Theory of the transverse momentum dependent (unintegrated) parton densities:

recent progress

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Based on collaboration with
N.G Stefanis *(RUHR-U., BOCHUM$_{\mathrm{DE}}$)*
and **A.I. Karanikas** *(ATHENS U.$_{\mathrm{GR}}$)*

- Transverse-momentum dependent (unintegrated) parton densities (TMD) accumulate information about intrinsic (longitudinal and transverse) motion of partons inside a hadron

- TMD can be defined as a natural generalization of the integrated collinear PDF: $\rightarrow$ `Collins; Soper (1979)`

$$P_{[\mathrm{collinear}]}(x, \mu) \ \rightarrow \ \mathcal{P}_{[\mathrm{tmd}]}(x, \boldsymbol{k}, \mu, \zeta)$$

—depends on the longitudinal $x = k^+/p^+$ and transverse $\boldsymbol{k}_\perp$ components of parton's momentum, as well as on the UV scale μ and rapidity cutoff ζ

- collinear PDFs (are expected to) restore after $\boldsymbol{k}_\perp$-integration

$$P_{[\mathrm{collinear}]}(x, \mu) \sim \int d^2k_\perp \ \mathcal{P}_{[\mathrm{tmd}]}(x, \boldsymbol{k}, \mu, \zeta)$$

Looks nice, but: the case is much more intricate than one could imagine 30 years ago!

WHY TMD?

- Nonperturbative domain of QCD: quark models, lattice, nonperturbative vacuum, instantons;
- Factorization and evolution;
- Gauge invariance;
- Structure of nucleon;
- Spin-related phenomena;
- Flavor structure

CURRENT and PLANNED "TMD" EXPERIMENTS

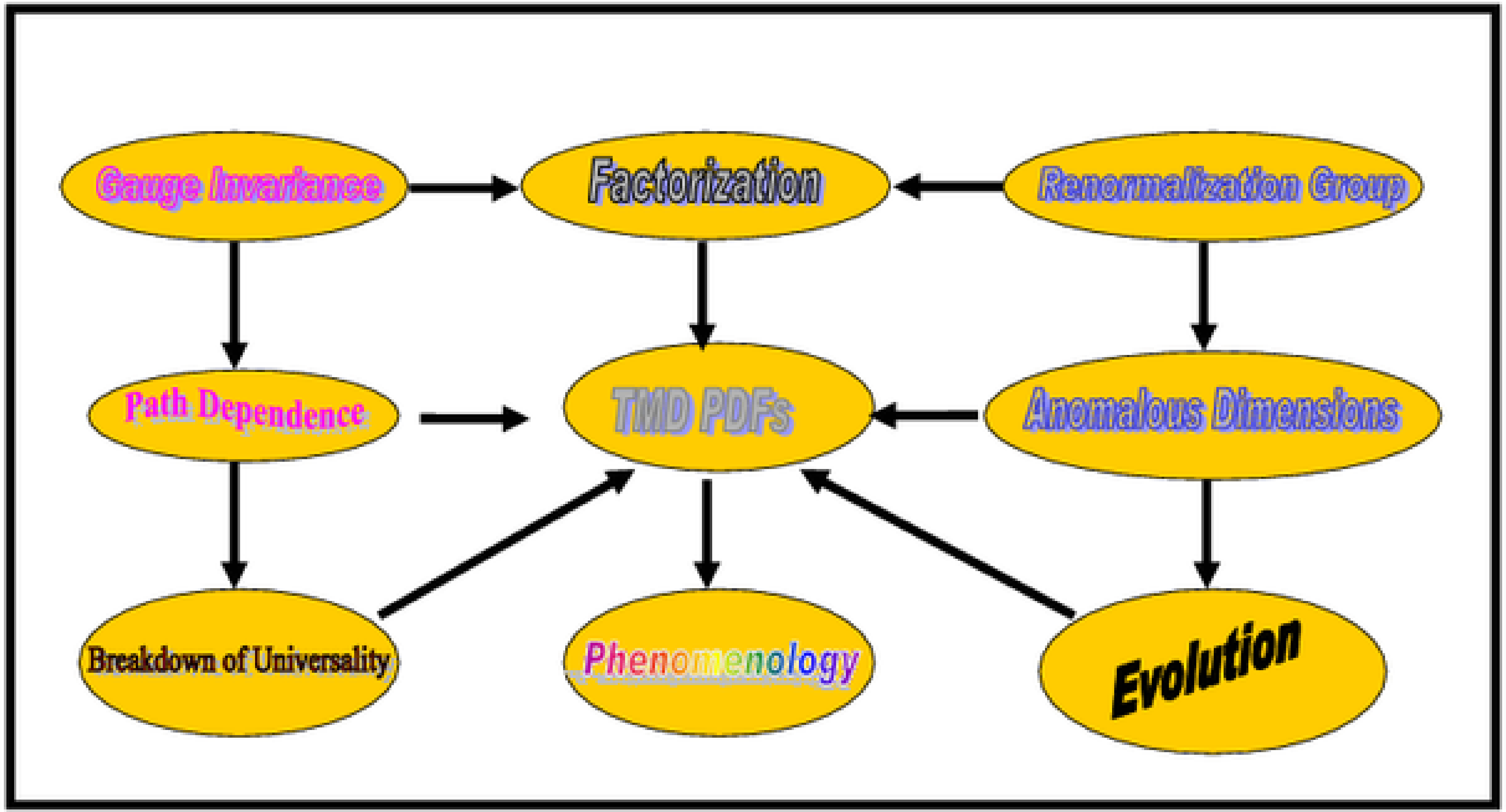
- **SIDIS process** $lH^{\uparrow} \to l'hX$: HERMES, COMPASS, JLab, EIC. To be studied: Sivers, Collins, transversity, Boer-Mulders, unpolarized X-sections, etc.

