Limiting soft particle emission

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- lessons from $e^+e^-$ annihilations
- soft particle spectra in $pp$ collisions
- nucleus nucleus interactions

with V.A. Khoze, M.G. Ryskin, EPJC 68, 141, 2010

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Limiting soft particle emission

inclusive production of particles in the limit $p \rightarrow 0; \ (p_T \rightarrow 0)$

$$I_0 \equiv E \frac{dN}{d^3p} \bigg|_{p\rightarrow0}$$

in this limit Born term in perturbative expansion dominates:
⇒ universal features for all processes:

1. inclusive spectra become energy independent
2. relative normalisation of spectra in different processes
given by relevant colour factors

This holds for QCD partons,
we assume the same is true for hadrons
qualitative picture:

- gluons of large wavelength do not resolve any detailed intrinsic jet structure
- coherent emission from all final partons
- they “see” only the colour charge of primary partons i.e. they are represented by the Born term for the minimal partonic process
$e^+ e^-$ annihilation
Quark and gluon jets in $e^+e^-$ annihilations

inclusive spectra in pQCD

particle energy $k$ and production angle $\Theta$

 evolution eqn. including angular ordering  

**Bassetto et al. ’83**  
**Dokshitzer et al. ’84**

most simple example:

**Double Logarithmic Approximation (DLA) at fixed $\alpha_s$**

$$\frac{dN_p}{dk d\Theta} = \frac{2}{\pi} \frac{C_p}{k^2} \alpha_s + \frac{4N_c}{\pi^2} \frac{C_p}{k^2} \alpha_s^2 \ln \frac{E}{k} \ln \frac{k_T}{Q_0} + \ldots$$

$\Rightarrow$ Born term $O(\alpha_s)$ dominates in soft limit ($k_T \rightarrow Q_0 \equiv k_T^{cut}$):

$\Rightarrow$ independent of jet energy $E$, proportional to colour factors $C_p = C_F, C_A$

more realistic case:

**Modified Leading Logarithmic Approximation (MLLA)**

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Limiting behaviour of $p_T$ spectra in $\sqrt{s}$

charged particles

analytic solution $O(\alpha_s^2)$ at $p \lesssim 1$ GeV based on MLLA+LPHD ($Q_0 \sim m_{had}$)

⇒ spectra become energy $\sqrt{s}$ independent for $p \to 0$

Khoze, Lupia, W.O. '97

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Colour factors in quark and gluon jets

For soft radiation
study particles \( N_\perp \) perpendicular to event plane in 2 and 3 jet events

aligned jets:

\[
\begin{align*}
qg & \leftrightarrow \bar{q} & \text{like} & \quad q & \leftrightarrow \bar{q} & \text{color factor} & \quad C_p = C_F \\
q\bar{q} & \leftrightarrow g & \text{like} & \quad g & \leftrightarrow g & \text{color factor} & \quad C_p = C_A
\end{align*}
\]
perpendicular radiation in $q\bar{q}g$ events

for general angles interpolate:

$$\frac{N_{qqg}}{N_{qq}} \equiv \frac{C_A}{C_F} r_t$$

$$r_t(\Theta_{ij}) =$$

$$= \frac{1}{4}[(1 - \cos \Theta_{qg}) + (1 - \cos \Theta_{\bar{q}g})] - \frac{1}{N_C^2}(1 - \cos \Theta_{qq})$$

only 2 angles are independent

Khoze, Lupia, W.O.’97

dashed: QCD prediction
full: fit with slope

$$C_A/C_F = 2.211 \pm 0.053$$

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pp/\bar{p}p scattering
Soft limit in $pp$ “minimum bias” events

minimal partonic process resolved: $2 \to 2 + g_s$ at small angles:
  gluon exchange (non-vanishing cross sections for large $s$)
examples:

\[
\begin{array}{c}
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\text{gluon exchange leads to radiating colour octet dipole, } pp \to 8 + 8
\end{array}
\end{array}
\]

$qq$ scattering with initial and final bremsstrahlung ($\sim$ two $q\bar{q}$ triplet dipoles)
$q - qq$ as color octet

\[
\Rightarrow \frac{I_0^{pp}}{I_0^{e^+e^-}} \sim \frac{dN_{pp\to8+8}}{dEd\eta} / \frac{dN_{e^+e^-\to\bar{q}q}}{dEd\eta} \to \frac{C_A}{C_F} \quad \text{for} \quad p \to 0
\]

compare with Brodsky, Gunion ‘76
Energy dependence of \( \frac{E}{d^3p} \) \( vs. \) \( p_T \) in \( pp \) collisions

630 - 1800 GeV  
27 - 1800 GeV  
900 - 7000 GeV

Extrapolation \( p_T \to 0 \) ?

