Excited QCD 2018

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Based on: S.S. Afonin and T.D. Solomko,
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The large-N_c masses of light mesons from QCD sum rules and the scalar sigma-meson

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Introduction

Large-N_c QCD $\Longrightarrow$ narrow resonances

In two-point correlators of quark currents:

Sum of narrow resonances $\overset{?}{\longleftrightarrow}$ Operator Product Expansion (OPE)

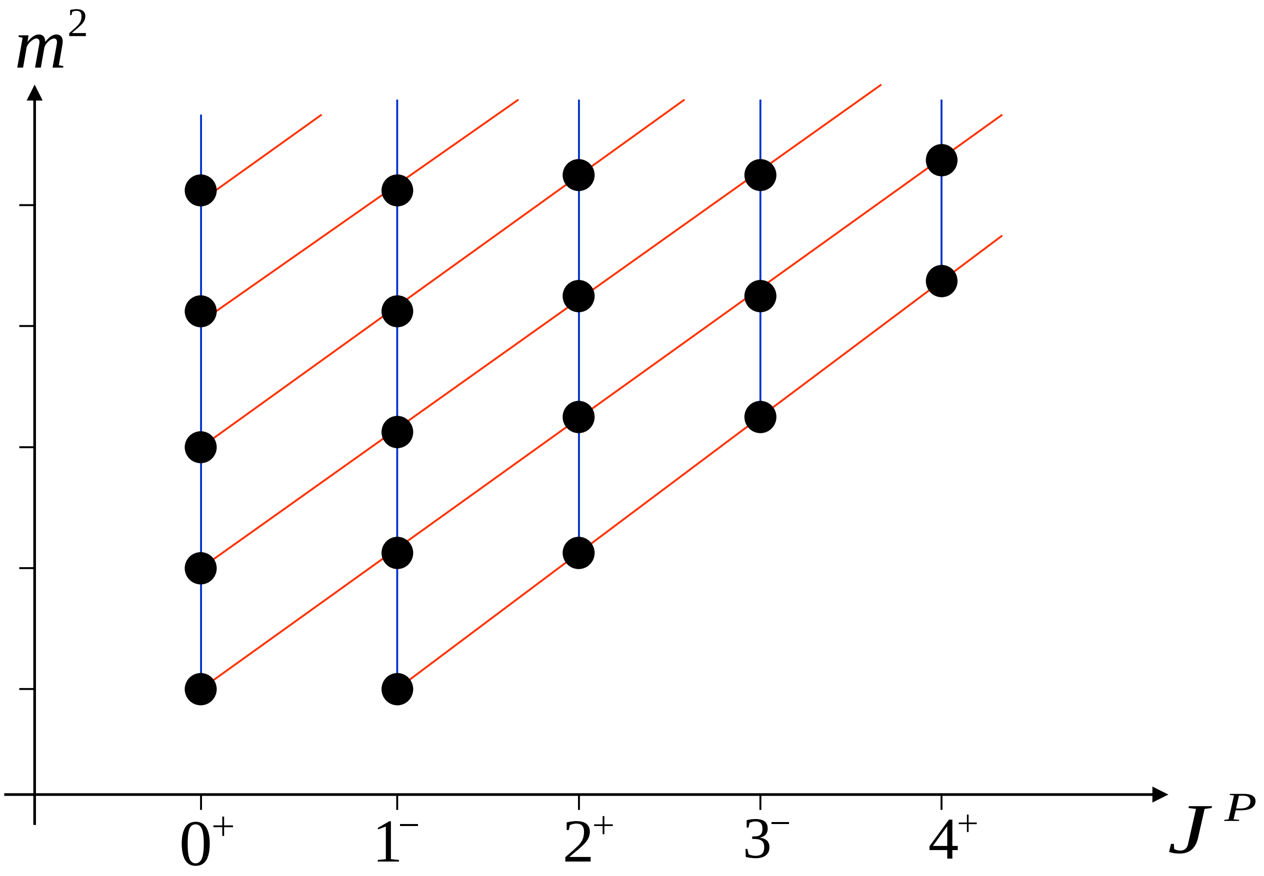
$$\langle J(q) J(-q) \rangle = \sum_n \frac{F_n^2}{q^2 - M_n^2}$$

$$M_n = \mathcal{O}(1) \qquad F_n^2 = \langle 0|J|n\rangle^2 = \mathcal{O}(N_c) \qquad \Gamma = \mathcal{O}(1/N_c)$$

Constraints on meson mass spectrum?

Dual models, hadron strings $\longleftrightarrow$ Linear mass spectrum with universal slope (in the light-flavour sector!)

