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# Phase transitions of QCD and QCD-like theories from Dyson-Schwinger equations

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UNI GRAZ
FWF Der Wissenschaftsfonds.
NAWI Graz Natural Sciences

eQCD 2018 Kopaonik: 12.03.2018

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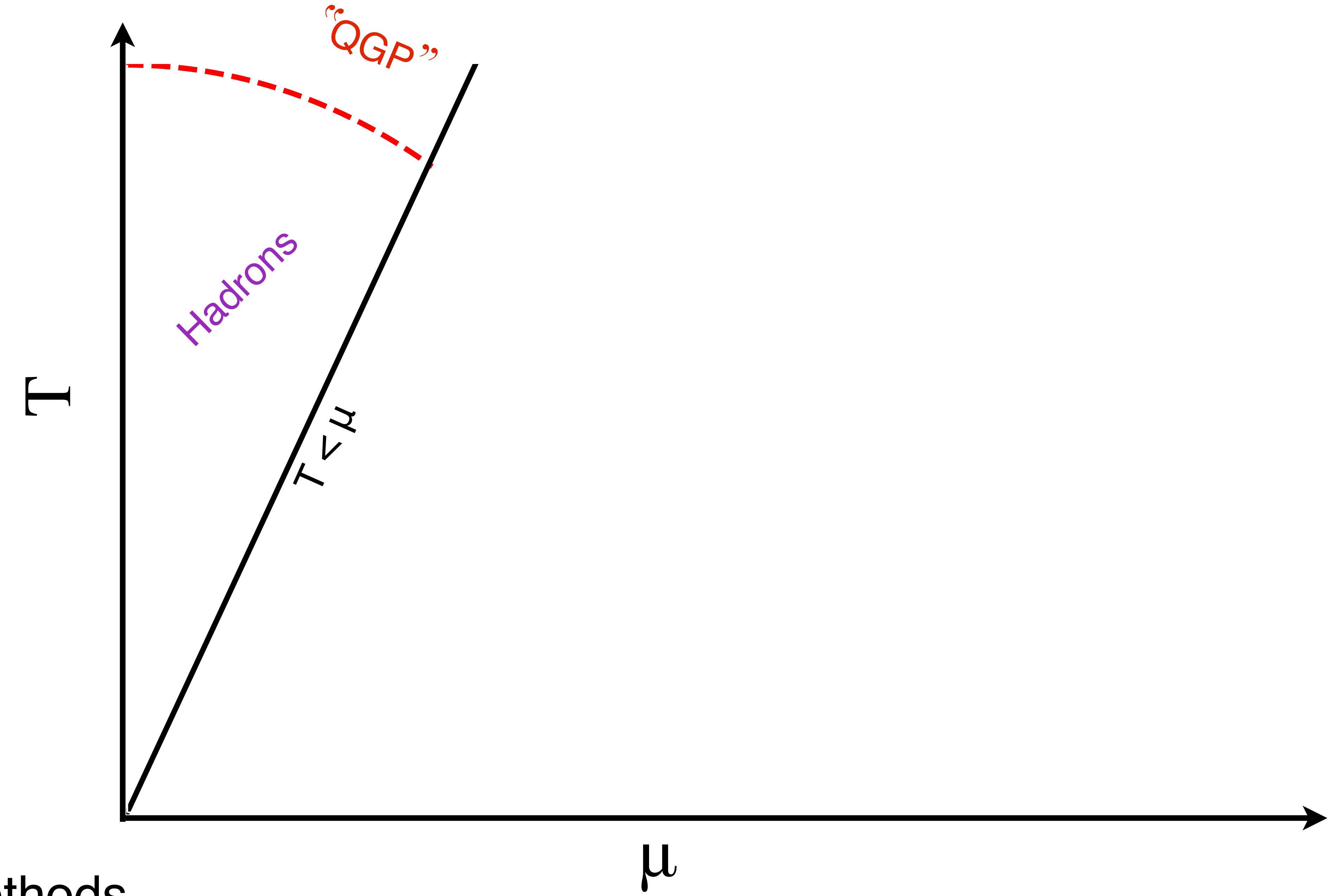
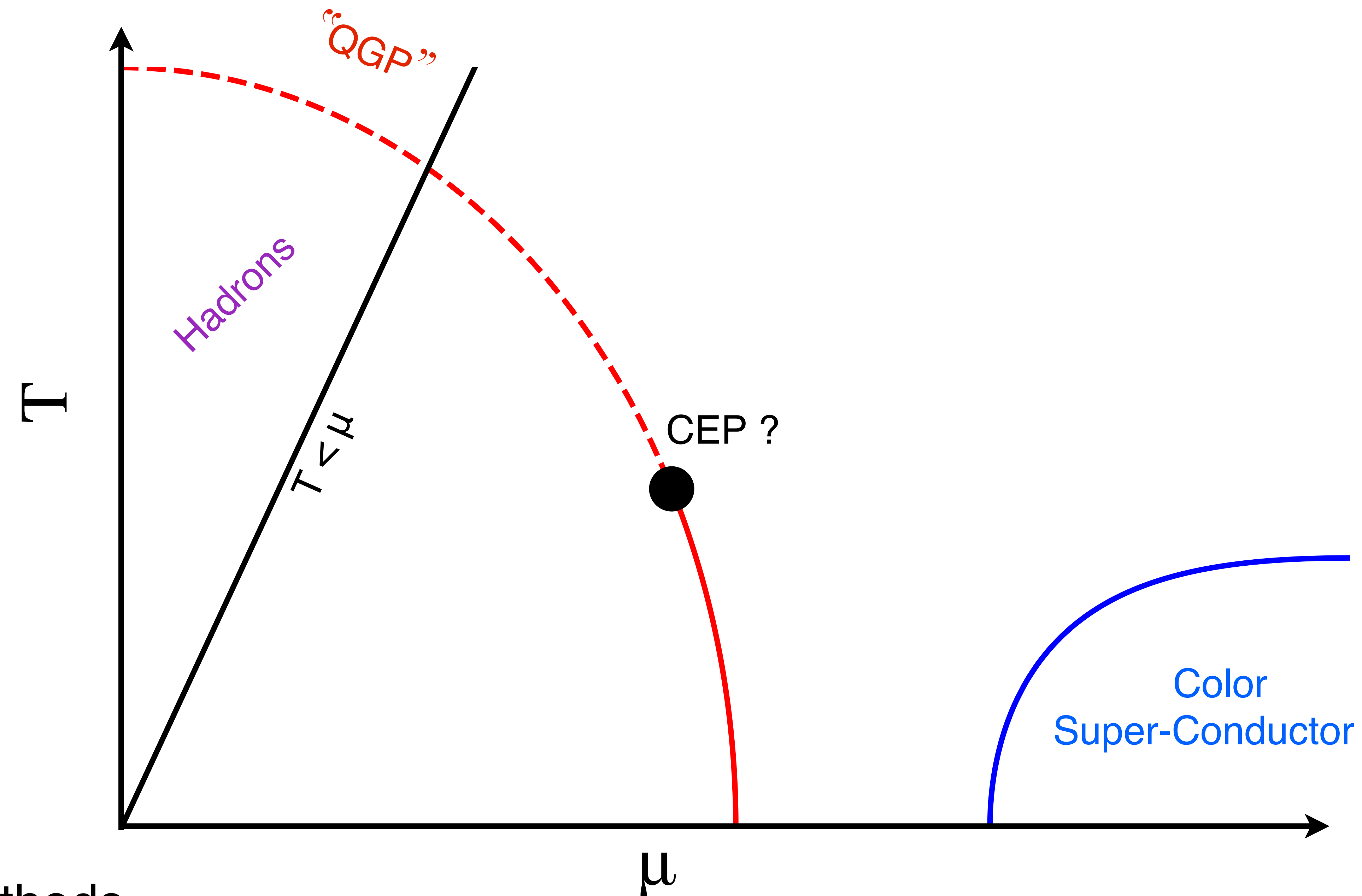
- The phase diagram of QCD




- Methods
  - Lattice QCD
  - → Sign problem

- The phase diagram of QCD




- Methods

  - Lattice QCD → Sign problem
  - Effective Models → Fixing parameters
  - Functional methods → Truncation and modeling

QCD

Lattice QCD ⟷ sign problem

Functional Methods ⟷ Truncation

## QCD-like

- A theory with dynamical mass generation
- Confinement and asymptotic freedom
- A positive fermion determinant

Minimal modification of QCD

Lattice QCD ⟷ ~~sign problem~~

Functional Methods ⟷ Truncation

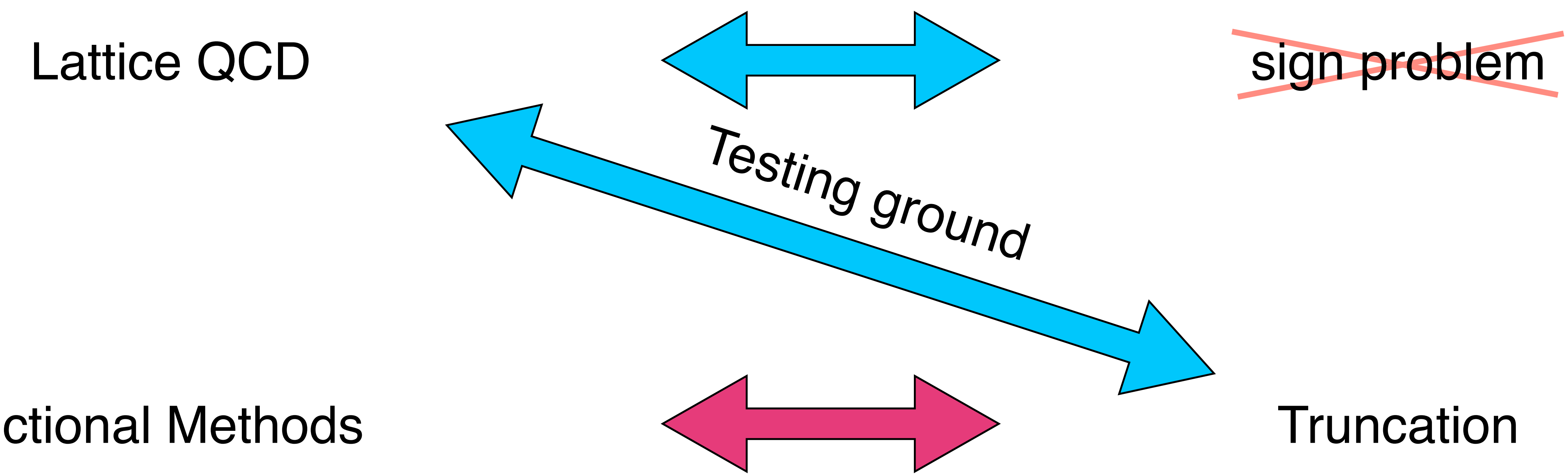
# Motivation for QCD-like theories



## QCD-like

- A theory with dynamical mass generation
- Confinement and asymptotic freedom
- A positive fermion determinant

Minimal modification of QCD




Apply the truncation to physical QCD

# 1 Introduction
## Motivation for QCD-like theories

- SU(2)

→ SU(2) for even number of degenerated quark flavors possesses a positive quark determinant

- $G_2$
  - Subgroup of SO(7) which satisfies an additional cubic constraint

→ All representations are real, allow lattice simulation at $\mu > 0$

# Motivation for QCD-like theories



- SU(3), SU(2) and $G_2$ have many properties in common

→ Asymptotically free, chiral symmetry breaking, confinement

→ Chiral and deconfinement transition coincide in the quenched case

- Similar functional equations

→ Different Casimir operators of the gauge groups

# Motivation for QCD-like theories



- SU(3), SU(2) and $G_2$ have many properties in common

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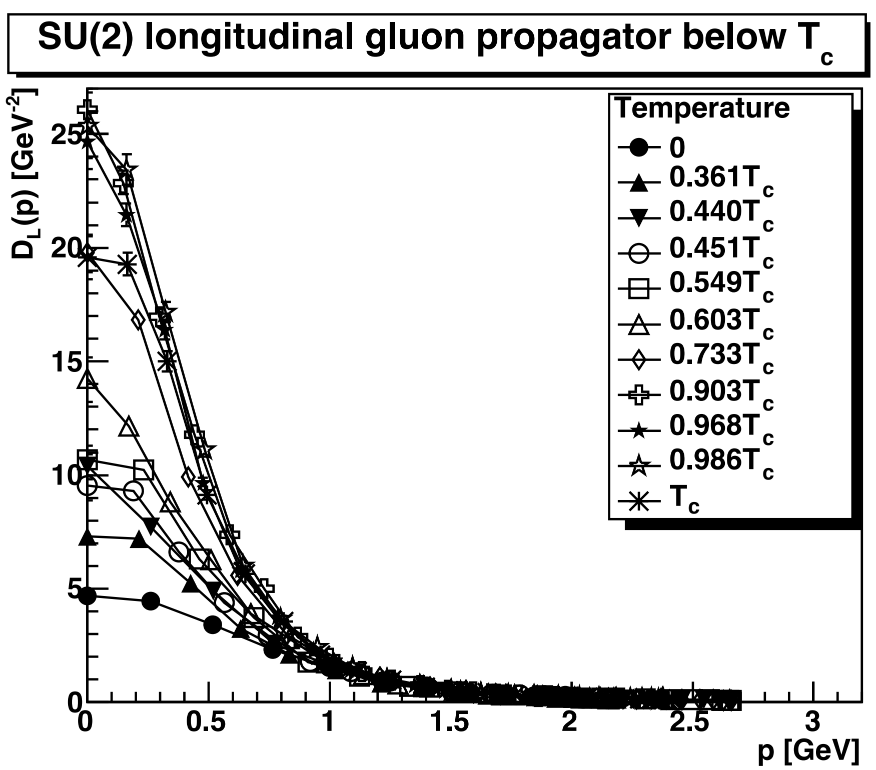
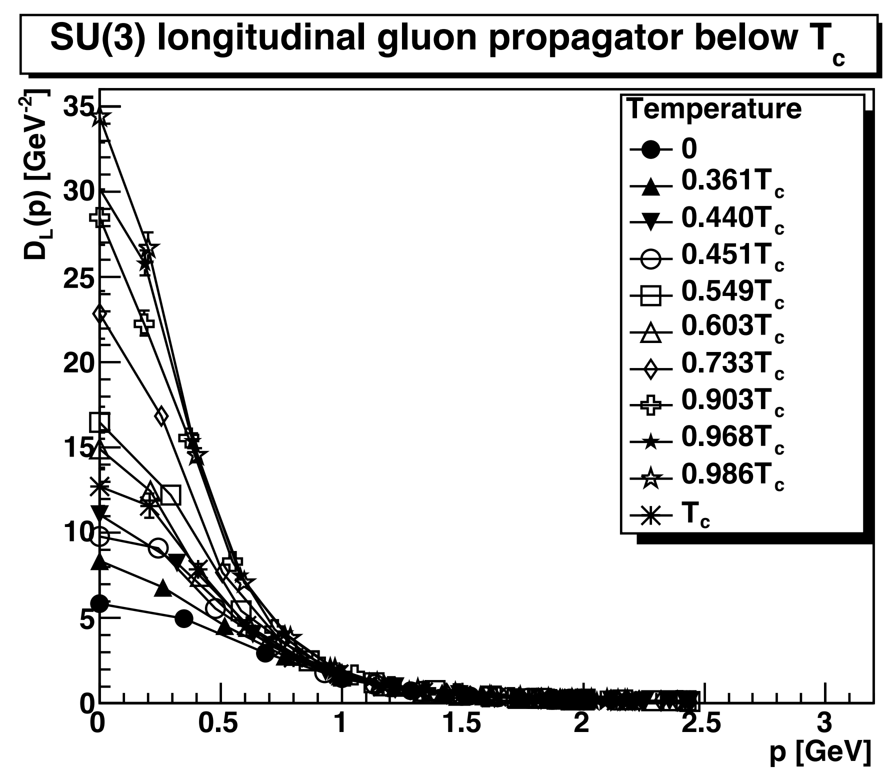
- Similar functional equations

  → Different Casimir operators of the gauge groups

- Similarities of the correlation function : SU(2) vs SU(3)




*[C.S. Fischer, A. Maas, J.A. Müller (2010)]*

# Synopsis

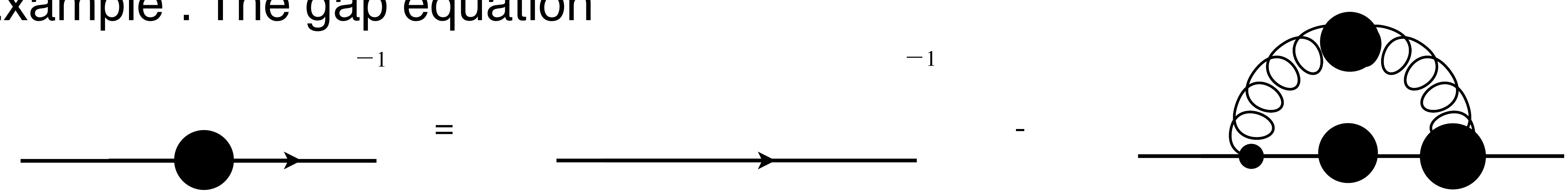
# Dyson-Schwinger Equation



$$Z = \int d\,[\Phi]\, \mathrm{Exp}\Big[-\int S[\Phi] + \Phi\, J\Big] \longrightarrow W[\Phi] = \mathrm{Log}[Z] \longrightarrow \Gamma[J]$$

Z : Partition function → *W :* Connected Diagrams → *Γ :* irreducible Diagrams

- Example : The gap equation




→ Describe all possible ways of propagation of a quark

# 2 Setup

## Dyson-Schwinger Equation

$$Z \quad = \int d[\Phi]\,\mathrm{Exp}\left[-\int S[\Phi] + \Phi J\right] \longrightarrow W[\Phi] = \mathrm{Log}[Z] \longrightarrow \Gamma[J]$$

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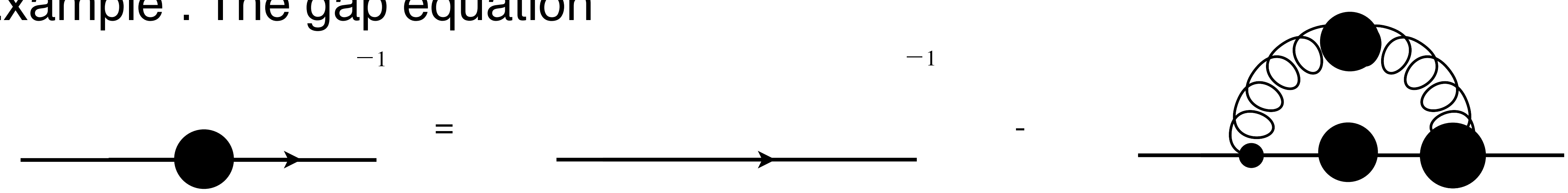


→ Describe all possible ways of propagation of a quark

- Contains perturbation theory
- Contains non-perturbative information
  - Confinement and dynamical chiral symmetry breaking
  - Bound state studies
  - ...

