Experimental astroparticle physics & cosmology

Observational cosmology

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LPSC

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Syllabus

1. Introduction
2. FRW Cosmology, cosmological parameters and inflation
3. CMB theory and observations
4. Probing dark matter and dark energy
5. Current cosmological results and constraints

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Lecture 1: Introduction

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References

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- A.R. Liddle & D.H. Lyth: Cosmological Inflation and Large-Scale Structure, Cambridge University Press, 2000
- S. Dodelson: Modern Cosmology,
- P.J.E. Peebles: Principles of physical cosmology, Princeton University Press, 1993
- Padmanabhan: Structure formation in the universe, Cambridge University Press, 1993
- An Introduction to Cosmology, W. Hu, `http://background.uchicago.edu/~whu/Courses/ast321_11.html`

Physical constants and parameters

Paremeters and units

Reduced Planck constant	$\hbar = 1.055 \times 10^{-27}$ cm^2.g.s^{-1}
Speed of light	$c = 2.998 \times 10^{10}$ cm.s^{-1}
Newton's constant	$G = 6.672 \times 10^{-8}$ cm^3.g^{-1}.s^{-2}
Reduced Planck mass	$M_{Pl} = 4.342 \times 10^{-6}$ g
	$= 2.436 \times 10^{18}$ GeV/c^2
Planck mass	$m_{Pl} = \sqrt{8\pi} M_{Pl} = 2.177 \times 10^{-5}$ g
Reduced Planck length	$L_{Pl} = 8.101 \times 10^{-33}$ cm
Reduced Planck time	$T_{Pl} = 2.702 \times 10^{-43}$ s
Boltzmann constant	$k_B = 1.381 \times 10^{-16}$ erg.K^{-1}
Thomson cross section	$\sigma_T = 6.652 \times 10^{-25}$ cm^2
Electron mass	$m_e = 0.511$ MeV/c^2
Neutron mass	$m_n = 939.6$ MeV/c^2
Proton mass	$m_p = 938.3$ MeV/c^2
Solar mass	$M_\odot = 1.99 \times 10^{33}$ g
Megaparsec	1 Mpc $= 3.086 \times 10^{24}$ cm
1 cm	$= 5.086 \times 10^{13}$ GeV^{-1}.$\hbar$
1 s	$= 1.519 \times 10^{24}$ GeV^{-1}.$\hbar$/c
1 g	$= 5.608 \times 10^{25}$ GeV/c^2
1 erg	$= 6.242 \times 10^{2}$ GeV
1 K	$= 8.618 \times 10^{-14}$ GeV/k_B

Parameters

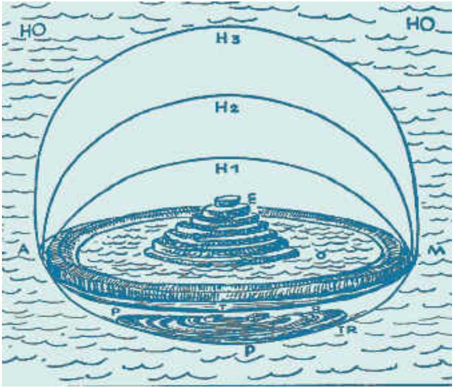
Hubble constant	$H_0 = 100\, h$ km.s^{-1}.Mpc^{-1}
Present Hubble distance	$cH_0^{-1} = 2998 h^{-1}$ Mpc
Present Hubble time	$H_0^{-1} = 9.78\, h^{-1}$ Gyr
Present critical density	$\rho_{c,0} = 1.88\, h^2 \times 10^{-29}$ g.cm^{-3}
	$= 2.775\, h^2 \times 10^{11}\, M_\odot/(\mathrm{Mpc})^3$
	$= \left(3.000 \times 10^{-3} \mathrm{eV}/c^2\right)^4 h^2$
Present photon density	$\Omega_{\gamma,0}\, h^2 = 2.48 \times 10^{-5}$
Present relativistic density	$\Omega_{R,0}\, h^2 = 4.17 \times 10^{-5}$
Baryon-to-photon ratio	$\eta = 2.68 \times 10^{-8}\, \Omega_b\, h^2$
Matter-radiation equality	$1 + z_{\mathrm{eq}} = 24000 \Omega_0\, h^2$
Hubble length at equality	$(a_{\mathrm{eq}} H_{\mathrm{eq}})^{-1} = 14 \Omega_0^{-1}\, h^{-2}$ Mpc
Top-hat filter/$10^{12}\, M_\odot$	$M(R) = 1.16\, h^{-1} \left(R/1 h^{-1}\mathrm{Mpc}\right)^3$
Gaussian filter/$10^{12}\, M_\odot$	$M(R) = 4.37\, h^{-1} \left(R/1 h^{-1}\mathrm{Mpc}\right)^3$

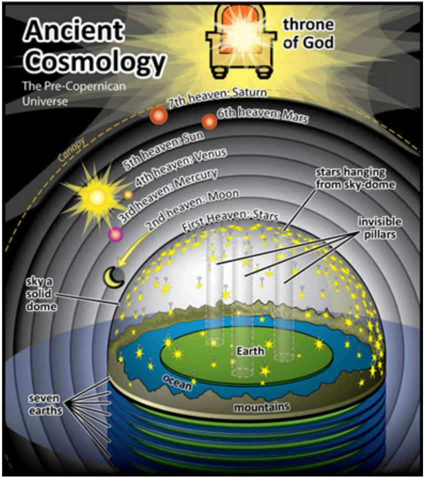
Cosmology in a nutshell

- Cosmology studies the formation and evolution of the universe as a whole in order to explain its origin, its current status and its future.
- Philosophy and religion were originally the main path to the understanding of the universe and their properties.
- Nowadays cosmology studies are mainly based on
  physical theories: general relativity, quantum physics, statistical physics, quantum field theory, quantum gravity, etc;
  mathematics: statistical description of fields and data;
  chemistry and biology: development of life
- Astrophysical observations of our galaxy, other external galaxies, cluster of galaxies and the Cosmic Microwave Background (CMB) are critical to understand our universe

Ancient cosmology

Explaining the universe as we observe it is very old human-kind concern



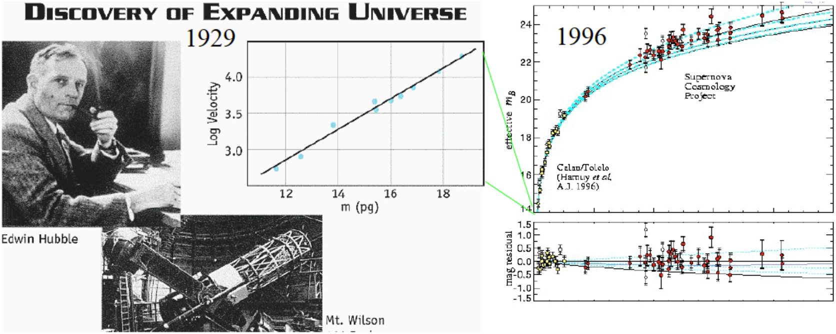


Recent physical cosmology history

- 1915 Einstein. Theory of general relativity
- 1922-1927 Friedmann-Lemaıtre. Expanding universe and Big Bang
- 1929 Hubble. Experimental proof of expansion of the universe
- 1933 Zwicky. First hints of dark matter problems in the Coma cluster
- 1940 Gamow. Prediction of primordial nucleosynthesis and cosmic microwave background
- 1948 Bondi, Gold & Hoyle. Stationary model
- 1965 Penzias & Wilson. CMB discovery
- 1970-1980s. Structure formation models
- 1981 Guth. Inflationary theory
- 1992 COBE satellite measures CMB anisotropies
- 1998 SNIa and accelerated expansion of the universe
- 2000s. Quintessence models for dark energy

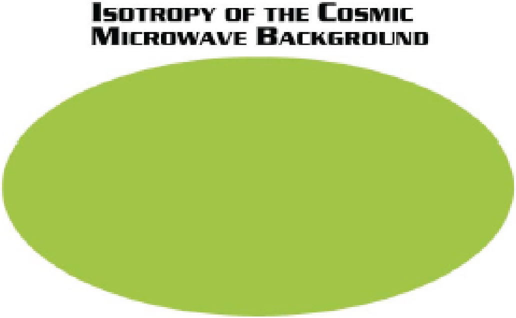
Expanding universe and dark energy

- Hubble in 1929 measured recession velocity of galaxies and showed that universe was expanding
- In 1998 the study of the luminosity of SN Ia showed the expansion of the universe is now accelerated



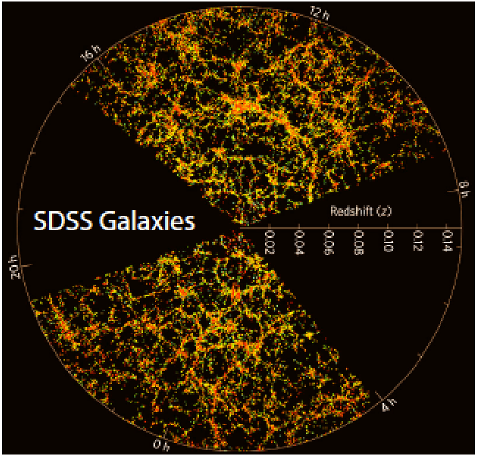
Cosmic Microwave Background

- Penzias & Wilson discovered in 1965 an isotropic and homogeneous radiation with a temperature of about 3 K as predicted by Gamow in 1940
- the COBE satellite in 1992 showed that the CMB has a black-body spectrum and fluctuations of about 10^{-5}



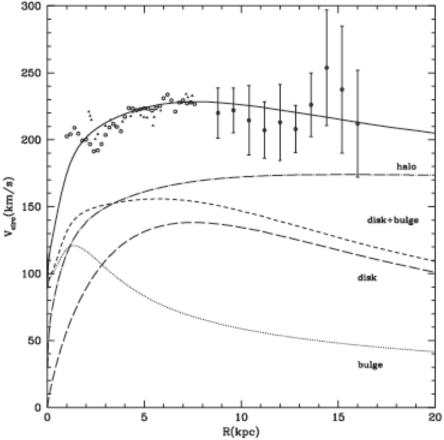
Large-scale structure

- galaxy surveys have shown the large-scale structure of the universe which is formed of voids, clusters of galaxies and filaments
- the universe is homogeneous for scales larger than 100 Mpc



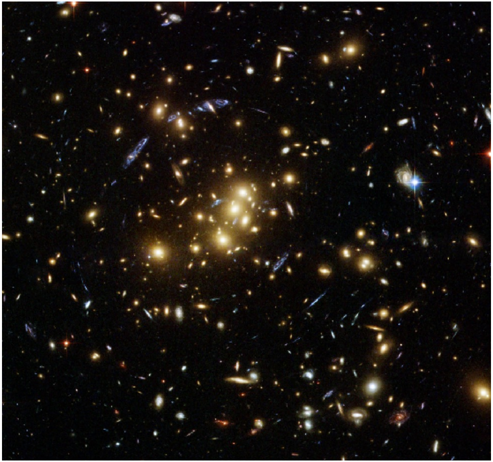
Dark matter

- Mass required to keep rotational curves flat is larger than expected from stars and gas
- In merging galaxy clusters the reconstructed matter distribution doest not peak where gas is observed

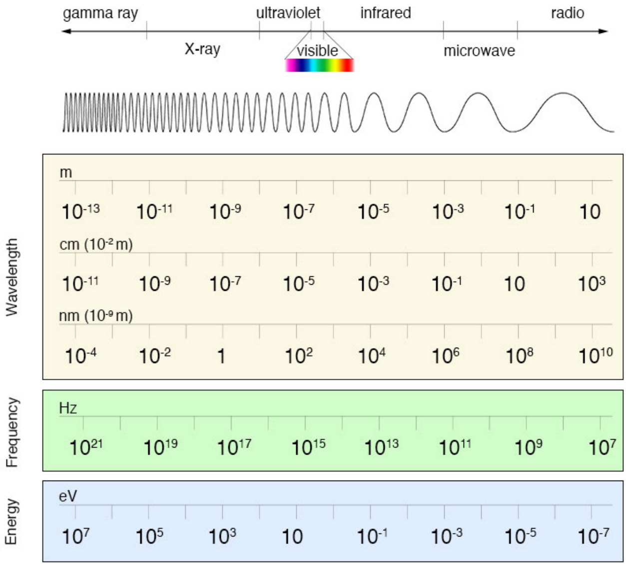


Strong lensing

Weak lensing



Electromagnetic spectrum



Summary of main observational facts today

- Galaxy distribution
  - the universe is expanding
  - small structures form first and combine to form larger ones
- Supernovae type Ia
  - currently expansion is accelerated: dark energy
- Cosmic Microwave Background (CMB)
  - the universe is isotropic and homogeneous
  - universe fully thermalized
  - density fluctuations of the order of 10^{-5}
- Abundance of light elements
  - Light elements form first from nucleosynthesis
- Dynamics of galaxies and of cluster of galaxies
  - Evidence for extra matter component: dark matter and/or modified gravity theory

Standard Cosmological Model in nut-shell

The *standard cosmological* model is based on:

1. Big Bang theory
   universe expands from a hot and dense initial point and cool down

   → primordial nucleosynthesis and CMB emission

2. Λ-CDM model
   describes universe energy density

   → photons, neutrinos, baryon, cold dark matter, dark energy, (may also be warm dark matter)

3. Inflation
   period of exponential expansion in the early universe

   → produces primordial fluctuations and solves horizon CMB problem

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Lecture 2: Expanding universe

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L. 2, Section 1: FLRW cosmology

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FLRW cosmology

The Friedmann-Lemaitre-Robertson-Walker (FRW) cosmology is based on:

1. The cosmological principle: the universe is isotropic and homogeneous on large scales
2. General Relativity (GR) theory:
    - A metric to describe the geometry of space-time: tells matter how to move
    - Einstein field equations: matter tells geometry how to curve
3. Multi-component energy density: photons, neutrinos, baryons, non-relativistic matter, dark energy and curvature

NB: Conceptually it is useful to separate geometry and dynamics to understand alternative cosmologies, e.g.