- **DY process** $H_1^{(\uparrow)} H_2^{\uparrow} \to l^+ l^- X$: COMPASS, PAX, GSI, RHIC. To be studied: distribution functions, transversity.

- **Hadron collisions** $H_1^{(\uparrow)} H_2^{\uparrow} \to l^+ l^- X$: RHIC

- $e^+ e^- \to h_1 h_2 X$: BELLE, BaBar

TMD ROADMAP (created by N. Stefanis)



- INCLUSIVE PROCESSES (DIS)

$$W_{\mu\nu} = \frac{1}{2\pi} \Im m \left[i \int d^4\xi \, \mathrm{e}^{iq\xi} \langle P | T\{ J_\mu(\xi) J_\nu(0) \} | P \rangle \right]$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x_B, Q^2) + \frac{1}{P \cdot q} \left(P_\mu - q_\mu \frac{P \cdot q}{q^2} \right) \left(P_\nu - q_\nu \frac{P \cdot q}{q^2} \right) F_2(x_B, Q^2)$$

- QCD FACTORIZATION in DIS

$$F(x_B, Q^2) = H(x_B, Q^2/\mu^2) \otimes F_D(\mu^2) = \sum_i \int_{x_B}^1 \frac{d\xi}{\xi} C_i \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2} \right) F_D^i(\xi, \mu^2)$$

- RENORMALIZATION PROPERTIES: DGLAP

$$\mu \frac{d}{d\mu} Q_{i/h}(x, \mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij} \left(\frac{x}{z} \right) Q_{j/h}(x, \mu)$$

- MOMENTS of collinear PDFs are related to matrix elements of the local twist-2 operators within the OPE

Completely gauge invariant (quark) density:

$$Q_{i/h}(x,\mu) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, \mathrm{e}^{-ik^+\xi^-} \langle h(P)|\bar{\psi}_i(\xi^-,\mathbf{0}_\perp)[\xi^-,0^-]\gamma^+\psi_i(0^-,\mathbf{0}_\perp)|h(P)\rangle$$

Gauge invariance is saved by the insertion of the gauge link

$$[y,x]_r = \mathcal{P}\exp\left[-ig\int_{\tau_1}^{\tau_2} d\tau r^\mu A_\mu^a(r\tau)t^a\right] \quad r^\mu\tau_1 = x\ ,\ r^\mu\tau_2 = y$$

Note: distinguish between longitudinal $[\ ,\ ]_{[n,\,v,\,v_0]}$ and transversal $[\ ,\ ]_{[\boldsymbol{l}]}$ gauge links!

QCD FACTORIZATION for TMD

$\longrightarrow$ Collins, Soper: NPB (1981);
$\longrightarrow$ Collins, Metz: PRL (2004);
$\longrightarrow$ Ji, Ma, Yuan: PRD (2005);
$\longrightarrow$ Bacchetta, et al: PRD (2005), EPJC (2006)

Standard factorization expected:

$$F(x_B, z_h, \boldsymbol{P}_{h\perp}, Q^2) = \sum_i e_i^2 \cdot H \otimes \mathcal{F}_D \otimes \mathcal{F}_F \otimes S$$

- Extra (rapidity) divergences;
- Complicated structure of gauge links: non-universality (generalized factorization):
  $\longrightarrow$ Bacchetta, Bomhof, Mulders, Pijlman
- Generalized factorization may fail: several counter-examples exist
  $\longrightarrow$ Collins, Qiu; Mulders, Rogers

"Naive" definition:

$$\mathcal{Q}_i(x, \boldsymbol{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \; \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp}.$$

$$\cdot \langle P, S | \bar{\psi}_i(\xi^-, \xi_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp;]^\dagger_{[n]} [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger_{\boldsymbol{l}} \cdot$$

$$\cdot \, \gamma^+ \, \cdot$$

$$\cdot [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp]_{\boldsymbol{l}} [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]_{[n]} \psi_i(0^-, 0_\perp) | P, S \rangle \, |_{\xi^+=0}$$

Formally:

$$\int d^2k_\perp \mathcal{Q}_i(x, \boldsymbol{k}_\perp) = Q_i(x)$$

CLASSIFICATION of SINGULARITIES

1. $\sim \frac{1}{\varepsilon}$ poles, usual UV-singularities: removed by the standard $R-$operation and are controlled by renormalization-group evolution equations (DGLAP in integrated case)

2. pure rapidity divergences: give rise to logarithmic and double-logarithmic terms of the form $\sim \ln \eta\ ,\ \ln^2 \eta$; have to be resumed

