Astroparticles

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Tutorials

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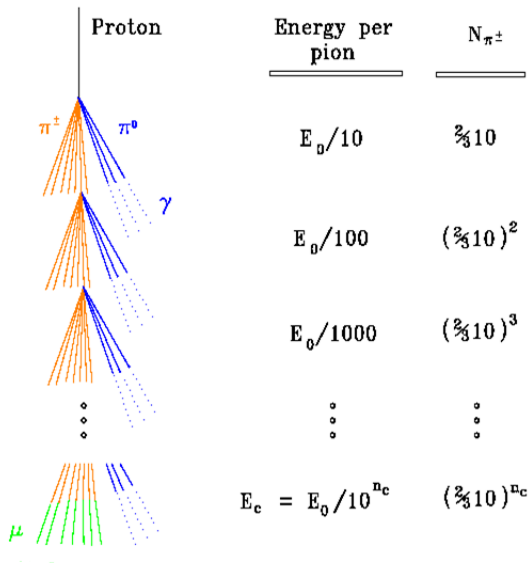
Menu:

A classical problem on the Heitler model of shower development.

1 Heitler model for EM showers



Using a simplified model à la Heitler, we want to model the EM and the muonic component of shower induced by a proton with an energy $E_0 = 10^{18}$eV and to study its longitudinal development.

One gives the critical energy to be (fixed) $\varepsilon_\pi = 100$ GeV and $\lambda_\pi = 120$ g/cm^2 is the pion interaction length.

We will do the following hypotheses:

1. At each fixed interaction length λ, all the non decayed hadrons with energy $> \varepsilon_\pi$, interact with a nucleus of the atmosphere.

2. Each interaction produces secondary hadrons (we assume here only pions) with multiplicity m, $2/3$ of them are $\pi^\pm$ and $1/3$ are π^0. One assume that the multiplicity is fixed and its value is $m = 10$.

3. Each of the m pions produced takes away a fraction of the parent energy $1/m$.

4. π^0 decay immediately in two γ that will feed the EM component.

5. When charged pions reach the critical energy ε_π, they all decay and produce each one muon that propagates to the ground. We will assume that the muons take away 1/2 of the critical energy.

6. Both the hadronic and the EM component reach their respective critical energy at about the same depth. After this point, charged pions will decay before they interact and the hadronic shower development will stop. Charged pions decay and produce muons. As for the EM component, ionisation losses of e^+, e^- start to dominate over the radiation processes and the development will also stop. The overall shower

will decrease in terms of number of particle. The depth at which both components reach critical energy is also where the number of particles in the shower reaches maximum. This depth is called X_p^{max} and the number of particles at that point is called N^{max} (also called the shower size).

Remark about the EM component development: the EM component, which is fed at each step of the hadronic shower by the $\pi_0 \to \gamma\gamma$ decays, is going to develop as well on its own (but instead with multiplicity 2 and interaction length X_0). As $\lambda_\pi \approx 4 \times X_0$, after every λ_π step of roughly $4 \times X_0$ the EM components energy is divided into $2^4 = 16$ parts which is not very different than the assumed pion multiplicity. Hence, we will make the assumption here that the hadronic and EM components reach their respective critical energy at about the same depth. Note that this coincidence can also be described by the following numerical equivalence:

$$\frac{\lambda_\pi}{X_0} \approx \frac{\ln(10)}{\ln(2)}.$$

Try to answer the following questions:

1. Find the number of interaction lengths before the hadronic component reaches its critical energy and give an approximate value using the given parameters.

Energy is divided by 10 at every steps so:

$$\varepsilon_\pi = \frac{E_0}{10^{n_c}}$$

hence:

$$n_c = \log_{10}\left(\frac{E_0}{\varepsilon_\pi}\right)$$

With $E_0 = 10^{18}$ eV and $E_c = 100$ GeV $= 10^{11}$ eV one immediately finds $n_c = 7$.

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2. Show that

$$X^{max} = \lambda_\pi \log_{10}\left(\frac{E_0}{\varepsilon_\pi}\right).$$

The extension length in this simple model is thus given by:

$$X^{max} = \lambda_\pi \times n_c = \lambda_\pi \times \log_{10}\left(\frac{E_0}{\varepsilon_\pi}\right)$$

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3. By how much does the depth of maximum X_p^{max} change when the primary energy E_0 changes by a $\times 10$?

Obviously, one decade in energy is one more interaction length to reach the maximum. The shower extension depends logarithmically on the primary energy.

$$X^{max}(10 \times E_0) = \lambda_\pi \times \log_{10}\left(\frac{10 \times E_0}{\varepsilon_\pi}\right) = \lambda_\pi \times \log_{10}\left(\frac{E_0}{\varepsilon_\pi}\right) + \lambda_\pi = X^{max}(E_0) + \lambda_\pi$$

Let's assume that the superposition principle holds, namely that a shower initiated by a nucleus with energy E_0 consisting of A nucleons can be described as the superposition of $A\times$ proton-induced showers each having an initial energy E_0/A.

4. Give the expression for the depth of the maximum development X_A^{max} of a shower initiated by a nucleus of mass A as a function of the interaction length λ_π and the depth of maximum for a proton shower X_p^{max}.