- Break homogeneity and isotropy assumptions under GR
- Modify gravity theory while keeping the geometry

General Relativity (GR)

Based on the equivalence principle that postulate that the laws of physics takes the same form in all reference frames (even those freely falling)

- Proper time is invariant and defines the metric $g_{\mu\nu}$

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu\,1}, \;\; x^{\mu} = (c\,dt, dx, dy, dz)$$

The metric defines the curvature of space-time

- The metric evolves accordingly to Einstein field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$$

where $R_{\mu\nu}$ and R are the Ricci tensor and scalar respectively and G the gravitational constant

- $T_{\mu\nu}$ is the stress-energy tensor that evaluates the effect of a given distribution of mass and energy on the space-time curvature

[1] We use here the repeated symbol sum convention $\sum_{\mu=0}^{3}\sum_{\nu=0}^{3}$

Robertson-Walker metric

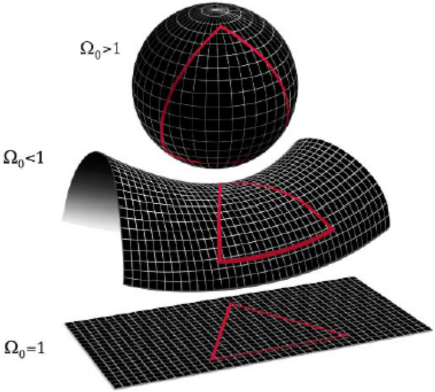
In 1930 Robertson and Walker independently showed that the most general metric possible for describing an expanding universe is

$$ds^2 = (c\,dt)^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

where (r, θ, ϕ) are spherical comoving coordinates and $a(t)$ is the scale factor

Spatial geometry is that of a constant curvature:

- $k = 0$ flat geometry universe
- $k = -1$ open universe
- $k = +1$ closed universe



Horizon

- Distance travelled by a photon in the whole lifetime of the universe defines the horizon
- For photons ds =0, so we have that

$$D_{horizon}(t) = \int_0^t \frac{dt'}{a(t)} = \eta(t)$$

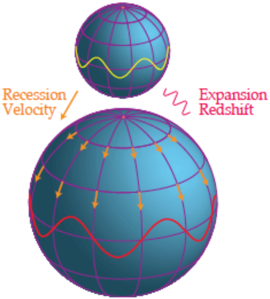
- $\eta(t)$ is also called the conformal time
- Two points in the universe are in casual contact if their distance is smaller than the horizon
- Horizon problem: why is the universe isotropic and homogeneous on large scales ? The observable universe is today larger than the horizon

Redshift

- Wavelength of light *stretches* with the scale factor
- Given a physical rest wavelength at emission λ_0, the observed wavelength today λ is

$$\lambda = \frac{1}{a(t)}\lambda_0 \equiv (1+z)\lambda_0$$

- Interpreting the redshift as a Doppler shift, objects recede in an expanding universe
- Today z = 0 and it increases back on time



Deceleration parameter and elapsed time

- The deceleration parameter q_0 is defined by the series

$$a(t) = a(t_0)\left[1 + H_0(t - t_0) - \frac{1}{2}H_0^2 q_0 (t - t_0)^2 + \dots\right]$$

- Taylor expanding a(t) we obtain

$$q_0 = -\frac{\ddot{a}(t_0)a(t_0)}{\dot{a}(t_0)}$$

- From above we deduce

$$1 + z = 1 + H_0(t - t_0) + H_0^2(t_0 - t)^2\left[1 + \frac{q_0}{2}\right] + \dots$$

- and inverting

$$t_0 - t = \frac{1}{H_0}\left[z - z^2\left(1 + \frac{q_0}{2}\right) + \dots\right]$$

Cosmological distances

- Proper distance, time for a photon to go from z to $z + dz$

$$d_{pr} = -cdt = -c\frac{da}{\dot{a}}$$

- Comobile distance between observer at z and emitter at z + dz

$$d_{com} = -c\frac{dt}{a} = -c\frac{da}{\dot{a}a}$$

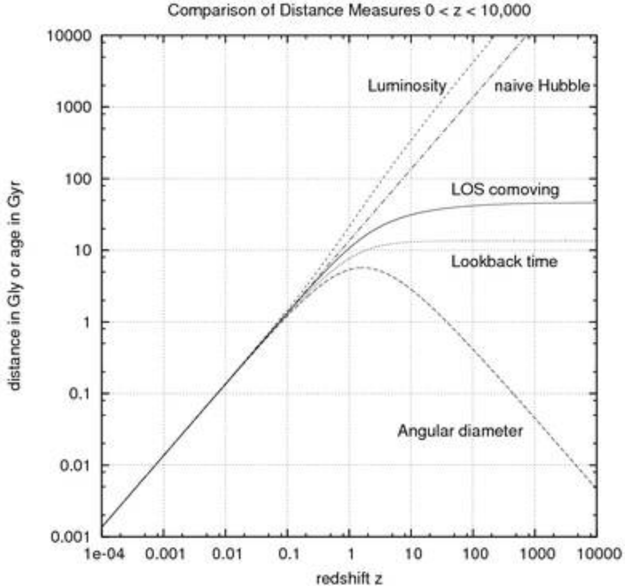
- Luminosity distance, d_L such that the observed flux, ℓ, of a source of absolute luminosity L is $\ell = \frac{L}{4\pi d_L^2}$,

$$d_L = \frac{c}{H_0}\left[z + \frac{1}{2}(1 - q_0)z^2 + \dots\right]$$

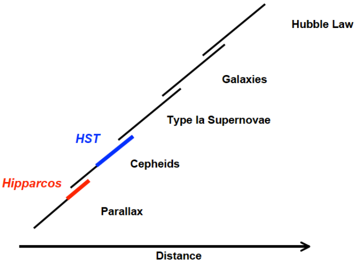
- Diameter angular distance, relates angular size $\Delta\theta$ and physical size, D of a source

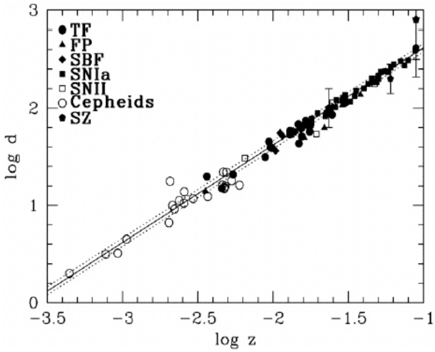
$$d_A = \frac{D}{\Delta\theta} = \frac{d_L}{(1+z)^2}$$

Cosmological distances

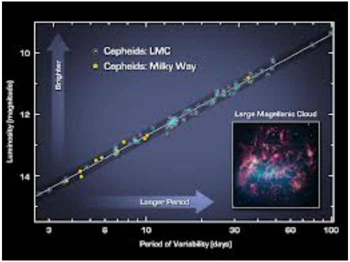


Cosmic Distance Ladder





Cepheids



- Parallax: Hipparcos 0-300 pc (GAIA 5 kpc)
- Cepheids: 100 pc - 20 Mpc (HST)
- Type Ia SNe: 20 - 400 Mpc

Friedmann-Lemaitre equations

Apply the Einstein field equations to the R-W metric

$$G_{\mu\nu} = -8\pi G\, T_{\mu\nu}$$

- From the LHS we obtain

$$G^0_0 = -\frac{3}{a^2}\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2}\right]$$

$$G^i_j = -\frac{1}{a^2}\left[2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2}\right]$$

- for the RHS isotropy demands that

$$T^0_0 = \rho$$

$$T^i_j = -p\delta^i_j$$

where ρ is the energy density and p the pressure

Dynamics of the universe

- Finally the FL equations stand

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2} = \frac{8\pi G}{3}a^2\rho$$

$$2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2} = -8\pi G a^2 p$$

- and can be combined into a single one

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = -\frac{4\pi G}{3}a^2\left(\rho + 3p\right) = a\frac{d^2a}{dt^2}$$

Curvature and critical density

- The first FL equation can be written as

$$H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \rho_k) \equiv \frac{8\pi G}{3}\rho_c$$

- ρ_c is the critical system and its value today is

$$\rho_c(z=0) = \frac{3H_0^2}{8\pi G} = 1.8788 \times 10^{-29} h^2 \text{g cm}^{-3}$$

- Curvature as an effective energy density component

$$\rho_K = -\frac{3}{8\pi G a^2 R^2}$$

Total energy density

- Energy density today can be given as a fraction of critical density

$$\Omega_{tot} \equiv \frac{\rho}{\rho_c(z=0)}$$

- Note that physical energy density is $\propto \Omega h^2$ (g cm^{-3})
- Likewise the radius of curvature is given by

$$\Omega_K = (1 - \Omega_{tot}) = \frac{1}{H_0^2 R^2} \rightarrow R = (H_0\sqrt{\Omega_{tot} - 1})^{-1}$$

- Ω value defines universe geometry
  - $\Omega_{tot} = 1$, flat universe
  - $\Omega_{tot} > 1$, positively curved
  - $\Omega_{tot} < 1$, negatively curved

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L. 2, Section 2: Λ-CDM model

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The multi-component universe

- We define the equation of state as $p = w\ \rho$
- Universe consists of multiple components:
  1. NR matter $\rho_m = m n_m \propto a^{-3}, w_m = 0$
  2. R radiation $\rho_r = E n_r \propto \nu n_r \propto a^{-4}, w_r = 1/3$
  3. curvature $\rho_k \propto a^{-2}, w_r = -1/3$
  4. (cosmological) constant energy density $\rho_\Lambda \propto a^0, w_\Lambda = -1$
- total energy density summed over all components

$$\rho(a) = \sum_i \rho_i(a) = \rho_c(a = 1; z = 0) \sum_i \Omega_i a^{-3(1+w_i)}$$

- density evolves as

$$\rho(a) = \rho_c(a = 1) \sum_i \Omega_i \exp^{-\int d \log a 3(1+w_i)}$$

- and the Hubble constant as

$$H^2(a) = H_0^2 \exp^{-\int d \log a 3(1+w_i)}$$

General solutions of FL equations

- Radiation domination

$$H^2 \propto a^{-4},\ a(t) \propto t^{1/2},\ H(t) = \frac{1}{2t},\ R_H = 2ct$$

- Matter domination

$$H^2 \propto a^{-3},\ a(t) \propto t^{2/3},\ H(t) = \frac{2}{3t},\ R_H = \frac{3}{2}ct$$

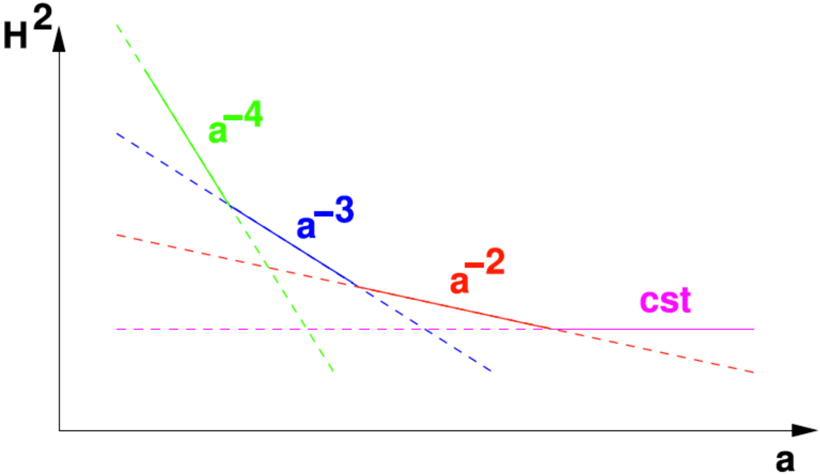
- Curvature domination $k < 0$

$$H^2 \propto a^{-2},\ a(t) \propto t,\ H(t) = \frac{1}{t},\ R_H = ct$$

- Dark energy domination

$$H^2 \rightarrow \text{constant},\ a(t) \propto exp(\Lambda t/3),\ H(t) = c/R_H = \sqrt{\Lambda/3}$$

Hubble constant evolution

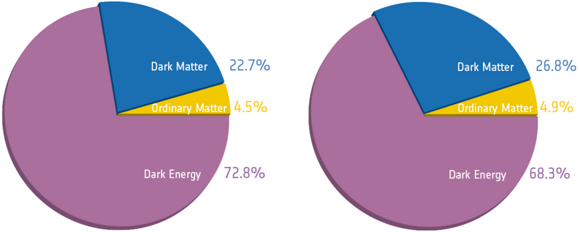


A first set of cosmological parameters and relations

H_0	Hubble constant
Ω_k	Curvature energy density
Ω_m	Matter density
Ω_Λ	Dark energy density
Ω_{CDM}	Cold Dark matter density
Ω_b	Baryonic matter density
Ω_γ	Photon density
Ω_ν	Neutrino density

- $(1 - \Omega_k) = \Omega_{tot} = \Omega_m + \Omega_\Lambda$
- $\Omega_m = \Omega_{CDM} + \Omega_b + \Omega_\gamma + \Omega_\nu$
- Deceleration parameter $q_0 = \frac{1}{2}\Omega_m^{NR} - \Omega_\Lambda$

$$H^2(z) = H_0^2(\Omega_m^R(1+z)^4 + \Omega_m^{NR}(1+z)^3 - \Omega_k(1+z)^2 + \Omega_\Lambda) = H_0^2 E(z)^2$$



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L. 2, Section 3: Inflation

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Motivations for inflation

Inflation was motivated by a set of problems encountered by Big Bang theory

- Flatness problem
  The universe is observed to be flat today to a great accuracy however the flat solution of the FL equations is unstable
- Relic abundances
  Phase transitions in the early universe will lead to relic particles like for example monopoles that are not observed today
- Horizon problem
  CMB temperature is uniform and isotropic all over the sky however regions of the sky separated by more than one degree were not in casual contact at the time of CMB formation
- Origin of cosmological fluctuations
  All observed structures in the universe were formed by the growth up of primordial fluctuations for which we have no explanation

Accelerated expansion

- To solve the horizon, flatness and relics problem we need

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \Rightarrow \ddot{a} > 0 \Rightarrow \rho + 3p < 0$$

