# Recent Progress in Lattice QCD

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lattice field theory talk

examples to reach the physical limit (physical mass & continuum)

SFB TR55

# Outline


1. Introduction
2. Hadron spectrum
3. Nonvanishing temperature
4. Summary

# The origin of mass of the visible Universe

source of the mass for ordinary matter (not a dark matter talk)

basic goal of LHC (Large Hadron Collider, Geneva Switzerland):

"to clarify the origin of mass"

e.g. by finding the Higgs particle, or by alternative mechanisms
order of magnitudes: 27 km tunnel and O(10) billion dollars

# The vast majority of the mass of ordinary matter

ultimate (Higgs or alternative) mechanism: responsible for the mass of the leptons and for the mass of the quarks

interestingly enough: just a tiny fraction of the visible mass (such as stars, the earth, the audience, atoms)
electron: almost massless, ≈1/2000 of the mass of a proton
quarks (in ordinary matter): also almost massless particles

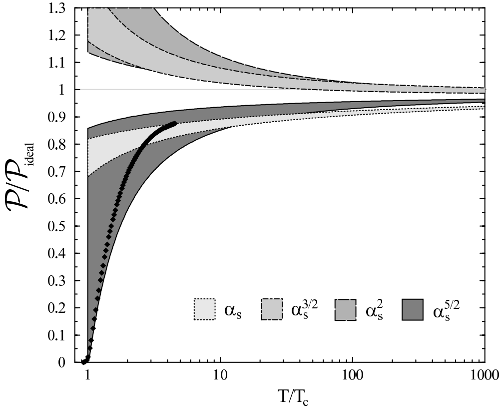
the vast majority (about 95%) comes through another mechanism
$\Longrightarrow$ this mechanism and this 95% will be the main topic of this talk

quantum chromodynamics (QCD, strong interaction) on the lattice

# QCD: need for a systematic non-perturbative method

in some cases: good perturbative convergence; in other cases: bad

pressure at high temperatures converges at T=$10^{300}$ MeV



# Lattice field theory

systematic non-perturbative approach (numerical solution):

quantum fields on the lattice

quantum theory: path integral formulation with $S=E_{kin}-E_{pot}$

quantum mechanics: for all possible paths add exp(iS)
quantum fields: for all possible field configurations add exp(iS)

Euclidean space-time ($t=i\tau$): exp(-S) sum of Boltzmann factors

we do not have infinitely large computers $\Rightarrow$ two restrictions

a. put it on a space-time grid (proper approach: asymptotic freedom)
formally: four-dimensional statistical system
b. finite size of the system (can be also controlled)

$\Rightarrow$ stochastic approach, with reasonable spacing/size: solvable

# Importance sampling

$$Z=\int\prod_{n,\mu}[dU_\mu(n)]e^{-S_g}\,\det(M[U])$$

we do not take into account all possible gauge configuration

each of them is generated with a probability $\propto$ its weight

importance sampling, Metropolis algorithm:
(all other algorithms are based on importance sampling)

$$P(U\to U')=\min\left[1,\exp(-\Delta S_g)\det(M[U'])/\det(M[U])\right]$$

gauge part: trace of 3$\times$3 matrices (easy, without M: quenched)
fermionic part: determinant of $10^6\times 10^6$ sparse matrices (hard)

more efficient ways than direct evaluation (Mx=a), but still hard

# Hadron spectroscopy in lattice QCD

Determine the transition amplitude between:
having a "particle" at time 0 and the same "particle" at time t
$\Rightarrow$ Euclidean correlation function of a composite operator $\mathcal{O}$:

$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

insert a complete set of eigenvectors $|i\rangle$

$$= \sum_i \langle 0|e^{Ht}\ \mathcal{O}(0)\ e^{-Ht}|i\rangle\langle i|\mathcal{O}^\dagger(0)|0\rangle = \sum_i |\langle 0|\mathcal{O}^\dagger(0)|i\rangle|^2\ e^{-(E_i - E_0)t},$$

where $|i\rangle$: eigenvectors of the Hamiltonian with eigenvalue $E_i$.

and $$\mathcal{O}(t) = e^{Ht}\ \mathcal{O}(0)\ e^{-Ht}.$$

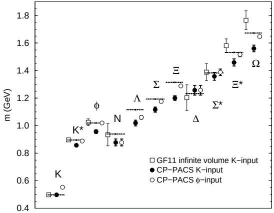
$t$ large $\Rightarrow$ lightest states (created by $\mathcal{O}$) dominate: $C(t) \propto e^{-M\cdot t}$
$t$ large $\Rightarrow$ exponential fits or mass plateaus $M_t$=log[C(t)/C(t+1)]

# Quenched results

QCD is 35 years old ⇒ properties of hadrons (Rosenfeld table)

non-perturbative lattice formulation (Wilson) immediately appeared
needed 20 years even for quenched result of the spectrum (cheap)
instead of det(M) of a $10^6 \times 10^6$ matrix trace of $3 \times 3$ matrices

always at the frontiers of computer technology:
GF11: IBM "to verify quantum chromodynamics" (10 Gflops, '92)
CP-PACS Japanese purpose made machine (Hitachi 614 Gflops, '96)




the ≈10% discrepancy was believed to be a quenching effect

# Ingredients to control systematics

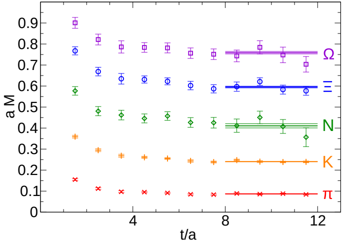
BMW Collaboration, Science 322:1224-1227,2008

- inclusion of det[M] with an exact $n_f$=2+1 algorithm
  action: universality class is known to be QCD (Wilson-quarks)
- spectrum: light mesons, octet & decuplet baryons (resonances)
  (three of these fix the averaged $m_{ud}$, $m_s$ and the cutoff)
- large volumes to guarantee small finite-size effects
  rule of thumb: $M_\pi L \gtrsim 4$ is usually used (correct for that)
- controlled interpolations & extrapolations to physical $m_s$ and $m_{ud}$
  (or eventually simulating directly at these masses)
  since $M_\pi \simeq$135 MeV extrapolations for $m_{ud}$ are difficult
  CPU-intensive calculations with $M_\pi$ reaching down to $\approx$200 MeV
- controlled extrapolations to the continuum limit ($a \to 0$)
  calculations are performed at no less than 3 lattice spacings

