





European School of Instrumentation in Particle & Astroparticle Physics

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Tracking version 0.0

O Hypothesis:

- → Two sensors
 - <u>perfect</u> positions
 - Infinitely thin
- → 1 straight tracks
 - 2 parameters (a,b)



O Estimation of track parameters
 Assuming track model is straight

 $a = \frac{x_1 - x_0}{z_1 - z_0}$, $b = \frac{x_0 z_1 - x_1 z_0}{z_1 - z_0}$

→ No uncertainty !

Tracking version 1.0

• Hypothesis:

- → Two sensors
 - Positions with UNCERTAINTY σ_{det}
 - Infinitely thin
- → 1 straight tracks
 - 2 parameters (a,b)



- O Estimation of track parameters
 Assuming track model is straight
 - Uncertainties from error propagation



Tracking version 1.1

• Hypothesis:

- → More than two sensors
 - Positions with uncertainty σ_{det}
 - Infinitely thin
- → 1 straight tracks
 - 2 parameters (a,b)



- Estimation of track parameters
 - Assuming track model is straight
 - Need FITTING PROCEDURE least square
 - Need covariance matrix of measurements (here diagonal)
 - → Uncertainties from error propagation
 - Detail depends on geometry
 - Both estimation & uncertainties improve

ESIPAP 2017 - Tracking - J.Baudot

$$a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2} , b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$$





See LSM on

straight tracks

later



Tracking version 2.0

• Hypothesis:

- → More than two sensors
 - Positions with uncertainty $\sigma_{
 m det}$
 - With some THICKNESS
 → physics effect
- → 1 straight tracks
 - 2 parameters (a,b)



- Assuming track model is straight
 - Need fitting procedure least square
 - Need covariance matrix of measurements physics effect → NON DIAGONAL terms
- → Uncertainties from error propagation
- same estimators but increased uncertainties

 $a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2} , b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$

Covariant matrix expression not analytic !

Tracking RELOADED

• Hypothesis:

- → More than two sensors
 - Positions with uncertainty σ_{det}
 - With some thickness
- → MANY straight tracks
 - Still 2 parameters (a,b)...per track!
- New step = FINDING
 - → Which hits to which tracks ?
 - Strongly depends on geometry
- Estimation of track parameters
 - → Happens after finder
 - → Same procedure as before





Lecture outline

- 1. Basic concepts
- 2. Position sensitive detectors
- 3. Standard algorithms
- 4. Advanced algorithms
- 5. Optimizing a tracking system
- 6. References

}	first lecture
}	second lecture
}	third lecture

practice

Motivations basic concepts

- O Motivations
- Types of measurements
- O The 2 main tasks
- Environmental considerations
- Figures of merit

ESIPAP 2017 - Tracking - J.Baudot

Motivations

- Understanding an event
 - → Individualize tracks ~ particles
 - → Measure their properties
 - → LHC: ~1000 particles per 25 ns "event"
- Track properties
 - → Momentum ⇔ curvature in B field
 - Reconstruct invariant masses
 - Contribute to jet energy estimation
 - → Energy ⇔ range measurement
 - Limited to low penetrating particle
 - → Mass ⇔ dE/dx measurement
 - → Origin ⇔ vertexing (connecting track)
 - Identify decays
 - Measure flight distance
 - → Extension ⇔ particle flow algorithm (pfa)
 - Association with calorimetric shower



8 jets event (tt-bar h) @ 1 TeV ILC

Momentum measurement

- Magnetic field curves trajectories
 - → Rewritten with position (x) and path length (l) → basic equation:

 $\frac{\mathrm{d}p}{\mathrm{d}t} = q\vec{v} \times \vec{B}$

- → In B=4T a 10 GeV/c particle will get a sagitta of 1.5 cm @ 1m
- Fixed-target experiments
 - Dipole magnet on a restricted path segment
 - Measurement of deflection (angle variation)
- O Collider experiment
 - → Barrel-type with axial B over the whole path
 - → Measurement of curvature (sagitta) $p_T(\text{GeV/c}) = 0.3 \cdot B(T) \cdot R(m)$
- Other arrangements
 - → Toroidal B... not covered
- Two consequences
 - Position sensitive detectors needed
 - Perturbation effects on trajectories limit precision on track parameters







Vertex measurements 1/3

- O Identifying through topology
 - Short-lived weakly decaying particles
 - Charm c τ ~ 120 μm
 - Beauty c τ ~ 470 μm
 - tau, strange/charmed/beauty particle
- Exclusive reconstruction
 - → Decay topology with secondary vertex
 - ➤ Exclusive = all particles associated
- O Inclusive "kink" reconstruction
 - Some particles are invisible (ν)







