Bose-Einstein Results from L3 and the Tau Model

W.J. Metzger

Radboud University Nijmegen

with T. Novák, T. Csörgő, W. Kittel

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BEC Introduction

$$R_2 = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)} = \frac{\rho_2(Q)}{\rho_0(Q)}$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$

$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a Gaussian with radius $r \Longrightarrow$

$$R_2(Q) = 1 + \lambda\, \mathrm{e}^{-Q^2 r^2}$$

intro

The L3 Data

- $e^+e^- \longrightarrow \mathrm{hadrons}$ at $\sqrt{s} \approx M_Z$
- about $36 \cdot 10^6$ like-sign pairs of well measured charged tracks from about $0.8 \cdot 10^6$ events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{\mathrm{cut}} = 0.006$
- about $0.3 \cdot 10^6 > 2$ jets, “3-jet events”
- use mixed events for reference sample, ρ_0

Previous Results: Elongation

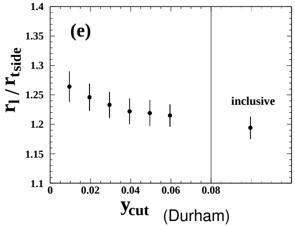
Results in LCMS frame: Longitudinal = Thrust axis ▸ LCMS

	$R_{\mathrm{L}}/R_{\mathrm{side}}$
L3	$1.25 \pm 0.03^{+0.36}_{-0.05}$
OPAL	$1.19 \pm 0.03^{+0.08}_{-0.01}$

(ZEUS finds similar results in ep)
∼25% elongation along thrust axis

OPAL:

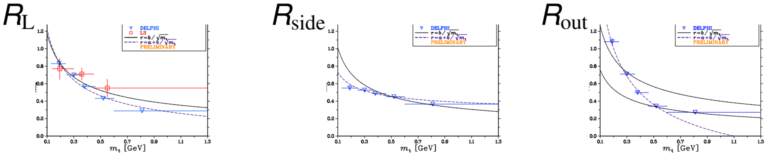
Elongation larger for narrower jets



(Durham)

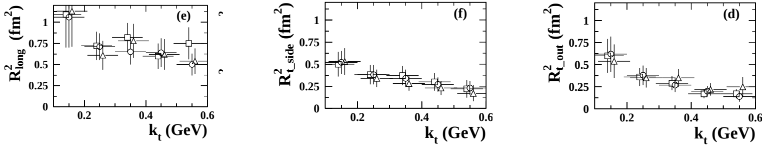
- Conclusion: Elongation, but it is relatively small.
- So: Ignore it. — Assume spherical.

Transverse Mass dependence of *r*



Smirnova&Lörstad,7th Int.Workshop on Correlations and Fluctuations (1996)

Van Dalen,8th Int.Workshop on Correlations and Fluctuations (1998)



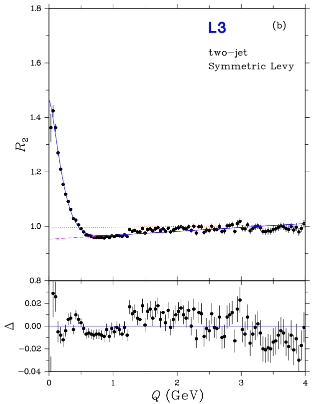
OPAL, Eur. Phys. J **C52** (2007) 787

r decreases with m_t (or k_t) for all directions

Results on Q from L3 Z decay

$$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$$

- Gaussian
  $G = \exp\left(-(rQ)^2\right)$
- Edgeworth expansion
  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$
  Gaussian if $\kappa = 0$
  $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $G = \exp\left(-|rQ|^{\alpha}\right)$
  $0 < \alpha \le 2$
  $\alpha = 1.34 \pm 0.04$



	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

Poor χ^2. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$

Summary

- BEC depend (approximately) only on Q, not its components.
- BEC depend on m_{t}.
- Gaussian and similar parametrizations do not fit.

Turn now to a model providing such dependence.

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:
$$\overline{x}^\mu(p^\mu) = a\tau p^\mu$$
where for 2-jet events, $a = 1/m_t$
$$\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2} \text{ is the “longitudinal” proper time}$$
$$\text{and } m_t = \sqrt{E^2 - p_z^2} \text{ is the “transverse” mass}$$
- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is
$$S(x,p) = \int_0^\infty \mathrm{d}\tau H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p)$$
- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.
$$\rho_2(p_1,p_2) = \int \mathrm{d}^4x_1 \mathrm{d}^4x_2 S(x_1,p_1)S(x_2,p_2)\left(1 + \cos\left([p_1 - p_2][x_1 - x_2]\right)\right)$$
- Assume $\delta_\Delta(x - a\tau p)$ is very narrow — a δ-function. Then
$$R_2(p_1,p_2) = 1 + \lambda \,\mathrm{Re}\widetilde{H}\left(\frac{a_1 Q^2}{2}\right)\widetilde{H}\left(\frac{a_2 Q^2}{2}\right), \qquad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction, $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\tfrac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\tfrac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right] \quad , \qquad \alpha \neq 1$$
  where
  - α is the index of stability
  - τ_0 is the proper time of the onset of particle production
  - $\Delta\tau$ is a measure of the width of the dist.
- Then, R_2 depends on Q, a_1, a_2

$$R_2(Q, a_1, a_2) = \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1+a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha}+a_2^{\alpha}}{2}\right]\right\} \cdot (1+\epsilon Q)$$

BEC in the τ-model

$$R_2(Q,a_1,a_2) = \gamma\left\{1+\lambda\cos\left[\frac{\tau_0 Q^2(a_1+a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^\alpha+a_2^\alpha}{2}\right]\right.$$
$$\left.\cdot\exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^\alpha+a_2^\alpha}{2}\right]\right\}\cdot(1+\epsilon Q)$$

Simplification:

- Particle production begins immediately, $\tau_0 = 0$
- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha}\frac{a_1^\alpha+a_2^\alpha}{2}$
- Then

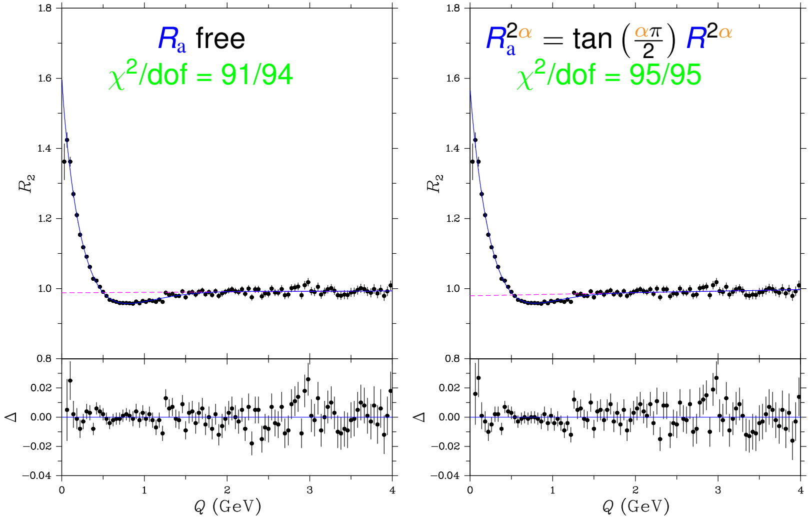
$$R_2(Q) = \gamma\left[1+\lambda\cos\left((R_a Q)^{2\alpha}\right)\exp\left(-(RQ)^{2\alpha}\right)\right]\cdot(1+\epsilon Q)$$

where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right)R^{2\alpha}$

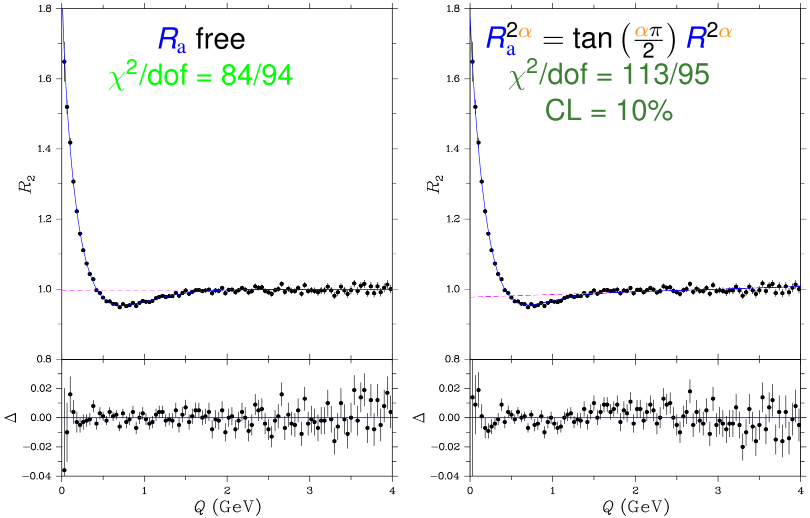
Compare to sym. Lévy parametrization:

$$R_2(Q) = \gamma\left[1+\lambda\qquad\qquad\exp\left[-|rQ|^{\alpha}\right]\right](1+\epsilon Q)$$