- So acceleration implies negative pressure $p < -1/3\rho$
- We define the number of e-folds as

$$N = \ln \frac{a_i}{a_f}$$

  where a_i and a_f correspond to the scale factors at beginning and end of the accelerated expansion period
- Notice that N represents some how the *amount expansion*
- To solve the horizon, flatness and relics problems we need $N \geq 60$

Scalar fields in cosmology

- For a FRWL universe the dynamics of a scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + V'(\phi) = 0$$

- For FRWL universe and assuming $\phi = \phi_0 + \delta\phi$ we obtain for the homogeneous field

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{(\nabla\phi)^2}{2a^2} + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{(\nabla\phi)^2}{6a^2} - V(\phi)$$

- So we can write FL equation

$$H^2 = \frac{8\pi G}{3}\rho_\phi - \frac{k^2}{2} \sim \frac{8\pi G}{3}\rho_\phi$$

Slow roll dynamics

- We can obtain accelerated expansion of the universe from the scalar field dynamics
  1. We neglect the term $\frac{\nabla^2\phi}{a^2}$ (somehow diluted by expansion)
  2. We assume $\frac{\dot{\phi}}{2} \ll V(\phi)$ we have $p_\phi \sim -\rho_\phi$ and thus

$$H^2 \sim \frac{8\pi G}{3} V(\phi)$$

  3. We assume $\ddot{\phi} \ll 3H\dot{\phi}$
- Thus :

$$H^2 \simeq \frac{8\pi G}{3} V$$

$$3H\dot{\phi} + V' \simeq 0$$

Slow roll parameters

- Net energy is dominated by potential energy and thus acts like a cosmological constant $w \to -1$
- First slow roll parameter

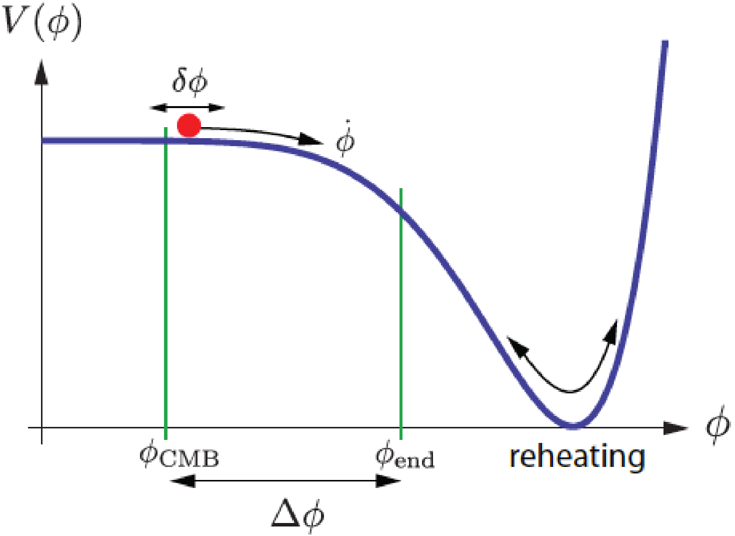
$$\epsilon \equiv \frac{3}{2}(1+w) = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$$

- Second slow roll parameter

$$\delta \equiv \frac{\ddot{\phi}}{\dot{\phi}}\left(\frac{\dot{a}}{a}\right) - 1 = \epsilon - \frac{1}{8\pi G}\frac{V''}{V} = \epsilon - \eta$$

- Slow roll conditions imply $\epsilon,\ \delta,\ |\eta| \ll 1$, corresponding to a very flat potential
- We normally define the reduced Planck mass as $M_P = \frac{1}{8\pi G}$

Potential slowly rolling down



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Lecture 3: CMB

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Cosmic Microwave Background

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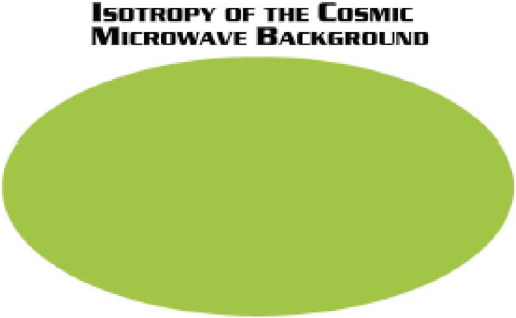
- the COBE satellite in 1992 showed that the CMB has a black-body spectrum and fluctuations of about 10^{-5}



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L. 3, Section 1: Thermal history of the Universe

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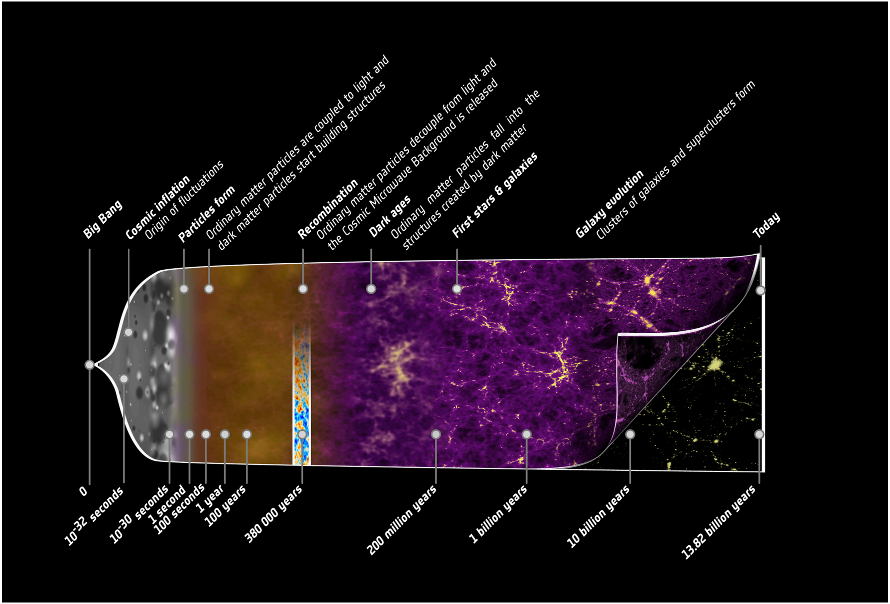
Cartoon thermal history of the universe



Detailed thermal history of the universe

Event	T (K)	kT (eV)	g_{eff}	z	t
Now	2.76	0.0002	3.43	0	13.6 Gyr
First Galaxies	16	0.001	3.43	6 (?)	$\sim$ 1 Gyr
Recombination	3000	0.3	3.43	1100	38000 yr
M-R equality	9500	0.8	3.43	3500	50000 yr
e^+-e^- pairs	$10^{9.7}$	$0.5\ 10^6$	11	$10^{9.5}$	3 s
Nucleosynthesis	10^{10}	$1\ 10^6$	11	10^{10}	1 s
Nucleon pairs	10^{13}	$1\ 10^9$	70	10^{13}	10^{-7} s
E-W unification	$10^{15.5}$	$25\ 10^{10}$	100	10^{15}	10^{-12} s
GUT	10^{28}	10^{24}	100 (?)	10^{28}	10^{-38} s
Quantum Gravitiy	10^{32}	10^{28}	100 (?)	10^{32}	10^{-43} s

3 Eras: radiation, matter and dark energy

- The energy density of radiation, matter and dark energy (DE) evolves differently

$$\text{radiation}: \ \rho_R \propto a^{-4}$$

$$\text{matter}: \ \rho_M \propto a^{-3}$$

$$\text{DE}: \ \rho_\Lambda = constant$$

- So, the total density of the universe can be written as

$$\rho = \rho_c \left(\Omega_R x^4 + \Omega_M x^3 + \Omega^\Lambda \right); \ x = 1 + z$$

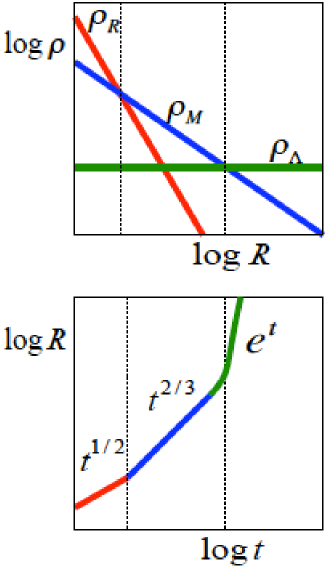
- Matter-radiation equality is obtained when $\rho_M = \rho_R$ at

$$z = \frac{\Omega_M}{\Omega_R} - 1 \sim 3402$$

- Matter-DE equality when $\rho_M = \rho_\Lambda$ at

$$z = \left(\frac{\Omega_\Lambda}{\Omega_M} \right)^{1/3} - 1 \sim 0.29$$

3 Eras: radiation, matter and dark energy



Experimental astroparticle physics & cosmology

L. 3, Section 2: Physics at recombination

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Thomson Scattering

- Thomson scattering of photons off of free electrons is the most important CMB process with a cross section (averaged over polarization states) of

$$\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} = 6.65 \times 10^{-25}\ \mathrm{cm}^2$$

- Density of free electrons in a fully ionized $x_e = 1$ universe is given by

$$n_e = (1 - Y_p/2)x_e n_b \approx 10^{-5}\Omega_b h^2(1+z)\mathrm{cm}^{-3}$$

,

- In general we can write the Thomson scattering rate as

$$\Gamma = \tau' = \sigma_t a n_e x_e$$

where τ is the medium optical depth

- The visibility function $g(\eta) = -\tau' e^{-\tau}$ indicates the probability that a CMB photon last scattered at conformal time η

Recombination

- When temperature drops to $\sim$ 1000 K it is thermodynamically favorable for the plasma to form atoms via

$$p + e^- \leftrightarrow H + \gamma$$

  This is called recombination.

- If thermal equilibrium hods then the number density of each species is

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T}\right)$$

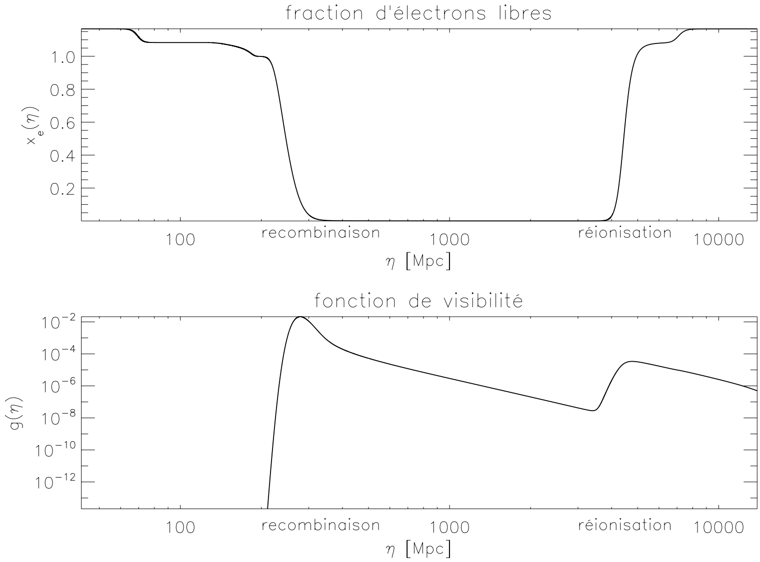
  and chemical equilibrium impose

$$\mu_e + \mu_p = \mu_H$$

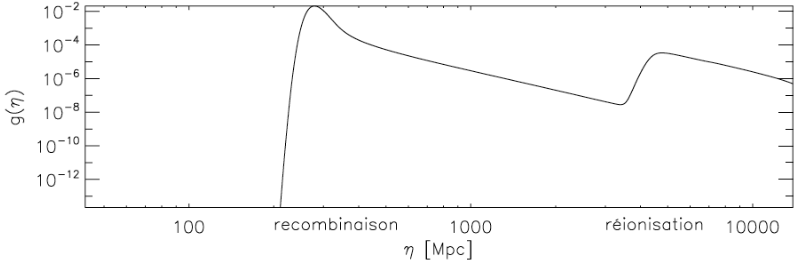
- As $m_H \sim m_p$ and defining $B_H = m_p + m_e - m_H = 13.6$ eV we have

$$n_H = \frac{f_H}{g_p g_e} n_e n_p \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(B_H/T\right)$$

Ionization fraction evolution

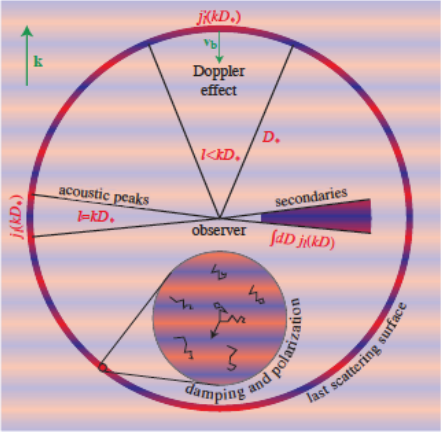


Recombination in a nutshell



- The Thomson scattering rate evolves as $\Gamma \propto a^{-2} x_e$
- The free electron fraction x_e starts from 1 at high redshift.
- Thus, before recombination $\Gamma \gg \frac{a'}{a}$ and the universe is opaque
- At recombination, about $z \sim 1080$, x_e decreases sharply and freezes at a very small value
- Then, after recombination $\Gamma \ll \frac{a'}{a}$ and the universe is transparent
- At reionization all electrons are free again, however because dilution n_e is small and Γ remains much smaller than $\frac{a'}{a}$ and so most photons do not interact any more

Last Scattering Surface



- Interaction between electrons and photons via Thomson scattering before recombination and after reionization
- Angular distribution of radiation is the 3D temperature field projected onto a shell - surface of last scattering
- Integrate along the line of sight in an expanding universe
- Describe radiation as an statistically isotropic temperature field with fluctuations

