Deconvolution of Photon Strength

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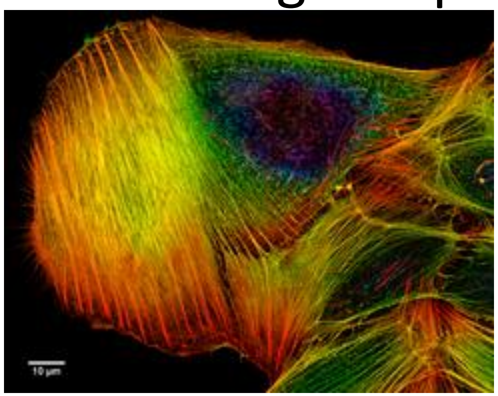
Deconvolution - a process of resolving something into its constituent elements or removing complication in order to clarify it.



Example of a deconvolved microscope image.

In mathematics, **deconvolution** is an algorithm-based process used to reverse the effects of convolution on recorded data

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Introduction

The photon strength function $f_{XL}\left(E_{\gamma)}\right)$ for multipolarity *XL* is commonly defined as the average reduced partial radiation width $\bar{\Gamma}_{XL}/E_{\gamma}^{2L+1}$ per unit energy interval*

$$f_{XL}\left(E_{\gamma}\right) = \bar{\Gamma}_{XL}/(D \cdot E_{\gamma}^{2L+1}) = \rho(E_x, J^{\pi}) \cdot \bar{\Gamma}_{XL}/E_{\gamma}^{2L+1}$$

for resonances of average spacing *D*, inverse of the level density ρ, at an excitation energy E_x. E_γ is the γ-ray energy for a transition from the GS to an excitation E_x in the nucleus.

Photon strength is a misnomer because it is defined as the product of a reduced transition width and a level density.

In this discussion I will deconstruct the photon strength into its two components.

*M. Uhl and J. Kopecky, *Gamma-ray strength models and their paramerization*, ECN-RX-94-099 (1994).

Reduced Transition Probabilities

For γ-ray transitions the reduced matrix elements, $B(\sigma L)$, are defined as

$$B(EL)\downarrow = \frac{\Gamma_\gamma(EL)\cdot L[(2L+1)!!]^2}{8\pi(L+1)e^2 b^L}\left(\frac{\hbar c}{E_\gamma}\right)^{2L+1} = C(EL)\cdot f_\gamma(EL)$$

$$B(ML)\downarrow = \frac{\Gamma_\gamma(ML)\cdot L[(2L+1)!!]^2}{8\pi(L+1){\mu_N}^2 b^{L-1}}\left(\frac{\hbar c}{E_\gamma}\right)^{2L+1} = C(ML)\cdot f_\gamma(ML)$$

where

$$B(E1)\downarrow = \Gamma_\gamma(E1)\frac{9.56\times 10^3}{E_\gamma^3}\ \mathrm{MeV}^{-2}$$

$$B(E2)\downarrow = \Gamma_\gamma(E2)\frac{1.24\times 10^8}{E_\gamma^5}\ \mathrm{MeV}^{-4}$$

$$B(M1)\downarrow = \Gamma_\gamma(M1)\frac{8.64\times 10^7}{E_\gamma^3}\ \mathrm{MeV}^{-2}$$

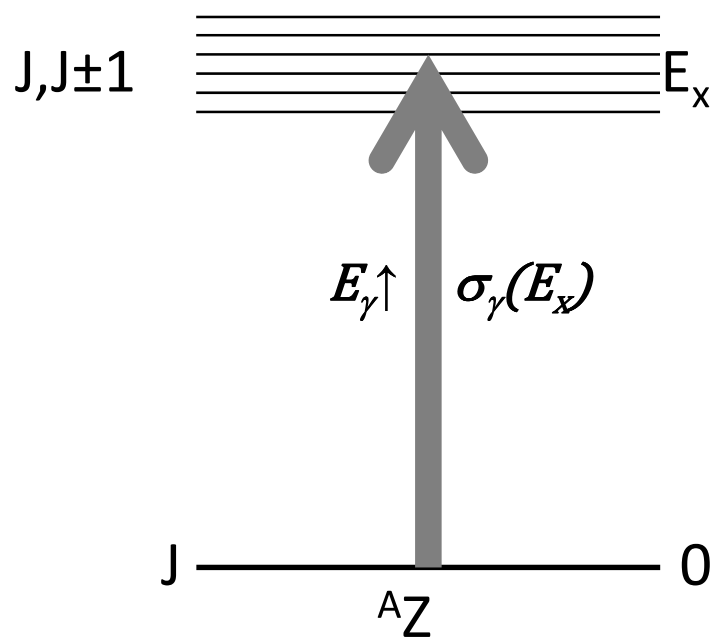
and

$$B(\sigma L)\uparrow = \frac{(2J_f+1)}{(2J_i+1)}B(\sigma L)\downarrow$$

These are the traditional definitions of single γ-ray transition strength

Photonuclear data

Photon strength $f_{E1}(E_\gamma)$ was originally defined to describe (γ,n) photonuclear cross section data which is dominated by *E1* multipolarity.



Photonuclear experiments measure the average γ-ray cross section $\sigma_\gamma(E_x)$ which is related to the average photon strength by detailed balance.

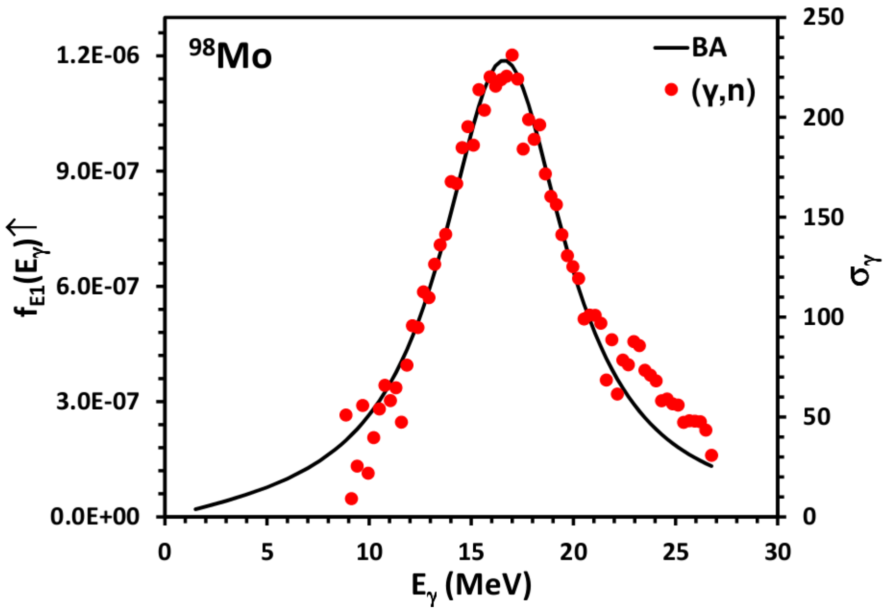
$$\overline{f_{E1}}(E_\gamma)\uparrow = \frac{\sigma_\gamma(E_x)}{3\pi^2\hbar^2c^2E_\gamma}\ MeV^{-3} = \overline{f_\gamma}(E1)\times\rho(E_x)$$

Brink-Axel Photon Strength

A major discovery by David Brink and Peter Axel showed that photonuclear photon strength can be completely described by the Lorentzian shape of the Giant Dipole Resonance (GDR).

$$\overline{f_{BA}^{E1}}(E_\gamma)\uparrow = \frac{1}{3(\pi\hbar c)^2}\sum_{i=1}^{i=2}\frac{\sigma_{G_i} E_\gamma \Gamma_{G_I}^2}{\left(E_\gamma^2 - E_{G_I}^2\right)^2 + E_\gamma^2 \Gamma_{G_i}^2}$$

Where E_G, Γ_G, and σ_G are the energy, width, and cross section of the GDR, respectively. For deformed nuclei the GDR is described by two resonance peaks.