Radial Regge trajectories

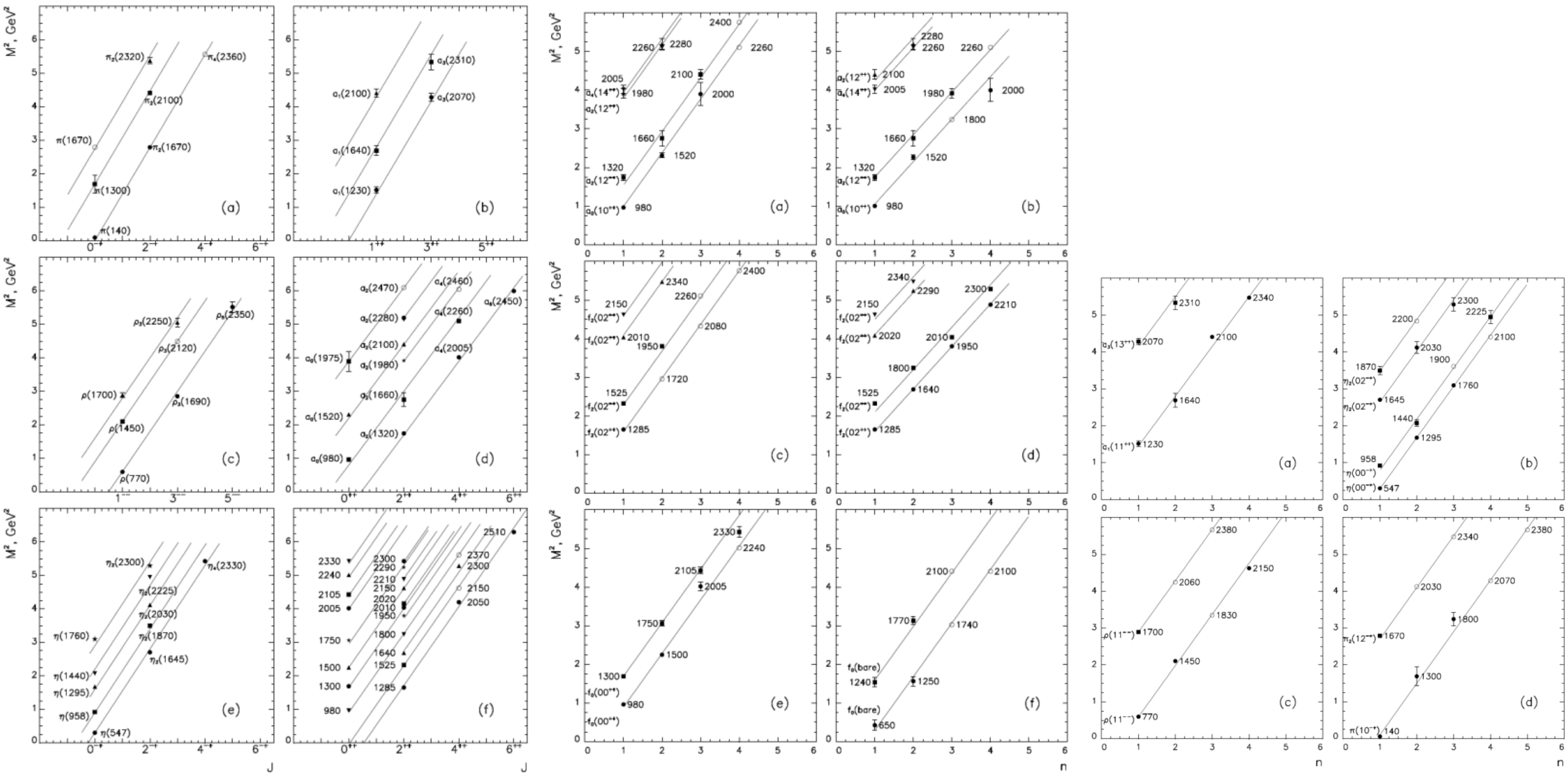


$$m^2(J) = m_0^2 + \alpha' J$$ – Regge trajectories

$$m^2(n) = m_0^2 + an$$ – Radial Regge trajectories

Linear Regge and radial trajectories: Experiment

(Anisovich et al. 2000)



Two-point correlators in Euclidean space (q denotes u- or d-quarks):

$$\Pi_J(Q^2) = \int d^4x \exp(iQx) \langle \bar{q}\Gamma q(x) \bar{q}\Gamma q(0) \rangle_{N_c \to \infty}$$

$$= \sum_n \frac{Z_J(n)}{Q^2 + m_J^2(n)} + D_0^J + D_1^J Q^2$$

$$J \equiv S, P, V, A; \ \Gamma = i, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5; \ D_0^J, D_1^J = \text{const.}$$

$$Z_{VA}(n) \equiv 2F_{VA}^2(n)$$ — residues — are related to some observables from $e^+e^- \to V, \tau \to \nu_\tau + \ldots$

$$Z_{SP}(n) \equiv 2G_{SP}^2(n) m_{SP}^2(n),$$

$$F_P^2(n) = \frac{2m_q G_P(n)}{m_P(n)}$$ — weak decay constants

Operator Product Expansion (chiral limit, large-N_c, LO of PT)

from the pert. theory — gluon condensate — four-quark condensate

$$\Pi_V(Q^2) = \frac{N_c}{12\pi^2}\ln\frac{\Lambda_{\text{cut}}^2}{Q^2} + \frac{\alpha_s}{12\pi}\frac{\langle G^2\rangle}{Q^4} - \frac{28}{9}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^6} + O\left(\frac{1}{Q^8}\right)$$

$$\Pi_A(Q^2) = \frac{N_c}{12\pi^2}\ln\frac{\Lambda_{\text{cut}}^2}{Q^2} + \frac{\alpha_s}{12\pi}\frac{\langle G^2\rangle}{Q^4} + \frac{44}{9}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^6} + O\left(\frac{1}{Q^8}\right)$$

$$\Pi_S(Q^2) = -\frac{N_c}{8\pi^2}Q^2\ln\frac{\Lambda_{\text{cut}}^2}{Q^2} + \frac{\alpha_s}{8\pi}\frac{\langle G^2\rangle}{Q^2} - \frac{22}{3}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^4} + O\left(\frac{1}{Q^6}\right)$$

$$\Pi_P(Q^2) = -\frac{N_c}{8\pi^2}Q^2\ln\frac{\Lambda_{\text{cut}}^2}{Q^2} + \frac{\alpha_s}{8\pi}\frac{\langle G^2\rangle}{Q^2} + \frac{11}{3}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^4} + O\left(\frac{1}{Q^6}\right)$$

After summing over resonances and comparing with the OPE (at each power of $1/Q^2$) one arrives at the so-called planar QCD sum rules.

