

Color Glass Condensate at NLO: Phenomenology at HERA, RHIC and the LHC

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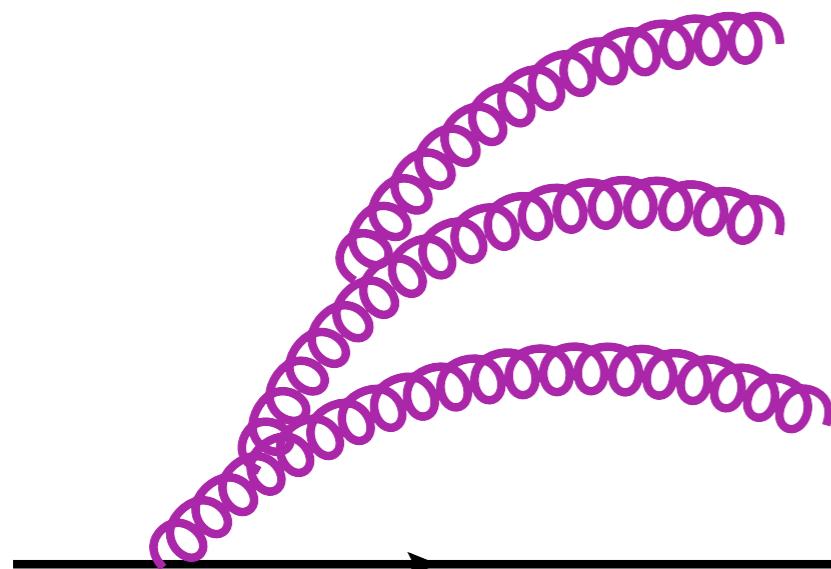
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XL ISMD



Outline

- ⇒ Motivation & Pocket Introduction to the CGC
- ⇒ Balitsky-Kovchegov equation including running coupling corrections
- ⇒ Structure functions in e+p collisions at HERA
- ⇒ Single inclusive hadron production in the CGC
- ⇒ Di-hadron correlations in d+Au collisions
- ⇒ Multiplicities in A+A collisions

At high energies, or small Bjorken-x, hadron's gluon densities are large



Probability of n-soft gluon emission

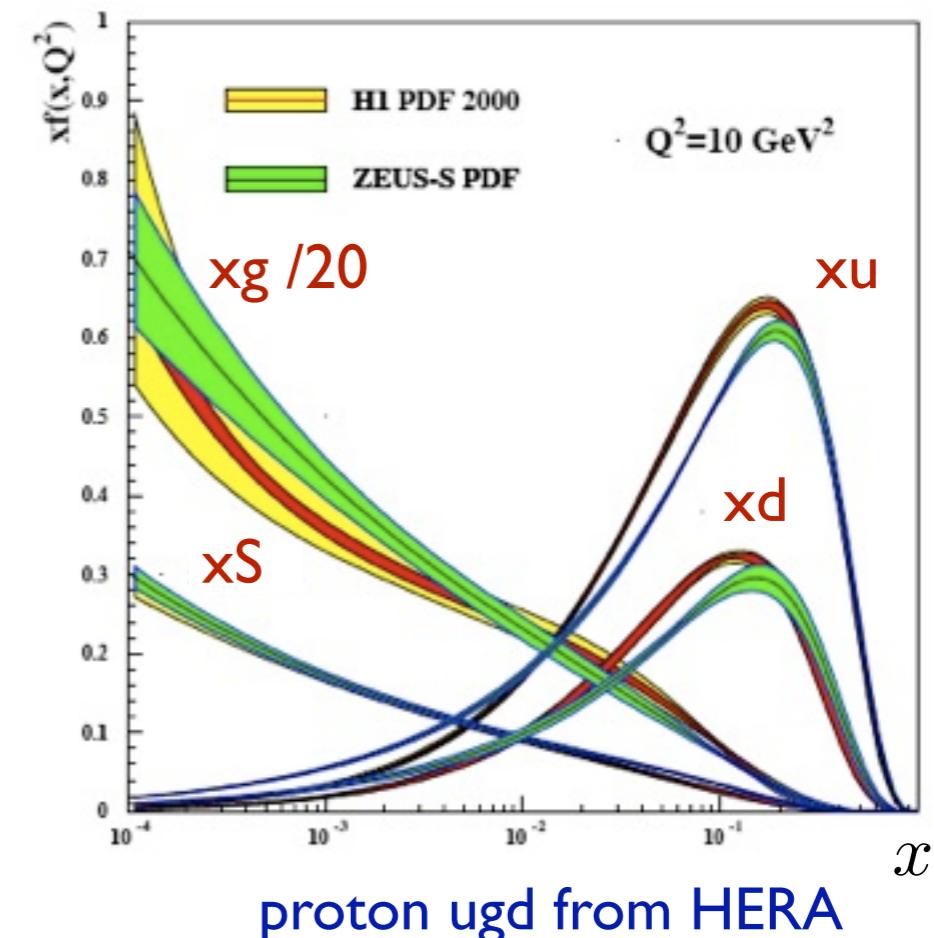
$$P \sim (\alpha_s \ln 1/x)^n$$

$$k_n^+ = x_n p^+$$

$$k_2^+ = x_2 p^+$$

$$k_1^+ = x_1 p^+$$

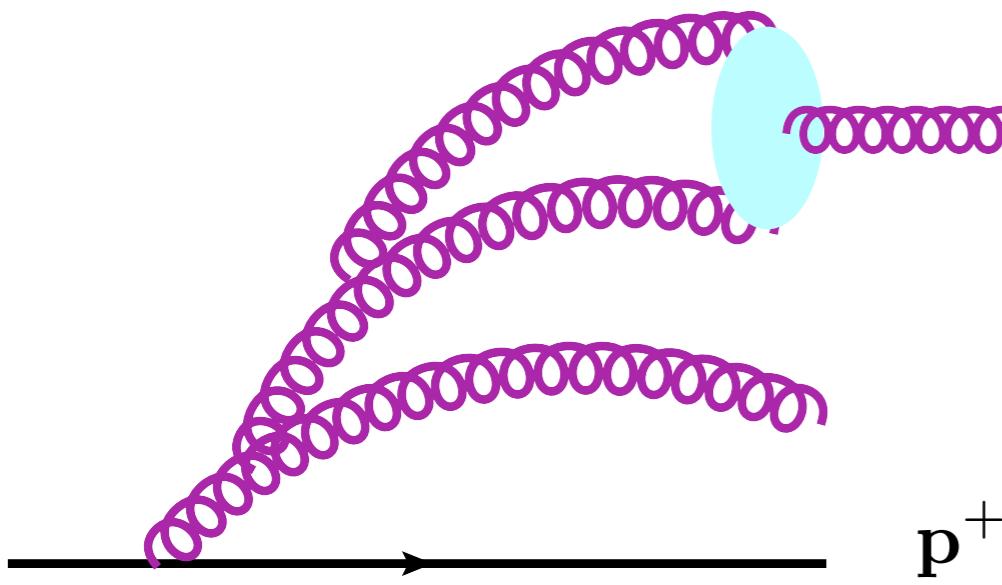
$$p^+$$



Multiple small-x gluon emissions are resummed by the BFKL equation

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k}_t)}{\partial \ln(\mathbf{x}_0/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k}_t)$$

Non-linear evolution: At small- x gluon both radiative and recombination processes

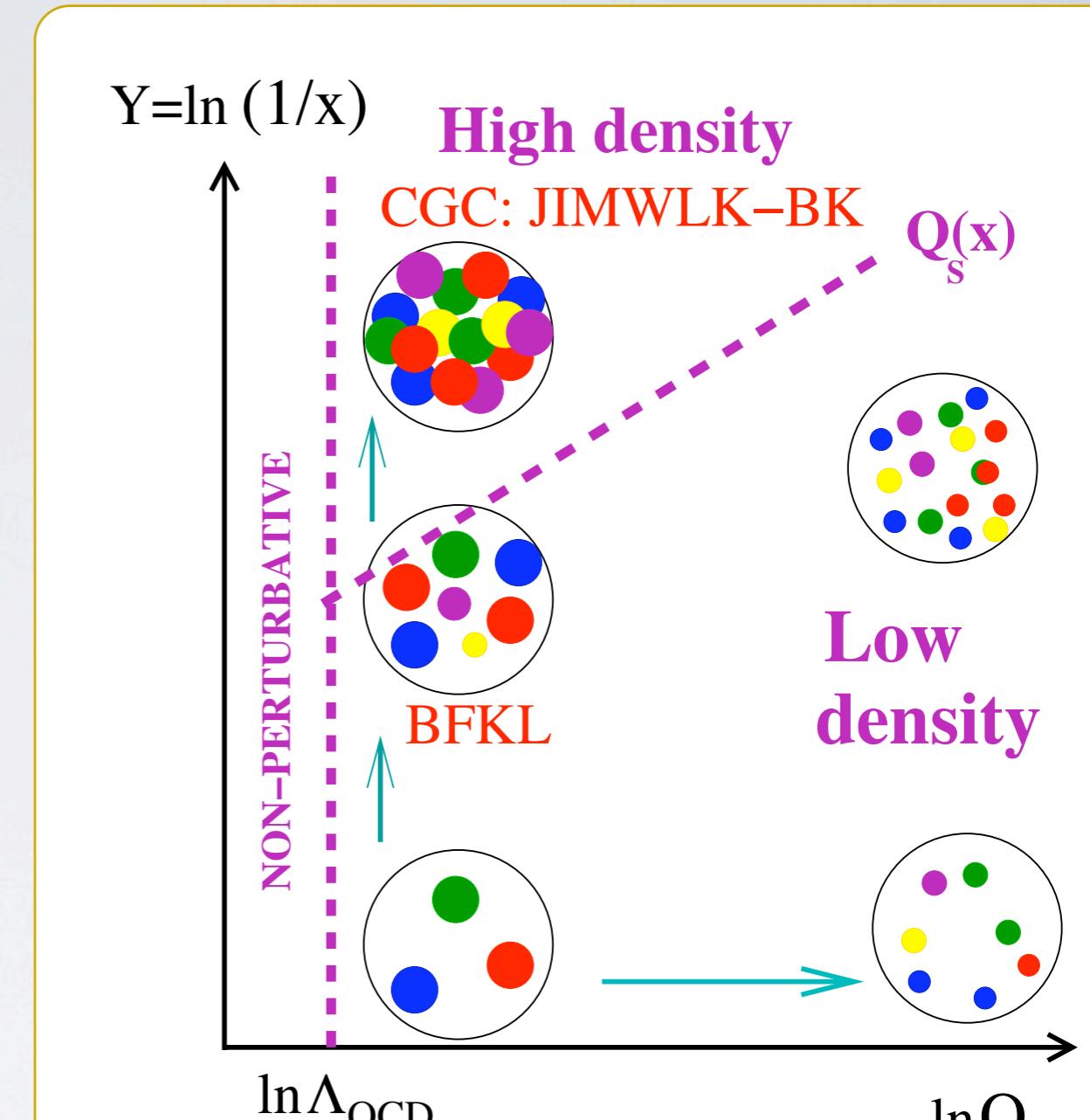


“BK-JIMWLK”

$$\frac{\partial \phi(x, k_t)}{\partial \ln(x_0/x)} \approx \mathcal{K} \otimes \phi(x, k_t) - \phi(x, k_t)^2$$



