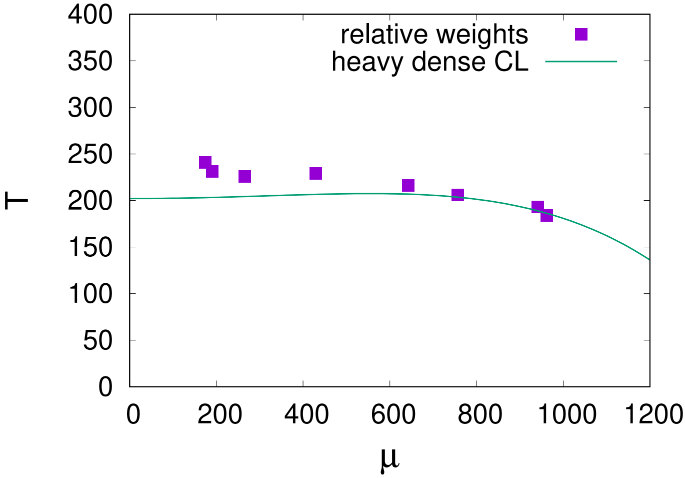
- analytical continuation from imaginary $\mu$ (M. d'Elia and M.-P. Lombardo, 2003)
  - four flavors of staggered quarks, $ma = 0.05$
  - $T(\mu) = T_c(1 - 0.021\frac{\mu^2}{2T_c^2})$, fit $T_c \approx 220\text{MeV}$
- heavy-dense complex Langevin (G. Aarts, F. Attanasio, B. Jäger, and D. Sexty, 2016)
  - two flavors of Wilson fermions, $\kappa = 0.04$ (heavy!)
  - $T(\mu) = 481(1 - \frac{\mu^2}{\mu_0^2}) - 279.3(1 - \frac{\mu^2}{\mu_0^2})^2$
  - $\mu_0 = -\log(2\kappa)$ motivated by hopping parameter expansion
  - take a $\mu_0$ to give the closest fit to our data

# Analytical Continuation from imaginary $\mu$



# Heavy-dense Complex Langevin



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- comparison to heavy dense complex Langevin and other methods tricky because of different lattice fermions, number of flavors and quark masses

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  - strong coupling effective PLA (G. Bergner, J. Langelage, O. Philipsen)

# Questions?

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**Thank You &**

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# QCD phase diagram from the lattice

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