2-jet Results on Simplified τ-model from L3 $\mathbf{Z}$ decay



3-jet Results on Simplified τ-model from L3 Z decay



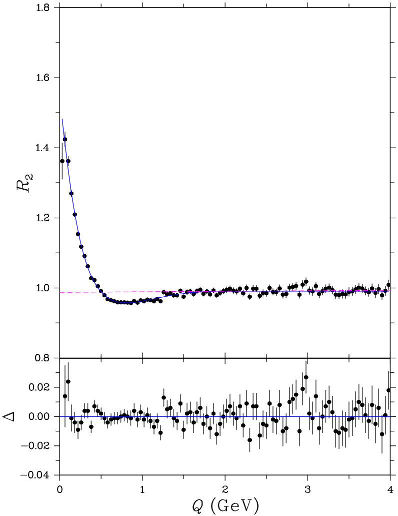
Summary of Simplified τ-model

	α	R (fm)	R_a (fm)	CL
2-jet	$0.41 \pm 0.02^{+0.04}_{-0.06}$	$0.79 \pm 0.04^{+0.09}_{-0.19}$	$0.69 \pm 0.04^{+0.21}_{-0.09}$	57%
3-jet	$0.35 \pm 0.01^{+0.03}_{-0.04}$	$1.06 \pm 0.05^{+0.59}_{-0.31}$	$0.85 \pm 0.04^{+0.15}_{-0.05}$	76%
3-jet	$0.41 \pm$ fixed	0.93 ± 0.03	0.76 ± 0.01	38%
2-jet	$0.44 \pm 0.01^{+0.05}_{-0.02}$	$0.78 \pm 0.04^{+0.09}_{-0.16}$	—	49%
3-jet	$0.42 \pm 0.01^{+0.02}_{-0.04}$	$0.98 \pm 0.04^{+0.55}_{-0.14}$	—	10%
3-jet	$0.44 \pm$ fixed	0.87 ± 0.01	—	3%

- consistent values of α
- $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$ to 0.5σ for 2-jet and to 1.5σ for 3-jet
- Simplified τ-model works well fitR fitRR
- *R* seems to be larger for 3-jet than for 2-jet events

Full τ-model for 2-jet events — $a = 1/m_t$

$$R_2(Q, m_{t1}, m_{t2}) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (m_{t1}+m_{t2})}{2(m_{t1} m_{t2})} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha}+m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}} \right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha}+m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}} \right] \right\} \cdot (1+\epsilon Q)$$

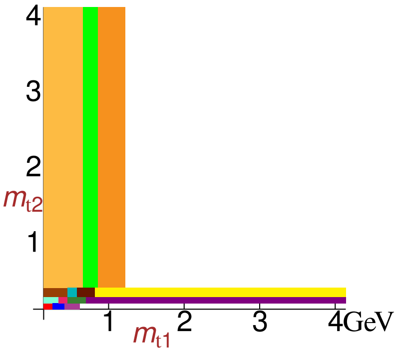


- Fit $R_2(Q)$ using avg m_{t1}, m_{t2} in each Q bin, $m_{t1} > m_{t2}$
- $\tau_0 = 0.00 \pm 0.02$ so fix to 0
- χ^2/dof = 90/95

fit

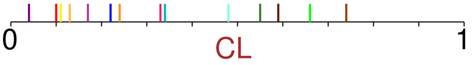
Full τ-model for 2-jet events

- τ-model predicts dependence on m_t, $R_2(Q, m_{t1}, m_{t2})$
- Parameters α, $\Delta\tau$, τ_0 are independent of m_t
- λ (strength of BEC) can depend on m_t



- divide m_{t1}-m_{t2} plane in regions (equal statistics)
- in each region fit $R_2(Q)$ using avg m_{t1}, m_{t2} in each Q bin with α, $\Delta\tau$, fixed to values found for entire plane and $\tau_0 = 0$

fits



Summary of τ-model

- τ-model with a one-sided Lévy proper-time distribution describes BEC well
  - in simplified form it provides a new parametrization of $R_2(Q)$ for both 2- and 3-jet events,
  - in full form for 2-jet events, $R_2(Q, m_{t1}, m_{t2})$
    - both Q- and m_t-dependence described correctly
    - Note: we found $\Delta\tau$ to be independent of m_t
      $\Delta\tau$ enters R_2 as $\Delta\tau Q^2/m_t$
      In Gaussian parametrization, r enters R_2 as $r^2 Q^2$
      Thus $\Delta\tau$ independent of m_t corresponds to $r \propto 1/\sqrt{m_t}$
- BUT, what about elongation?

Elongation?

- Previous elongation results used fits of Gaussian or Edgeworth
- But we find that Gaussian and Edgeworth fit $R_2(Q)$ poorly
- τ-model predicts no elongation and fits the data well
- Could the elongation results be an artifact of an incorrect fit function?
  or is the τ-model in need of modification?
- So, we modify *ad hoc* the τ-model description to allow elongation and see what we get

Elongation in the Simplified τ-model?

LCMS: $$Q^2 = Q_\mathrm{L}^2 + Q_\mathrm{side}^2 + Q_\mathrm{out}^2 - (\Delta E)^2$$
$$= Q_\mathrm{L}^2 + Q_\mathrm{side}^2 + Q_\mathrm{out}^2\left(1-\beta^2\right)\ , \qquad \beta = \frac{p_{1\mathrm{out}}+p_{2\mathrm{out}}}{E_1+E_2}$$

Replace $$R^2 Q^2 \Longrightarrow A^2 = R_\mathrm{L}^2 Q_\mathrm{L}^2 + R_\mathrm{side}^2 Q_\mathrm{side}^2 + R_\mathrm{out}^2 Q_\mathrm{out}^2$$

Then in τ-model,

$$R_2(Q_\mathrm{L}, Q_\mathrm{side}, Q_\mathrm{out}) = \gamma\left[1+\lambda\cos\left(\tan\left(\frac{\alpha\pi}{2}\right)A^{2\alpha}\right)\exp\left(-A^{2\alpha}\right)\right]$$
$$\cdot\left(1+\epsilon_\mathrm{L} Q_\mathrm{L} + \epsilon_\mathrm{side} Q_\mathrm{side} + \epsilon_\mathrm{out} Q_\mathrm{out}\right)$$

for 2-jet events:

	$R_\mathrm{side}/R_\mathrm{L}$ (fm)	χ^2/dof	CL
τ-model	0.61 ± 0.02	14847/14921	66%
Edgeworth	0.64 ± 0.02	14891/14919	56%
	consistent		

fit

Elongation is real

Direct Test of Q^2-only Dependence

1. $Q^2 = Q^2_{\mathrm{LE}} + Q^2_{\mathrm{side}} + Q^2_{\mathrm{out}}$ where $Q^2_{\mathrm{LE}} = Q^2_{\mathrm{L}} - (\Delta E)^2$ inv. boosts along thrust axis
2. $Q^2 = Q^2_{\mathrm{L}} + Q^2_{\mathrm{side}} + q^2_{\mathrm{out}}$ where $q_{\mathrm{out}} = Q_{\mathrm{out}}$ boosted (β) along out direction to rest frame of pair

In τ-model, for case 1

$$R_2(Q_{\mathrm{LE}}, Q_{\mathrm{side}}, Q_{\mathrm{out}}) = \gamma \left[1 + \lambda \cos\left(\tan\left(\frac{\alpha\pi}{2}\right) B^{2\alpha}\right) \exp\left(-B^{2\alpha}\right)\right] b$$

where $B^2 = R^2_{\mathrm{LE}} Q^2_{\mathrm{LE}} + R^2_{\mathrm{side}} Q^2_{\mathrm{side}} + R^2_{\mathrm{out}} Q^2_{\mathrm{out}}$

$$b = 1 + \epsilon_{\mathrm{LE}} Q_{\mathrm{LE}} + \epsilon_{\mathrm{side}} Q_{\mathrm{side}} + \epsilon_{\mathrm{out}} Q_{\mathrm{out}}$$

and comparable expression for case 2, $R_2(Q_{\mathrm{L}}, Q_{\mathrm{side}}, q_{\mathrm{out}})$

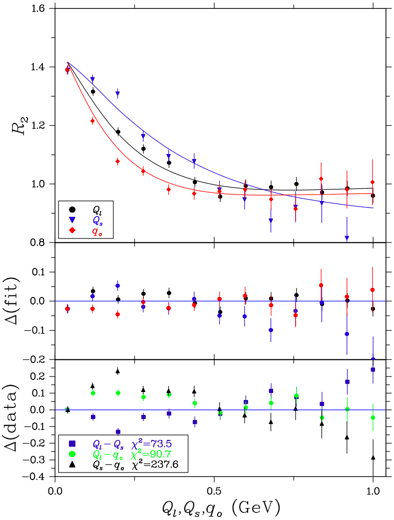
Direct Test of Q^2-only Dependence

Compare fits with all 'radii' free
to fits with all 'radii' constrained to be equal

case 1	α	0.46 ± 0.01	0.46 ± 0.01	fits1
	$R_{\rm LE}$ (fm)	0.84 ± 0.04	0.71 ± 0.04	
	$R_{\rm side}/R_{\rm LE}$	0.60 ± 0.02	1	
	$R_{\rm out}/R_{\rm LE}$	0.986 ± 0.003	1	difference
	χ^2/DoF	$14590/14538$	$14886/14540$	$\Delta\chi^2 = 296/2$
	CL	38%	2%	≈ 0
case 2	α	0.41 ± 0.01	0.44 ± 0.01	fits2
	$R_{\rm L}$ (fm)	0.96 ± 0.05	0.82 ± 0.04	
	$R_{\rm side}/R_{\rm L}$	0.62 ± 0.02	1	
	$r_{\rm out}/R_{\rm L}$	1.23 ± 0.03	1	difference
	χ^2/DoF	$10966/10647$	$11430/10649$	$\Delta\chi^2 = 464/2$
	CL	2%	10^{-7}	≈ 0

Dependence on components of Q is strongly preferred.

$\boldsymbol{Q}$ Dependence



case 2, $R_2(Q_\mathrm{L}, Q_\mathrm{side}, q_\mathrm{out})$ *vs.*

Q_L for $Q_\mathrm{side}, q_\mathrm{out} < 0.08\,\mathrm{GeV}$

Q_side for $Q_\mathrm{L}, q_\mathrm{out} < 0.08\,\mathrm{GeV}$

q_out for $Q_\mathrm{L}, Q_\mathrm{side} < 0.08\,\mathrm{GeV}$

Dependence on components of *Q* is preferred.

