

# Lectures on calorimetry

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Lecture 4



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## Note

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If you want any mistake or want to ask a question, please contact me at:  
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# Plan of lectures

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## Lecture 1

Why/what calorimeters ?

Physics of EM & HAD showers

Calorimeter Energy Resolution

## Lecture 2

ATLAS & CMS calorimeters

Calorimeter Objects

Triggering

## Lecture 3

Example of calorimeters (suite)

Future of calorimetry

## Lecture 4

Tutorial  
Exercises

# Useful Formulas (EM showers) [1]

Radiation Length:

$$X_0 \approx \frac{180A}{Z^2} \text{ (g.cm}^{-2}\text{)}$$

Radiation Length for composite material:

$$\frac{1}{X_0} = \sum \frac{w_j}{X_j}$$

$w_j$ : fraction of material  $j$   
 $X_j$ : radiation length of material  $j$   
(in g.cm<sup>-2</sup>)

Moliere Radius:

$$R_M = \frac{21\text{MeV}}{E_C} X_0$$

Moliere Radius for composite material:

$$\frac{1}{R_M} = \sum \frac{w_j}{R_{Mj}}$$

$w_j$ : fraction of material  $j$   
 $R_{Mj}$ : Moliere Radius of material  $j$   
(in g.cm<sup>-2</sup>)

Energy Resolution:

$$\frac{\sigma}{E} = \frac{S}{\sqrt{E}} \oplus \frac{N}{E} \oplus C$$

$\oplus$  : quadratic sum  
S: Stochastic  
N: noise  
C: constant

## Useful Formulas (EM showers) [2]

$$E_C(\text{solid}) = \frac{610 \text{ MeV}}{Z+1.24}$$

$E_C$ : critical energy

$$E_C(\text{liquid}) = \frac{710 \text{ MeV}}{Z+0.92}$$

**Shower maximum**

$$t_{\max} = \frac{\ln E_0 / E_C}{\ln 2}$$

$$N(t_{\max}) \approx \frac{E_0}{E_C}$$

Longitudinal containment:

$$t_{95\%} = t_{\max} + 0.08Z + 9.6$$

$$\frac{\sigma_E}{E} = 3.2\% \sqrt{\frac{E_C [\text{MeV}] \cdot t_{\text{abs}}}{F \cdot E [\text{GeV}]}}$$

(stochastic contribution)

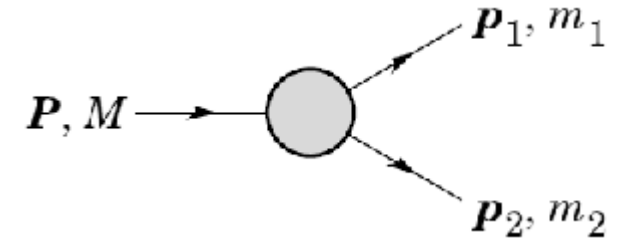
$t_{\text{abs}}$ : thickness of absorber (in units of  $X_0$ )

F: factor (~0.2 for liquid noble gaz, 0.06 for Si, ~1 for scintillators)

