Nucleon-deuteron collision as a probe of the partonic distributions

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The one-body partonic distributions in the hadrons are well investigated using electromagnetic or weak interactions. The events with multiple electromagnetic or weak interactions on the same hadron are the very rare. Thus the double parton distributions can be investigated in the events with double QCD hard scattering of partons of the same hadron: it depends not only on the rapidities (or fractional momenta) of the partons but also on a transverse variable. The nucleon-deuteron collision can be a good candidate for this kind of investigations because of the known structure of the deuteron wave function which provides a probe, absent in hadron-hadron collision.

1 Motivations

The one-body partonic distributions in the hadrons are well investigated using electromagnetic or weak interactions.
In order to study events with hard double scattering of partons of the same hadron, one can observe events with hard QCD double scattering:[1][2][3][4] such events become more and more abundant as the energy of the colliding hadrons grows. In fact at very high energies even the parton at small fractional momentum x may suffer collisions with momentum transfer large enough to allow a perturbative treatment.
A global parameter describing the effect of multiple interactions is the effective cross section: $\sigma_{\rm eff} = {\sigma_S}^2/(2 \cdot \sigma_D)$; where σ_S is the integrated inclusive cross section for one hard scattering and σ_D is the integrated inclusive cross section for two hard scatterings. At first sight $\sigma_{\rm eff}$ defines the size of the hadron; this is not the whole story, also the correlations among the partons and the multiplicity distribution have their role in buiding up $\sigma_{\rm eff}$. One can note that it is also possible to consider less global characteristics performing only a limited integration: $x_o - \Delta x < x < x_o + \Delta x \;\; x'_o - \Delta x < x' < x'_o + \Delta x \to \sigma_{\rm eff}\big|_{x_o x'_o}$.
We could go on further and, for $K-$hard scatterings, define the dimensionless parameters τ_K through [5][6]

$$\sigma_K = \frac{(\sigma_S)^K}{K!(\sigma_{\rm eff})^{K-1}\tau_K} \qquad\qquad \tau_2 = 1\ .$$

The relevance of the multiplicity distribution on the effective cross section is shown working out a short example where two distributions are compared:

1-Uncorrelated Poissonian distribution.

$$\Gamma(x_1, b_1, \ldots, x_n, b_n) = \frac{1}{n!} D(x_1, b_1) \ldots D(x_n, b_n) \exp\Big[- \int D(x, b) dx d^2 b \Big]$$

If furthermore: $D(x, b) = g(x) f(b)$ with

$$\int f(b) d^2 b = 1 \quad \text{and} \quad F(\beta) = \int d^2 b f(b) f(b - \beta) \quad \text{then}$$

$$\sigma_{\rm eff} = \frac{1}{\int d^2\beta F^2(\beta)}$$

2-Negative-binomial distribution[7].

$$\Gamma(x_1, b_1, \ldots, x_n, b_n) = \frac{(\nu)_n}{n!} D(x_1, b_1) \ldots D(x_n, b_n) \Big[1 - \int D(x, b) dx d^2 b \Big]^{\nu}$$

With the same procedure

$$\sigma_{\rm eff} = \frac{1}{\int d^2\beta F^2(\beta)} \Big[\frac{\nu + 1}{\nu} \Big]^2$$

In an analogous, but more complicated way, a not factored distribution yields a different result for $\sigma_{\rm eff}$, it depends now explicitly on the two-body correlations.

2 New features in Nucleon-Deuteron scattering

2.1 Description of the process

The typical feature of the Nucleon-Deuteron scattering is that there are two different processes that act, in principle, coherently:
I: Only one nucleon suffers hard interaction:
With respect to the Nucleon-Nucleon interaction nothing essentially new happens.
II: Both nucleons of the deuteron suffer hard interaction.
The deuteron $w.f.$ (wave function) enlightens deeper details of the nucleon structure.

The processes are described by Feynman graphs (see next pages), so the kinematics is correct and the relevant singularities are apparent.
Since the final states of the two processes are different, the former contains two fragmented nucleons of the deuteron, the latter a fragmented and an unbroken nucleon there is no interference term.