\( pp \) data: “exponential \( p_T \)” fits ( \( p_T > 0.1 \) GeV (BS)) (bad analyticity)

\( AA \) data: “thermal” fits in \( m_T = \sqrt{m^2 + p_T^2} \) (\( p_T \gtrsim 30 \) MeV (PHOBOS))
Soft limit \( I_0 = \left. \frac{E^2 dN}{d^3 p} \right|_{p_T \to 0} \) in \( pp \) collisions

\[ I_0 \approx (7 \pm 1) \text{ GeV}^{-2} \]

Data suggest rather flat energy dependence (contrast: \( \frac{dN}{dy} \) rise by factor 2)

to compare with \( e^+ e^- \) annihilation we use instead:

**Non-diffractive \( pp/p\bar{p} \) collisions (exp. fit):** \( I_0 \approx (8 \pm 1) \text{ GeV}^{-2} \)

**Non-diffractive \( pp/p\bar{p} \) collisions (therm. fit):** \( I_0 \approx (6 \pm 1) \text{ GeV}^{-2} \) \((-25\%)\)
Comparison of $pp$ with $e^+e^-$ collisions

1. spectrum $E \frac{dN}{d^3p}$ vs $p_T$ using sphericity-jet axis in $e^+e^-$ annihilations:
   - $e^+e^-$: TPC Coll. at SLAC, 1987
   - $pp$: British-Scandinavian Coll. at ISR, 1975

spectra are falling more steeply in $pp$ collisions; also different shapes

for pions: $r \approx \frac{I_0^{pp}}{I_0^{e^+e^-}} \approx 2.7$ for exp. extrapol. of $pp$

   $r \approx 2.0$ for thermal fit.

for kaons: $r \approx \frac{I_0^{pp}}{I_0^{e^+e^-}} \approx 2.0$ for exp. extrap.

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2. fits to the momentum spectra \((p, \text{ not } p_T)\) in \(e^+e^-\) annihilations

fits exponential in energy, \(\sqrt{s} = 10-34\) GeV \hspace{1cm} \text{ARGUS, TASSO, TPC/2\(\gamma\)}

\[
I_0^{e^+e^-} \approx (3.3 \pm 0.5) \text{ GeV}^{-2} \quad (p \gtrsim 0.05\) GeV \hspace{1cm} \text{ARGUS,TPC/2\(\gamma\)}
\]

\[
\Rightarrow \frac{I_0^{pp}}{I_0^{e^+e^-}} \approx \frac{(1.8 \pm 0.4)}{(2.4 \pm 0.5)}
\]

thermal exponential \(p_T\) extrapolation of \(pp\) data.

expect \[
I_0^{pp}/I_0^{e^+e^-} = C_A/C_F = 2.25
\]

(thermal fit is more realistic from theoretical standpoint)

\Rightarrow \text{existing data are consistent with expectations from soft bremsstrahlung:}

independence of \(I_0\) on energy, color factors

\Rightarrow \text{What will happen at LHC? Incoherent multiple scattering? Rising } I_0\text{?}
AA scattering
Spectra at low $p_T$ in AA collisions

limiting cases:

- pointlike interactions, $R_{AA}^{N_{coll}} = 1$

$$R_{AA}^{N_{coll}} = \frac{1}{N_{coll}} \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T}$$  

$n_{coll}$ No of nucleon nucleon collisions (Glauber)

- Soft particle production:
  expect reduced rate for coherent production over range $R \sim 1/p_T$

Phenomenology: normalization by
No of participating nucleons $N_{part}$ or “wounded nucleons”