Example: Vector and axial-vector mesons

$$\langle J_\mu^V(q) J_\nu^V(-q)\rangle = (q_\mu q_\nu - g_{\mu\nu} q^2)\Pi_V(q^2)$$

$$J_\mu^V = \bar{q}\gamma_\mu q$$

Linear ansatz:

$$M_V^2(n) = \Lambda^2 n + M_V^2, \quad n = 0, 1, 2, \ldots .$$

$$\Pi_V(Q^2) = \frac{F_\omega^2}{Q^2 + M_\omega^2} + \sum_{n=0}^{\infty} \frac{F^2}{Q^2 + \Lambda^2 n + M_V^2}$$

$$Q^2 = -q^2$$

OPE:

$$\Pi_V(Q^2) = -\frac{C_0}{8\pi^2}\log\frac{Q^2}{\mu^2} + \frac{\alpha_s}{24\pi}\frac{\langle G^2\rangle}{Q^4} - \frac{14}{9}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^6} + \cdots$$

Introduce the dimensionless variables:

$$m_v = \frac{M_V}{\Lambda}, \quad m_\omega = \frac{M_\omega}{\Lambda}, \quad f = \frac{F}{\Lambda}, \quad f_\omega = \frac{F_\omega}{\Lambda}$$

After summation and expansion

$$\Pi_V(Q^2) = -f^2\log\frac{Q^2}{\mu^2} + \frac{\Lambda^2}{Q^2}\left[f_\omega^2 - f^2\left(m_v^2 - \frac{1}{2}\right)\right] + \frac{\Lambda^4}{Q^4}\left[-f_\omega^2 m_\omega^2 + \frac{1}{2}f^2\left(m_v^4 - m_v^2 + \frac{1}{6}\right)\right]$$

$$+ \frac{\Lambda^6}{Q^6}\left[f_\omega^2 m_\omega^4 - \frac{1}{3}f^2 m_v^2\left(m_v^2 - \frac{1}{2}\right)\left(m_v^2 - 1\right)\right] + \cdots$$

The axial-vector case is similar:

$$\Pi_A(Q^2) = \frac{f_\pi^2}{Q^2} + \sum_{n=0}^{\infty} \frac{F^2}{Q^2 + \Lambda^2 n + M_A^2}$$

The final set of sum rules

6 equations for 6 variables

$\Lambda^2,\ m_v^2,\ m_\omega^2,\ m_a^2,\ f^2$, and f_ω^2

$$f^2 = \frac{1}{8\pi^2},$$

$$f^2\left(m_v^2 - \frac{1}{2}\right) = f_\omega^2,$$

$$\Lambda^2 f^2\left(m_a^2 - \frac{1}{2}\right) = f_\pi^2,$$

$$\Lambda^4\left[-f_\omega^2 m_\omega^2 + \frac{1}{2} f^2\left(m_v^4 - m_v^2 + \frac{1}{6}\right)\right] = \frac{\alpha_s}{24\pi}\langle G^2\rangle,$$

$$\Lambda^4 f^2\left(m_a^4 - m_a^2 + \frac{1}{6}\right) = \frac{\alpha_s}{12\pi}\langle G^2\rangle,$$

$$f_\omega^2 m_\omega^4 - \frac{1}{3} f^2 m_v^2\left(m_v^2 - \frac{1}{2}\right)(m_v^2 - 1) = \frac{7}{33} f^2 m_a^2\left(m_a^2 - \frac{1}{2}\right)(m_a^2 - 1)$$

The numerical solution

$$\frac{\alpha_s}{\pi}\langle G^2\rangle = (360 \pm 20\,\mathrm{MeV})^4$$

	$f_\pi = 93$ MeV	$f_\pi = 87$ MeV
Λ	1.43 (2)	1.32 (2)
M_V	1.60 (4)	1.45 (4)
M_A	1.31 (1)	1.21 (1)
M_ω	0.79 (3)	0.69 (3)
F	0.16	0.15
F_ω	0.14	0.13
$(-\langle\bar{q}q\rangle)^{\frac{1}{3}}$	0.30 (1)	0.27 (1)

n	0	1	2
$f_\pi = 93$ MeV			
$M_V(n)$	0.79	1.60	2.15
$M_A(n)$	1.31	1.93	2.41
$f_\pi = 87$ MeV			
$M_V(n)$	0.69	1.45	1.96
$M_A(n)$	1.21	1.79	2.22

Scalar mesons

$$\Pi_S(q^2) = \langle J^S(q) J^S(-q) \rangle = \sum_n \frac{G_n^2 M_S^2(n)}{q^2 - M_S^2(n)}$$

$$J^S = \bar{q}q$$

$$\langle 0 | J^S | n \rangle = G_n M_S(n)$$

$$M_S^2(n) = \Lambda^2 n + M_S^2, \quad n = 0, 1, 2, \ldots$$

Consider

$$\Pi_S^{(\mathrm{II})}(Q^2) = \frac{G_\sigma^2 M_\sigma^2}{Q^2 + M_\sigma^2} + \sum_{n=1}^{\infty} \frac{G^2(\Lambda^2 n + M_S^2)}{Q^2 + \Lambda^2 n + M_S^2}$$

One obtains for the lowest scalar

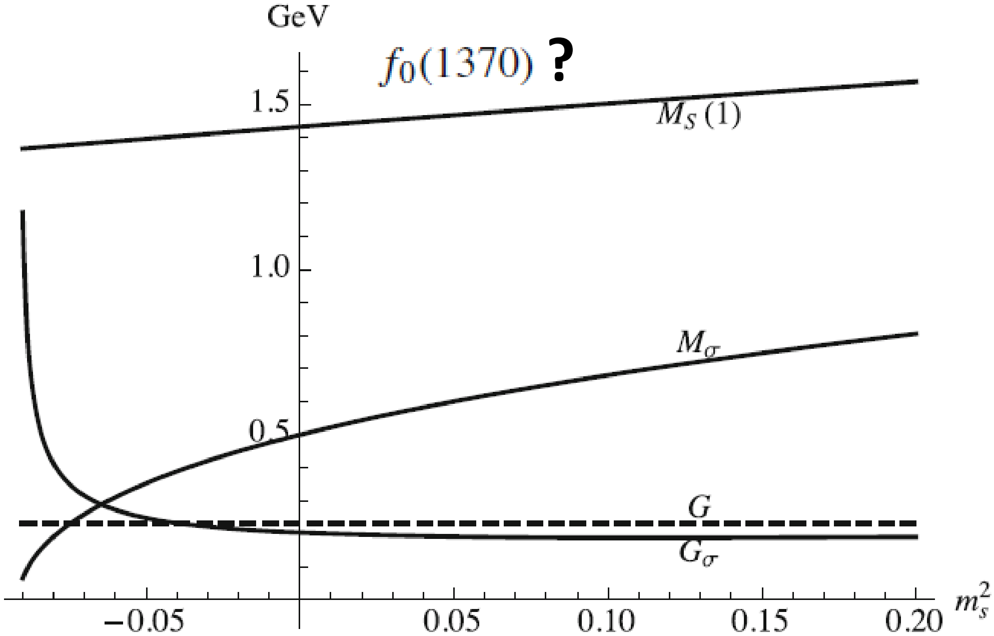
$$M_\sigma^2 = \frac{\frac{C_0}{16\pi^2}\Lambda^6 m_s^2 \left(m_s^2 + \frac{1}{2}\right)\left(m_s^2 + 1\right) + \frac{11}{3}\pi\alpha_s \langle \bar{q}q \rangle^2}{\frac{3C_0}{32\pi^2}\Lambda^4 \left(m_s^4 + m_s^2 + \frac{1}{6}\right) + \frac{\alpha_s}{16\pi}\langle G^2 \rangle}$$