Experimental astroparticle physics & cosmology

L. 3, Section 3: Observing the CMB

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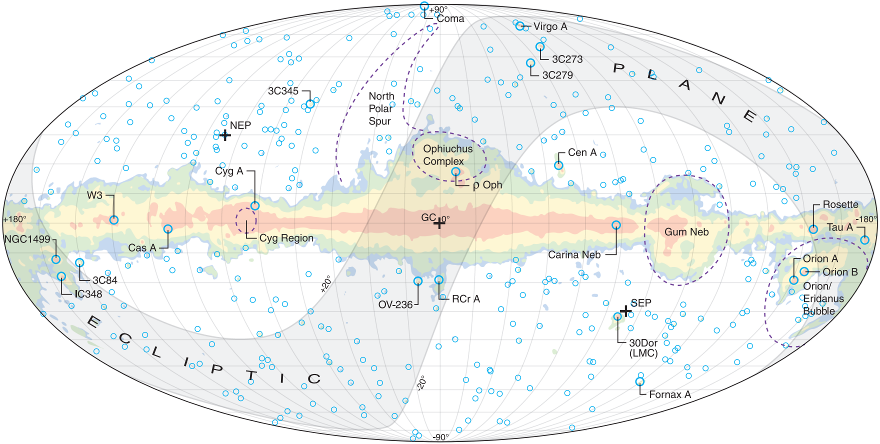
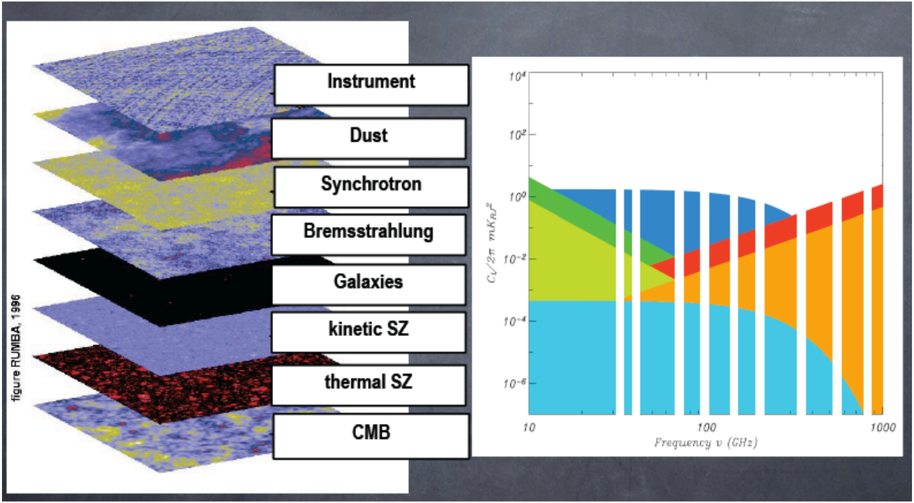
Brief history of CMB observations

- Penzias & Wilson discovered in 1965 an isotropic and homogeneous radiation with a temperature of about 3 K
- In 1992 the COBE satellite demonstrated that the CMB has a black-body spectrum and fluctuations of about 10^{-5}
- In 1998 Boomerang and Maxima measured the so-called acoustic peaks in the CMB power spectrum
- The WMAP satellite, launched in 2001, provided first CMB polarization precise measurements
- The Planck satellite 2013 results has provided best possible CMB temperature anisotropies measurements and much more (polarization analysis expected in 2014)
- Late 2013 the South Pole telescope and the PolarBear experiment reported first observation of B-lensing modes

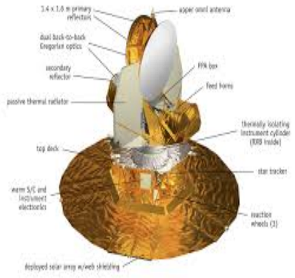
Observing the sky

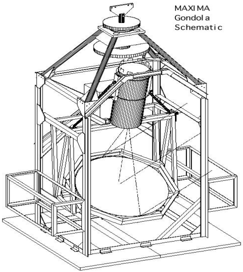


Foregrounds



CMB instruments

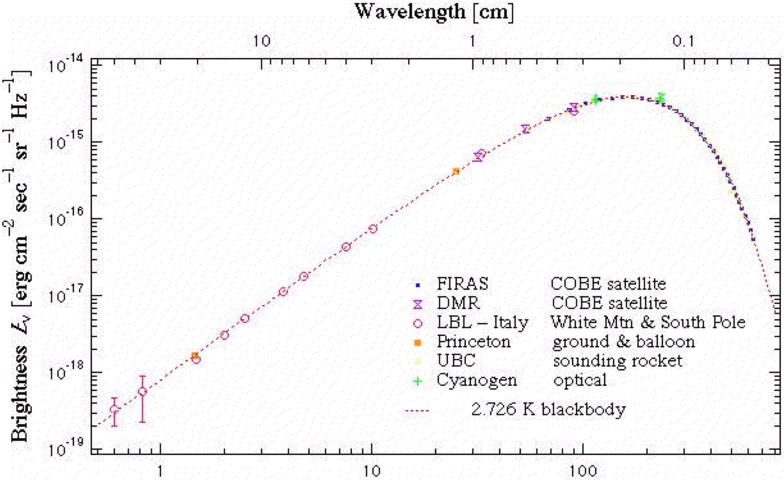




	Radio	mm
Telescopes	dish and horns	dish and horns
Detectors	HEMT + square law detectors	bolometer and/or KIDs
Cooling	18-50 K	100-300 mK
Observing mode	Ground, satellite	ground, balloon, satellite

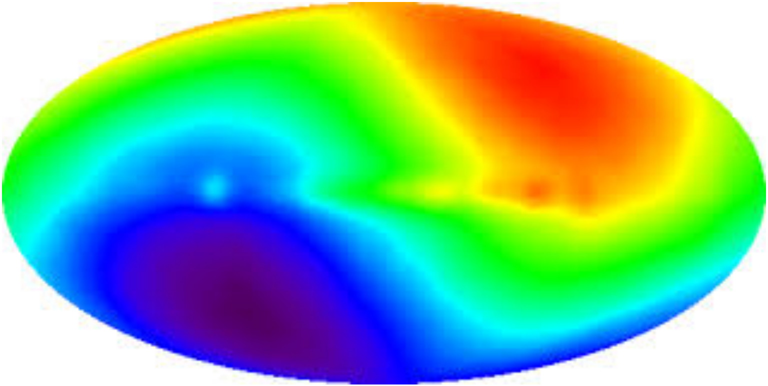
CMB black-body spectrum

- Compton scattering of photons with electrons is very efficient to thermalize photons
- In 1994 the FIRAS spectrograph in the COBE satellite measured the CMB temperature: $T_{\text{CMB}} = 2.726 \pm 0.001$ K



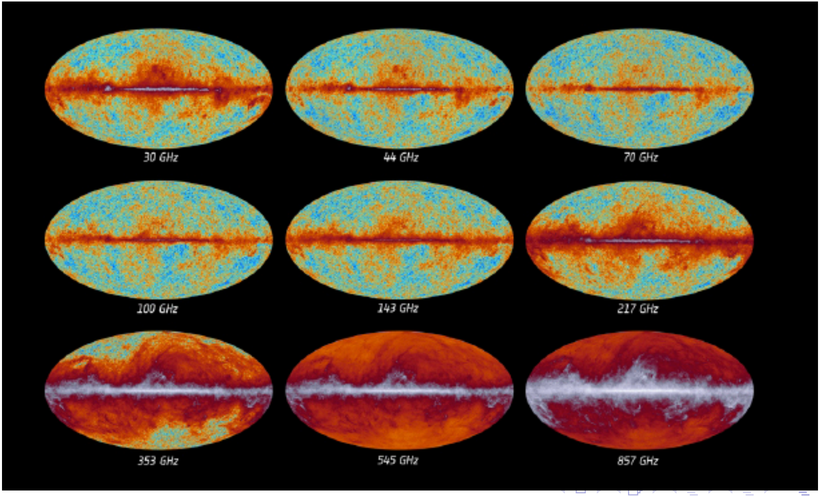
Measured CMB anisotropies I

- Dipole anisotropy induced by Doppler effect (relative motion of the observer with respect to the CMB rest frame)
- First measured by the COBE satellite in 1992 with an amplitude of $3.358 \pm 0.001 \pm 0.023$ mK in the direction of (l,b)=($264.31 \pm 0.04 \pm 0.16$,$+48.05 \pm 0.02 \pm 0.09$) degrees



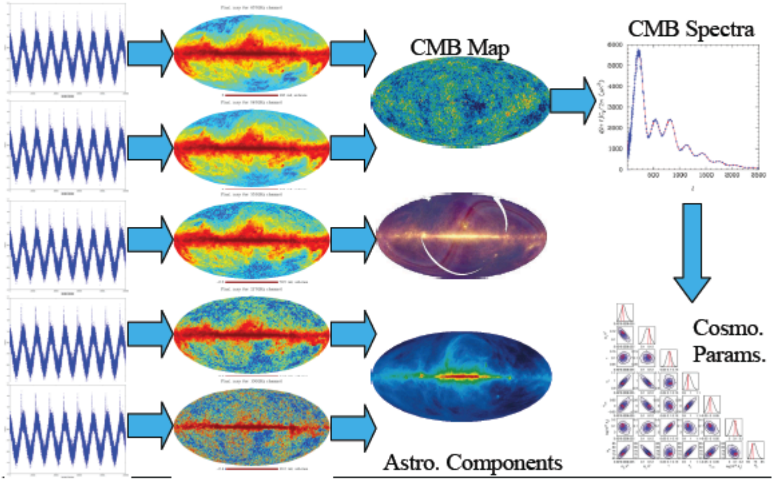
The micro-wave and mm sky

- We observe a mixture of components: CMB, galactic thermal dust, synchrotron and free-free emissions, extragalactic emission from dusty and radio galaxies



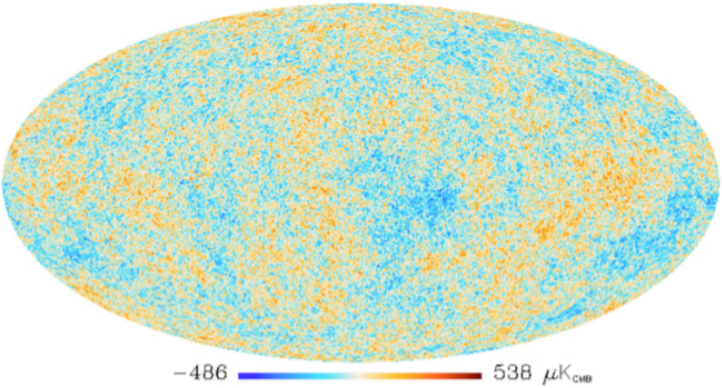
From sky observations to CMB maps

- Component separation algorithms are used to recover the CMB emission



Measured CMB anisotropies II

- Temperature fluctuations of the order of 10^{-5}
- Planck satellite 2013 results: most precise measurements of the CMB temperature anisotropies



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L. 3, Section 4: Physics of CMB anisotropies

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Spherical harmonics and power spectrum

- Any scalar field on the sphere, $A(\theta, \phi)$ can be decomposed into spherical harmonics

$$A(\theta, \phi) = \sum_{\ell} \sum_{m=-\ell}^{+\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

- We can define the power spectrum as

$$C_\ell = < a_{\ell m} a^*_{\ell m} > = \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2$$

- For a Gaussian random field then

$$< a_{\ell m} a^*_{\ell' m'} > = C_\ell \delta_{\ell \ell'} \delta_{m m'}$$

The Boltzmann equation

- Photons decouple from baryons at recombination so we can not describe them with fluid equations
- Need to solve the Boltzmann equation for the photon space-phase distribution

$$\frac{d}{d\eta} f_\gamma(\eta, \mathbf{x}, \mathbf{q}) = C[f_\gamma(\eta, \mathbf{x}, \mathbf{q}), f_e(\eta, \mathbf{x}, \mathbf{q})]$$

  at first order in perturbation
- Notice that as discussed above electrons and baryon are so tightly coupled that it makes no difference to think in terms of photon-electron coupling or photon-baryon coupling
- In thermal equilibrium the space-phase photon distribution function behaves as a Bose-Einstein distribution

$$f_\gamma(\eta, \mathbf{x}, \mathbf{q}) = \frac{1}{e^{\frac{q}{T(\eta, \mathbf{x})}} - 1}$$

Perturbations

- We expand the photon space-phase distribution function as a background part and first order perturbation $f_\gamma = \bar{f}_\gamma + \delta f_\gamma$ and so

$$\bar{f}_\gamma(\eta, \mathbf{x}, \mathbf{q}) = \frac{1}{e^{\frac{q}{\bar{T}(\eta)+\delta T(\eta)}} - 1}$$

and

$$\delta f_\gamma(\eta, \mathbf{x}, \mathbf{q}) = \frac{d\bar{f}_\gamma}{d \log q} \frac{\delta T(\eta, \mathbf{x})}{\bar{T}(\eta)}$$

- Therefore, we can replace $f_\gamma(\eta, \mathbf{x}, \mathbf{q})$ by the brightness function

$$\Theta(\eta, \mathbf{x}) \equiv \frac{\delta T(\eta, \mathbf{x})}{\bar{T}(\eta)}$$

- In an inhomogeneous universe photons travelling on different geodesic (line-of-sights) experience different redshifts so

$$\Theta(\eta, \mathbf{x}, \mathbf{n}) \equiv \frac{\delta T(\eta, \mathbf{x}, \mathbf{n})}{\bar{T}(\eta)}$$

Spherical harmonic decomposition

- The brightness function can be decomposed in Fourier modes such that

$$\Theta(\eta, \mathbf{x}, \mathbf{n}) = \int \frac{dk^3}{(2\pi)^3} \Theta(\eta, \mathbf{k}, \mathbf{n}) e^{i\mathbf{k}.\mathbf{x}}$$

with power spectrum

$$< \Theta(\eta, \mathbf{k}, \mathbf{n}) \Theta^*(\eta, \mathbf{k}', \mathbf{n}) >= (2\pi)^3 P_{\Theta(\eta,\mathbf{n})}(k)$$

- Finally Fourier modes can be decomposed in spherical harmonics taking into account the fact that the propagation direction of photons is $-\mathbf{n}$

$$\Theta(\eta, \mathbf{k}, \mathbf{n}) = \sum_{\ell,m} (-1)^\ell \Theta_{\ell,m}(\eta, \mathbf{k}) Y_{\ell.m}(\mathbf{n})$$

or equivalently in Legendre polynomials

$$\Theta(\eta, \mathbf{k}, \mathbf{n}) = \sum_{\ell} (-1)^\ell (2\ell + 1) \Theta_\ell(\eta, \mathbf{k}) P_\ell(\mathbf{k}.\mathbf{n}/k)$$