Summary

- R_2 depends, to some degree, separately on components of Q, *i.e.*, on $\vec{Q}$
- contradicts τ-model, where dependence is on Q, not on $\vec{Q}$
- Nevertheless, τ-model with a one-sided Lévy proper-time distribution succeeds:
  - Simplified, provides a new parametrization of $R_2(Q)$ which works well
  - $R_2(Q, m_{t1}, m_{t2})$ successfully fits R_2 for 2-jet events both Q- and m_t-dependence described correctly
- But dependence of R_2 on components of Q implies τ-model is in need of modification
  Perhaps, a should be different for transverse/longitudinal

$$\overline{x}^{\mu}(p^{\mu}) = a\tau p^{\mu}, \qquad a = 1/m_t \text{ for 2-jet}$$

Emission Function of 2-jet Events.

In the τ-model, the emission function in configuration space is

$$S(\vec{x},\tau) = \frac{1}{\bar{n}}\frac{\mathrm{d}^4 n}{\mathrm{d}\tau \mathrm{d}\vec{x}} = \frac{1}{\bar{n}}\left(\frac{m_\mathrm{t}}{\tau}\right)^3 H(\tau)\rho_1\left(\vec{p} = \frac{m_\mathrm{t}\vec{x}}{\tau}\right)$$

For simplicity, assume $\rho_1(\vec{p}) = \rho_\mathrm{y}(y)\rho_{p_\mathrm{t}}(p_\mathrm{t})/\bar{n}$

(ρ_1, ρ_y, ρ_{p_t} are inclusive single-particle distributions)

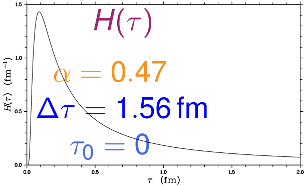
expt. – Factorization OK

Then $S(\vec{x},\tau) = \frac{1}{\bar{n}^2} H(\tau)G(\eta)I(r)$

Strongly correlated $x, p \Longrightarrow$

$$\eta = y \qquad\qquad r = p_\mathrm{t}\tau/m_\mathrm{t}$$

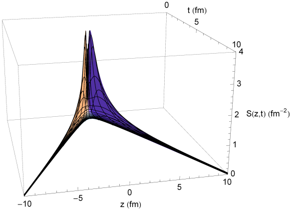
$$G(\eta) = \rho_\mathrm{y}(\eta) \qquad I(r) = \left(\frac{m_\mathrm{t}}{\tau}\right)^3 \rho_{p_\mathrm{t}}(rm_\mathrm{t}/\tau)$$



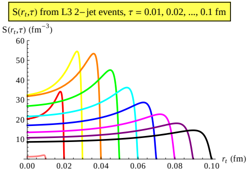
So, using experimental $\rho_\mathrm{y}(y)$, $\rho_{p_\mathrm{t}}(p_\mathrm{t})$ dists.
and $H(\tau)$ from BEC fits,
we can reconstruct S.

Emission Function of 2-jet Events.

Integrating over r,



Integrating over z,



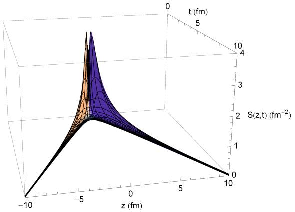
"Boomerang" shape

Particle production is close to the light-cone

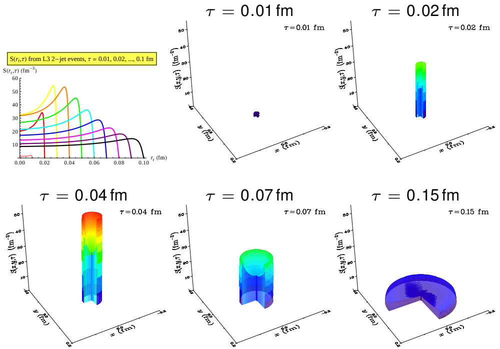
Emission Function of 2-jet Events.

Integrating over z,

Integrating over r,



“Boomerang” shape



Expanding ring

Particle production is close to the light-cone

α_s

- LLA parton shower leads to a fractal in momentum space fractal dimension is related to α_s Gustafson *et* al.
- Lévy dist. arises naturally from a fractal, or random walk, or anomalous diffusion Metzler and Klafter, Phys.Rep.**339**(2000)1.
- strong momentum-space/configuration space correlation of τ-model $\Longrightarrow$ fractal in configuration space with same α
- generalized LPHD suggests particle dist. has same properties as gluon dist.
- Putting this all together leads to Csörgő *et* al.

$$\alpha_s = \frac{2\pi}{3}\alpha^2$$

- Using our value of $\alpha = 0.47 \pm 0.04$ yields $\alpha_s = 0.46 \pm 0.04$
- This value is reasonable for a scale of $1\text{–}2\,\mathrm{GeV}$, where production of hadrons takes place
  cf., from τ decays $\alpha_s(m_\tau \approx 1.8\,\mathrm{GeV}) = 0.34 \pm 0.03$ PDG

BEC Introduction

q-particle density $\quad \rho_q(p_1,\ldots,p_q) = \frac{1}{\sigma_{\rm tot}} \frac{\mathrm{d}^q\sigma_q(p_1,\ldots,p_q)}{\mathrm{d}p_1\ldots\mathrm{d}p_q}$

2-particle correlation: $\quad \dfrac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)}$

To study only BEC, not all correlations,
let $\rho_0(p_1,p_2)$ be the 2-particle density if no BEC
($= \rho_2$ of the 'reference sample') and define

$$R_2(p_1,p_2) = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \cdot \frac{\rho_1(p_1)\rho_1(p_2)}{\rho_0(p_1,p_2)} = \frac{\rho_2(p_1,p_2)}{\rho_0(p_1,p_2)}$$

Since 2-π BEC only at small Q

$$Q = \sqrt{-(p_1-p_2)^2} = \sqrt{M_{12}^2 - 4m_\pi^2}$$

integrate over other variables: $\quad R_2(Q) = \dfrac{\rho(Q)}{\rho_0(Q)}$

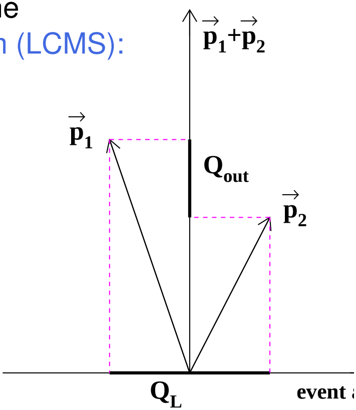
LCMS

The usual parametrization assumes a symmetric Gaussian source

But, there is no reason to expect this symmetry in $e^+e^- \to q\bar{q}$.

Therefore, do a 3-dim. analysis in the

Longitudinal Center of Mass System (LCMS):



Boost each π-pair along event axis

(thrust or sphericity) $p_{L1} = -p_{L2}$

$\vec{p}_1 + \vec{p}_2$ defines 'out' axis

$Q_{side} \perp (Q_L, Q_{out})$

advantage

LCMS

Advantages of LCMS:

$$
\begin{aligned}
Q^2 &= Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 - (\Delta E)^2 \\
&= Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 (1 - \beta^2) \qquad \text{where } \beta \equiv \frac{p_{\mathrm{out}\,1} + p_{\mathrm{out}\,2}}{E_1 + E_2}
\end{aligned}
$$

Thus, the energy difference,
and therefore the difference in emission time of the pions
couples only to the out-component, Q_{out}.
Thus,
Q_{L} and Q_{side} reflect only spatial dimensions of the source
Q_{out} reflects a mixture of spatial and temporal dimensions.

Fit Results Simplified τ-model

parameter	two-jet	three-jet
λ	$0.63 \pm 0.03^{+0.08}_{-0.35}$	$0.92 \pm 0.05^{+0.06}_{-0.48}$
α	$0.41 \pm 0.02^{+0.04}_{-0.06}$	$0.35 \pm 0.01^{+0.03}_{-0.04}$
R (fm)	$0.79 \pm 0.04^{+0.09}_{-0.19}$	$1.06 \pm 0.05^{+0.59}_{-0.31}$
R_a (fm)	$0.69 \pm 0.04^{+0.21}_{-0.09}$	$0.85 \pm 0.04^{+0.15}_{-0.05}$
ϵ (GeV^{-1})	$0.001 \pm 0.002^{+0.005}_{-0.008}$	$0.000 \pm 0.002^{+0.001}_{-0.007}$
γ	$0.988 \pm 0.005^{+0.026}_{-0.012}$	$0.997 \pm 0.005^{+0.019}_{-0.002}$
χ^2/DoF	91/94	84/94
confidence level	57%	76%

Fit Results Simplified τ-model

parameter	two-jet	three-jet
λ	$0.61 \pm 0.03^{+0.08}_{-0.26}$	$0.84 \pm 0.04^{+0.04}_{-0.37}$
α	$0.44 \pm 0.01^{+0.05}_{-0.02}$	$0.42 \pm 0.01^{+0.02}_{-0.04}$
R (fm)	$0.78 \pm 0.04^{+0.09}_{-0.16}$	$0.98 \pm 0.04^{+0.55}_{-0.14}$
ϵ (GeV^{-1})	$0.005 \pm 0.001 \pm 0.003$	$0.008 \pm 0.001 \pm 0.005$
γ	$0.979 \pm 0.002^{+0.009}_{-0.003}$	$0.977 \pm 0.001^{+0.013}_{-0.008}$
χ^2/DoF	95/95	113/95
confidence level	49%	10%