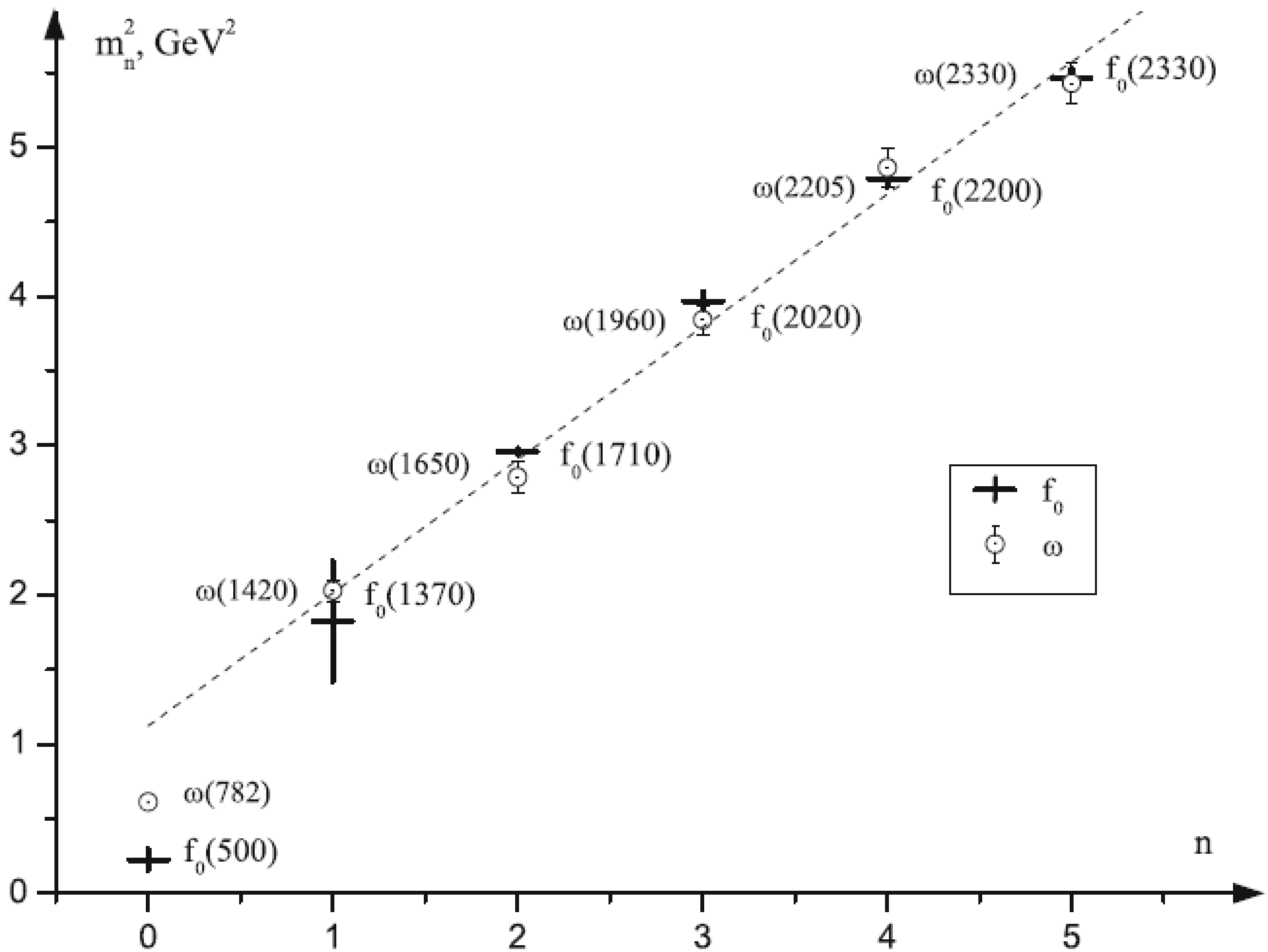
$$m_s = \frac{M_S}{\Lambda}$$

A natural interval

$$M_\sigma = 500 \pm 100\,\mathrm{MeV}$$

$f_0(500)$ **?**





A presumable spectrum of non-strange ω (*circles*) and f_0 (*crosses*) mesons

Borelized sum rules in the large-N_c limit

(extension of **Shifman-Vainstein-Zaharov** sum rules)

Consider the vector correlator:

$$\Pi_{\mu\nu} = i \int d^4x\, e^{ipx} \langle 0 | \mathrm{T}\{j_\mu(x), j_\nu(0)\} | 0 \rangle \qquad j_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$$

$$\Pi_{\mu\nu} = (q_\mu q_\nu - g_{\mu\nu} q^2)\Pi(Q^2) \qquad q^2 = -Q^2$$

OPE:

$$\Pi(Q^2) = \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right)\ln\frac{\mu^2}{Q^2} + \frac{\langle m_q \bar{q} q\rangle}{Q^4} + \frac{1}{24Q^4}\left\langle \frac{\alpha_s}{\pi}\left(G^a_{\mu\nu}\right)^2\right\rangle - \frac{14}{9}\frac{\pi\alpha_s}{Q^6}\langle\bar{q}q\rangle^2$$