Power spectrum of the CMB anisotropies

- We want to compute the power spectrum of the temperature field today as observed from our position, $\mathbf{x} = \mathbf{0}$, today $\eta = \eta_0$

$$\frac{\delta T}{\bar{T}}(\mathbf{n}) = \Theta(\eta_0, \mathbf{0}, -\mathbf{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\mathbf{n})$$

- Using previous results and Legendre polynomials to spherical harmonic relations we can write

$$a_{\ell m} = \frac{4\pi}{(2\pi)^3}(i)^\ell \int d^3\mathbf{k} Y_{\ell m}(\mathbf{k}) \Theta_\ell(\eta_0, \mathbf{k})$$

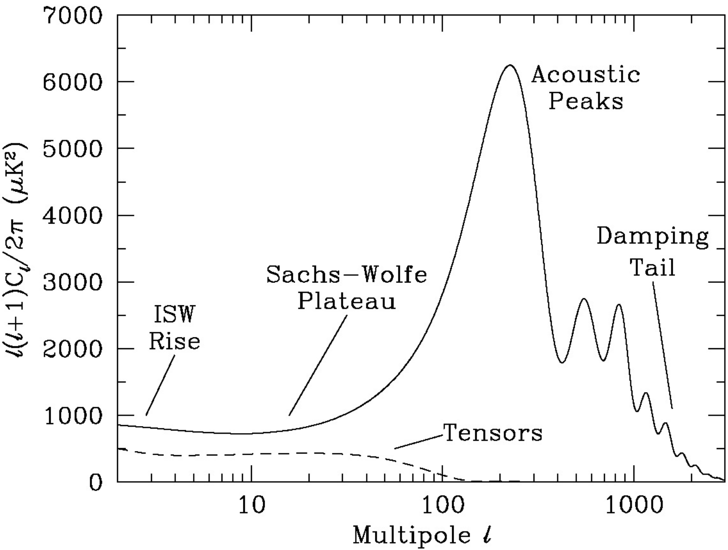
- Using the orthonormality of spherical harmonics we can write

$$C_\ell = 4\pi \int_0^\infty \Delta^2_{\Theta_\ell}(\eta_0, k) \frac{dk}{k}$$

and using the transfer function we obtain

$$C_\ell = 4\pi \int_0^\infty T^2_{\Theta_\ell}(k) \Delta^2_R(k) \frac{dk}{k}$$

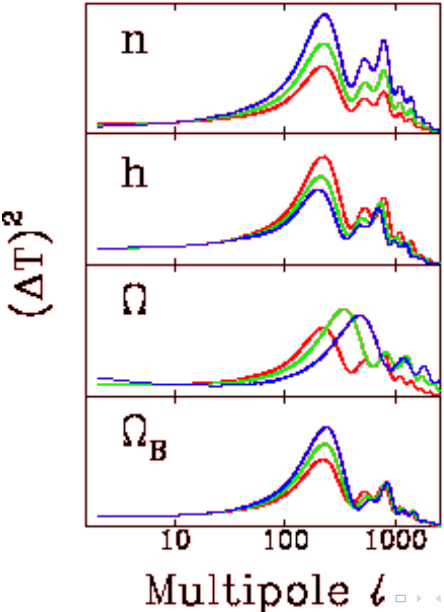
CMB temperature power spectrum



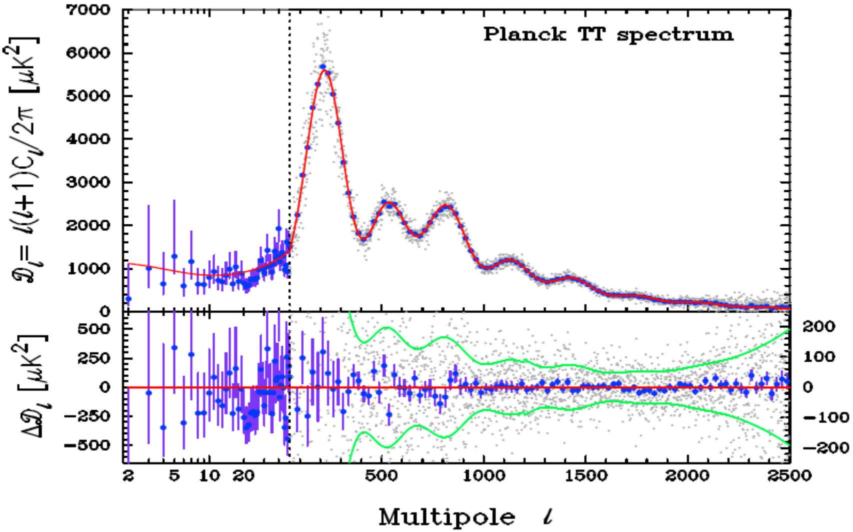
CMB power spectrum and cosmological parameters

(P1)	Peak Scale	$\Omega_m,\ \Omega_b,\ \Omega_\Lambda$
(P2)	Odd/even peak amplitude ratio	Ω_b
(P3)	Overall peak amplitude	Ω_m
(P4)	Damping enveloppe	$\Omega_m,\ \Omega_b,\ \Omega_\Lambda$
(P5)	Global Amplitude	A_s
(P6)	Global tilt	n_s
(P7)	Additional SW plateau tilting via ISW	Ω_Λ
(P8)	Amplitude for $l > 40$ only	τ_{reio}

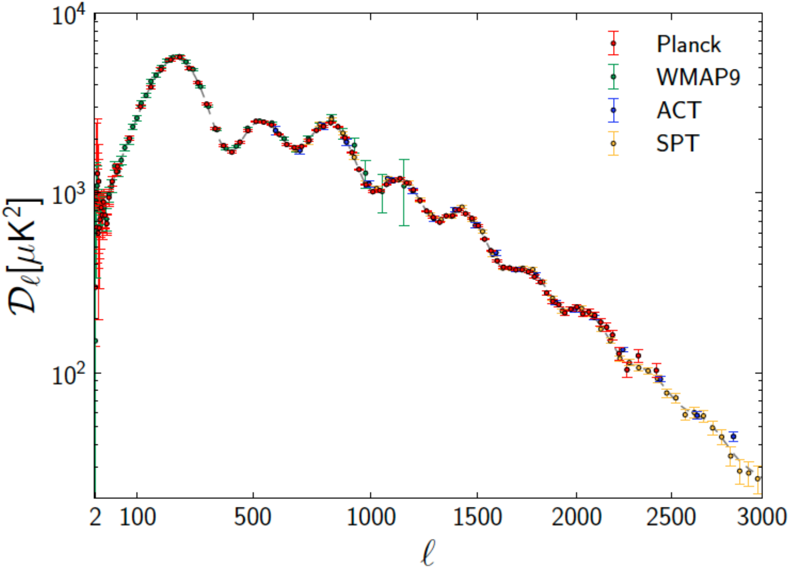
CMB temperature power spectrum and parameters



Planck measured CMB temperature spectrum



Measured CMB temperature spectrum at small angular scales



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L. 3, Section 5: Secondary CMB temperature anisotropies

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Main secondary temperature anisotropies

We call secondary CMB anisotropies those that are generated after recombination either by gravitational effects of interaction of photons with electrons:

- Integrated Sachs Wolfe (ISW) effect: Sachs-Wolfe effect originated by changes in the gravitational potentials along the line-of-sight. The non-linear contribution is generally called Vishniac effect.
- Gravitational Lensing: gravitational lensing induced by mass distribution along the line-of-sight
- Sunyaev-Zel'dovich effect: Compton inverse between CMB photons and hot free electrons on clusters of galaxies
- Reionization: Thomson interaction of CMB photons with free electrons at the time global reionization of the universe when first star form.

Gravitational lensing in a nutshell

- Gravitational potentials along the line of sight **n** to some source at comoving distance D_s gravitationally lens the image
- We can define an effective potential

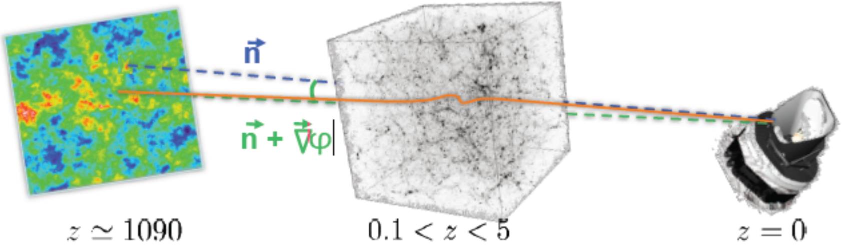
$$\phi(\mathbf{n}) = 2 \int dD \frac{D_s - D}{D D_s} \Phi(D\mathbf{n}, \eta(D))$$

such that the image is remapped as

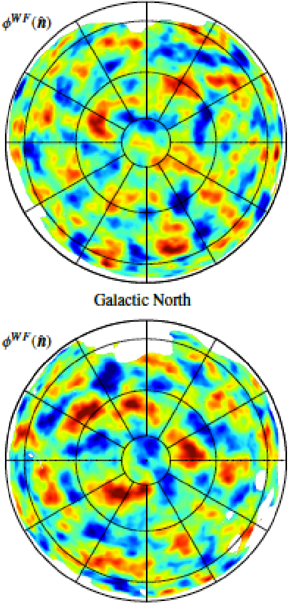
$$\mathbf{n}^I = \mathbf{n}^S + \nabla_{\mathbf{n}} \phi(\mathbf{n})$$

- In the case of CMB lensing we are in the weak lensing regime and we expect small distortions of the image
- In particular we can observe that the convergence is simply the projected mass

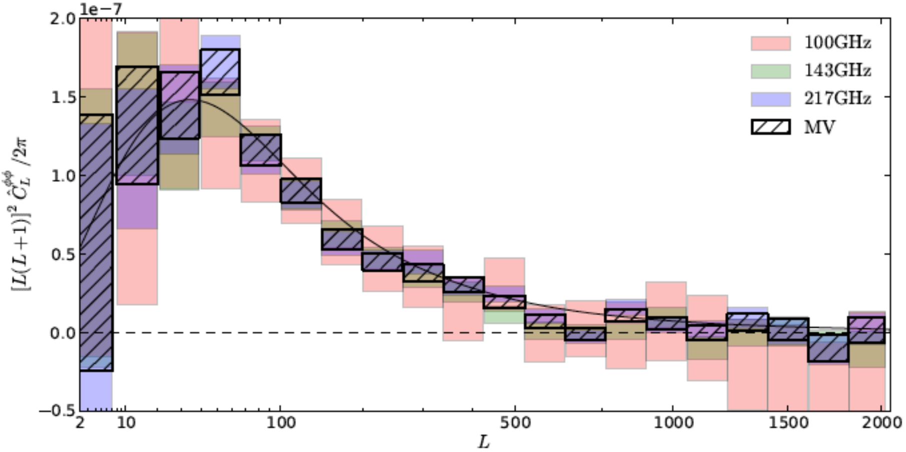
CMB lensing cartoon



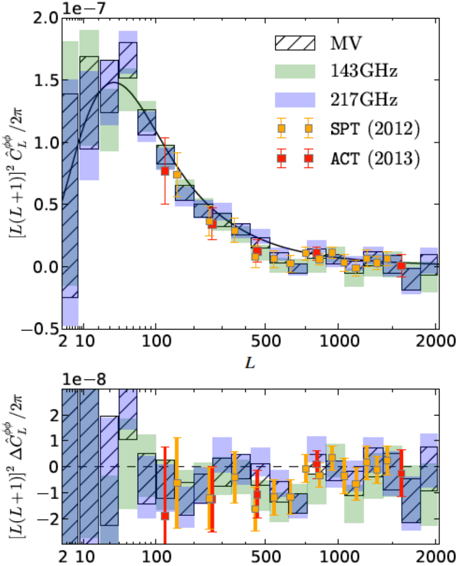
Integrated gravitational potential



Lensing power spectrum



Lensing power spectrum



Sunyaev-Zeldovich (SZ) effect

- Thermal (t)SZ effect corresponds to a small spectral distortion of the CMB spectrum

$$\frac{\Delta T_{tSZ}}{T_{CMB}} = f(x)y = f(x)\int n_e \frac{k_B T_e}{m_e c^2}\sigma_T d\ell$$

