Hard diffraction

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Overview

• What is hard diffraction:
  – Diffractive DIS, hadron-hadron

• Old models:
  – pomeron model, Soft Color Interactions

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• Our approach:
  – Diffraction from rescattering of final state partons

What is hard diffraction?

In 10–20% of DIS events, the proton escapes intact, keeping a large fraction of its initial momentum — this leads to a large rapidity gap between the proton and the produced particles (the X-system)

The net $t$-channel exchange must be color singlet — a pomeron?
The pomeron formalism

Assuming DIS on a hadronic “pomeron” radiated from the proton, the **dиффрактивная структурная функция** is Regge factorized

\[
\frac{d\sigma}{dx \ dQ^2 \ dx_{IP} \ dt} = \frac{4\pi \alpha^2}{\beta Q^4} \left( 1 - y + \frac{y^2}{2} \right) F_2^{D(4)}
\]

\[
F_2^{D(4)}(x, Q^2, x_{IP}, t) = f(x_{IP}, t) \ F_2^{IP}(\beta, Q^2)
\]

**IP flux**  
**IP structure**

Taken from Regge theory

Fitted
The pomeron formalism

The pomeron gives a good description of HERA data
But the fit (pomeron flux and structure) fails completely when applied to Tevatron data!

- QCD factorization doesn't hold for diffractive pp!
- Regge factorization questionable

Also: this model doesn't tell us what's going on in diffraction from “real” QCD
Soft Color Interaction model (SCI)

- Phenomenological MC model by Edin, Ingelman, Rathsman
- Soft color exchanges in final state “after” hard process
- Changes color topology
Soft Color Interaction model (SCI)

SCI model has been compared to data with good agreement:

- diffractive DIS

- hard diffraction in hadron–hadron coll. at the Tevatron

The SCI model reproduces diffractive rates in both DIS and hadron-hadron!

But it is not theoretically well-founded.
Soft rescattering in QCD

• The SCI model is just a simple phenomenological ansatz, and we would like to go further and understand from QCD why it seems to work.

• Study soft gluon exchanges in the final state

• Some first steps were taken in Brodsky, RE, Hoyer, Ingelman, hep-ph/0409119

• Here: explicit model constructed with resummation of soft gluon exchanges
Diffractive DIS from rescattering

**Hard part:**
- conventional pQCD, color octet exchange

**Soft part:**
- resum soft multigluon exchange (non-perturbative), color screening

**Overall exchange is color singlet!**
Similar ideas

• Hautmann and Soper: soft rescattering of dipole

• Brodsky, RE, Hoyer, Ingelman

• Hebecker et al.

• Peschanski et al.
Final state rescattering

The first, hard gluon carries $x_p$, the soft gluons $x'$

- we factorize into a hard part and a soft part
- We use $k_t$-factorization at the proton

$x_p \ll 1$, $M_X \ll W$, $|t| \ll Q^2$, $M^2_X$, $x' \ll x_p$
kt-factorization

Replace coupling to quark line by PDF

Off-diagonal unintegrated PDF:

\[ f_g^{\text{off}}(x_P, x', \Delta^2_\perp, \Delta'^2_\perp, \mu_F^2) \]

\[ \mathcal{F}_g^{\text{off}} \approx \sqrt{\mathcal{F}_g(x_P, \Delta^2_\perp, \mu_F^2) \mathcal{F}_g(x', \Delta'^2_\perp, \mu_{\text{soft}}^2)} \]

\[ \frac{f_g(x, \Delta^2_\perp)}{\Delta^2_\perp} = \mathcal{F}(x, \Delta^2_\perp) \to \text{const}, \quad \Delta^2_\perp \to 0 \]