Vertex measurements 2/3

- Inclusive reconstruction
 - Selecting parts of the daughter particles
 = flavor tagging
 - → based on impact parameter (IP)
 - $\rightarrow \sigma_{\rm IP} \sim 20-100 \,\mu{\rm m}$ requested
- Definition of impact parameter (IP)
 - Also DCA = distance of closest approach from the trajectory to the primary vertex
 - → Full 3D or 2D (transverse plane d_{ρ}) +1D (beam axis)
 - → Sign extremely useful for flavor-tagging



Sign defined by charge + traj. Position /VP





Vertex measurements 3/3

- Finding the event origin
 - → Where did the collision did occur?
 - = Primary vertex
 - → (life)Time dependent measurements
 - CP-asymmetries @ B factories ($\Delta z^{260-120} \mu m$)
 - → Case of multiple collisions / event
 - >> 10 vertex @ LHC



• Remarks for collider

- → Usually no measurement below 1-2 cm / primary vertex
 - Due to beam-pipe maintaining vacuum
- → Requires extrapolation → expect uncertainties

Energy measurement

- O Usually not a tracker task
 - → CALORIMETERs (see lecture by Isabelle)
 - \rightarrow Indeed calorimeters gather material to stop particles while trackers try to avoid material (multiple scattering)
 - → however...calorimetry tries to improve granularity
- Particle flow algorithm
 - → LHC / ILC
- Energy evaluation by counting particles
 - → Clearly heretic for calorimetry experts
 - NOTCOVERED - Requires to separate $E_{deposit}$ in dense environment
- Range measurement for low energy particles
 - → Stack of tracking layers
 - → Modern version of nuclear emulsion

Multiple scattering - 1/4

- Reminder on the physics (see other courses)
 - → Coulomb scattering mostly on nuclei
 - → Molière theory description as a centered gaussian process
 - the thinner the material, the less true → large tails



Multiple scattering -2/4

- O <u>In-space</u> description (defined by fixed x/y axes)
 → Corresponds to (θ_x, θ_y) with $p_{out,T}^2 = p_{out,x}^2 + p_{out,y}^2 \begin{cases} p_{out} \sin \theta_x \approx p_{out} \theta_x \\ p_{out} \sin \theta_y \approx p_{out} \theta_y \end{cases}$ $\theta^2 = \theta_x^2 + \theta_y^2$

 - $\rightarrow \theta x$ and θy are independent gaussian processes

 $\sigma_{\theta}^2 = \sigma_{\theta x}^2 + \sigma_{\theta y}^2$ and $\sigma_{\theta x} = \sigma_{\theta y} = \frac{\sigma_{\theta}}{\sqrt{2}}$

z



- Important remark when combining materials
 - → Total thickness T = ΣT_i , each material (i) with $X_0(i)$
 - → Definition of effective radiation length ➡

$$X_{0,eff} = \frac{\sum T_i \times X_0(i)}{T}$$

→ Consider single gaussian process

$$\sigma_{eff} \propto \sqrt{\frac{T}{X_{0,eff}}}$$

and never do variance addition (which minimize deviation)



Multiple scattering -4/4

Remember

this simple case

- O Impact on tracking algorithm
 - → The track parameters evolves along the track !
 - \rightarrow May drive choice of reconstruction method

• Photon conversion

→ Alternative definition of radiation length probability for a high-energy photon to generate a pair over a path dx:

CMS Preliminary 2010

- $\rightarrow \gamma \rightarrow e^+e^- = \text{conversion vertex}$
- → Generate troubles :
 - Additional unwanted tracks
 - Decrease statistics for electromagnetic calorimeter



(silicon tracker only)

z, (first point)

z, (last po

The two main tasks - 1/2

The collider paradigm

- Basic inputs from detectors
 - Succession of 2D or 3D points (or track segments)
 - ➡ Who's who ?
- O 2 steps process
 - → Step 1: track identification = finding = pattern recognition
 - Associating a set of points to a track
 - → Step 2: track fitting
 - Estimating trajectory parameters → momentum
- O Both steps require
 - Track model (signal, background)
 - → Knowledge of measurement uncertainties
 - Knowledge of materials traversed (Eloss, mult. scattering)
- Vertexing needs same 2 steps
 - Identifying tracks belonging to same vertex
 - Estimating vertex properties (position + 4-vector)





The two main tasks - 2/2

- Telescope mode
 - → Single particle at a time
 - Sole nuisance = background
 - → Trigger from beam
 - Often synchronous
 - → Goal = get the incoming direction
- The astroparticle way
 - → Similar to telescope mode
 - ➤ No synchronous timing
 - → Ex: deep-water ν telescopes