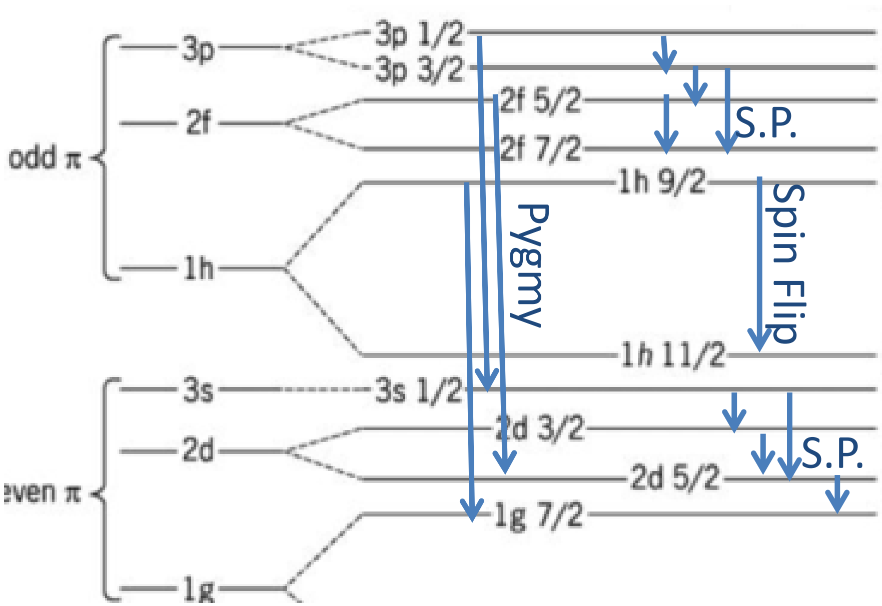


^{98}Mo GDR data are fit to the parameters E_G=17.0 MeV, Γ_G=7.4 MeV, and σ_G=230 mb. Data fall below the BA prediction at low energies where γ-ray emission competes with neutron decay and above BA predictions at high energy where multiple neutron emission occurs.

These are average E1 photon strengths whose values vary in a Porter-Thomas distribution.

Other simple γ-ray transitions

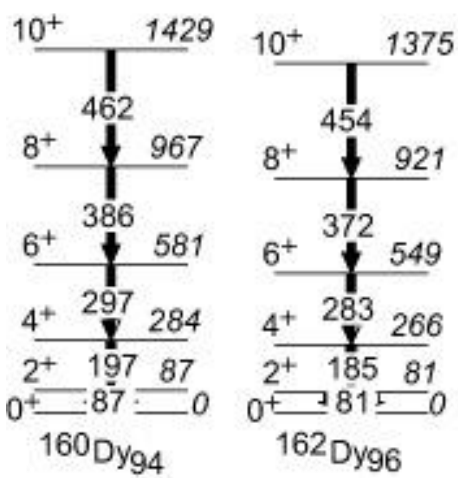
Near and below the neutron separation energy, S_n, simple γ-ray transitions can be observed. ***These transitions do not follow a Porter-Thomas distribution.***



Shell model transitions

Single particle M1 shell model transitions occur in shells neat the GS and S_n.

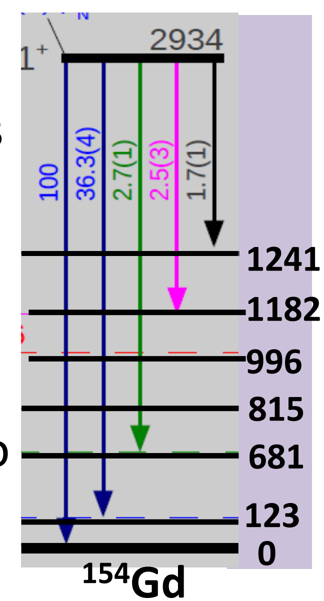
E1 (pygmy) and M1 (spin-flip) transitions occur between adjacent shells. Their strength is divided into many transitions between multiparticle configurations.



Collective transitions

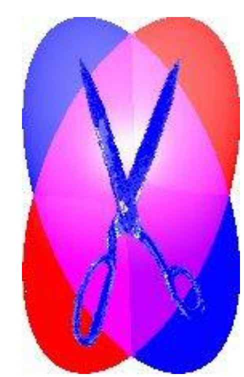
Rotational

Deformed nuclei have strong M1, E2 transitions confined to rotational bands.



Scissors

Counterrotation of a proton/neutron fluid.



Level Density Formulas

I will use the Gilbert-Cameron formulation to deconvolute the average γ-ray strength from photonuclear data for the $^{92\text{-}101}$Mo isotopes.

The constant temperature model CT) is applied for levels below 10 MeV.

$$\rho(E, J^{\pi})_{CT} = \frac{f(J^{\pi})}{T} \exp\left(\frac{E_{\gamma} - E_0}{T}\right)$$

The Back Shifted Fermi Gas model (BSFG) is used for levels above 10 MeV.

$$\rho(E, J^{\pi})_{BSFG} = f(J^{\pi}) \frac{\exp\left[2\sqrt{a(E_{\gamma} - E_1)}\right]}{12\sqrt{2}\sigma_c a^{0.25}(E_{\gamma} - E_1)^{1.25}}$$

The spin distribution function *f(J)* model is often defined as

$$f(J) = \frac{2J+1}{2\sigma_c{}^2} exp\left[-\frac{(J+1/2)^2}{2\sigma_c^2}\right]$$

The level density parameters *E0, T, E1*, and *a* are chosen to match level densities at S_n. The spin cutoff parameter is given as $\sigma_c = 0.98A^{0.29}$ for the CT model and

$$\sigma_c{}^2 = 0.0146A^{5/3} \frac{1 + \sqrt{1 + 4a(E_{\gamma} - E_1)}}{2a}$$

for the BSFG model.

The spin distribution function is only valid near S_n

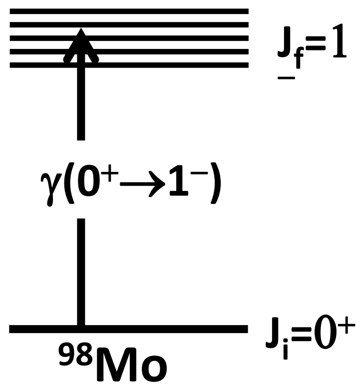
Spin distribution

Photonuclear reactions populate a small subset of spins and parities so we need to determine *$f(J^{\pi})$*. At low excitation energies the spin distribution function is invalid so a new model is needed.

1. **RIPL-3 HFB Calculations** - nuclear level density obtained within the HFB plus combinatorial model* at the ground state deformation or all J^{π}.
2. **CT-JPI Model (RBF)** – separate CT or BSFG model equation or each J^{π} value where $E_0=E_1=E_{yrast}(J^{\pi})$ and $T(J^{\pi})$ can vary for each spin and parity. $T(J^{\pi})$ is constrained so that the s-wave and p-wave level spacing at Sn are consistent with D0 and D1.

* S. Goriely, S. Hilaire, and A.J. Koning, *Improved microscopic nuclear level densities within the Hartree-Fock-Bogoliubov plus combinatorial method*, Phys. Rev. **C78** (2008) 064307.