Gaussian ansatz for \( k_t \)-dependence:

\[ \sqrt{x_P} \mathcal{F}_g^{\text{off}} \approx \sqrt{x_P g(x_P, \mu_F^2) x' g(x', \mu_{\text{soft}}^2)} f_G(\Delta^2_\perp), \]

\[ f_G(\Delta^2_\perp) = \frac{1}{(2\pi \rho_0^2)} \exp \left( -\frac{\Delta^2_\perp}{2\rho_0^2} \right), \]
Hard-soft factorization

\[ M(\delta) \sim \int d^2 b e^{-i\delta b} \hat{M}^{\text{hard}}(b) \cdot \hat{M}^{\text{soft}}(b) \]

\[ \delta \equiv \sqrt{-t} = |\Delta_\perp + \Delta'_\perp| \]

\[ M^{\text{hard}}_{L,T}(\Delta_\perp, k'_\perp) = \]

\[ M^{\text{soft}}(\Delta'_\perp, k_\perp) = \]

DIS amplitude
The soft amplitude

Large-N_c limit: only consider planar diagrams

\[
e^{-i\mathbf{k}'_\perp} M^\text{soft}_1 = A e^{-i\mathbf{k}'_\perp} \frac{1}{\Delta'_{\perp}} \left[e^{-i\mathbf{r}\Delta'_{\perp}} - 1\right],
\]

\[
e^{-i\mathbf{k}'_\perp} M^\text{soft}_2 = \frac{A^2}{2!} e^{-i\mathbf{k}'_\perp} \times \int \frac{d^2\Delta'_{2\perp}}{(2\pi)^2} \frac{1}{\Delta'_{\perp} \Delta'_{2\perp}^2} \left[e^{-i\mathbf{r}\Delta'_{\perp}} - e^{-i\mathbf{r}\Delta'_{2\perp}} - e^{-i\mathbf{r}\Delta'_{1\perp}} + 1\right]
\]

Fourier transform to \((b, r)\):

\[
e^{-i\mathbf{k}'_\perp} \hat{M}^\text{soft}_1 = e^{-i\mathbf{k}'_\perp} A \cdot \mathcal{W}(b, r),
\]

\[
e^{-i\mathbf{k}'_\perp} \hat{M}^\text{soft}_2 = e^{-i\mathbf{k}'_\perp} \frac{A^2 \cdot \mathcal{W}(b, r)^2}{2!}
\]

The series exponentiates:  

\[
e^{-i\mathbf{k}'_\perp} \hat{M}^\text{soft}(b, r) = -e^{-i\mathbf{k}'_\perp} \left(1 - e^{A \mathcal{W}(b, r)}\right)
\]

where

\[
A = i g_s^2 C_F / 2 \quad \mathcal{W}(b, r) = \frac{1}{2\pi} \ln \frac{|b - r|}{|b|}
\]

[inspired by Brodsky et al., PRD 65, 114025 (2002)]
Final result

qq dipole:

\[ x_P F_L^{D(4)} = S Q^4 M_X^2 \int_{z_{\text{min}}}^{1/2} d z (1 - 2z) z^2 (1 - z)^2 |J_L|^2 \]

\[ x_P F_T^{D(4)} = 2S Q^4 \int_{z_{\text{min}}}^{1/2} d z (1 - 2z) \left\{ (1 - z)^2 + z^2 \right\} |J_T|^2, \]

where

\[ J_L = i \alpha_s (\mu_F^2) \int d^2 r d^2 b \; e^{-i \delta b} e^{-i r_{\perp}} K_0 (\varepsilon r) \]
\[ \times \mathcal{V}(b, r) \left[ 1 - e^{A W} \right], \]
\[ J_T = i \alpha_s (\mu_F^2) \int d^2 r d^2 b \; e^{-i \delta b} e^{-i r_{\perp}} \varepsilon K_1 (\varepsilon r) \]
\[ \times \frac{r_x \pm i r_y}{r} \mathcal{V}(b, r) \left[ 1 - e^{A W} \right]. \]