Environmental conditions - 1/2

- Life in a real experiment is tough (for detectors of course)
 - → Chasing small cross-sections → large luminosity and/or energy
 - → Short interval between beam crossing
 - LHC: 25 ns (and >10 collisions / crossing)
 - CLIC: 5 ns (but not continuous)
 - → Large amount of particles (could be > 10^7 part/cm²/s) → background, radiation
 - makes the finding more complicated
 - → Vacuum could be required (space, very low momentum particles (CBM, LHCb))
- Radiation tolerance
 - → Two types of energy loss
 - Ionizing (generate charges): dose in Gy = 100 Rad
 - Non-ionizing (generate defects in solid): fluence in n_{eq}(1MeV)/cm²
 - → The more inner the detection layer, the harder the radiation (radius² effect)
 - → Examples for most inner layers:
 - LHC: 10^{15} to $< 10^{17} n_{eq} (1 MeV) / cm^2$ with 50 to 1 MGy
 - ILC: $<10^{12} n_{eq}(1 \text{MeV})/\text{cm}^2$ with 5 kGy

Environmental conditions - 2/2

- O Timing consideration
 - Integration time drives occupancy level (important for finding algorithm)
 - → Time resolution offers time-stamping of tracks
 - Tracks in one "acquisition event" could be associated to their proper collision event if several have piled-up
 - Key question = triggered ot not-triggered experiment?

• Heat concerns

→ Spatial resolution → segmentation → many channels Readout speed → power dissipation/channel



- Efficient cooling techniques exist BUT add material budget and may not work everywhere (space)
- Summary
 - → Tracker technology driven by environmental conditions: hadron colliders (LHC)
 - → Tracker technology driven by physics performances: lepton colliders (B factories, ILC), heavy-ion colliders (RHIC, LHC)
 - → Of course, some intermediate cases: superB factories, CLIC

• For detection layer

- → Detection efficiency
 - Mostly driven by Signal/Noise
- → Intrinsic spatial resolution
 - Driven by segmentation (not only)
 - Useful tracking domain $\sigma < 1$ mm
- Linearity and resolution on dE/dx
- → Material budget
- → "Speed" (integration time, time resolution, ...)
- For detection systems (multi-layers)
 - ➤ Two-track resolution
 - Ability to distinguish two nearby trajectories
 - Mostly governed by signal spread / segments
 - ➤ Momentum resolution
 - → Impact parameter resolution
 - Sometimes called "distance of closest approach" to a vertex

DELPHI 26024 / 1730 0.0 cm 7.5 c DELPHI 26024 / 1730 2.0 c 0.0 cn

Figures of Merit

2. Detection technologies

O Intrinsic resolution

- O Single layer systems
 - Silicon, gas sensors, scintillator
- Multi-layer systems
 Drift chamber and TPC
- O Tentative simplistic comparison
- O Magnets
- O Leftovers
- Practical considerations

Intrinsic resolution

+HV

n⁺ backplane

particle track

O Position measurement comes from segmentation \rightarrow Pitch $\sigma = \frac{\text{pitch}}{\sqrt{12}}$ Digital resolution Signal generated Sensitive segments O Improvement from signal sharing \rightarrow Position = charge center of gravity pitch $\sigma \propto$ signal/noise \rightarrow Effects generated by Signals generated Secondary charges spread inside volume Sensitive segments • Inclined tracks (however, resol. limited at large angles) → Potential optimization of segmentation / sharing • Work like signal sampling theory (Fourier transform) O B field p⁺ implant → Warnings: ٥V holes • Lorentz force from B mimic the effect n-type E counterproductive / 2-track resolution silicon electrons

Silicon sensors: strips

- Basic sensitive element
 - → E-h pairs are generated by ionization in silicon
 - 3.6 eV needed
 - 300 µm thick Si generates ~ 22000 charges for MIP BUT beware of Landau fluctuation
 - → Collection: P-N junction = diode
 - Full depletion (10 to 0.5 kV) generates a drift field (10⁴ V/cm)
 - Collect time ~ 15 ps/µm
- Silicon strip detectors
 - → sensor "easily" manufactured with pitch down to $^{\sim}25 \,\mu\text{m}$
 - → 1D if single sided
 - → Pseudo-2D if double-sided
 - Stereo-angle useful against ambiguities
 - Difficult to go below 100 μ m thickness
 - → Speed and radiation hardness: LHC-grade





Silicon sensors: hybrid-pixels

O Concept

- → Strips → pixels on sensor
- → One to one connection from electronic channels to pixels

• Performances

- Real 2D detector
 & keep performances of strips
 - Can cope with LHC rate (speed & radiation)
- Pitch size limited by physical connection and #transistors for treatment
 - minimal (today): 50x50 μm²
 typical: 100x150/400 μm²
 - spatial resolution about10 μm
- → Material budget
 - Minimal(today): 100(sensor)+100(elec.) μm
- → Power budget: 10 μ W/pixel





CMOS Pixel Sensor (CPS)