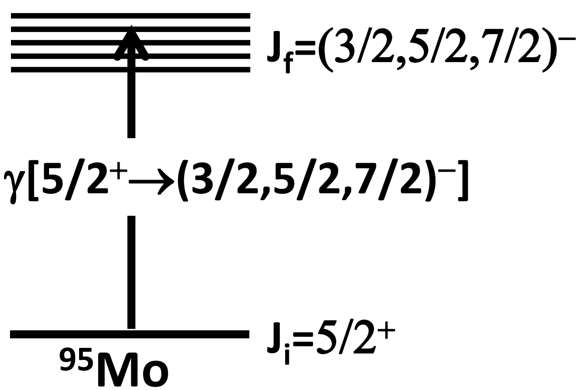
Deconvoluting the γ-ray strength



Even-Z, even-N photonuclear reactions only populate $J^\pi=1^-$ levels by E1 transitions.

$$f_\gamma(E_\gamma)\uparrow = f_{BA}^{E1}(E_\gamma)\Big/\rho(1^-)$$

$$f_\gamma(E_\gamma)\downarrow = \frac{2J_i+1}{2J_f+1} f_\gamma(E_\gamma)\uparrow = f_\gamma(E_\gamma)/3$$



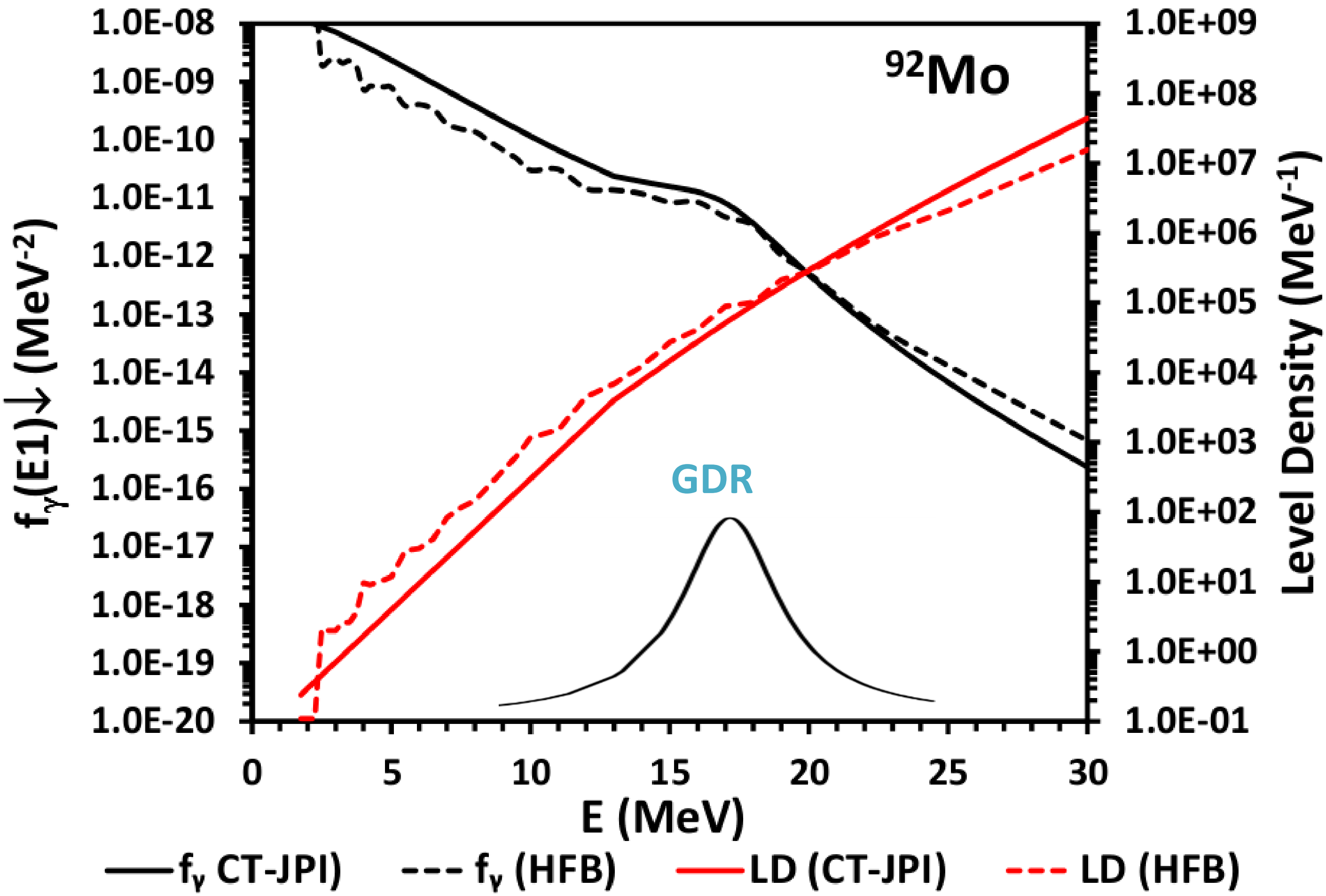
Even-Z, odd-N photonuclear reactions only populate $J_i, J_i\pm1$ levels by E1 transitions

$$f_\gamma(E_\gamma)\uparrow = \frac{f_{BA}^{E1}(E_\gamma)}{\sum_{k=3/2^-}^{k=7/2^-}\rho(J_{f_k})}$$

$$f_\gamma(E_\gamma)\downarrow = f_\gamma(E_\gamma)\uparrow \sum_{J_f=3/2^-}^{7/2^-}\frac{2J_i+1}{x_k(2J_f+1)}$$

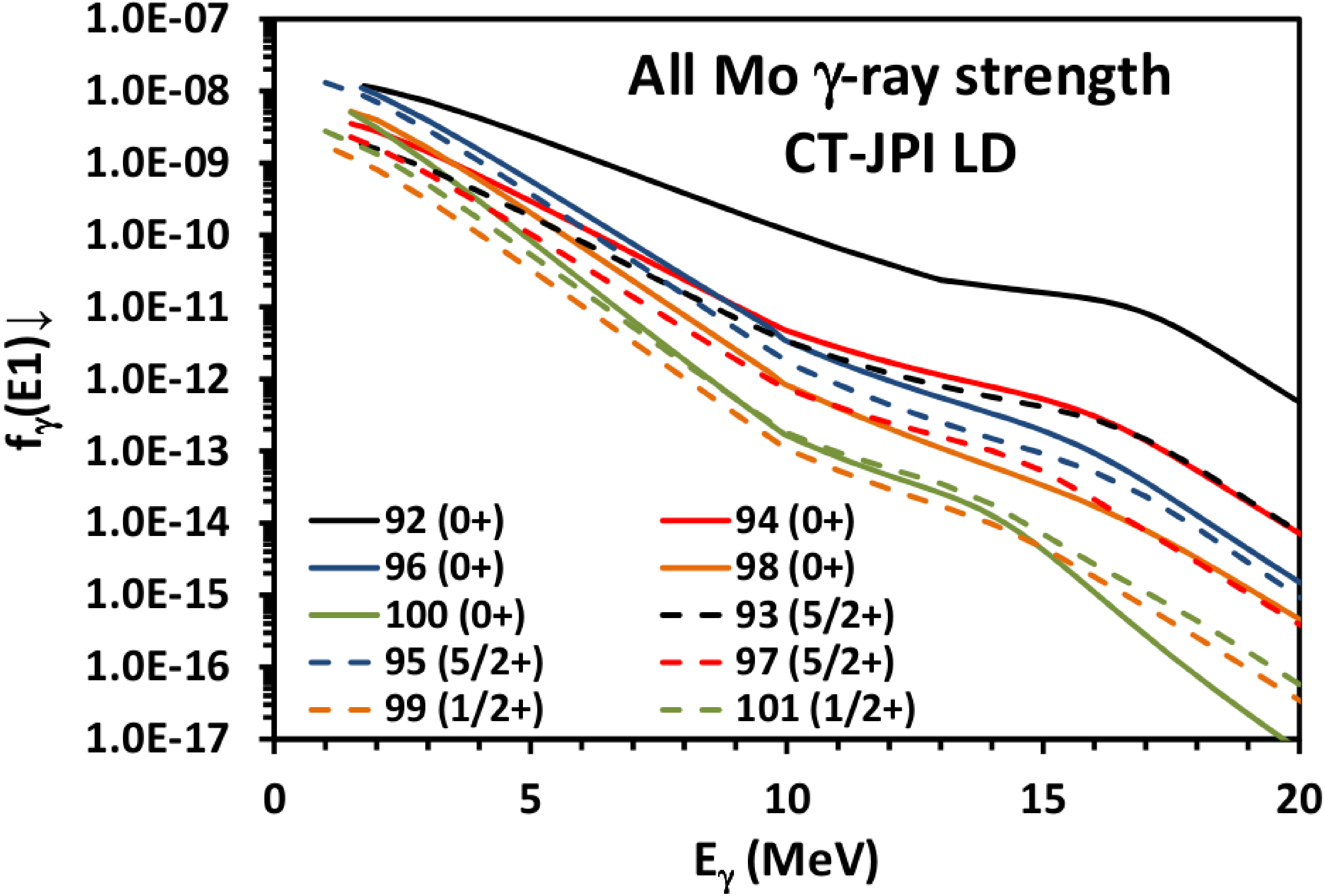
where $\boldsymbol{x_k}$ is fraction of final levels with spin $\boldsymbol{J_f}$.