3. overlapping divergences: contain both UV and soft singularities simultaneously $\sim \frac{1}{\varepsilon} \ln \eta$; highly undesirable—depend on the parameters of the chosen gauge; prevents the removal of *all* UV-singularities by the standard $R-$procedure; a special *generalized* renormalization procedure is needed

EXAMPLE: three different definitions of a unintegrated quark distribution:

A. *pure light-cone* $\mathcal{F}_{[n]}$: $[\, n^2 = 0 \;,\; n^+ = 0 \;,\; \boldsymbol{n}_\perp = 0]$

$$\mathcal{F}_{[n]}\,(x, \boldsymbol{k}_\perp; \mu, \eta) =$$

$$= \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2}\, \mathrm{e}^{-ik\xi} \cdot \langle\, h\,|\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp)\, [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]_n^\dagger [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]_{\boldsymbol{l}}^\dagger \gamma^+$$

$$[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp]_{\boldsymbol{l}} [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]_n\, \psi(0^-, \mathbf{0}_\perp)\,|\, h\rangle$$

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B. *off-light-cone* $\mathcal{F}_{[v]}$: $\left[\, v^2 > 0 \;,\; v^- \gg v^+ \;,\; \boldsymbol{v}_\perp = 0 \right] \;,\; \zeta = \frac{4(P\cdot v)^2}{v^2}$

$$\mathcal{F}_{[v]}\left(x, \boldsymbol{k}_\perp; \mu, \zeta\right) =$$

$$= \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \, \mathrm{e}^{-ik\xi} \langle\, h \,|\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp) \, [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]_v^\dagger [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]_{\boldsymbol{l}}^\dagger \gamma^+$$

$$[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{0}_\perp]_{\boldsymbol{l}} [\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp]_v \, \psi(0^-, \boldsymbol{0}_\perp) \,|\, h\rangle$$

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Γ. *direct link* $\mathcal{F}_{[v_0]}$: $\left[\, v_0^2 = \boldsymbol{v}_0^2 < 0 \;,\; v^+ = 0 \;,\; ,\; \zeta = \frac{4(P\cdot v_0)^2}{v_0^2} \right]$

$$\mathcal{F}_{[v_0]}\,(x, \boldsymbol{k}_\perp; \mu, \zeta_0) =$$

$$= \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2}\, \mathrm{e}^{-ik\xi} \langle\, h\,|\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp)\,\gamma^+\,[\xi^-, \boldsymbol{\xi}_\perp; 0^-, \mathbf{0}_\perp]_{v_0}\,\psi(0^-, \mathbf{0}_\perp)\,|\,h\rangle$$

ONE – GLUON ORDER ANOMALOUS DIMENSIONS

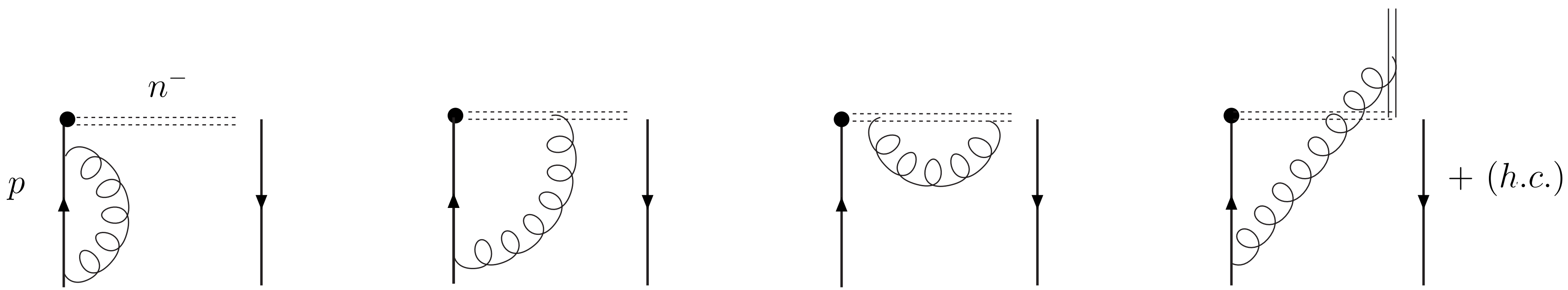
→ `ICh, Stefanis: 2008,2009`

Tree approximation (distribution of *quark in a quark*):

$$\mathcal{F}^{(0)}(x, \boldsymbol{k}_\perp) =$$

$$= \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} \langle p | \bar{\psi}(\xi^-, \xi_\perp) \gamma^+ \psi(0^-, 0_\perp) | p \rangle =$$
$$= \delta(1-x)\delta^{(2)}(\boldsymbol{k}_\perp)$$

One-gluon exchanges, contributing to the UV-divergences:



(a) (b) (c) (d)

Source of the uncertainties and extra divergences: pole in the gluon propagator

$$D^{\mu\nu}_{\rm LC}(q) = \frac{1}{q^2}\left[g^{\mu\nu} - \frac{q^\mu n^{-\nu}}{[q^+]} - \frac{q^\nu n^{-\mu}}{[q^+]}\right]$$