The Borel transform:

$$L_M \Pi(Q^2) = \lim_{\substack{Q^2, n\to\infty \\ Q^2/n = M^2}} \frac{1}{(n-1)!}(Q^2)^n \left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)$$

$$L_M \Pi(Q^2) = \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right) + \frac{\langle m_q \bar{q} q\rangle}{M^4} + \frac{1}{24M^4}\left\langle \frac{\alpha_s}{\pi}\left(G^a_{\mu\nu}\right)^2\right\rangle - \frac{7}{9}\frac{\pi\alpha_s}{M^6}\langle\bar{q}q\rangle^2 \qquad \textbf{(1)}$$

$$\Pi(q^2) = \frac{1}{\pi}\int_{4m_q^2}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{s - q^2 + i\varepsilon} + \Pi(0) \qquad \Pi(q^2) = \sum_n \frac{F_n^2}{q^2 - m_n^2 + i\varepsilon} \qquad \mathrm{Im}\,\Pi(q^2) = \sum_n \pi F^2 \delta(q^2 - m_n^2)$$

$$\Rightarrow \quad L_M \Pi(Q^2) = \frac{1}{\pi M^2}\int_0^{\infty} e^{-s/M^2}\,\mathrm{Im}\,\Pi(s) ds = \frac{F^2}{M^2}\sum_n e^{-m_n^2/M^2}$$

For linear spectrum $$m_n^2 = an + m_0^2, \qquad n = 0, 1, 2, \ldots$$

$$L_M \Pi(Q^2) = \frac{F^2}{M^2} \frac{e^{-m_0^2/M^2}}{1 - e^{-a/M^2}} \qquad \textbf{(2)}$$

(2)=(1):

$$\frac{F^2 e^{-m_0^2/M^2}}{1 - e^{-a/M^2}} = \frac{M^2}{8\pi^2} \left[1 + \frac{\alpha_s}{\pi} + \frac{8\pi^2}{M^4} \langle m_q \bar{q} q \rangle + \frac{\pi^2}{3M^4} \left\langle \frac{\alpha_s}{\pi} \left(G^a_{\mu\nu} \right)^2 \right\rangle - \frac{56}{9} \frac{\pi^3 \alpha_s}{M^6} \langle \bar{q} q \rangle^2 \right] \qquad \textbf{(3)}$$

Consider $$-\frac{d(3)}{d(1/M^2)} \Big/ (3) \quad \Rightarrow \quad m_0^2 = M^2 \frac{h_0 - \frac{h_2}{M^4} - \frac{2h_3}{M^6}}{h_0 + \frac{h_1}{M^2} + \frac{h_2}{M^4} + \frac{h_3}{M^6}} - \frac{a}{e^{a/M^2} - 1}$$

correction to SVZ formula

In the scalar sector: $$m_0^2 = \frac{a}{1 - e^{a/M^2}} + \frac{\mathbf{L}}{2} \pm \frac{\sqrt{\mathbf{L}^2 \left(e^{a/M^2} - 1 \right)^2 / 4 - a^2 e^{a/M^2}}}{e^{a/M^2} - 1}$$

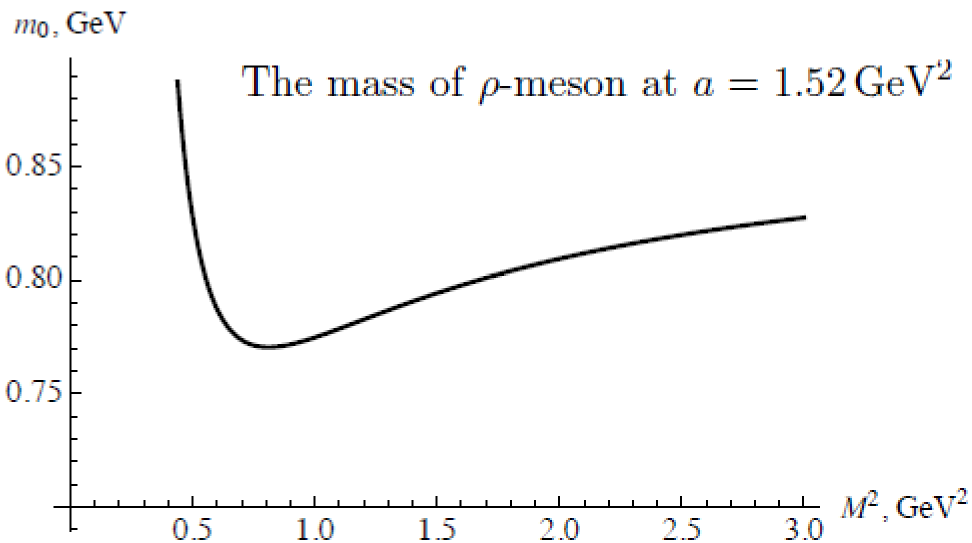
2 solutions!

$$\mathbf{L} \equiv M^2 \frac{2h_0 + \frac{h_3}{M^6}}{h_0 + \frac{h_2}{M^4} - \frac{h_3}{M^6}}$$

Table 1: The condensate contributions h_i in different meson channels.

