# Recent developments in NLO QCD calculations

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# Outline

- General motivation for NLO QCD calculations
- State-of-the-art of NLO QCD calculations @ LHC
- **HELAC-NLO** in a nutshell
- $t\bar{t}b\bar{b}$ & $t\bar{t}jj$ @ NLO QCD
  - motivations
  - integrated and differential cross sections
- Summary & Outlook

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# Introduction

- 8-10 partons in the final state @ LO, well separated to avoid divergences
- On the market automatic parton level tools which are completely self contained
- Provide amplitudes and integrators on their own
- Standard Model and beyond tools @ tree level (just few examples)

  - **ALPGEN, AMEGIC++/SHERPA, COMIX/SHERPA, HELAC-PHEGAS, MADGRAPH/MADEVENT, O'MEGA/WHIZARD, ...**

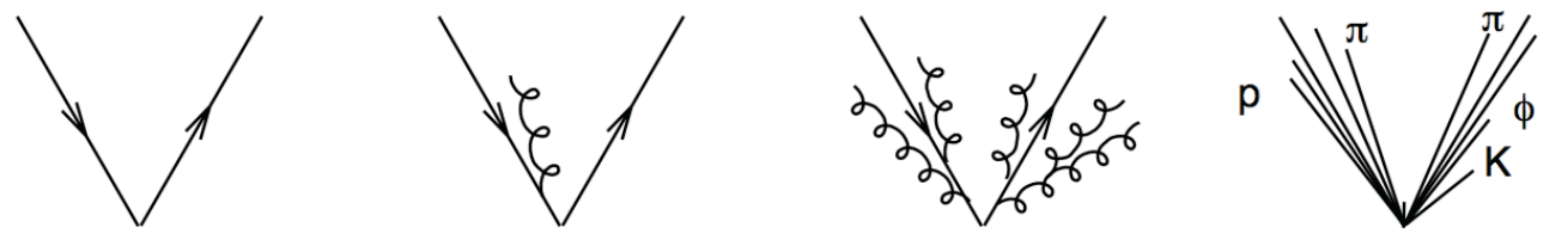
- General purpose Monte Carlo programs (parton shower, hadronisation, multiple interactions, hadrons decays, etc.)

  - **HERWIG, HERWIG++, PYTHIA 6.4, PYTHIA 8.1, SHERPA, ...**

- High sensitivity to unphysical input scales, to improve accuracy of prediction higher order calculations are needed

# Motivation for NLO

- Stabilizing the scale in the QCD input parameters most notably the strong coupling constant and PDFs
- Normalization and shape of distributions first known at NLO
- Many scale processes: V+ jets, VV + jets, ttH, tt + jets, njets ...
- Sometimes dynamical scales seem to work better for some observables
- How do we know which scale to choose ?
- Improved description of jets




*Jets:* *LO* *NLO* *Parton Shower* *Hadron Level*

# Les Houches NLO Wishlist

- NLO QCD corrections to $\mathbf{2 \to 4}$ processes is current technical frontier