# Resolution

- Two-body decay. Ex:  $H \rightarrow \gamma\gamma$

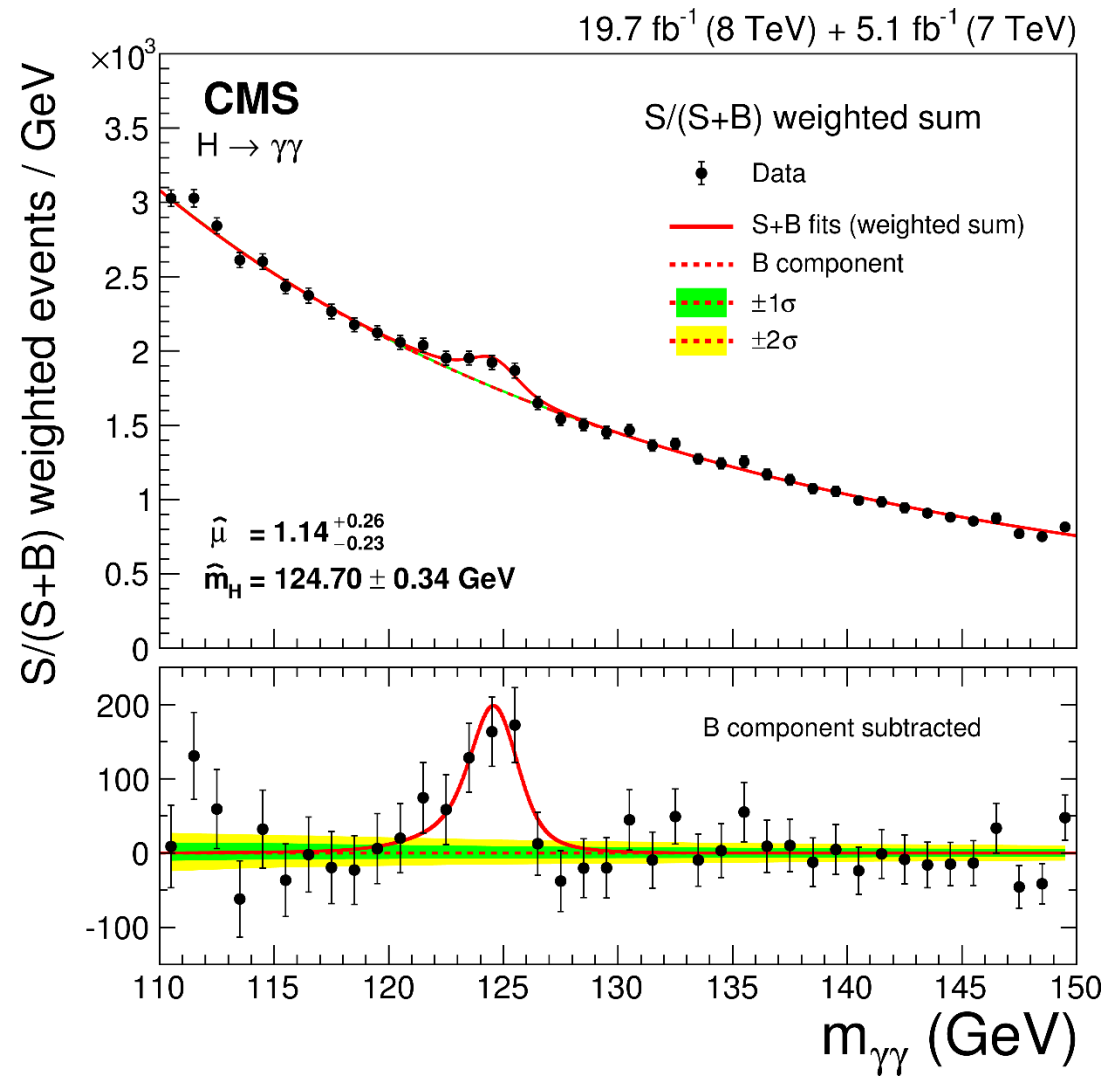
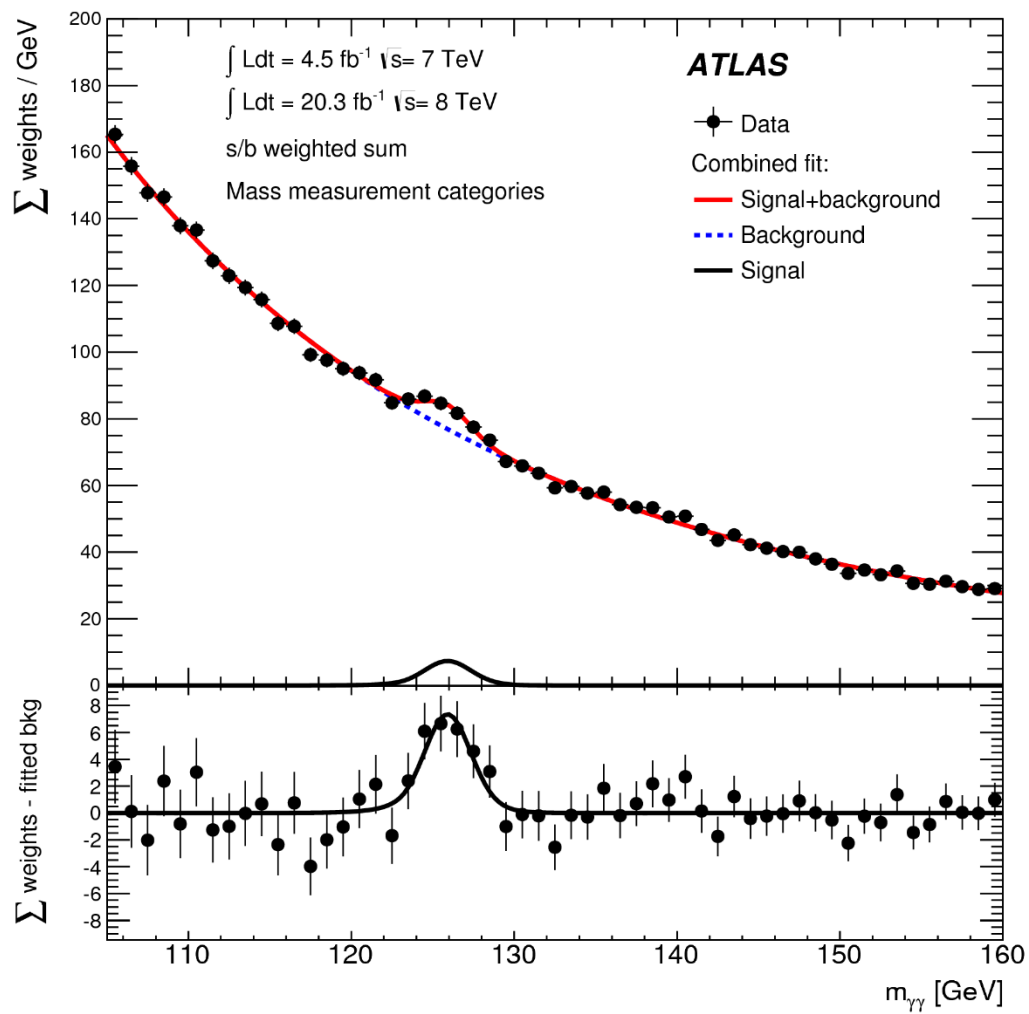
$$m_{\gamma\gamma} = 2E_1E_2(1 - \cos\theta_{\gamma\gamma})$$