→ Multi-scale problems feasible

→ No sign problem

# Dyson-Schwinger Equation

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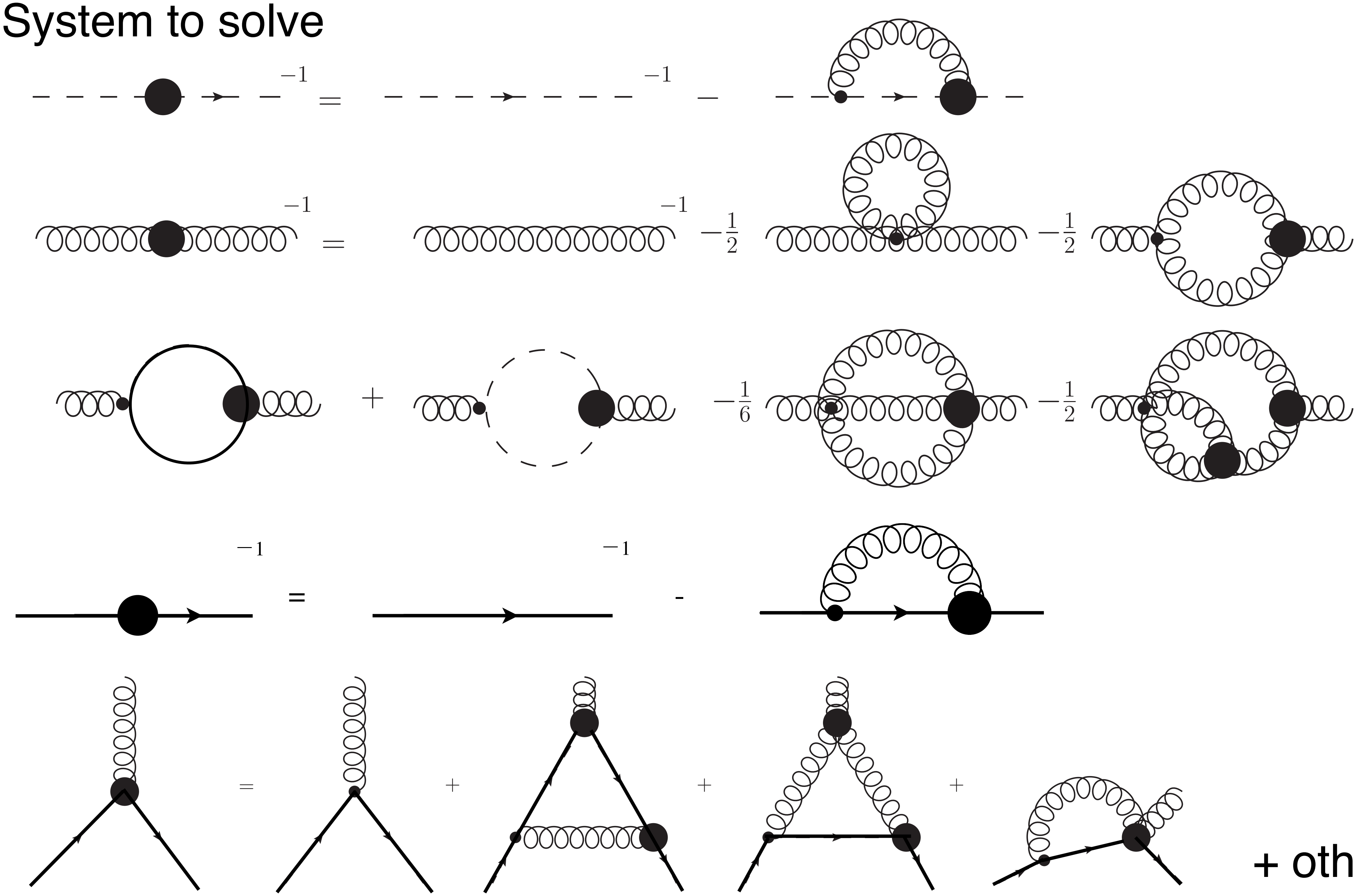
Z : Partition function → *W :* Connected Diagrams → *Γ :* irreducible Diagrams

- System to solve

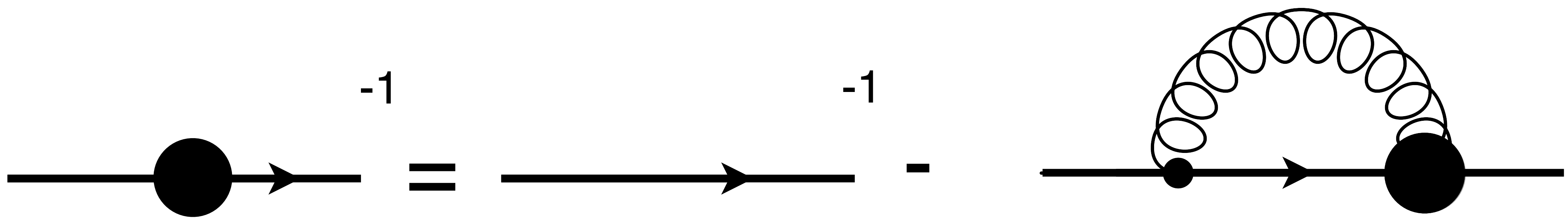


+ other equations

- System to solve
  - Truncations are mandatory

→ We want a realistic temperature dependence of the gluon

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \cancel{\omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)}$$

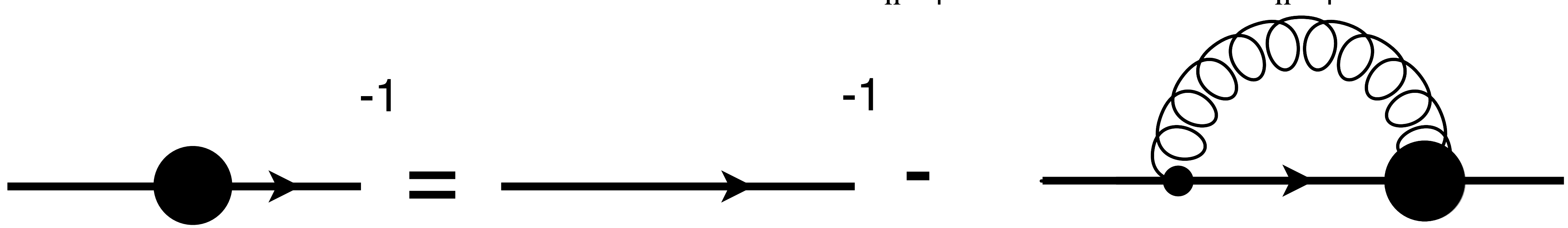


# 2 Setup
## The truncation and modeling

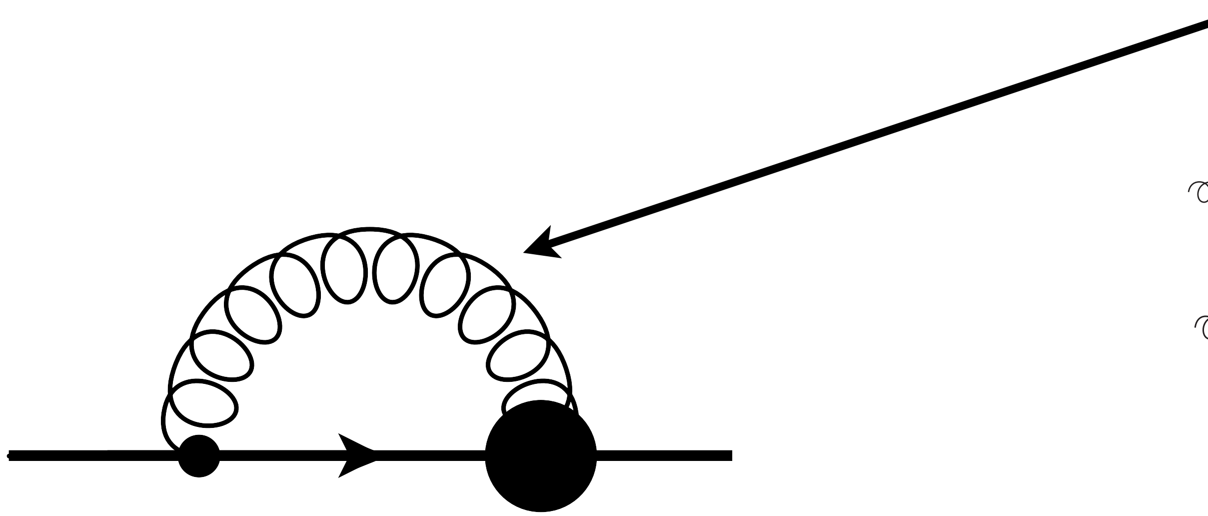
- System to solve
  - Truncations are mandatory

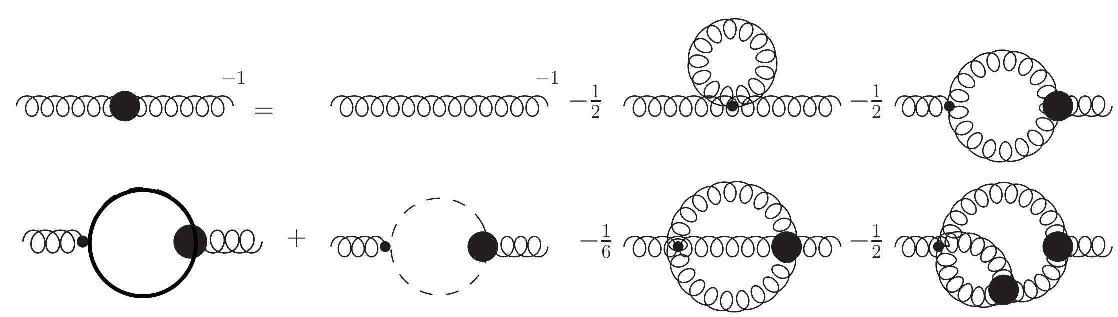
→ We want a realistic temperature dependence of the gluon

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \cancel{\omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)}$$




$$D_{\mu\nu}(p) = \frac{1}{p^2}\left(Z_T(p)P^T_{\mu\nu} + Z_L(p)P^L_{\mu\nu}\right)$$






→ Spuriously divergent terms

→ 2-loops diagrams

→ Accessible on lattice

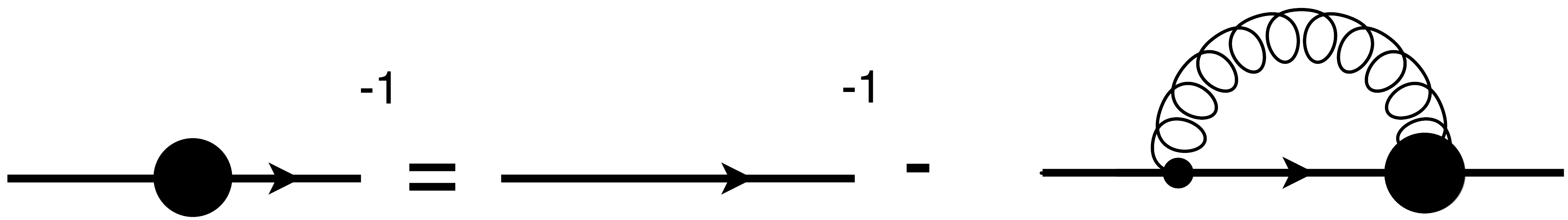
# The truncation and modeling



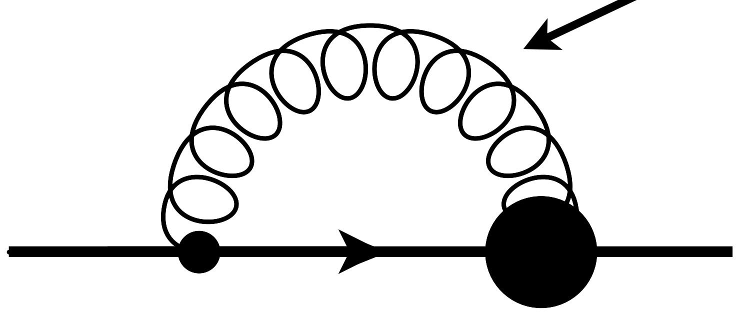
- System to solve
  - Truncations are mandatory

→ We want a realistic temperature dependence of the gluon

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \cancel{\omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)}$$




$$Z_{T,L}(x) = \frac{x}{(x+1)^2}\left(\left(\frac{c}{x+a_{T,L}(T)}\right)^{b_{T,L}(T)} + x\left(\frac{\beta_0\alpha(\mu)}{4\pi}\ln(x+1)\right)^{\gamma}\right)$$



Coefficients are fitted to reproduce lattice data

*[A. Maas, J.M Pawlowski, L. von Smekal, D. Spielmann (2012)]*

*[C.S. Fischer, A. Maas, J.A. Müller (2010)]*

# 2 Setup
## The truncation and modeling

- System to solve
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→ We want a realistic temperature dependence of the gluon

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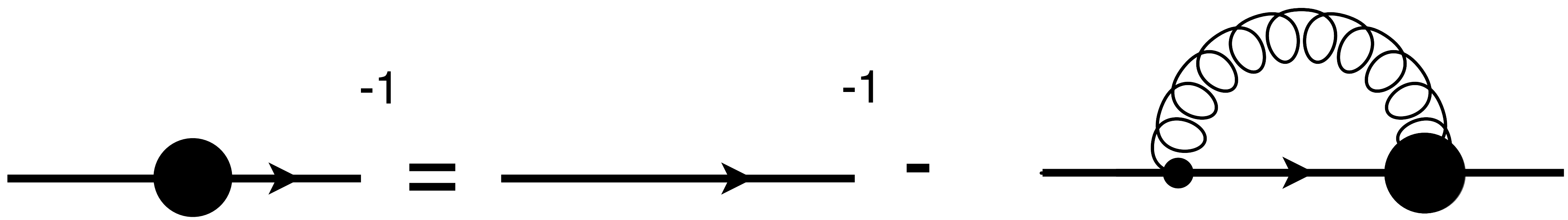
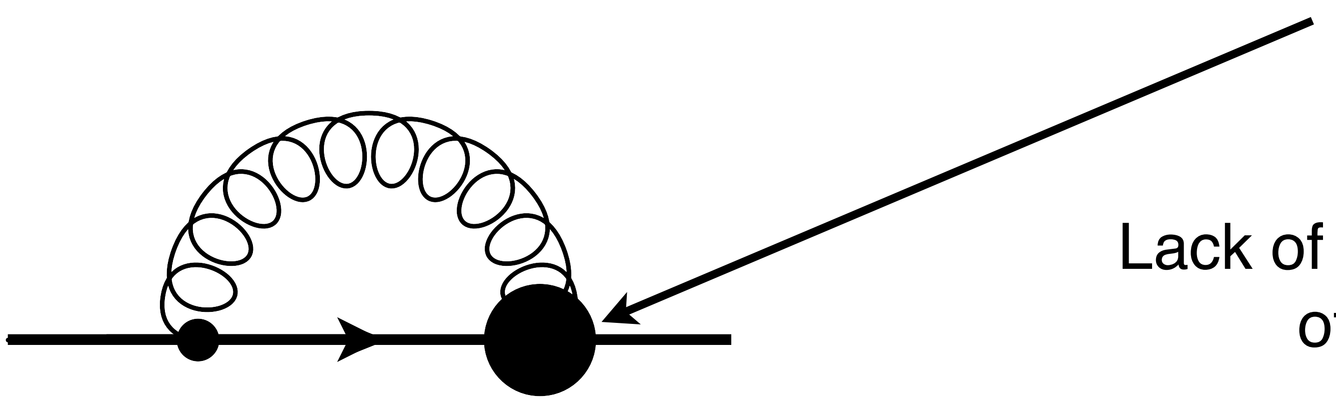



$\Gamma_{q\text{-}gl}(p,q,l)$

24 tensor parts

Difficult to obtain from lattice



Lack of information of the temperature dependence of this quantity from continuum studies

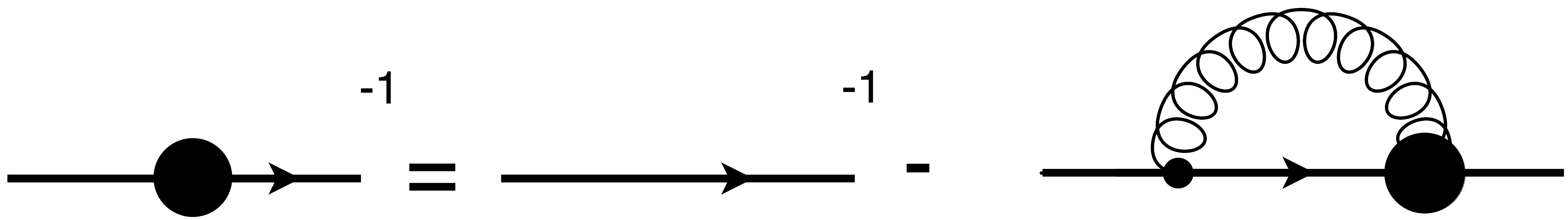
# The truncation and modeling

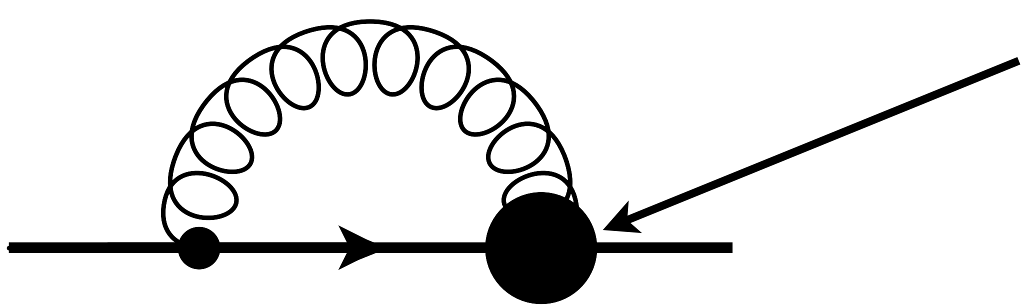


- System to solve
  - Truncations are mandatory

→ We want a realistic temperature dependence of the gluon

$$S^{-1}(\vec{p}, \omega_n) = A(\vec{p}, \omega_n)\, \vec{\gamma}\, \vec{p} + C(\vec{p}, \omega_n)\, \omega_n \gamma_4 + B(\vec{p}, \omega_n) + \cancel{\omega_n \gamma_4\, \vec{p}\, \vec{\gamma}\, D(\vec{p}, \omega_n)}$$



$$\Gamma_{q\text{-}gl}(p, q, l) = \left( \frac{A(p)+A(q)}{2} \vec{\gamma}, \frac{C(p)+C(q)}{2} \gamma_4 \right) W(p,q,l)$$

$$W(p,q,l) = \boxed{\frac{d_1}{d_2+l^2}} + \frac{l^2}{1+l^2} \left( \frac{\beta_0\, \alpha(\mu)}{4\pi} \ln(l^2+1) \right)^{2\delta}$$

*[C.S. Fischer (2009)]*

# 2 Setup
## The truncation and modeling

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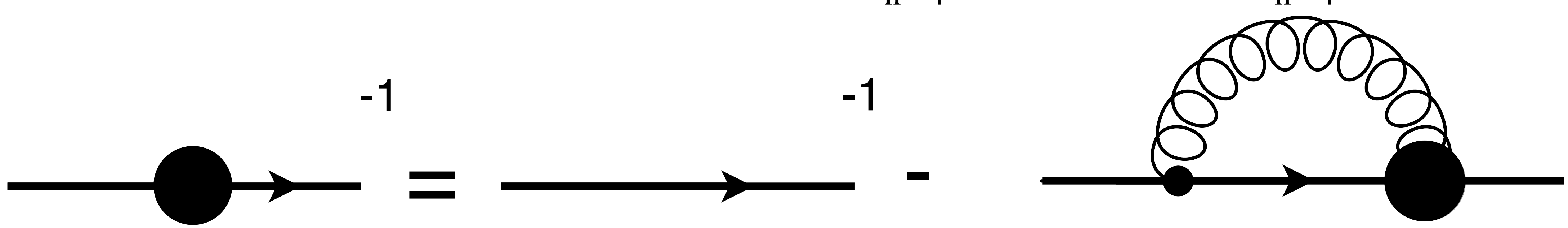
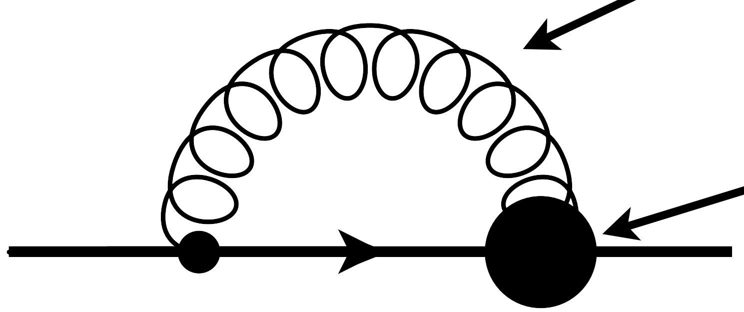



$$Z_{T,L}(p^2) = \frac{p^2}{(1+p^2)^2}\left(\left(\frac{c}{p^2+a_{T,L}(T)}\right)^{b_{T,L}(T)} + p^2\left(\frac{\beta_0\,\alpha(\mu)}{4\pi}\ln(p^2+1)\right)^{\gamma}\right)$$



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→ The system can be solved

# (Pseudo)-order parameter



- Chiral Symmetry Breaking

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \cancel{\omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)}$$

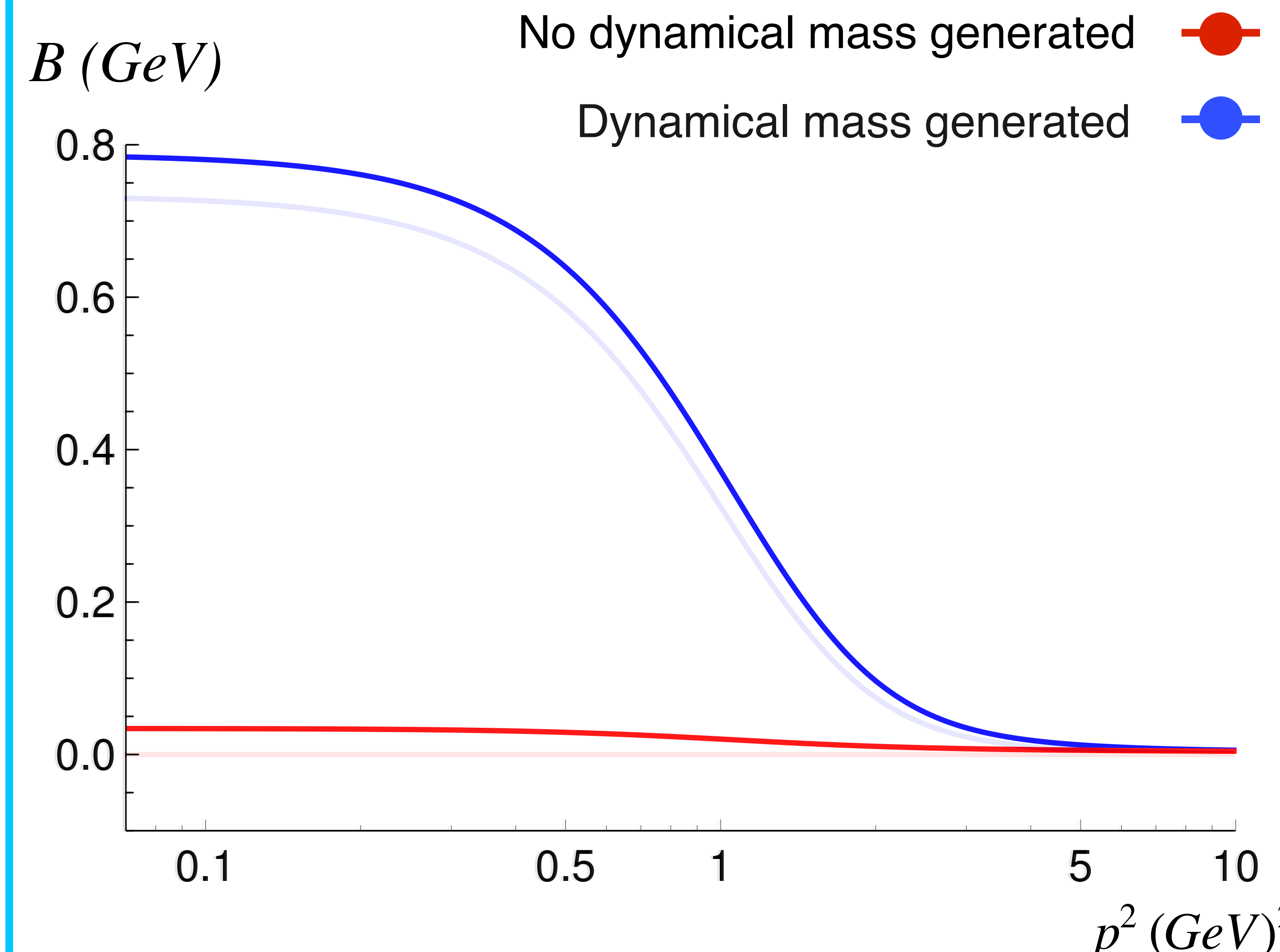
$$\Delta_\pi(T) = Z_2(Z_m)N_c\,T\sum_{\omega_n}\int_{\vec{p}}\frac{d^3\vec{p}}{(2\pi)^3}\mathrm{Tr}[S(\vec{p},\omega_n)]$$