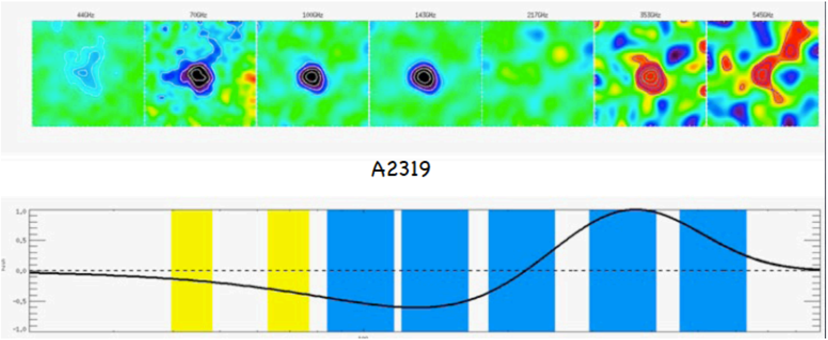
where $x = \frac{h\nu}{k_B T}$ and

$$f(x) = \left(x\frac{e^x+1}{e^x-1} - 4 \right)$$

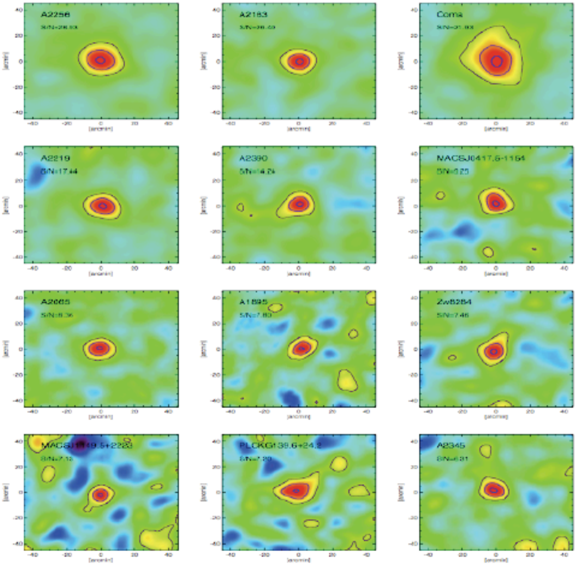
- Kinetic (k)SZ effect If clusters are moving with respect to the CMB frame there is an additional spectral distortion due to the Doppler effect of the cluster bulk velocity on the scattered CMB photons. In the non-relativistic limit the kSZ is just a thermal distortion

$$\frac{\Delta T_{kSZ}}{T_{CMB}} = -\tau_e \left(\frac{v_{pec}}{c}\right) = -\int n_e \sigma_T \left(\frac{v_{pec}}{c}\right) d\ell$$

tSZ effect with Planck

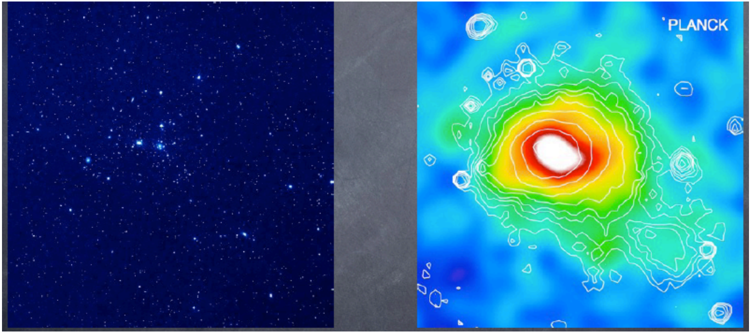


Examples of cluster of galaxies observed via the tSZ effect



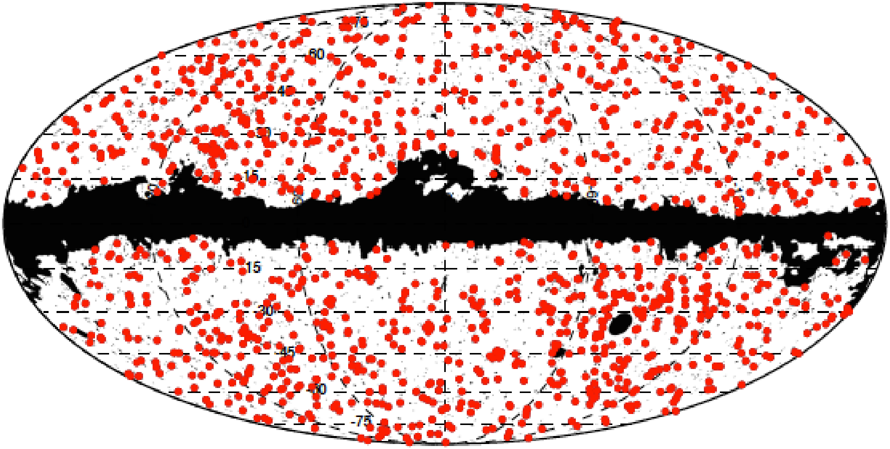
The COMA cluster

- Detailed observations of the Coma cluster including the outskirts
- Direct observation of compression shocks on the tSZ data



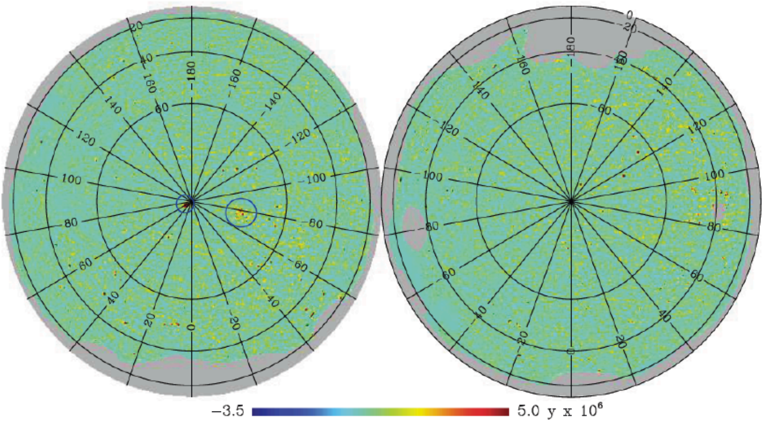
The Planck cluster sample

- 1227 cluster candidates: 861 clusters and 366 candidates being confirmed



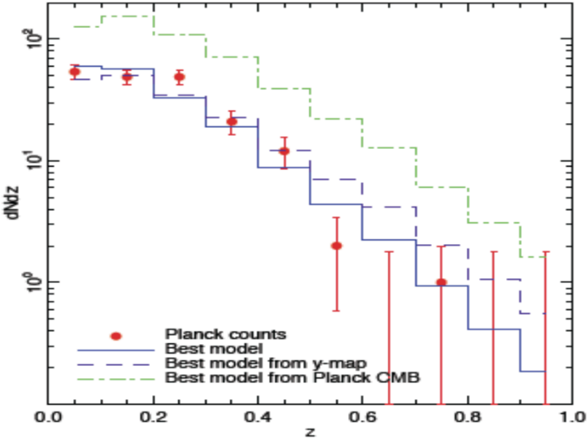
Compton parameter map

- All-sky map of cluster of galaxies and maybe filaments
- Unfortunately foreground contribution is important, more work needed, keep tuned next year.



Cluster number counts and cosmology

- Clusters of galaxies are the largest gravitational bound structures in the universe and can be assimilated to dark matter halos
- The number of cluster of galaxies in terms of their mass and redshift is very sensitive to cosmological parameters and non-linear physics



Experimental astroparticle physics & cosmology

Lecture 4: CMB polarization

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Experimental astroparticle physics & cosmology

L. 4, Section 1: polarization power spectra

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Stokes parameters

- Polarised light can be described using Stokes parameters
- For a light beam propagating on the z direction, the polarization plane is defined by $x - y$ plane
- The electric field can be decomposed as

$$\mathbf{E}(t,z) = E_x(t,z)\mathbf{e_x} + E_y(t,z)\mathbf{e_y}$$

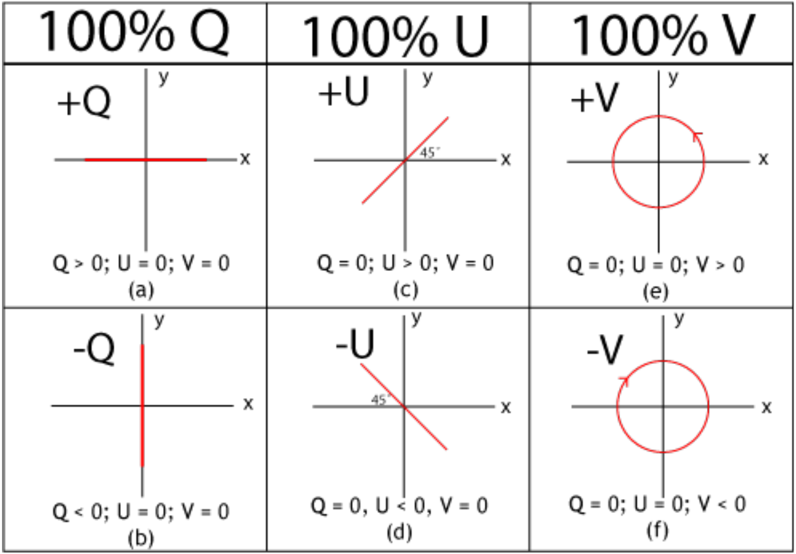
where $E_x(t,z)$ and $E_y(t,z)$ are plane waves

$$E_x(t,z) = A_x e^{\phi_x} e^{i(kz-wt)}$$
$$E_y(t,z) = A_y e^{\phi_y} e^{i(kz-wt)}$$

- Stokes parameters are defined are

$$I =< E_x E_x^* + E_y E_y^* >= A_x^2 + A_y^2$$
$$Q =< E_x E_x^* - E_y E_y^* >= A_x^2 - A_y^2$$
$$U =< E_x E_y^* + E_y E_x^* >= 2A_x A_y \cos(\phi_y - \phi_x)$$
$$V = -i < E_x E_y^* - E_y E_x^* >= 2A_x A_y \sin(\phi_y - \phi_x)$$

Stokes parameters II



Stokes parameters III: some special cases

1. Right-handed (left handed) circularly polarised light, $E_x = E_y$ and $\cos(\phi_y - \phi_x) = \pm\frac{\pi}{2}$

$$
\begin{aligned}
I &= S \\
Q &= 0 \\
U &= 0 \\
V &= \pm S
\end{aligned}
$$

2. Linearly polarized light $\cos(\phi_y - \phi_x) = 0$

$$
\begin{aligned}
I &= S \\
Q &= pS\cos(2\psi) \\
U &= pS\sin(2\psi) \\
V &= 0
\end{aligned}
$$

where $p = \frac{\sqrt{Q^2+U^2}}{I}$ and ψ are the degree and polarization angle.

Linear polarization properties

- In the case of linearly polarised light a change of reference frame modify the Stokes parameters as follows

$$\begin{aligned} I' &= I \\ Q' &= Q\cos(2\theta) + U\sin(2\theta) \\ U' &= -Q\sin(2\theta) + U\cos(2\theta) \end{aligned}$$

- So we can form a spin ± 2 object $Q \pm iU$ that transforms as

$$Q' \pm iU' = e^{\mp 2i\theta}[Q \pm iU]$$

- Thus, Stokes parameters on the sphere can be decomposed as

$$\begin{aligned} T(\mathbf{n}) &= \sum_{\ell m} a^T_{\ell m} Y_{\ell m}(\mathbf{n}) \\ [Q \pm iU] &= \sum_{\ell m} [a^E_{\ell m} \pm i a^B_{\ell m}]\ {}_{\pm 2}Y_{\ell m}(\mathbf{n}) \end{aligned}$$

polarization power spectra

- We can define three scalar fields T, E, B which are independents of the chosen reference frame
- Using those we can form 3 auto-power spectra

$$C_\ell^{TT} = \frac{1}{2\ell+1} \sum_m |a_{\ell m}^T|^2$$
$$C_\ell^{EE} = \frac{1}{2\ell+1} \sum_m |a_{\ell m}^E|^2$$
$$C_\ell^{BB} = \frac{1}{2\ell+1} \sum_m |a_{\ell m}^B|^2$$

  and 3 cross-spectra

$$C_\ell^{TE} = \frac{1}{2\ell+1} \sum_m (a_{\ell m}^T a_{\ell m}^{E\ *})$$
$$C_\ell^{TB} = \frac{1}{2\ell+1} (\sum_m (a_{\ell m}^T a_{\ell m}^{B\ *})$$
$$C_\ell^{EB} = \frac{1}{2\ell+1} \sum_m (a_{\ell m}^E a_{\ell m}^{B\ *})$$

- C^{TB} and C^{EB} vanish if parity is conserved

Experimental astroparticle physics & cosmology

L. 4, Section 2: CMB polarization physics

J.F. Macías-Pérez

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Thomson scattering

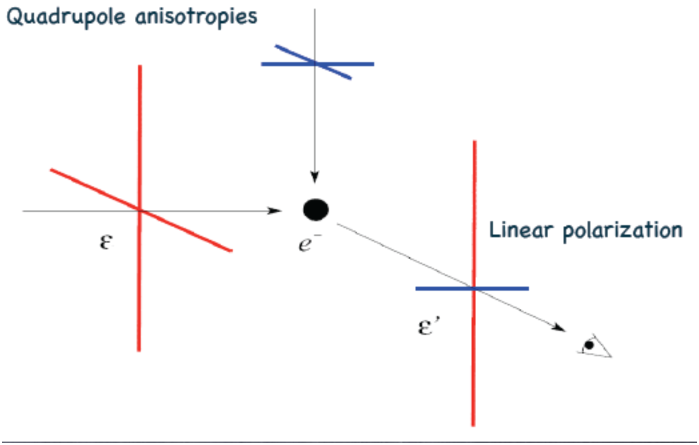
- As discussed before polarization state of radiation along the line-of-sight is described by the components of the electric field **E**
- The differential cross section of Thomson scattering is given by

$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi}|\mathbf{E}'.\mathbf{E}|^2$$

  where $\mathbf{E}'$ and $\mathbf{E}$ are the incoming and outgoing directions of the electric field
- To get final polarization state along the line-of-sight **n** we sum over angle and incoming polarization

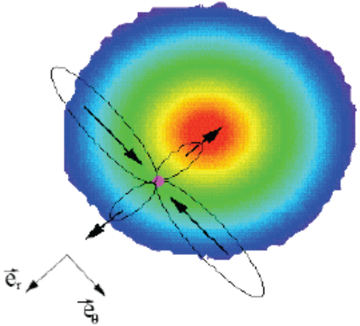
$$\sum_{i=1,2} \int d\mathbf{n}' \frac{d\sigma}{d\Omega}$$

Cartoon polarization generation



- Only quadrupole anisotropies generate polarization

Local quadrupole perturbations

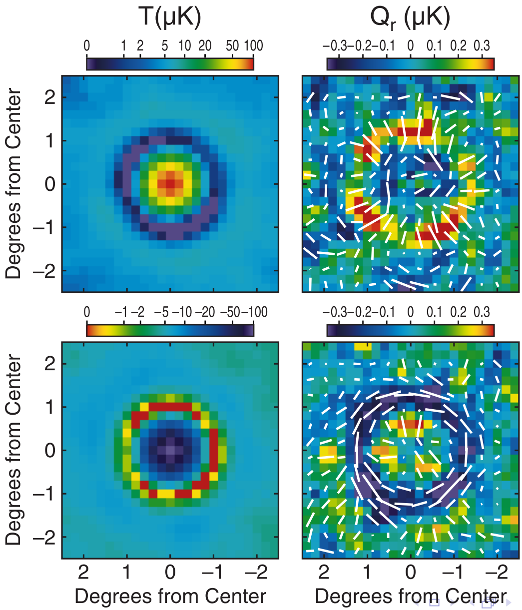


- In hot and cold spots electrons observes local quadrupoles
- Density, scalar, perturbations produce Q_r polarisation corresponding to E modes