## Scale setting and masses in lattice QCD

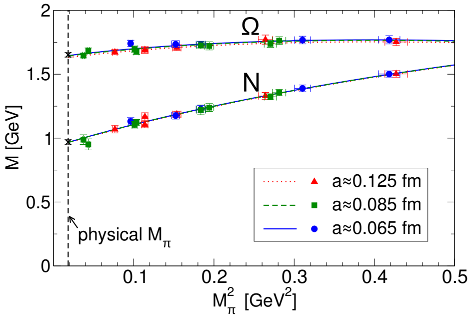
in meteorology, aircraft industry etc. grid spacing is set by hand
in lattice QCD we use g, $m_{ud}$ and $m_s$ in the Lagrangian ('a' not)
measure e.g. the vacuum mass of a hadron in lattice units: $M_\Omega a$
since we know that $M_\Omega$=1672 MeV we obtain 'a'

masses are obtained by correlated fits (choice of fitting ranges)
illustration: mass plateaus at the smallest $M_\pi \approx$190 MeV (noisiest)




volumes and masses for unstable particles: avoided level crossing
decay phenomena included: in finite V shifts of the energy levels

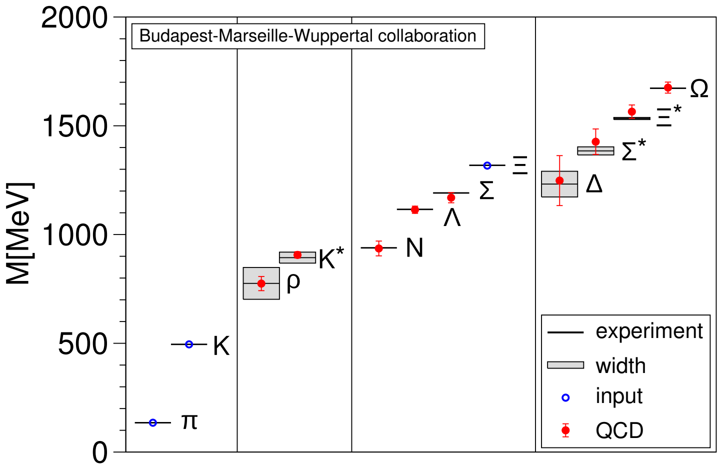
altogether 15 points for each hadrons




smooth extrapolation to the physical pion mass (or $m_{ud}$)
small discretization effects (three lines barely distinguishable)

continuum extrapolation goes as $c \cdot a^n$ and it depends on the action
in principle many ways to discretize (derivative by 2,3... points)
goal: have large $n$ and small $c$ (in this case $n = 2$ and $c$ is small)

# Final result for the hadron spectrum



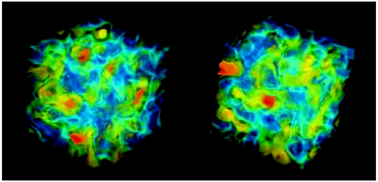
## Breakthrough of the Year

# Proton's Mass 'Predicted'

STARTING FROM A THEORETICAL DESCRIPTION OF ITS INNARDS, physicists precisely calculated the mass of the proton and other particles made of quarks and gluons. The numbers aren't new; experimenters have been able to weigh the proton for nearly a century. But the new results show that physicists can at last make accurate calculations of the ultracomplex strong force that binds quarks.

In simplest terms, the proton comprises three quarks with gluons zipping between them to convey the strong force. Thanks to the uncertainties of quantum mechanics, however, myriad gluons and quark-antiquark pairs flit into and out of existence within a proton in a frenzy that's nearly impossible to analyze but that produces 95% of the particle's mass.

To simplify matters, theorists from France, Germany, and Hungary took an approach known as "lattice quantum chromodynamics."

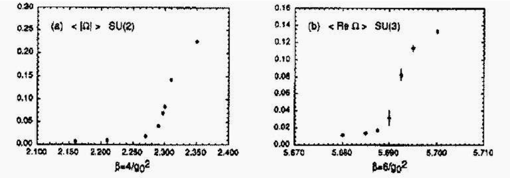


They modeled continuous space and time as a four-dimensional array of points—the lattice—and confined the quarks to the points and the gluons to the links between them. Using supercomputers, they reckoned the masses of the proton and other particles to a precision of about 2%—a tenth of the uncertainties a decade ago—as they reported in November.

In 2003, others reported equally precise calculations of more-esoteric quantities. But by calculating the familiar proton mass, the new work signals more broadly that physicists finally have a handle on the strong force.

# Finite-size scaling theory

problem with phase transitions in Monte-Carlo studies
Monte-Carlo applications for pure gauge theories ($V = 24^3 \cdot 4$)
existence of a transition between confining and deconfining phases:
Polyakov loop exhibits rapid variation in a narrow range of $\beta$



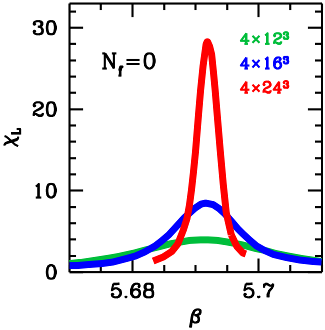

- theoretical prediction: SU(2) second order, SU(3) first order

$\Longrightarrow$ Polyakov loop behavior: SU(2) singular power, SU(3) jump

data do not show such characteristics!

# Finite size scaling in the quenched theory

look at the susceptibility of the Polyakov-line
first order transition (Binder) $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V




finite size scaling shows: the transition is of first order

# The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

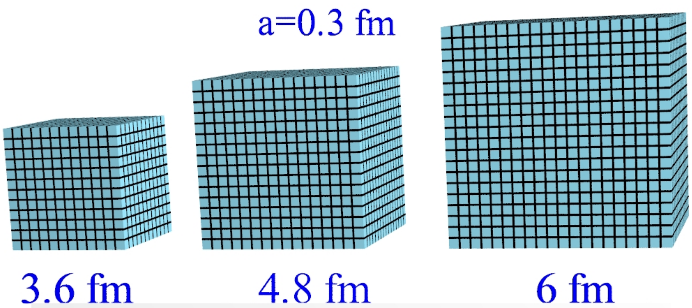
finite size scaling study of the chiral condensate (susceptibility)

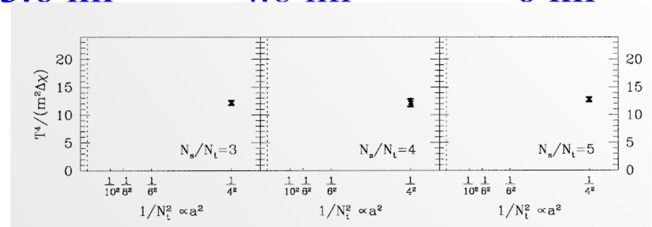
$$\chi=(T/V)\partial^2\log Z/\partial m^2$$

phase transition: finite V analyticity $V\rightarrow\infty$ increasingly singular
(e.g. first order phase transition: height $\propto$ V, width $\propto$ 1/V)
for an analytic cross-over $\chi$ does not grow with V

two steps (three volumes, four lattice spacings):
a. fix V and determine $\chi$ in the continuum limit: a=0.3,0.2,0.15,0.1fm
b. using the continuum extrapolated $\chi_{max}$: finite size scaling