- $m_{bare} > 0$

→ $B \neq 0$, formation of a chiral condensate

*Chiral symmetry broken*

Pseudo-order parameter



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- Dual quark condensate

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \cancel{\omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)}$$

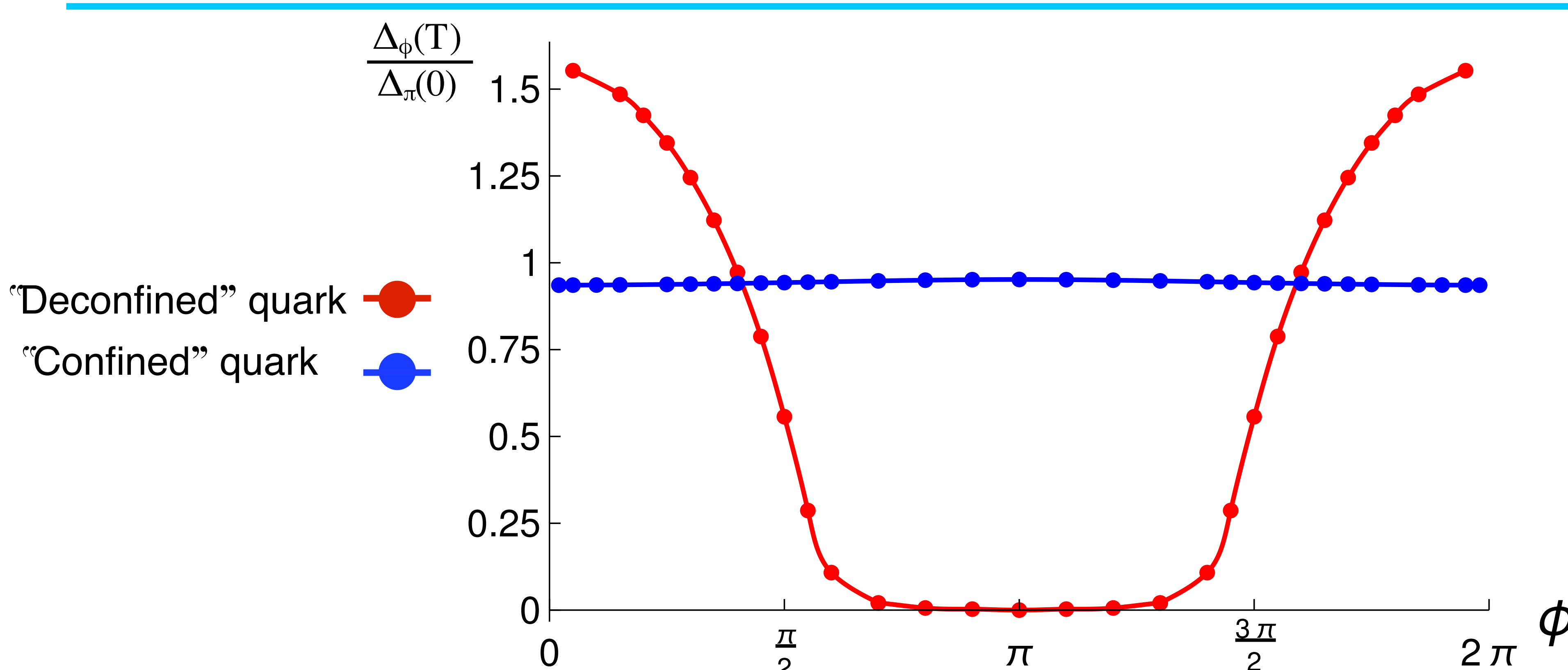
$$\Delta_\phi(T) = Z_2(Z_m)N_c\,T\sum_{\omega_n(\phi)}\int_{\vec{p}}\frac{d^3\vec{p}}{(2\pi)^3}\mathrm{Tr}[S(\vec{p},\omega_n(\phi))]$$

We introduce a phase dependence : $\omega_n = \pi\,T\left(2n + \frac{\phi}{\pi}\right)$

$$\boxed{\Sigma_1 = \int_0^{2\pi} e^{i\phi}\,d\phi\;\Delta_\phi(T)}$$

<u>The dual quark condensate is proportional to the Polyakov Loop</u>

*[E. Bilgici, F. Bruckmann, C. Gattringer, and C. Hagen (2008)]*
*[C.S. Fischer (2009)]*



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- Dual quark condensate in NJL model (non-confining theory) ‘mimics’ the chiral transition

*[T.K Mukherjee, H. Chen, and M. Huang (2010)]*
*[S. Benić (2013)]*

- The dual quark condensate for adjoint QCD is able to distinguish between chiral and deconfinement transition for light quark mass

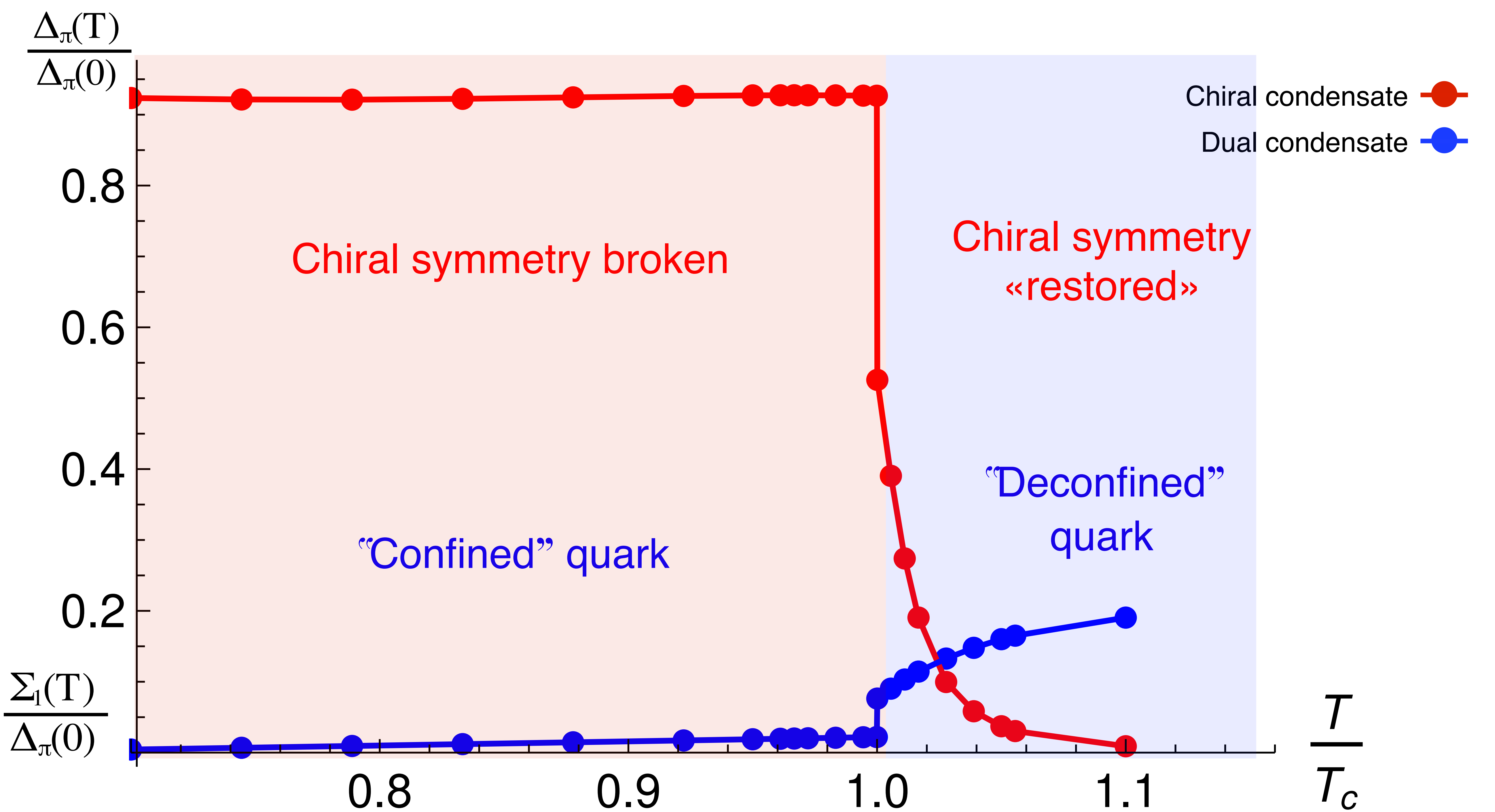
*[E. Bilgici, C. Gattringer, E.M Ilgenfritz and A. Maas (2009)]*

- Can be compared with lattice computations
  - Gauge invariant

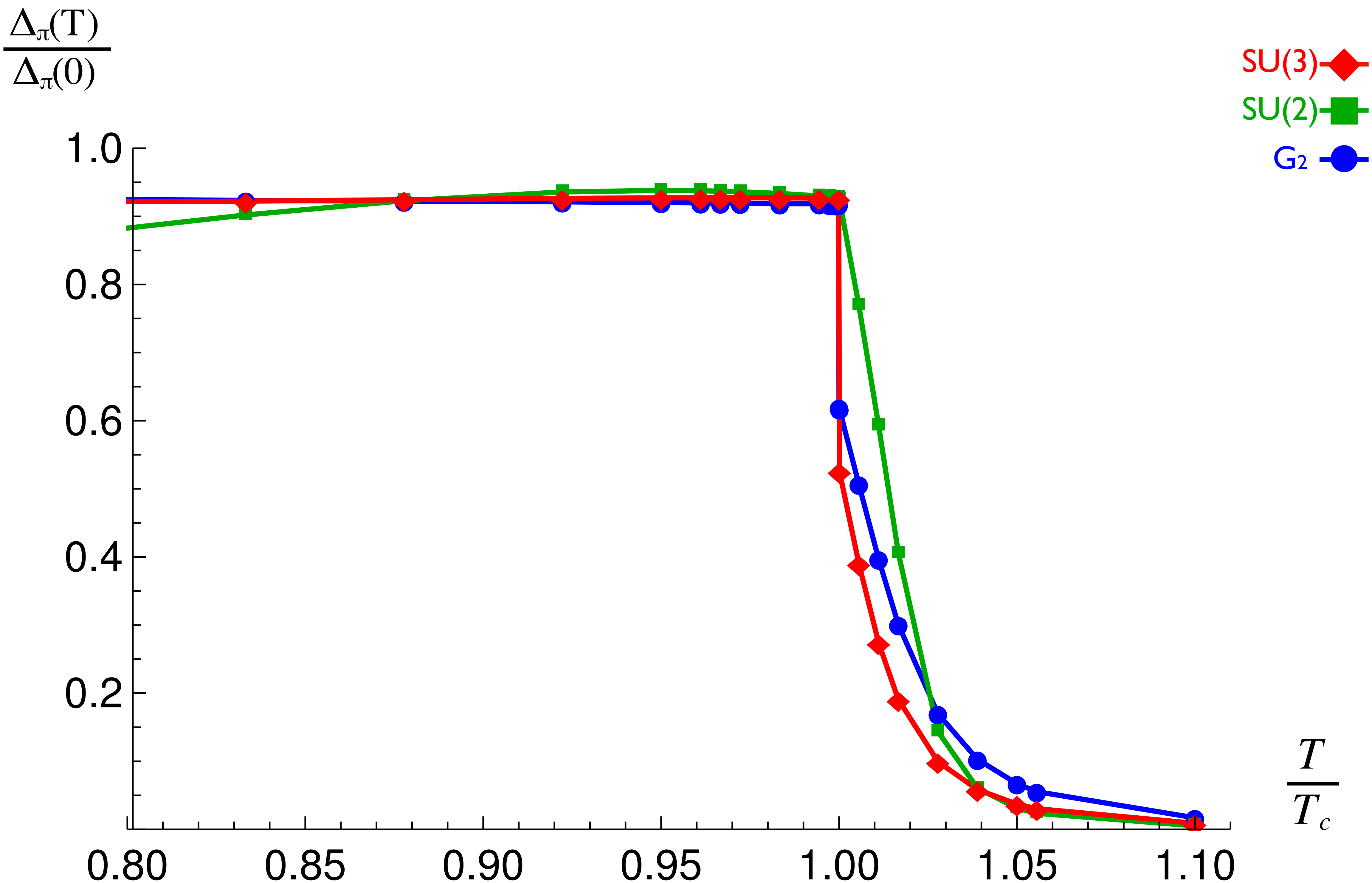
# Order parameters



Chiral Condensate and dual condensate



Chiral condensate



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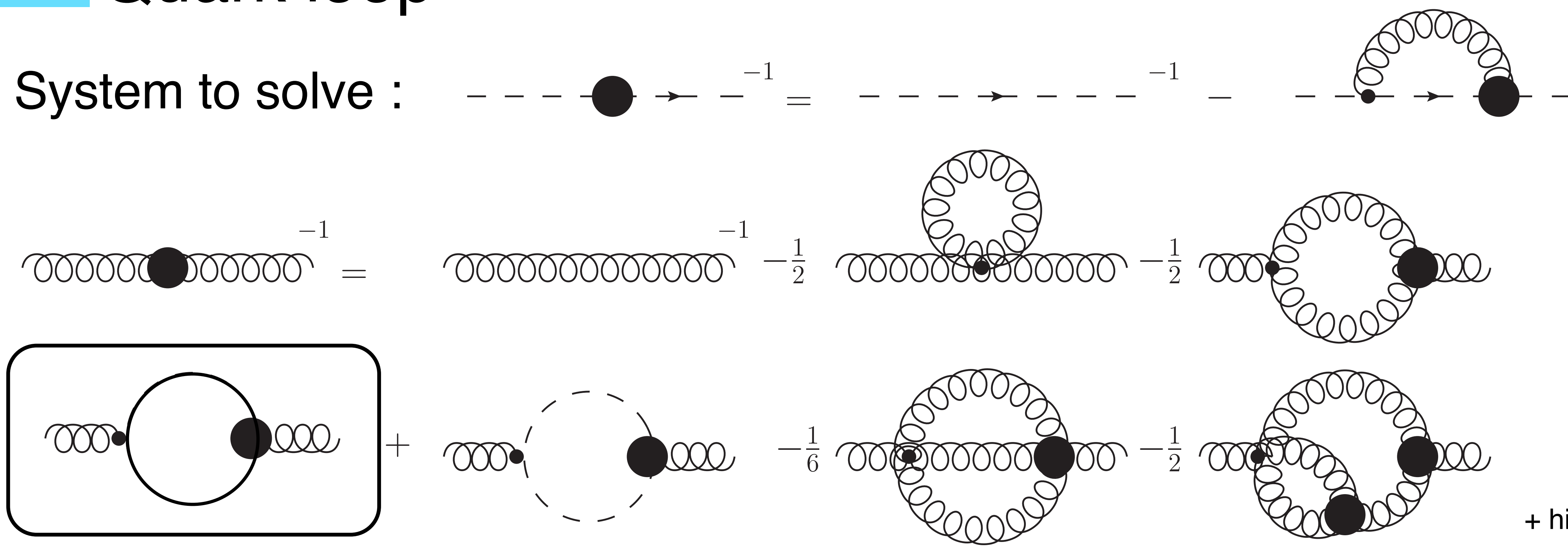
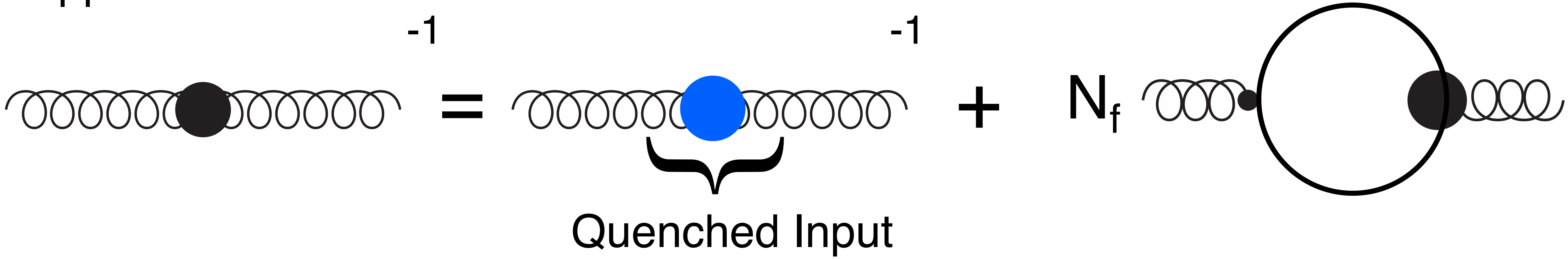
- System to solve :




+ higher equations ...

- Approximation :

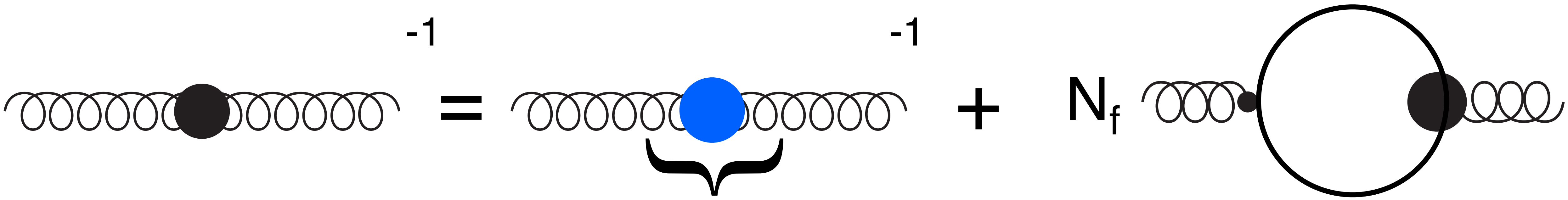



*[C.S Fischer , J. Luecker (2012)]*

Neglect all indirect quark contributions in the gluon dressing

# 4 Unquenching
## Quark loop

- Adding the quark loop

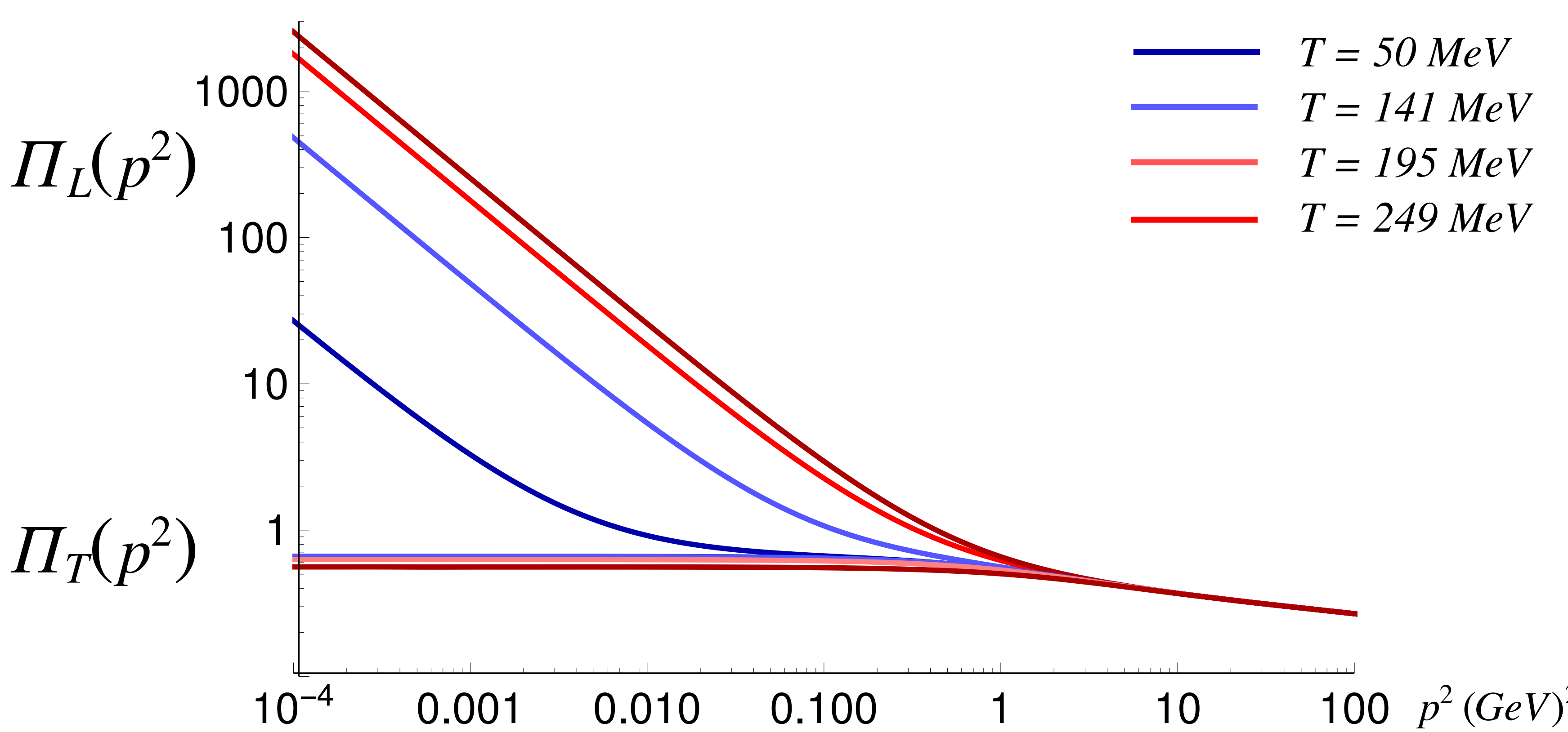



[C.S Fischer , J. Luecker (2012)]

