Color Glass Condensate at NLO: Phenomenology at HERA, RHIC and the LHC

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Antwerp, Belgium 21-25 2010

XL ISMD
Outline

⇒ Motivation & Pocket Introduction to the CGC
⇒ Balitsky-Kovchegov equation including running coupling corrections
⇒ Structure functions in e+p collisions at HERA
⇒ Single inclusive hadron production in the CGC
⇒ Di-hadron correlations in d+Au collisions
⇒ Multiplicities in A+A collisions
At high energies, or small Bjorken-x, hadron’s gluon densities are large

\[ k_1^+ = x_1 p^+ \]
\[ k_2^+ = x_2 p^+ \]
\[ k_n^+ = x_n p^+ \]
\[ P \sim (\alpha_s \ln 1/x)^n \]

Probability of n-soft gluon emission

Multiple small-x gluon emissions are resummed by the BFKL equation

\[ \frac{\partial \phi(x, k_t)}{\partial \ln(x_0/x)} \approx \mathcal{K} \otimes \phi(x, k_t) \]

proton ugd from HERA

\[ xg /20 \]
\[ xu \]
\[ xS \]
\[ xd \]
Non-linear evolution: At small-$x$ gluon both radiative and recombination processes are demanded by UNITARITY.

Non-linear recombination corrections

\[ \frac{\partial \phi(x, k_t)}{\partial \ln(x_0/x)} \approx K \otimes \phi(x, k_t) - \phi(x, k_t)^2 \]

CGC: JIMWLK–BK

Low density

BFKL

High density

\[ Y = \ln \left( \frac{1}{x} \right) \]

\[ Q_s(x) \]

\[ \ln \Lambda_{QCD} \]

\[ \ln Q \]

Saturation scale: transverse momentum scale which marks the onset of non-linear corrections

\[ K \otimes \phi(x, Q_s) \approx \phi(x, Q_s)^2 \]

Nuclear enhancement:

\[ Q_{sA}^2 \approx A^{1/3} Q_{sp}^2 \]
DIS in the dipole model

Probing your hadron with a photon (color dipole). Eikonal scattering

\[ S(x, y; Y) = \frac{1}{N_c} \langle \text{tr} \{ U_x U_y^\dagger \} \rangle_Y = 1 - \mathcal{N}(x, y; Y) \]

unintegrated gluon distribution:

\[ \varphi(x, k_t) = \int \frac{d^2 r}{2\pi r^2} e^{i k \cdot r} \mathcal{N}(r, x) \]

\[
\sigma_{\gamma^* h}^{T,L}(x, Q^2) = \sum_{\text{flavours}} \int d^2 r \int_0^1 dz \left| \Psi^{f,\gamma^* \rightarrow q\bar{q}}(z, r, Q^2) \right|^2 \sigma_{\text{dip}}(r, x) \\
\text{QED piece} \quad \text{Strong interactions are here} \\
\Rightarrow \text{dipole cross section:} \quad \sigma_{\text{dip}}(r, x) = 2 \int d^2 b \mathcal{N}(b, r, x) \approx \sigma_0 \mathcal{N}(b, r, x) \]
CGC evolution: The BK equation

Balitsky 96, Kovchegov 99

\( \frac{1}{x} \sim \ln s \sim Y \)

( large-N_c limit of full JIMWLK evolution)

\[
\begin{align*}
\frac{\partial \mathcal{N}(r, x)}{\partial \ln(x_0/x)} &= \int d^2r_1 K(r, r_1, r_2) \left[ \mathcal{N}(r_1, x) + \mathcal{N}(r_2, x) - \mathcal{N}(r, x) - \mathcal{N}(r_1, x)\mathcal{N}(r_2, x) \right] \\
S(x, y; Y) &= \frac{1}{N_c} \langle \text{tr}\{U_x U_y^\dagger\}\rangle_y = 1 - \mathcal{N}(x, y; Y)
\end{align*}
\]

unintegrated gluon distribution:

\[
\varphi(x, k_t) = \int \frac{d^2r}{2\pi r^2} e^{ik\cdot r} \mathcal{N}(r, x)
\]

Increase the collision energy and resum small-x gluon radiation

\[
\Rightarrow \text{The kernel: probability of small-x gluon emission at leading-logarithmic accuracy}
\]

\[
K(x, y, z) = \frac{\alpha_s N_c}{2\pi^2} \frac{(x - y)^2}{(x - z)^2(z - y)^2}
\]

+ all possible permutations
NLO corrections to BK-JIMWLK equations have been calculated recently (Balitsky-Chirilli; Kovchegov-Weigert, Gardi et al). Phenomenological tool: The BK equation including only running coupling corrections in Balitsky’s scheme grasps most of the NLO corrections (JLA-Kovchegov).