Continuous γ-ray strength function

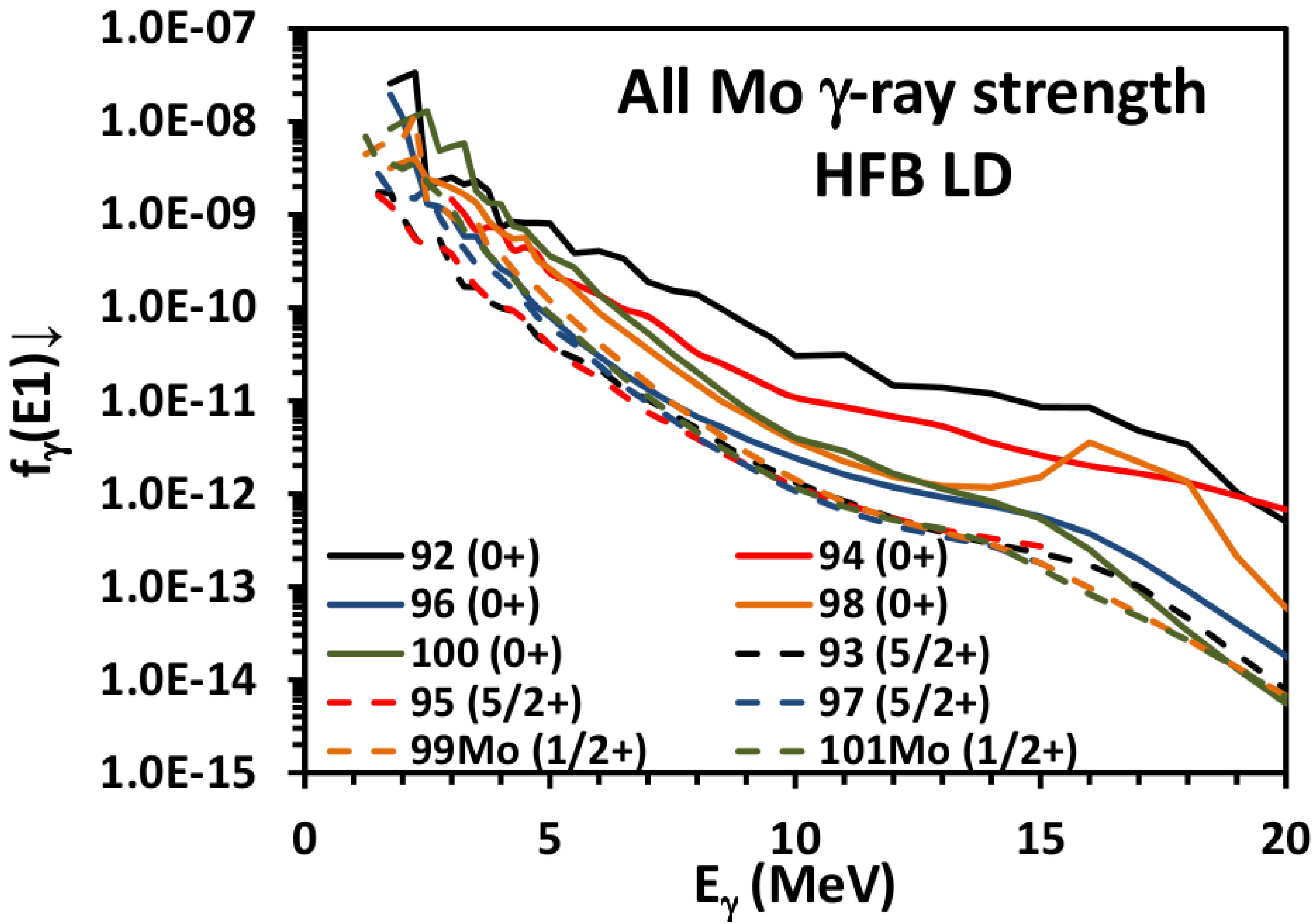


Association of GDR with γ-ray strength is an artifact of this analysis.

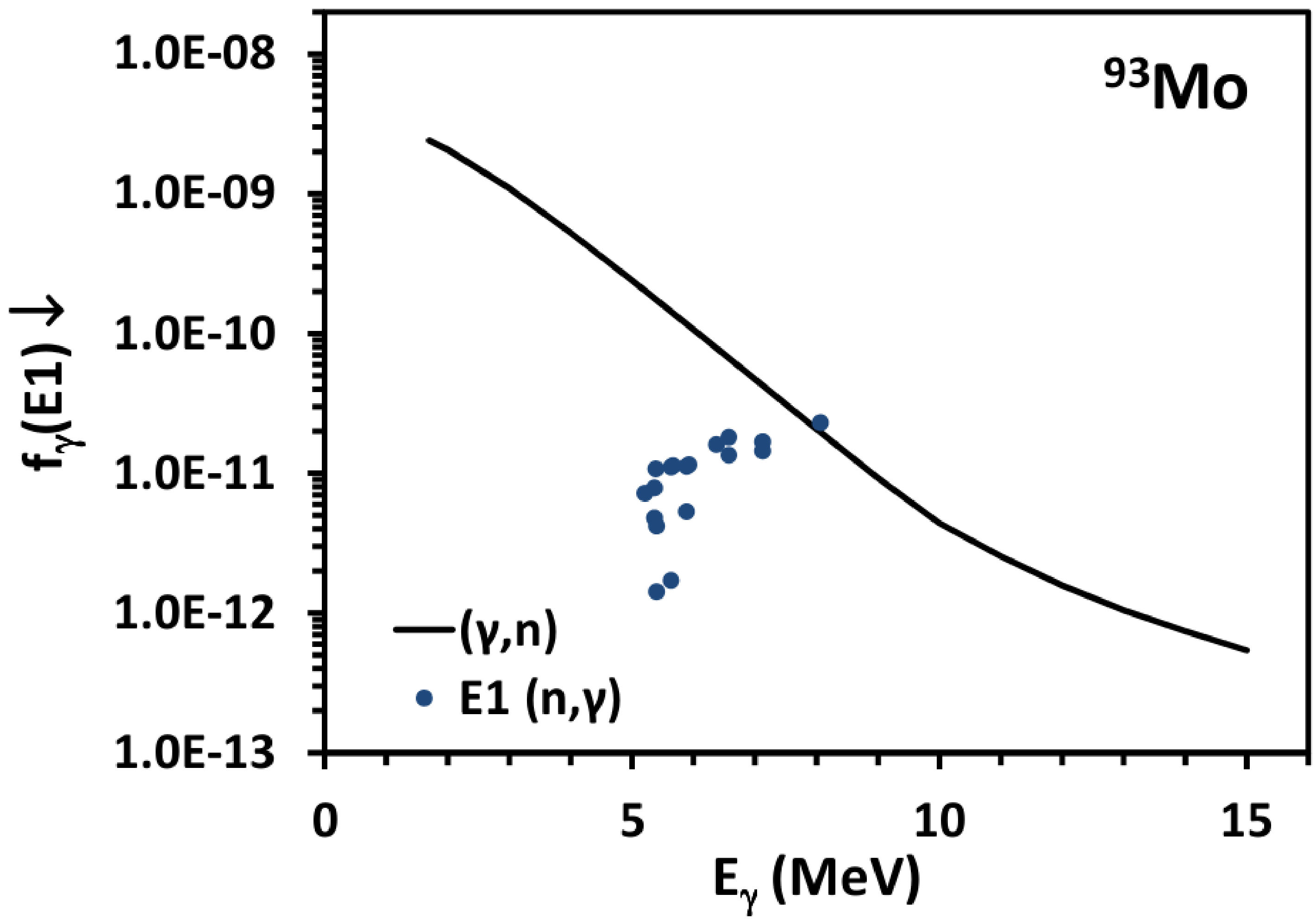
Mo γ-ray strength with CT-JPI level density