# Approaching the continuum limuit



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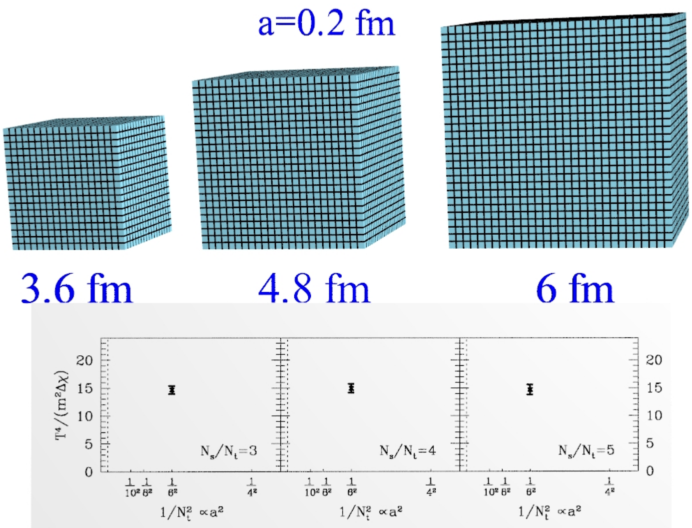

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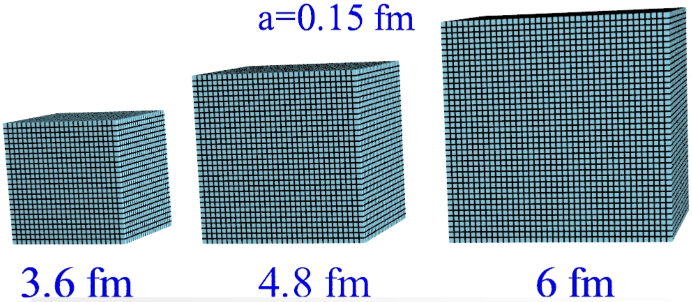


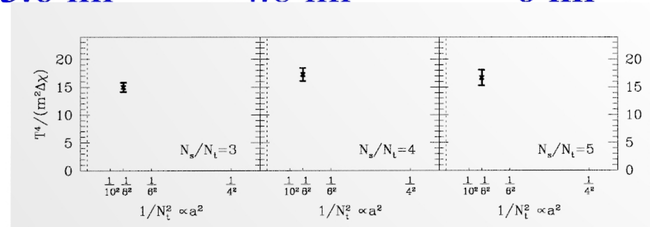

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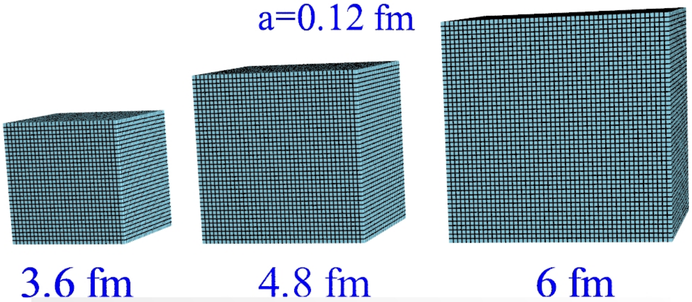


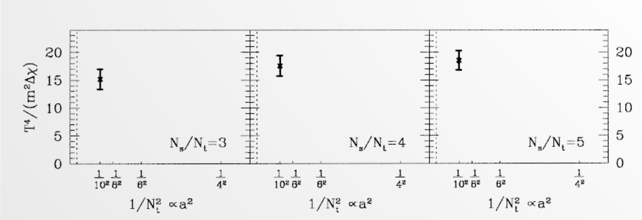

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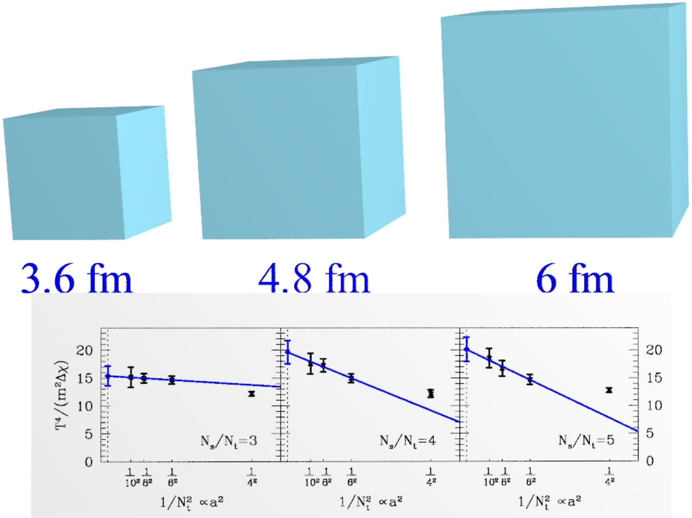
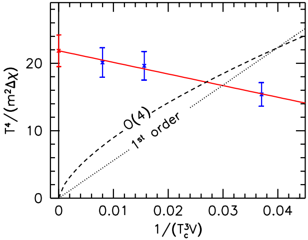


# The nature of the QCD transition: analytic

- finite size scaling analysis with continuum extrapolated $T^4/m^2\Delta\chi$




the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range

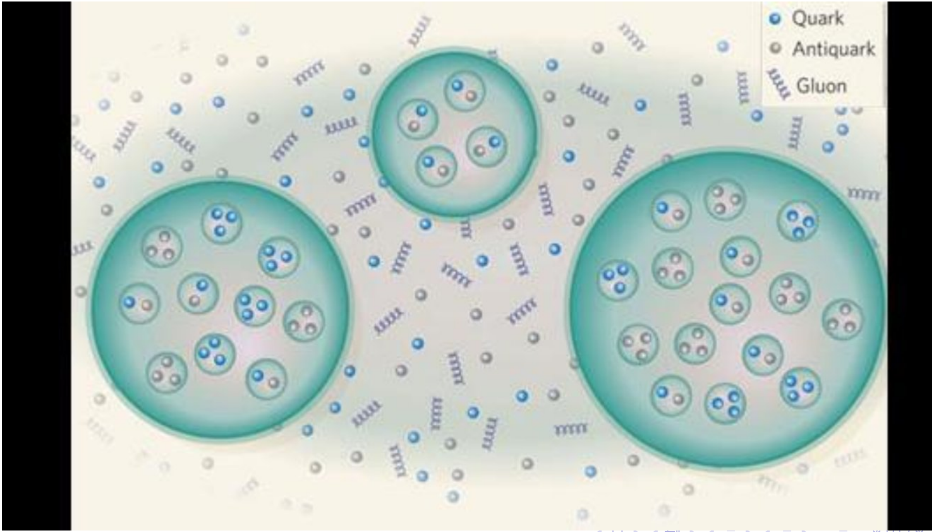
chance probability for 1/V is $10^{-19}$ for O(4) is $7 \cdot 10^{-13}$

continuum result with physical quark masses in staggered QCD:

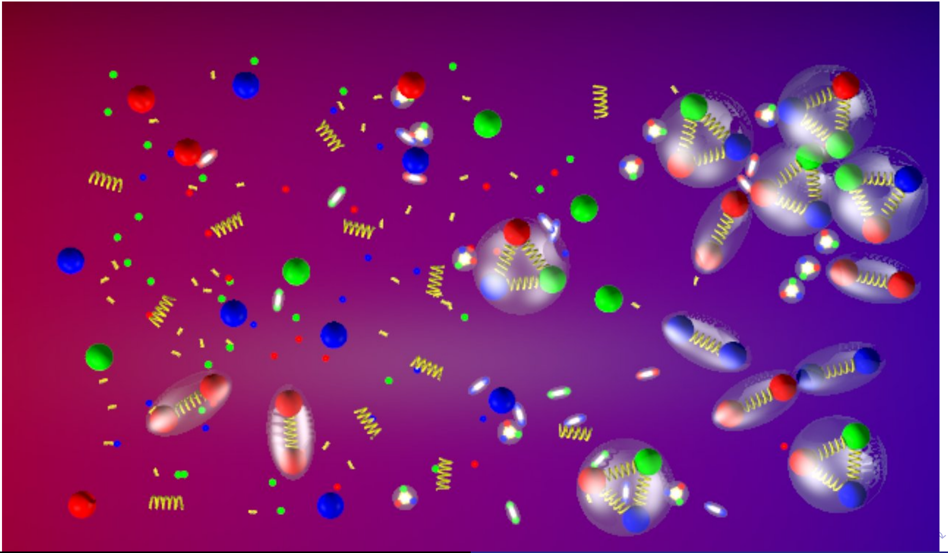
the QCD transition is a cross-over

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

# Possible first order scenario with critical bubbles



# Reality: smooth analytic transition (cross-over)



# Literature: discrepancies between $T_c$

Bielefeld-Brookhaven-Riken-Columbia Collaboration:

M. Cheng et.al, Phys. Rev. D74 (2006) 054507

$T_c$ from $\chi_{\bar{\psi}\psi}$ and Polyakov loop, from both quantities:

$T_c$=192(7)(4) MeV

Bielefeld-Brookhaven-Riken-Columbia merged with MILC: 'hotQCD'

Wuppertal-Budapest group: WB

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46

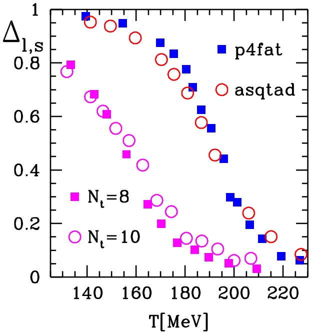
chiral susceptibility: $T_c$=151(3)(3) MeV
Polyakov and strange susceptibility: $T_c$=175(2)(4) MeV

'chiral $T_c$': ≈40 MeV; 'confinement $T_c$': ≈15 MeV difference

both groups give continuum extrapolated results with physical $m_\pi$

# Literature: discrepancies between T dependencies

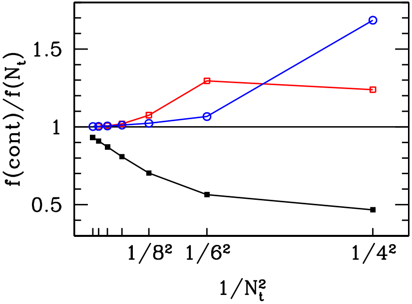
Reason: shoulders, inflection points are difficult to define?
Answer: no, the whole temperature dependence is shifted




for $\Delta_{l,s}$ ≈35 MeV; for the strange susceptibility ≈15 MeV
this discrepancy would appear in all quantities (eos, fluctuations)

# Examples for improvements, consequences

how fast can we reach the continuum pressure at T=$\infty$?




p4 action is essentially designed for this quantity T$\gg T_c$

asqtad designed mostly for T=0 physics (but good at high T, too)

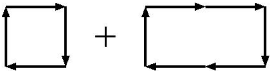
stout-smeared one-link converges slower but in the $a^2$ scaling regime (e.g. extrapolation from $N_t$=8,10 provides a result within about 1%)

# Choice of the action

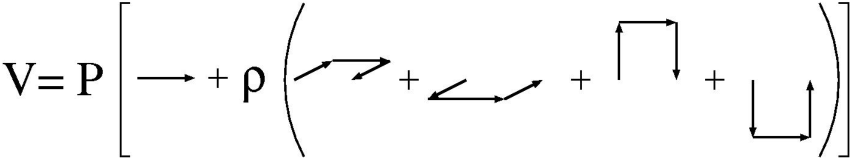
no consensus: which action offers the most cost effective approach

Aoki, Fodor, Katz, Szabo, JHEP 0601, 089 (2006) [arXiv:hep-lat/0510084]

our choice: tree-level $O(a^2)$-improved Symanzik gauge action



2-level (stout) smeared improved staggered fermions

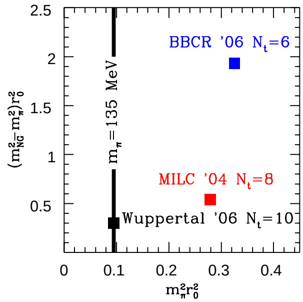


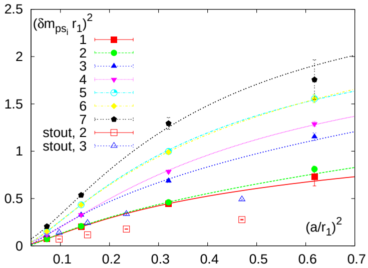

best known way to improve on taste symmetry violation

# Chiral symmetry breaking and pions

transition temperature for remnant of the chiral transition:
balance between the f's of the chirally broken & symmetric sectors
chiral symmetry breaking: 3 pions are the pseudo-Goldstone bosons

staggered QCD: 1 ($\frac{3}{16}$) pseudo-Goldstone instead of 3 (taste violation)
staggered lattice artefact ⇒ disappears in the continuum limit
WB: stout-smeared improvement is designed to reduce this artefact