**BK eqn:**
\[
\frac{\partial \mathcal{N}(r, x)}{\partial \ln(x_0/x)} = \int d^2 r_1 K(r, r_1, r_2) [\mathcal{N}(r_1, x) + \mathcal{N}(r_2, x) - \mathcal{N}(r, x) - \mathcal{N}(r_1, x)\mathcal{N}(r_2, x)]
\]

**Running coupling kernel:**
\[
K^{\text{run}}(r, r_1, r_2) = \frac{N_c \alpha_s(r^2)}{2\pi^2} \left[ \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left( \frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]
\]

LO: \(\alpha_s \ln(1/x)\)
small-x gluon emission

“NLO”: \(\alpha_s N_f\)
Quark loops resummed to all orders

Gluon contribution: \(N_f \rightarrow -6\pi\beta_2\)
Running coupling corrections are large, rendering evolution compatible with experimental data.

\[ \lambda(Y) = \frac{d \ln Q_s(Y)}{dY} \]

\[ \lambda^{LO} \approx 4.8 \alpha_s \]

MV Initial conditions:  
\[ \mathcal{N}(r, x = x_0) = 1 - \exp \left[ -\frac{r^2 Q_0^2}{4} \ln \left( \frac{1}{r \Lambda} + e \right) \right] \]
**Fitting structure functions**

⇒ **Normalization**

\[ \int d^2b \rightarrow \sigma_0 \]

⇒ **Initial Conditions**

GBW: \[ N^{GBW}(r, x_0 = 10^{-2}) = 1 - \exp \left[ - \left( \frac{r^2 Q^2}{4} \right)^\gamma \right] \]

MV: \[ N^{MV}(r, x_0 = 10^{-2}) = 1 - \exp \left[ - \left( \frac{r^2 Q^2}{4} \right)^\gamma \ln \left( \frac{1}{r \Lambda_{QCD}} \right) \right] \]

⇒ **IR regularization and FT**

\[ \alpha_s(r^2) = \frac{12 \pi}{(11 N_c - 2 N_f) \ln \left( \frac{4 C^2}{r^2 \Lambda_{QCD}} \right)} \] for \( r < r_{fr} \), with \( \alpha_s(r^2_{fr}) \equiv \alpha_{fr} = 0.7 \)

3 (4) free parameters:

⇒ **Experimental data:** ZEUS, H1 (HERA), NMC (CERN-SPS) and E665 (Fermilab) coll.

- \( 0.045 < Q^2 < 800 \text{ GeV}^2 \) 847 data points
- \( x \leq 10^{-2} \)
- \( 0.045 < Q^2 < 50 \text{ GeV}^2 \) 703 data points

Fits are stable when large \( Q^2 \) data are not included in the fit.
### Table

<table>
<thead>
<tr>
<th>Initial condition</th>
<th>( \sigma_0 ) (mb)</th>
<th>( Q^2_{80} ) (GeV(^2))</th>
<th>( C^2 )</th>
<th>( \gamma )</th>
<th>( \chi^2/d.o.f. )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBW</td>
<td>31.59</td>
<td>0.24</td>
<td>5.3</td>
<td>1 (fixed)</td>
<td>916.3/844=1.086</td>
</tr>
<tr>
<td>MV</td>
<td>32.77</td>
<td>0.15</td>
<td>6.5</td>
<td>1.13</td>
<td>906.0/843=1.075</td>
</tr>
</tbody>
</table>

### Figures

- **Figure 1**: Graphs of \( F_2 \) and \( F_L \) vs. \( Q^2 \) for different initial conditions (GBW and MV) and \( Q^2 \) values (0.11 GeV\(^2\), 0.5 GeV\(^2\), 1.5 GeV\(^2\), 2.5 GeV\(^2\), 5 GeV\(^2\), 10 GeV\(^2\), 20 GeV\(^2\), 50 GeV\(^2\), 80 GeV\(^2\), 120 GeV\(^2\), 250 GeV\(^2\), 450 GeV\(^2\)).
- **Figure 2**: Data from H1 (PLB665, 139; x-averaged) showing \( F_2 \) and \( F_L \) with solid lines for GBW initial conditions and dotted lines for MV initial conditions.

**Legend**
- \( \chi^2/d.o.f. \) values (1.086 and 1.075).
- \( \gamma \) values (1 for GBW and 1.13 for MV).
- \( Q^2_{80} \) values (0.24 for GBW and 0.15 for MV).
- \( \sigma_0 \) values (31.59 for GBW and 32.77 for MV).