$$\Pi_L(p)p^2 \xrightarrow{p \to 0} (m_{th})^2$$

Debye Screening
of the chromo-electric
charge

$$\Pi_T(p)p^2 \xrightarrow{p \to 0} 0$$



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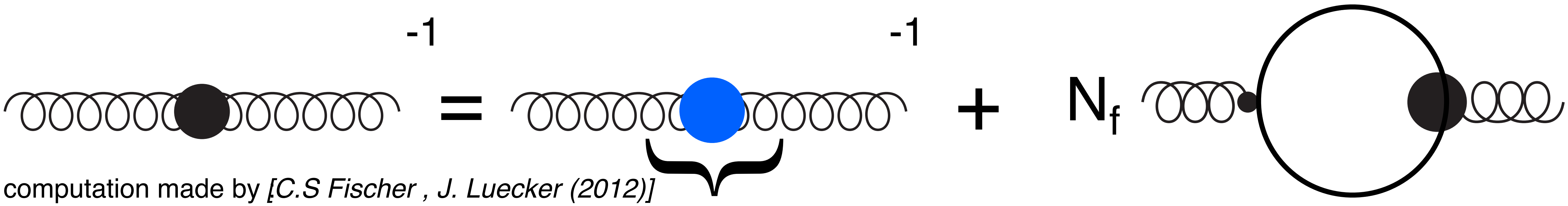
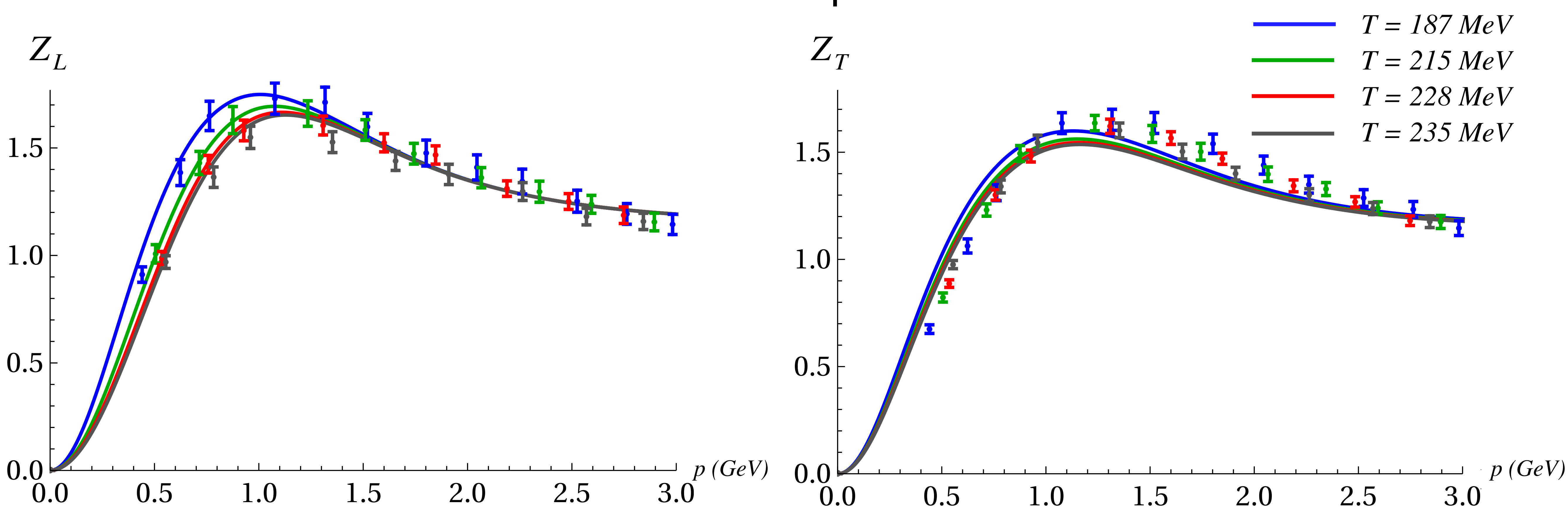



First computation made by [C.S Fischer , J. Luecker (2012)]

Quenched Input



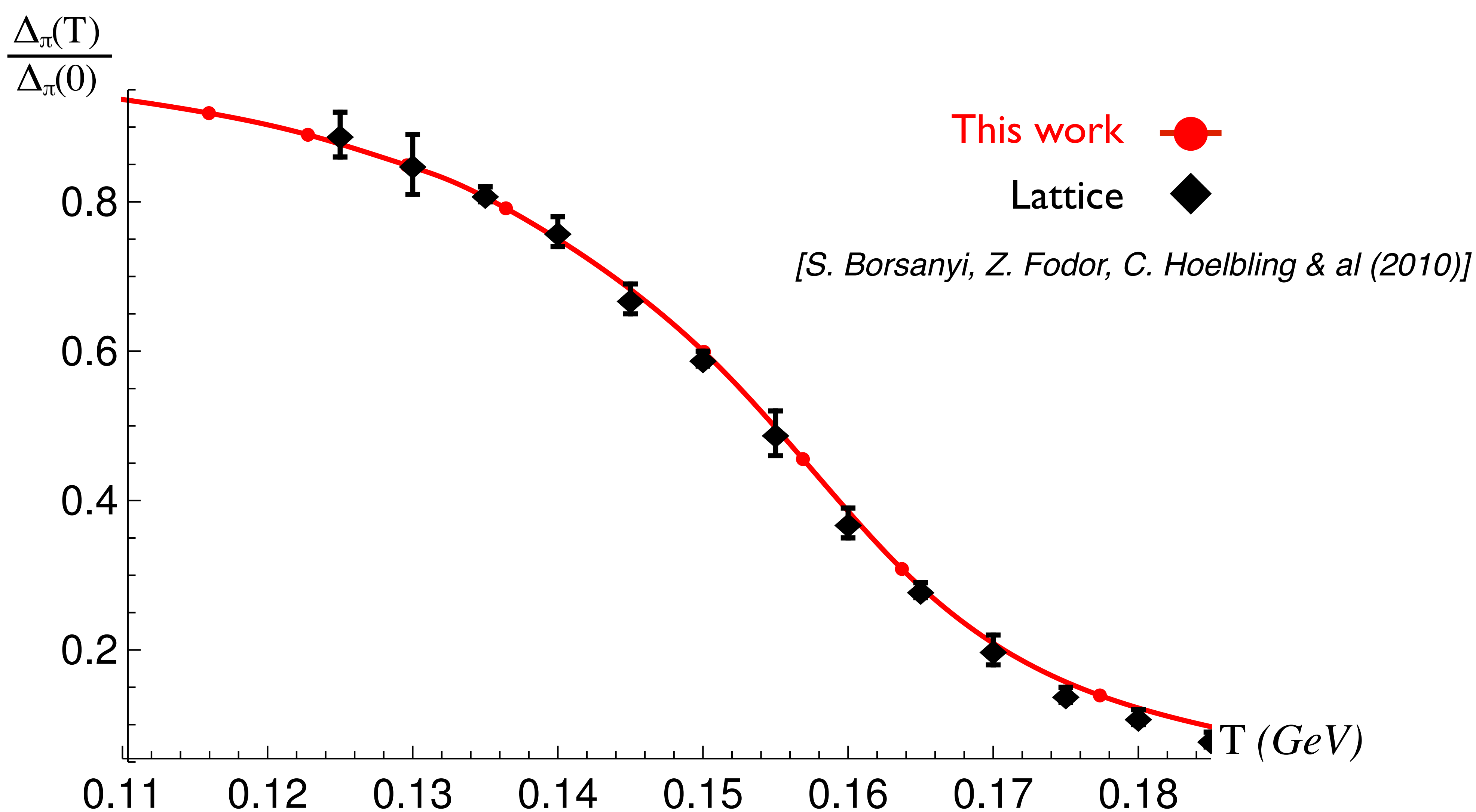

Compared to : [R.Aouane, F. Burger E.-M. Illgenfritz & al (2012)]

→ Very small quark mass dependence

## Chiral condensate

For 2 light flavors and a strange quark, comparison with available lattice results is possible

First computation made by : *[C.S Fischer , J. Luecker (2012)]*

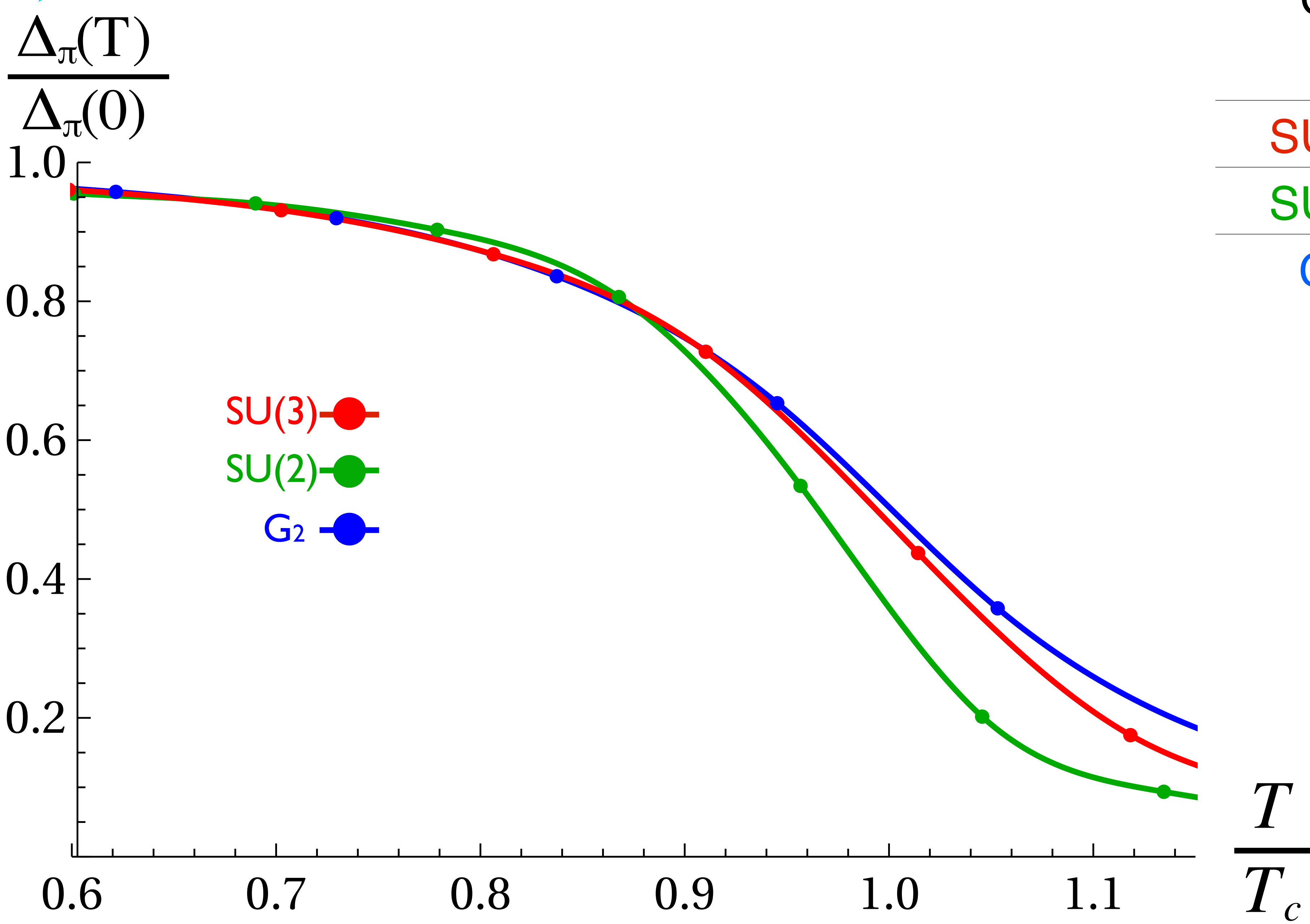


# Order parameters



## Chiral condensate

For 2 light flavors :

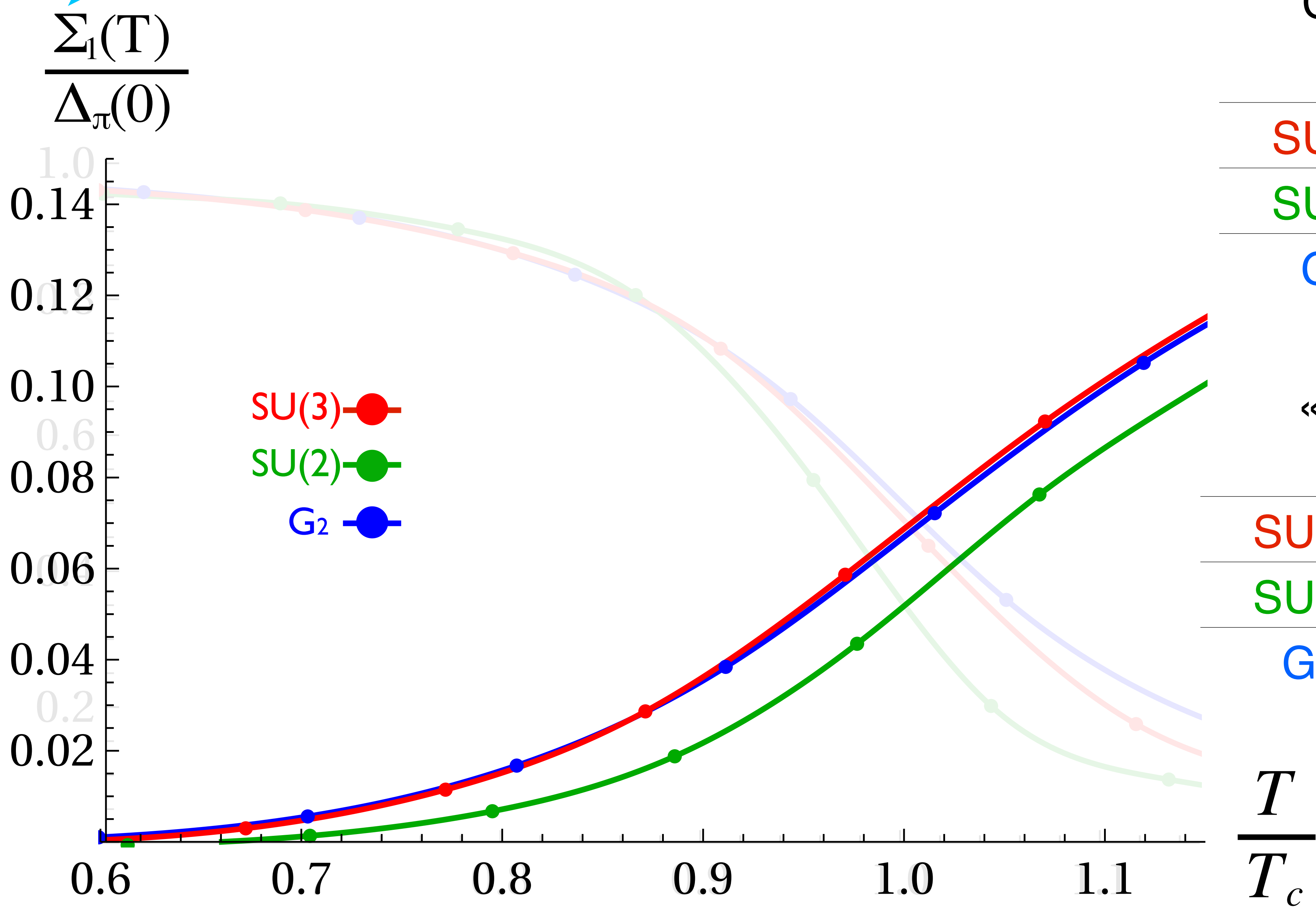



Chiral «restoration» $T_c$ $(MeV)$

| | quenched | 2 flavors |
|---|---|---|
| SU(3) | 277 | 174 |
| SU(2) | 303 | 218 |
| G2 | 255 | 155 |

## Dual condensate

For 2 light flavors :




Chiral «restoration» $T_c$ $(MeV)$

| | quenched | 2 flavors |
|---|---|---|
| SU(3) | 277 | 174 |
| SU(2) | 303 | 218 |
| G2 | 255 | 155 |

«Deconfinement» $(MeV)$

| | quenched | 2 flavors |
|---|---|---|
| SU(3) | 277 | 182 |
| SU(2) | 303 | 222 |
| G2 | 255 | 160 |

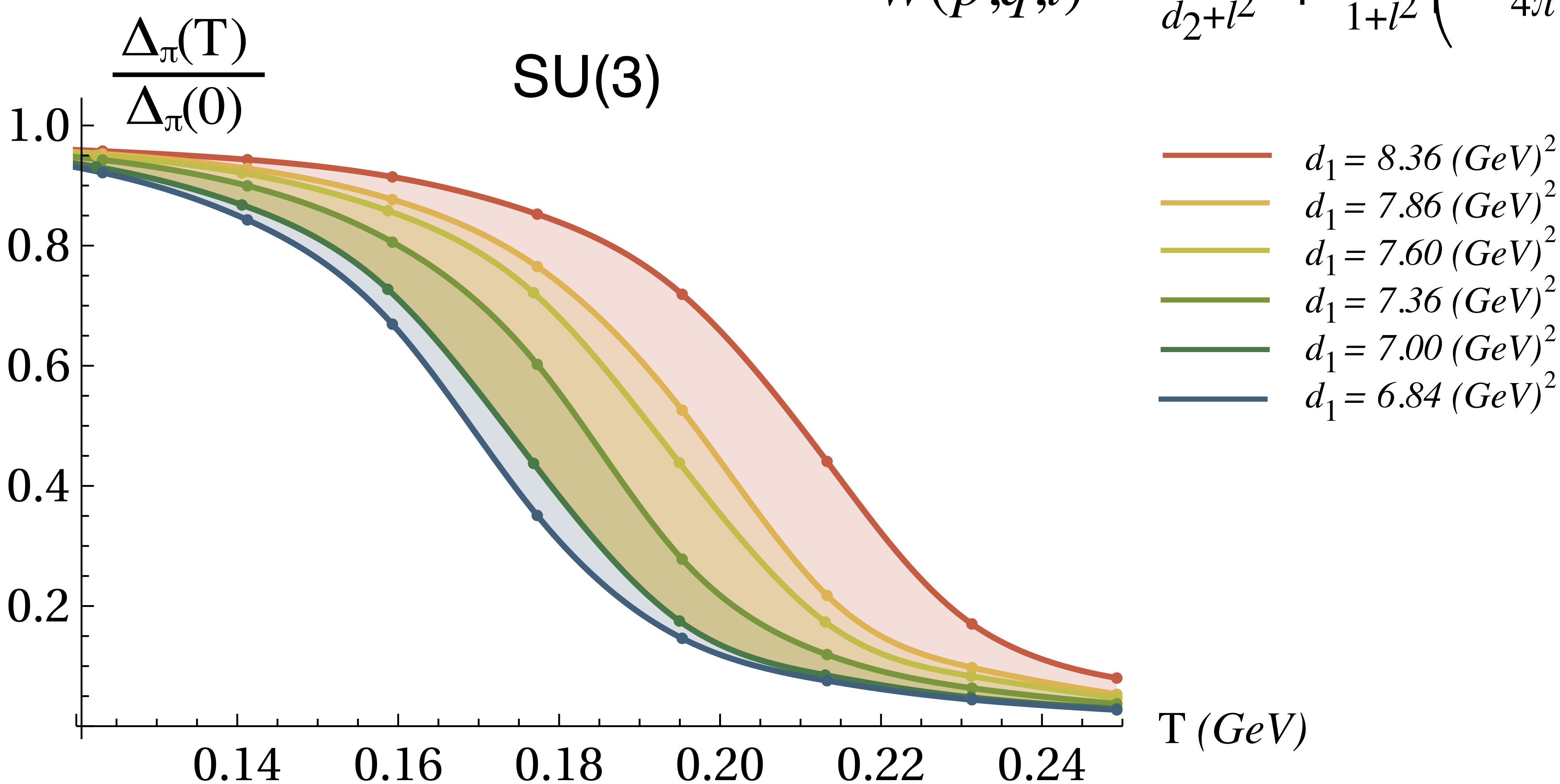
The employed setup behaves universally

# 4 Unquenching
## Parameter variation

$$\Gamma_{q\text{-}gl}(p,q,l) = \left( \frac{A(p)+A(q)}{2}\vec{\gamma}, \frac{C(p)+C(q)}{2}\gamma_4 \right) W(p,q,l)$$

$$W(p,q,l) = \boxed{\frac{d_1}{d_2+l^2}} + \frac{l^2}{1+l^2}\left( \frac{\beta_0\,\alpha(\mu)}{4\pi}\ln(l^2+1) \right)^{2\delta}$$