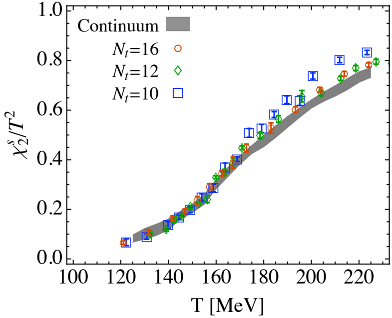


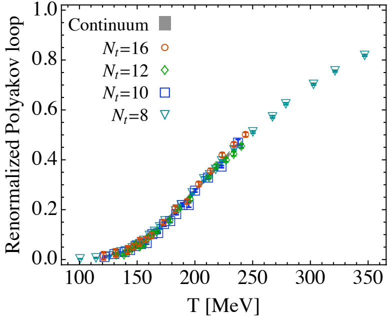



# strange quark number susceptibility and Polyakov-loop

strange susceptibility: $\chi_2^s = (T/V) \cdot \partial^2 \ln Z / \partial \mu_s^2$
Polyakov-loop renormalization procedure: Aoki, Fodor, Katz, Szabo: PLB643 46 (2006)

continuum behaviour can be given for both observables




overall scale (lattice spacing, thus also T) is set by $f_K$

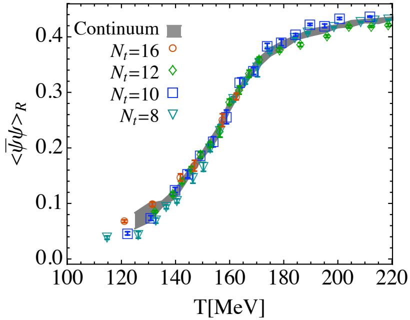
# renormalized chiral condensate

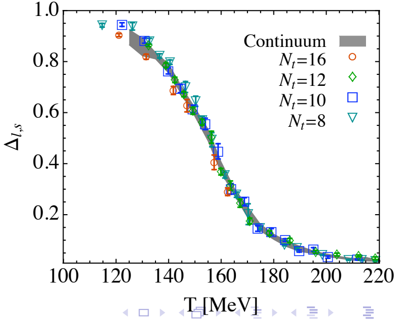
$$\langle\bar\psi\psi\rangle_R = -\left[\langle\bar\psi\psi\rangle_{l,T} - \langle\bar\psi\psi\rangle_{l,0}\right]\frac{m_l}{X^4}$$

X can be chosen as $m_\pi$

$$\Delta_{l,s} = \frac{\langle\bar\psi\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,T}}{\langle\bar\psi\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,0}}$$

$\Delta_{l,s}$ (strange subtraction)



# $T_c$ summary of the Wuppertal-Budapest group

list of pseudocritical temperatures (various observables)

| | $\chi_{\bar\psi\psi}/T^4$ | $\Delta_{l,s}$ | $\langle\bar\psi\psi\rangle_R$ | $\chi_2^s/T^2$ | $\epsilon/T^4$ | ($\epsilon$-3p)/$T^4$ |
|---|---|---|---|---|---|---|
| WB'10 | 147(2)(3) | 157(3)(3) | 155(3)(3) | 165(5)(3) | 157(4)(3) | 154(4)(3) |
| WB'09 | 146(2)(3) | 155(2)(3) | - | 169(3)(3) | - | - |
| WB'06 | 151(3)(3) | - | - | 175(2)(4) | - | - |

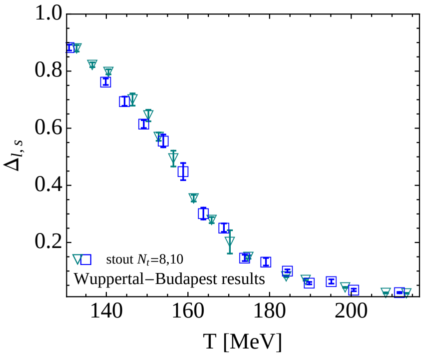
all numbers (in a given coloumn) are in complete agreement
different variables give different pseudocritical $T_c$-s: 147–165 MeV
reason: the transition is a broad one with 30-40 MeV broadness

3% shift to lower values between 2006 and 2009
reason: 3% experimental change in $f_K$ (no change in lattice results)

# progress in the transition temperature

Wuppertal-Budapest: physical quark masses ($m_s/m_{ud} \approx$28)
gauge configs: $N_t$=8,10 in 2006 $\Rightarrow$ $N_t$=12 in 2009 $\Rightarrow$ $N_t$=16 in 2010

hotQCD 2009: realistic quark masses ($m_s/m_{ud}$ =10)
hotQCD 2010 preliminary: physical quark masses ($m_s/m_{ud}$ =20)



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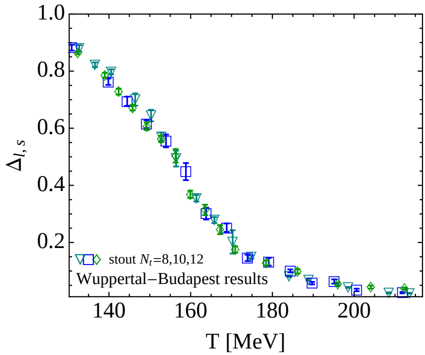

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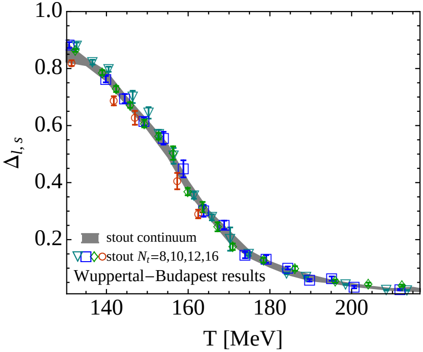

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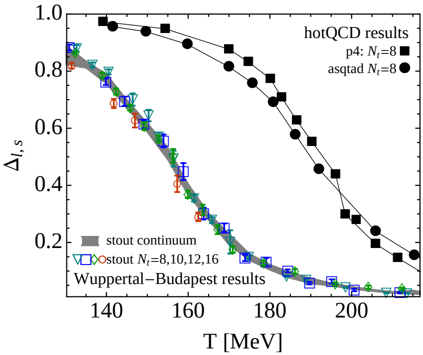

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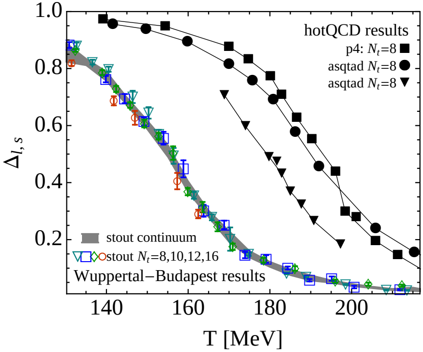

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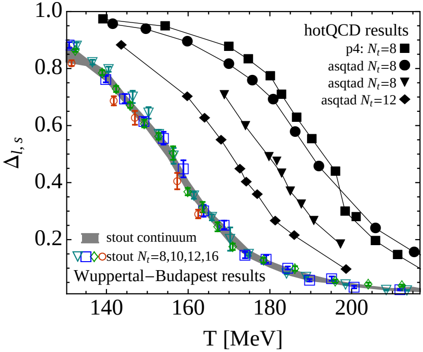
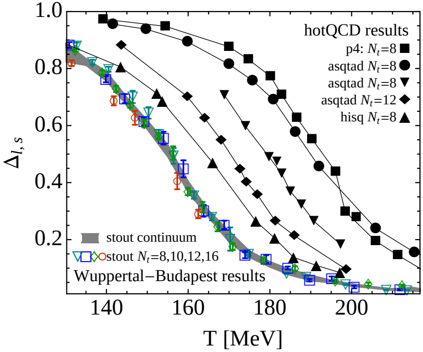
hotQCD 2010 preliminary: physical quark masses ($m_s/m_{ud}$ =20)