**Details**
- The data is labeled as \( J / \psi \) (PLB665, 139; x-averaged).
- The initial conditions are solid for GBW and dotted for MV.
- The graphs cover a range of \( Q^2 \) values from 0.11 GeV\(^2\) to 450 GeV\(^2\).

**Notes**
- The AAMS 1.0 was mentioned in the context of the data.
Preliminary results AAMQs 1.0

✓ Good fits to data on reduced cross sections from combined analysis by H1 and ZEUS coll (much smaller error bars!). Fit parameters stable wrt to AAMS 1.0 analysis

\[ \chi^2 / d.o.f \approx 1 \div 1.5 \]

✓ Inclusion of charm and beauty

\[ \sigma_0^{\text{light}} > \sigma_0^{\text{charm}} \]

✓ Fits to new HERA data on reduced cross sections
d+Au and p+p collisions at RHIC

RHIC Kinematics:

- single particle production: Small-x ~ forward production

\[ x_{1(2)} \sim \frac{m_t}{\sqrt{s}} \exp(\pm y_h) \]

- double inclusive production: Small-x ~ two particles in the forward region!

\[ x_p = \frac{|k_1| e^{y_1} + |k_2| e^{y_2}}{\sqrt{s}} \]
\[ x_A = \frac{|k_1| e^{-y_1} + |k_2| e^{-y_2}}{\sqrt{s}} \]

At RHIC energies, forward measurements needed to isolate small-x (<0.01) effects
Forward hadron production in the CGC
(Dumitru, Jalilian-Marian)

\[ \frac{dN_h}{dy_h \, d^2 p_t} = \left( \frac{K}{(2\pi)^2} \right) \sum_q \int_{x_F}^1 \frac{dz}{z^2} \left[ x_1 f_q / p(x_1, p_t^2) \tilde{N}_F \left( x_2, \frac{p_t}{z} \right) D_{h/q}(z, p_t^2) \right. \\
\left. + x_1 f_g / p(x_1, p_t^2) \tilde{N}_A \left( x_2, \frac{p_t}{z} \right) D_{h/g}(z, p_t^2) \right] \]

large-x parton from proj. (pdf) \hspace{1cm} small-x glue from target (CGC)

Unintegrated gluon from running coupling BK

MV Initial conditions:

JLA & C. Marquet 10

\[ \tilde{N}_{F(A)}(x, k) = \int d^2 r \, e^{-i k \cdot r} \left[ 1 - N_{F(A)}(r, Y = \ln(x_0/x)) \right] \]

\[ N(r, x = x_0) = 1 - \exp \left[ -\frac{r^2 Q_0^2}{4} \ln \left( \frac{1}{r \Lambda} + e \right) \right] \]

Two free parameters: \((x_0, Q_0)\)

We use CTEQ6 pdf’s and de Florian-Sassot ff’s

Alternative approaches: Modelization of quantum corrections
(Dumitru-JalilianMarian-Hayashigaki; De Boer-Utermann-Wessels; Goncalves et al; Kharzeev-Kovchegov-Tuchin)
Comparison to RHIC forward data [JLA, C. Marquet ’10]

- Very good description of forward yields in proton+proton and d+Au collisions

- K=1 for h^-. K=0.4 (0.3) for neutral pions in p+p (d+Au) ??

\[
\begin{align*}
0.005 \leq x_0 \leq 0.01 \\
Q_{s0}^2 &= 0.2 \text{ GeV}^2 \\
0.01 \leq x_0 \leq 0.025 \\
Q_{s0}^2 &= 0.4 \text{ GeV}^2 \\
Q_{s0, \text{gluon}}^2 &= 0.9 \text{ GeV}^2 \\
0.005 \leq x_0 \leq 0.01 \\
Q_{s0}^2 &= 0.5 \text{ GeV}^2 \\
Q_{s0, \text{gluon}}^2 &= 1.125 \text{ GeV}^2
\end{align*}
\]
- ...by simply taking the ratio of d+Au and p+p spectra we get a good description of the nuclear modification factor (not a trivial statement!!)