Non-linear recombination corrections
are demanded by UNITARITY



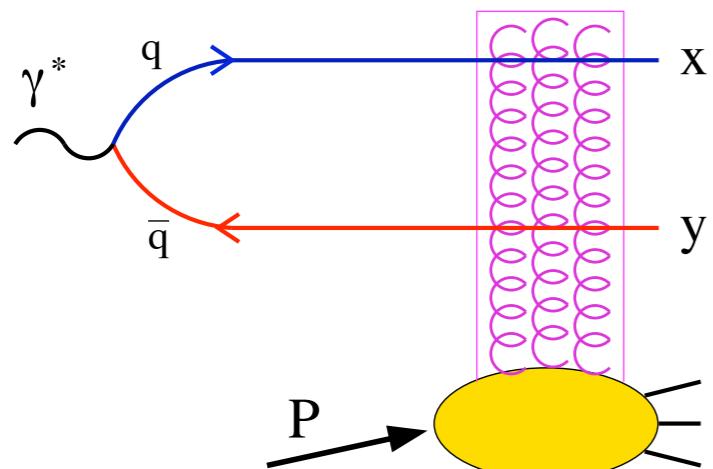
Saturation scale: transverse momentum scale which marks the onset of non-linear corrections

$$\mathcal{K} \otimes \phi(x, Q_s) \approx \phi(x, Q_s)^2$$

$$\text{Nuclear enhancement: } Q_{sA}^2 \approx A^{1/3} Q_{sp}^2$$

DIS in the dipole model

Probe your hadron with a photon (color dipole). Eikonal scattering



$$S(\underline{x}, \underline{y}; Y) = \frac{1}{N_c} \langle \text{tr}\{U_{\underline{x}} U_{\underline{y}}^\dagger\} \rangle_Y = 1 - \mathcal{N}(\underline{x}, \underline{y}; Y)$$

unintegrated gluon distribution:

$$\varphi(x, k_t) = \int \frac{d^2 r}{2\pi r^2} e^{i\mathbf{k} \cdot \mathbf{r}} \mathcal{N}(r, x)$$

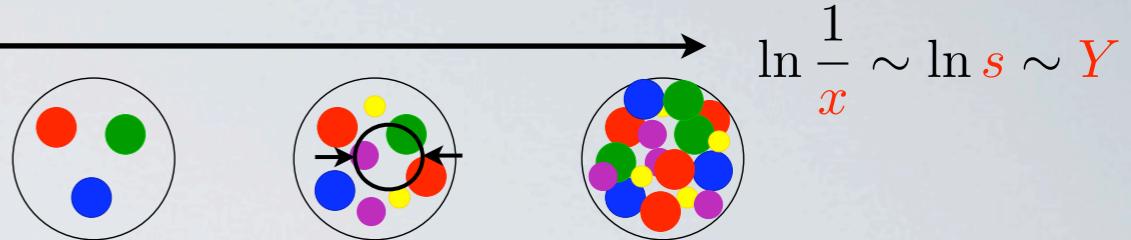
$$\sigma_{T,L}^{\gamma^* h}(\underline{x}, Q^2) = \sum_{flavours} \int d^2 r \int_0^1 dz \left| \Psi_{T,L}^{f, \gamma^* \rightarrow q\bar{q}}(z, r, Q^2) \right|^2 \sigma^{dip}(r, \underline{x})$$

↓ ↓
QED piece Strong interactions
 are here

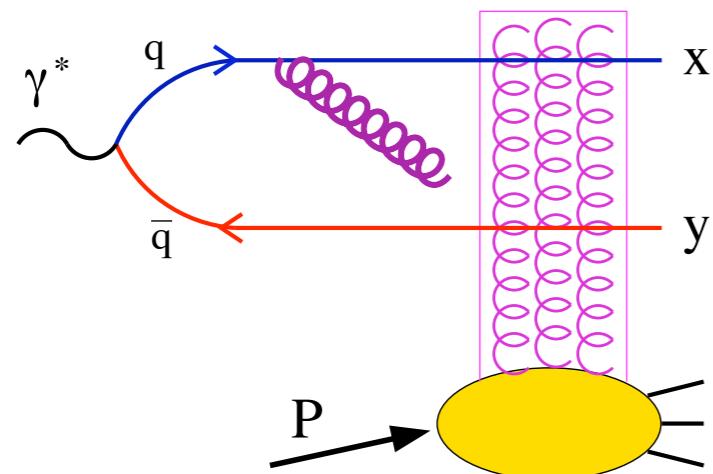
⇒ dipole cross section: $\sigma^{dip}(r, \underline{x}) = 2 \int d^2 b \mathcal{N}(b, r, \underline{x}) \approx \sigma_0 \mathcal{N}(b, r, \underline{x})$

CGC evolution: The BK equation

Balitsky 96, Kovchegov 99



(large- N_c limit of full JIMWLK evolution)



$$S(\underline{x}, \underline{y}; Y) = \frac{1}{N_c} \langle \text{tr}\{U_{\underline{x}} U_{\underline{y}}^\dagger\} \rangle_Y = 1 - \mathcal{N}(\underline{x}, \underline{y}; Y)$$

unintegrated gluon distribution:

$$\varphi(x, k_t) = \int \frac{d^2 r}{2\pi r^2} e^{i\mathbf{k}\cdot\mathbf{r}} \mathcal{N}(r, x)$$

Increase the collision energy and resum small- x gluon radiation

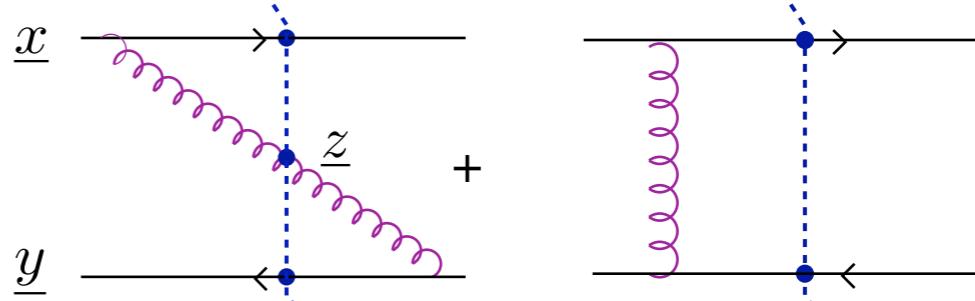
$$\frac{\partial \mathcal{N}(r, x)}{\partial \ln(x_0/x)} = \int d^2 r_1 K(r, r_1, r_2) [\mathcal{N}(r_1, x) + \mathcal{N}(r_2, x) - \mathcal{N}(r, x) - \mathcal{N}(r_1, x)\mathcal{N}(r_2, x)]$$

perturbative kernel non-linear term

⇒ The kernel: probability of small- x gluon emission at leading-logarithmic accuracy

in $\alpha_s \ln(1/x)$:

$$K(\underline{x}, \underline{y}, \underline{z}) = \frac{\alpha_s N_c}{2\pi^2} \frac{(\underline{x} - \underline{y})^2}{(\underline{x} - \underline{z})^2 (\underline{z} - \underline{y})^2} =$$



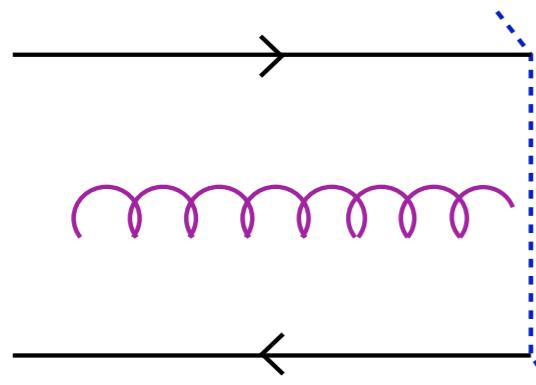
+ all possible permutations

✓ NLO corrections to BK-JIMWLK equations have been calculated recently (Balitsky-Chirilli; Kovchegov-Weigert, Gardi et al). Phenomenological tool: The BK equation including only running coupling corrections in Balitsky's scheme grasps most of the NLO corrections (JLA-Kovchegov)

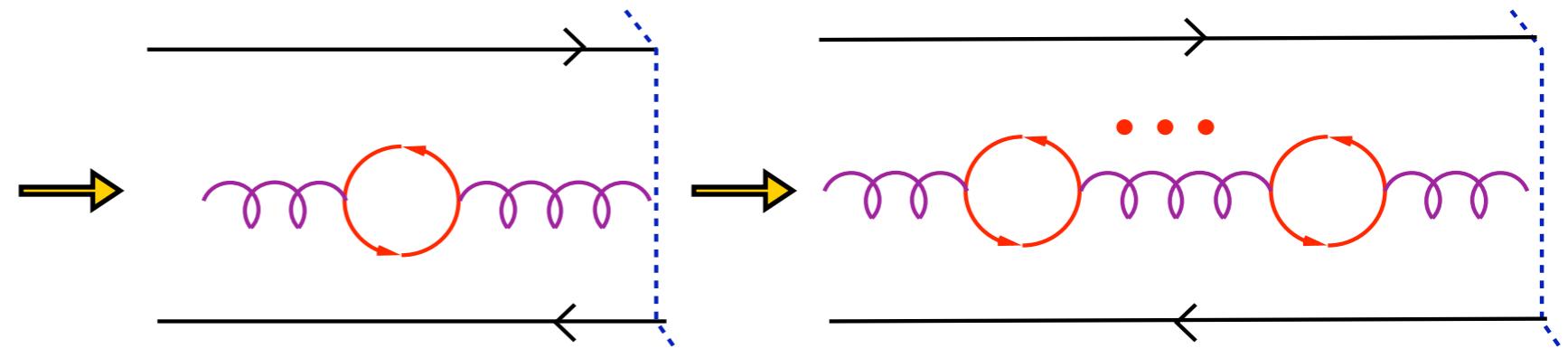
BK eqn: $\frac{\partial \mathcal{N}(r, x)}{\partial \ln(x_0/x)} = \int d^2 r_1 K(r, r_1, r_2) [\mathcal{N}(r_1, x) + \mathcal{N}(r_2, x) - \mathcal{N}(r, x) - \mathcal{N}(r_1, x)\mathcal{N}(r_2, x)]$

Running coupling kernel: $K^{\text{run}}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \frac{N_c \alpha_s(r^2)}{2\pi^2} \left[\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$