# progress in transition temperature

Wuppertal-Budapest: physical quark masses ($m_s/m_{ud} \approx$28)
gauge configs: $N_t$=8,10 in 2006 ⇒ $N_t$=12 in 2009 ⇒ $N_t$=16 in 2010

hotQCD 2009: realistic quark masses ($m_s/m_{ud}$ =10)
hotQCD 2010 preliminary: physical quark masses ($m_s/m_{ud}$ =20)



# Equation of state: integral method

J. Engels et al., Phys. Lett. B252 (1990) 625

on the lattice the dimensionless pressure is given by

$$p^{\rm lat}(\beta, m_q) = (N_t N_s^3)^{-1} \log \mathcal{Z}(\beta, m_q)$$

not accessible using conventional algorithms, only its derivatives

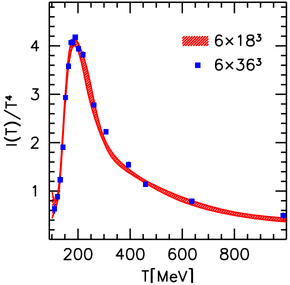
$$p^{\rm lat}(\beta, m_q) - p^{\rm lat}(\beta^0, m_q^0) = (N_t N_s^3)^{-1} \int_{(\beta^0, m_q^0)}^{(\beta, m_q)} \left( d\beta \frac{\partial \log \mathcal{Z}}{\partial \beta} + dm_q \frac{\partial \log \mathcal{Z}}{\partial m_q} \right)$$

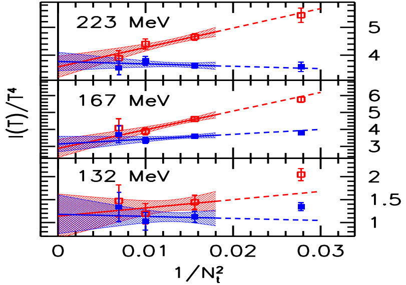
first term: gauge action & second term: chiral condensate

the pressure has to be renormalized: subtraction at T=0 (or T$>$0)

T$\neq$0 simulations can't go below T$\approx$100 MeV (lattice spacing is large)
physical HRG gives here 5% contribution of SB $\Rightarrow$
path of $M_\pi$=720 MeV $\Rightarrow$ distorted HRG no contribution at T=100 MeV

# Finite volume and discretization effects



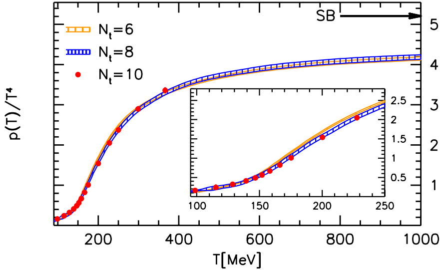



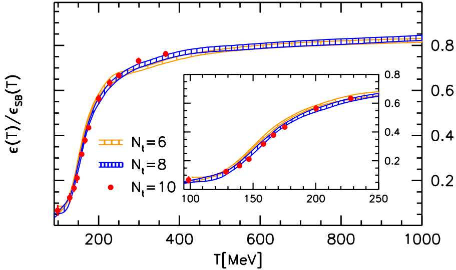

finite V: $N_s/N_t$=3 and 6 (8 times larger volume): no sizable difference

finite a: improvement program of lattice QCD (action & observables)
tree-level improvement for p (thermodynamic relations fix the others)
trace anomaly for three T-s: high T, transition T, low T
continuum limit $N_t$=6,8,10,12: same with or without improvement

improvement strongly reduces cutoff effects: slope $\approx$0 (1-2$\sigma$ level)

# Pressure and energy density

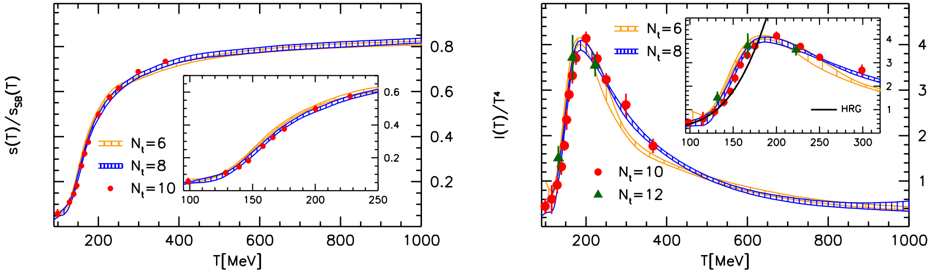



$\epsilon$ normalized to the Stefan-Boltzmann limit: $\epsilon(T \rightarrow \infty)$=15.7
at 1000 MeV still 20% difference to the Stefan-Boltzmann value

essentially perfect scaling, lines/points are lying on top of each other

# Entropy and trace anomaly



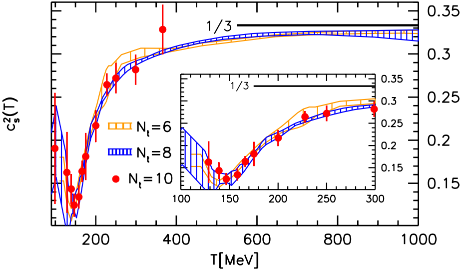

good agreement with the HRG model up to the transition region

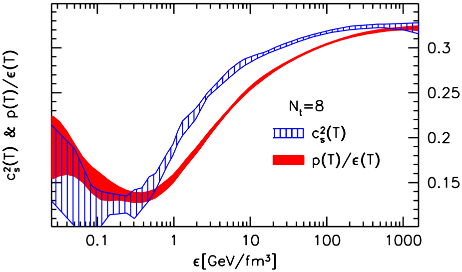
$T_c$ can be defined as the inflection point of the trace anomaly

| | |
|---|---|
| Inflection point of $I(T)/T^4$ | 154(4) MeV |
| $T$ at the maximum of $I(T)/T^4$ | 187(5) MeV |
| Maximum value of $I(T)/T^4$ | 4.1(1) |

agreement with Aoki, Fodor, Katz, Szabo, JHEP 0601, 089 (2006) [arXiv:hep-lat/0510084]