O Concept

- → Use industrial CMOS process
 - Implement an array of sensing diode
 - Amplify the signal with transistors near the diode
- → Benefit to
 - granularity: pixel pitch down to \sim 10 μ m
 - material: sensitive layer thickness as low as 10-20 μm
- → Known as Monolitic Active Pixel Sensors (MAPS)

Sensitive layer

- → If undepleted & thin (10-20 μ m)
 - Slow (100 ns) thermal drift of charges
 - non-ionizing rad. tolerance $\leq 10^{13} n_{eq(1MeV)}/cm^2$
- → If fully depleted (from 10 to 100 μ m)
 - Fast (few ns) field-driven drift of charges
 - non-ionizing rad. tolerance > $10^{15} n_{eq(1MeV)}/cm^2$



CMOS Pixel Sensor

O Concept

- → Use industrial CMOS process
 - Implement an array of sensing diode
 - Amplify the signal with transistors near the diode
- Gain in granularity: pitch down to $\sim 10 \,\mu m$
- → Gain in sensitive layer thickness ~ 10-20 µm
- → For undepleted thin sensitive layer
 - Slow (100 ns) thermal drift of charges
 - non-ionizing rad. tolerance $\leq 10^{13} n_{eq(1MeV)}/cm^2$
- → For fully depleted thin to thick sensitive layer
 - Fast (few ns) field-driven drift of charges
 - non-ionizing rad. tolerance > $10^{15} n_{eq(1MeV)}/cm^2$

• Performances

- Spatial resolution 1-10 μm (in 2 dimensions)
- → Material budget: ≤ 50 µm
- Power budget: $\leq \mu W/pixel$
- → Integration time ~5-100 µs demonstrated
 - $\sim 1 \ \mu s$ in development
- → Timestamping @ ns level in development





Mimosa resolution vs pitch

Other active pixel sensors

O DEPFET



- → Fully depleted sensitive layer
- → Large amplification
- → Still require some read-out circuits
 - Not fully monolithic
 - Possibly limited in read-out speed

O Silicon On Insulator (SOI)



- → Fully depleted sensitive layer
- → Fully monolithic
- → Electronics similar to MAPS

Wire chambers

• Basic sensitive element

- → Metallic wire, 1/r effect generated an avalanche
- Signal depends on gain (proportional mode) typically 10⁴
- → Signal is fast, a few ns
- Gas proportional counters
 - → Multi-Wire Proportional Chamber
 - Array of wires
 - 1 or 2D positioning depending on readout
 - Wire spacing (pitch) limited to 1-2 mm
 - → Straw or drift tube
 - One wire in One tube
 - Extremely fast (compared to Drift Chamber)
 - Handle high rate
 - Spatial resolution <200 μm
 - Left/right ambiguity





Electric fields line around anode wires

Wire chambers "advanced"

- O Micro-pattern gas multipliers
 - → MSGC
 - Replace wires with lithography micro-structures
 - Smaller anodes pitch 100-200 μm
 - BUT Ageing difficulties due to high voltage and manufacturing not so easy
 - → GEM
 - Gain 10⁵
 - Hit rate 10⁶ Hz/cm2



Wire chambers "advanced"

- O Micro-pattern gas multipliers
 - → MSGC
 - Replace wires with lithography micro-structures
 - Smaller anodes pitch 100-200 μm
 - BUT Ageing difficulties due to high voltage and manufacturing not so easy
 - → GEM
 - Gain 10⁵
 - Hit rate 10⁶ Hz/cm2
 - → MICROMEGAS
 - Even smaller distance anode-grid
 - Hit rate 10⁹ Hz/cm2
 - → More development
 - Electron emitting foil working in vacuum!



Drift chambers

• Basic principle

- → Mix field and anode wires
 - Generate a drift
- → Pressurize gas to increase charge velocity (few atm)
- → 3D detector
 - 2D from wire position
 - 1D from charge sharing at both ends
- O Spatial Resolution
 - → Related to drift path

 $\sigma \propto \sqrt{\text{drift length}}$

- → Typically 100-200 µm
- 0 Remarks
 - → Could not go to very small radius



Time Projection Chambers 1/2

• Benefits

- → Large volume available
- → Multi-task: tracking + Part. Identification
- Basic operation principle
 - → Gas ionization → charges
 - → Electric field → charge drift along straight path
 - → Information collected
 - 2D position of charges at end-cap
 - 3rd dimension from drift time
 - Energy deposited from #charges
 - → Different shapes:
 - rectangles (ICARUS)
 - Cylinders (colliders)
 - Volumes can be small or very large