Fit Results Full τ-model for 2-jet events

m_t regions (GeV) m_{t1}	m_{t2}	average m_t (GeV) $Q < 0.4$	all	confidence level (%)	λ
0.14 – 0.26	0.14 – 0.22	0.19	0.19	10	0.39 ± 0.02
0.14 – 0.34	0.22 – 0.30	0.27	0.27	48	0.76 ± 0.03
0.14 – 0.46	0.30 – 0.42	0.37	0.37	74	0.83 ± 0.03
0.14 – 0.66	0.42 – 4.14	0.52	0.52	13	0.97 ± 0.04
0.26 – 0.42	0.14 – 0.22	0.25	0.26	22	0.53 ± 0.02
0.34 – 0.46	0.22 – 0.30	0.32	0.33	33	0.80 ± 0.03
0.46 – 0.58	0.30 – 0.42	0.43	0.44	34	0.91 ± 0.04
0.66 – 0.86	0.42 – 4.14	0.65	0.65	66	1.01 ± 0.05
0.42 – 0.62	0.14 – 0.22	0.34	0.34	17	0.41 ± 0.03
0.46 – 0.70	0.22 – 0.30	0.41	0.41	55	0.64 ± 0.03
0.58 – 0.82	0.30 – 0.42	0.52	0.52	59	0.70 ± 0.04
0.86 – 1.22	0.42 – 4.14	0.80	0.81	24	0.66 ± 0.05
0.70 – 4.14	0.22 – 0.30	0.59	0.65	4	0.37 ± 0.04
0.82 – 4.14	0.30 – 0.42	0.71	0.76	11	0.56 ± 0.05

Fit Result $R_2(Q, m_{t1}, m_{t2})$

parameter	
λ	$0.58 \pm 0.03^{+0.08}_{-0.24}$
α	$0.47 \pm 0.01^{+0.04}_{-0.02}$
$\Delta\tau$ (fm)	$1.56 \pm 0.12^{+0.32}_{-0.45}$
ϵ (GeV^{-1})	$0.001 \pm 0.001 \pm 0.003$
γ	$0.988 \pm 0.002^{+0.006}_{-0.002}$
χ^2/DoF	90/95
confidence level	62%

Fit Results elongation in τ-model for 2-jet events

λ	0.49 ± 0.02
α	0.46 ± 0.01
R_{L} (fm)	0.85 ± 0.04
$R_{\mathrm{side}}/R_{\mathrm{L}}$	0.61 ± 0.02
$R_{\mathrm{out}}/R_{\mathrm{L}}$	0.66 ± 0.02
ϵ_{L} (GeV^{-1})	0.001 ± 0.001
ϵ_{side} (GeV^{-1})	-0.076 ± 0.003
ϵ_{out} (GeV^{-1})	-0.029 ± 0.002
γ	1.011 ± 0.002
χ^2/DoF	14847/14921
CL	66%

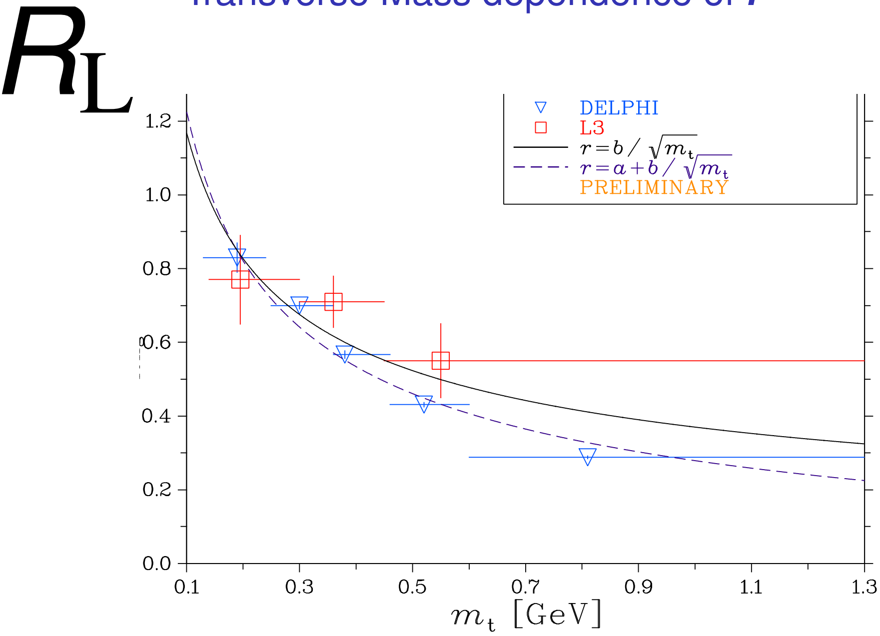
Fit Results of direct tests for 2-jet events

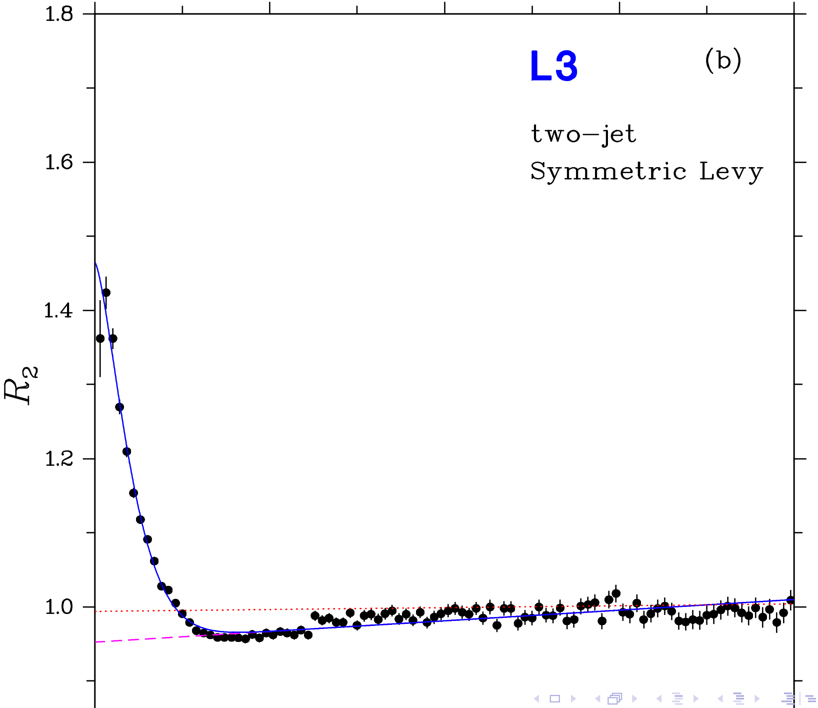
case 1	λ	0.51 ± 0.03	0.49 ± 0.03
	α	0.46 ± 0.01	0.46 ± 0.01
	R_{LE} (fm)	0.84 ± 0.04	0.71 ± 0.04
	$R_{\mathrm{side}}/R_{\mathrm{LE}}$	0.60 ± 0.02	1
	$R_{\mathrm{out}}/R_{\mathrm{LE}}$	0.986 ± 0.003	1
	ϵ_{LE} (GeV^{-1})	0.001 ± 0.001	0.000 ± 0.001
	ϵ_{side} (GeV^{-1})	-0.069 ± 0.003	-0.064 ± 0.003
	ϵ_{out} (GeV^{-1})	-0.032 ± 0.002	-0.035 ± 0.002
	γ	1.010 ± 0.002	1.012 ± 0.002
	χ^2/DoF	$14590/14538$	$14886/14540$
	CL	38%	2%

Fit Results of direct tests for 2-jet events

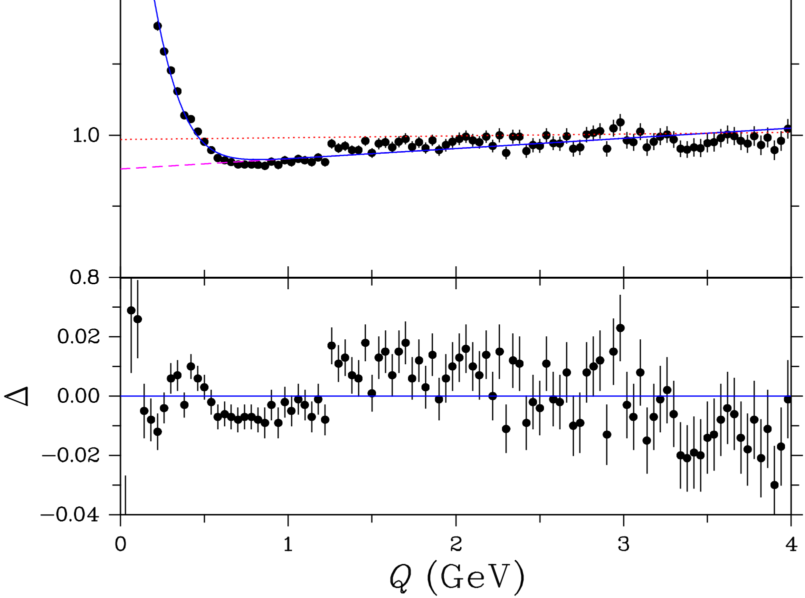
case 2	λ	0.65 ± 0.03	0.57 ± 0.03
	α	0.41 ± 0.01	0.44 ± 0.01
	R_{L} (fm)	0.96 ± 0.05	0.82 ± 0.04
	$R_{\mathrm{side}}/R_{\mathrm{L}}$	0.62 ± 0.02	1
	$r_{\mathrm{out}}/R_{\mathrm{L}}$	1.23 ± 0.03	1
	ϵ_{L} (GeV^{-1})	0.004 ± 0.001	0.003 ± 0.001
	ϵ_{side} (GeV^{-1})	-0.067 ± 0.003	-0.059 ± 0.003
	ϵ_{out} (GeV^{-1})	-0.022 ± 0.003	-0.029 ± 0.002
	γ	1.000 ± 0.002	1.003 ± 0.002
	χ^2/DoF	$10966/10647$	$11430/10649$
	CL	2%	10^{-7}