Mo γ-ray strength with HB level density

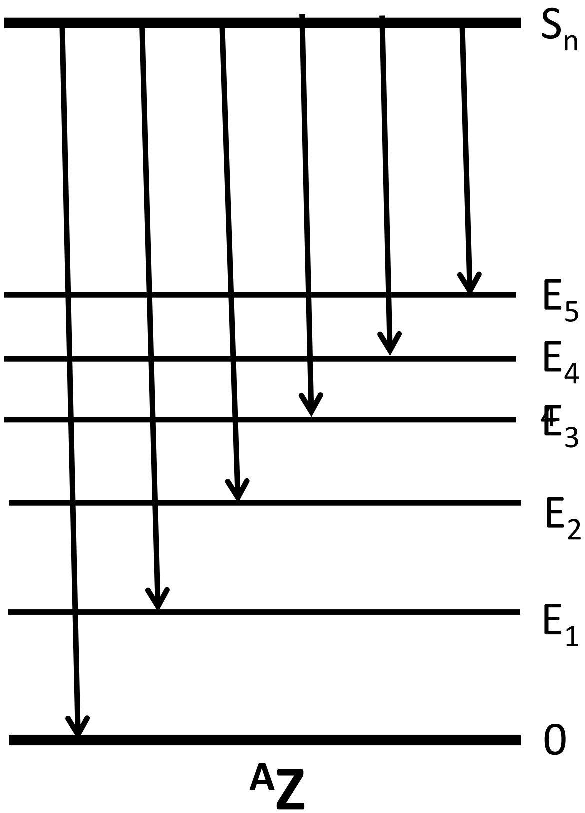


Comparison with (n,γ) primary γ-ray strength



Thermal (n,γ) data is clearly inconsistent photonuclear γ-ray strength.

(n,γ) primary γ-ray strengths



Primary (n,γ) γ-rays can be binned to give average γ-ray strengths.

All primary γ-rays deexcite the level density at S_n

$$f_\gamma(E1)_{n,\gamma}\downarrow = \frac{(2J_i+1)f_{BA}^{E1}(E_\gamma)\uparrow}{(2J_f+1)\rho(S_n)}$$

where

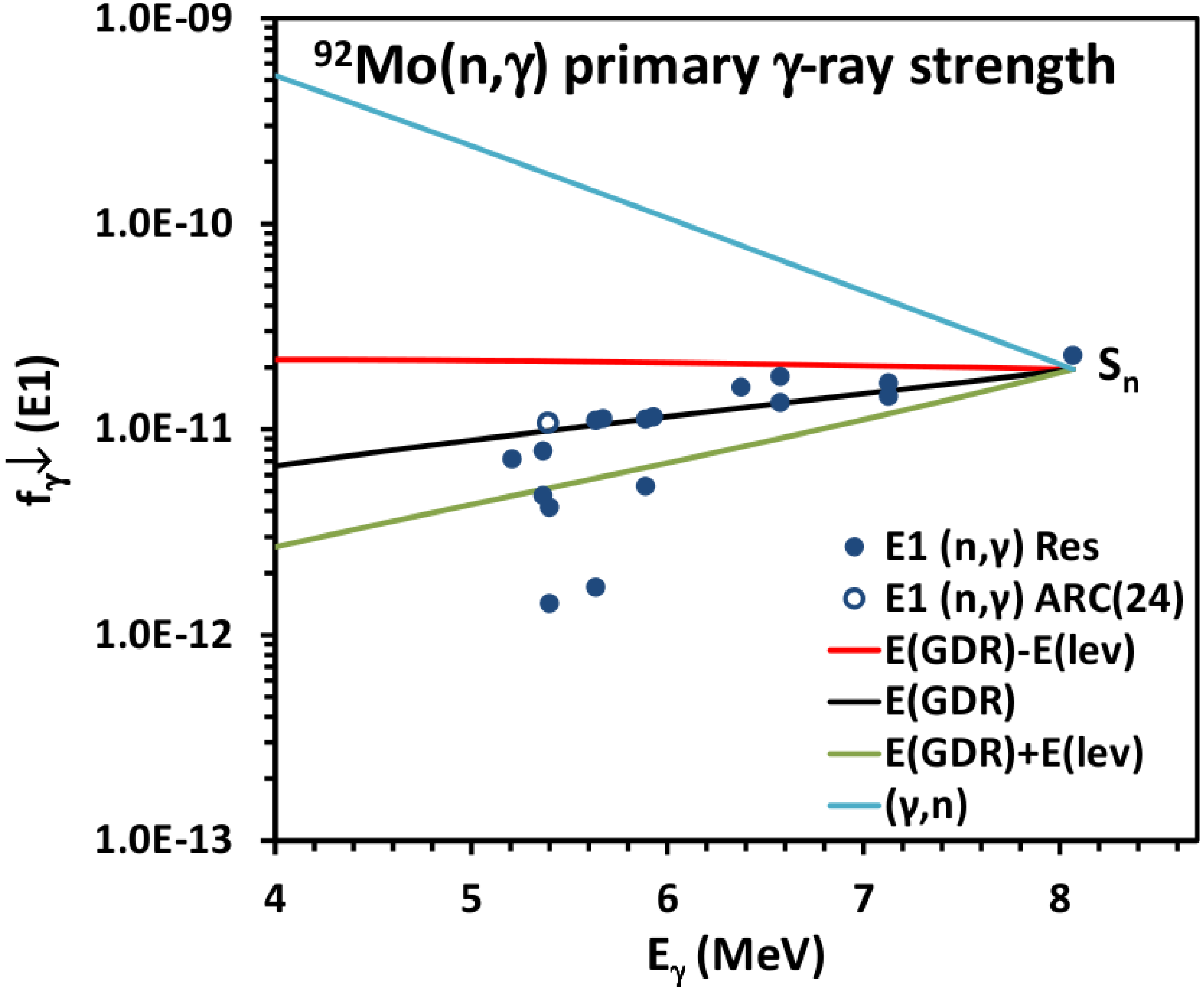
$$\overline{f_{BA}^{E1}}(E_\gamma)\downarrow = \frac{1}{3(\pi\hbar c)^2}\sum_{i=1}^{i=2}\frac{\sigma_{G_i}E_\gamma\Gamma_{G_I}^2}{\left(E_\gamma^2 - E_{G_I}^2\right)^2 + E_\gamma^2\Gamma_{G_i}^2}$$

What value should we use for the energy of the GDR?

1. GDR is built on each level: E_1=E(GDR)+E_i
2. GDR is fixed relative to GS: E_1=E(GDR)
3. GDR is fixed relative to level energy:

   E_1=E(GDR)-E_i

(n,γ) primary γ-ray strengths



Consistent with E(GDR) energy fixed with respect to GS. Anomalous low values are unexplained.