$$\frac{\sigma_m}{m_{\gamma\gamma}} = \frac{1}{2} \sqrt{\left(\frac{\sigma_{E1}}{E_1}\right)^2 + \left(\frac{\sigma_{E2}}{E_2}\right)^2 + \left(\frac{\sigma_\theta}{\text{tg}\theta/2}\right)^2}$$

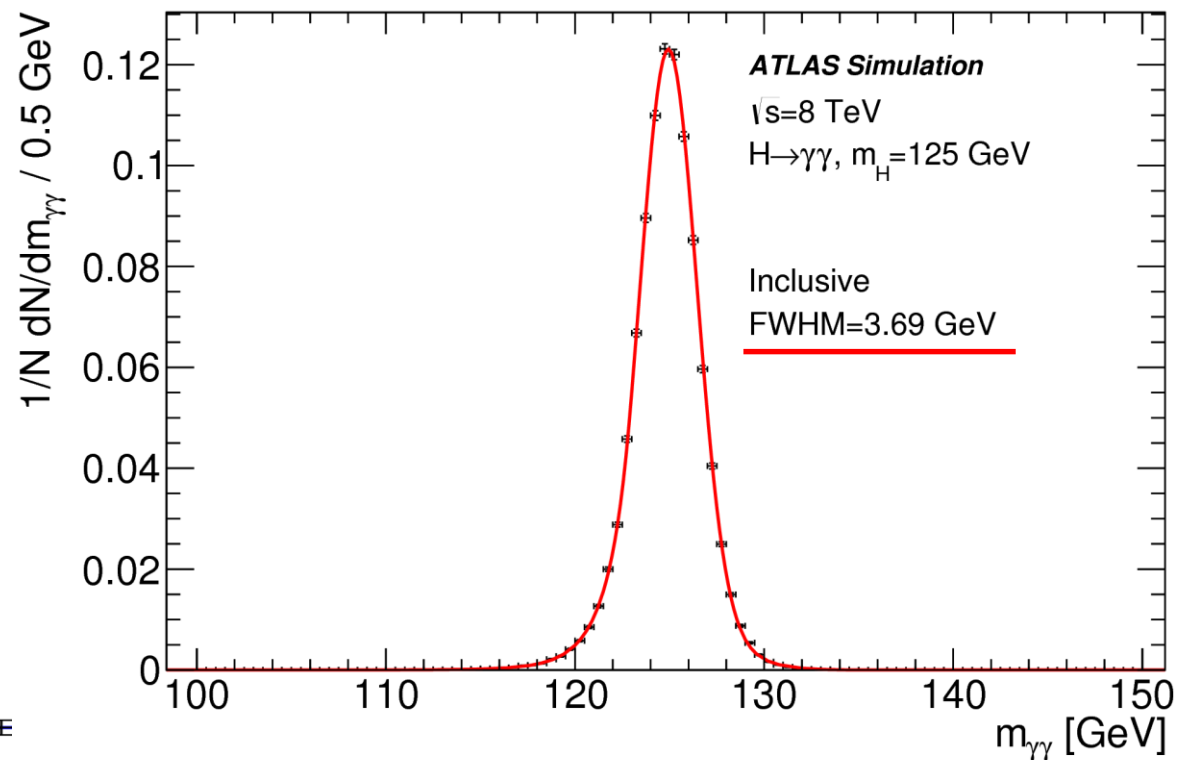
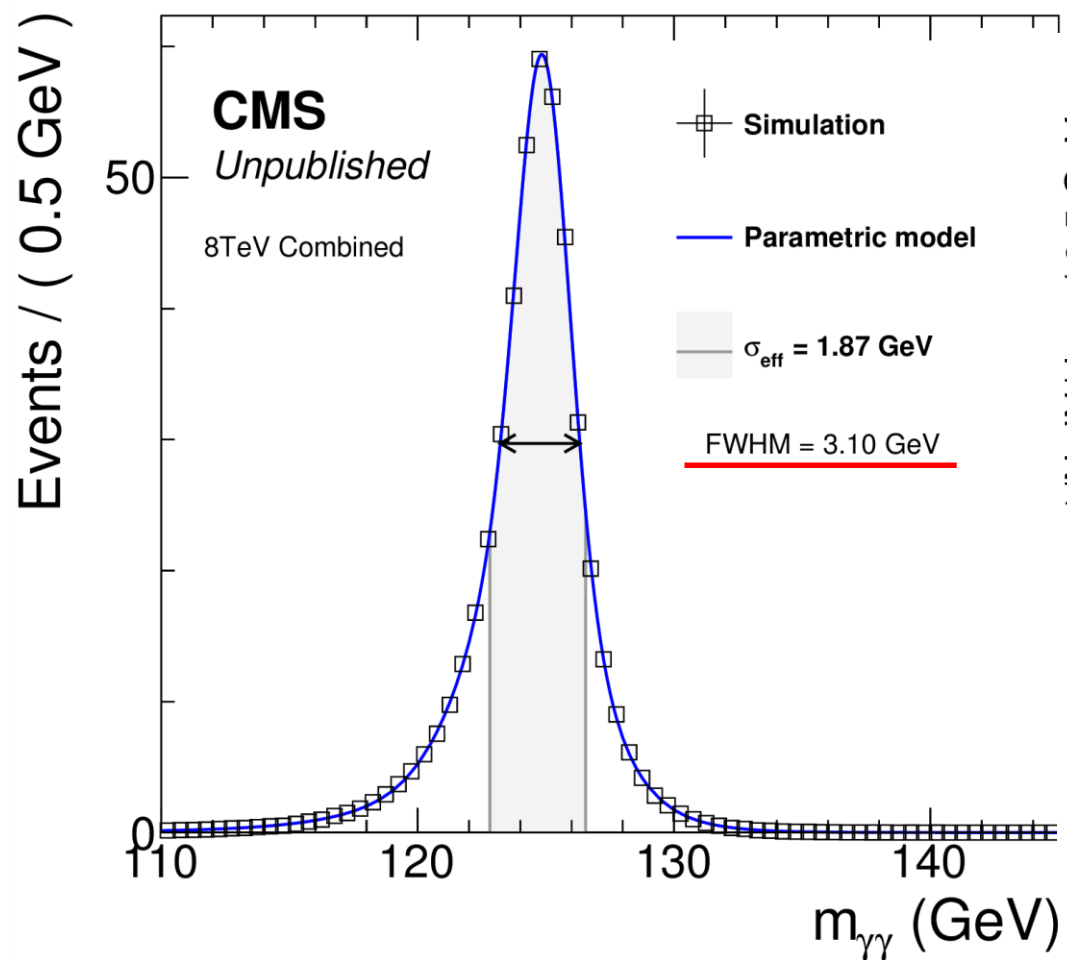
- Resolution on E comes from calorimeters
- How do we measure position of photons ? (in CMS and ATLAS)