Transverse Mass dependence of *r*

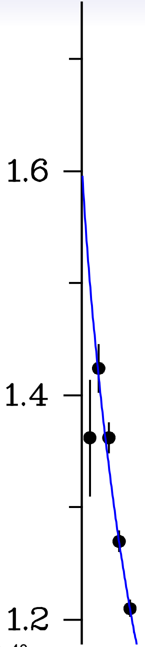




L3
(b)
two−jet
Symmetric Levy
R_2
1.8
1.6
1.4
1.2
1.0

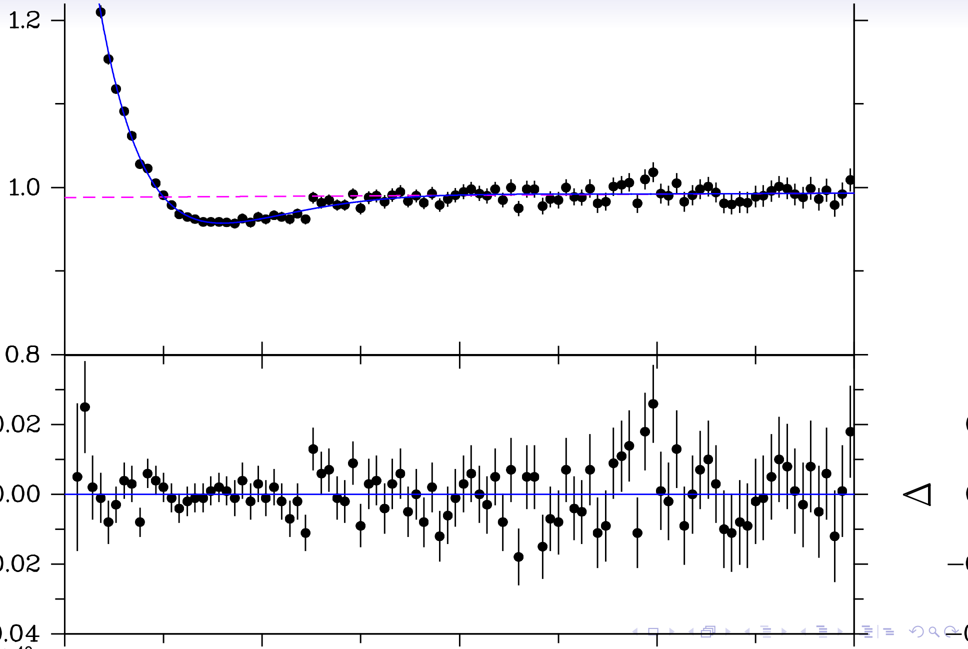


	Gauss	Edgew	Lévy
CL :	10^{-15}	10^{-5}	10^{-8}



R_a free

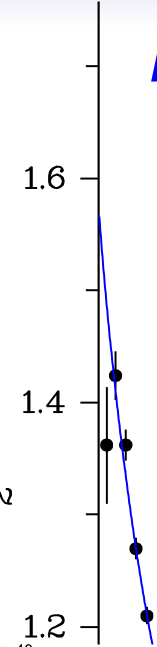
$\chi^2/\mathrm{dof} = 91/94$

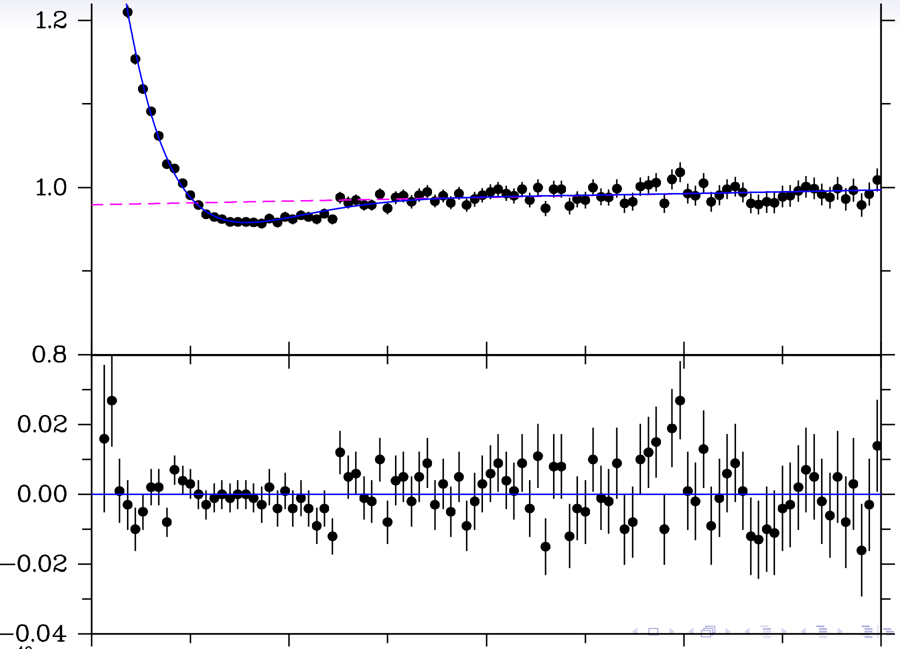


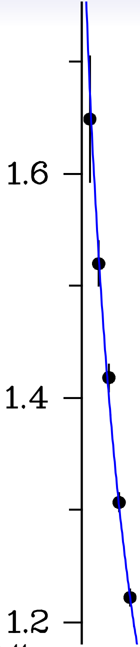
1.2
1.0
0.8
0.02
0.00
0.02
0.04
Δ

$$R_{\mathrm{a}}^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$$

$$\chi^2/\mathrm{dof} = 95/95$$

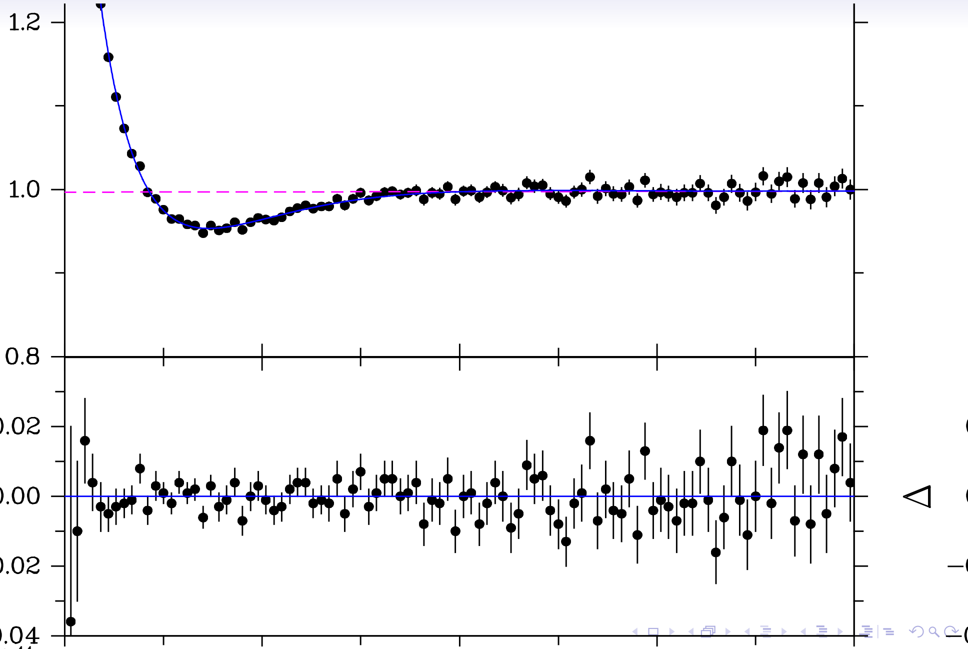


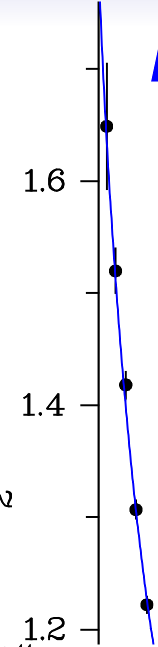

1.2
1.0
0.8
0.02
0.00
−0.02
−0.04



R_a free

χ^2/dof = 84/94

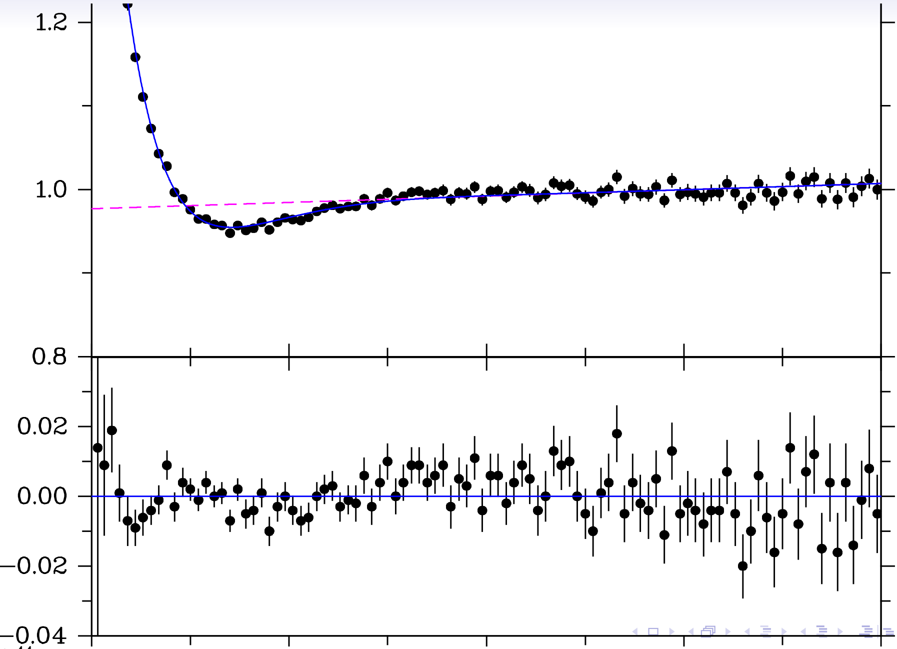

1.2
1.0
0.8
0.02
0.00
0.02
0.04
Δ

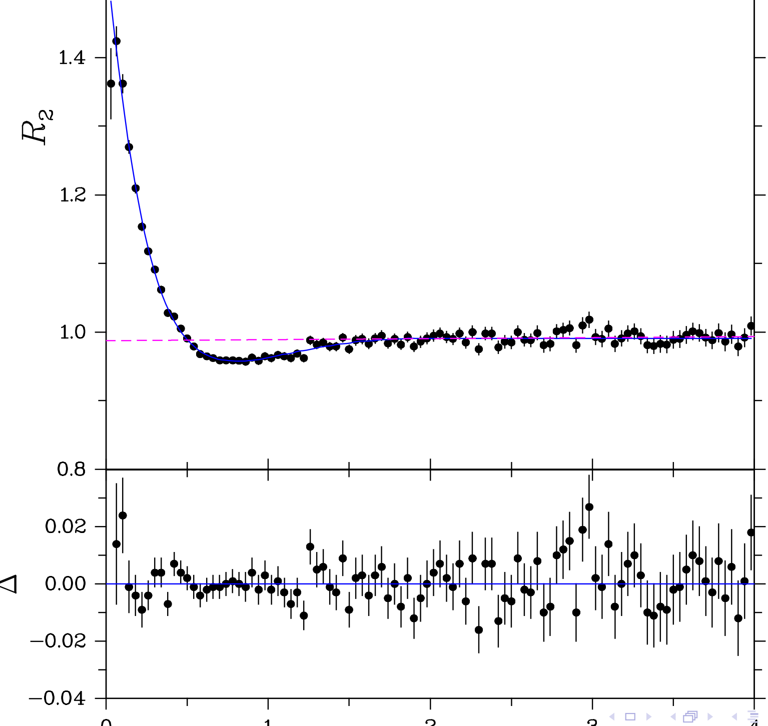


$$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$$

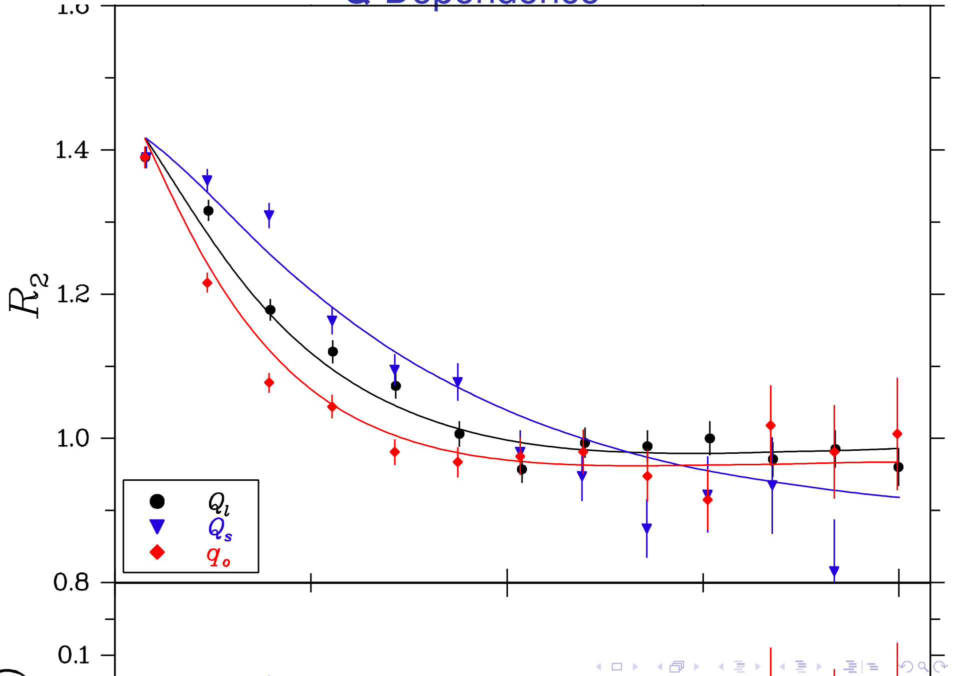
$$\chi^2/\mathrm{dof} = 113/95$$

$$\mathrm{CL} = 10\%$$

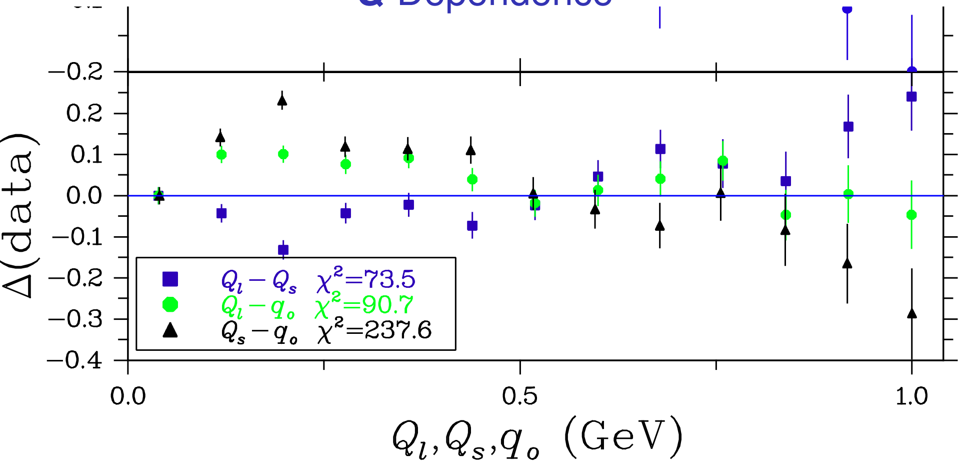

1.2
1.0
0.8
0.02
0.00
−0.02
−0.04


R_2
1.4
1.2
1.0
0.8
Δ
0.02
0.00
−0.02
−0.04
0
1
2
3
4

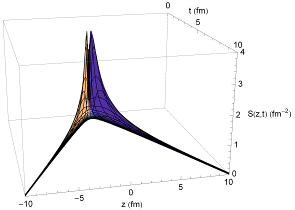