LO: $\alpha_s \ln(1/x)$
small-x gluon emission



“NLO”: $\alpha_s N_f$
Quark loops resummed to all orders

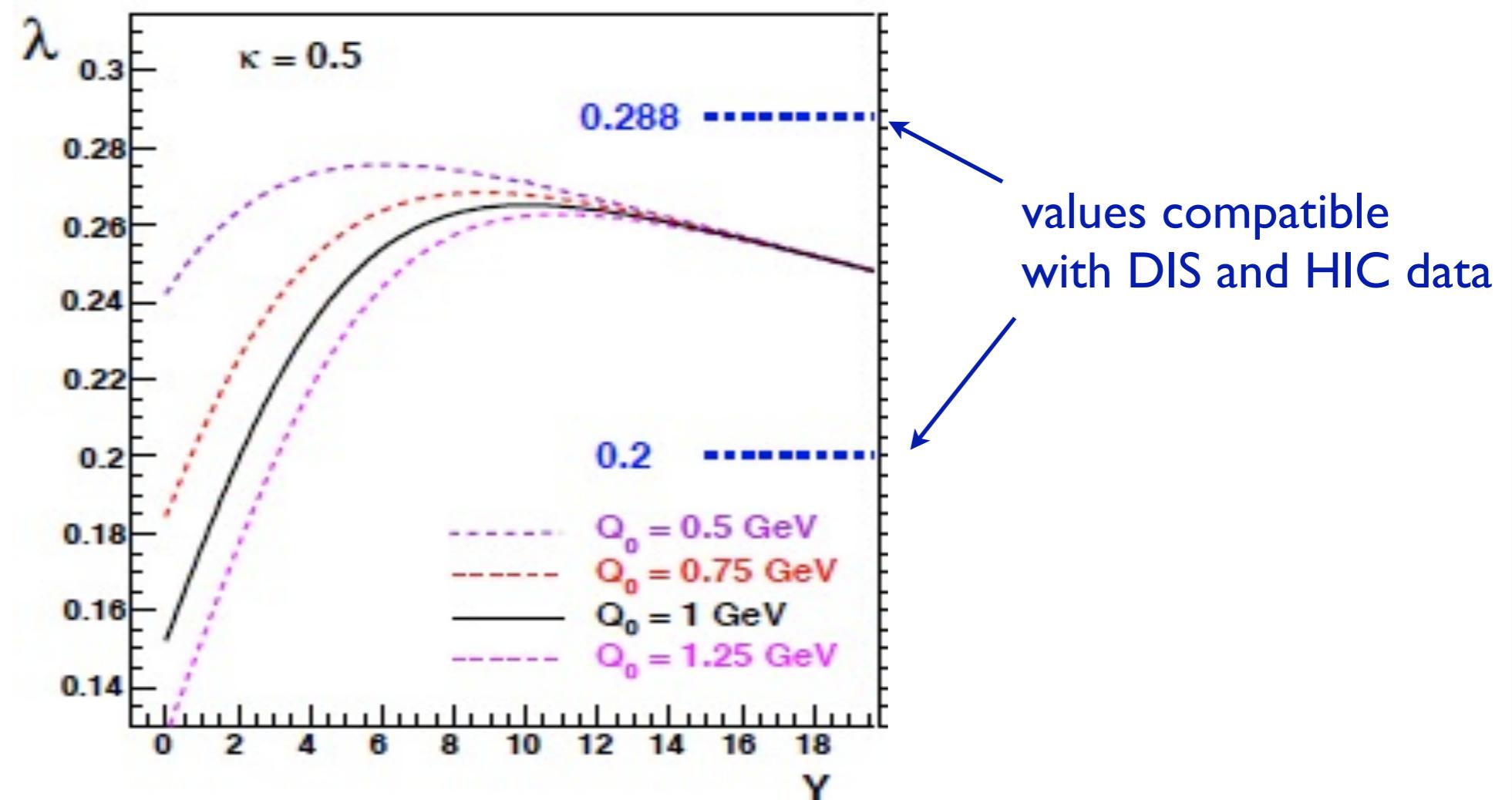


Gluon contribution: $N_f \rightarrow -6\pi\beta_2$

Running coupling corrections are large, rendering evolution compatible with experimental data.

$$\lambda(Y) = \frac{d \ln Q_s(Y)}{dY}$$

$$\lambda^{LO} \approx 4.8 \alpha_s$$



MV Initial conditions:

$$\mathcal{N}(r, x = x_0) = 1 - \exp \left[-\frac{r^2 Q_0^2}{4} \ln \left(\frac{1}{r \Lambda} + e \right) \right]$$

Fitting structure functions

• JLA, N. Armesto, J.G. Milhano, C. Salgado
Phys.Rev.D80:034031,2009;

⇒ Normalization

$$\int d^2 b \rightarrow \sigma_0$$

GBW: $\mathcal{N}^{GBW}(r, x_0 = 10^{-2}) = 1 - \exp \left[- \left(\frac{r^2 Q_{s0}^2}{4} \right)^\gamma \right]$

⇒ Initial Conditions

MV: $\mathcal{N}^{MV}(r, x_0 = 10^{-2}) = 1 - \exp \left[- \left(\frac{r^2 Q_{s0}^2}{4} \right)^\gamma \ln \left(\frac{1}{r \Lambda_{QCD}} \right) \right]$

⇒ IR regularization and FT

$$\alpha_s(r^2) = \frac{12 \pi}{(11 N_c - 2 N_f) \ln \left(\frac{4 C^2}{r^2 \Lambda_{QCD}} \right)} \quad \text{for } r < r_{fr}, \text{ with } \alpha_s(r_{fr}^2) \equiv \alpha_{fr} = 0.7$$

3 (4) free parameters:

⇒ Experimental data: ZEUS, HI (HERA), NMC (CERN-SPS) and E665 (Fermilab) coll.

$$0.045 < Q^2 < 800 \text{ GeV}^2$$

847 data points

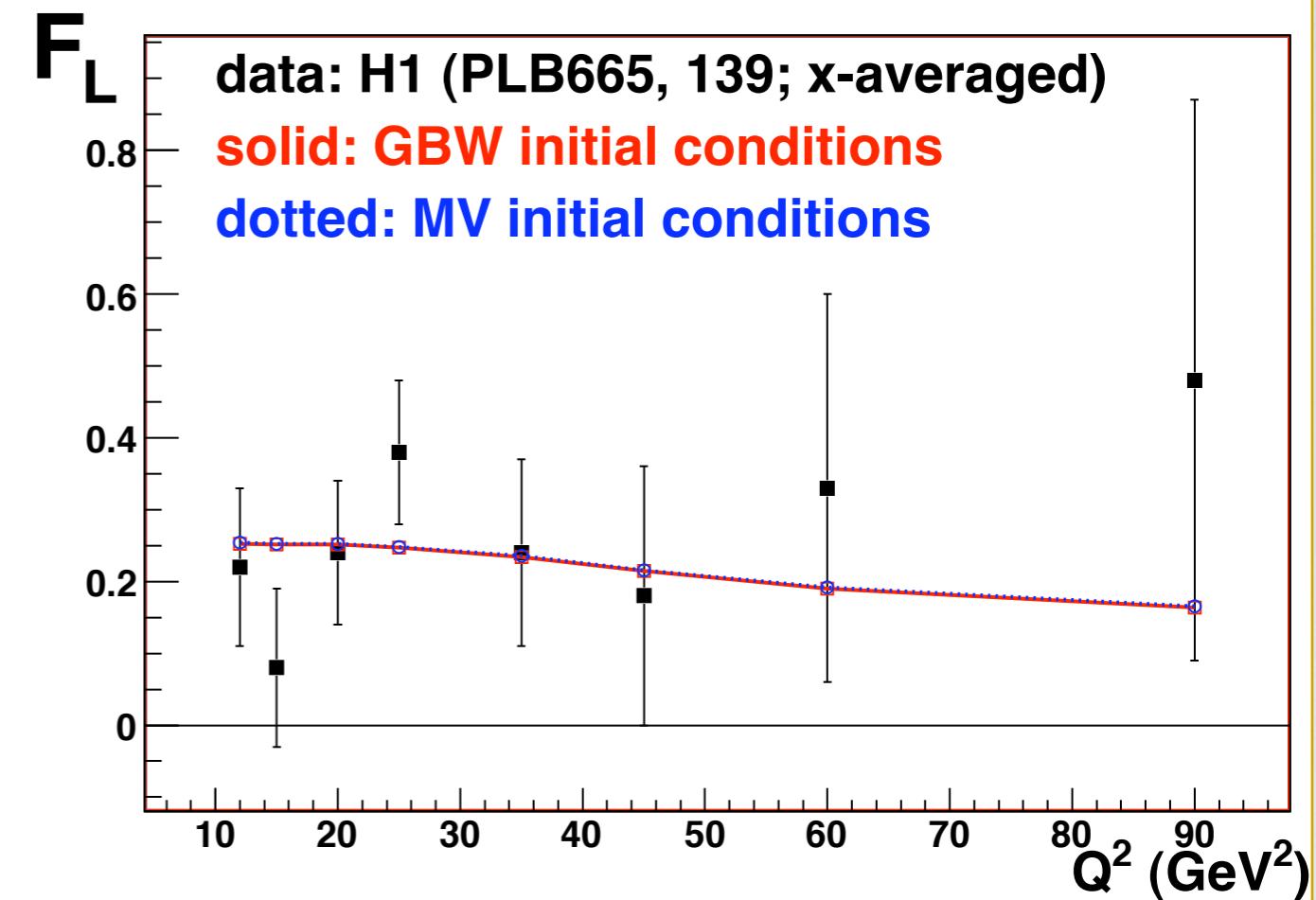
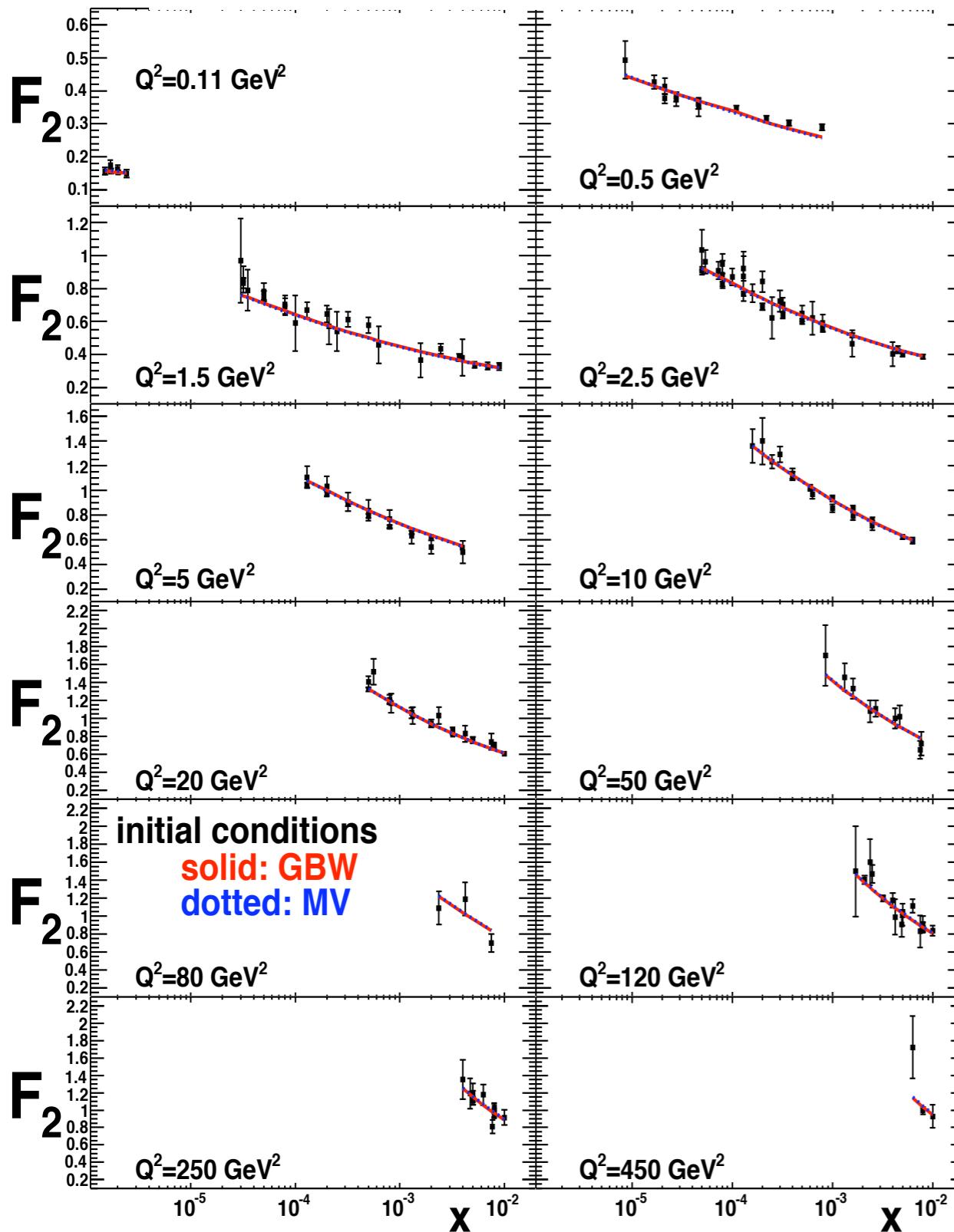
$$x \leq 10^{-2}$$

$$0.045 < Q^2 < 50 \text{ GeV}^2$$

703 data points

Fits are stable when large Q^2 data are not included in the fit

Initial condition	σ_0 (mb)	Q_{s0}^2 (GeV 2)	C^2	γ	$\chi^2/\text{d.o.f.}$
GBW	31.59	0.24	5.3	1 (fixed)	916.3/844=1.086
MV	32.77	0.15	6.5	1.13	906.0/843=1.075