Outline of the calculation
In order to go from the general expression corresponding to the Feynman graphs to simpler expressionhe suited for our purposes the following procedure is used:
1. Whenever possible, the *small* $\pm$-components are neglected with respect to the *large* $\pm$-components.
2. The *large* $\pm$-components are integrated taking into account the dominant singularities.
3. On the transverse components the *Fourier transformation* is performed, so we end with an expression in terms of fractional longitudinal momenta and transverse coordinates. The hard

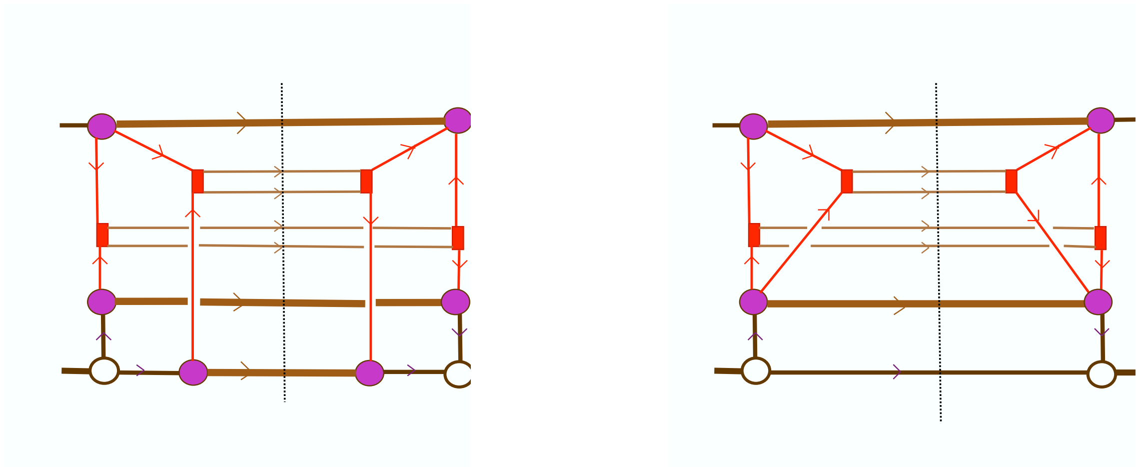

Figure 1: I-Both nucleons interact once. II-One nucleon interacts twice

interaction is local in comparison with the hadron size.
In this way the two addenda of the cross section for the double scattering $\sigma^{(2)}(x_1, x_2, x_1', x_2')$ acquires an explicit form:

When both nucleon interact the expression for the cross section is:

$$
\begin{aligned}
\sigma_{1,1} &= \frac{1}{(2\pi)^3} \int \Gamma(x_1, x_2, b_1, b_2) \Gamma(x_1'/Z, \beta_1) \Gamma(x_2'/(2-Z), \beta_2) \\
&\times \frac{d\hat{\sigma}(x_1, x_1')}{d\Omega_1} \frac{d\hat{\sigma}(x_2, x_2')}{d\Omega_2} \frac{|\Psi_D(Z, B)|^2}{Z(2-Z)} \\
&\times dBdZ \prod_{i=1,2} db_i d\beta_i dx_i dx_i' d\Omega_i \delta(B + b_1 - b_2 - \beta_1 + \beta_2)
\end{aligned}
$$

When only one nucleon interacts (twice) the expression for the cross section becomes:

$$
\begin{aligned}
\sigma_{2,0} &= \frac{1}{(2\pi)^3} \int \Gamma(x_1, x_2, b_1, b_2) \Gamma(x_1'/Z, x_2'/Z, \beta_1, \beta_2) \\
&\times \frac{d\hat{\sigma}(x_1, x_1')}{d\Omega_1} \frac{d\hat{\sigma}(x_2, x_2')}{d\Omega_2} \frac{|\Psi_D(Z, B)|^2}{Z(2-Z)} \\
&\times dBdZ \prod_{i=1,2} db_i d\beta_i dx_i dx_i' d\Omega_i \delta(b_1 - b_2 - \beta_1 + \beta_2)
\end{aligned}
$$

Note that now in the argument of the $\delta-$function the coordinates of the deuteron $w.f.$ are absent.