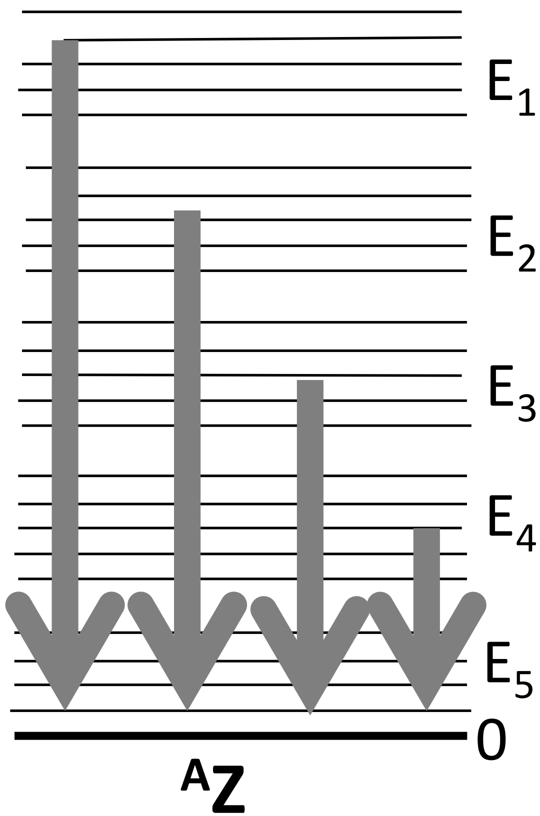
Comparison with Γ_0 and Γ_1 at S_n

For CT-JPI LD model the level densities are fit to exactly reproduce D0 and D1. This is not true for the HFB LD model.

	Sn	D0	D1	Γ_0 (eV)			Γ_1 (eV)	
	(MeV)	(eV)		Atlas	Calc	$M1_{exp}$*	Atlas	Calc
^{93}Mo	8.08981	2800	780	0.155	0.031	0.069	0.240	0.038
^{95}Mo	7.3691	1690	508	0.128	0.076	0.050	0.188	0.039
^{96}Mo	9.15432	81	37.7	0.162	0.019	0.056	0.210	0.175
^{97}Mo	6.82125	1170	324	0.096	0.057	0.036	0.125	0.026
^{98}Mo	8.6426	46.5	21.9	0.130	0.054	0.050	-	0.317
^{99}Mo	5.92544	970	286	0.072	0.029	0.033	0.107	0.011
^{101}Mo	5.39824	617	236	0.064	0.016	0.013	0.093	0.007

* Observed (n,γ) M1 width.

Deconvolution of Reaction (Oslo) Data



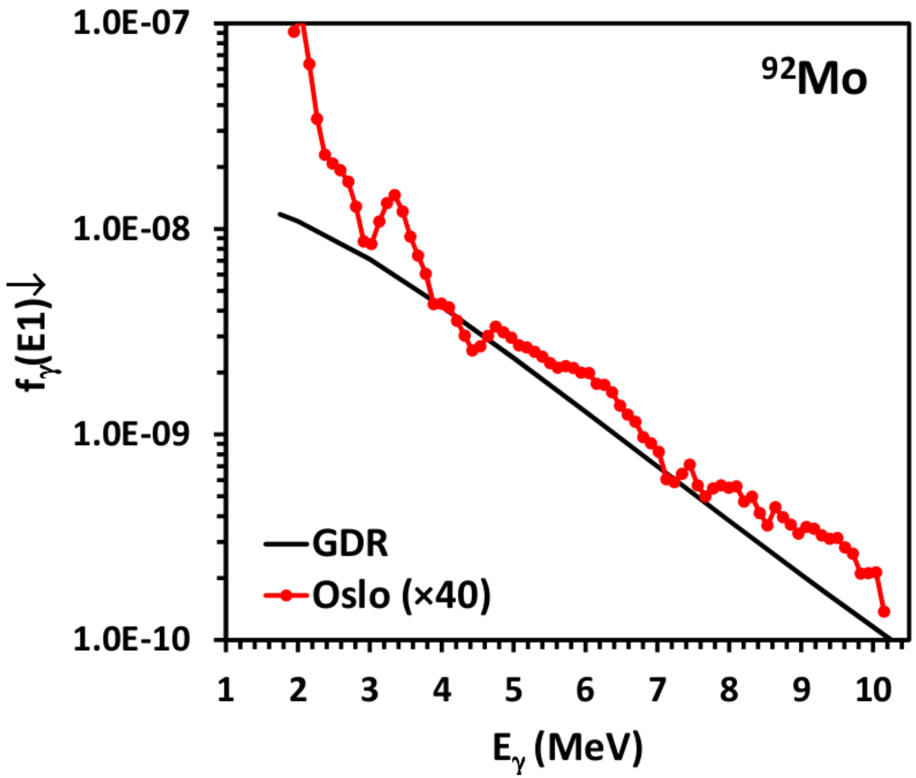
Oslo experiments measure average total primary GS γ-ray photon strengths $\overline{f(E_\gamma)}$ and level densities $\rho(E_i)$.

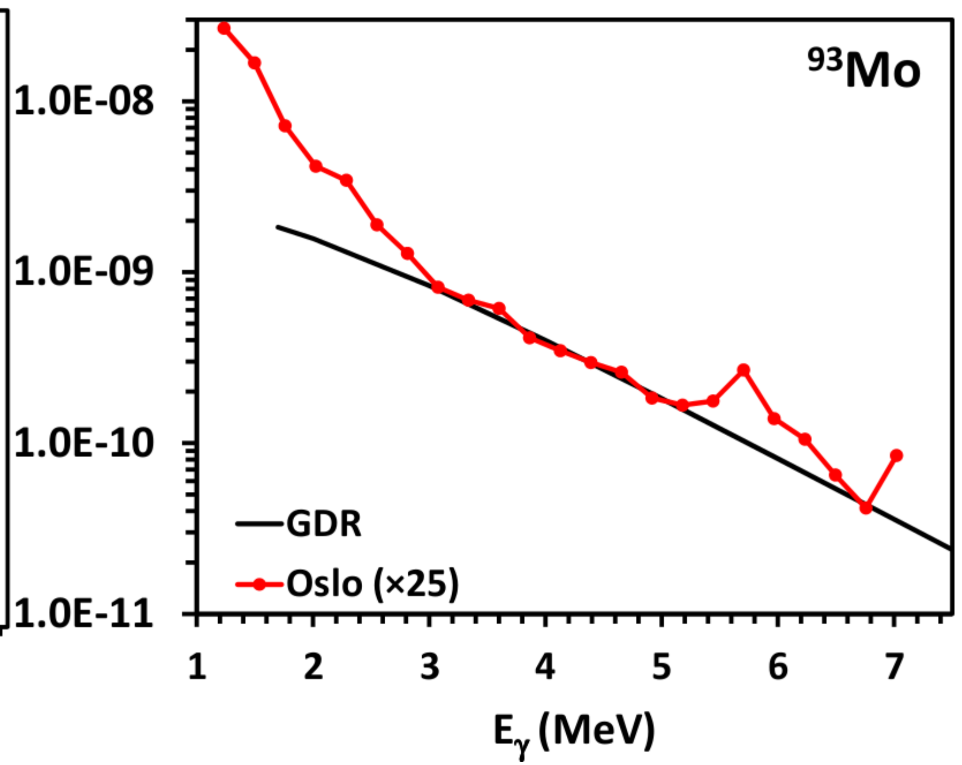
These photon strengths are often normalized to photonuclear data however this introduces problems if the two data sets correspond to different level density regimes.

Oslo data is complimentary to photonuclear data because it measures both the sum of GDR E1 γ-ray strength and other transitions described earlier.