# ATLAS/CMS Results (1)





# ATLAS/CMS Results (2)





# Resolution (again)

## CMS

$$\frac{\sigma(E)}{E} = \frac{0.03}{\sqrt{E(\text{GeV})}} \oplus \frac{0.3}{E(\text{GeV})} \oplus 0.005$$

(test beam)

## ATLAS

$$\frac{\sigma(E)}{E} = \frac{0.1}{\sqrt{E(\text{GeV})}} \oplus \frac{0.3}{E(\text{GeV})} \oplus 0.007$$

(test beam)

- Fill the table for both calorimeters
- Comment ?

	10 GeV	1 TeV
Stochastic (GeV)		
Noise (GeV)		
Constant (GeV)		
$\sigma(E)$ (GeV)		
$\sigma(E) / E$ (%)		

# Resolution (again) [SOLUTION]

## CMS

$$\frac{\sigma(E)}{E} = \frac{0.03}{\sqrt{E(\text{GeV})}} \oplus \frac{0.3}{E(\text{GeV})} \oplus 0.005$$

(test beam)

## ATLAS

$$\frac{\sigma(E)}{E} = \frac{0.1}{\sqrt{E(\text{GeV})}} \oplus \frac{0.3}{E(\text{GeV})} \oplus 0.007$$

(test beam)

- Fill the table for both calorimeters
- Comment ?

To compute, for instance, the contribution of the stochastic term to the resolution, do:  $\sigma_{\text{stochastic}}(\text{GeV}) = E * S / \text{sqrt}(E)$ , where S is given in the formula (0.03 for CMS, 0.1 for ATLAS) and E=10 or 1000 GeV.

10 GeV	CMS	ATLAS
Stochastic (GeV)	0,095	0,316
Noise (GeV)	0,300	0,300
Constant (GeV)	0,050	0,070
sigmaE(GeV)	0,32	0,44
sigmaE/E (%)	3,19	4,41

1000 GeV	CMS	ATLAS
Stochastic (GeV)	0,949	3,162
Noise (GeV)	0,300	0,300
Constant (GeV)	5,000	7,000
sigmaE(GeV)	5,10	7,69
sigmaE/E (%)	0,51	0,77

- A few comments:

- At low energy, noise dominates CMS measurement while stochastic and noise competes in ATLAS
- Constant term overcome all other contributions at high energy
- CMS has always a better energy resolution than ATLAS (be careful, these are test beam results... in real life, with tracker, B-field, pile-up,... everything gets more complicated !)