We also include qqg, see papers for details
Comparison with $F_2^D$ — high Q2

<table>
<thead>
<tr>
<th>$Q^2 [\text{GeV}^2]$</th>
<th>$M_X = 20 \text{ GeV}$</th>
<th>$M_X = 13 \text{ GeV}$</th>
<th>8 GeV</th>
<th>5 GeV</th>
<th>3 GeV</th>
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<tr>
<td>16</td>
<td>$\beta = 0.038$</td>
<td>$\beta = 0.068$</td>
<td>$\beta = 0.200$</td>
<td>$\beta = 0.390$</td>
<td>$\beta = 0.640$</td>
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<td>22</td>
<td>$\beta = 0.052$</td>
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<td>$\beta = 0.151$</td>
<td>$\beta = 0.319$</td>
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<td>$\beta = 0.769$</td>
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<tr>
<td>40</td>
<td>$\beta = 0.091$</td>
<td>$\beta = 0.191$</td>
<td>$\beta = 0.365$</td>
<td>$\beta = 0.615$</td>
<td>$\beta = 0.816$</td>
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<td>50</td>
<td>$\beta = 0.111$</td>
<td>$\beta = 0.228$</td>
<td>$\beta = 0.439$</td>
<td>$\beta = 0.667$</td>
<td>$\beta = 0.847$</td>
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<tr>
<td>65</td>
<td>$\beta = 0.140$</td>
<td>$\beta = 0.278$</td>
<td>$\beta = 0.504$</td>
<td>$\beta = 0.722$</td>
<td>$\beta = 0.878$</td>
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<tr>
<td>85</td>
<td>$\beta = 0.175$</td>
<td>$\beta = 0.335$</td>
<td>$\beta = 0.570$</td>
<td>$\beta = 0.773$</td>
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<td>110</td>
<td>$\beta = 0.216$</td>
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<td>140</td>
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<td>255</td>
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<td>$\beta = 0.799$</td>
<td>$\beta = 0.911$</td>
<td>$\beta = 0.966$</td>
</tr>
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</table>

R. Enberg: Hard Diffraction
Comparison with $F_2^D$ — lower $Q^2$

Here the curves are for different parametrizations of $g(x,Q^2)$

Uncertainty from PDFs at low $Q^2$
PDF uncertainties

There are large differences in parametrizations for small $x$ and $Q^2$
Coupling of soft gluons

• Soft gluon coupling to quarks in dipole:
  \[ \alpha_s(\mu^2_{\text{soft}}) = 0.7 \]
  from Analytic Perturbation Theory

• Soft gluon coupling to proton remnant through the off-diagonal gluon distribution:

  \[ \sqrt{x_P F_{g}^{\text{off}}} \simeq \sqrt{x_P g(x_P, \mu_F^2) x' g(x', \mu_{\text{soft}}^2)} f_G(\Delta_{\perp}^2), \]

  Thus we define a “soft gluon PDF”:

  \[ \sqrt{x_P F_{g}^{\text{off}}} \simeq R_g(x', \mu_{\text{soft}}^2) \sqrt{x_P g(x_P, \mu_F^2)} f_G(\Delta_{\perp}^2) \]

  \( R_g \) is remarkably constant \( \sim 1 \) almost everywhere
Coupling of soft gluons

Thus we define a “soft gluon PDF”:

$$\sqrt{x_P F_g^{\text{off}}} \simeq \bar{R}_g(x', \mu_{\text{soft}}^2) \sqrt{x_P g(x_P, \mu_F^2)} f_G(\Delta_1^2)$$

\( \bar{R}_g \) is remarkably constant \( \sim 1 \) almost everywhere
Physical parameters

• IR regulator: gluon mass = $\Lambda_{\text{QCD}}$ (only in “NLO” correction)

• Effective “constituent” quark mass in dipole $m^\text{eff}$
Hadron-hadron

- We would like to test this model for different processes (diffractive and non-diffractive)
- In particular in hadron-hadron collisions—not completely straightforward
- We are making a Monte Carlo implementation
- Contributes to underlying event! (cf. Rick Field's talk, Hannes Jung's talk)

In particular, this is color reconnection where the "probability" depends on the dynamics!
Summary

- Hard diffraction is important to understand QCD dynamics
- Rapidity gaps related to underlying event
- New model for soft rescattering → diffraction
- Could be important in hadron-hadron collisions