Q Dependence



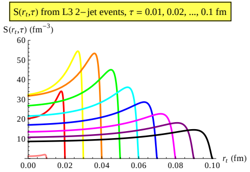
Q Dependence



Integrating over r,



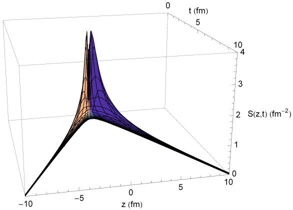
Integrating over z,



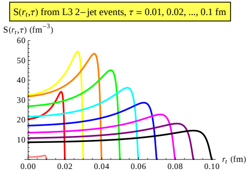
"Boomerang" shape

Particle production is close to the light-cone

Integrating over r,



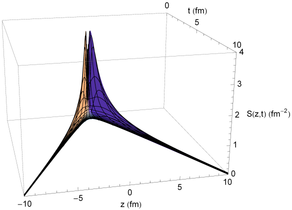
Integrating over z,



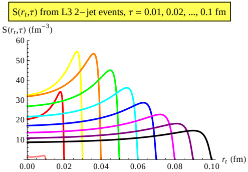
“Boomerang” shape

Particle production is close to the light-cone

Integrating over *r*,



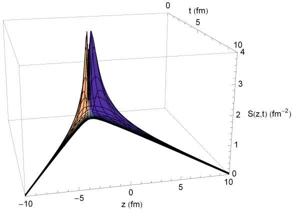
Integrating over *z*,



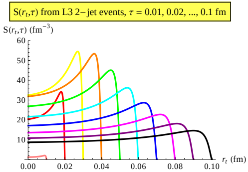
"Boomerang" shape

Particle production is close to the light-cone

Integrating over r,



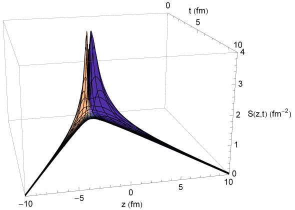
Integrating over z,



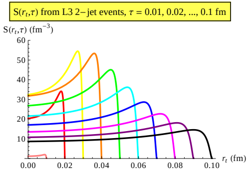
"Boomerang" shape

Particle production is close to the light-cone

Integrating over r,



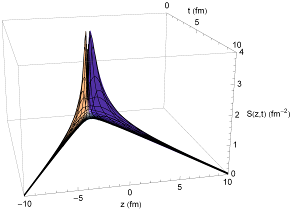
Integrating over z,



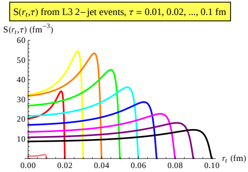
"Boomerang" shape

Particle production is close to the light-cone

Integrating over r,



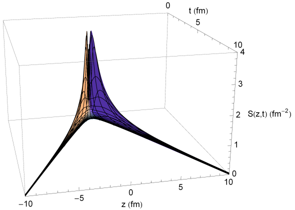