For 2 light flavors :




Similar results for the QCD-like theories

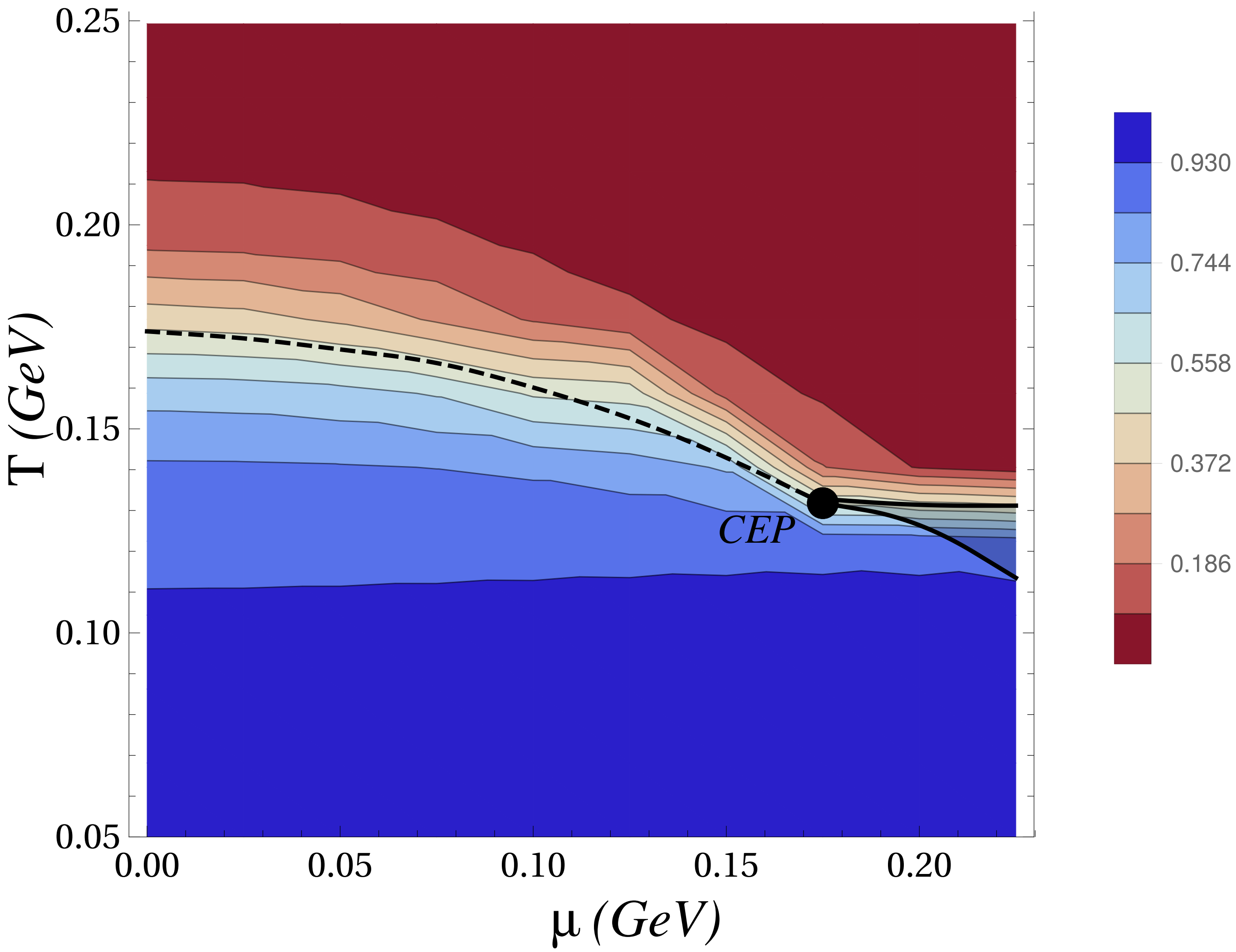
A large parameter dependence of the quark-gluon vertex

# Finite $\mu$
## Chiral transition



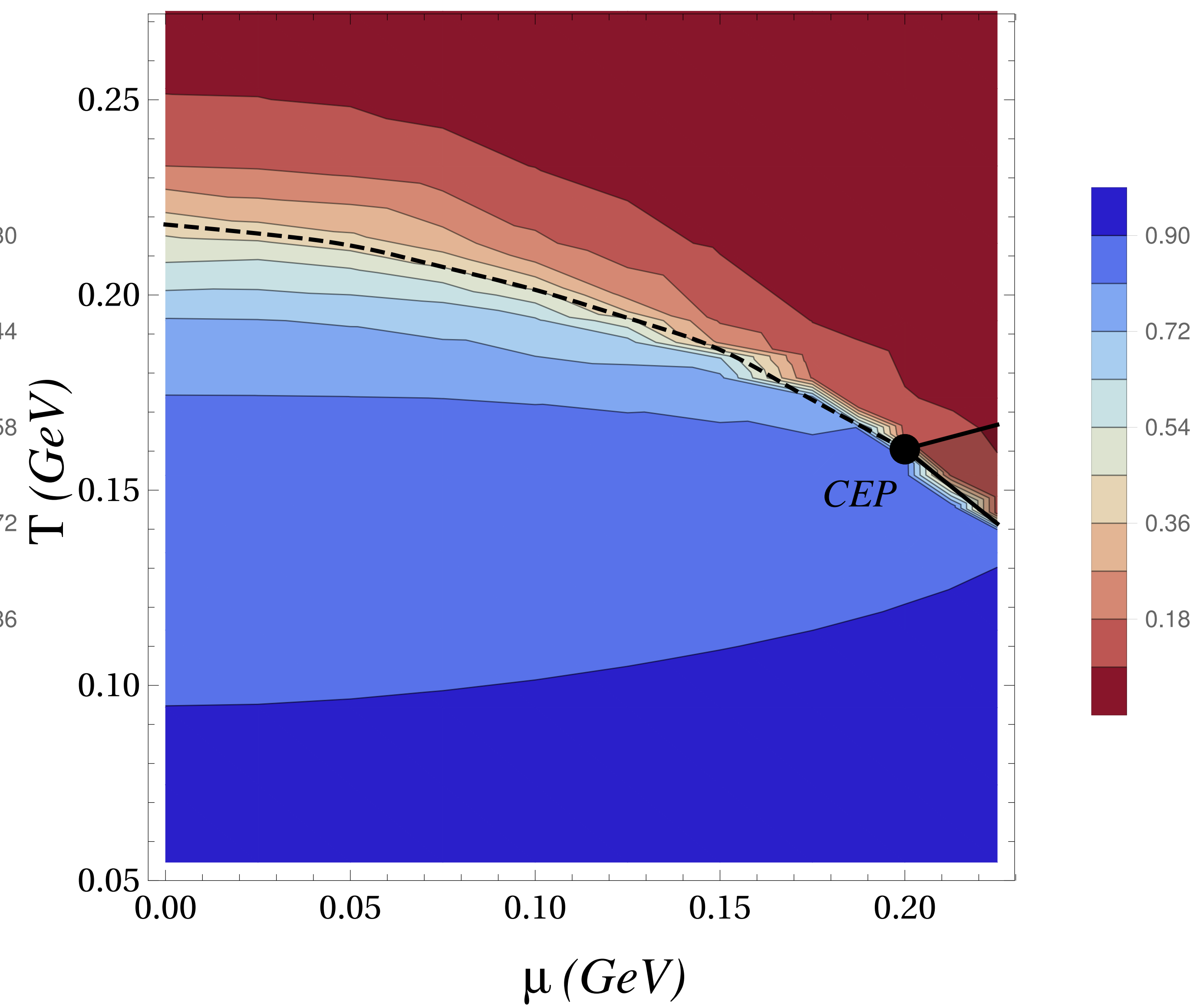
- Shift $\omega_n$ to $\omega_n + i\,\mu$
- For 2 light flavors

SU(3)




CEP ≈ (175, 132) MeV

SU(2)

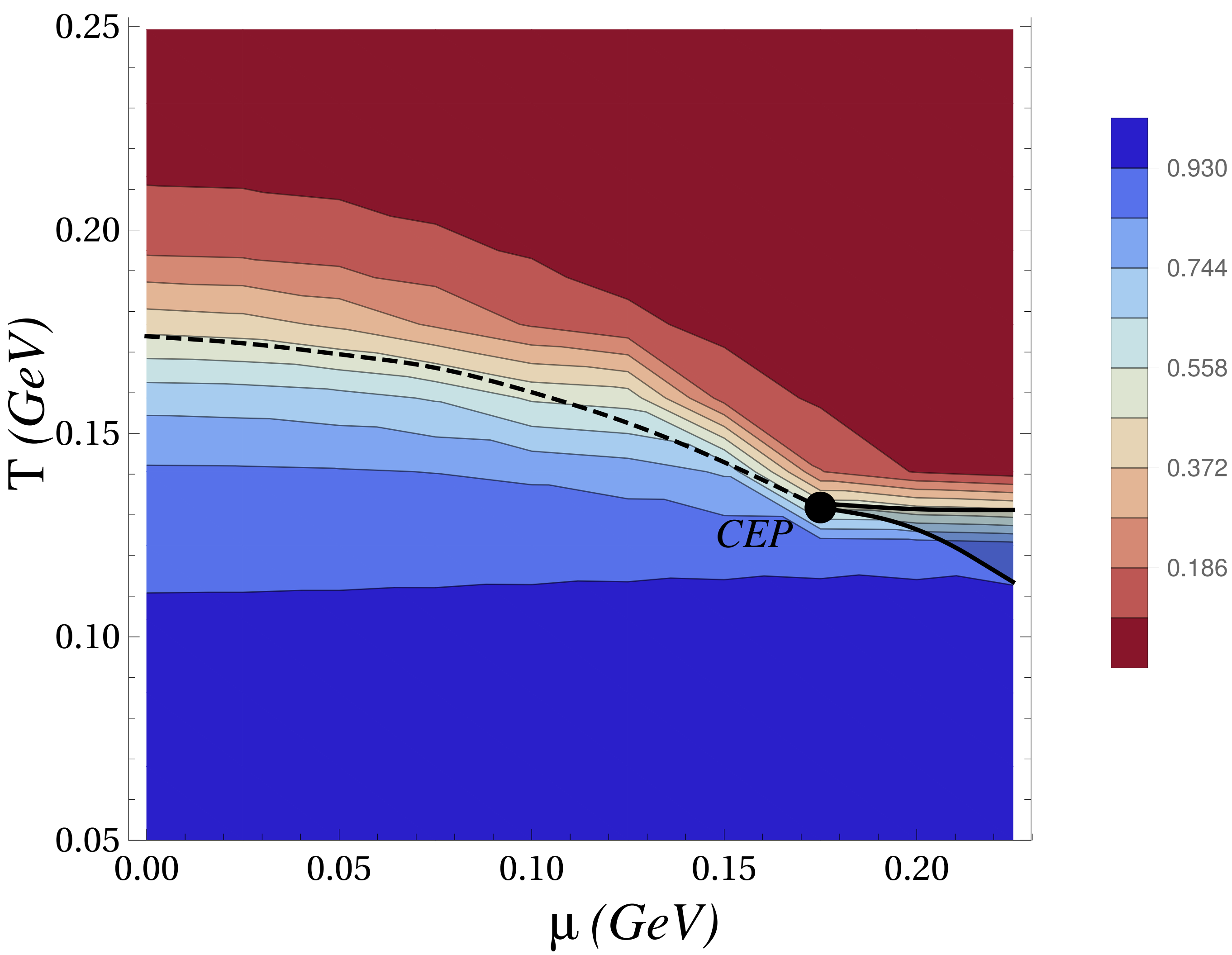



CEP ≈ (200, 160) MeV

# 5 Finite $\mu$
## Chiral transition

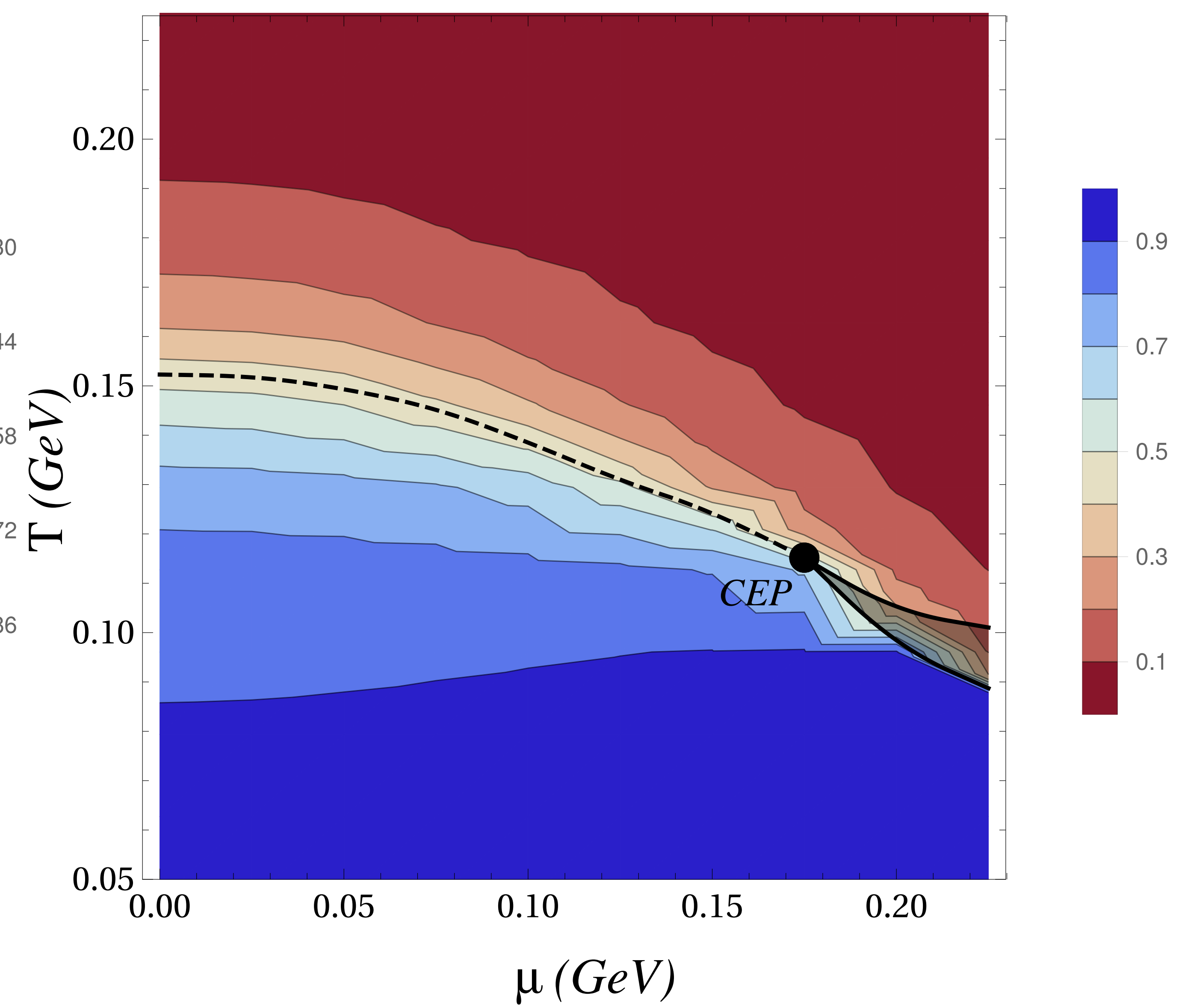
- Shift $\omega_n$ to $\omega_n + i\,\mu$
- For 2 light flavors

SU(3)




CEP ≈ (175, 132) MeV

G2

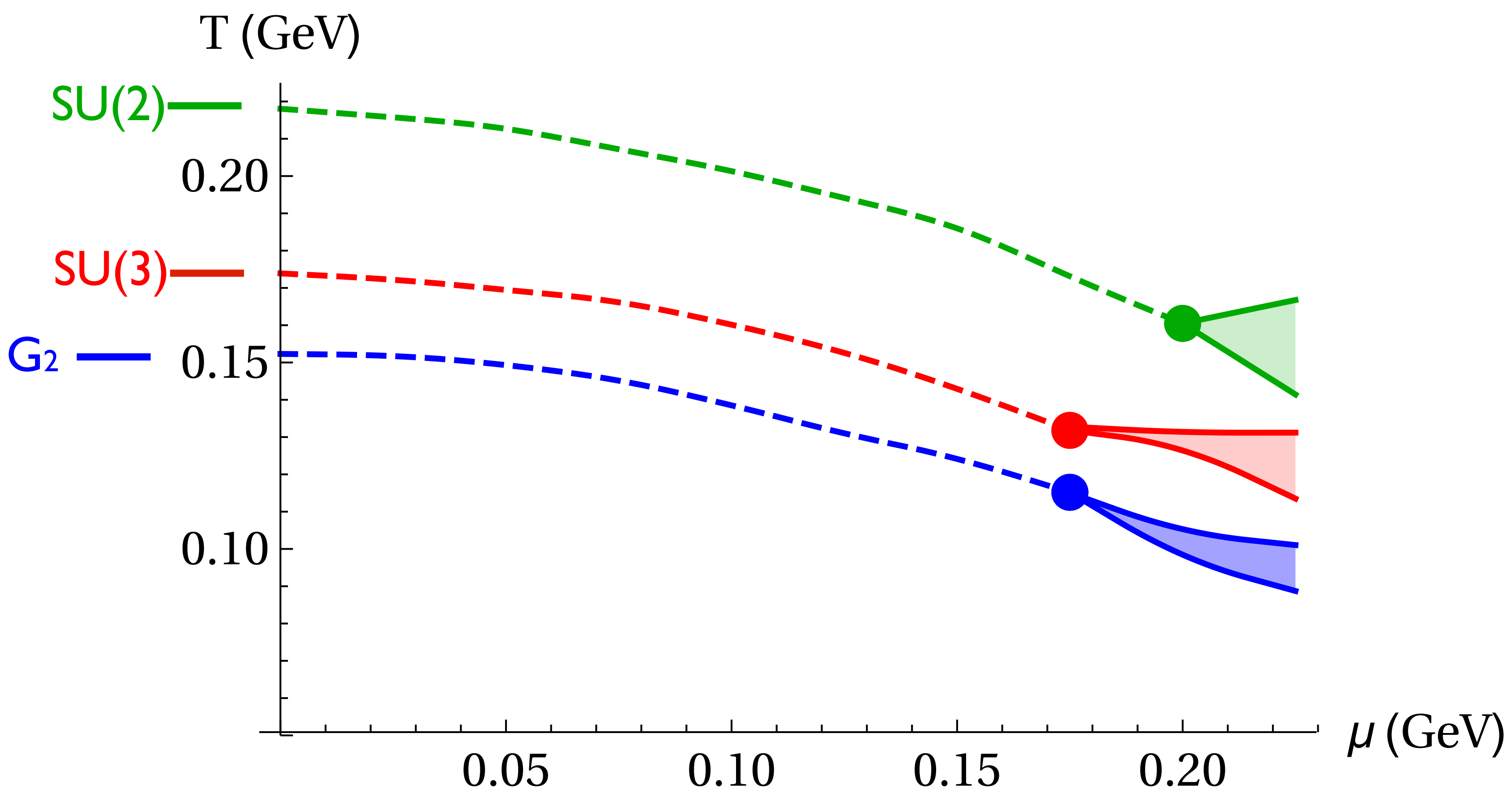



CEP ≈ (175, 115) MeV

# Summary



**The employed setup behaves universally**

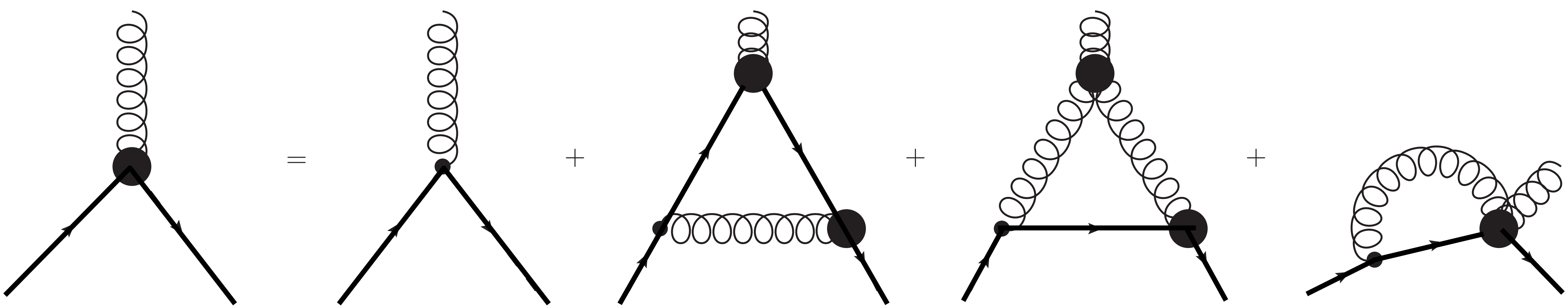


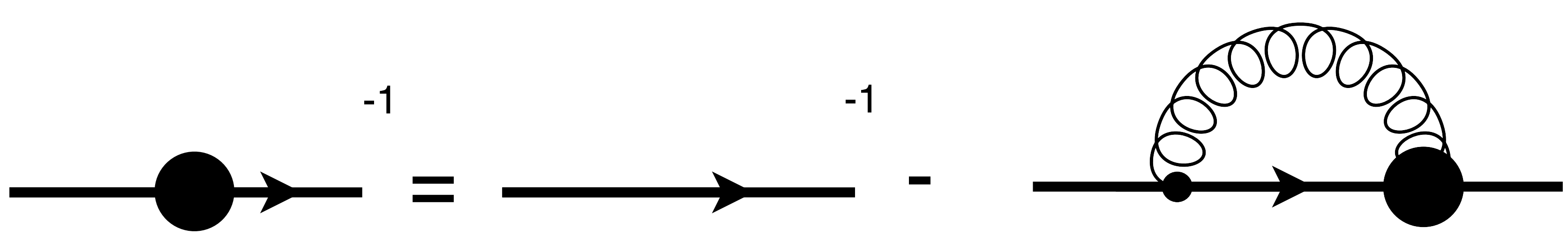

- The bending of the crossover line does not agree with lattices results for low μ
- Enlarging the truncation to assess universality