Possible pole prescriptions:

$$d^{\mu\nu}_{\rm PV}(q) = -(q^\mu n^{-\nu} + q^\nu n^{-\mu})\frac{1}{2}\left(\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta}\right)$$

$$d^{\mu\nu}_{\rm Adv/Ret}(q) = -(q^\mu n^{-\nu} + q^\nu n^{-\mu})\frac{1}{q^+ \mp i\eta}$$

Mandelstam-Leibbrand prescription

$$\frac{1}{[q^+]_{\rm ML}} = \begin{cases} \frac{1}{q^+ + i0q^-} \\ \frac{q^-}{q^+q^- + i0} \end{cases}$$

UV divergent part reads:

$$\Sigma_{\rm left}^{UV}(p,\alpha_s;\epsilon) =$$

$$= -\frac{\alpha_s}{\pi} C_{\rm F}\, \frac{1}{\epsilon}\left[-\frac{3}{4} - \ln\frac{\eta}{p^+} + \frac{i\pi}{2} + i\pi\, C_\infty\right] + \alpha_s C_{\rm F}\ \frac{1}{\epsilon}\left[iC_\infty\right] =$$

$$= -\frac{\alpha_s}{\pi} C_{\rm F}\, \frac{1}{\epsilon}\left[-\frac{3}{4} - \ln\frac{\eta}{p^+} + \frac{i\pi}{2}\right]$$

prescription dependence is canceled

$$C_\infty = \left\{ \begin{array}{l} 0\ ,\ \ \text{Advanced}: \frac{1}{[q^+]} = \frac{1}{q^+ - i\eta} \\ -1\ ,\ \ \text{Retarded}: \frac{1}{[q^+]} = \frac{1}{q^+ + i\eta} \\ -\frac{1}{2}\ ,\ \ \text{PrincipalValue}: \frac{1}{[q^+]} = \frac{1}{2}\left(\frac{1}{q^+ - i\eta} + \frac{1}{q^+ + i\eta}\right) \end{array} \right.$$

Full UV divergent part:

$$\Sigma_{\text{tot}}(p, \alpha_s(\mu); \epsilon) = \Sigma_{\text{left}} + \Sigma_{\text{right}} =$$

$$= -\frac{\alpha_s}{4\pi} C_{\text{F}} \frac{2}{\epsilon} \left(-3 - 4 \ln \frac{\eta}{p^+} \right)$$

Dependence on η remains:

- gauge invariance is not complete
- AD doesn't coincide with AD_{2q}

LO anomalous dimension is defined via the renormalization constant

$$\gamma = \frac{1}{2}\,\frac{1}{Z^{(1)}}\,\mu\frac{\partial \alpha_s(\mu)}{\partial \mu}\,\frac{\partial Z^{(1)}(\mu,\alpha_s(\mu);\epsilon)}{\partial \alpha_s}$$

$$\gamma_{\rm LC} = \gamma_{\rm smooth} - \delta\gamma \quad , \quad \gamma_{\rm smooth} = \frac{3}{4}\frac{\alpha_s}{\pi}C_{\rm F} + O(\alpha_s^2)$$

Defect of anomalous dimension

$$\delta\gamma = -\frac{\alpha_s}{\pi}C_{\rm F}\;\ln\frac{\eta}{p^+}$$

contains undesirable p^+-dependent term: must be removed by a consistent procedure.

$\delta\gamma$ is nothing else, but the cusp anomalous dimension:

$$p^+ = (p \cdot n^-) \sim \cosh \chi$$

defines an angle χ between the direction of the quark momentum p_μ and the light-like vector n^-. In the large χ limit:

$$\ln p^+ \to \chi, \ \chi \to \infty$$

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Renormalization of the Wilson operators with obstructions requires extra renormalization factor
$\to$ Korchemsky, Radyushkin: 1987

$$Z_\chi = \left[\langle 0 | \mathcal{P} \exp \left[ig \int_\chi d\zeta^\mu \ \hat{A}^a_\mu(\zeta) \right] |0\rangle \right]^{-1}$$

Generalized renormalization

$$\mathcal{O}_{\rm ren}(\chi, ...) = Z_\chi Z_{\rm R} \, \mathcal{O}(\chi, ...)$$