Bialas, Bleszynski and Czyz ’76

$$R_{AA}^{N_{part}} = \frac{1}{N_{part}/2} \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T}$$  

$\Leftrightarrow$ don’t count nucleon rescatterings

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$p_T$ distributions in $AuAu$ collisions

upper data points: $\sqrt{s} = 62.4$ GeV; lower data points: $\sqrt{s} = 200$ GeV
centralities: peripheral $\leftrightarrow$ central

$R^{N_{coll}}_{AA}$

$R^{N_{part}}_{AA}$

$R^{N_{part}}_{PC}$

$\Rightarrow 1.$ $R$ has weak energy dependence for $p_T \to 0$ ($AA$ behaves like $pp$)
$\Rightarrow 2.$ Normalization for $p_T \to 0$: $R^{N_{part}}_{AA} \to 1$

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Comparison of $AA$ and $pp$ collisions

Thermal extrapolation of $pp$ and $AuAu$ spectra to $p_T = 0$ at 200 GeV:

$pp$ collisions \quad I_0 \approx 5.9 \text{ GeV}^{-2} \quad p_T > 0.4 \text{ GeV} \quad \text{STAR}

$AuAu$ central \quad I_0 \approx (950 \pm 100) \text{ GeV}^{-2} \quad p_T > 0.03 \text{ GeV} \quad \text{PHOBOS, STAR}

\Rightarrow \quad \frac{I_0^{AA}}{I_0^{pp}} \approx 160 \pm 17

compare with

$N_{coll} = 1040$ \quad and \quad $N_{part}/2 = 172 \ (\pm 15\%)$ \quad (Glauber model calculation)

conclude:

\Rightarrow \quad I_0^{AuAu} \approx \frac{N_{part}}{2} I_0^{pp} \quad \text{works very well, but only for } p_T \to 0
Color antenna pattern in nuclear collisions

each nucleon after repeated rescatterings produces colour octet state.

\[ N_{\text{coll}} = 1 \]
\[ N_{\text{part}}/2 = 1 \]

\[ N_{\text{coll}} = 3 \]
\[ N_{\text{part}}/2 = 2 \]

simple example: gluon exchange between quarks & diquarks \( (p \rightarrow qqq) \)

compare Bialas and Bzdak

BFKL at large \( s \): color 8 exchange dominates over color 12, 27 \ldots Bronzan

This mechanism results in “participant scaling”
Universal composition of soft particles?

\[K^-/\pi^-\] ratios converge for \(p_T \to 0\)
in \(e^+e^-\), \(pp\) and in peripheral and central \(AA\) collisions

very soft particles emitted as from quarks
stay behind expanding plasma and decouple from thermalisation
Summary on limiting soft particle emission

- Universal features of particle production for $p, p_T \to 0$
  (dominance of QCD gluon bremsstrahlung)
  a) energy independence of $I_0$
  b) intensity $I_0$ determined by color factors

- $e^+ e^-$ annihilation: $I_0(\text{gluon jet})/I_0(\text{quark jet}) = C_A/C_F$
- $pp$ scattering: $I_0(pp)/I_0(e^+e^-) = C_A/C_F$
- $AA$ scattering: $I_0(\text{AAA})/I_0(pp) = (N_{part}/2)C_A/C_A$

- all particle ratios vs. $p_T$ converge to those from quark jets
  Soft hadrons decouple from equilibration

- LHC:
  New incoherent sources: $I_0(pp)$ rising with energy?
Back up
Extrapolated $I_0$ in $AA$ collisions

$\pi$ spectra at very low $p_T \gtrsim 30$ MeV measured by PHOBOS ’04
common fit with data from PHENIX ’04 at $p_T \gtrsim 300$ MeV

“thermal fit” to spectrum in central $AuAu$ collisions

$$E \frac{dN}{d^3p} = \frac{A}{(\exp(m_T/T) - 1)}; \quad m_T = \sqrt{m^2 + p_T^2}$$

with $T=0.229$ GeV
Normalisation as measured by STAR ’2003 for $\sqrt{s} = 200$ GeV

smooth extrapolation from $p_T > 0.4$ GeV to $R_{AA} = N_{part}/2$ for $p_T \to 0$