AAMS 1.0

Preliminary results AAMQs 1.0

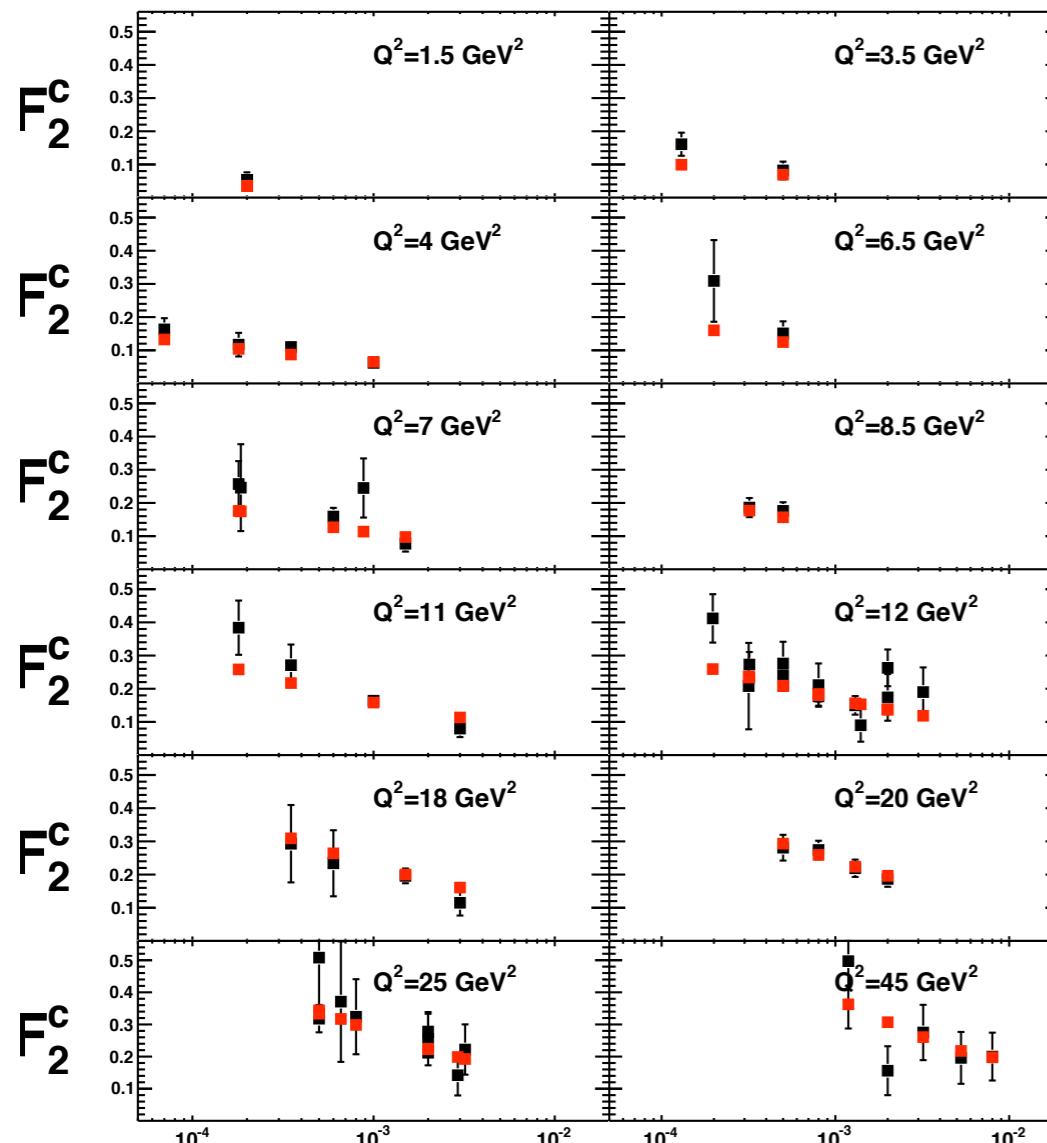
AAMS+P. Quiroga in preparation

- ✓ Good fits to data on reduced cross sections from combined analysis by H1 and ZEUS coll (much smaller error bars!). Fit parameters stable wrt to AAMS 1.0 analysis

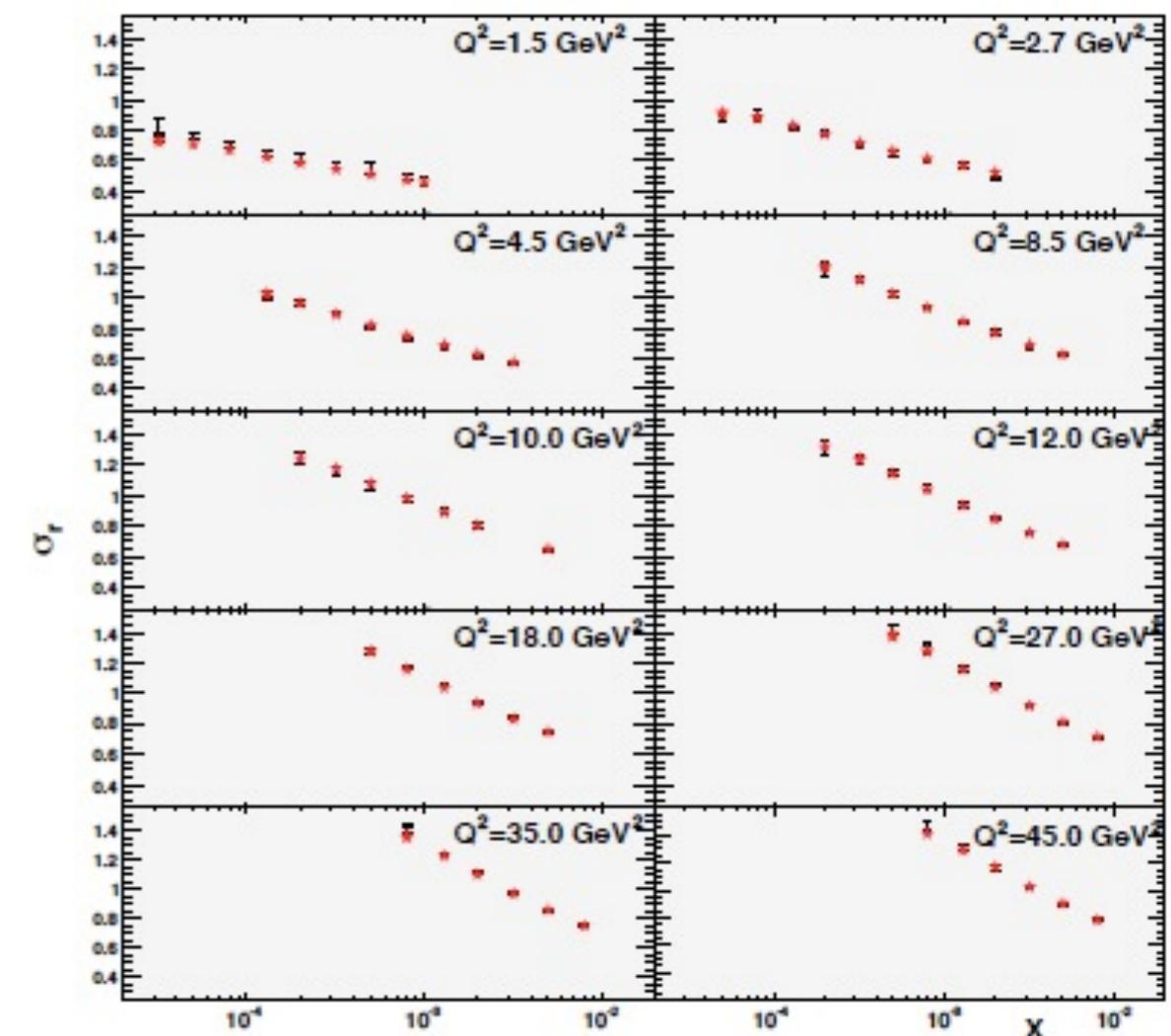
$$\chi^2/d.o.f \approx 1 \div 1.5$$

- ✓ Inclusion of charm and beauty

$$\sigma_0^{light} > \sigma_0^{charm}$$



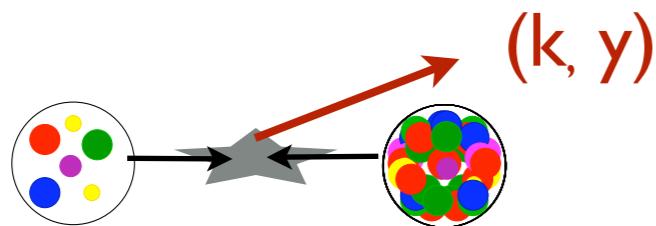
- ✓ Fits to new HERA data on reduced cross sections



d+Au and p+p collisions at RHIC

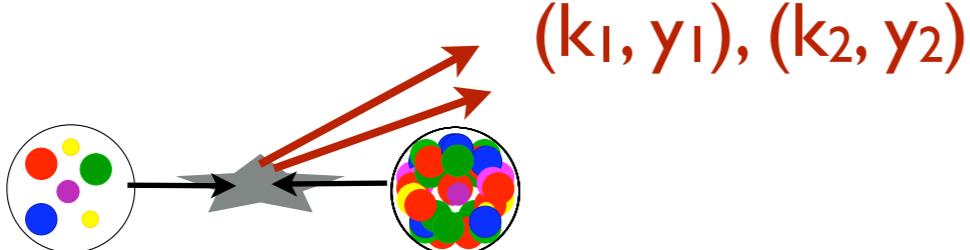
RHIC Kinematics:

- single particle production: Small- $x \sim$ forward production



$$x_{1(2)} \sim \frac{m_t}{\sqrt{s}} \exp(\pm y_h)$$

- double inclusive production: Small- $x \sim$ two particles in the forward region!



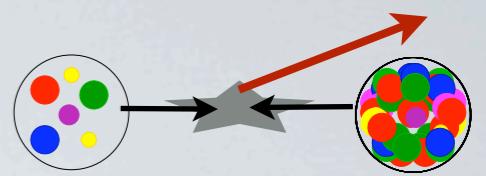
$$x_p = \frac{|k_1| e^{y_1} + |k_2| e^{y_2}}{\sqrt{s}}$$

$$x_A = \frac{|k_1| e^{-y_1} + |k_2| e^{-y_2}}{\sqrt{s}}$$

At RHIC energies, forward measurements needed to isolate small- x
(<0.01) effects

⇒ Forward hadron production in the CGC

(Dumitru, Jalilian-Marian)



$$\frac{dN_h}{dy_h d^2p_t} = \frac{K}{(2\pi)^2} \sum_q \int_{x_F}^1 \frac{dz}{z^2} \left[x_1 f_{q/p}(x_1, p_t^2) \tilde{N}_F \left(x_2, \frac{p_t}{z} \right) D_{h/q}(z, p_t^2) \rightarrow \text{fragmentation} \right. \\ \left. + x_1 f_{g/p}(x_1, p_t^2) \tilde{N}_A \left(x_2, \frac{p_t}{z} \right) D_{h/g}(z, p_t^2) \right]$$

Unintegrated gluon from running coupling BK

MV Initial conditions:

JLA & C. Marquet 10

$$\tilde{N}_{F(A)}(x, k) = \int d^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} [1 - \mathcal{N}_{F(A)}(r, Y = \ln(x_0/x))]$$

$$\mathcal{N}(r, x = x_0) = 1 - \exp \left[-\frac{r^2 Q_0^2}{4} \ln \left(\frac{1}{r \Lambda} + e \right) \right]$$

Two free parameters: (x_0, Q_0)