Mesons	h_0	h_1	h_2	h_3
ρ	$1 + \frac{\alpha_s}{\pi}$	0	$8\pi^2 \langle m_q \bar{q} q \rangle + \frac{\pi^2}{3} \left\langle \frac{\alpha_s}{\pi} \left(G^a_{\mu\nu} \right)^2 \right\rangle$	$-\frac{56}{9} \pi^3 \alpha_s \langle \bar{q} q \rangle^2$
a_1	$1 + \frac{\alpha_s}{\pi}$	$-8\pi^2 f_\pi^2$	$-8\pi^2 \langle m_q \bar{q} q \rangle + \frac{\pi^2}{3} \left\langle \frac{\alpha_s}{\pi} \left(G^a_{\mu\nu} \right)^2 \right\rangle$	$\frac{88}{9} \pi^3 \alpha_s \langle \bar{q} q \rangle^2$
f_0	$1 + \frac{11}{3} \frac{\alpha_s}{\pi}$	0	$8\pi^2 \langle m_q \bar{q} q \rangle + \frac{\pi^2}{3} \left\langle \frac{\alpha_s}{\pi} \left(G^a_{\mu\nu} \right)^2 \right\rangle$	$\frac{176}{9} \pi^3 \alpha_s \langle \bar{q} q \rangle^2$

Perturbative approach: step 1 – calculate the ground state mass from the classical SVZ sum rules; step 2 – find the slope *a* using this mass



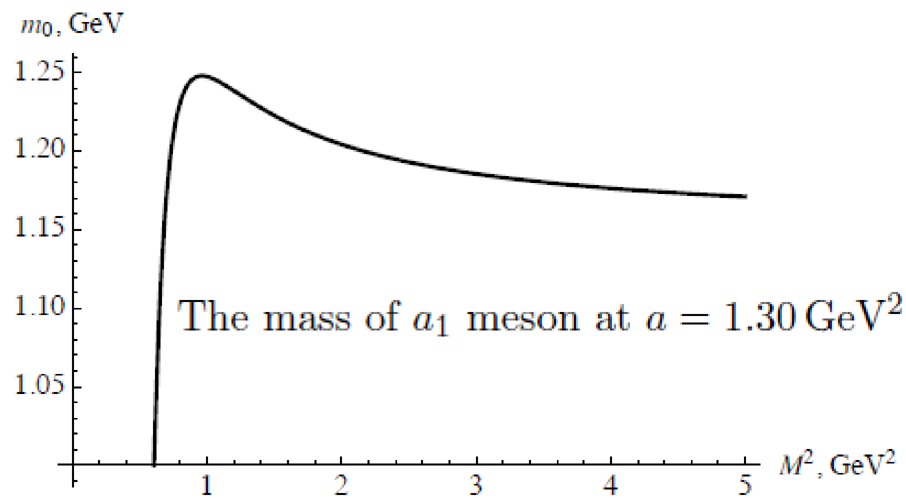


Table 3: The radial spectrum of ρ-mesons for the slope $a = 1.52 \pm 0.07\,\mathrm{GeV}^2$. The masses are given in MeV. The first 4 predicted states are tentatively assigned to the resonances $\rho(770)$, $\rho(1450)$, $\rho(1900)$, and $\rho(2270)$ [11] which presumably form the *S*-wave radial trajectory.

n	0	1	2	3	4
m_ρ (th)	770 ± 10	1450 ± 20	1910 ± 40	2230 ± 50	2580 ± 50
m_ρ (exp)	775	1465 ± 25	1870–1920	2265 ± 40	—

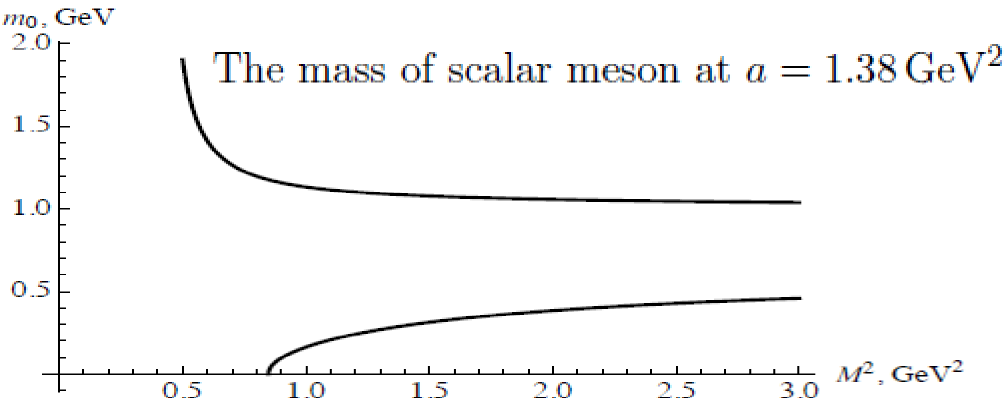
Table 4: The radial spectrum of a_1-mesons for the slope $a = 1.30 \pm 0.18\,\mathrm{GeV}^2$. The first 4 predicted states are tentatively assigned to the resonances $a_1(1230)$, $a_1(1640)$, $a_1(1930)$, and $a_1(2270)$ [11].

n	0	1	2	3	4
m_{a_1} (th)	1150 ± 40 1250	1620 ± 60 1690 ± 50	1980 ± 90 2040 ± 90	2280 ± 120 2330 ± 120	2550 ± 140 2600 ± 140
m_{a_1} (exp)	1230 ± 40	1647 ± 22	1930^{+30}_{-70}	2270^{+55}_{-40}	—

The scalar sector

The standard SVZ method gives $m_{f_0} = 1.00 \pm 0.03$ GeV

$\Rightarrow$ $a_{f_0} = 1.38 \pm 0.07$ GeV^2 $\Rightarrow$ $m_{f_0} \approx 0.62$ GeV (the second solution)



$M^2 \to \infty$

$$m_0^2 = \frac{a}{2} \pm \frac{1}{2}\sqrt{\frac{a^2}{3} - 8h_2}$$

Table 5: The radial spectrum of the first f_0-trajectory for the slope $a = 1.38 \pm 0.07\,\text{GeV}^2$. The first 5 predicted states are tentatively assigned to the resonances $f_0(980)$, $f_0(1500)$, $f_0(2020)$, $f_0(2200)$, and $X(2540)$ [11].

n	0	1	2	3	4
m_{f_0} (th 1)	1000 ± 30	1540 ± 20	1940 ± 40	2270 ± 50	2560 ± 50
m_{f_0} (exp 1)	990 ± 20	1504 ± 6	1992 ± 16	2189 ± 13	$2539 \pm 14^{+38}_{-14}$