In the previous expressione x_j are, as usual, the fractional large momenta of the partons (the + or the - component) Z is the fractional momentum of the nucleon in the deuteron normalized in such a way that $0 < Z < 2$, the two-dimensional vectors b_j, B, β are transverse coordinates, the Γ are the one-parton or two-parton structure function of the nucleons and Ψ is the three-dimensional wave function of the deuteron. Finally the expressions $d\hat{\sigma}/d\Omega$ are the differential cross sections of the elastic scattering of the partons with respect to the scattering angles.

2.2 Covariant form for the deuteron wave function

Since the deuteron wave-function has a role in the present analysis something must be added about it: In a fully relativistic treatment, but neglecting the spin structure, we should start with the Bethe-Salpeter function $\chi_D(p)$ satisfying the equation[8][9]

$$\chi_D(p) = \frac{1}{(D/2+p)^2 - m^2 + i\epsilon}\,\frac{1}{(D/2-p)^2 - m^2 + i\epsilon}\int \frac{ig^2}{q^2-\mu^2}\chi_D(p+q)\frac{d^4q}{(2\pi)^4}$$

Correspondingly the effective vertex is

$$\Phi_D(p) = \int \frac{ig^2}{q^2-\mu^2}\chi_D(p+q)\frac{d^4q}{(2\pi)^4}$$

The relative motion of the two nucleon is nonrelativistic, in the frame $\mathbf{D} = 0$, $q^2 - \mu^2 \approx -(\mathbf{q}^2 + \mu^2)$ and we can integrate over q_o, that is we consider static interactions, thus we define

$$\phi_D(\mathbf{p}+\mathbf{q}) = \int \Psi_D(p+q)\frac{dq_o}{2\pi}\ .$$

The binding energy B is defined by: $M_D^2 = (2m-B)^2 \approx 4m(m-B)$
At first order in kinetic and binding energies from the original equation we get the Schrödinger equation:

$$\Big(\frac{\mathbf{p}^2}{m} + B\Big)\phi_D(\mathbf{p}) = \frac{1}{4m^2}\int \frac{g^2}{\mathbf{q}^2+\mu^2}\phi_D(\mathbf{p}+\mathbf{q})\frac{d^3q}{(2\pi)^3}$$

and an effective vertexis defined by:

$$\phi_D = \frac{i\Phi_D}{E(4E^2 - M_D^2)} \qquad E = \sqrt{\mathbf{p}^2 + m^2}$$

The normalization is fixed by the overall conservation of charge.
A scalar wave function depends on $\mathbf{p}^2$, this in turn is expressible in covariant form and in light-cone variables:

$$\mathbf{p}^2 + m^2 = \frac{1}{4{M_D}^2}\big[(D-p)^2 - m^2 - M_D^2\big]^2 \qquad \mathbf{p}^2 + m^2 = \frac{1}{4{M_D}^2}\Big[\frac{Z}{2}M_D^2 + \frac{2}{Z}m_t^2\Big]^2$$

So we can use the well known non relativistic wave function and still have a covariant expression.

2.3 A model worked out completely

This is a model which is oversimplified, but it can be worked out in a transparent way:
One-parton distribution:

$$\Gamma_1 = \frac{\mathcal{G}(x)}{\pi R^2}\exp[-b^2/R^2]$$

Two-parton distribution:

$$\Gamma_2 = K \frac{\mathcal{G}(x_1)\mathcal{G}(x_2)}{(\pi R^2)^2(1-\lambda^2)} \exp[-(b_1^2 + b_2^2 + 2\lambda b_1 b_2)/R^2(1-\lambda^2)]$$

Since

$$\int \mathcal{G}(x)dx = n \quad \text{it} \quad \text{results}: \quad \int \Gamma_2 dx_2 db_2 = Kn\Gamma_1$$