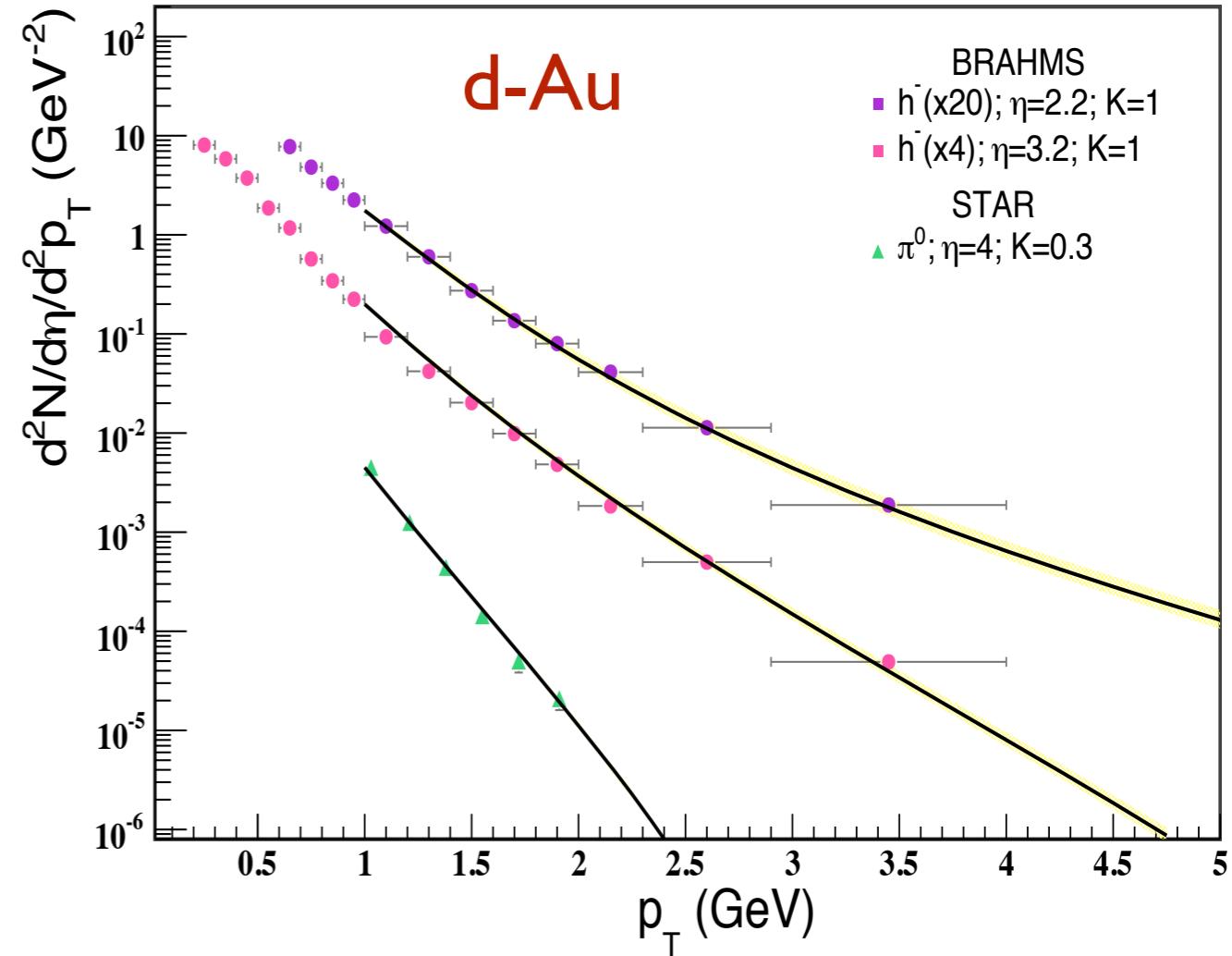
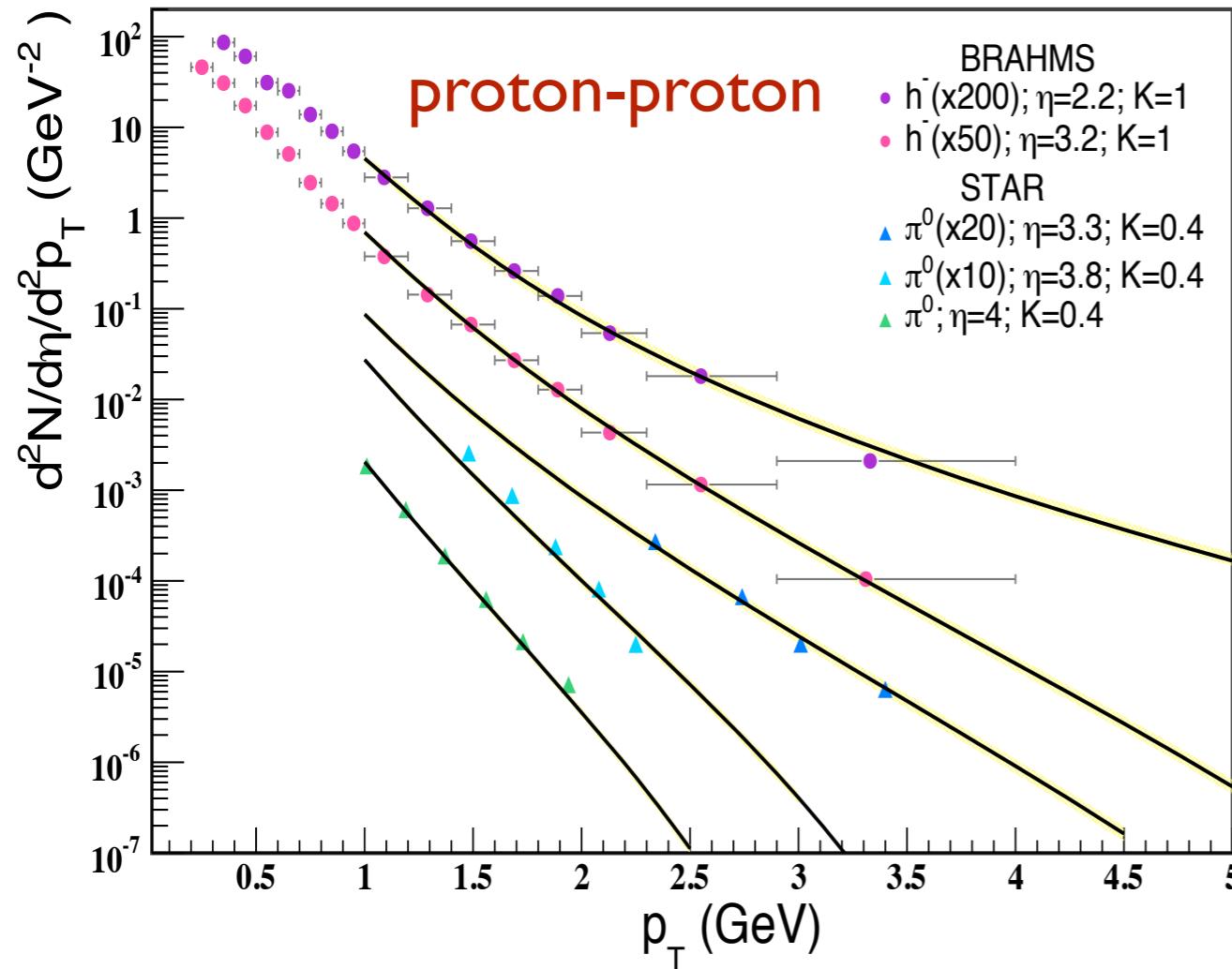
We use CTEQ6 pdf's and de Florian-Sassot ff's

Alternative approaches: Modelization of quantum corrections

(Dumitru-JalilianMarian-Hayashigaki; De Boer-Utermann-Wessels; Goncalves et al;
Kharzeev-Kovchegov-Tuchin)

Comparison to RHIC forward data [JLA, C. Marquet '10]

- Very good description of forward yields in proton+proton and d+Au collisions
- $K=1$ for h^- . $K=0.4$ (0.3) for neutral pions in p+p (d+Au) ??



$$0.005 \leq x_0 \leq 0.01$$

$$Q_{s0}^2 = 0.2 \text{ GeV}^2$$

$$0.01 \leq x_0 \leq 0.025$$

$$Q_{s0}^2 = 0.4 \text{ GeV}^2$$

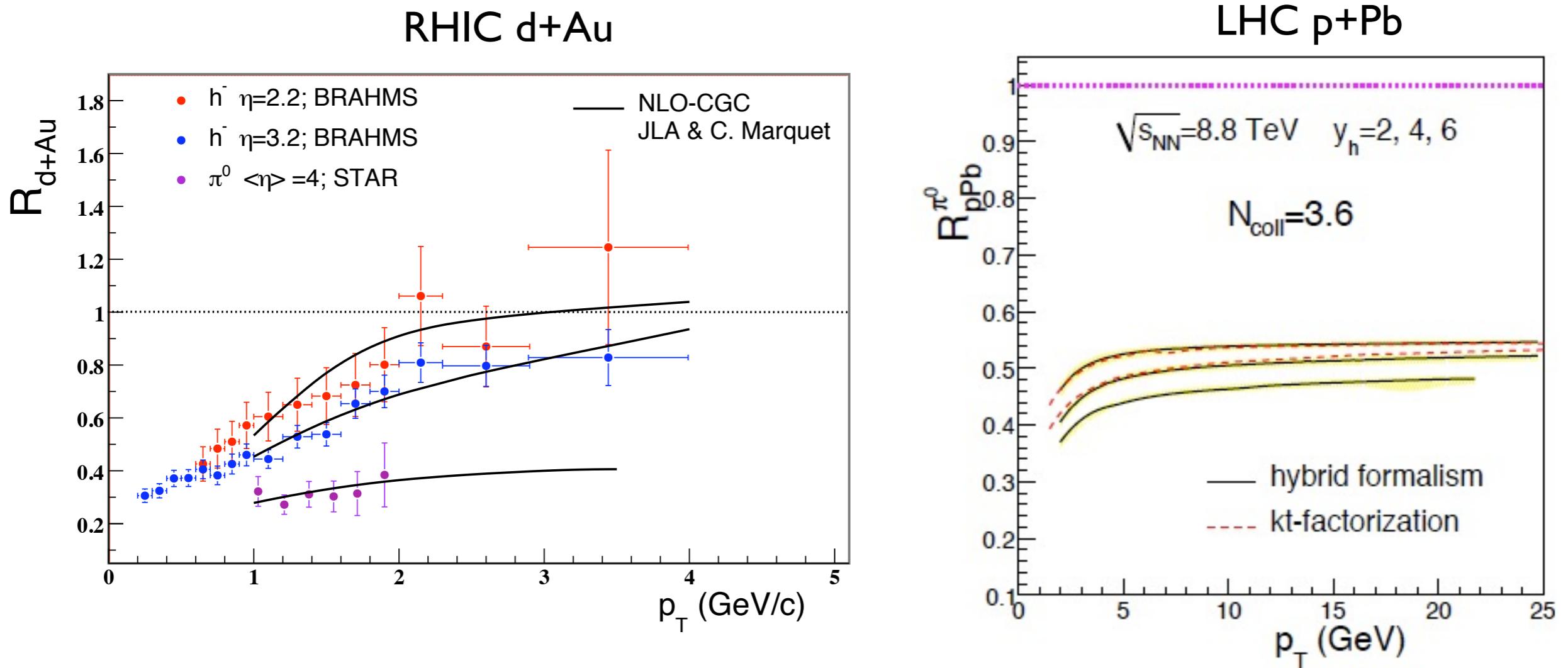
$$Q_{s0, gluon}^2 = 0.9 \text{ GeV}^2$$

$$0.005 \leq x_0 \leq 0.01$$

$$Q_{s0}^2 = 0.5 \text{ GeV}^2$$

$$Q_{s0, gluon}^2 = 1.125 \text{ GeV}^2$$

- ...by simply taking the ratio of d+Au and p+p spectra we get a good description of the nuclear modification factor (**not a trivial statement!!**)