## Resolution (again and again)

The ATLAS LAr calorimeter has Pb absorber plates of 1.53mm.

- What will be the expected contribution to the stochastic term ?
- Comparison with test beam ?

$$E_c(\text{Pb}) = 7,43 \text{ MeV}$$
$$X_0(\text{Pb}) = 5,6 \text{ mm}$$

a) Use the formula on slide 5 (bottom)

$$\sigma E/E = \sqrt{(7,43 * \text{tabs} / 0.2) / \sqrt{E}}$$

Where tabs is the absorber thickness in units of  $X_0$ .

If 1  $X_0$  (Pb) = 5,6mm, then tabs = 1.53/5.6  $X_0$  = 0,27  $X_0$

Then,  $\sigma E/E = 10,2\% / \sqrt{E}$

b) In slide 9, we see that the test beam is giving  $\sim 10\% / \sqrt{E}$  for the stochastic term of the ATLAS Liquid ARgon ECAL, well in agreement with what we found.

# Exercise: Crystal Calorimeter



	Atomic Mass	$X_0$ (g.cm <sup>-2</sup> )	$R_M$ (g.cm <sup>-2</sup> )
Cs	132.9	8.31	15.53
I	126.9	8.48	15.75

- 1) Compute the radiation length of a CsI crystal (g.cm<sup>-2</sup>)
- 2) Given its density (4.51 g.cm<sup>-3</sup>), give  $X_0$  in cm
- 3) Given the critical Energy  $E_C=11.17$  MeV, deduce the Moliere Radius (g.cm<sup>-2</sup> and cm)
- 4) Compute the Moliere Radius with the formula for composite material. Compare to 3).

## Exercise: Crystal Calorimeter [SOLUTION]

	Atomic Mass	X <sub>0</sub> (g.cm <sup>-2</sup> )	R <sub>M</sub> (g.cm <sup>-2</sup> )
Cs	132.9	8.31	15.53
I	126.9	8.48	15.75

1) Use the formula from slide 3 for composite material.

$$w(\text{Cs}) = 132.9 / (132.9 + 126.9) = 0.511$$

$$w(\text{I}) = 126.9 / (132.9 + 126.9) = 0.489$$

$$X_0 = 1 / [w(\text{Cs})/8.31 + w(\text{I})/8.48] = 8.39 \text{ g.cm}^{-2}$$

$$\frac{1}{X_0} = \sum \frac{w_j}{X_j}$$

2)  $X_0(\text{cm}) = X_0(\text{g.cm}^{-2}) / \text{density} = 1.86 \text{ cm}$

3) Use the formula from the lectures :

$$R_M = 21 / 11.17 * 8.39 = 15.77 \text{ g.cm}^{-2}$$

$$R_M = \frac{21 \text{ MeV}}{E_C} X_0$$

4) Use the formula from slide 3

$$R_M = 1 / [w(\text{Cs})/15.53 + w(\text{I})/15.75] = 15.64$$

$$\frac{1}{R_M} = \sum \frac{w_j}{R_{Mj}}$$

## Exercise: EM showers in various materials

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Take e- with  $E=100$  GeV and  $E=1$  TeV going through Cu ( $Z=29$ ) and W( $Z=74$ )

- 1) Compute the critical energy  $E_c$  for each material.
- 2) For each material and energy, where does the shower max occurs (in unit of  $X_0$ )
  - Use the formula:  $t_{\max} = \ln(E/E_c) - t_1$ ,  $t_1=1$  for e-, 0.5 for  $\gamma$
- 3) Compute the 95% longitudinal containment (in unit of  $X_0$ ) in each case
- 4) Compute the Moliere Radius of each material. ( $X_0=1.436$  cm for Cu, 0.35cm for W)
- 5) Which material would you choose to build an EM calorimeter. Why ?

# Exercise: EM showers in various materials [SOLUTION]

1) Use the formula from slide 4

$$E_c(\text{Cu}) = 610 / (29+1.24) = 20.17 \text{ MeV}$$

$$E_c(\text{W}) = 610 / (74+1.24) = 8.1 \text{ MeV}$$

$$E_c(\text{solid}) = \frac{610 \text{ MeV}}{Z+1.24}$$

2)

$$t_{\max}(\text{Cu}, 100 \text{ GeV}) = \ln(100 \cdot 10^9 / 20.17 \times 10^6) - 1 = 7.5$$

$$t_{\max}(\text{Cu}, 1000 \text{ GeV}) = \ln(100 \cdot 10^{12} / 20.17 \times 10^6) - 1 = 14.4$$

$$t_{\max}(\text{W}, 100 \text{ GeV}) = \ln(100 \cdot 10^9 / 8.1 \times 10^6) - 1 = 8.4$$

$$t_{\max}(\text{W}, 1000 \text{ GeV}) = \ln(100 \cdot 10^{12} / 8.1 \times 10^6) - 1 = 15.3$$