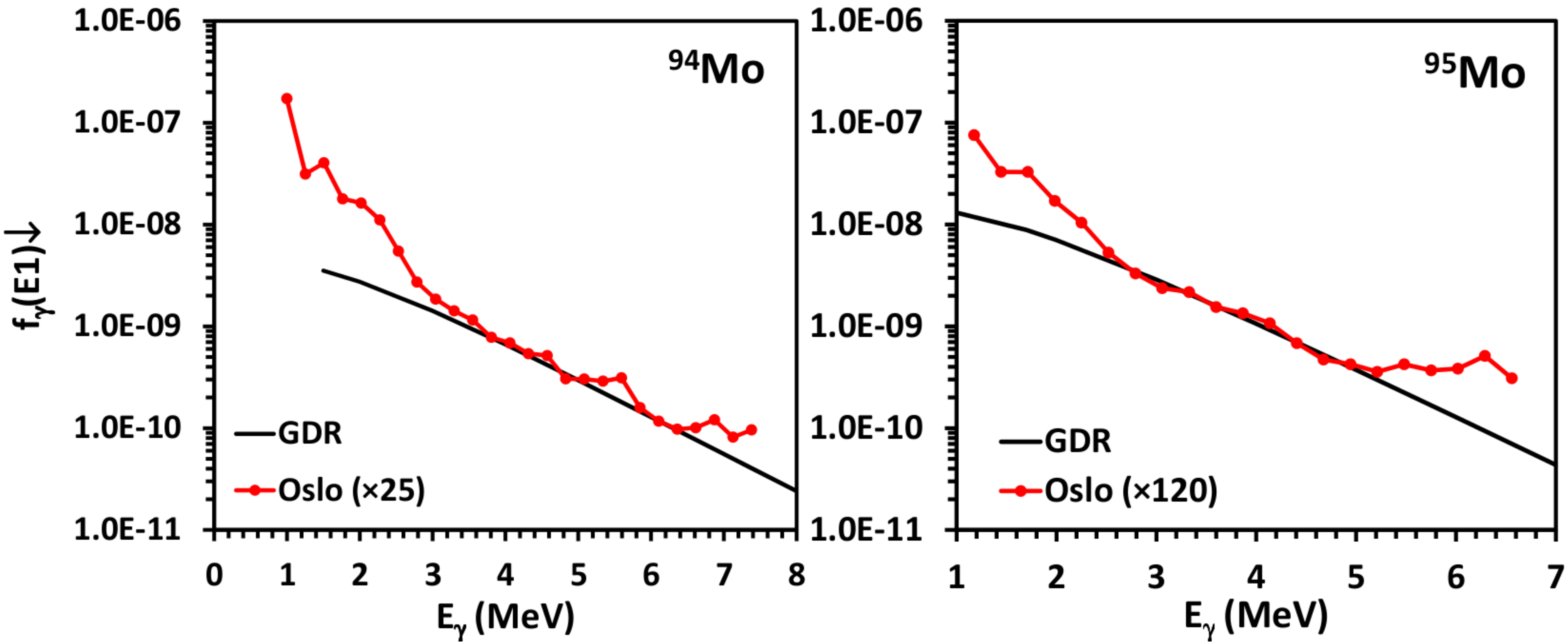
Deconvolution of the Oslo data simply requires dividing the photon strengths initial Oslo level densities. $f\left(E_\gamma\right) = {\overline{f(E_\gamma)}}/{\rho(E_i)}$.