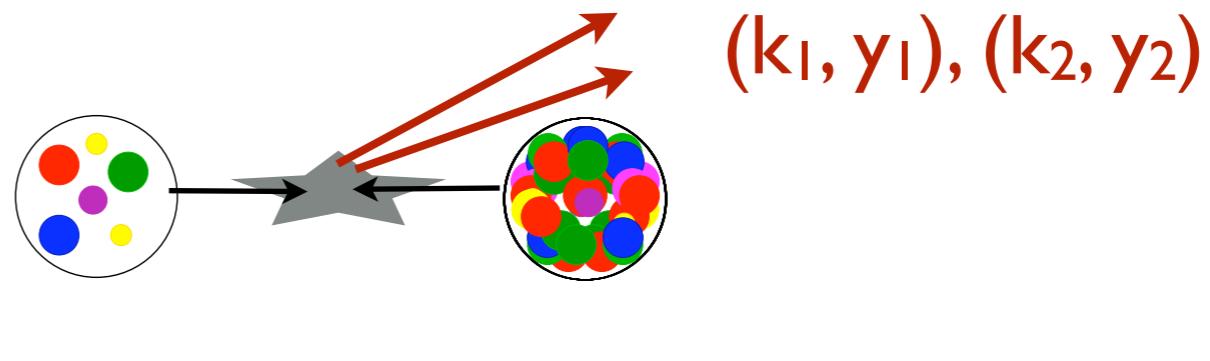


- We predict a similar suppression in p+Pb collisions at the LHC already at central rapidities

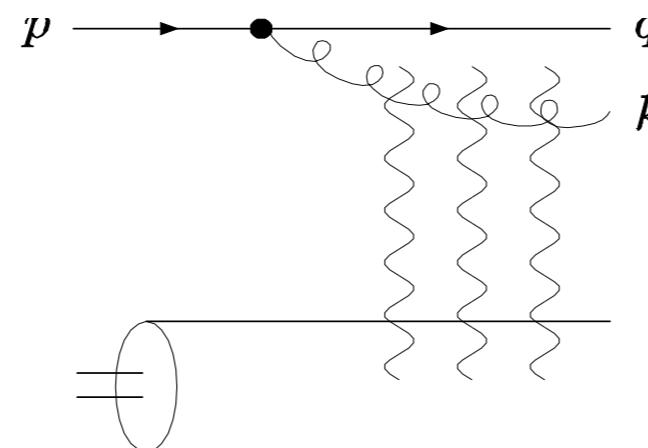
⇒ Double Inclusive forward hadron production in the CGC

$$x_p = \frac{|k_1| e^{y_1} + |k_2| e^{y_2}}{\sqrt{s}}$$

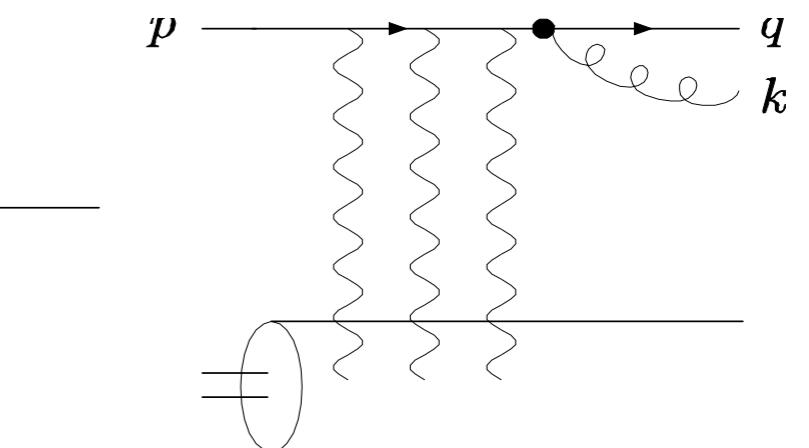
$$x_A = \frac{|k_1| e^{-y_1} + |k_2| e^{-y_2}}{\sqrt{s}}$$



Cyrille Marquet 07:



hard quark initiating scattering



Fourier transform coordinate space to momentum

$q \rightarrow qg$ splitting (pQCD)

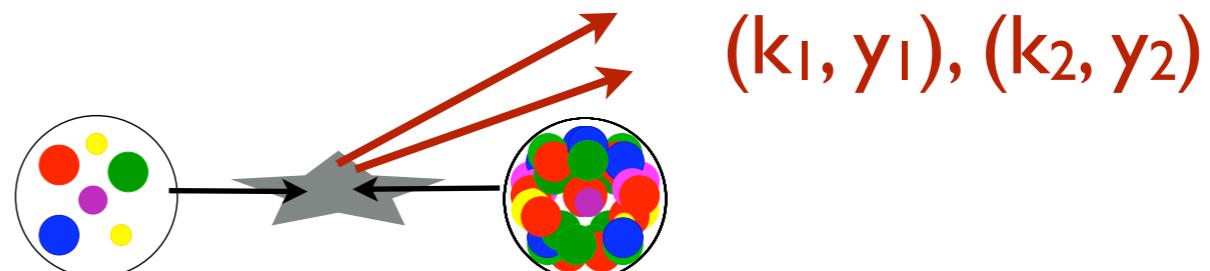
Scattering of the 2-parton system with the CGC target

Involves more than 3 and 4 point functions. Calculated in the large N_c limit

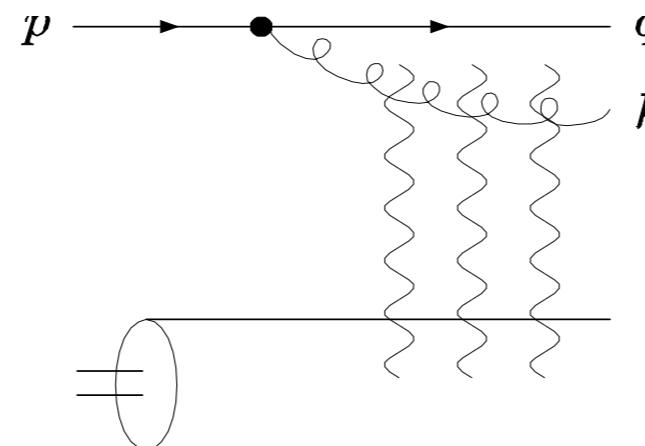
⇒ Double Inclusive forward hadron production in the CGC

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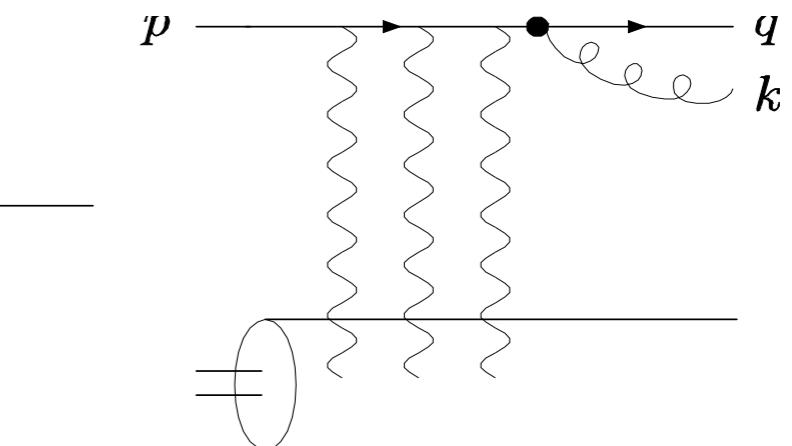
$$x_A = \frac{|k_1| e^{-y_1} + |k_2| e^{-y_2}}{\sqrt{s}}$$



Cyrille Marquet 07:



hard quark initiating scattering



Fourier transform coordinate space to momentum

$$\frac{d\sigma^{dAu \rightarrow qgX}}{d^2 k_\perp dy_k d^2 q_\perp dy_q} = \alpha_S C_F N_c x_d q(x_d, \mu^2) \int \frac{d^2 x}{(2\pi)^2} \frac{d^2 x'}{(2\pi)^2} \frac{d^2 b}{(2\pi)^2} \frac{d^2 b'}{(2\pi)^2} e^{ik_\perp \cdot (\mathbf{x}' - \mathbf{x})} e^{iq_\perp \cdot (\mathbf{b}' - \mathbf{b})}$$

$$|\Phi^{q \rightarrow qg}(z, \mathbf{x} - \mathbf{b}, \mathbf{x}' - \mathbf{b}')|^2 \left\{ S_{qg\bar{q}g}^{(4)}[\mathbf{b}, \mathbf{x}, \mathbf{b}', \mathbf{x}'; x_A] - S_{qg\bar{q}}^{(3)}[\mathbf{b}, \mathbf{x}, \mathbf{b}' + z(\mathbf{x}' - \mathbf{b}'); x_A] \right.$$

q->qg splitting (pQCD)

$$\left. - S_{\bar{q}gq}^{(3)}[\mathbf{b} + z(\mathbf{x} - \mathbf{b}), \mathbf{x}', \mathbf{b}'; x_A] + S_{q\bar{q}}^{(2)}[\mathbf{b} + z(\mathbf{x} - \mathbf{b}), \mathbf{b}' + z(\mathbf{x}' - \mathbf{b}'); x_A] \right\}$$

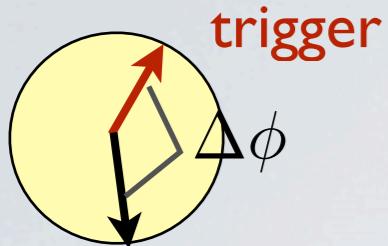
Scattering of the 2-parton system with the CGC target

$$z = \frac{|k_\perp| e^{y_k}}{|k_\perp| e^{y_k} + |q_\perp| e^{y_q}}$$

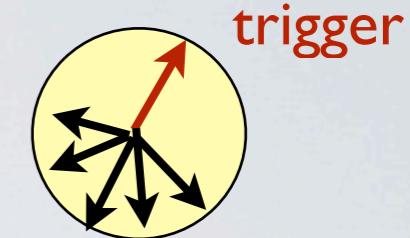
Involves more than 3 and 4 point functions. Calculated in the large N_c limit