3) Use the formula from slide 4

$$t_{95\%}(\text{Cu}, 100 \text{ GeV}) = 7.5 + 0.08 \times 29 + 9.6 = 19.4$$

$$t_{95\%}(\text{Cu}, 1000 \text{ GeV}) = 14.4 + 0.08 \times 29 + 9.6 = 26.3$$

$$t_{95\%}(\text{W}, 100 \text{ GeV}) = 8.4 + 0.08 \times 74 + 9.6 = 23.9$$

$$t_{95\%}(\text{W}, 1000 \text{ GeV}) = 15.3 + 0.08 \times 74 + 9.6 = 30.8$$

Longitudinal containment:

$$t_{95\%} = t_{\max} + 0.08Z + 9.6$$

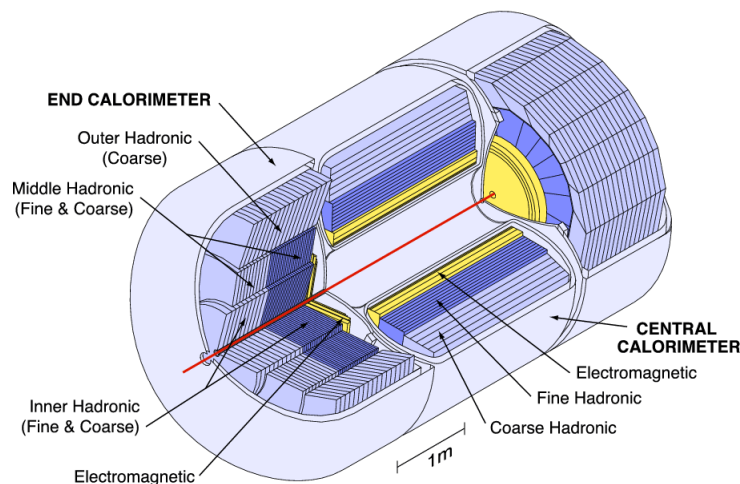
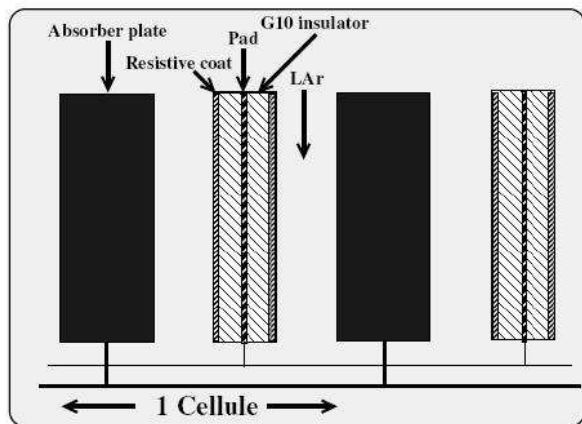
$$4) \text{RM}(\text{Cu}) = (21 / 20.17) \times 1.436 = 1.46 \text{ cm}$$

$$\text{RM}(\text{Cu}) = (21/8.1) \times 0.35 = 0.9 \text{ cm}$$

5) W has a smaller radiation length. Although it seems more X0 are needed to stop particles, an EM calorimeter using W will be more compact than one using Cu. Moreover, W has a much smaller Moliere Radius. A calorimeter using W as absorber will have the capability to better separate showers.



# Exercise: DØ Calorimeter



	Z	X0 (g.cm <sup>-2</sup> )	ρ (g.cm <sup>-3</sup> )
U	92	6	19
LAr	18	19.6	1.4

		ηxφ
EM1	2 X0	0.1 x 0.1
EM2	2 X0	0.1 x 0.1
EM3	6.8 X0	0.05 x 0.05
EM4	9.8 X0	0.1 x 0.1

One cell of the U/LAr central EM calorimeter of DØ is made of a sandwich of 3mm U plate and 2x2.3mm LAr gap.

- 1) Compute the X0 for the sandwich (in g.cm<sup>-2</sup>)
- 2) Compute the average density of the sandwich
- 3) Give X0 in cm
- 4) Compute the position of the shower max (in units of X0) for an electron with E=45 GeV, given E<sub>c</sub>=6.65 MeV.
- 5) The EM part has four sections with different granularity and X0.

Comment wrt to the result on question 4.

- 6) During RunII, a magnet was added before the calorimeters as well as a pre-shower (Pb/scintillating fibers). What is the impact on the shower max ? What are the consequences on the calorimetric performance ? What is the role of the pre-shower ?