The shower is a superposition of A showers having each an initial energy E_0/A, so, using the above results:

$$X_A^{max} = \lambda_\pi \times \log_{10}\left(\frac{E_0/A}{\varepsilon_\pi}\right) = X_p^{max} - \lambda_\pi \log_{10}(A) \approx X_p^{max} - X_0 \frac{\ln(A)}{ln(2)}$$

5. What would be the statistical difference between the depth of maximum of iron showers and of proton showers?

For $A = 56$, one gets:

$$X_{\text{Fe}}^{max} = X_p^{max} - \lambda_\pi \log_{10}(A) \approx X_p^{max} - 209\text{g/cm}^2$$

6. Given that the resolution on the X^{max} measured by the Pierre Auger Observatory using the fluorescence telescopes techniques reaches $\Delta X^{max} = 20$ g/cm^2, what is the resolution achieved on $\ln(A)$?

$$\Delta X^{max} = \lambda_\pi \Delta(\ln(A)) = \frac{\lambda_\pi}{\ln(10)}\Delta(\ln(A))$$

so:

$$\Delta(\ln(A)) = \frac{\ln(10) \times \Delta X^{max}}{\lambda_\pi} = 38\%$$

Note that in reality, the X^{max} measurements are affected by very large shower to shower fluctuations (mostly due to the broad distribution of the depths of first-interactions) that spoil the resolution on $\ln(A)$.

Let now study the energy balance un muons and in the EM component.

7. Compute what is the fraction of the total energy transferred to the EM component after n_c steps.

$$
\begin{aligned}
E_{\rm EM}/E_0 &= \frac{1}{E_0}\sum_{n=1}^{n_c}\left(\frac{E_0}{10^n}\right)\left(\frac{2}{3}\times 10\right)^{n-1}\left(\frac{1}{3}\times 10\right) = \sum_{n=1}^{n_c}\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^n \\
&= \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{1-(2/3)^7}{1-2/3}\right) = 1 - 5.85\times 10^{-2} = 94\%
\end{aligned}
$$

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8. Compute the number of muons produced at the end of the development. What fraction of energy do they take away.

The number of muons is equal to the number of pions at the critical energy:

$$
\begin{aligned}
N_\mu &\approx N^c_{\pi^\pm} = \left(\frac{2}{3}\times 10\right)^{n_c} \\
\log_{10} N_\mu &\approx \left(1+\log_{10}\left(\frac{2}{3}\right)\right) n_c \\
&= 0.82\log_{10}(E_0/E_c) \\
\Rightarrow N_\mu &\approx \left(\frac{E_0}{E_c}\right)^{0.82}
\end{aligned}
$$

The energy of each muon is assumed to be $E_c/2$ and hence:

$$
E_\mu = \frac{1}{2}E_c\left(\frac{2}{3}\times 10\right)^{n_c} = \frac{1}{2}E_0\left(\frac{2}{3}\right)^{n_c} = \frac{1}{2}\times 5.85\times 10^{-2} \approx 3\%
$$

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9. Compare the EM, muonic and initial energy. Do you observed missing energy? What is taking this missing energy away? (this will help you with

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Obviously when imposing half of the pion energy is transferred to the muon, one takes into account the energy transferred to neutrinos ! Thus approximately 3% of the energy is missing and that missing is energy carried away by neutrinos.

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10. Suppose now that the incident particle is an iron nucleus with the same total energy than the proton. Assuming superposition principle holds, what is now the muons energy fraction?

Superposition principle again: a nucleus ^AN is equivalent to A protons.

Given:

$$N_\mu \approx \left(\frac{E_0}{E_c}\right)^{0.82}$$

at equivalent total energy, the superposition principle says:

$$N_\mu^A(E) \propto A\left(\frac{E_0}{AE_c}\right)^{0.82}$$

Thus:

$$\begin{aligned} N_\mu^A(E) &\approx A^{(1-0.82)} \times N_\mu^p(E) \approx A^{0.18} \times N_\mu^p(E) \\ N_\mu^{Fe} &\approx 2 \times N_\mu^p(E) \end{aligned}$$

In fact one observes 80% more muons
for a ^{56}Fe primary compared to a proton with the same total energy.

Conclude if this muons energy fraction can be used to deduce the nature of the incident particle.

Measuring total energy from the sum of muons energy + EM energy and computing the fraction of it the muonic component, one obtains a rather reliable measurement of the $\log_{10}(A)$ of the primary nuclei. The problem as usual comes from shower to shower fluctuations (so that makes it difficult on an event by event basis) and from the fact that separating experimentally the muon component from the EM is not trivial.

A few useful numerical values and formulae:
$(2/3)^7 \approx 6 \times 10^{-2}$
$\log_{10}(x) = \ln(x)/\ln(10)$
$\log_2(x) = \ln(x)/\ln(2)$
$\ln(10) \approx 2.3$
$\ln(2) \approx 0.7$
$\log_{10}(56) \approx 1.75$
$\log_2(56) \approx 5.8$
$\sum_{k=1}^{n} r^k = r\frac{1-r^k}{1-r}$