| Process ($V \in \{Z, W, \gamma\}$) | Comments |
|---|---|
| Calculations completed since Les Houches 2005 | |
| 1. $pp \to VV$jet | $WW$jet completed by Dittmaier/Kallweit/Uwer [4,5]; Campbell/Ellis/Zanderighi [6]. $ZZ$jet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [7] |
| 2. $pp \to$ Higgs+2jets | NLO QCD to the $gg$ channel completed by Campbell/Ellis/Zanderighi [8]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [9,10] |
| 3. $pp \to V\,V\,V$ | $ZZZ$ completed by Lazopoulos/Melnikov/Petriello [11] and $WWZ$ by Hankele/Zeppenfeld [12] (see also Binoth/Ossola/Papadopoulos/Pittau [13]) |
| 4. $pp \to t\bar{t}\,b\bar{b}$ | relevant for $t\bar{t}H$ computed by Bredenstein/Denner/Dittmaier/Pozzorini [14,15] and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek [16] |
| 5. $pp \to V$+3jets | calculated by the Blackhat/Sherpa [17] and Rocket [18] collaborations |
| Calculations remaining from Les Houches 2005 | |
| 6. $pp \to t\bar{t}$+2jets | relevant for $t\bar{t}H$ computed by Bevilacqua/Czakon/Papadopoulos/Worek [19] |
| 7. $pp \to VV\,b\bar{b}$, | relevant for VBF $\to H \to VV$, $t\bar{t}H$ |
| 8. $pp \to VV$+2jets | relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [20–22] |
| NLO calculations added to list in 2007 | |
| 9. $pp \to b\bar{b}b\bar{b}$ | $q\bar{q}$ channel calculated by Golem collaboration [23] |
| NLO calculations added to list in 2009 | |
| 10. $pp \to V$+4 jets | top pair production, various new physics signatures |
| 11. $pp \to Wb\bar{b}j$ | top, new physics signatures |
| 12. $pp \to t\bar{t}t\bar{t}$ | various new physics signatures |

*Report of the SM and NLO Multileg Working Group for the Workshop "Physics at TeV Colliders", Les Houches, France, 8–26 June, 2009*

$\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$

$\mathrm{pp} \to \mathrm{t\bar{t}jj}$

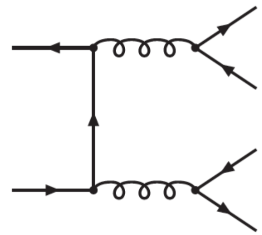
$\mathrm{pp} \to \mathrm{Vjjj}$

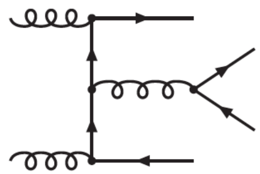
$\mathrm{pp} \to \mathrm{b\bar{b}b\bar{b}}$

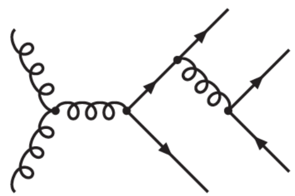
$\mathrm{pp} \to \mathrm{VVjj}$

# State-of-the-Art

- Several $\mathbf{2 \to 4}$ processes have recently been calculated by different groups using different methods

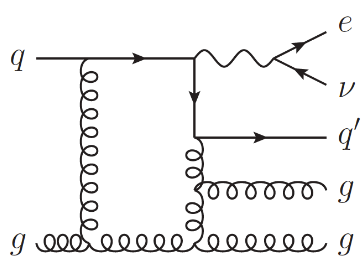






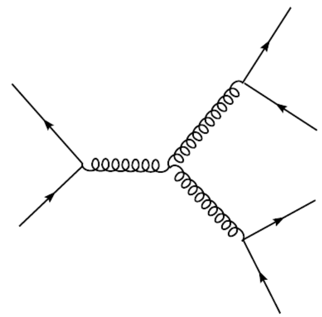
- Two calculations for $\mathbf{pp \to t\bar{t}b\bar{b}}$
  - *Bredenstein, Denner, Dittmaier, Pozzorini ['08, '09, '10]*
    based on Feynman diagrams and tensor integrals
  - *Bevilacqua, Czakon, Papadopoulos, Pittau, Worek ['09]*
    based on OPP reduction, Dyson-Schwinger recursion

- Two calculations for $\mathbf{pp \to W^{\pm} + 3j}$
  - *Ellis, Melnikov, Zanderighi ['09]*
    based on D-dimensional unitarity methods, LC
  - *Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maître ['09]*
    based on unitarity methods