# Speed of sound & parametrization



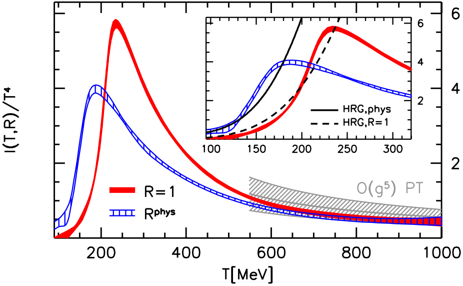



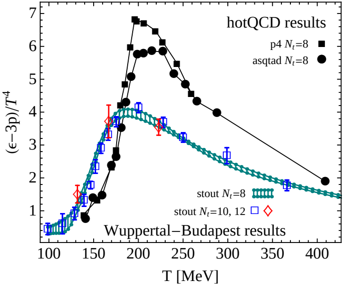

$c_s$ minimum value is about 0.13 at T≈145 MeV
'smaller than error' parametrization T=100...1000 MeV (t=T/200 MeV)

$$\frac{I(T)}{T^4} = \exp(-h_1/t - h_2/t^2) \cdot \left( h_0 + \frac{f_0 \cdot [\tanh(f_1 \cdot t + f_2) + 1]}{1 + g_1 \cdot t + g_2 \cdot t^2} \right)$$

| $h_0$ | $h_1$ | $h_2$ | $f_0$ | $f_1$ | $f_2$ | $g_1$ | $g_2$ |
|---|---|---|---|---|---|---|---|
| 0.1396 | -0.1800 | 0.0350 | 2.76 | 6.79 | -5.29 | -0.47 | 1.04 |

# Equation of state: I(T)=$\epsilon$-3p




two pion masses: $M_\pi \approx$720 MeV (R=1) and $M_\pi$=135 MeV ($R^{phys}$)
good agreement with the HRG model up to the transition region
quark mass dependence disappears for high T
good agreement with perturbation theory

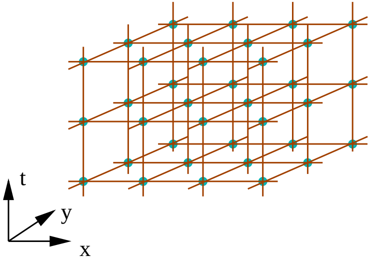
comparison with the published results of the hotQCD collaboration
discrepancy: peak at ≈20 MeV larger T and ≈50% higher

# Summary

- lattice QCD arrived to the "productive phase"
  algorithmic developments: Berlin Wall has fallen
  physical $m_q$ ($M_\pi$) are possible with a=0.05 fm & L=6 fm
  we can control all systematic uncertainites

- full result for the light hadron spectrum
  all systematic uncertainties are controlled:
  action, algorithm, a$\rightarrow$0, L$\rightarrow\infty$, $M_\pi\rightarrow$135 MeV
  both stable particles & resonances can be appropriately treated

- QCD at nonvanishing temperatures: analytic cross-over
  long standing discrepancy in the literature
  overall scale clarified (equation of state needs more time)




fine lattice to resolve the structure of the proton ($\lesssim$0.1 fm)
few fm size is needed
50-100 points in ‘xyz/t’ directions
$a \Rightarrow a/2$ means 100-200$\times$CPU

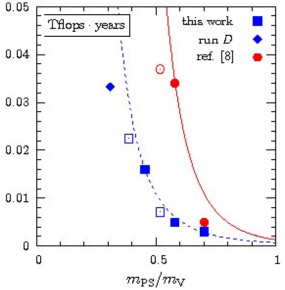
mathematically
$10^9$ dimensional integrals

advanced techniques,
good balance and
several Tflops are needed

# Difficulties of full dynamical calculations

though the quenched result can be qualitatively correct
uncontrolled systematics ⇒ full "dynamical" studies
by two-three orders of magnitude more expensive (balance)
present day machines offer several hundreds of Tflops

no revolution but evolution in the algorithmic developments
Berlin Wall '01: it is extremely difficult to reach small quark masses:



# Discretization errors in the transition region

we always have discretization errors: nothing wrong with it as long as

a. result: close enough to the continuum value (error subdominant)
b. we are in the scaling regime ($a^2$ in staggered)

various types of discretization errors $\Rightarrow$ we improve on them (costs)

we are speaking about the transition temperature region
interplay between hadronic and quark-gluon plasma physics
smooth cross-over: one of them takes over the other around $T_c$

both regimes (low T and high T) are equally important
improving for one: T$\gg T_c$, doesn't mean improving for the other: T$<T_c$

example: 'expansion' around a Stefan-Boltzman gas (van der Waals)
for water: it is a fairly good description for T$\gtrsim 300^o$
claculate the boiling point: more accuracy needed for the liquid phase

# Further advantages of the action

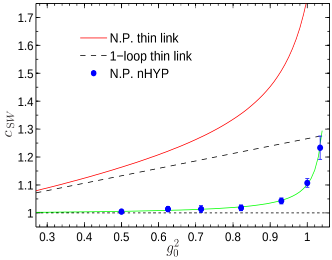
smallest eigenvalue of M: small fluctuations
⇒ simulations are stable (major issue of Wilson fermions & speedup)

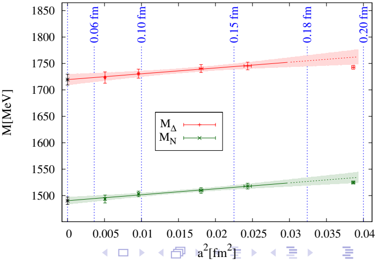
non-perturbative improvement coefficient: ≈ tree-level (smearing)

R. Hoffmann, A. Hasenfratz, S. Schaefer, PoS **LAT2007** (2007) 1 04

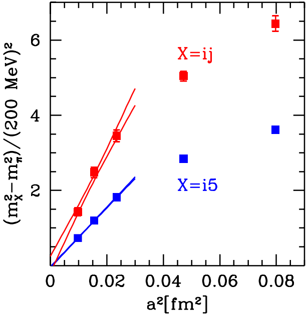
good $a^2$ scaling of hadron masses ($M_\pi/M_\rho$=2/3) up to $a$≈0.2 fm

S. Dürr et al. [Budapest-Marseille-Wuppertal Collaboration] Phys. Rev. D79, 014501 (2009)



# Scaling for the pion splitting




scaling regime is reached if $a^2$ scaling is observed

asymptotic scaling starts only for $N_t \gtrsim 8$ (a $\lesssim$ 0.15 fm): two messages

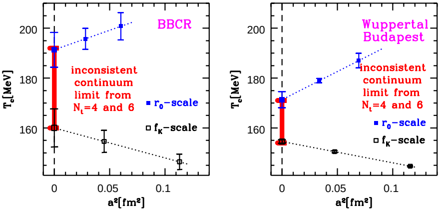
a. $N_t$=8,10 extrapolation gives 'p' on the $\approx$1% level: good balance

b. stout-smeared improvement is designed to reduce this artefact

most other actions need even smaller '$a$' to reach scaling

# Consequences of the non-scaling behaviour

for large '$a$' no proper $a^2$ scaling (e.g. due to large $m_\pi$ splitting)
how do we monitor it, how to be sure being in the scaling regime?
dimensionless combinations in the a→0 limit:
$T_c r_0$ or $T_c/f_K$ for the remnant of the chiral transition