K controls the parton multiplicity, λ their spatial correlation.
For the single σ_1 and double σ_2 scattering we have:

$$\sigma_1 = \int \hat{\sigma}\mathcal{G}(x)dx\mathcal{G}(x')dx' \quad , \quad \sigma_2 = \Big[\int \hat{\sigma}\mathcal{G}(x)dx\mathcal{G}(x')dx'\Big]^2 \frac{1}{\pi R^2}\frac{K^2}{4(1+\lambda)}$$

Hence the ratio σ_{eff} is given by

$$\sigma_{\text{eff}} = \frac{2\pi R^2(1+\lambda)}{K^2}$$

Now represent deuteron $|w.f.|^2$ as

$$\mathcal{D}(Z)\frac{1}{\pi S^2}\exp[-s^2/S^2] \qquad 0 < Z < 2$$

$w.f.$ nonrelativistic, $Z \approx 1$. One-parton distribution within deuteron:

$$\bar{\Gamma}_1 = \int \frac{\mathcal{G}_1(x/Z)}{\pi R^2}\exp[-(b-s)^2/R^2]\frac{\mathcal{D}(z)}{\pi S^2}\exp[-s^2/S^2]dZds$$

using $Z \approx 1$:

$$\bar{\Gamma}_1 = \frac{\mathcal{G}_1(x)}{\pi(R^2+S^2)}\exp[-b^2/(R^2+S^2)]$$

Collision with a free nucleon, simple scattering: $\sigma_1 = \int \hat{\sigma}\mathcal{G}(x)dx\mathcal{G}(x')dx'$, the form is the same as for the Nucleon-Nucleon case, the Deuteron structure does not appear.
Collision with a free nucleon, double scattering. There are two possibilities already discussed:
Only one interacting nucleon:

The integration over the spectator $w.f.$ eliminates S.

$$\sigma_2' = 2 \times \Big[\int \hat{\sigma}\mathcal{G}(x)dx\mathcal{G}(x')dx'\Big]^2 \frac{1}{\pi R^2}\frac{K^2}{4(1+\lambda)}$$

Both interacting nucleons:

Here the size of the deuteron S is relevant

$$\sigma"_2 = \Big[\int \hat{\sigma}\mathcal{G}(x)dx\mathcal{G}(x')dx'\Big]^2 \frac{1}{\pi R^2}\frac{K}{(2+\lambda)+(S/R)^2}$$

so, for the effective cross sections we get:

$$\sigma_{\text{eff}}^{(N)} = \frac{2\pi R^2(1+\lambda)}{K^2} \qquad \sigma_{\text{eff}}^{(d)} = 2\sigma_{\text{eff}}^{(N)}\Big[1 + \frac{2(1+\lambda)/K}{(2+\lambda)+(S/R)^2}\Big]$$

By determinig $\sigma_{\text{eff}}^{(N)}$ and $\sigma_{\text{eff}}^{(d)}$ we know separately K and λ, assuming that S is known since the deuteron $w.f.$ is known.

3 Conclusions and outlook

The knowledge of the deuteron $w.f.$ is a help in investigating the structure of the hadron, in particular the transverse two-body correlations.
Refinements of the treatment here presented could be:
Taking into account the spin properties both of the free nucleons and of the deuteron.
Taking into account the possible soft rescattering of the particles that underwent hard interaction with the remnants. One could say that either the rescattering can happen very soon after the primary interaction, in this case the particles are very near each other and a large momentum can be exchanged, the rescattering is simply a next-order correction in the perturbative expansion or the rescattering happens when the primary interaction is already finshed the particles are free and the interaction introduces an $\mathcal{S}-$matrix which is unitary, in the sum over the final state we simply use $\mathcal{S}\mathcal{S}^+ = 1$ and sum over the original states. Since the soft fragments of the spectator nucleon are unobserved, the on shell configuration of the spectator includes also final state interactions.

A more detailed presentation can be found on arXiv 1009,5881

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