⇒ “Monojets” in d+Au collisions at RHIC at forward rapidity

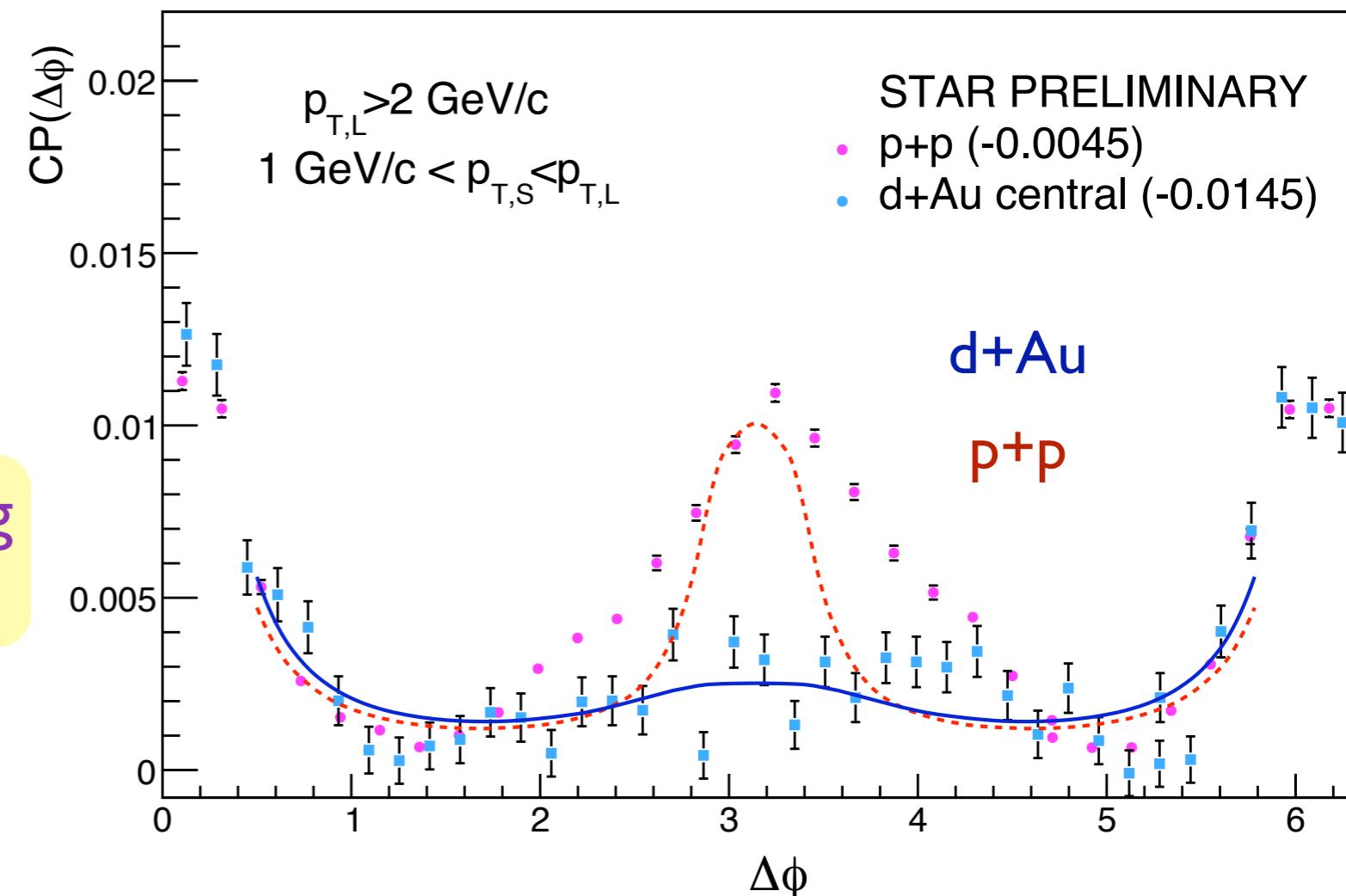
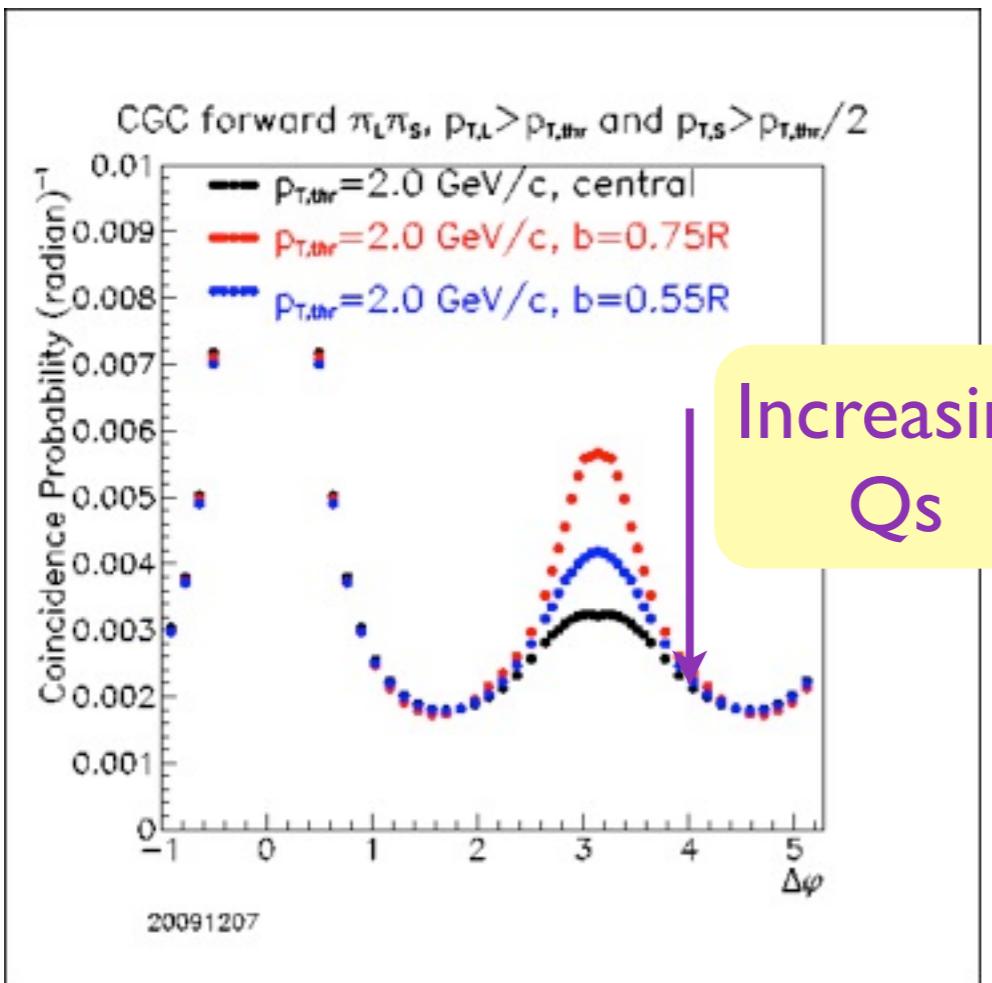
► “Coincidence probability” measured by STAR Coll. at forward rapidities:



$$CP(\Delta\phi) = \frac{1}{N_{trig}} \frac{dN_{pair}}{d\Delta\phi}$$

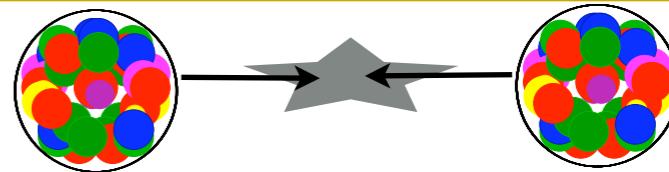


► Dependence on the saturation scale of the target (centrality)



Parameter free!!: All info about nucleus w.f. from single inclusive analysis

⇒ Multiparticle production in A+A coll.



RHIC multiplicities smaller than expected.

Most of particles produced in RHIC Au+Au collisions are small-x gluons

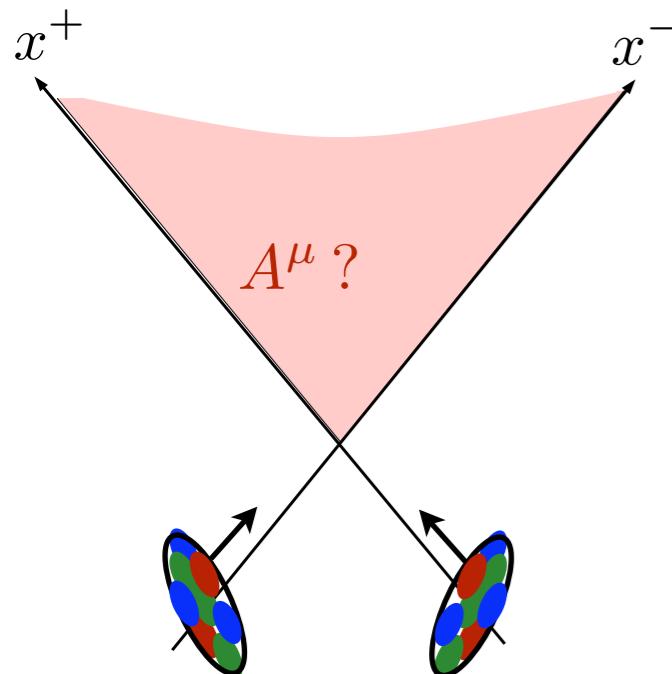
produces particles \sim # scattering centers

Two alternative approaches to describe multiparticle production within the CGC:

⇒ **k_t -factorization** (Valid in p+A coll. Violated in A+A collisions). Starting point to the Kharzeev-Levin-Nardi model

$$\frac{dN_{AB}^g}{d\eta} = \frac{4\pi N_c}{N_c^2 - 1} \int \frac{d^2 p}{p^2} \int d^2 k \alpha_s(Q^2) \varphi_A(\textcolor{red}{x}_1, k) \varphi_B(\textcolor{red}{x}_2, |p - k|)$$

⇒ **Classical Yang-Mills (CYM)** Kovner, McLerran, Weigert.



$$D_\mu F^{\mu\nu} = J^\nu \quad \text{with} \quad J^\pm \sim \rho_{A(B)}[Q_s(\textcolor{red}{x})] \delta(x^\pm)$$

More rigorous, but requires numerical implementation

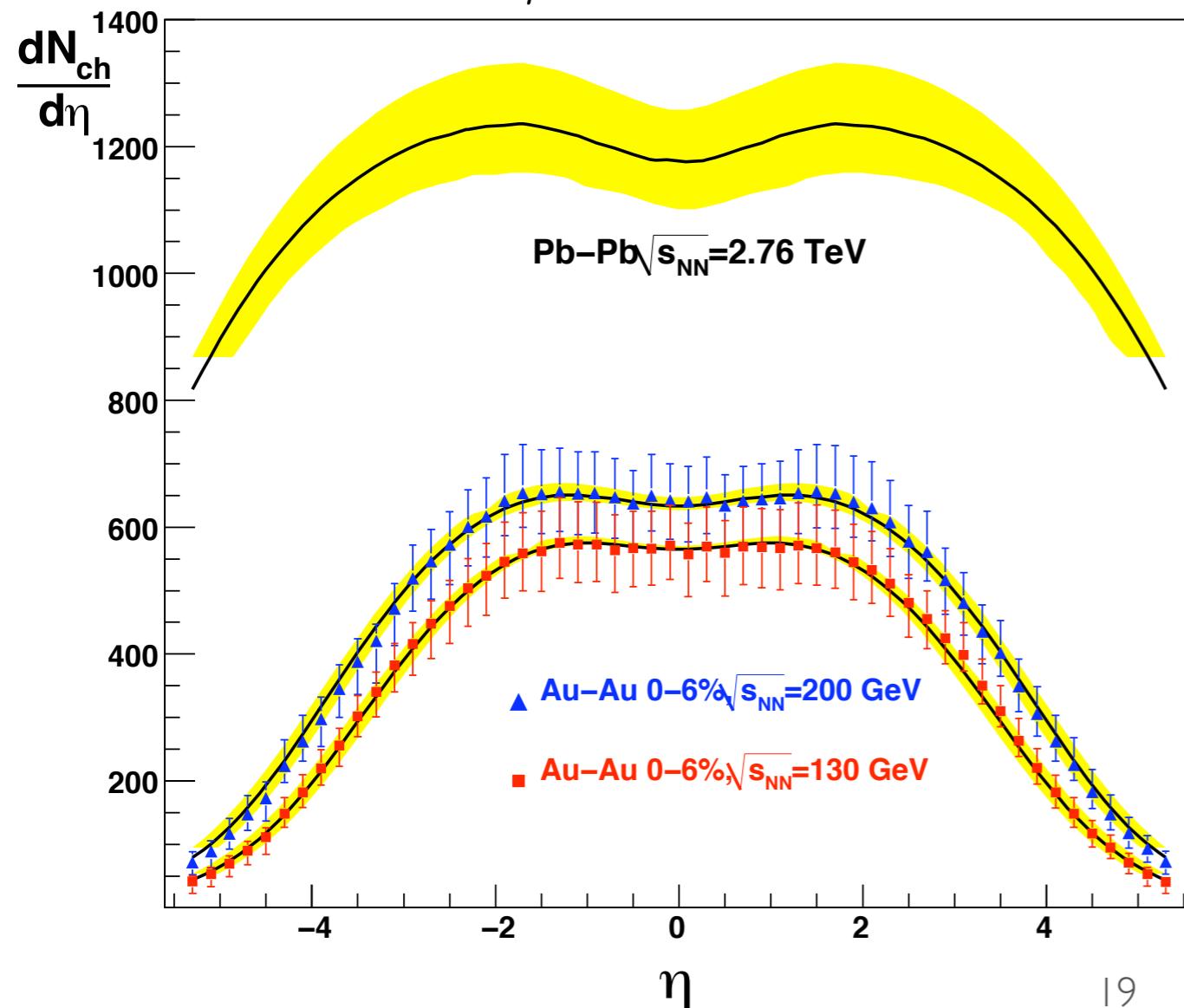
Local parton-hadron duality

$\varphi(x, k) \Rightarrow$ Solutions of BK with running coupling $\times (1 - x)^4$