- We predict a similar suppression in p+Pb collisions at the LHC already at central rapidities
Double Inclusive forward hadron production in the CGC

$$x_p = \frac{|k_1| e^{y_1} + |k_2| e^{y_2}}{\sqrt{s}}$$

$$x_A = \frac{|k_1| e^{-y_1} + |k_2| e^{-y_2}}{\sqrt{s}}$$

Cyrille Marquet 07:

- hard quark initiating scattering
- Fourier transform coordinate space to momentum
- q -> qg splitting (pQCD)

Scattering of the 2-parton system with the CGC target

Involves more than 3 and 4 point functions. Calculated in the large $N_c$ limit
Double Inclusive forward hadron production in the CGC

\[ x_p = \frac{|k_1| e^{y_1} + |k_2| e^{y_2}}{\sqrt{s}} \]

\[ x_A = \frac{|k_1| e^{-y_1} + |k_2| e^{-y_2}}{\sqrt{s}} \]

Cyrille Marquet 07:

\[ \frac{d\sigma}{d^2k_\perp dy_1 d^2q_\perp dy_2} = \alpha S C_F N_c x_d q(x_d, \mu^2) \int \frac{d^2x}{(2\pi)^2} \frac{d^2x'}{(2\pi)^2} \frac{d^2b}{(2\pi)^2} \frac{d^2b'}{(2\pi)^2} e^{ik_\perp \cdot (x' - x)} e^{iq_\perp \cdot (b' - b)} \]

|\Phi^{q \rightarrow qg}(z, x - b, x' - b')| \left\{ S_{qgg}^{(4)}[b, x, b', x'; x_A] - S_{qgg}^{(3)}[b, x, b' + z(x' - b'); x_A] \right\}

- S_{qgg}^{(3)}[b + z(x - b), x', b'; x_A] + S_{qg}^{(2)}[b + z(x - b), b' + z(x' - b'); x_A] \]

q\rightarrow qg splitting (pQCD)

Involves more than 3 and 4 point functions. Calculated in the large Nc limit.
“Monojets” in d+Au collisions at RHIC at forward rapidity

“Coincidence probability” measured by STAR Coll. at forward rapidities:

\[ CP(\Delta \phi) = \frac{1}{N_{trig}} \frac{dN_{pair}}{d\Delta \phi} \]

- Dependence on the saturation scale of the target (centrality)

Parameter free!!: All info about nucleus w.f. from single inclusive analysis
Multiparticle production in A+A coll.

RHIC multiplicities smaller than expected.

Most of particles produces in RHIC Au+Au collisions are small-x gluons

\[ \# \text{ produces particles} \sim \# \text{ scattering centers} \]

Two alternative approaches to describe multiparticle production within the CGC:

\[ k_t \text{-factorization} \] (Valid in p+A coll. Violated in A+A collisions). Starting point to the Kharzeev-Levin-Nardi model

\[ \frac{dN^g_{AB}}{d\eta} = \frac{4\pi N_c}{N_c^2 - 1} \int \frac{d^2 p}{p^2} \int d^2 k \alpha_s(Q^2) \varphi_A(x_1,k) \varphi_B(x_2,|p-k|) \]

\[ \text{Classical Yang-Mills (CYM)} \] Kovner, McLerran, Weigert.

\[ D_\mu F^{\mu\nu} = J^\nu \quad \text{with} \quad J^\pm \sim \rho_{A(B)}[Q_s(x)] \delta(x^\pm) \]

More rigorous, but requires numerical implementation
Local parton-hadron duality

\[ \varphi(x, k) \Rightarrow \text{Solutions of BK with running coupling} \times (1 - x)^4 \]

\[ JLA\ 07 \]

\[ m_h \approx 0.25 \text{ GeV}, \quad 0.75 \leq Q_0 \leq 1.25 \text{ GeV}, \quad 0.05 \leq x_0 \leq 0.1 \]

\[ \frac{dN_{ch}^{Pb-Pb}(\sqrt{s} = 2.75 \text{ TeV}, \eta = 0)}{d\eta} \approx 1100 \div 1250 \]
Local parton-hadron duality

\[ \varphi(x, k) \Rightarrow \text{Solutions of BK with running coupling} \times (1 - x) \]

\[ JLA\ 07 \]

\[ m_h \approx 0.25 \text{ GeV}, \ 0.75 \leq Q_0 \leq 1.25 \text{ GeV}, \ 0.05 \leq x_0 \leq 0.1 \]

\[ \frac{dN_{Pb-Pb}}{d\eta} (\sqrt{s} = 2.75 \text{ TeV}, \eta = 0) \approx 1100 \div 1250 \]

CGC prediction features the same power-law dependence as p+p LHC data
Conclusions

⇒ NLO corrections bring the CGC to a new period of quantitative and predictive phenomenology
⇒ Good description of latest e+p data, including heavy flavour
⇒ Good description of forward particle production @ RHIC
⇒ Still, many things remain to be done to refine the CGC as a practical precise tool...

Thanks!!!
Back up slides