Comparison of Oslo data with GDR E1

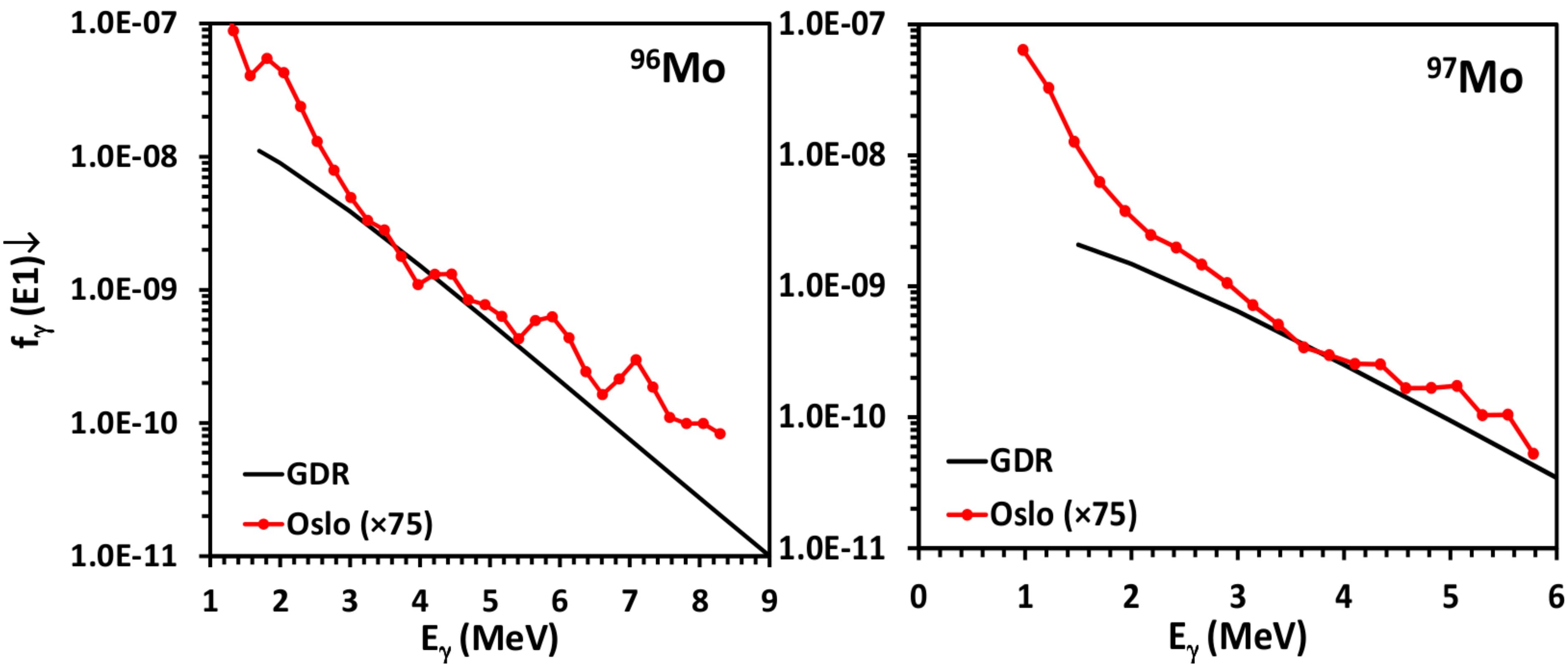




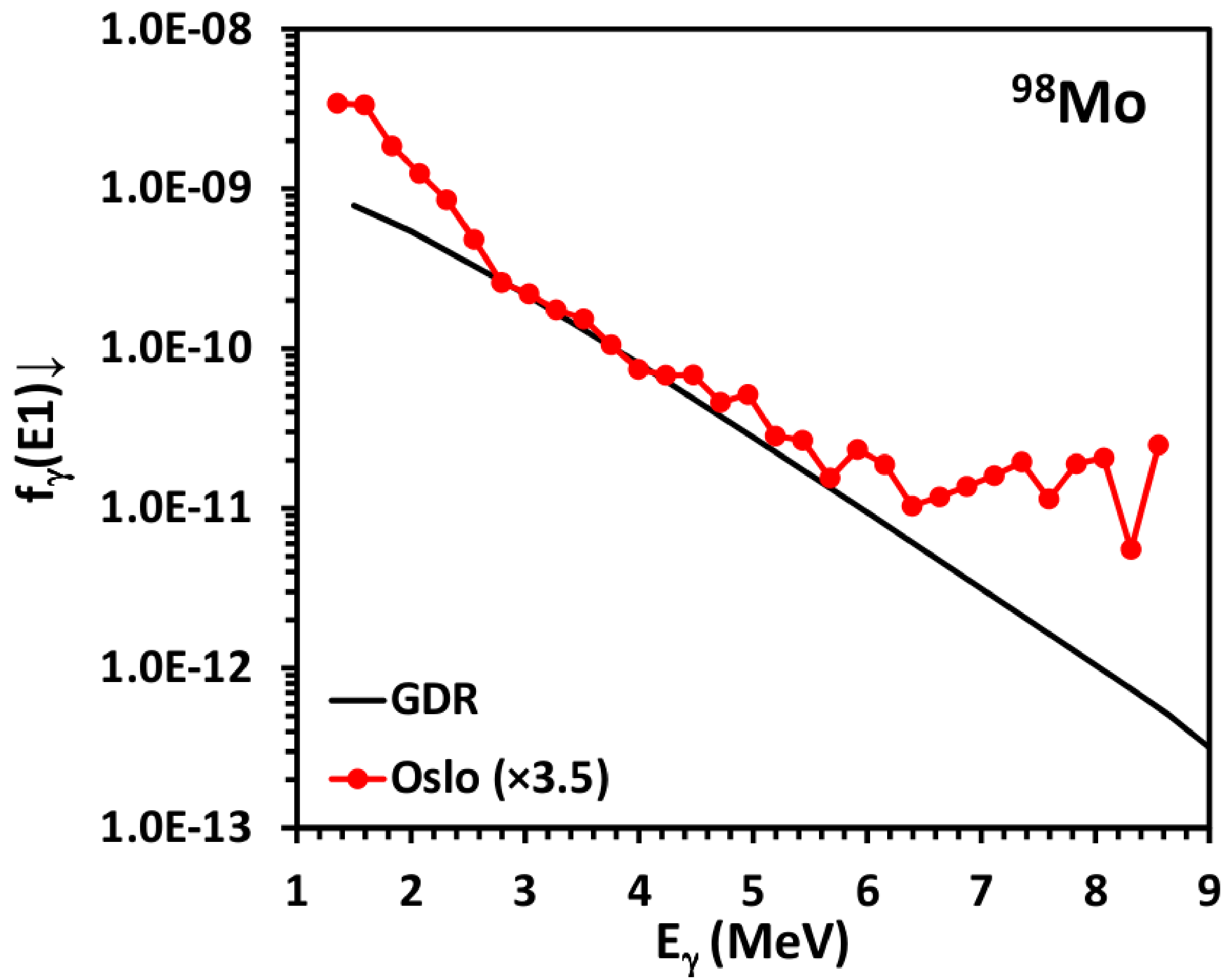
Comparison of Oslo data with GDR E1



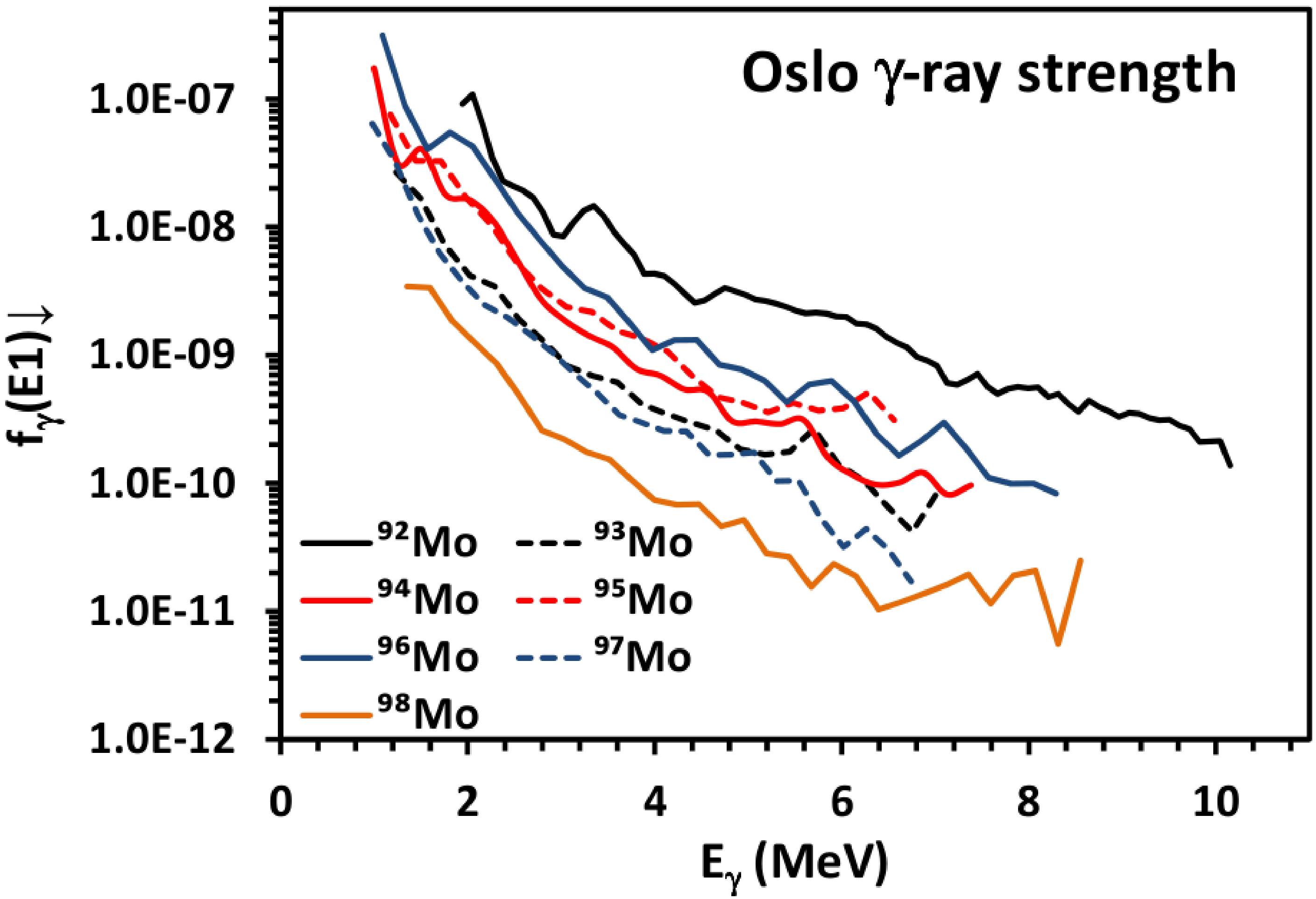
Comparison of Oslo data with GDR E1



Comparison of Oslo data with GDR E1



Oslo γ-ray strengths



Conclusions

- The GDR can be deconvoluted into the product of a continuous γ-ray strength function and a continuous level density function.
- The average, GDR based E1 γ-ray strength depopulating levels at an excitation energy E_i to a level at energy E_f is given by

$$f_\gamma(E1)\downarrow = \frac{2J_f+1}{2J_i+1} f_{BA}(E1)\uparrow/\rho(E_x,J^\pi)$$

- The energy of the GDR is fixed with respect to the GS or all γ-rays.
- Individual GDR based E1 transitions will have a Porter-Thomas distribution of γ-ray strength at each energy.
- Other shell model and collective E1, M1, and E2 transitions do not follow a Porter-Thomas distribution of γ-ray strengths.
- Oslo data can be deconvoluted by dividing the photon strength $f_{Oslo}(E_\gamma)\downarrow$ by the level density $\rho_{Oslo}(E_x)$ and renormalizing to the GDR E1 γ-ray strength $f_\gamma(E1)\downarrow$.

Thank you for your attention

HFB LD Model fails to reproduce D0, D1

	$\rho0=1/D0$		$\rho1=1/D1$	
	Atlas	HFB	Atlas	HFB
^{93}Mo	357	551	1282	1601
^{95}Mo	590	1630	1969	4860
^{96}Mo	12346	66554	26525	127400
^{97}Mo	855	882	3086	2445
^{98}Mo	21505	39216	45662	72085
^{99}Mo	1031	490	3497	1351
^{101}Mo	1521	637	4237	1710