Table 6: The radial spectrum of the second f_0-trajectory for the slope $a = 1.38 \pm 0.07\,\text{GeV}^2$. The first 5 predicted states are tentatively assigned to the resonances $f_0(500)$, $f_0(1370)$, $f_0(1710)$, $f_0(2100)$, and $f_0(2330)$ [11].

n	0	1	2	3	4
m_{f_0} (th 2)	620	1330 ± 30	1780 ± 40	2130 ± 50	2430 ± 60
m_{f_0} (exp 2)	400–550	1200–1500	1723^{+6}_{-5}	2101 ± 7	2300–2350

<u>Finally</u>

Our estimate for averaged slope of light meson radial trajectories

$$a = 1.4 \pm 0.1\,\text{GeV}^2$$

Phenomenology

$$a = 1.25 \pm 0.15\,\text{GeV}^2$$

CONCLUSION

$$M_\sigma = 500 \pm 100\,\mathrm{MeV}$$

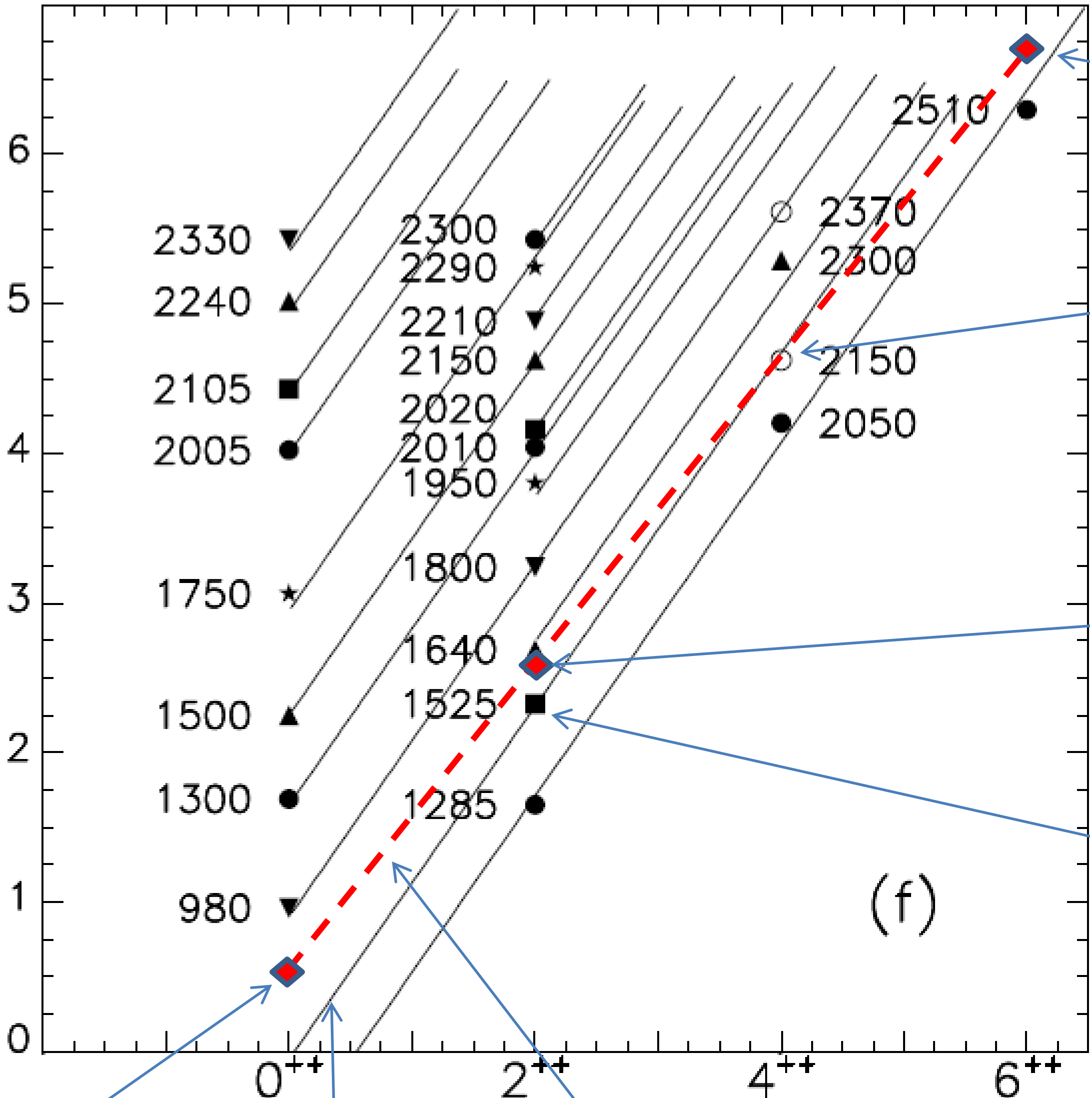
may be obtained naturally in QCD sum rules in the large-N_c limit

Answers to some questions

Angular Regge trajectory with the sigma-meson on the top? (f_2, f_4, f_6, … companions of $f_0(500)$?)

Systematics of $q\bar{q}$-states in the (n, M^2) and (J, M^2) planes

A.V. Anisovich, V.V. Anisovich and A.V. Sarantsev, PRD (2000)



prediction
$m_{f_6} \approx 2.60 \div 2.71$ GeV

prediction
$m_{f_4} \approx 2.13 \div 2.23$ GeV
$f_J(2220)$ of PDG? (J=2,4)

$f_2(1565)$
a natural candidate from PDG!

$f_2'(1525)$
but it is produced in reactions with K-mesons!

$f_0(500)$

this trajectory!
$1.1 \div 1.2$ GeV^2

corrected trajectory

Who also arrived at a similar conclusion for sigma-meson?

Similar result from the sum rules in the large-N_c limit:

E.R. Arriola, W. Broniowski, Phys. Rev. D **81**, 054009 (2010)

$$m_\sigma \approx 450\text{–}600\ \text{MeV}$$

But! It was obtained for the 2-point correlator of energy-momentum tensor

Mixing with other states?