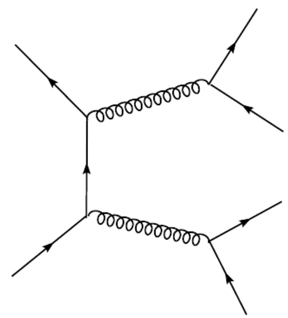


# State-of-the-Art

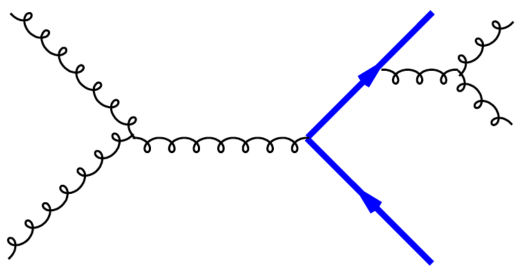
- One calculation for $\mathbf{pp(q\bar{q}) \to b\bar{b}b\bar{b}}$
  - *Binoth, Greiner, Guffanti, Guillet, Reiter, Reuter ['09]*
    based on Feynman diagrams and tensor integrals



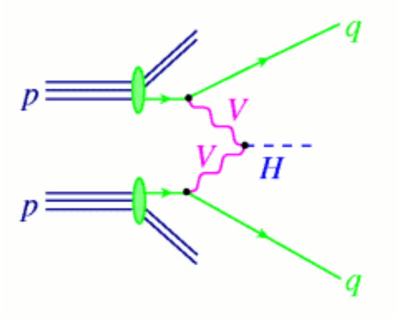
- One calculation for $\mathbf{pp \to t\bar{t}jj}$
  - *Bevilacqua, Czakon, Papadopoulos, Worek ['10]*
    based on OPP reduction, Dyson-Schwinger recursion



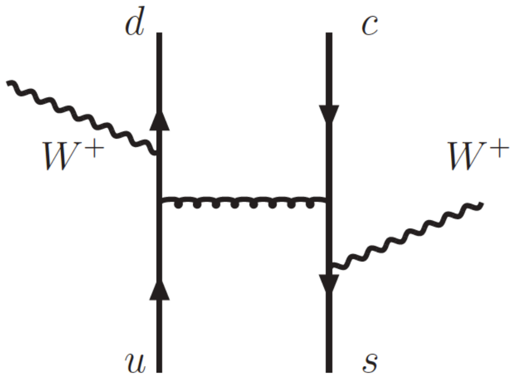
- One calculation for $\mathbf{pp \to Z/\gamma^* + 3j}$
  - *Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maître ['10]*
    based on unitarity methods



# *State-of-the-Art*




- One calculation for VBF processes $\mathbf{pp \to VV + 2j}$
  - *Bozzi, Jager, Oleari, Zeppenfeld ['06, '07, '09]* based on Feynman diagrams and PV reduction
  - approximation used, t-channel diagrams only, no color exchange between upper and lower quark lines, loop diagrams up to pentagons only
  - implemented in **VBFNLO** program
  - $\mathbf{pp \to W^+W^-jj},\ \mathbf{pp \to ZZjj},\ \mathbf{pp \to W^{\pm}Zjj},\ \mathbf{pp \to W^+W^+jj}$

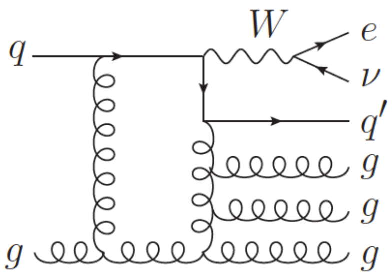


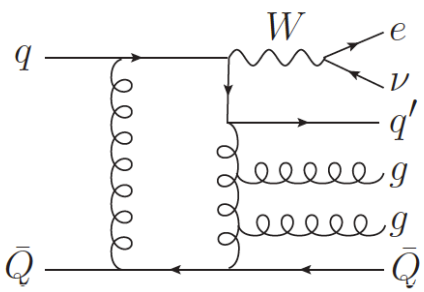

- One calculation for process $\mathbf{pp \to W^+W^+ + 2j}$
  - *Melia, Melnikov, Rontsch, Zanderighi ['10]* based on D-dimensional unitarity methods