# 6 Conclusion
## Outlook

- The quark-gluon vertex

→ New system to solve :



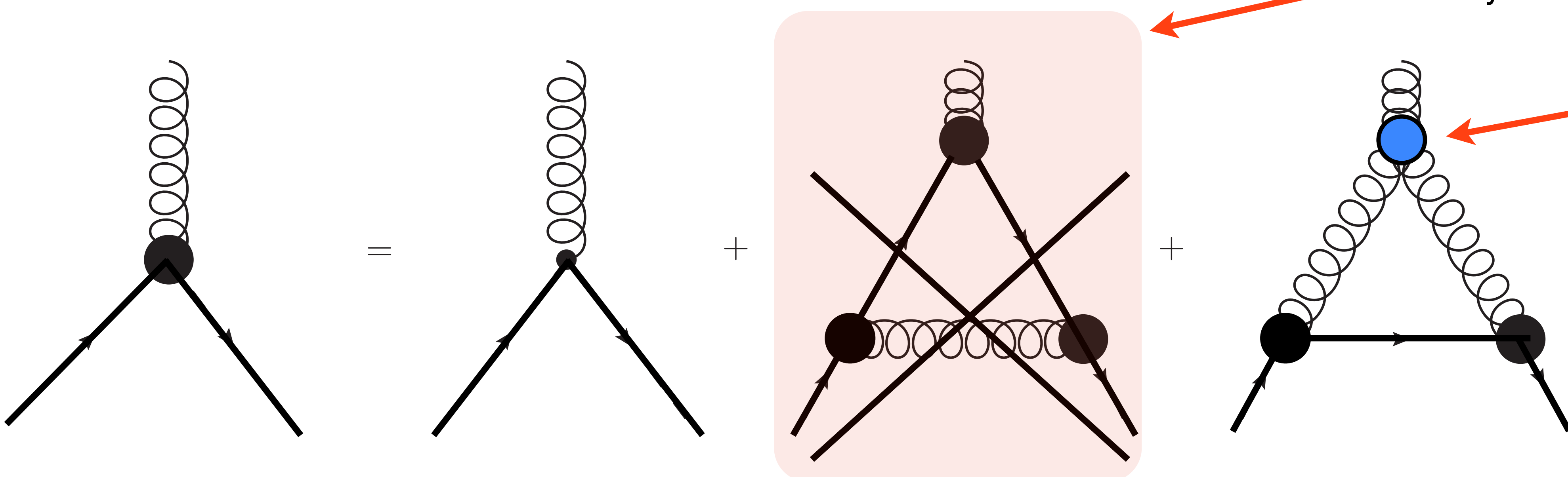
# 6 Conclusion
## Outlook

- The quark-gluon vertex

→ New system to solve :

Relative color factor still negligible ?
Dynamical suppression after Tc ?

New correlation function



- Follow the approximation used in vacuum study

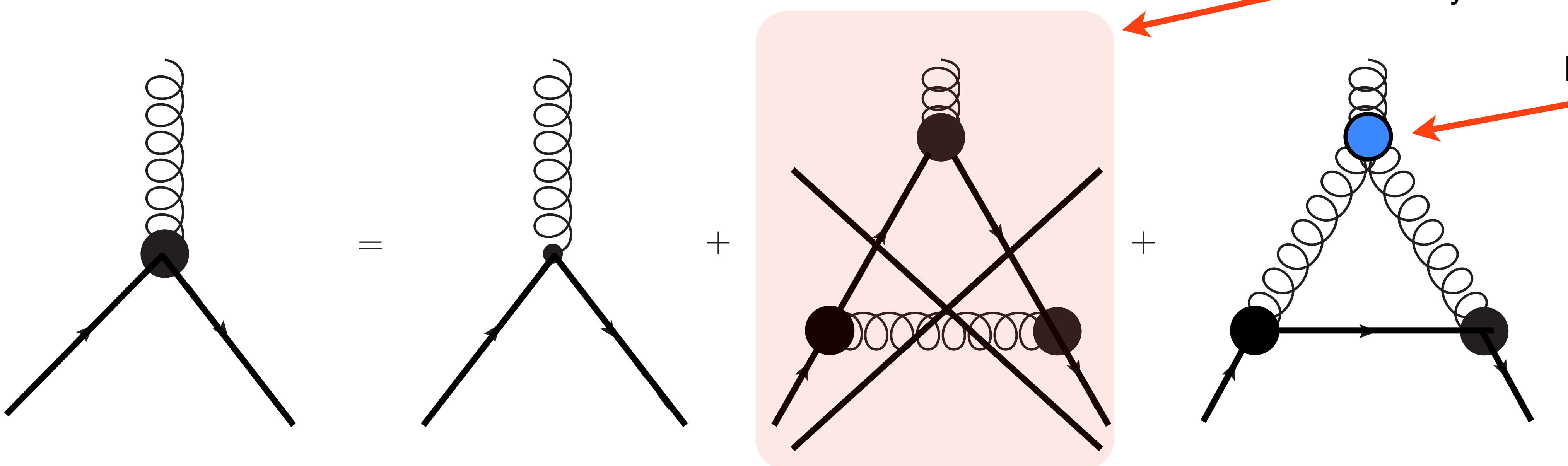
*[R.Williams (2015)]*

This approximation is still acceptable : - for QCD-like theory ?
- in medium ?

# 6 Conclusion
## Outlook

- The quark-gluon vertex

→ New system to solve :




- Follow the approximation used in vacuum study

*[R.Williams (2015)]*

- In Landau gauge, 24 dressings functions

$$\vec{\gamma} h_1 + u^\mu \gamma^4 h_2 + i\vec{\gamma}\vec{\not{l}} h_3 + i\vec{\gamma}\vec{\not{k}}_3 h_4 + \vec{\gamma}[\vec{\not{l}},\vec{\not{k}}_3] h_5 + i\vec{l} h_6 + \vec{l}\vec{\not{l}} h_7 + \vec{l}\vec{\not{k}}_3 h_8 + i\vec{l}[\vec{\not{l}},\vec{\not{k}}_3] h_9 + iu^\mu h_{10} + u^\mu \vec{\not{l}} h_{11} + u^\mu \vec{\not{k}}_3 h_{12}$$

$$+iu^\mu[\vec{\not{l}},\vec{\not{k}}_3] h_{13} + i\vec{\gamma}\gamma^4 h_{14} + \vec{l}\gamma^4 h_{15} + \vec{\gamma}[\vec{\not{l}},\not{u}] h_{16} + \vec{\gamma}[\vec{\not{k}}_3,\not{u}] h_{17} + i\vec{l}[\vec{\not{l}},\not{u}] h_{18} + i\vec{l}[\vec{\not{k}}_3,\not{u}] h_{19}$$

$$+iu^\mu[\vec{\not{l}},\not{u}] h_{20} + iu^\mu[\vec{\not{k}}_3,\not{u}] h_{21} + i\vec{\gamma}\vec{\not{l}}\vec{\not{k}}_3\not{u} h_{22} + \vec{l}\vec{\not{l}}\vec{\not{k}}_3\not{u} h_{23} + u^\mu \vec{\not{l}}\vec{\not{k}}_3\not{u} h_{24}$$

# 6 Conclusion

## Outlook

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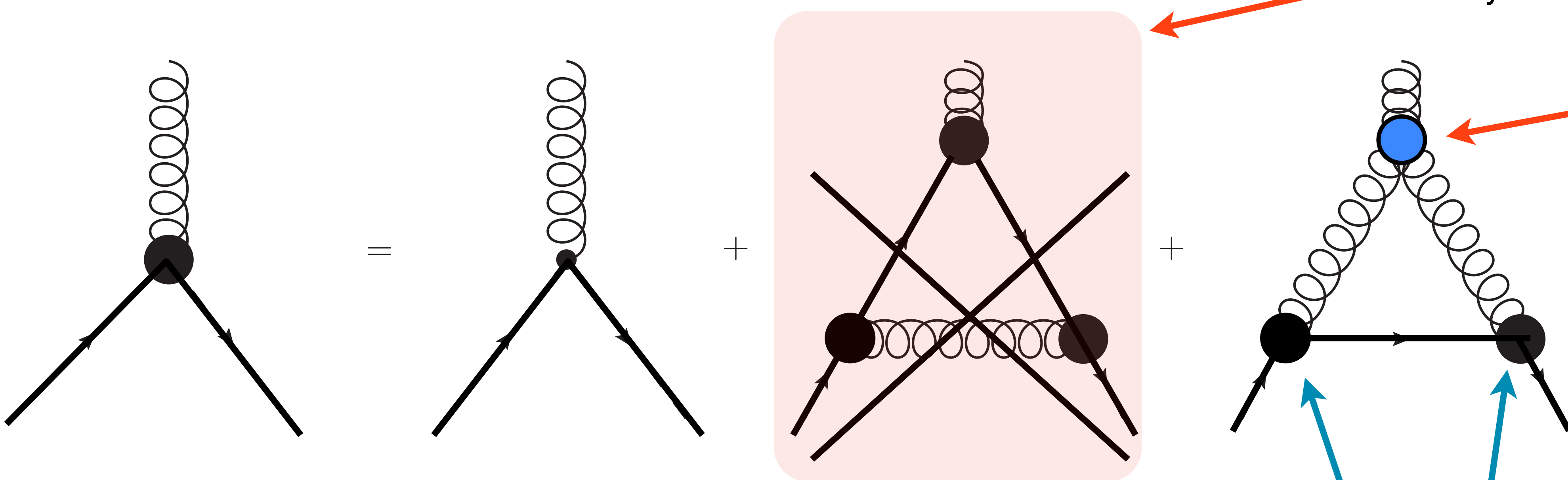

- Follow the approximation used in vacuum study

*[R.Williams (2015)]*

x Projection → 13 824 terms ...

- In Landau gauge, 24 dressings functions

$$\vec{\gamma}h_1 + u^\mu\gamma^4 h_2 + i\vec{\gamma}\vec{\not l} h_3 + i\vec{\gamma}\vec{\not k}_3 h_4 + \vec{\gamma}[\vec{\not l},\vec{\not k}_3]h_5 + i\vec{l}h_6 + \vec{l}\vec{\not l}h_7 + \vec{l}\vec{\not k}_3 h_8 + i\vec{l}[\vec{\not l},\vec{\not k}_3]h_9 + iu^\mu h_{10} + u^\mu\vec{\not l}h_{11} + u^\mu\vec{\not k}_3 h_{12}$$

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# 6 Conclusion
## Outlook

- The quark-gluon vertex

→ Semi-perturbative : Quenched $T = 250\ MeV$

$$\boxed{\vec{\gamma} h_1} + u^\mu \gamma^4 h_2 + i\vec{\gamma}\vec{l} h_3 + i\vec{\gamma}\vec{k}_3 h_4 + \boxed{\vec{\gamma}[\vec{l}, \vec{k}_3] h_5} + i\vec{l} h_6 + \boxed{\vec{l}\vec{l} h_7} + \boxed{\vec{l}\vec{k}_3 h_8} + i\vec{l}[\vec{l}, \vec{k}_3] h_9 + i u^\mu h_{10} + \boxed{u^\mu \vec{l} h_{11}} + \boxed{u^\mu \vec{k}_3 h_{12}}$$

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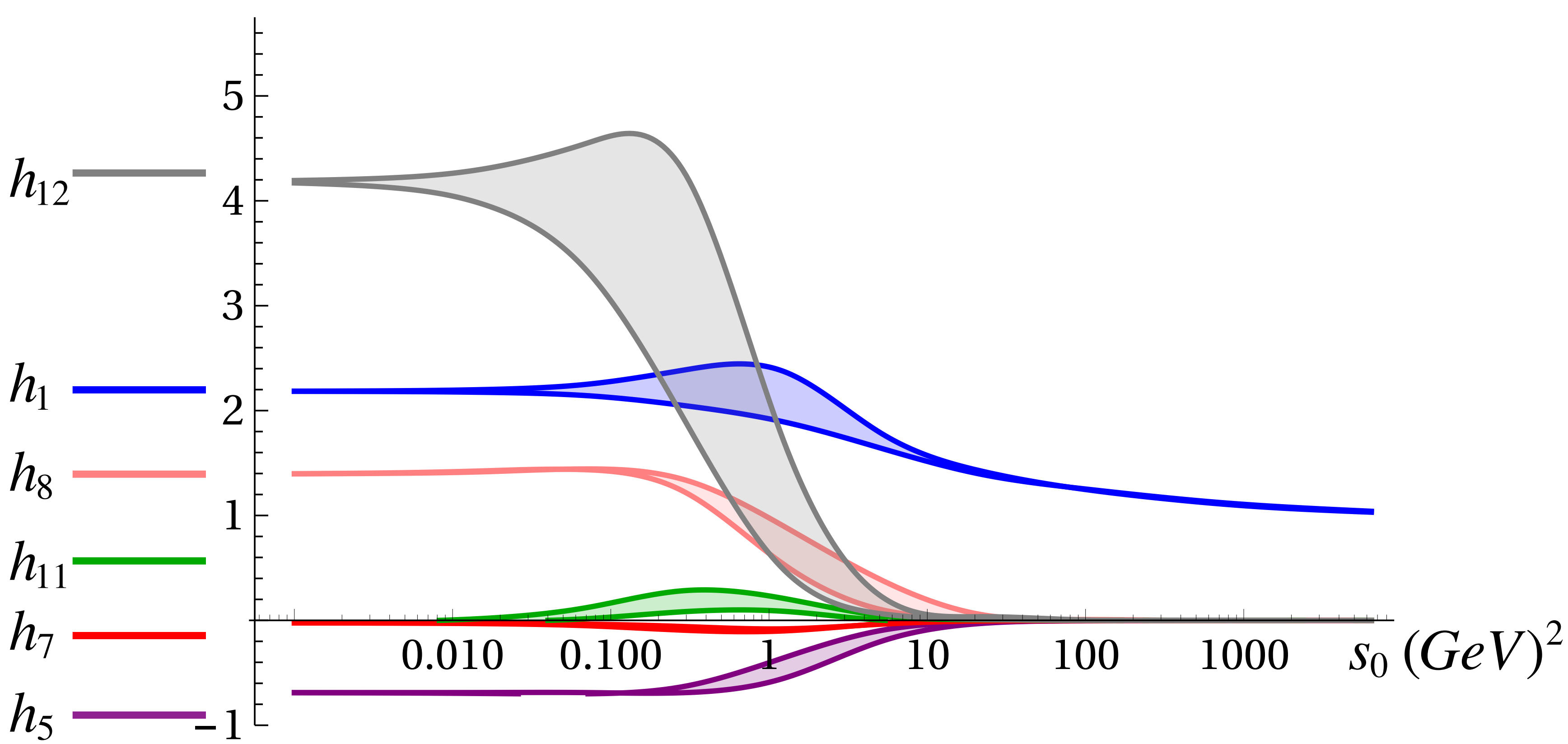

Contant Romain



- The quark-gluon vertex

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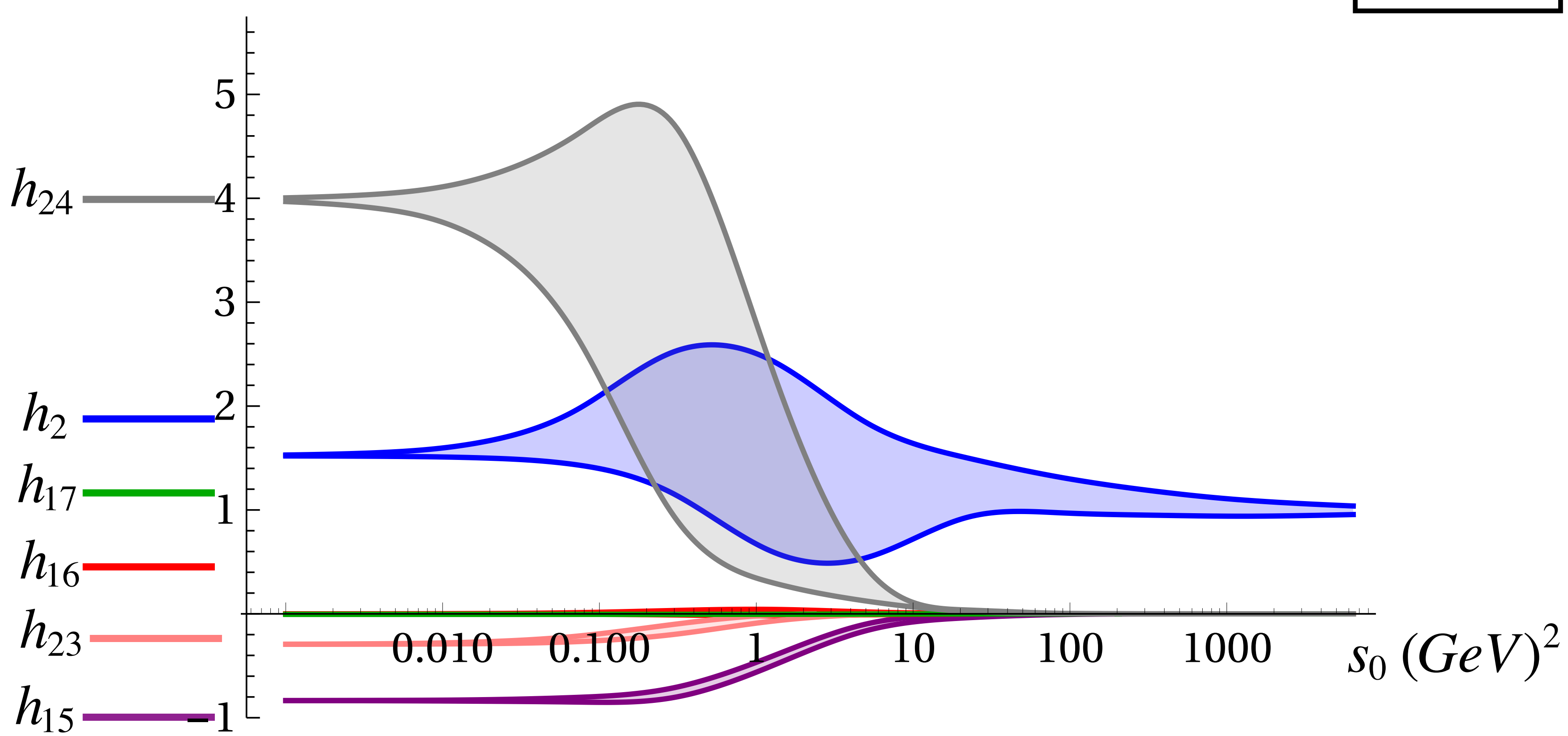

Contant Romain



# 6 Conclusion

## Outlook

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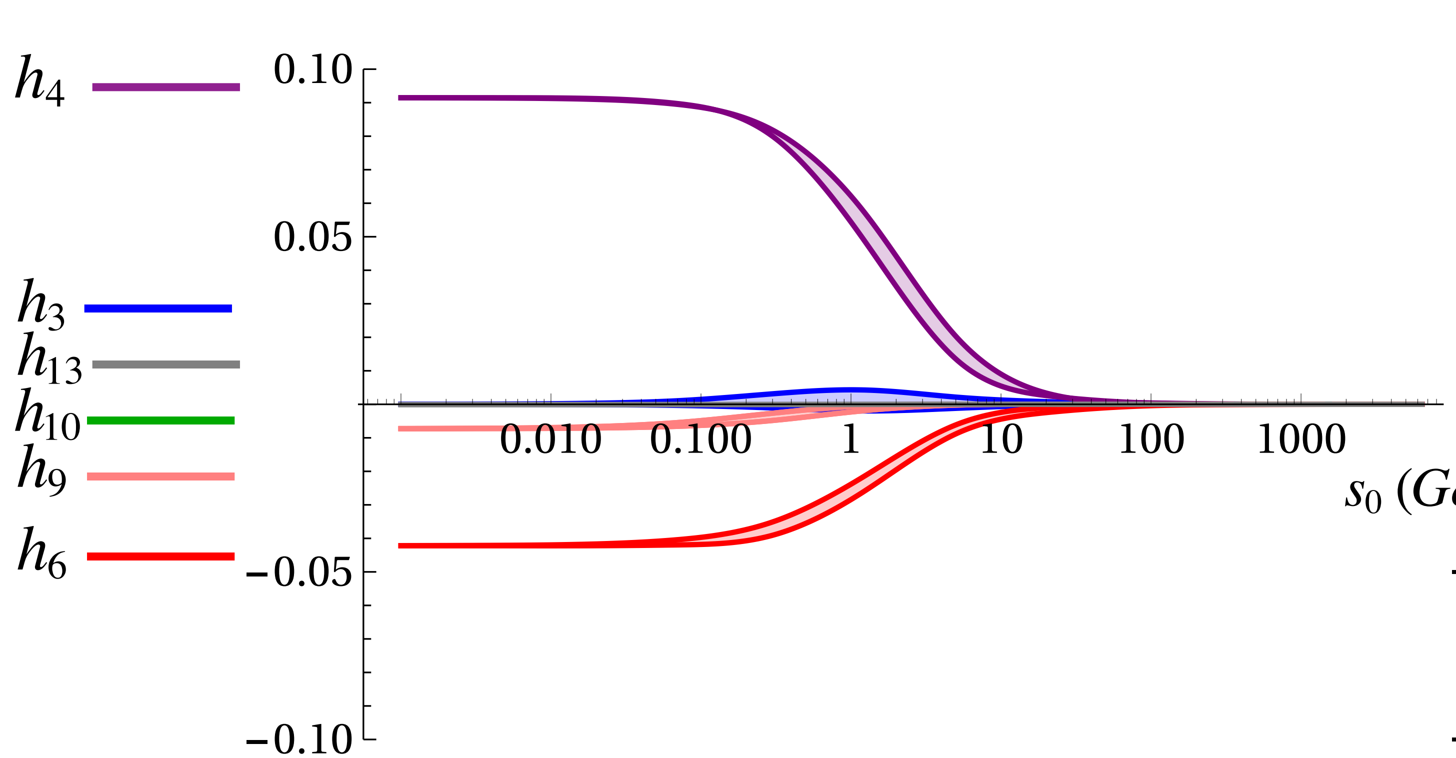