Time Projection Chambers 2/2

- End cap readout
 - → Gas proportional counters
 - Wires+pads, GEM, Micromegas
- Performances
 - → Two-track resolution ~ 1cm
 - Transverse spatial resolution \sim 100 200 μ m
 - → Longitudinal spatial resolution ~0.2 1 mm
 - → Longitudinal drift velocity: 5 to 7 cm/µs
 - ALICE TPC (5m long): 92 µs drift time
 - → Pro
 - Nice continuously spaced points along trajectory
 - Minimal multiple scattering (inside the vessel)
 - → Cons
 - Limiting usage with respect to collision rate




Tentative "simplistic" comparison



Magnets

Solenoid

- → Field depends on current I, length L, # turns N
 - on the centerline

$$B = \frac{\mu_0 NI}{\sqrt{L^2 + 4R^2}}$$

- Typically: 1 T needs 4 to 8 kA

 superconducting metal to limit heat
- → Field uniformity needs flux return (iron structure)
 - Mapping is required for fitting (remember B(x)?)
 - Usually performed with numerical integration
- → Calorimetry outside → limited material → superconducting
- → Fringe field calls for compensation



	Field (T)	Radius (m)	Length (m)	Energy (MJ)
ALICE	0.5	6		150
ATLAS	2	2.5	5.3	700
CMS	4	5.9	12.5	2700
ILC	4	3.5	7.5	2000

• Supercondiction

- → cryo-operation → quenching possible !
- Magnetic field induces energy: $E \propto B^2 R^2 L$
 - Cold mass necessary to dissipate heat in case of quench

Practical considerations

- From a detection principle to a detector
 - → Build large size or many elements
 - Manufacture infrastructures
 - Characterization capabilities
 - Production monitoring
 - → Integration in the experiment
 - Mechanical support
 - Electrical services (powering & data transmission)
 - Cooling (signal treatment dissipates power)
 - → Specific to trackers
 - Internal parts of multi-detectors experiment
 → limited space
 - Material budget is ALWAYS a concern
 - \Rightarrow trade-offs required





Leftovers

- Silicon drift detectors
 - → Real 2D detectors made of strips
 - → 1D is given by drift time
- Diamond detectors
 - Could replace silicon for hybrid pixel detectors
 - → Very interesting for radiation tolerance
- Plasma sensor panels
 - Derived from flat television screen
 - → Still in development
- O Charge Coupled Devices (CCD)
 - → Fragile/ radiation tolerance
- O Signal generation
 → see Ramo's theorem

- Nuclear emulsions
 - One of the most precise $\sim 1 \mu m$
 - → No timing information → very specific applications
- O Scintillators
 - → Extremely fast (100 ps)
 - Could be arranged like straw tubes
 - But quite thick ($X_0 \sim 2 \text{ cm}$)

• Finders

- First evaluation of momentum resolution
- 0 Fitters
- O Alignment

FINDING : 2 strategies

• Global methods

- → Transform the coordinate space into pattern space
 - "pattern" = parameters used in track model
- → Identify the "best" solutions in the new phase space
- → Use all points at a time
 - No history effect
- → Well adapted to evenly distributed points with same accuracy
- O Local methods
 - Start with a track seed = restricted set of points
 - Could require good accuracy from the beginning
 - → Then extrapolate to next layer-point
 - And so on...iterative procedure
 - "Wrong" solutions discarded at each iteration
 - Possibly sensitive to "starting point"
 - Well adapted to redundant information

- A simple example
 - → Straight line in 2D: model is $x = a^*z + b$
 - → Track parameters (a,b); N measurements x_i at z_i (i=1..N)
- A more complex example
 - → Helix in 3D with magnetic field
 - Track parameters (ϕ , z, D, tan λ ,C)
 - \rightarrow Measurements (ϕ , z)
- Generalization
 - → Parameters: P-vector p
 - → Measurements: N-vector c
 - → Model: function f (\Re^{P} → \Re^{N})

 $f(p) = c \iff propagation$





- Another view of the helix
 - \rightarrow s = track length
 - \rightarrow h = sense of rotation
 - $\rightarrow \lambda = \text{dip angle}$
 - → Pivot point (s=0):
 - position (x_0, y_0, z_0)
 - orientation ϕ_0

->

$$x(s) = x_o + R \left[\cos \left(\Phi_o + \frac{hs \cos \lambda}{R} \right) - \cos \Phi_o \right]$$

$$y(s) = y_o + R \left[\sin\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_o \right]$$

$$z(s) = z_o + s \sin \lambda$$

Local method 1/2

- Track seed = initial segment
 - → Made of few (2 to 4) points
 - One point could be the expected primary vtx
 - Allows to initialize parameter for track model
 - → Choose <u>most precise</u> layers first
 - usually inner layers
 - → But if high hit density
 - Start farther from primary interaction
 <u>a lowest density</u>
 - Limit mixing points from different tracks
- Extrapolation step
 - → Out or inward (=toward primary vtx) onto the next layer
 - Not necessarily very precise, especially only local model needed
 - Extrapolation uncertainty ≤ layer point uncertainty
 - Computation speed important
 - → Match (associate) nearest point on the new layer
 - Might skip the layer if point missing
 - Might reject a point: if worst track-fit or if fits better with another track