One-gluon exchanges for the generalized multiplicative renormalization factor

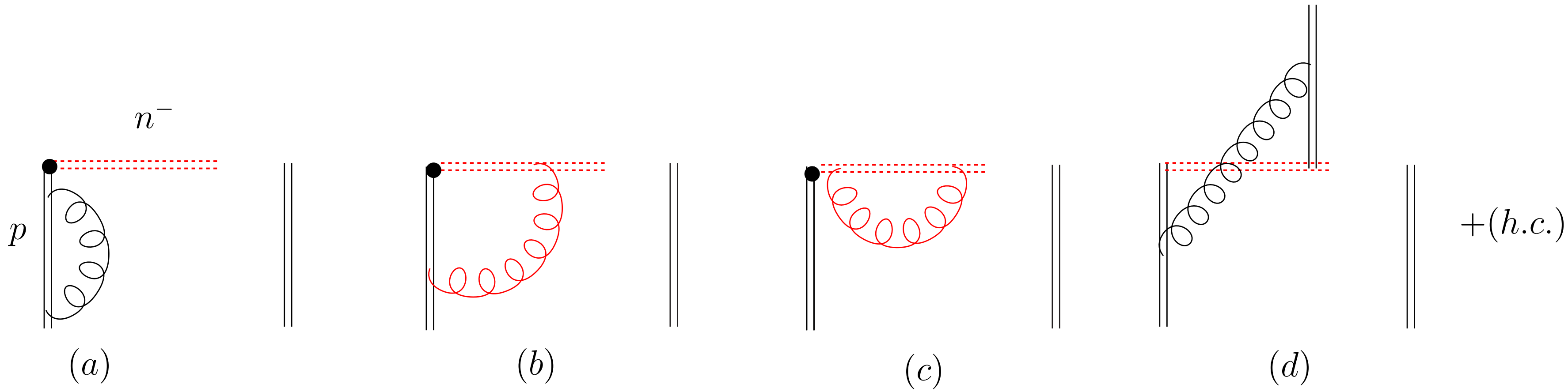
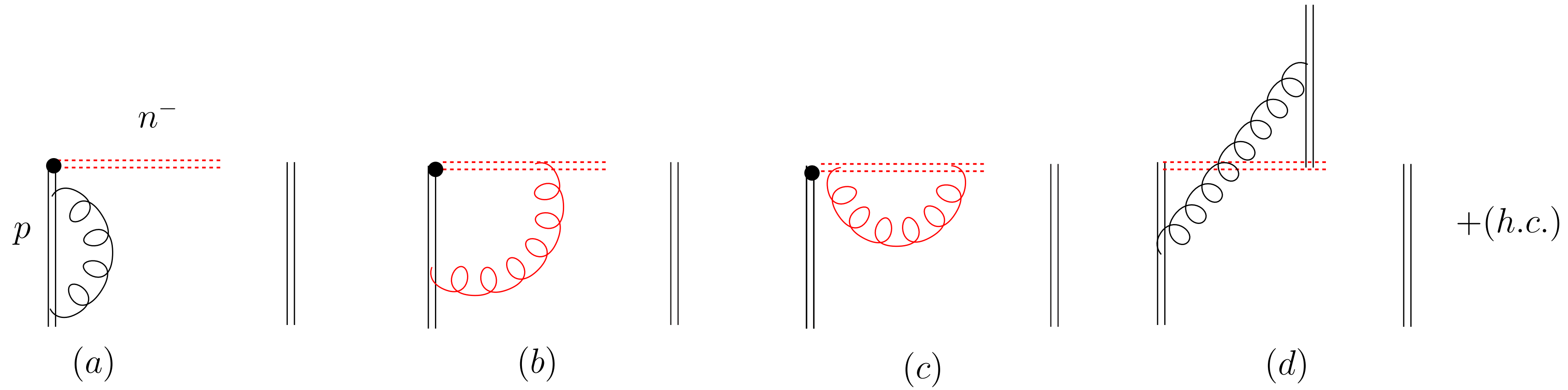


Figure 1: One-gluon exchanges for the generalized multiplicative renormalization factor

One-gluon exchanges for the generalized multiplicative renormalization factor



Generalized renormalization constant reads

$$\hat{Z}_{\rm mod} = 1 + \frac{\alpha_s}{4\pi} C_{\rm F} \frac{2}{\epsilon} \left(-3 - 4 \ln \frac{\eta}{p^+} + 4 \ln \frac{\eta}{p^+} \right) = 1 - \frac{3\alpha_s}{4\pi} C_{\rm F} \frac{2}{\epsilon}$$

so that

$$\frac{1}{2} \mu \frac{d}{d\mu} \ln \hat{Z}_{\rm mod}(\mu, \alpha_s, p^+) = \frac{3}{4} \frac{\alpha_s}{\pi} C_{\rm F} + O(\alpha_s^2)$$

Renormalization group equations

$\mathbf{B}, \mathbf{\Gamma}$. *off-light-cone* and *direct link*

$$\mu \frac{d}{d\mu} \mathcal{F}_{[v\,,\,v_0]} = \gamma_0\, \mathcal{F}_{[v\,,\,v_0]} \quad , \quad \gamma_0 = \frac{3}{4}\, \frac{\alpha_s C_{\rm F}}{\pi} + O(\alpha_s^2)$$

extraction of the soft factor

$$\mathcal{F}_{[\,v\,]} \to \mathcal{F}_{[\,v\,]} \cdot R_v^{-1}$$

$$\mu \frac{d}{d\mu} \left[\mathcal{F}_{[\,v\,]} \cdot R_v^{-1}\right] = (\gamma_0 - \gamma_{\rm R}) \left[\mathcal{F}_{[\,v\,]} \cdot R_v^{-1}\right]$$

$\gamma_{\rm R}$—anomalous dimension of the soft factor

A. *pure light-cone*

$$\mu \frac{d}{d\mu} \mathcal{F}_{[\,n\,]} = (\gamma_0 - \gamma_{\rm cusp}) \, \mathcal{F}_{[\,n\,]}$$

generalized renormalization “restores” the anomalous dimension

$$\mathcal{F}_{[n]}(\eta) \longrightarrow \mathcal{F}_{[n]}(\eta) \cdot R^{-1}(\eta)$$

$$\mu \frac{d}{d\mu} \left[\mathcal{F}_{[\,n\,]} \cdot R_n^{-1} \right] = \gamma_0 \left[\mathcal{F}_{[\,n\,]} \cdot R_n^{-1} \right]$$