## Exercise: DØ Calorimeter [SOLUTION]

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1)

$$w(U) = 3 / (7.6) = 0.39$$

$$w(LAr) = 4.6 / 7.6 = 0.61$$

$$X_0 = 1/[0.39/6 + 0.61 / 19] \sim 10.5 \text{ g.cm}^{-2}$$

$$2) \langle \text{density} \rangle = 0.39 \cdot 19 + 0.61 \cdot 1.4 = 8.25 \text{ g.cm}^{-3}$$

$$3) X_0 \text{ (cm)} = X_0(\text{g.cm}^{-2}) / \langle \text{density} \rangle \sim 1.27 \text{ cm}$$

$$4) t_{\text{max}} = \ln(45 \cdot 10^9 / 6.65 \cdot 10^6) - 1 \sim 7.8$$

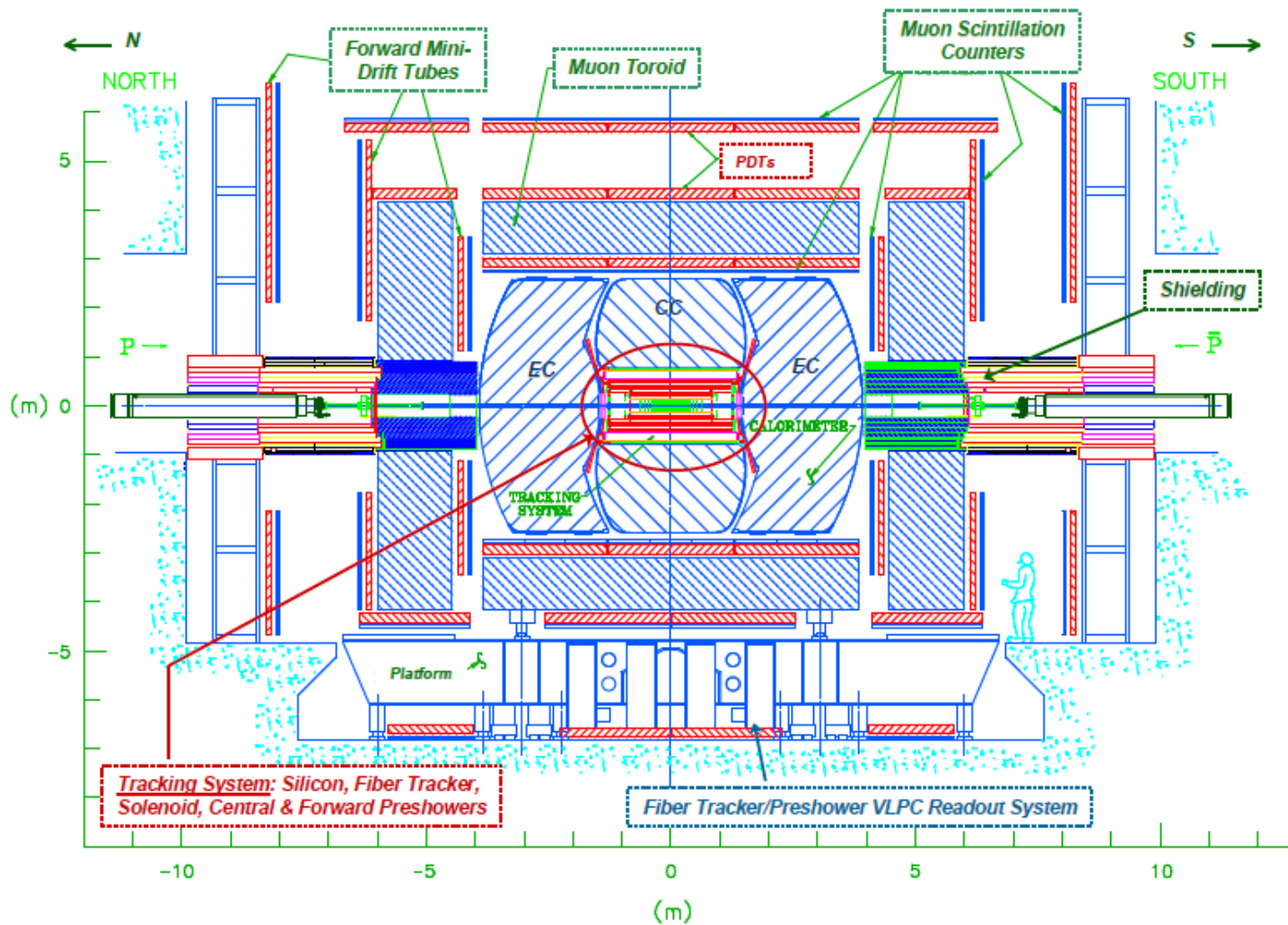
(ie, the shower max will occur after 7.8  $X_0$ )

5) The shower max occurs in EM3, where the granularity is the finer. This was designed on purpose to sample the shower max more efficiently and achieve the best resolution.