Integrating over z,



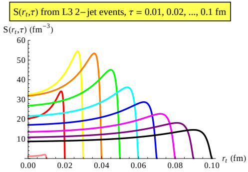
"Boomerang" shape

Particle production is close to the light-cone

Integrating over r,



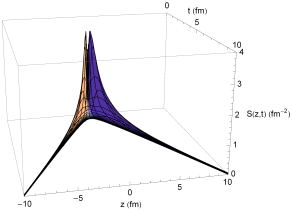
Integrating over z,



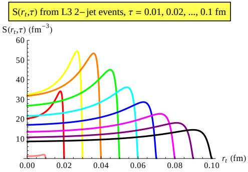
"Boomerang" shape

Particle production is close to the light-cone

Integrating over r,



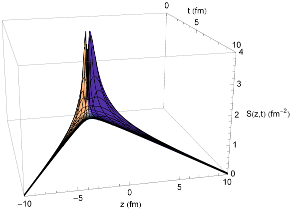
Integrating over z,



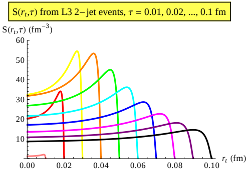
“Boomerang” shape

Particle production is close to the light-cone

Integrating over *r*,

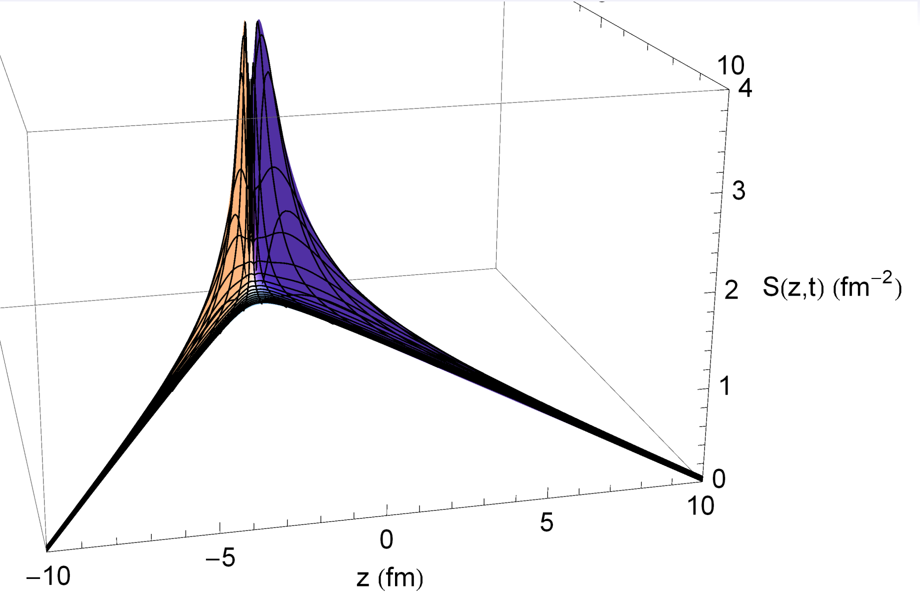


Integrating over *z*,



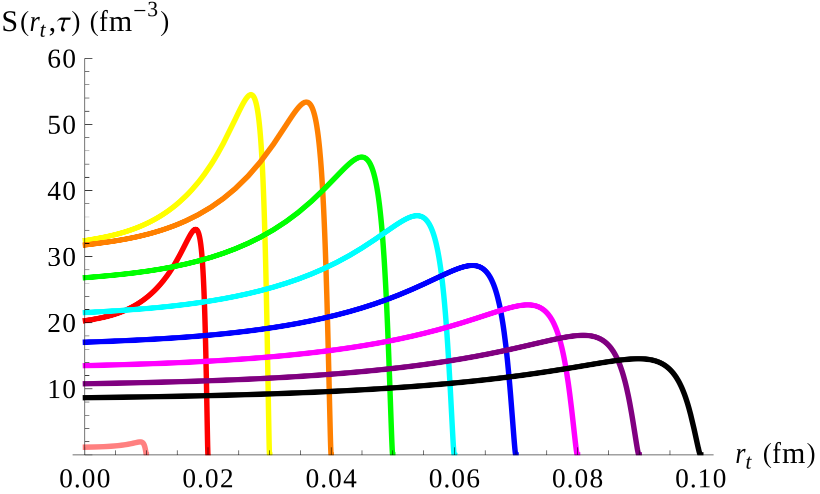
“Boomerang” shape

Particle production is close to the light-cone



10
4
3
2
1
0
S(z,t) (fm^{-2})
-10
-5
0
5
10
z (fm)

$S(r_t,\tau)$ from L3 2−jet events, τ = 0.01, 0.02, ..., 0.1 fm

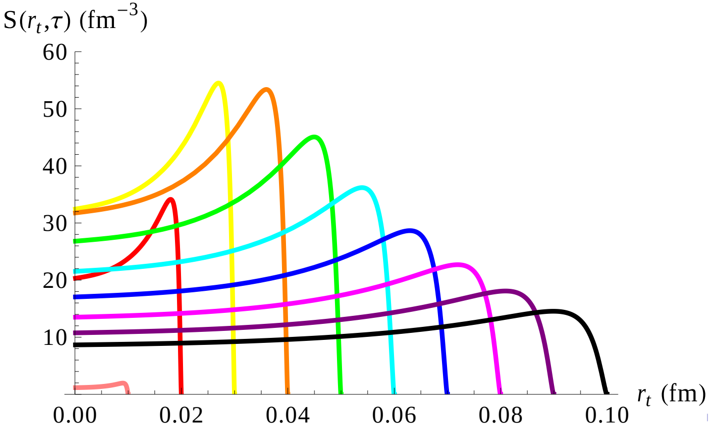


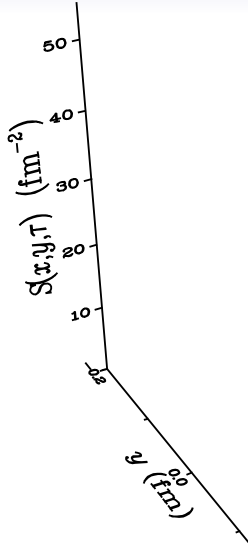
m^−2)

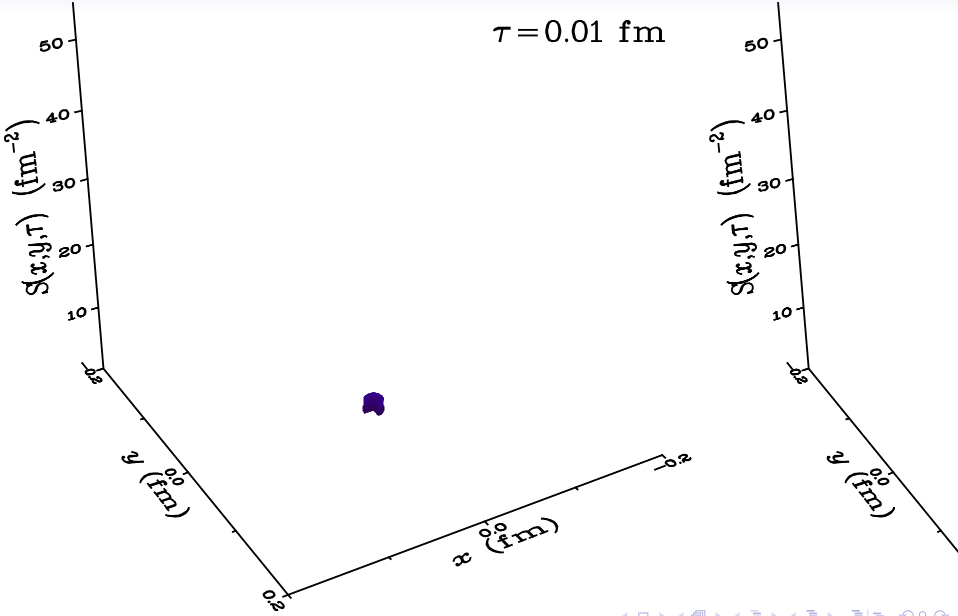
(Loading movie...)

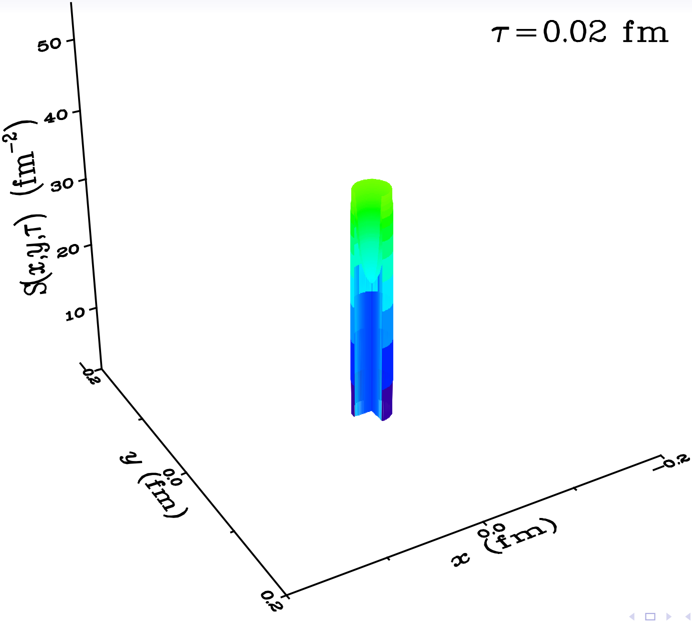
)

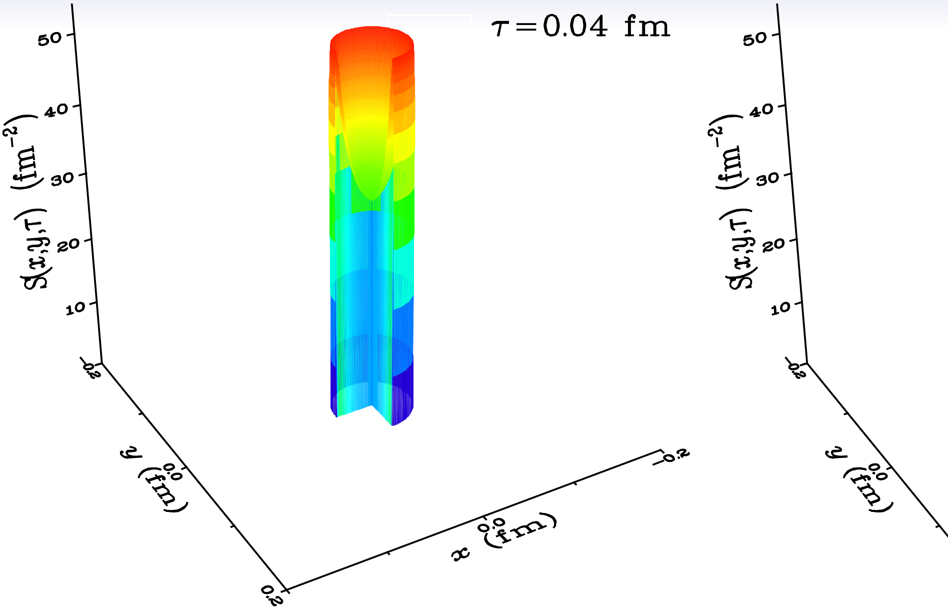
$S(r_t,\tau)$ from L3 2−jet events, τ = 0.01, 0.02, ..., 0.1 fm