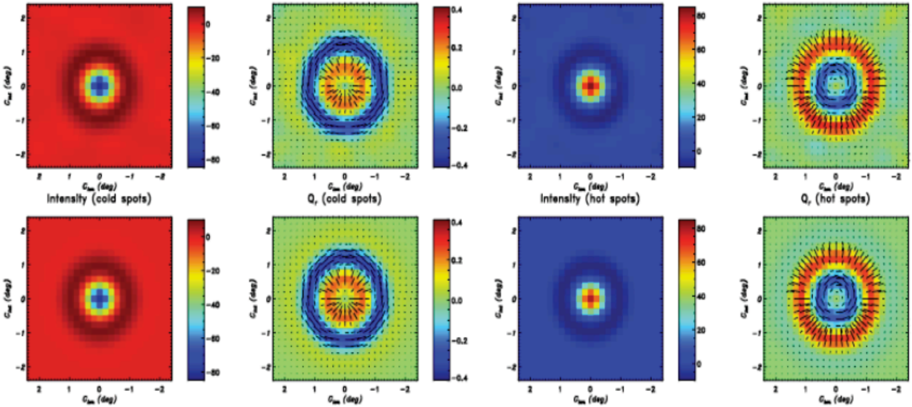
Local quadrupole perturbations and gravitational waves

- Gravitational waves distort the polarization pattern and induce also U_r polarization which corresponds to E and B modes

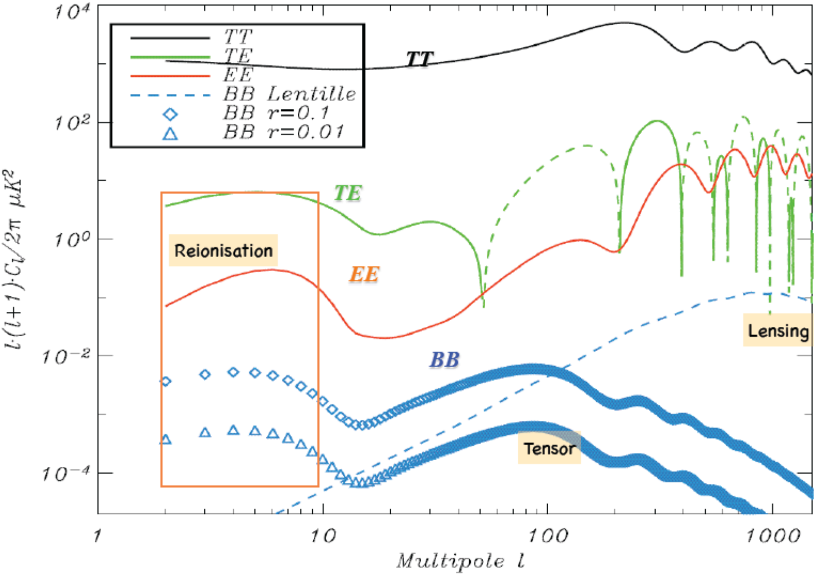
WMAP hot and cold spots polarization



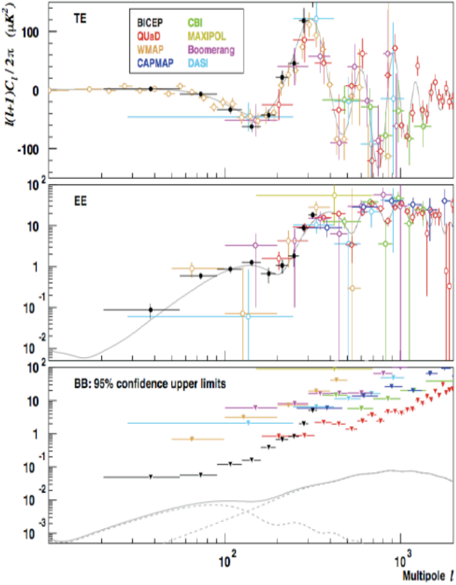
Planck hot and cold spots polarization



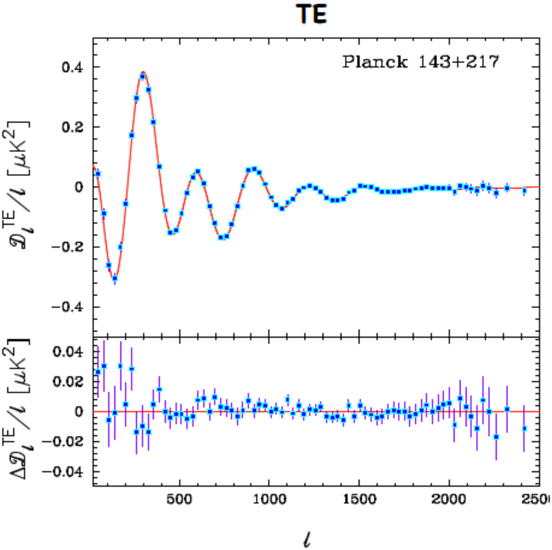
Expected CMB polarization power spectra



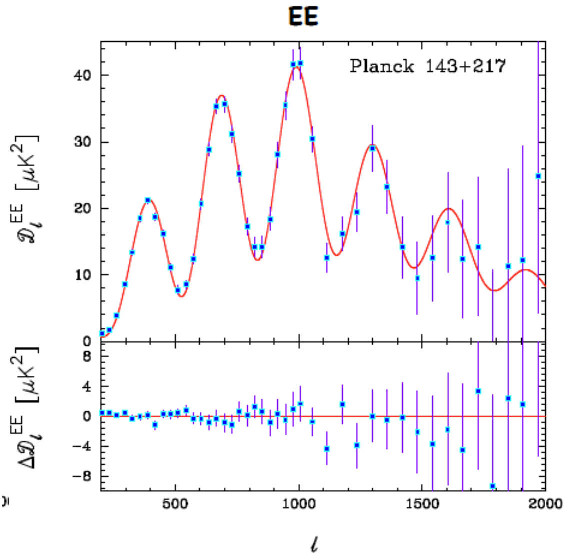
Measured CMB polarisation power spectra before planck



Planck measured CMB polarisation power spectra



Planck measured CMB polarisation power spectra



Experimental astroparticle physics & cosmology

Lecture 5: Linear Cosmological Perturbation Theory

J.F. Macías-Pérez

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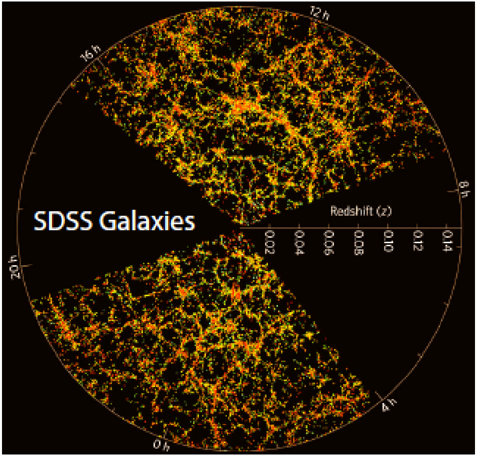
Large-scale structure

- galaxy surveys have shown the large-scale structure of the universe which is formed of voids, clusters of galaxies and filaments
- the universe is homogeneous for scales larger than 100 Mpc



Experimental astroparticle physics & cosmology

L. 5, Section 1: Linear Perturbation Theory

J.F. Macías-Pérez

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Inhomogeneous Universe

- Inflationary theory predicts small curvature and tensor fluctuations
- Inhomogeneities in the matter-energy distribution grow via gravitational instability
- In the expanding universe, growth rate is a power law
- Follow general principles of FRW/ Thermal History but drop homogeneity and isotropy
  - Matter evolves in a perturbed geometry, conserving stress-energy tensor
  - Matter curves geometry, cosmological Poisson equation generates gravitational potential from density perturbations
  - Use linear perturbation theory to derive evolution equations
  - Use extra closure relations in addition to average equation of state

Inhomogeneous Fields

- As for homogeneous cosmology, a full description of matter is given through the phase space distribution

$$f(\mathbf{x}, \mathbf{q}, t)$$

where momentum dependence **q** describes bulk motion of particles

- Thus, energy density and pressure are functions of position

$$\rho(\mathbf{x}, t) = g \int \frac{d^3q}{(2\pi)^3} f(\mathbf{x}, \mathbf{q}, t) E$$

and

$$p(\mathbf{x}, t) = g \int \frac{d^3q}{(2\pi)^3} f(\mathbf{x}, \mathbf{q}, t) \frac{|\mathbf{q}|^2}{3E}$$

and can be considered as low order moments of the distribution function

Inhomogeneous Boltzmann Equation

- Evolution of density inhomogeneities is governed by the Boltzmann equation as in the homogenous case
- We work now on comoving representation: conformal time η, comoving coordinates x and retains physical momentum
- Then we have as before

$$f' + \mathbf{q}' \frac{\partial f}{\partial \mathbf{q}} + x' . \frac{\partial f}{\partial x} = C(f)$$

  where $'$ corresponds to derivative with respect to conformal time and $C(f)$ is the collision term
- These formulation will be important mainly for photons and baryons and cold dark matter although fully decouple can be consider as a perfect fluid to first order approximation

Summary of homogeneous and isotropic universe results

- We have perfect fluids such that $p = w\rho$
- Energy conservation

$$\dot{\rho}a^3 + 3(\rho + p)\dot{a}a^2 = 0$$

- FL equations

$$H^2 = \frac{8\pi G}{3}\rho$$

- Solutions of the FL equations

	w	$\rho(a)$	$a(t)$	$H(t)$
radiation	$1/3$	a^{-4}	$t^{1/2}$	$\frac{1}{2}t^{-1}$
matter	0	a^{-3}	$t^{2/3}$	$\frac{2}{3}t^{-1}$
Λ	$-1+\epsilon$	H_0	$e^{H_0 t}$	H_0

Linear Perturbation Theory

- We assume perturbations are small enough to be in the linear regime so for example

$$\rho(x,t) =< \rho(x,t) > +\delta\rho(x,t) = \rho_0(t) + \delta\rho(x,t)$$

where ρ_0 is the background density (homogeneous like)

- The evolution of the background term is given by the FL equations studied in Lecture 2
- We can also define contrast quantities, as for example the density contrast

$$\delta_\rho = \frac{\delta\rho(x,t)}{\rho_0(t)}$$

- Linear perturbation theory can applied to all physical quantities and in particular to the metric and the stress-energy tensor

$$g_{\mu\nu} = g^{RW}_{\mu\nu}(t) + \delta g_{\mu\nu}(\mathbf{x},t)$$

$$T_{\mu\nu} = T^{hom}_{\mu\nu}(t) + \delta T_{\mu\nu}(\mathbf{x},t)$$

where *RW* stands for the Robertson-Walker metric and *hom* for the homogenous stress-energy tensor

Metric perturbations

- The perturbed metric $\delta g_{\mu\nu}(\mathbf{x}, t)$ is a symmetric 4x4 tensor and therefore will have 10 degrees of freedom
- Bardeen in 1980 proved that these can be described on the basis of scalar, vectors and tensors perturbations
- In a general form, for a flat universe and using conformal time we can write for an homogenous space

$$ds^2 = a^2(\eta)(d^2\eta - dx^2 - dy^2 - dz^2)$$

and thus the perturbed version reads

$$ds^2 = a^2(\eta)[(1 + 2\phi)d^2\eta + B_i dx^i d\eta - \{(1 - 2\psi)\delta_{ij} + h_{ij}\}dx^i dx^j]$$

with $\sum_i h_{ii} = 0$

Metric perturbations: degrees of freedom

- The generalized gravitational potential, ψ (1 scalar dof)
- Local distortions of the average scale factor, ϕ (1 scalar dof)
- Longitudinal and transverse components of $B_i = B_i^{||} + B_i^{\perp}$
  - longitudinal $B_i^{||} = \frac{\partial b}{\partial i} = \vec{\nabla} b$ (1 scalar dof)
  - transverse $B_i^{\perp}$ (2 vectorial dof)
- We can also decompose tensors as $h_{ij} = h_{ij}^T + h_{ij}^{||} + h_{ij}^{\perp}$
  - Transverse h_{ij}^T with $\partial_i h_{ij}^T = 0$ (2 tensor dof)
  - divergence longitudinale $h_{ij}^{||} = 2(\partial_i \partial_j - \frac{1}{3}\nabla^2 \mu$ (1 scalar dof)
  - divergence transverse $h_{ij}^{\perp} = \partial_i A_j + \partial_j A_i$ (2 vector dof)
- So we have in total 10 dof : 4 scalars + 4 vectors + 2 tensors)
- We do not consider vector modes that decay very rapidly
- To many degrees of freedoms, need to have close relations

Stress-energy tensor perturbations

- For a perfect fluid we have

$$T^{\mu\nu} \equiv -pg^{\mu}_{\nu} + (p+\rho)U^{\mu}U_{\nu}$$

- Perturbing it to first order with $U^{\mu} = (1, v^i, v^i, v^i)$ and v^i small

$$T^0_0 = \rho = \bar{\rho} + \delta\rho \ \ (1 \text{ dof})$$

$$\partial_i T^0_i = (\bar{\rho} + \bar{p})v_i \ \ (2+1 \text{ dof})$$

$$T^i_j = -p\delta_{ij} = -(\bar{p} + \delta p)\delta_{ij} \ \ (1 \text{ dof})$$

- As before $v_i = v_i^{||} + v_i^{\perp}$, the scalar degree of freedom is obtained from

$$\theta = \partial^i v_i$$

- An extra scalar degree of freedom is hidden in the tensor component of the perturbation $\Sigma_{ij}^{||} = (\partial i \partial j - \frac{1}{3}\nabla^2\delta_{ij}\bar{\sigma}$ from which we define the anisotropic stress

$$(\bar{\rho} + \bar{p})\nabla^2\sigma = -\partial_i\partial_j - \frac{1}{3}\nabla^2\delta_{ij}\Sigma^i_j \ \ (1 \text{ dof})$$

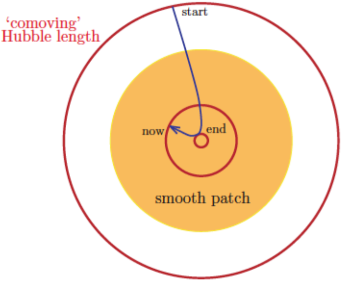
Stress-energy tensor perturbations degrees of freedom