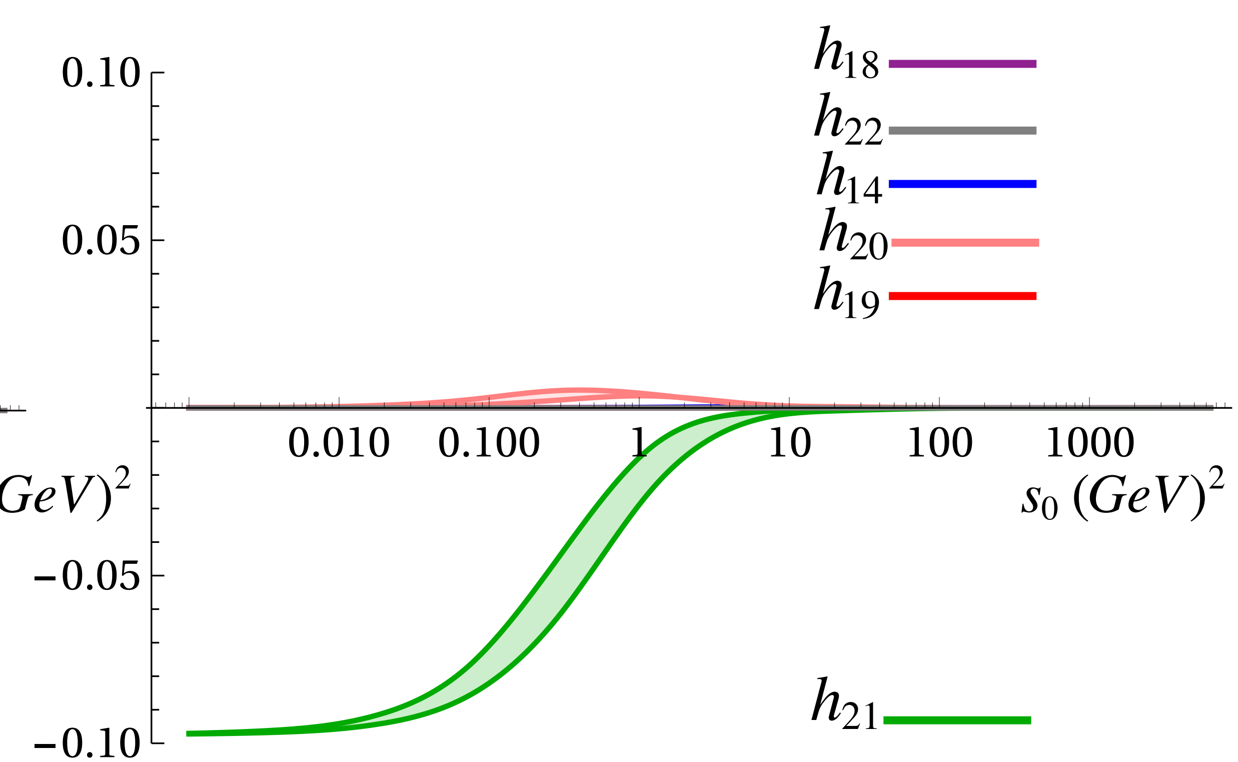

## Summary



- The quenched results show the expected behavior

---

- An unquenching procedure is possible
  - The qualitative behavior of the order parameters is respected
  - The qualitative behavior remains the same for different quark-gluon vertex parameters
  - The (pseudo)-critical temperature for chiral and dual chiral are close to each other
  - A quantitative comparison for the gluon dressing function and chiral condensate

---

- At finite $\mu$
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---

- Quark gluon vertex computation in progress

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→ At finite $\mu$

- The qualitative behavior of the order parameters is respected
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---

→ Quark gluon vertex computation in progress

## 6 Conclusion



- The employed setup behaves universally
- Qualitative and quantitative comparison with lattice results at vanishing $\mu$

- Sensitivity to the model parameters

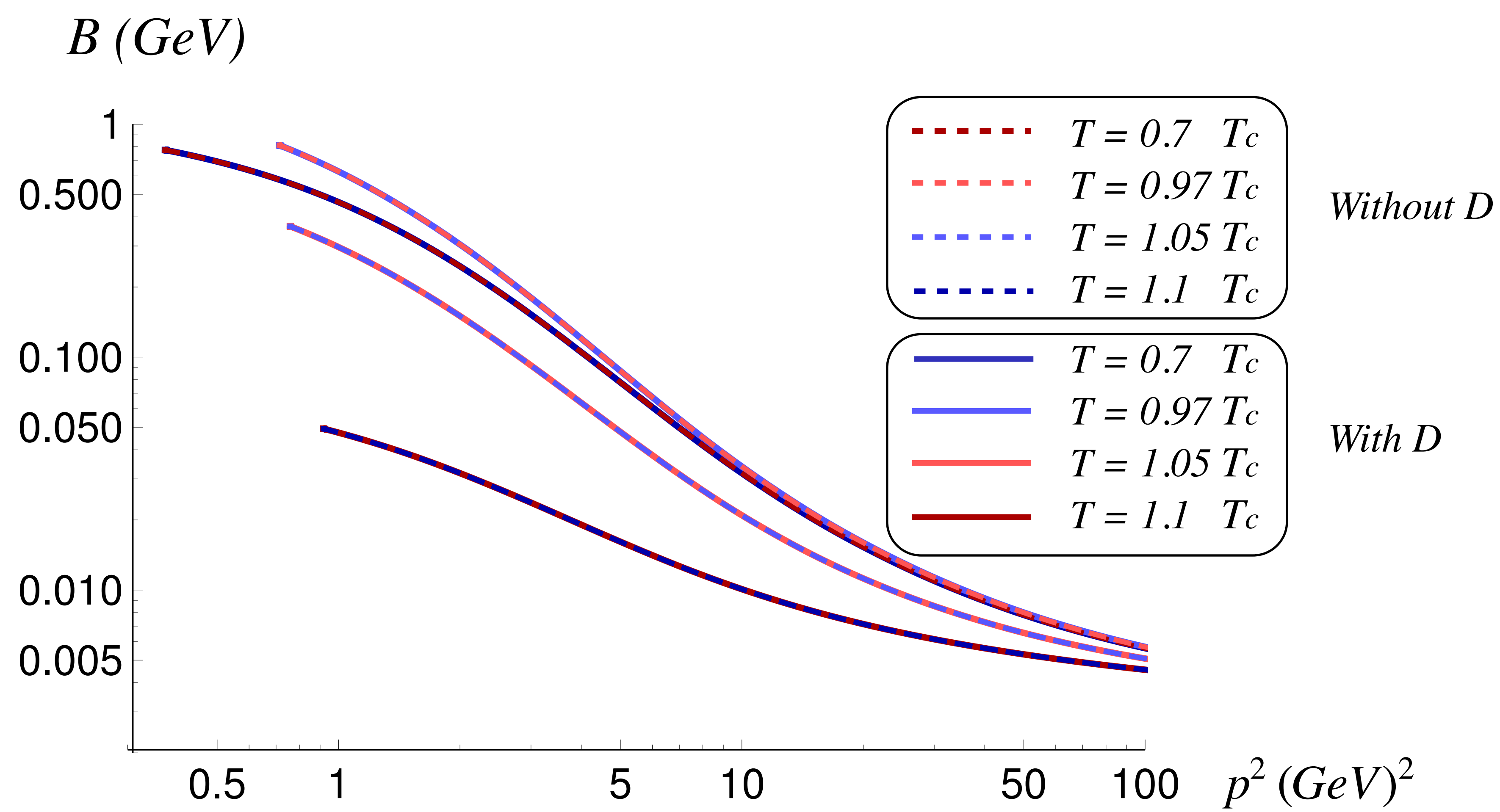
- Improvement of the truncation in progress

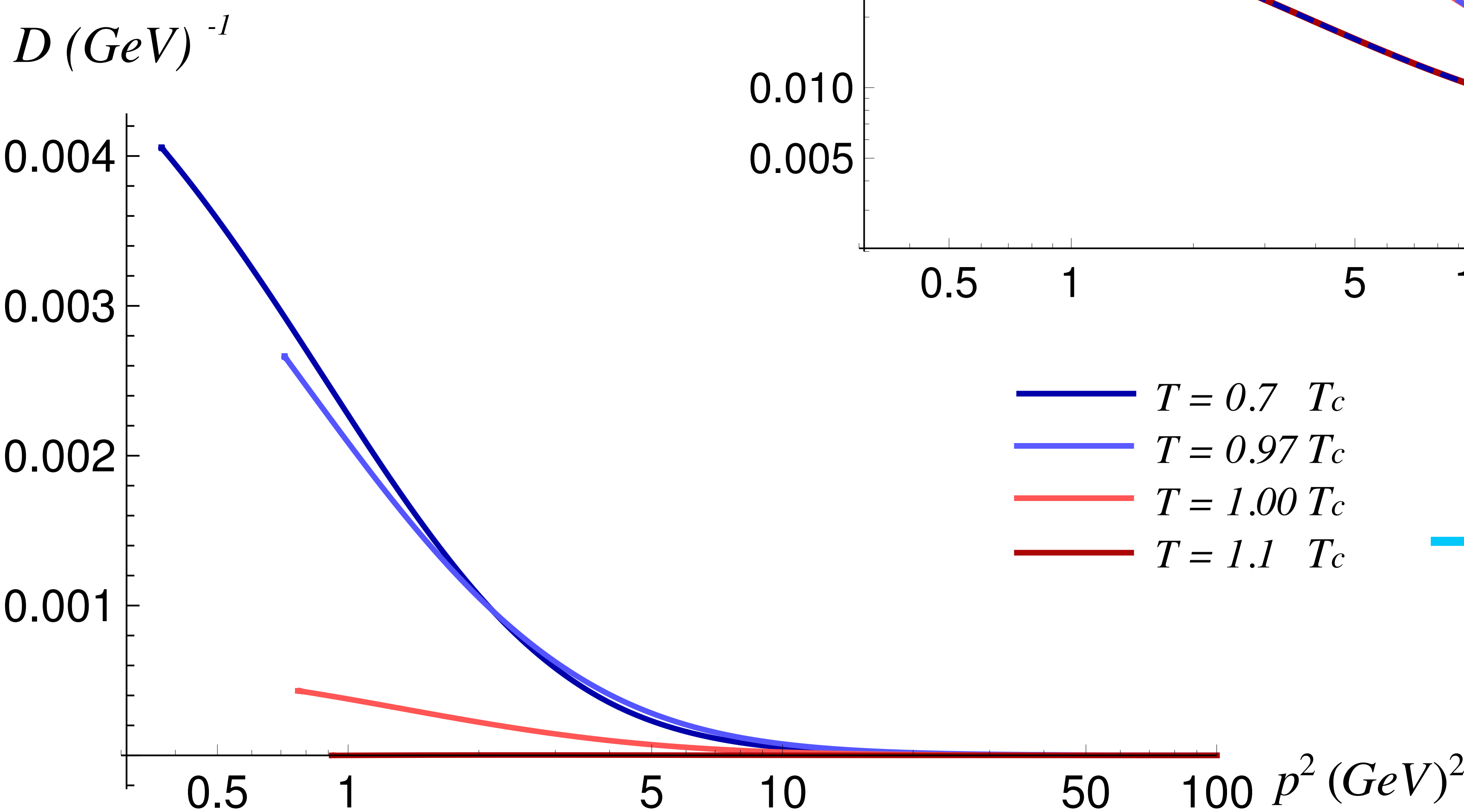
# **Thank you**

# Effects of the *D* function



$$S^{-1}(p,\omega_0) = A(p,\omega_0)\,\gamma\, p + C(p,\omega_0)\,\omega_0\gamma_4 + B(p,\omega_0) + \omega_0\gamma_4\, p\,\gamma\, D(p,\omega_0)$$




$$p^2 = (\omega_0)^2 + \vec{p}^{\,2}$$

Relative contribution :

→ < 0.01 % to the *B* dressing

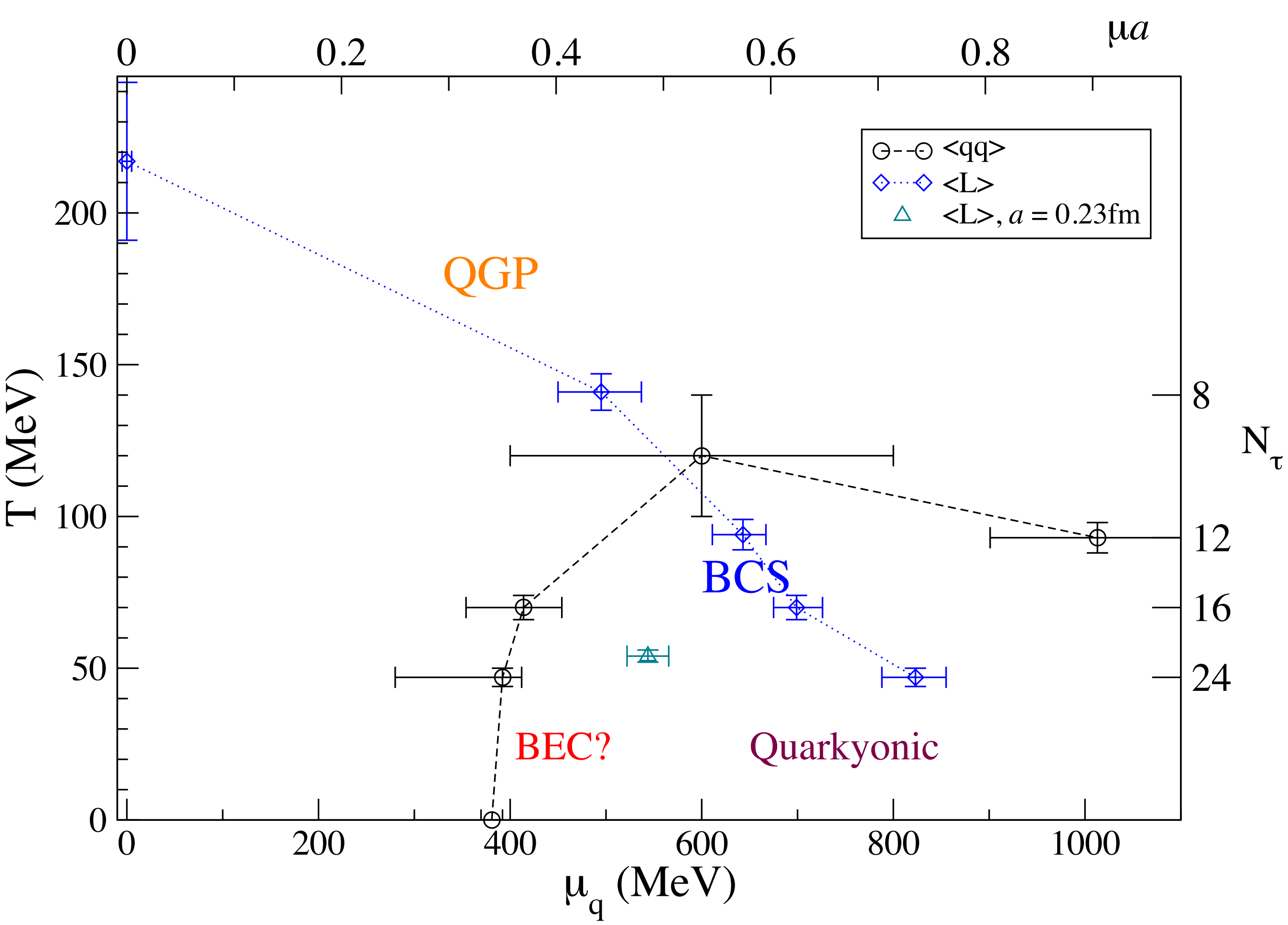
# 7 Back up

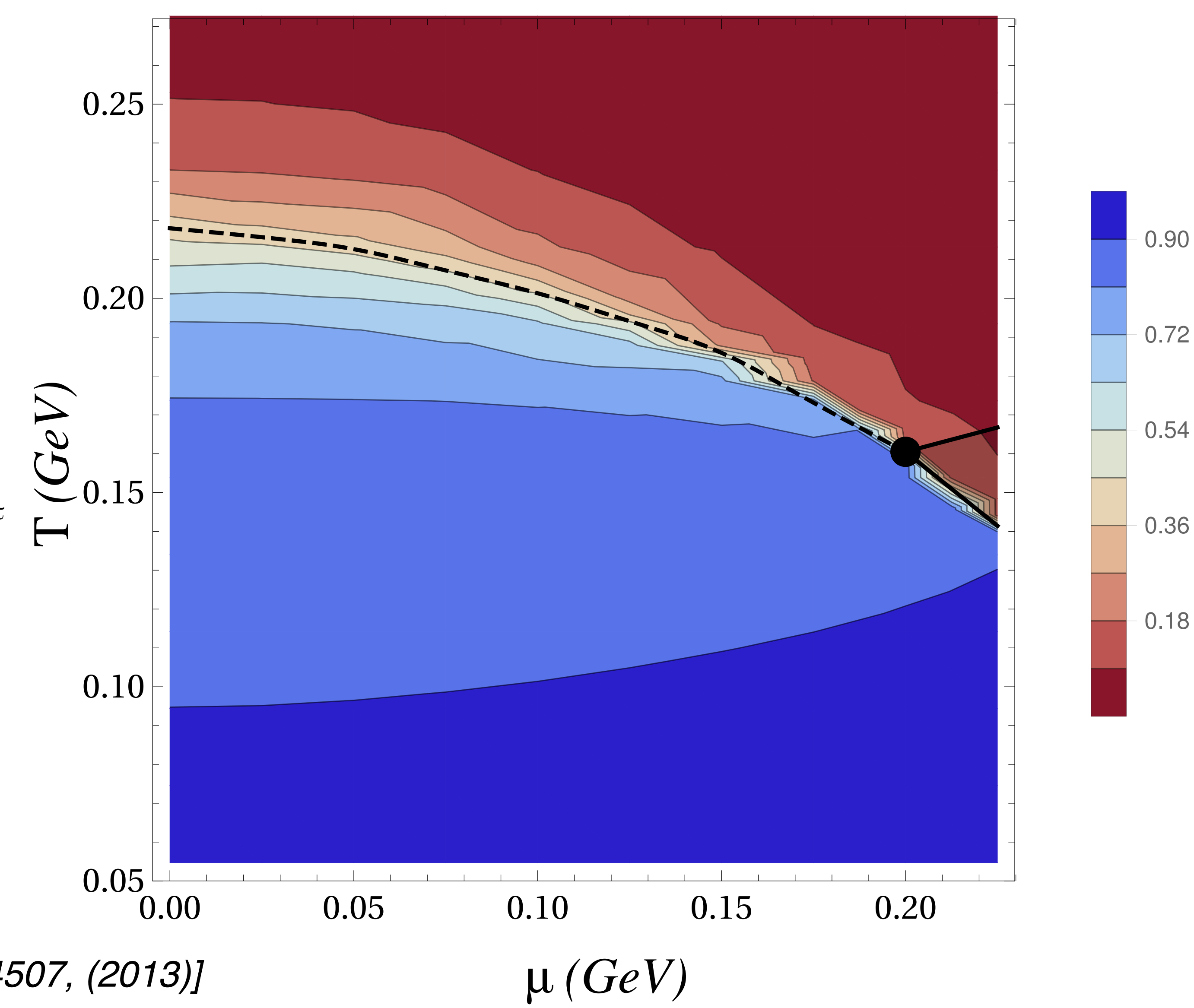
Contant Romain

Kopaonik : 12.03.2018

Computation on-going

SU(2)




Diquark condensate : *[S. Cotter et al. Phys.rev., vol. D87, pp. 034507, (2013)]*

Polyakov loop : *[S. Hands et al. Eur.phys.j., vol. C48, pp. 193, (2006)]*

CEP ≈ (200, 160) MeV

→ Scale fixing procedure ?