# State-of-the-Art

- **First $2 \to 5$ process has recently been calculated !**

- One calculation for process $\mathbf{pp \to W^{\pm} + 4j}$
  - *Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre ['10]*
  - based on unitarity methods
  - leading-color approximation
  - accurate to 3% for W production with fewer jets
  - matrix elements based on on-shell methods



# (Incomplete) List of NLO Tools

- Libraries of specific processes based on analytic calculations
  - **MCFM** *[Campbell, Ellis]*
  - **MC@NLO** *[Frixone, Webber]*
  - **VBFNLO** *[Zeppenfeld et al.]*

- Tools based on PV reduction of Feynman diagrams
  - *Highly-refined methods for tensor integral reduction [Denner, Dittmaier]*
  - **FormCalc/LoopTools** *[Hahn]*
  - **FeynCalc** *[Mertig, Orellana]*
  - **GOLEM** *[Binoth, Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers]*

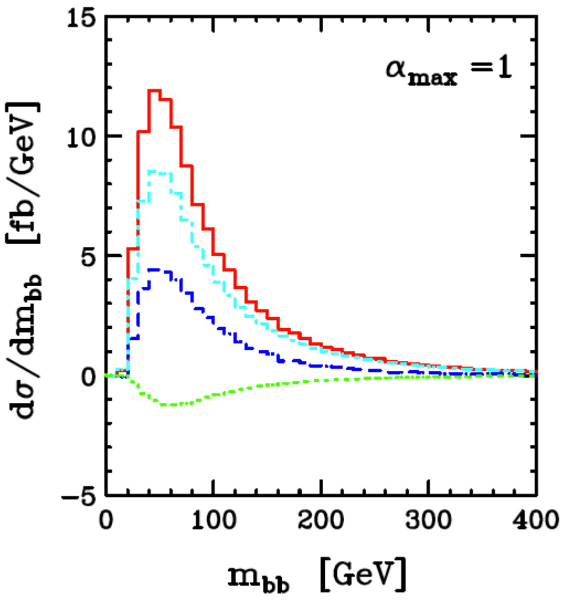
- Tools based on OPP reduction / unitarity-based methods
  - **BlackHat/SHERPA** *[Berger, Bern, Dixon, Kosower, Maitre, et al.]*
  - **Rocket/MCFM** *[Ellis, Giele, Kunszt, Melnikov, Zanderighi, et al.]*
  - *C++ implementation of DDU [Lazopoulos]*
  - **HELAC-NLO** ➔ *This talk is focused on the HELAC approach*

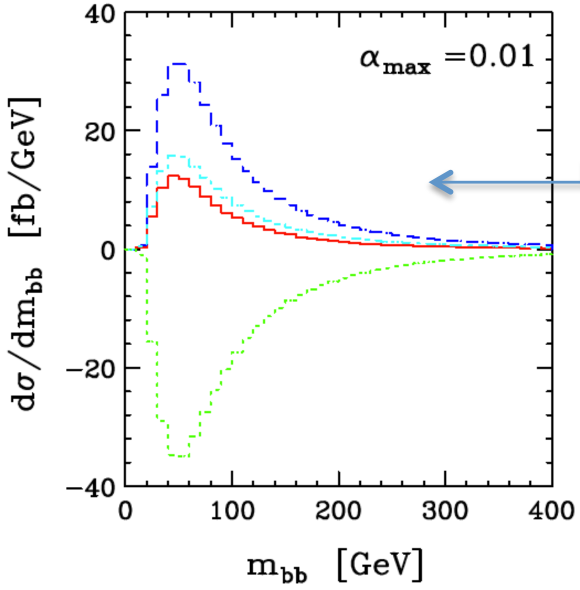
# *HELAC-NLO in a Nutshell*

- **HELAC-PHEGAS**
  - Event generator for all parton level processes @ LO
- **HELAC-1LOOP**
  - Evaluation of virtual one-loop amplitudes, based on **HELAC**
- **CUTTOOLS**
  - Reduction of tensor integrals and determination of coefficients via OPP reduction method
- **ONELOOP**
  - Evaluation of scalar integrals (divergent and finite scalar integrals)
- **HELAC-DIPOLES**
  - Catani-Seymour dipole subtraction for massless and massive cases
  - Phase space integration of subtracted real radiation and integrated dipoles
  - Arbitrary polarizations & phase space restriction on dipoles contribution