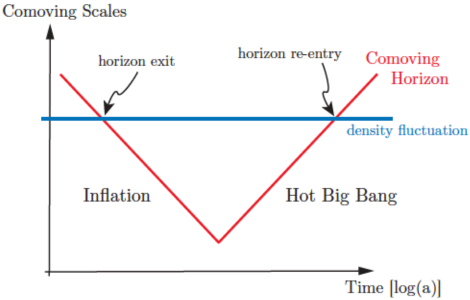
- Finally we have the following scalars degrees of freedom
  1. $$T^0_0 = \bar{\rho}(1+\delta)$$
  2. $$\partial_i T^0_i = (\bar{\rho}+\bar{p})\theta$$
  3. $$T^i_i = -3(\bar{p}+3\delta p)$$
  4. $$-\partial_i\partial_j - \frac{1}{3}\nabla^2\delta_{ij}T^i_j = (\bar{\rho}+\bar{p})\nabla^2\sigma$$
- Anisotropic stress is generally neglected so $\sigma = 0$
- We will consider no pure vector perturbations neither
- Tensors perturbations comes only from the metric

A few words on gauges

- For an idealized FLRW universe there is only a single choice of time slicing compatible with homogeneity
- For a perturbed universe there is an infinity of time slice choices compatible with linear perturbation hypothesis
- As $\delta\rho(t, \mathbf{x}) = \rho(t, \mathbf{x}) - \bar{\rho}(t)$, we observe that the perturbation value would depend on the time slicing
- A gauge is a choice of time slicing.
- Gauge transformations are induced by coordinates transformations of the form $x_\mu \leftarrow x_\mu + \epsilon_\mu$ that maps the points of one time slicing to another
- Physics should not depend on gauge transformations and so we can fix some degrees of freedoms: 2 for scalar perturbations
- We can define gauge invariant quantities as the Bardeen potentials Φ_A and Φ_H
- Either we work with gauge invariant quantities or with particular gauge choice

Back to inflation





Working on Fourier space

- We define the comoving wavelength λ_{com} and wave number k as

$$\lambda_{com} = \frac{2\pi}{k} = \frac{\lambda}{a}$$

where λ is the physical wavelength of the perturbation

- For perturbations outside the horizon we have

$$k < 2\pi aH$$

and inside the horizon

$$k > 2\pi aH$$

- As we did before we define the power spectrum as

$$< \delta_A(\mathbf{k}_1, \eta)\delta_A^*(\mathbf{k}_2, \eta) >= P_A(k, \eta)\delta(\mathbf{k}_1 - \mathbf{k}_2)$$

Transfer function

- Before we have seen that for super Hubble modes the perturbations remain constant and then for any perturbation $A(\eta, \mathbf{x})$ we can write

$$< A(\eta, \mathbf{k}_1) A(\eta, \mathbf{k}_2) >= \delta(\mathbf{k}_2 - \mathbf{k}_1) P_A(k)$$

- As physics is linear we can imagine a linear function such that

$$A(\eta, \mathbf{k}) = T_A(k, \eta) A(\eta_0, \mathbf{k}) = T_A(k, \eta) A(\mathbf{k})$$

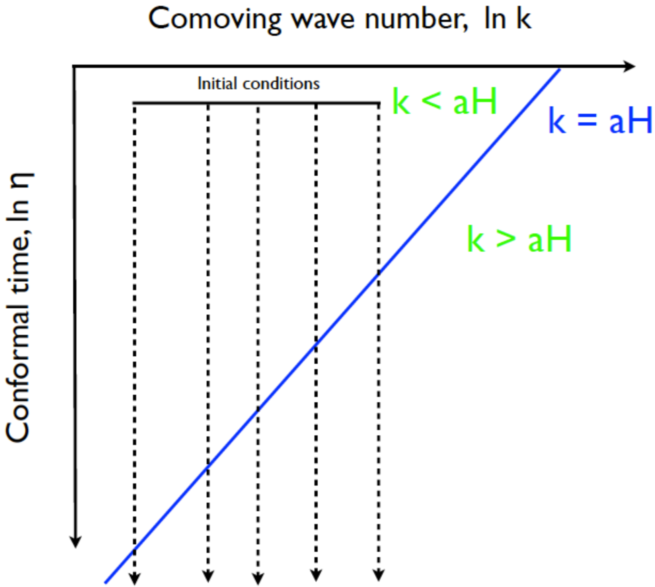
and so

$$P_A(\eta, \mathbf{k}) = T_A^2(k, \eta) P_A(\mathbf{k})$$

- In the case of adiabatic conditions we can set a common initial perturbation using the Baardeen curvature $\mathcal{R} = \phi - \frac{1}{3} \frac{\delta \rho_{tot}}{\bar{\rho}_{tot} + \bar{p}_{tot}}$ such that

$$P_A(\eta, \mathbf{k}) = T_{A,\mathcal{R}}^2(k, \eta) P_{\mathcal{R}}(\mathbf{k}) = \frac{2\pi}{k^3} T_{A,\mathcal{R}}^2(k, \eta) \Delta_{\mathcal{R}}^2(k)$$

Cartoon evolution of perturbations



Experimental astroparticle physics & cosmology

L. 5, Section 2: Dark matter power spectrum

J.F. Macías-Pérez

LPSC

Matter power spectrum definition

- We are interested in computing the power spectrum of the non-relativistic matter density perturbation

$$\delta_m = \frac{\delta\rho_m}{\bar{\rho}_m} = \frac{\delta\rho_b + \delta\rho_{CDM}}{\bar{\rho}_b + \bar{\rho}_{CDM}}$$

- Thus, the matter power spectrum is

$$< \delta_m(\eta, \mathbf{k}_1), \delta_m(\eta, \mathbf{k}_2) >= \delta_D(\mathbf{k}_2 - \mathbf{k}_{21}) P(\eta, k)$$

- Accounting for adiabatic initial conditions and using the curvature power spectrum we can write

$$P(\eta, k) = \frac{2\pi}{k^3} A_S \left(\frac{k}{k_*}\right)^{n_s - 1} T^2_{\delta_m}(\eta, k)$$

where A_S is a normalization factor for $k = k_*$

Computing the evolution of the transfer function

- Let's assume CDM dominates the matter density $\Omega_b \ll \Omega_{CDM}$ and so $\delta_m \approx \delta_{CDM}$
- Using the continuity and Euler equations for CDM perturbations (we saw before CDM behaves like pressureless perfect fluid, $\sigma = w = 0$)

$$\delta''_{CDM} + \frac{a'}{a}\delta'_{CDM} = -k^2\psi + 3\phi'' + 3\frac{a'}{a}\phi'$$

- For an expanding universe the clustering rate will depend on the expansion rate
- For $k < aH$ (super Hubble) the perturbations remain constant
- For $k > aH$ we neglect dilation terms and then we can deduce the Mészáros equation

$$\delta''_{CDM} + \frac{a'}{a}\delta'_{CDM} - \frac{3}{2}\left(\frac{a'}{a}\right)^2 \Omega_{CDM}(a)\delta_{CDM} = 0$$

- The Mészáros equation is obtained by combining previous equation with (00) component of the Einstein equations and the FL equations

Solutions to Mészáros equation

1. For a radiation dominated universe $a \propto \eta$ and $\Omega_{CDM} \ll 1$ so we can neglect the las term in the equation and so

$$\delta_{CDM} = \text{ constant or } \delta_{CDM} \propto \log(\eta)$$

so perturbation growth logarithmically

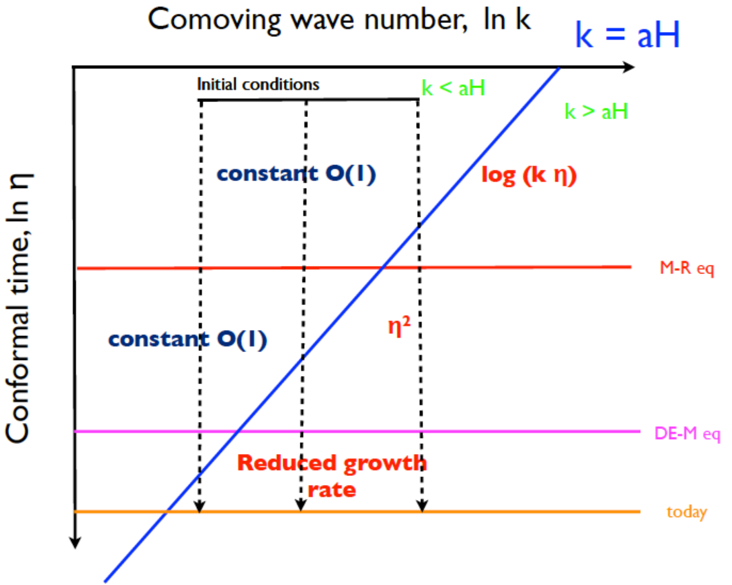
2. For a matter dominated universe $a \propto \eta^2$ and $\Omega_{CMB} \simeq 1$ and so the solutions are

$$\delta_{CDM} \propto \eta^{-3} \text{ or } \delta_{CDM} \propto \eta^2$$

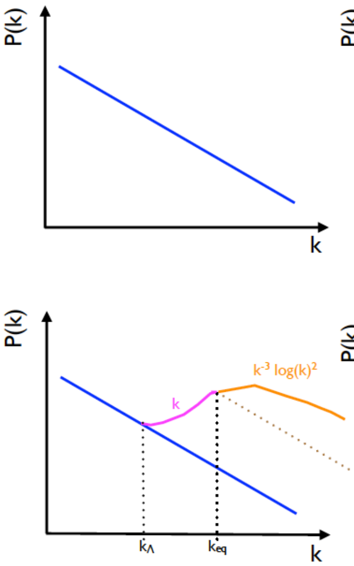
so it growth quadratically with η

3. For dark energy dominated universe δ_{CDM} growths at smaller rate than for matter domination (i.e. slower than η^2 and this reduction of the growth rate does not depends on k

Cartoon matter fluctuations evolution



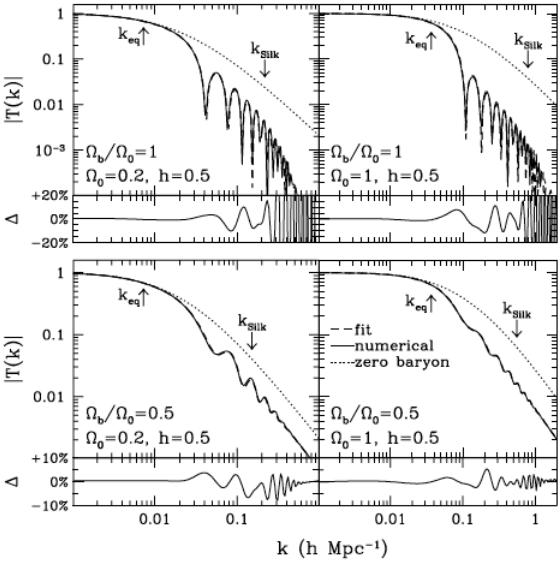
Cartoon matter power spectrum evolution



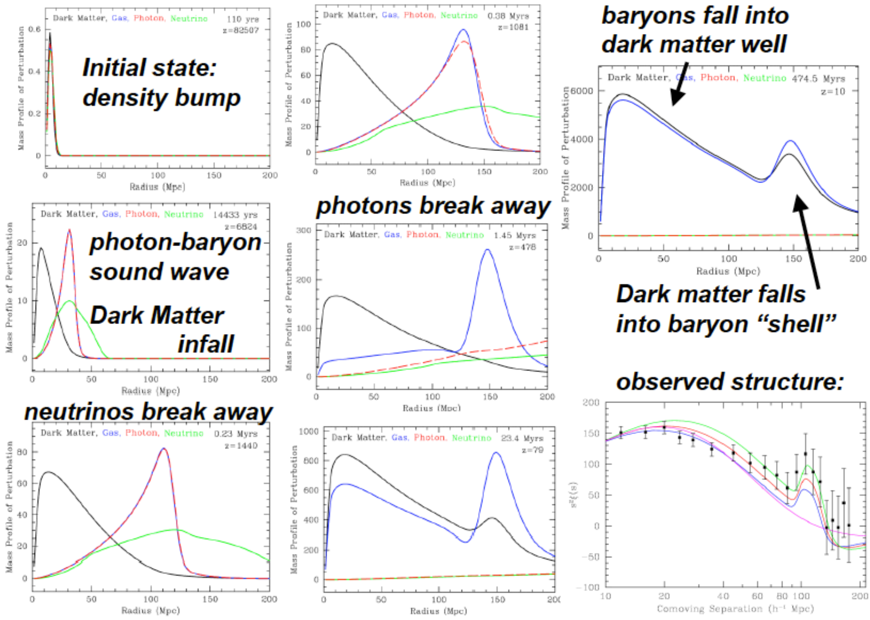
Baryon corrections to the matter power spectrum

- Baryons modify the shape of the power spectrum introducing baryon acoustic oscillations (BAO) and power suppression at $k > k_{eq}$
- BAO are produced by the Thomson interaction of photons and electrons before decoupling. The photon pressure will counter balance gravitational collapse.
- BAOs can be observed both on CMB and Large Scale Structure however the mean time of formation of the oscillations is not the same and so neither their characteristic scale.
- For CMB BAO are frozen at decoupling while for baryons they are frozen at baryon drag (last time baryons interacted)
- Full study of BAOs requires to solve the Boltzmann equation. We will do this for CMB next lecture.

BAO in the matter power spectrum



Baryon Acoustic Oscillations



Parameter dependence of the matter power spectrum

(P1) The time of equality determines the peak of the spectrum.

(P2) Baryon abundance (relative to CDM) determines suppression at $k > k_{eq}$ and also BAOs features

(P3) The baryon drag scale $r_s(\eta_{drag})$ depends mainly on Ω_b

(P4) The global amplitude of the spectrum depends on the primordial spectrum amplitude A_s but also on Ω_Λ because of growth suppression

(P5) The global tilt of the spectrum depends on the primordial spectrum tilt, n_s

Observed matter power spectrum