Local method 2/2

- Variant with track segments
 - → First build "tracklets" on natural segments
 - Sub-detectors, or subparts with same resolution
 - → Then match segments together
 - → Typical application:
 - Segments large tracker (TPC) with vertex detector (Si)
 - → layers dedicated to matching
- Variant with track roads
 - → Full track model used from start
- O Variant with Kalman filter
 - → See later
- Figure of merit

$$\sigma_{eff,\phi} \times \sigma_{eff,z} \times \rho_{bckgrnd}$$

- → $\sigma_{\text{eff}} = \sigma(\text{sensor}) \oplus \sigma(\text{track extrapolation}) = \text{effective spatial resolution}$
- $\rightarrow \rho$ = background hit density



Global methods 1/2

- Brute force = combinatorial way
 - → Consider all possible combination of points to make a track
 - → Keep only those compatible with model
 - → Usually too time consuming...
- Hough transform
 - → Example straight track:
 - Coord. space $y = a^*x + b \iff pattern space b = y x^*a$
 - Each point (y,x) defines a line in pattern space
 - All lines, from points belonging to same straight-track, cross at same point (a,b)
 - In practice: discretize pattern space and search for maximum
 - Applicable to circle finder
 - needs two parameters as well (r, ϕ of center) if track is assumed to originate from (0,0)
 - → More difficult for more than 2 parameters...





Global methods 2/2

- O Conformal mapping
 - → Helix transverse projection = Circle
 - $(x-a)^2 + (y-b)^2 = r^2$
 - Transform to $u = x/(x^2+y^2)$, $v = y/(x^2+y^2)$
 - Then: v = -(a/b) u + (1/2b)

• Figure of merit



$$\sigma_{eff,\phi} \times \sigma_{eff,z} \times \rho_{bckgrnd}$$

FITTING

- Why do we need to fit?
 - → Measurement error
 - → Multiple scattering error
- O Global fit
 - → Assume knowledge of:
 - all track points
 - full correlation matrix
 - → difficult if $\sigma_{\text{mult. scatt.}} \gtrsim \sigma_{\text{meas.}}$
 - → Least square method
- O Iterative fit
 - → Iterative process:
 - points included in the fit one by one
 - could be merged with finder step
 - → Kalman filter

FITTING drives track extrapolation & momentum res.

Least Square Method (LSM)

- O Linear model hypothesis
 - → P track parameters p, with N measurements c

$$\vec{c} = \vec{c}_s + A(\vec{p} - \vec{p}_s) + \vec{\varepsilon}$$

p_s = known starting point (pivot), A = track model NxP matrix,
 ε = error vector corresponding to V = covariance NxN matrix

"N measurements" means:

- K points (or layers)
- D coordinates at each point

• N = KxD

• Sum of squares: $\sum \frac{(\text{model} - \text{measure})^2}{\text{uncertainty}^2}$

$$S(\vec{p}) = (\vec{c}_s + A(\vec{p} - \vec{p}_s) - \vec{c})^T V^{-1} (\vec{c}_s + A(\vec{p} - \vec{p}_s) - \vec{c})$$

• Best estimator (minimizing variance)

$$\frac{\mathrm{d}S}{\mathrm{d}\vec{p}}(\vec{p}) = 0 \implies \vec{p} = \vec{p}_s + \left(A^T V^{-1} A\right)^{-1} A^T V^{-1} \left(\vec{c} - \vec{c}_s\right)$$

Variance (= uncertainty) of the estimator:

$$\underline{V_{\vec{p}}} = \left(A^T V^{-1} A\right)^{-1}$$

- Estimator p follows a χ^2 law with N-P degrees of freedom

• Problem \Leftrightarrow inversion of a PxP matrix (A^TV^1A)

- → But real difficulty could be computing V (NxN matrix)
 - ← layer correlations if multiple scattering non-negligible if $\sigma_{\text{mult. scatt.}} \gtrsim \sigma_{\text{meas}}$

LSM on straight tracks

- O Straight line model
 - → 2D case → D=2 coordinates (z,x)
 - → 2 parameters: a = slobe, b = intercept at z=0
- O General case
 - → K+1 detection planes (i=0...k)
 - located at z_i
 - Spatial resolution $\boldsymbol{\sigma}_{i}$
 - → Useful definitions

$$S_{1} = \sum_{i=0}^{K} \frac{1}{\sigma_{i}^{2}} , S_{z} = \sum_{i=0}^{K} \frac{z_{i}}{\sigma_{i}^{2}} , S_{xz} = \sum_{i=0}^{K} \frac{x_{i}z_{i}}{\sigma_{i}^{2}} , S_{z^{2}} = \sum_{i=0}^{K} \frac{z_{i}^{2}}{\sigma_{i}^{2}}$$