JLA 07

$m_h \approx 0.25 \text{ GeV}, \quad 0.75 \leq Q_0 \leq 1.25 \text{ GeV}, \quad 0.05 \leq x_0 \leq 0.1$

$$\frac{dN_{ch}^{Pb-Pb}(\sqrt{s} = 2.75 \text{ TeV}, \eta = 0)}{d\eta} \approx 1100 \div 1250$$



kt-factorization approaches yield a good description of energy, rapidity and geometry dependence of RHIC data

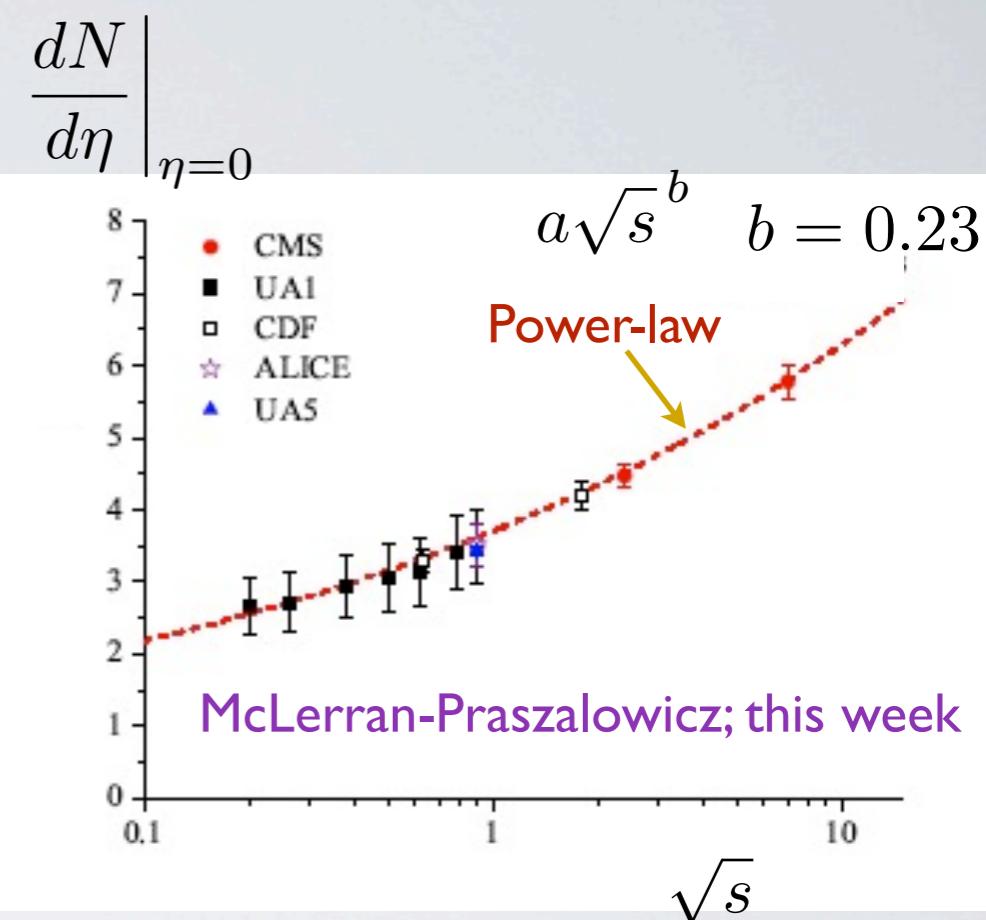
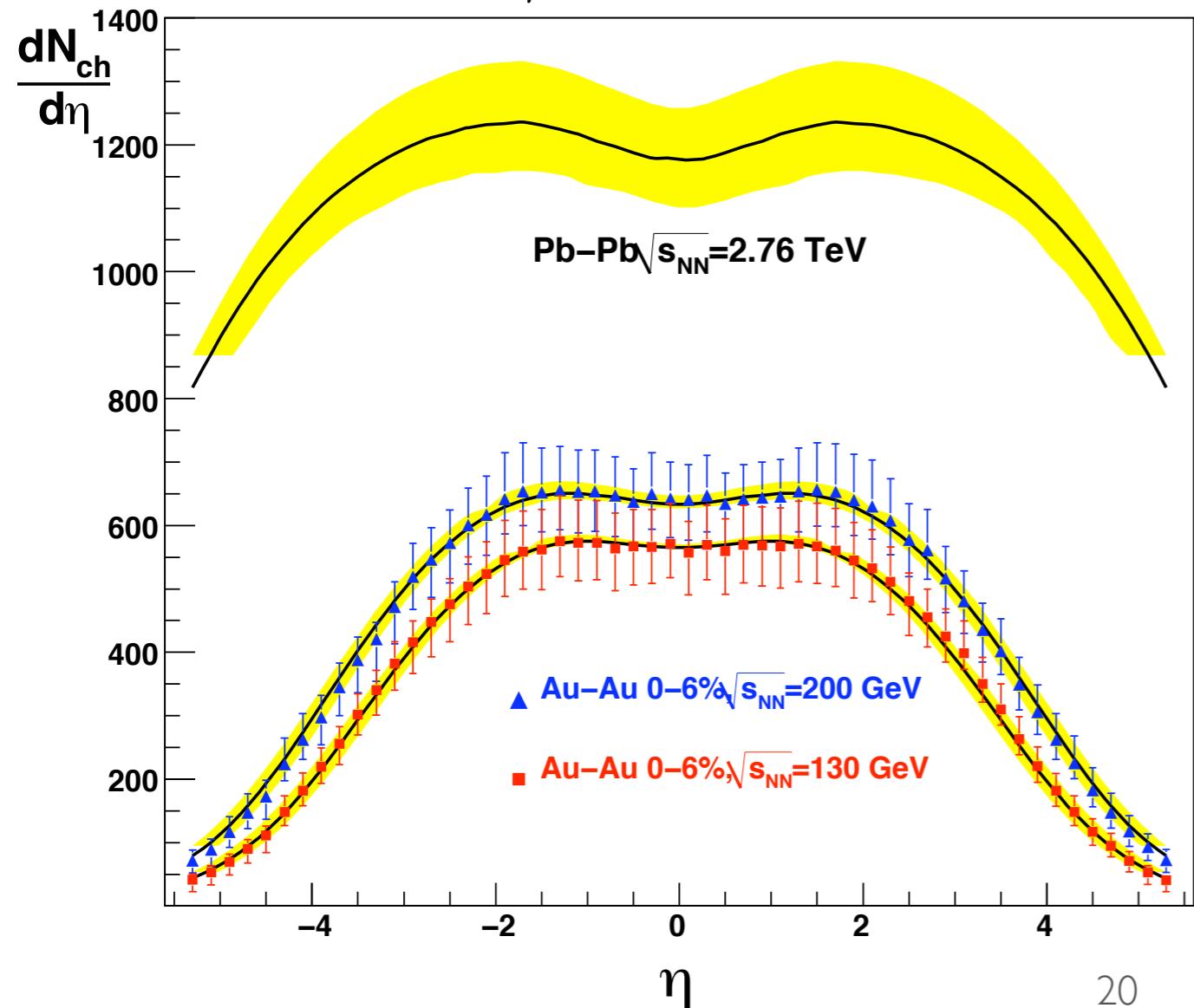
Local parton-hadron duality

$\varphi(x, k) \Rightarrow$ Solutions of BK with running coupling $\times (1 - x)$

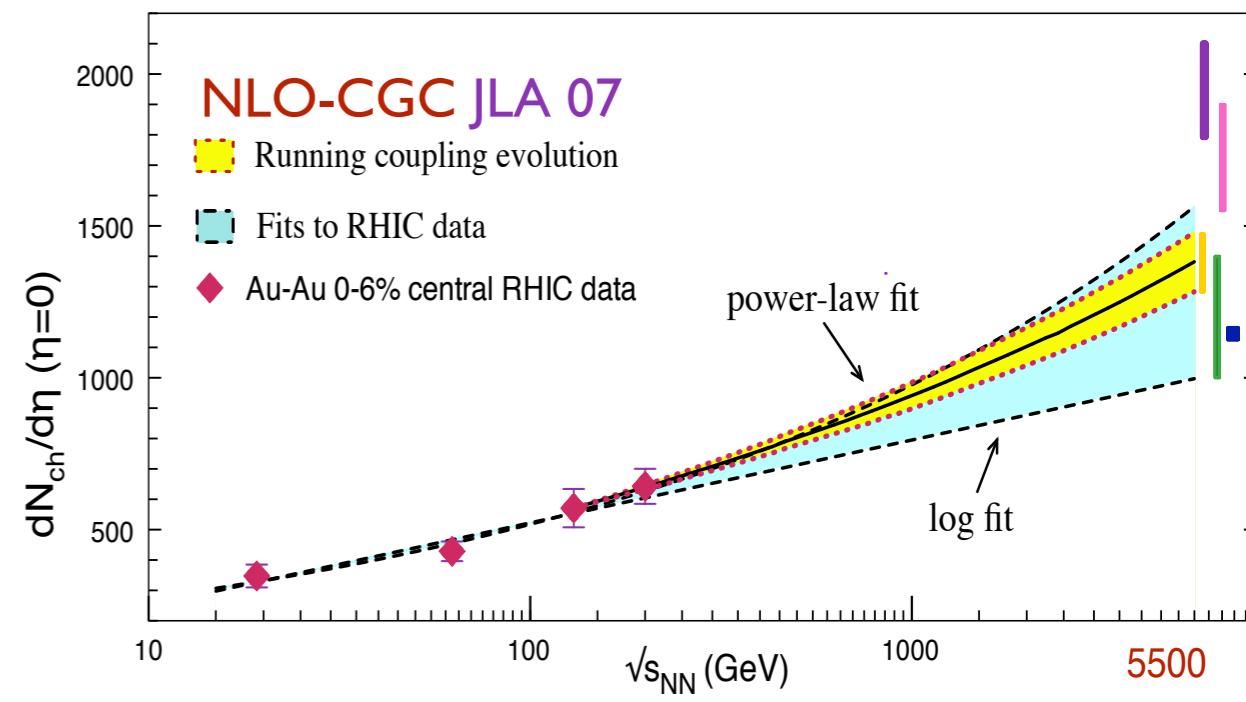
JLA 07

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$$\frac{dN_{ch}^{Pb-Pb}(\sqrt{s} = 2.75 \text{ TeV}, \eta = 0)}{d\eta} \approx 1100 \div 1250$$



CGC prediction features the same power-law dependence as p+p LHC data



Conclusions

- ⇒ NLO corrections bring the CGC to a new period of quantitative and predictive phenomenology
- ⇒ Good description of latest e+p data, including heavy flavour
- ⇒ Good description of forward particle production @ RHIC
- ⇒ Still, many things remain to be done to refine the CGC as a practical precise tool...

Thanks!!!

Back up slides