A popular phenomenological tool – K-matrix approach.

It could be reasonable to compare our predictions with K-matrix "bare states".

Take for instance

A.V. Anisovich, V.V. Anisovich, A.V. Sarantsev, Phys. Rev. D **62**, 051502(R) (2000)

$f_0(1370) \quad \Rightarrow \quad m_{f_0(\text{bare})} = 1240 \pm 50 \text{ MeV}$

$\Lambda^2 \approx 1.38 \text{ GeV}^2$ - the slope of "bare" radial trajectory

$$\Rightarrow \quad m_\sigma \approx \sqrt{m^2_{f_0(\text{bare})} - \Lambda^2} \approx 400 \pm 100 \text{ MeV}$$

Why overlooked in the QCD sum rules, including the large-N_c ones?

S.S. Afonin, Int. J. Mod. Phys. A **31**, 1650164 (2016)

Lower and upper bounds on the mass of light quark–antiquark scalar resonance in the SVZ sum rules

The calculation of the mass of light scalar isosinglet meson within the Shifman–Vainshtein–Zakharov (SVZ) sum rules is revisited. We develop simple analytical methods for estimation of hadron masses in the SVZ approach and try to reveal the origin of their numerical values. The calculations of hadron parameters in the SVZ sum rules are known to be heavily based on a choice of the perturbative threshold. This choice requires some important *ad hoc* information. We show analytically that the scalar mass under consideration has a lower and upper bound which are independent of this choice: $0.78 \lesssim m_s \lesssim 1.28$ GeV.

In the large-N_c sum rules:

S.S. Afonin, A.A. Andrianov, V.A. Andrianov, D. Espriu, JHEP **0404**, 039 (2004)

S.S. Afonin, D. Espriu, JHEP **0609**, 047 (2006)

A consequence of ad hoc ansatz.

STOP

SPECULATIONS!

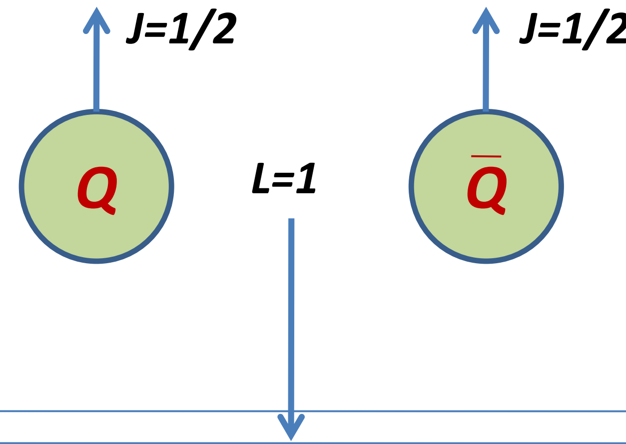
SURE?

Nature of sigma-meson?

<u>A possible scenario:</u>

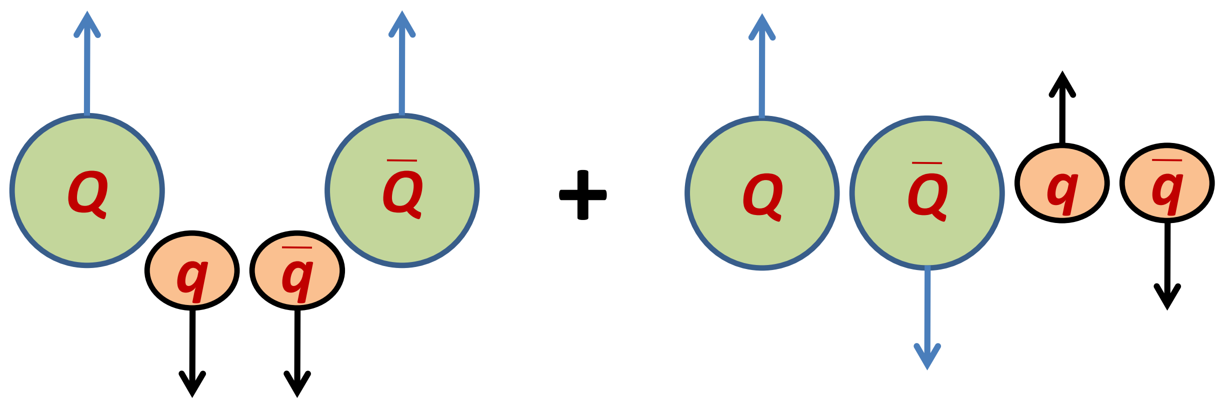
The scalar meson in the quark model:

P-wave state



In reality: seems to be a

<u>Tetraquark</u>



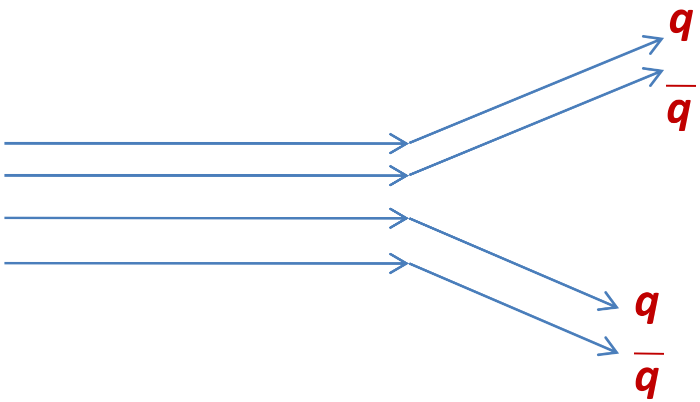
But! Containing 2 constituent and 2 current quarks!

S-wave state!

A hint from the NJL model: $m_\sigma^2 = 4M^2 + m_\pi^2$

Consequences:

- Decay to pions is OZI superallowed → large width



-The Coulomb part of the confinement potential is not small
→ the ground state mass is decreased (similarly to rho-meson)

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r,$$