• Solutions
$$a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2}$$
, $b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$





- Case of uniformly distributed (K+1) planes $z_{i+1} - z_i = L/K$ et $\sigma_i = \sigma$ $\forall i$
 - → $S_z = 0$ → a,b uncorrelated

$$\sigma_a^2 = \frac{12K}{(K+2)L^2} \frac{\sigma^2}{K+1} , \ \sigma_b^2 = \left(1 + 12\frac{K}{K+2}\frac{z_c^2}{L^2}\right) \frac{\sigma^2}{K+1}$$

- → Uncertainties :
 - $\boldsymbol{\sigma}_{a}$ and $\boldsymbol{\sigma}_{b}$ improve with $1/\sqrt{(K+1)}$
 - $\boldsymbol{\sigma}_{a}$ and $\boldsymbol{\sigma}_{b}$ improve with 1/L
 - $\boldsymbol{\sigma}_{\rm b}$ improve with $z_{\rm c}$

LSM on fixed target geometry

B

K/4 det.

K/4 det.

Δα

P_{out}

• Hypothesis

- K detectors, each with σ single point accuracy
- Uniform field over L from dipole
 - Trajectory: $\Delta \alpha = \frac{0.3qBL}{p}$
 - Bending: $\Delta p = p \Delta \alpha$
- → Geometrical arrangement optimized for resolution
 - Angular determination on input and output angle:

$$\sigma_{\alpha}^2 = \frac{16 \sigma^2}{K l^2}$$

B

K/4 det.

• Without multiple scattering

→ Uncertainty on momentum

$$\frac{\sigma_p}{p} = \frac{8}{0.3q} \frac{1}{BL} \frac{\sigma}{l\sqrt{K}} p$$

 \mathbf{P}_{in}

K/4 det.

- Note proportionality to p!
- O Multiple scattering contribution
 - → Additional term on σ_{α} almost directly from smult.scatt $\sigma_{\theta} = \frac{13.6 \text{ (MeV/c)}}{\beta p} z$

LSM on collider geometry

- O Hypothesis
 - K detectors uniformly distributed
 each with σ single point accuracy
 - → Uniform field over path length L
- Without multiple scattering
 - Uncertainty on transverse momentum (Glückstern formula)

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sqrt{720}}{0.3q} \frac{1}{BL^2} \frac{\sigma}{\sqrt{K+6}} p_T$$

- Works well with large K > 20



K detection cylindrical layers

Kalman filter 1/2

O Dimensions

- → P parameters for track model
- → D "coordinates" measured at each point (usually D<P)
- → K measurement points (# total measures: N = KxD)
- O Starting point
 - → Initial set of parameters: first measurements
 - → With large uncertainties if unknowns
- O Iterative method
 - Propagate to next layer = prediction
 - Using the system equation
- $\vec{p}_k = G \, \vec{p}_{k-1} + \vec{\omega}_k$
- G = PxP matrix, ω = perturbation associated with covariance PxP matrix V_{ω}
- Update the covariance matrix with additional uncertainties (ex: material budget between layers)
- Add new point to update parameters and covariance, using the measure equation
 - *H*=DxP matrix, $\boldsymbol{\varepsilon}$ = measure error associated with diagonal covariance DxD matrix V_m
 - Weighted means of prediction and measurement using variance $\Leftrightarrow \chi^2$ fit
- ➤ Iterate...



$$V_{k|k-1} = V_{k-1} + V_{\omega_k}$$

$$\vec{m}_k = H \vec{p}_k + \vec{\varepsilon}_k$$

$$\vec{p}_{k} = \left(V_{k|k-1}^{-1}\vec{p}_{k|k-1} + H^{T}V_{m_{k}}^{-1}\vec{m}_{k}\right) \cdot \left(V_{k|k-1}^{-1} + H^{T}V_{m_{k}}^{-1}H\right)^{-1}$$

Kalman filter 2/2

- Forward and backward filters
 - → Forward estimate of p_k : from 1→k-1 measurements
 - → Backward estimate of p_k : from k+1→K measurements
 - → Independent estimates → combination with weighted mean = smoother step
- O Computation complexity
 - \rightarrow only PxP, DxP or DxD matrices computation (\ll NxN)
- Mixing with finder
 - → After propagation step: local finder
 - Some points can be discarded if considered as outliers in the fit (use χ^2 value)
- Include exogenous measurements
 - \rightarrow Like dE/dx, correlated to momentum
 - → Additional measurement equation