A. *pure light-cone* with Mandelstam-Leibbrandt prescription

$$\mu \frac{d}{d\mu} \left[\mathcal{F}_{[n]}^{\mathrm{ML}} \cdot R_n^{-1} \right] = \mu \frac{d}{d\mu} \, \mathcal{F}_{[n]}^{\mathrm{ML}} =$$

$$\gamma_0 \left[\mathcal{F}_{[n]}^{\mathrm{ML}} \cdot R_n^{-1} \right] = \gamma_0 \, \mathcal{F}_{[n]}^{\mathrm{ML}}$$

anomalous dimension without light-cone artifacts from the very beginning!

Generalized definition of TMD:

$$\mathcal{F}_{[n]}\left(x, \boldsymbol{k}_\perp\right) \cdot R_n^{-1} =$$

$$\frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} \langle h|\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger_{[n]}$$

$$\times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger_{[\boldsymbol{l}]} \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{0}_\perp]_{[\boldsymbol{l}]} [\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp]_{[n]} \psi(0^-, \boldsymbol{0}_\perp)|h\rangle \times$$

$$\times \Big[\text{SOFT FACTOR}\Big]^{-1}$$

SOFT FACTOR $=$

$$\langle 0|\ \mathcal{P} \exp\left[ig \int_{\mathcal{C}_{\text{cusp}}} d\zeta^\mu\ t^a A^a_\mu(\zeta)\right] \cdot \mathcal{P}^{-1} \exp\left[-ig \int_{\mathcal{C}'_{\text{cusp}}} d\zeta^\mu\ t^a A^a_\mu(\xi + \zeta)\right] |0\rangle$$

Generalized-to-generalized definition of TMD: $Pauli\ Spin\ Interactions$

ICh, A.I. Karanikas, N.G. Stefanis: 2010

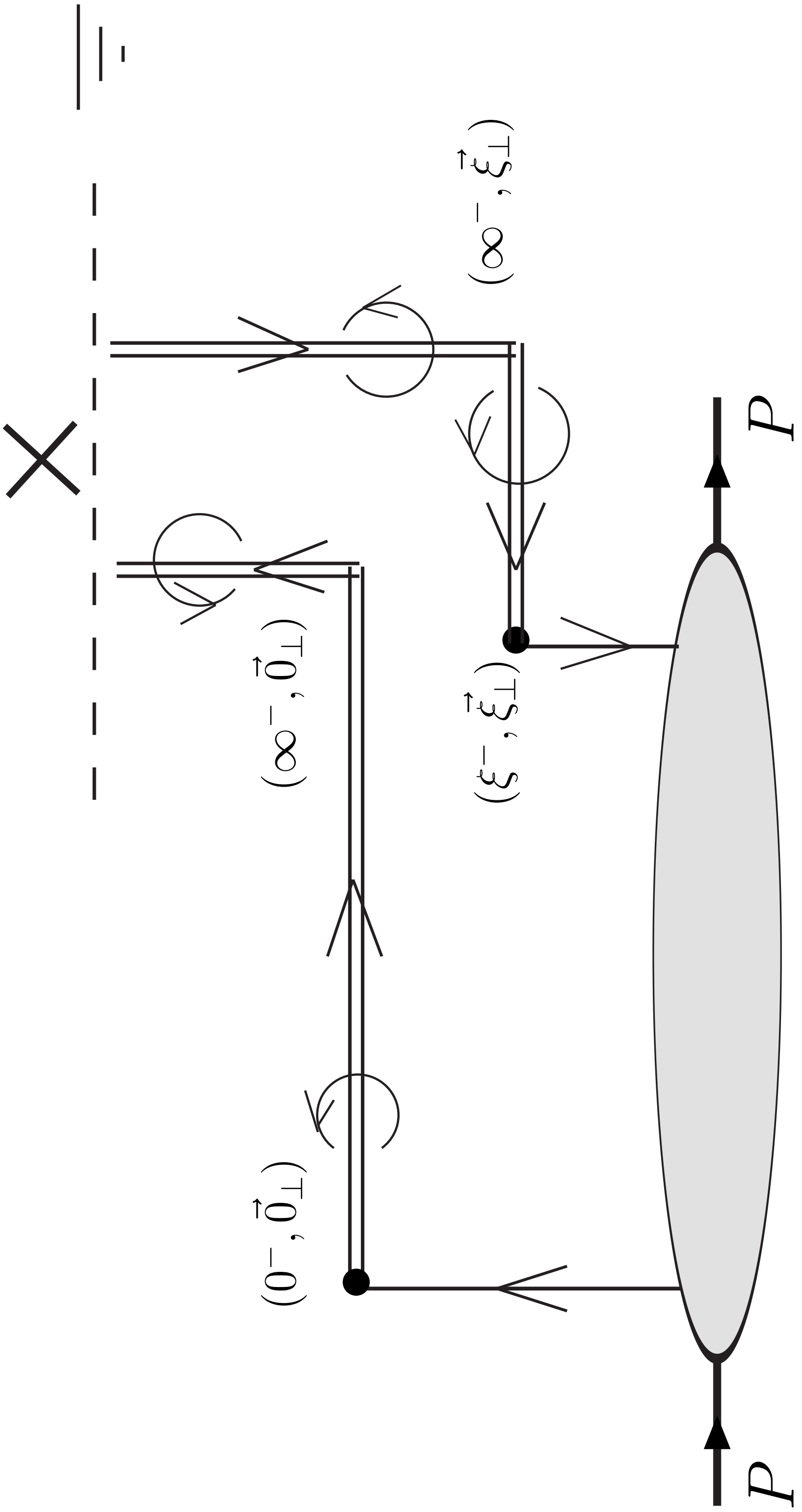
Gauge potential A_μ^a is *spin-blind* $\longrightarrow$ generalized conception of gauge invariance has to include coupling to spinning particles in terms of the Pauli term $\sim F^{\mu\nu} S_{\mu\nu}$, with the spin operator $S_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]$:

- Enhanced lightlike gauge link with Pauli term:

$$[[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]] = \mathcal{P} \exp\left[-ig\int_0^\infty d\sigma\ u_\mu A_a^\mu(u\sigma)t^a - ig\int_0^\infty d\sigma\ S_{\mu\nu} F_a^{\mu\nu}(u\sigma)t^a\right]$$

- Enhanced transverse gauge link with Pauli term:

$$[[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp]] = \mathcal{P} \exp\left[-ig\int_0^\infty d\tau \boldsymbol{l}_\perp \cdot \boldsymbol{A}_\perp^a(\boldsymbol{l}\tau)t^a - ig\int_0^\infty d\tau S_{\mu\nu} F_a^{\mu\nu}(\boldsymbol{l}\tau)t^a\right]$$



$(\infty^-, \vec{\xi}_\perp)$
$(\xi^-, \vec{\xi}_\perp)$
$(\infty^-, \vec{0}_\perp)$
$(0^-, \vec{0}_\perp)$
P
P

- Collinear PDF from TMDs:

  Definition **A** reproduces the DGLAP evolution after integration:

$$\tilde{\int} d^2\boldsymbol{k}_\perp \, \mathcal{F}_{[\,n\,]}(x, \boldsymbol{k}_\perp, \mu) = F_{[\,n\,]}(x, \mu)$$

$$\mu \frac{d}{d\mu} \, F_{[\,n\,]} = \mathcal{K}_{\mathrm{DGLAP}} \otimes F_{[\,n\,]}$$

  Definition **B** fails to reproduce the DGLAP evolution after integration:

$$\tilde{\int} d^2\boldsymbol{k}_\perp \, \mathcal{F}_{[\,v\,]}(x, \boldsymbol{k}_\perp, \mu) = F_{[\,v\,]}(x, \mu)$$

$$\mu \frac{d}{d\mu} \, F_{[\,v\,]} = \mathcal{K}_v \otimes F_{[\,v\,]} \quad , \quad \mathcal{K}_v \neq \mathcal{K}_{\mathrm{DGLAP}}$$

FULL (?) SET of the EVOLUTION EQUATIONS for TMD

- UV-evolution (in the integrated case—DGLAP)

$$\mu \frac{d}{d\mu}\, \mathcal{P}(x, \boldsymbol{k}_\perp, \mu, \zeta) = \mathcal{K}_{\rm UV} \otimes \mathcal{P}(x, \boldsymbol{k}_\perp, \mu, \zeta)$$

- rapidity evolution (Collins-Soper)

$$\zeta \frac{d}{d\zeta}\, \mathcal{P}(x, \boldsymbol{k}_\perp, \mu, \zeta) = \mathcal{K}_{\rm CS} \otimes \mathcal{P}(x, \boldsymbol{k}_\perp, \mu, \zeta)$$

- BFKL evolution $\to$ Collins-Soper?

$$x \frac{d}{dx}\, \mathcal{P}(x, \boldsymbol{k}_\perp, \mu, \zeta) = \mathcal{K}_{\rm BFKL} \otimes \mathcal{P}(x, \boldsymbol{k}_\perp, \mu, \zeta)$$

- ... ?

Current problems:

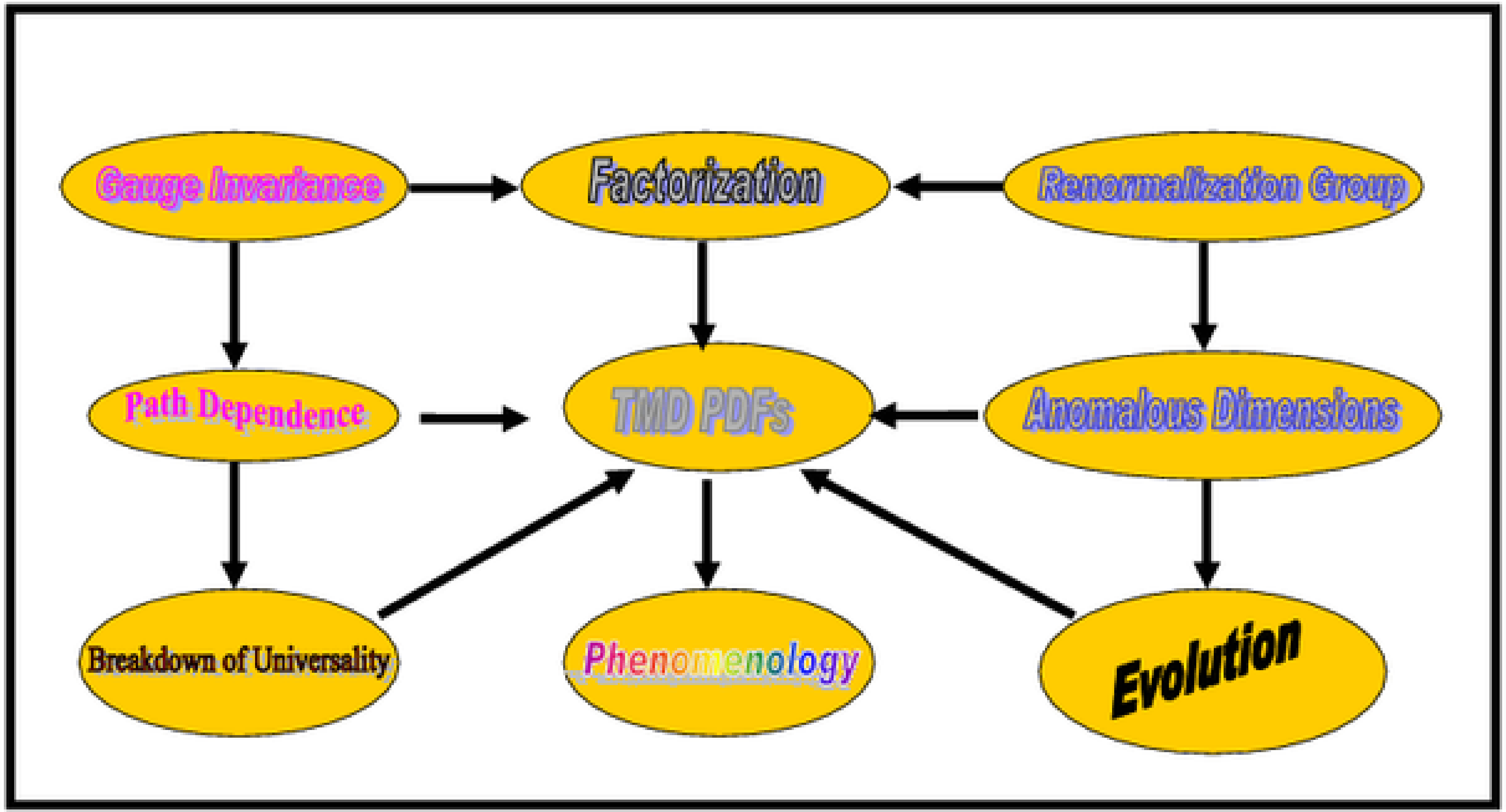
- QCD factorization within the TMD approach:

$$F_{\text{structure function}} = H_{\text{hard}} \otimes \mathcal{F}_{\text{distribution}} \otimes \mathcal{F}_{\text{fragmentation}} \otimes S_{\text{soft}}$$

—expected, but not proved so far; (some counter-examples are known)

- Operator definition of TMDs: extra $[\text{overlapping} = \text{UV} \otimes \text{rapidity}]$ divergences $\longrightarrow$ satisfactory in the one-loop order (generalized renormalization); higher orders to be studied
- Very complicated structure of gauge links $\longrightarrow$ non-universality of TMDs
- Evolution equations for TMD: (i) UV evolution, (ii) rapidity Collins-Soper evolution, (iii) small-X BFKL [?] evolution—full set of the evolution equations: `[not known so far]`
- Relationship between (unintegrated) TMDs and collinear (integrated) PDFs: `[not known so far]`
- Role of the soft factor: different within different frameworks
- Lattice simulations: problems with light-like gauge links
- ...A LOT OF WORK IN FRONT OF US!

TMD ROADMAP


Gauge Invariance
Factorization
Renormalization Group
Path Dependence
TMD PDFs
Anomalous Dimensions
Breakdown of Universality
Phenomenology
Evolution

Field-theoretic analysis of

T M D

is in progress!

ICh., A. Karanikas, N. Stefanis:

- **Nucl. Phys. B** 840 (2010) 379

ICh., N. Stefanis:

- **AIP Conf. Proc.** 1105 (2009) 327
- **Mod. Phys. Lett A** 24 (2009) 2913
- **Phys. Rev. D** 80 (2009) 054008
- **Nucl. Phys. B** 802 (2008) 146
- **Phys. Rev. D** 77 (2008) 094001
- `arXiv: 1008.0725 [hep-ph]`
- `arXiv: 0911.1031 [hep-ph]`
- `arXiv: 0811.4357 [hep-ph]`
- `arXiv: 0809.5235 [hep-ph]`
- `arXiv: 0809.1315 [hep-ph]`
- `arXiv: 0808.3390 [hep-ph]`