$N_t$=4,6: inconsistent continuum limit
$N_t$=6,8,10: consistent continuum limit (stout-link improvement)

independently which quantity is taken one obtains the same $T_c$
signal: extrapolation is safe, we are in the $a^2$ scaling regime

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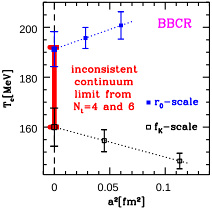


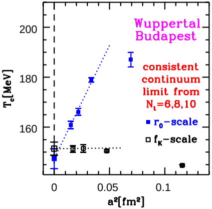


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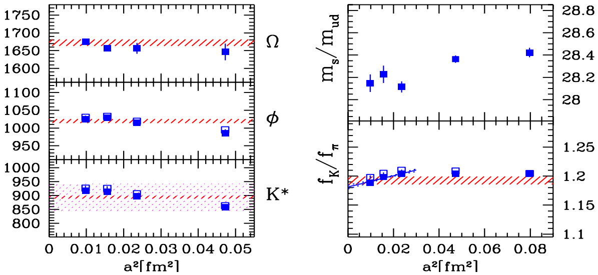
independently which quantity is taken one obtains the same $T_c$
signal: extrapolation is safe, we are in the $a^2$ scaling regime

# Setting the scale in lattice QCD

in meteorology, aircraft industry etc. grid spacing is set by hand

in lattice QCD we use g,$m_{ud}$ and $m_s$ in the Lagrangian ('a' not)
measure e.g. the vacuum mass of a hadron in lattice units: $M_\Omega a$
since we know that $M_\Omega$=1672 MeV we obtain 'a' and T=1/$N_t$a

Y.Aoki et al. [Wuppertal-Budapest Collaboration] arXiv:0903.4155

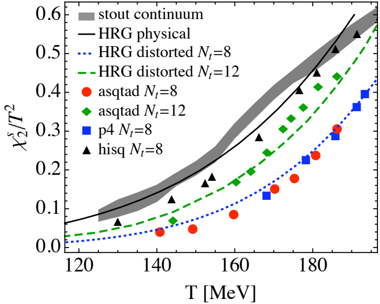



independently which quantity is taken (we used physical masses)

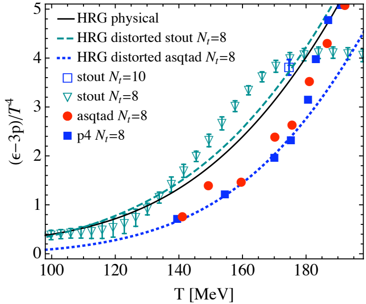
⇒ one obtains the same '*a*' and T, result is safe

# compare with the hadron resonance gas model: HRG

strange quark number susceptibility




trace anomaly




Wuppertal-Budapest: test of HRG (agrees with the continuum result)

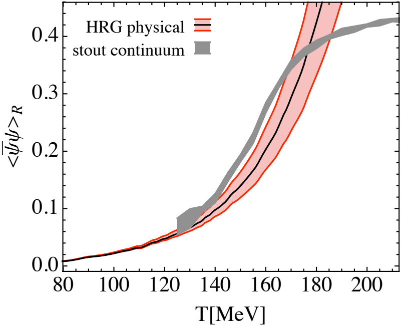
P. Huovinen, P. Petreczky, arXiv:1005.0324 use heavier than physical hadrons in HRG

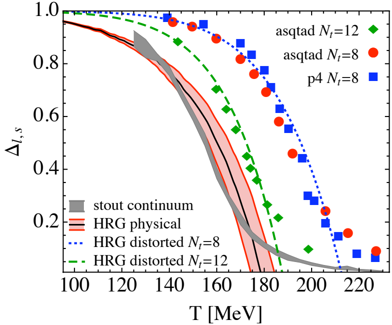
hotQCD: agreement only with the distorted spectrum

(our splittings and hadron spectrum gives minimal change: EoS)

though their results are gradually getting closer to ours

# temperature dependence of the chiral condensate




Wuppertal-Budapest: good agreement with the physical HRG

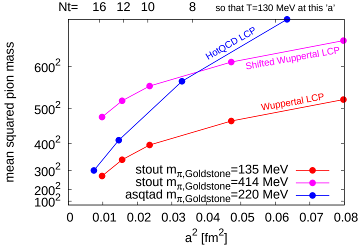
Borsanyi, Fodor, Hoelbling, Katz, Krieg, Ratti, Szabo, arXiv:1005.3508

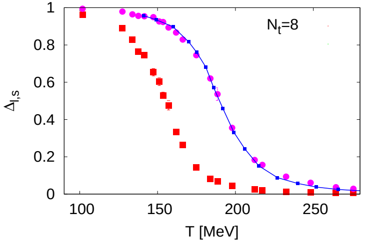
hotQCD: agreement only with the distorted spectrum
though their results are gradually getting closer to ours

# Illustration: lattice artefacts due to pion splitting

we have seen: our action (WB) has less unphysical pion splitting than the asqtad (MILC) and far less than the p4 (Bielefeld) action

in the continuum limit: no problem; at $a \neq 0$ it mimics larger $M_\pi$ "reproduce" the result of hotQCD with larger $M_\pi$ (asqtad is better)

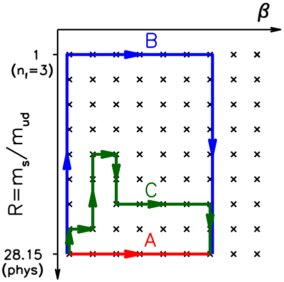



$M_\pi \approx 220$ MeV (hotQCD) "corresponds" to $M_\pi \approx 410$ MeV (WB) asqtad (MILC) needs finer p4 (Bielefeld) needs much finer lattices in order to handle physical quark masses

# All path approach

goal: determine the equation of state for several pion masses
reduce the uncertainty related to the choice of $\beta^0$
give the uncertainty related to the integration path



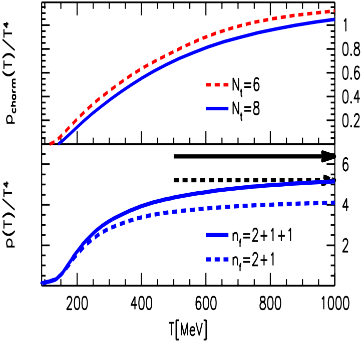

conventional path: A, though B, C or any other paths are possible
generalize: take all paths into account (use derivatives of p)
two-dimensional spline function gives p for any ($\beta$,R=$m_s/m_{ud}$)
technically: solution of a large system of linear equations

# Charm contribution

perturbative indications: important already at $2{\cdot}T_c$

M. Laine and Y. Schroder, Phys. Rev. D73 (2006) 085009

determine it within the partially quenched framwork: $m_c/m_s$=11.85




charm contribution is indeed non-negligeble from 200 MeV

one has to extend this observation to the dynamical case