**http://helac-phegas.web.cern.ch/helac-phegas/**

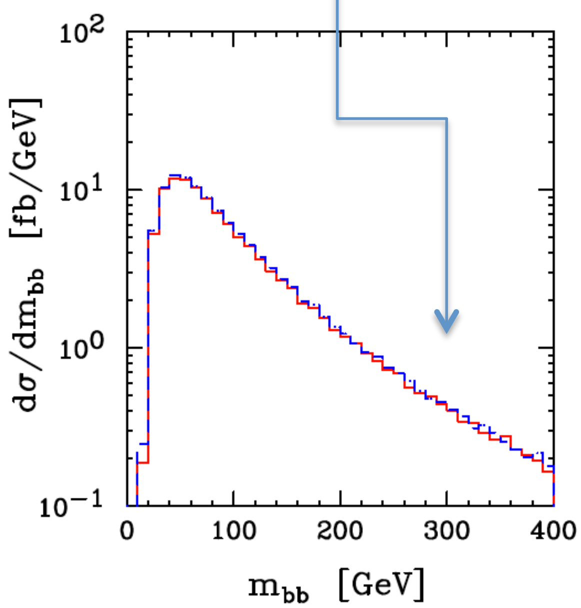
# Real Emission




*Subtracted real emission*
*K + P operators*
*I operators*
*Full result*

*Cutoff independence !!!*




- Phase space restriction on the dipoles phase space $\alpha_{\mathbf{max}} \in (\mathbf{0}, \mathbf{1}]$
- Less dipole subtraction terms per event
- Increased numerical stability
- Reduced missed binning problem
- Large cancellations between subtracted real radiation and integrated dipoles

*Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09*

# Virtual Corrections

- One loop n-particle amplitude

$$A = \sum_{I \in \{1,2,\ldots,n\}} \int \frac{\mu^{4-d} d^d \bar{q}}{(2\pi)^d} \frac{\bar{N}_I(\bar{q})}{\prod_{i \in I} \bar{D}_i(\bar{q})} \qquad \bar{D}_i(\bar{q}) = (\bar{q} + p_i)^2 - m_i^2, \quad i = 1,2,\ldots,n$$

$$A = \sum_i d_i \; Box_i + \sum_i c_i \; Triangle_i + \sum_i b_i \; Bubble_i + \sum_i a_i \; Tadpole_i + R$$

- Can be expressed in basis of known integrals such 4, 3, 2, 1-point scalar integrals
- In order to calculate one loop amplitude three main building blocks are needed
  - Evaluation of numerator function N(q) → **HELAC-1LOOP**
  - Determination of coefficients via reduction method → **OPP, CUTTOOLS**
  - Evaluation of scalar functions → **ONELOOP**

# Virtual Corrections

- Reduction at integrand level – **OPP** method imptemented in **CUTTOOLS**

- Computing numerator functions for specific values of loop momenta that are solutions of equations

  $D_i(q) = 0 \quad \text{for} \quad i = 0, \ldots, M\text{-}1$

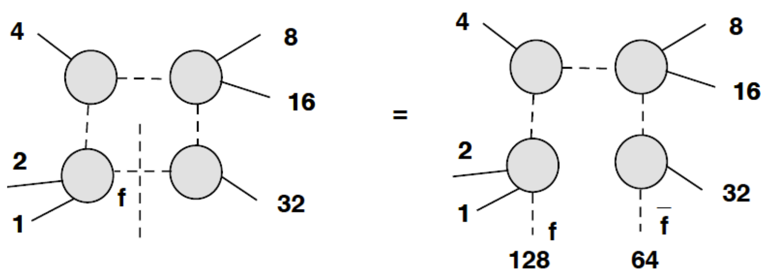
- It is customary to refer to these equations as quadruple (M = 4), triple (M = 3), double (M = 2) and single (M = 1) cuts