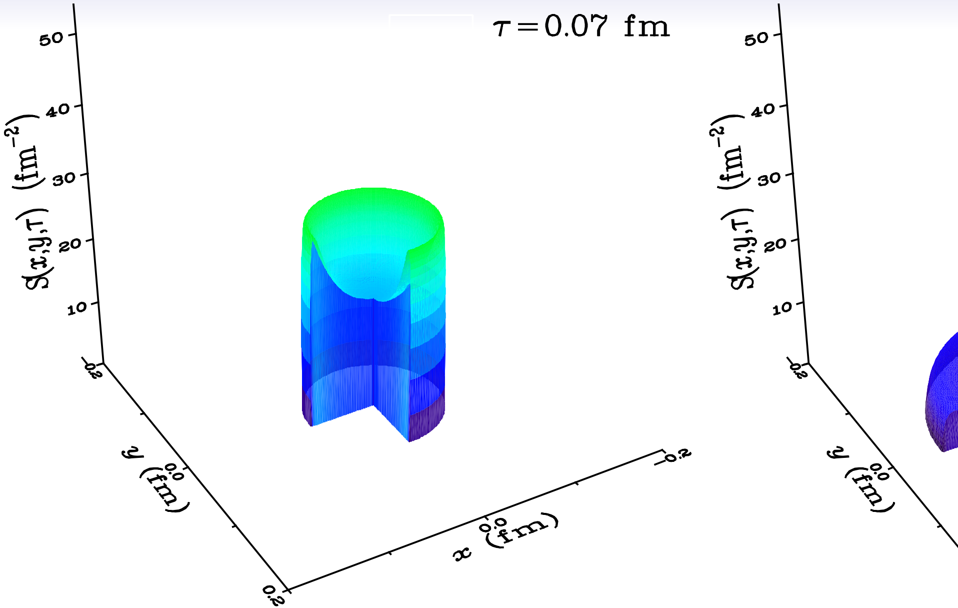




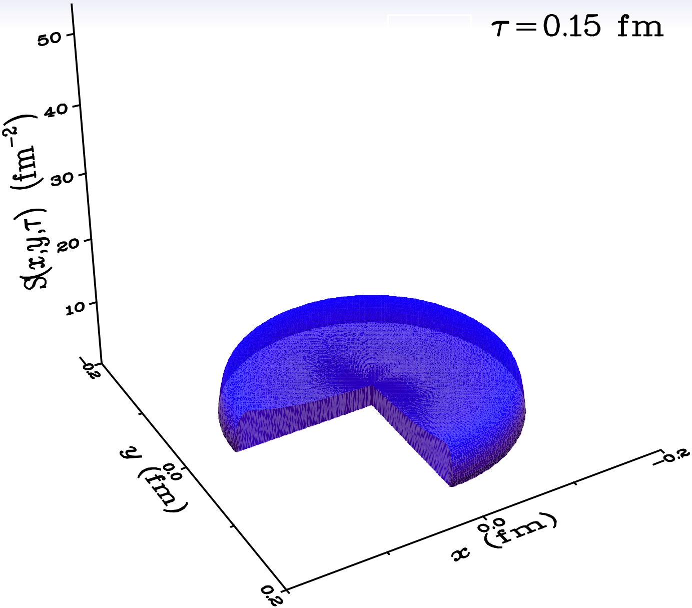

$\tau = 0.01$ fm
$S(x,y,\tau)$ (fm^{-2})
50
40
30
20
10
0.2
0.0
0.2
y (fm)
x (fm)


$\tau = 0.02$ fm
$S(x,y,\tau)$ (fm^{-2})
50
40
30
20
10
0.2
0.0
0.2
y (fm)
0.2
0.0
0.2
x (fm)

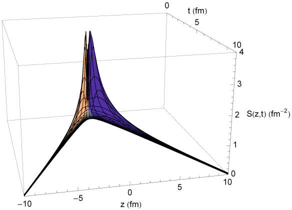

$\tau = 0.04$ fm
$S(x,y,\tau)$ (fm^{-2})
50
40
30
20
10
0.2
0.0
y (fm)
x (fm)



$\tau = 0.07$ fm
S(x,y,τ) (fm^{-2})
50
40
30
20
10
0.2
0.0
0.2
0.0
y (fm)
x (fm)

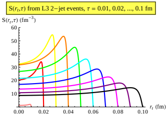

$\tau = 0.15$ fm
50
40
30
20
10
S(x,y,τ) (fm^{-2})
0.2
0.0
y (fm)
0.2
0.0
x (fm)
0.2

Integrating over r,

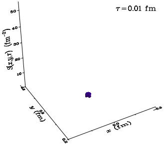


“Boomerang” shape

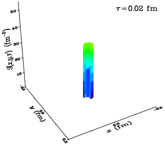
Integrating over z,



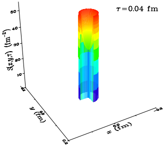
$\tau = 0.01$ fm



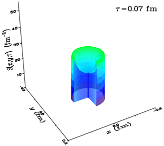
$\tau = 0.02$ fm



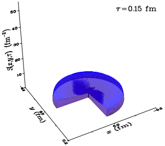
$\tau = 0.04$ fm



$\tau = 0.07$ fm



$\tau = 0.15$ fm

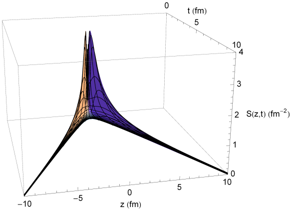


Expanding ring

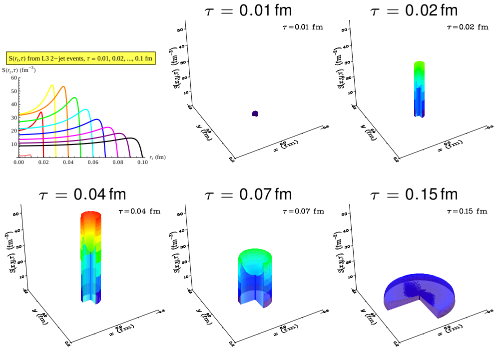
Particle production is close to the light-cone

Integrating over z,

Integrating over r,



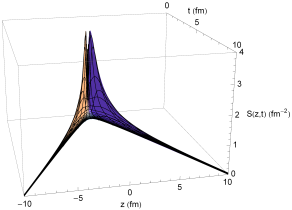
“Boomerang” shape



Expanding ring

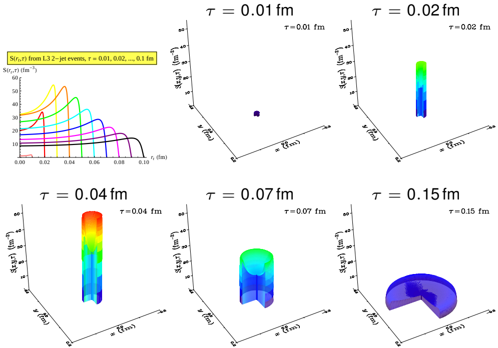
Particle production is close to the light-cone

Integrating over r,



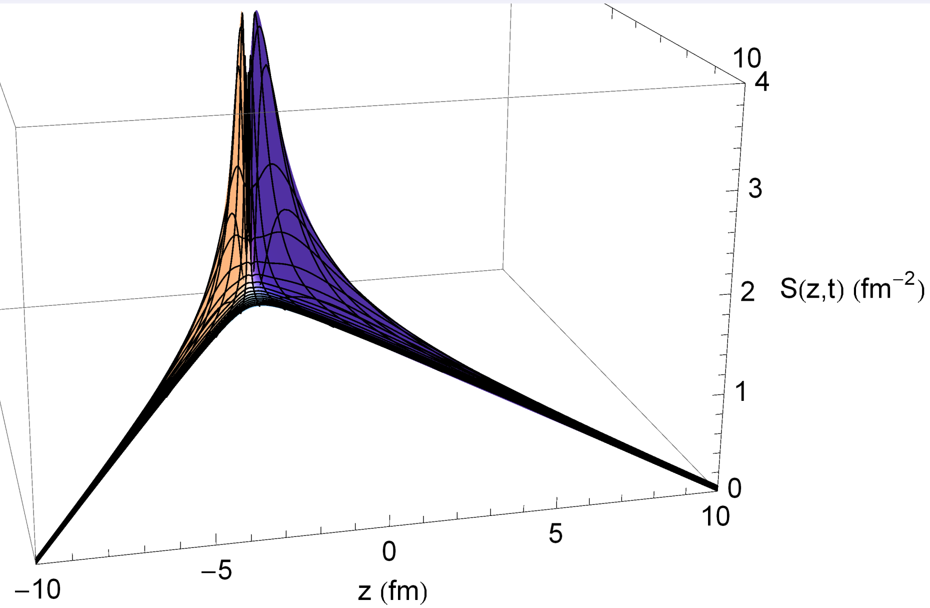
“Boomerang” shape

Integrating over z,



Expanding ring

Particle production is close to the light-cone


10
4
3
2
S(z,t) (fm^{-2})
1
0
10
5
0
−5
−10
z (fm)