Contant Romain
Kopaonik : 12.03.2018

# Back up

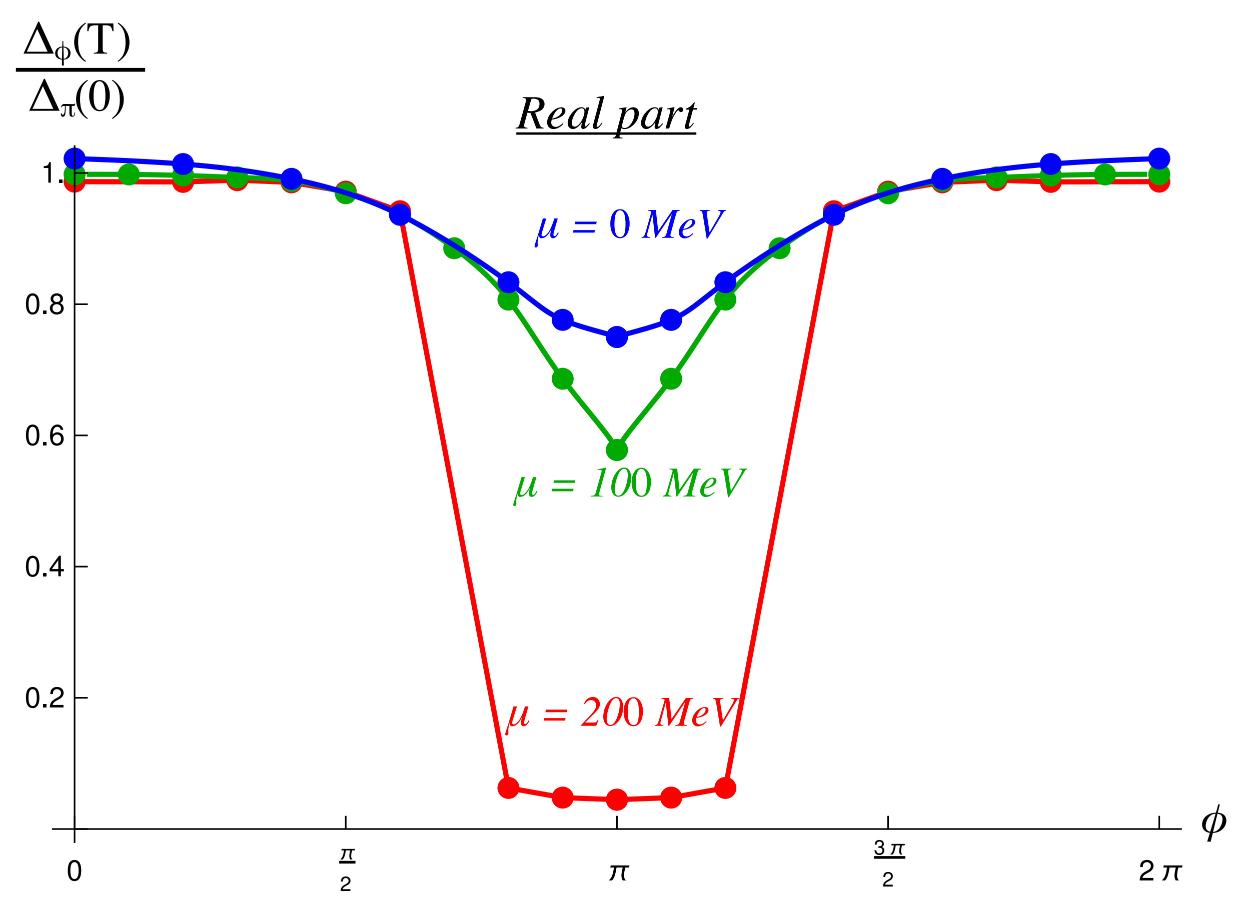
Computation on-going

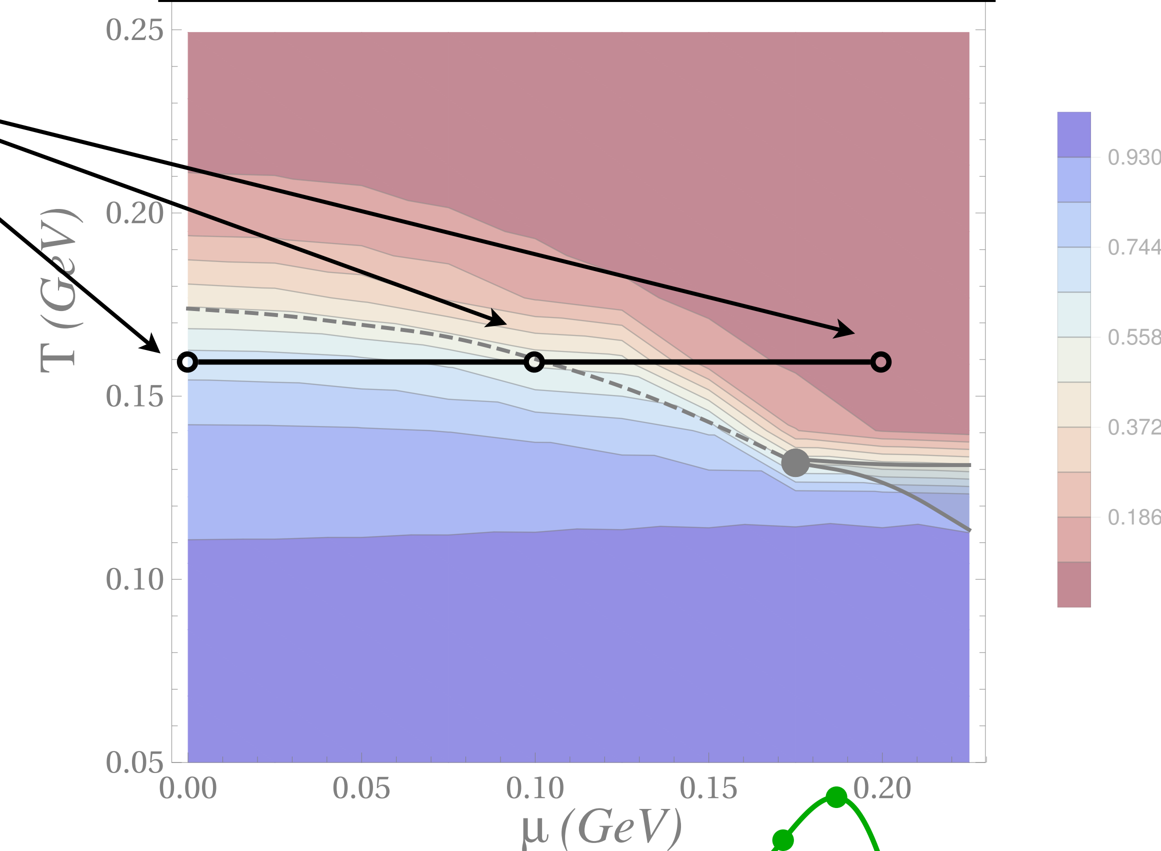
→ Chemical potential effects on the dual condensate

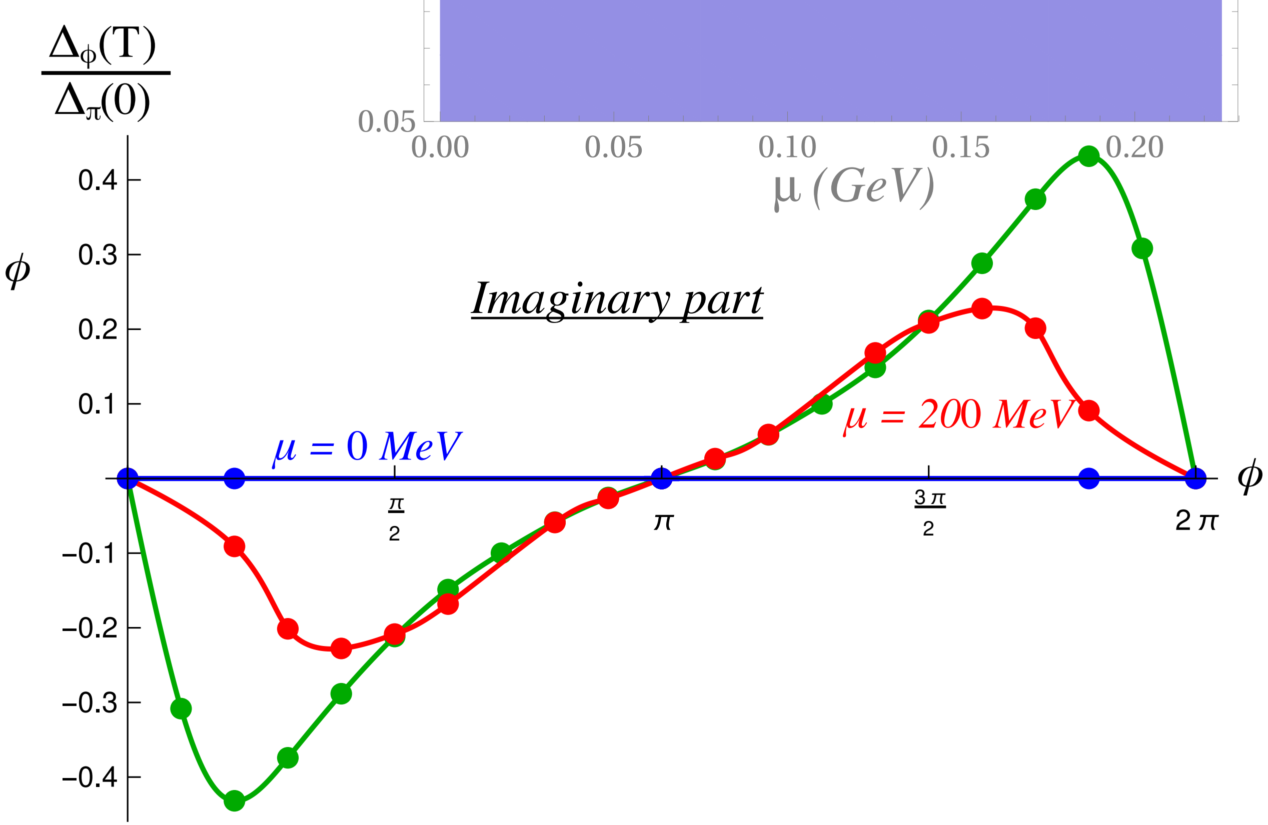
$$\Delta_\phi(T) = Z_2(Z_m)N_c\, T \sum_{\omega_n(\phi)} \int_p \frac{d^3\vec{p}}{(2\pi)^3} \mathrm{Tr}[S(\vec{p},\omega_n(\phi))]$$

$$\omega_n = \pi\, T \left(2n + \frac{\phi}{\pi}\right)$$

- Follow the line $T = 159\ MeV$ for selected $\mu$




$\mu = 0\ MeV$

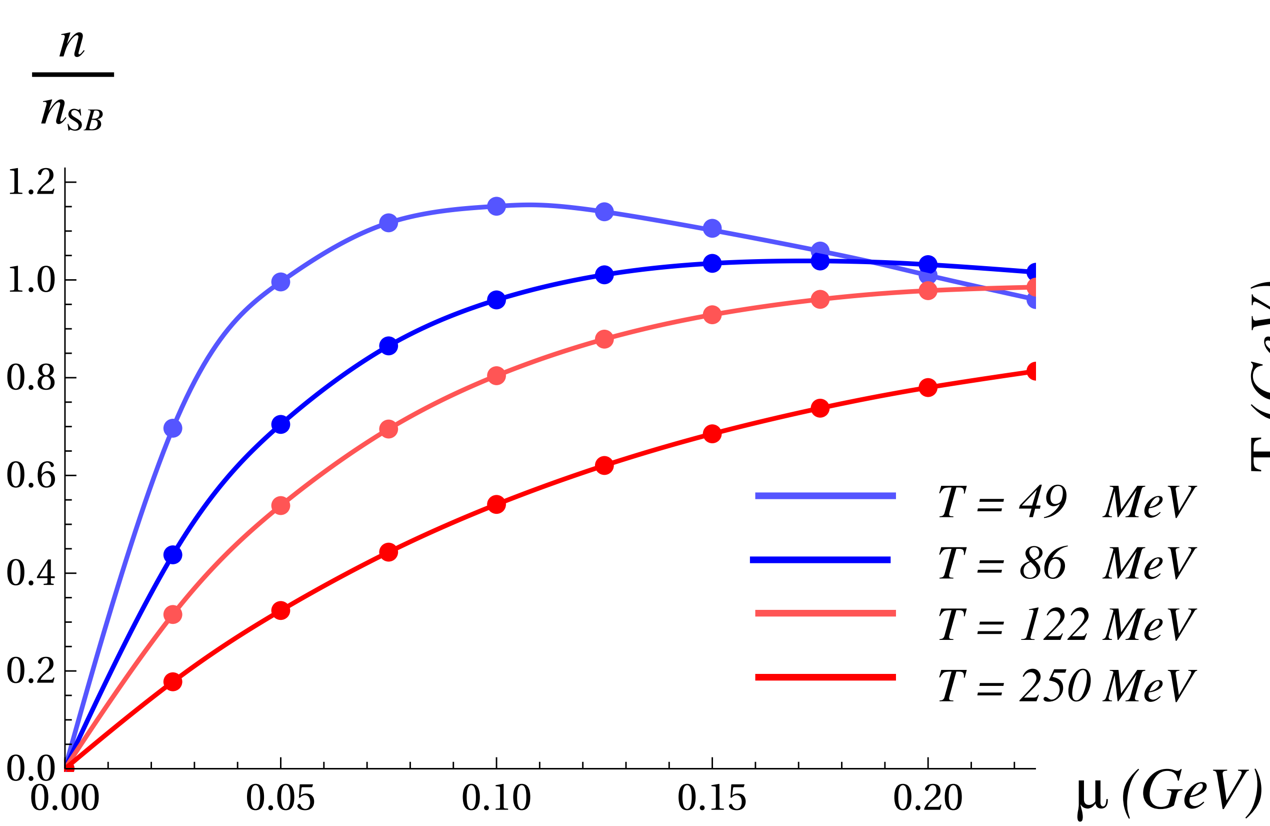
$\mu = 100\ MeV$

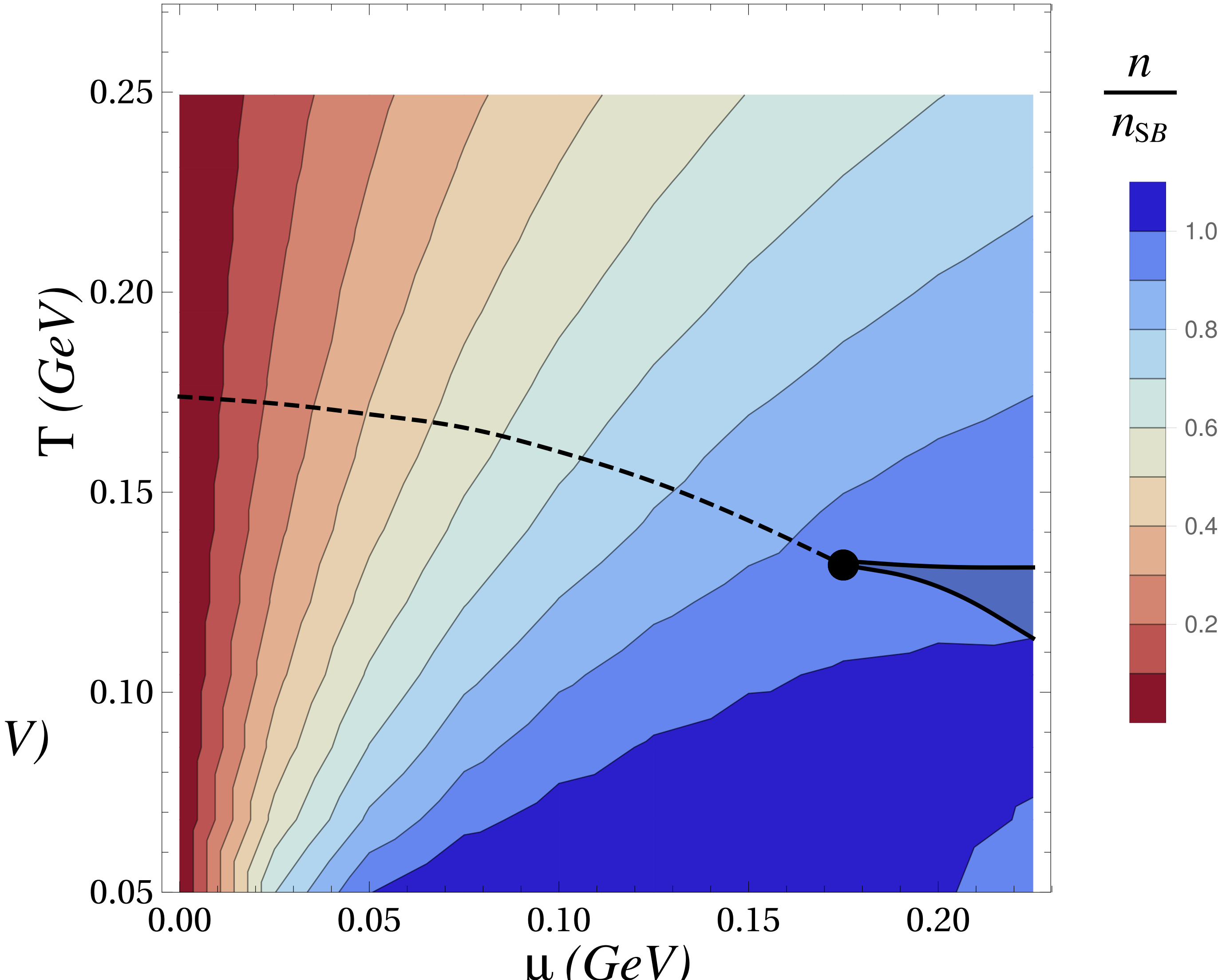
$\mu = 200\ MeV$

Computation on-going

$$S^{-1}(\vec{p},\omega_n) = A(\vec{p},\omega_n)\,\vec{\gamma}\,\vec{p} + C(\vec{p},\omega_n)\,\omega_n\gamma_4 + B(\vec{p},\omega_n) + \omega_n\gamma_4\,\vec{p}\,\vec{\gamma}\,D(\vec{p},\omega_n)$$

$$n_q = Z_2 \sum_{\omega_n} \int \frac{d^3\vec{p}}{(2\pi)^3}\, Tr[\gamma_4\, S(\vec{p},\omega_n)]$$



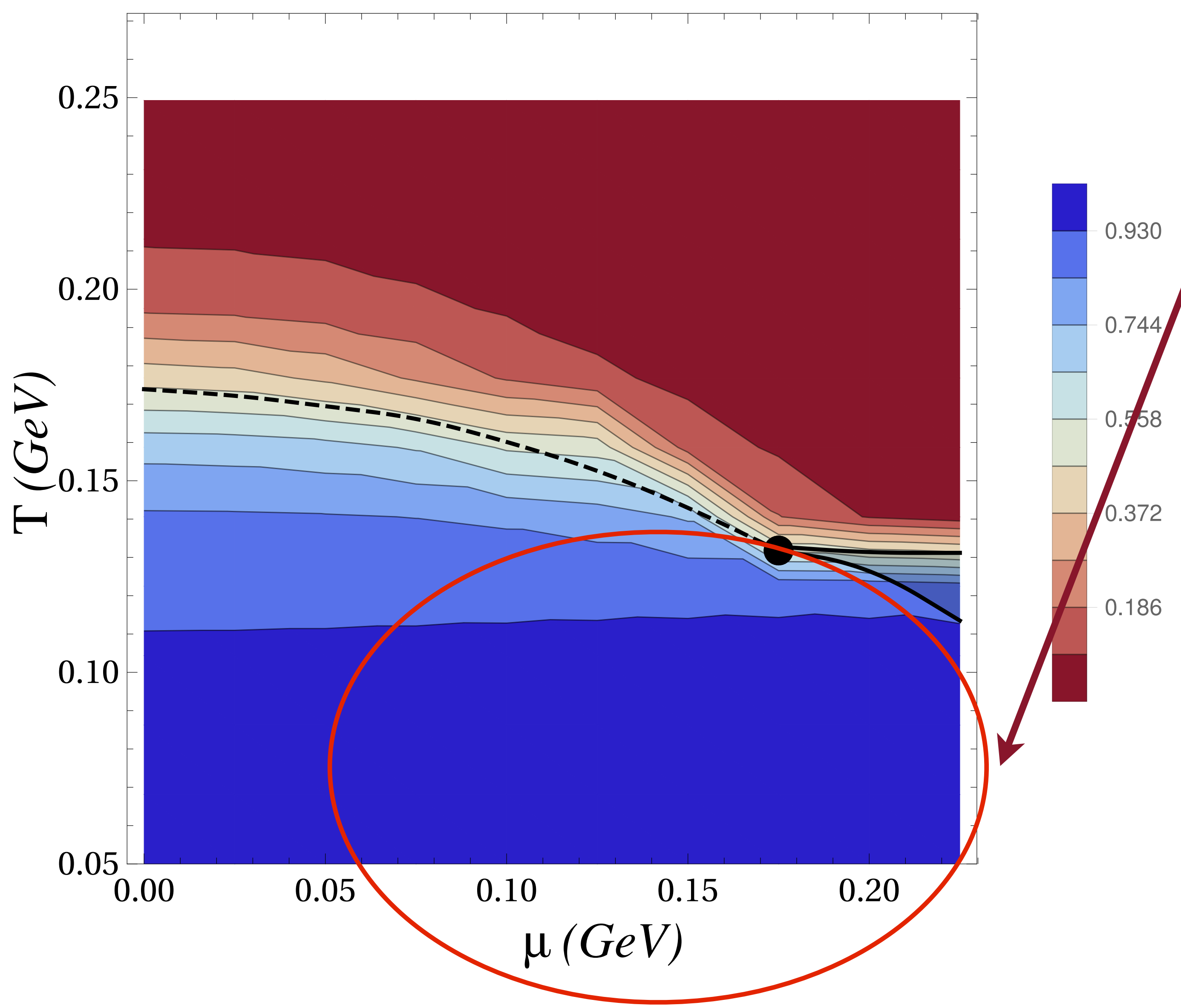



# Back up

Contant Romain



Computation on-going




Require an additional "ingredient"

- Nambu-Gorkov off-diagonal terms

  → 2SC, CFL, ....

  *[D. Müller, M. Buballa and J. Wambach (2013)]*

- Inhomogeneous phases

  *[D. Müller, M. Buballa and J. Wambach (2013)]*