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[ d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
& + \sum_{i_0<i_1<i_2}^{m-1} \left[ c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
& + \sum_{i_0<i_1}^{m-1} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
& + \sum_{i_0}^{m-1} \left[ a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i \\
& + \tilde{P}(q) \prod_{i}^{m-1} D_i \,.
\end{aligned}
$$

*Ossola,, Papadopoulos, Pittau '07, '08*

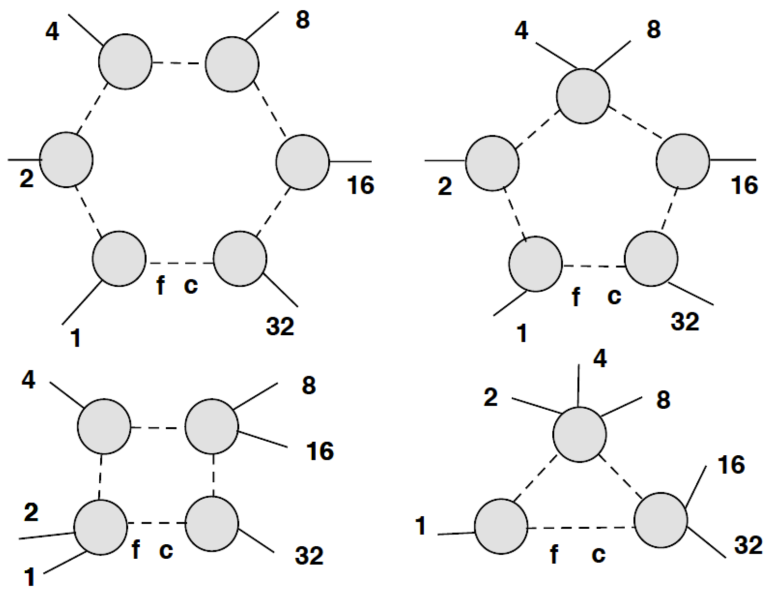
# Virtual Corrections

- Calculating numerator function for specific values of loop momenta
- Possibility to use tree level amplitudes as building blocks
- Collecting all contributions with given loop propagator via **HELAC-1LOOP**
- Calculated as part of tree level amplitude with n+2 particles (in 4 dimensions)




Constrain: attached blobs contain no propagator depending on loop momenta, no denominator used for internal loop propagators

*Hameren, Papadopoulos, Pittau '09*



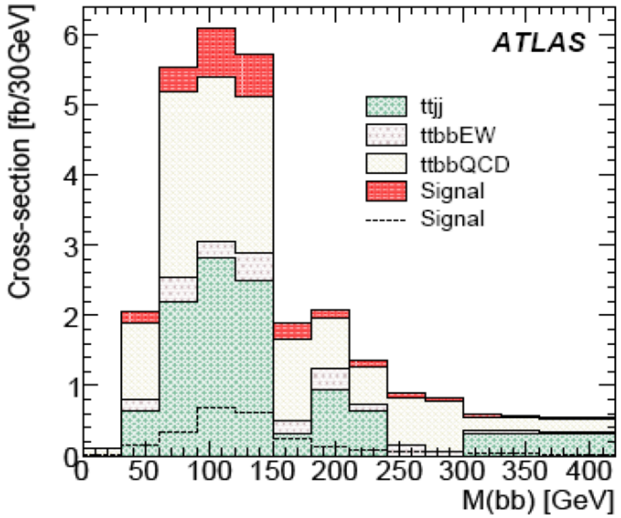

Typical collections of possible contributions

# Motivations for ttbb and ttjj

- $pp \rightarrow t\bar{t}H$ potential discovery channel
  - $H \rightarrow b\bar{b}$
  - $m_H \leq 135$ GeV
- top & bottom Yukawa coupling
- Large QCD backgrounds: $t\bar{t}b\bar{b}$ & $t\bar{t}jj$