6) During RunII, the shower max was displaced to EM2 due to the new material in front that made the shower begin before reaching the calorimeters. This induced in particular a loss of resolution due to this dead material in front..

Pre-shower detectors were added between the magnet coil and the ECAL. Their role is in particular to allow the derivation of dead-material corrections as well as providing e/hadrons separation.

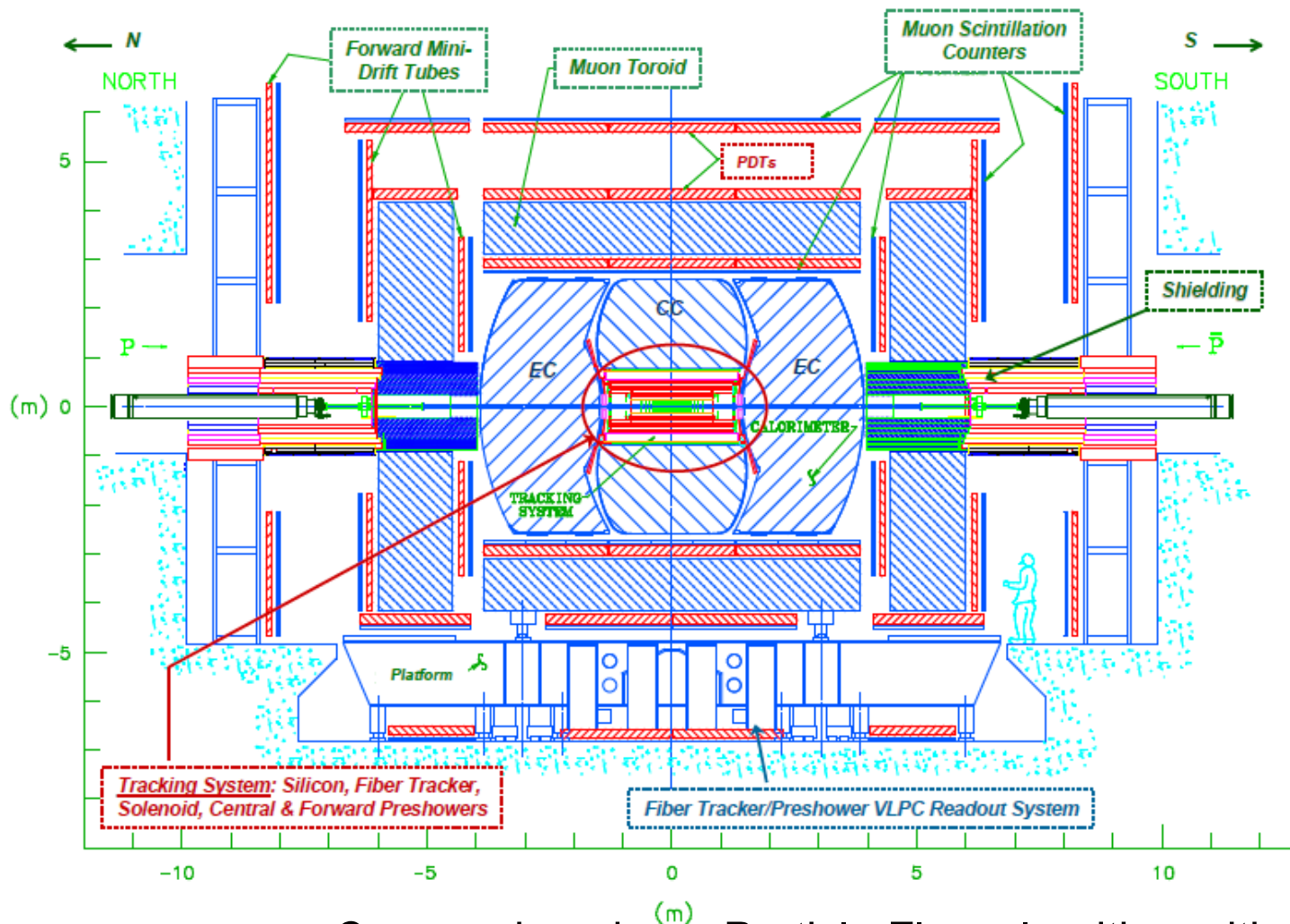
# Particle Flow & DØ



B-field = 2 T (solenoid)  
ECAL radius: 0.8 m

Can you imagine a Particle Flow algorithm with this detector? Why?

# Particle Flow & DØ [SOLUTION]



B-field = 2 T (solenoid)  
ECAL radius: 0.8 m

Can you imagine a Particle Flow algorithm with this detector? Why?

The B-field integral is small ( $2 \times 0.8 = 1.6$  T.m (to be compared to  $\sim 5$  for CMS !))

There is material in front the calorimeter (2 X0 at normal incidence)

Granularity of both tracking and calorimeters are not sufficient, given the particle rates at ppbar colliders.

“Energy flow” technics were tried but were not successful enough to become standard.

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# **BACK UP SLIDES**