 $\vec{m}'_k = H' \vec{p}_k + \vec{\varepsilon}'_k$

 $\vec{p}_{k} = \left(V_{k|k-1}^{-1}\vec{p}_{k|k-1} + H^{T}V_{m_{k}}^{-1}\vec{m}_{k} + H^{T}V_{m_{k}'}^{-1}\vec{m}_{k}'\right) \cdot \left(V_{k|k-1}^{-1} + H^{T}V_{m_{k}}^{-1}H + H^{T}V_{m_{k}'}^{-1}H'\right)^{-1}$

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Alignment strategy

- Let's come back to one initial & implicit hypothesis
 - → "We know were the point are located."
 - → True to the extent we know were the detector is!
 - → BUT, mechanical instability (magnetic field, temperature, air flow...) and also drift speed variation (temperature, pressure, field inhomogeneity...) limit our knowledge
 - Periodic determination of positions and deformations needed = alignment
- Methods
 - Track model depends on new "free" parameters, i.e. the alignment
 - ➤ Global alignment:
 - Fit the new params. to minimize the overall χ^2 of a set of tracks (Millepede algo.)
 - Beware: many parameters could be involved (few 10³ can easily be reached)
 - ➤ Local alignment:
 - Use tracks reconstructed with reference detectors
 - Align other detectors by minimizing the "residual" (track-hit distance) width
 - Use a set of well know tracks and tracking-"friendly" environment to avoid bias





4. Advanced methods

O Why?

• (Gaussian sum filter: not treated yet)

O Neural network

O Cellular automaton

4. Advanced methods

Adaptive methods

- Shall we do better?
 - Higher track/vertex density, less efficient the classical method
 - → Allows for many options and best choice
- O Adaptive features
 - Dynamic change of track parameters during finding/fitting
 - → Measurements are weighted according to their uncertainty
 - Allows to take into account several "normally excluded" info
 - Many hypothesis are handled simultaneously
 - But their number decrease with iterations (annealing like behavior)
 - → Non-linearity
 - Often CPU-time costly (is that still a problem?)
- O Examples
 - Neural network, Elastic nets, Gaussian-sum filters, Deterministic annealing



Denby-Peterson net

Elastic Tracking

4. Advanced methods

Cellular automaton

O Cellular automaton

- → Initialization
 - built any cell (= segment of 2 points)
- → Iterative step
 - associate neighbour cells (more inner)
 - Raise "state" with associated cells
 - Kill lowest state cells



5. Deconstructing some tracking systems

O CMS (colliders)

O AMS, ANTARES (telescopes)

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CMS



CMS

• The trackerS



CMS

• Alignment residual width



• Taking a picture of the material budget

→ Using secondary vertices from γ → e⁺e⁻



CMS

• Measuring it by data/simulation comparison



CMS

• Tracking algorithm = multi-iteration process



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CMS

• Tracking efficiency



66

CMS



CMS



Impact parameter resolution



AMS









Fig. 5. The effective position resolution (weighted average of two Gaussian widths) in the *y*-coordinate for different inclination angles (top), the Maximum Detectable Rigidity (MDR, 100% rigidity measurement error) as a function of the inclination angle estimated for 1TV proton incidence with the simulation (middle), and the inclination angle distribution in the geometric acceptance of the tracker (bottom).

ANTARES







2000
Summary

- Fundamental characteristics of any tracking & vertexing device:
 - (efficiency), granularity, material budget, power dissipation, "timing", radiation tolerance
 - → All those figures are intricated: each technology has its own limits
- Many technologies available
 - → None is adapted to all projects (physics + environment choose, in principle)
 - → Developments are ongoing for upgrades & future experiments
 - Goal is to extent limits of each techno. → convergence to a single one?
- Reconstruction algorithms
 - → Enormous boost (variety and performances) in the last 10 years
 - Each tracking system has its optimal algorithm
- Development trend
 - Always higher hit rates call for more data reduction
 - → Tracking info in trigger → high quality online tracking/vertexing
- O Link with:
 - → PID: obvious with TPC, TRD, topological reco.
 - → Calorimetry: Particle flow algorithm, granular calo. using position sensors

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Was not discussed

- Particle interaction with matter
- The readout electronics
- O Cooling systems
- The magnets to produce the mandatory magnetic field for momentum measurement
- O Vertexing



W



OPAL drift chamber





ALICE - TPC



(ALICE) TPC dE/dx



ICARUS - TPC



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NA-50 fixed target



ATLAS tracking setup



ATLAS tracking setup



ALICE setup



CMS

0



More position sensitive detectors



DEPFET



MICROMEGAS



CCD



Was not discussed

- Particle interaction with matter
- The readout electronics
- O Cooling systems
- The magnets to produce the mandatory magnetic field for momentum measurement
- O Vertexing