- *Problem 1:* combinatorial background of b-jets:
  - bb pair can be chosen incorrectly, lack of distinctive kinematic feature of Higgs decay jets
- *Problem 2:* b-tagging efficiency:
  - two b-jets for Higgs candidate can arise from mistagged QCD light jets
- *Goal:* Backgrounds need to be controlled

*ATLAS TDR, CERN-OPEN-2008-020*



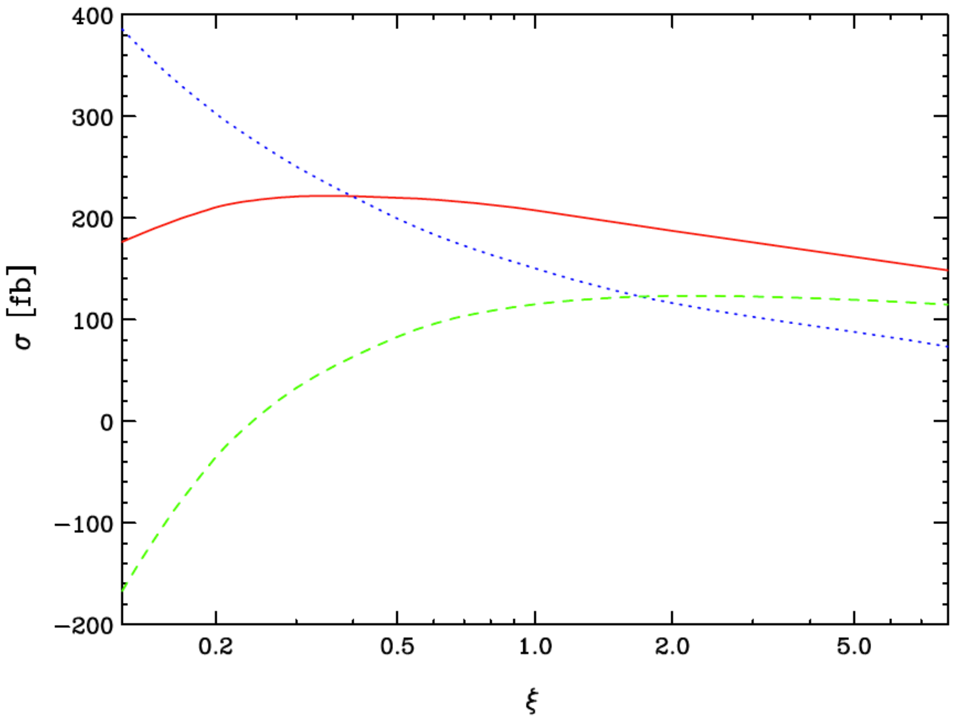

S/B ~ 1/9

| Summary table | Significance loose/tight | Luminosity |
|---|---|---|
| ATLAS (Lepton+jets) | 2.2 | 30 $fb^{-1}$ |
| CMS (Lepton+jets) | 2.5/1.9 | 60 $fb^{-1}$ |
| CMS(Combined) | 3.9/3.3 | 60 $fb^{-1}$ |

*G. Aad, J. Steggemann, ATLAS & CMS @ TOP 2008*

# pp -> ttH -> ttbb @ LHC

- Scale dependence and integrated cross sections




$\mathbf{m_H = 130\ GeV}$

$$\sigma_{\rm LO} = (150.375 \pm 0.077)\ \text{fb}$$

$$\sigma_{\rm NLO} = (207.268 \pm 0.150)\ \text{fb}$$

$$\sigma_{\rm NLO}^{\rm veto} = (114.880 \pm 0.152)\ \text{fb}$$

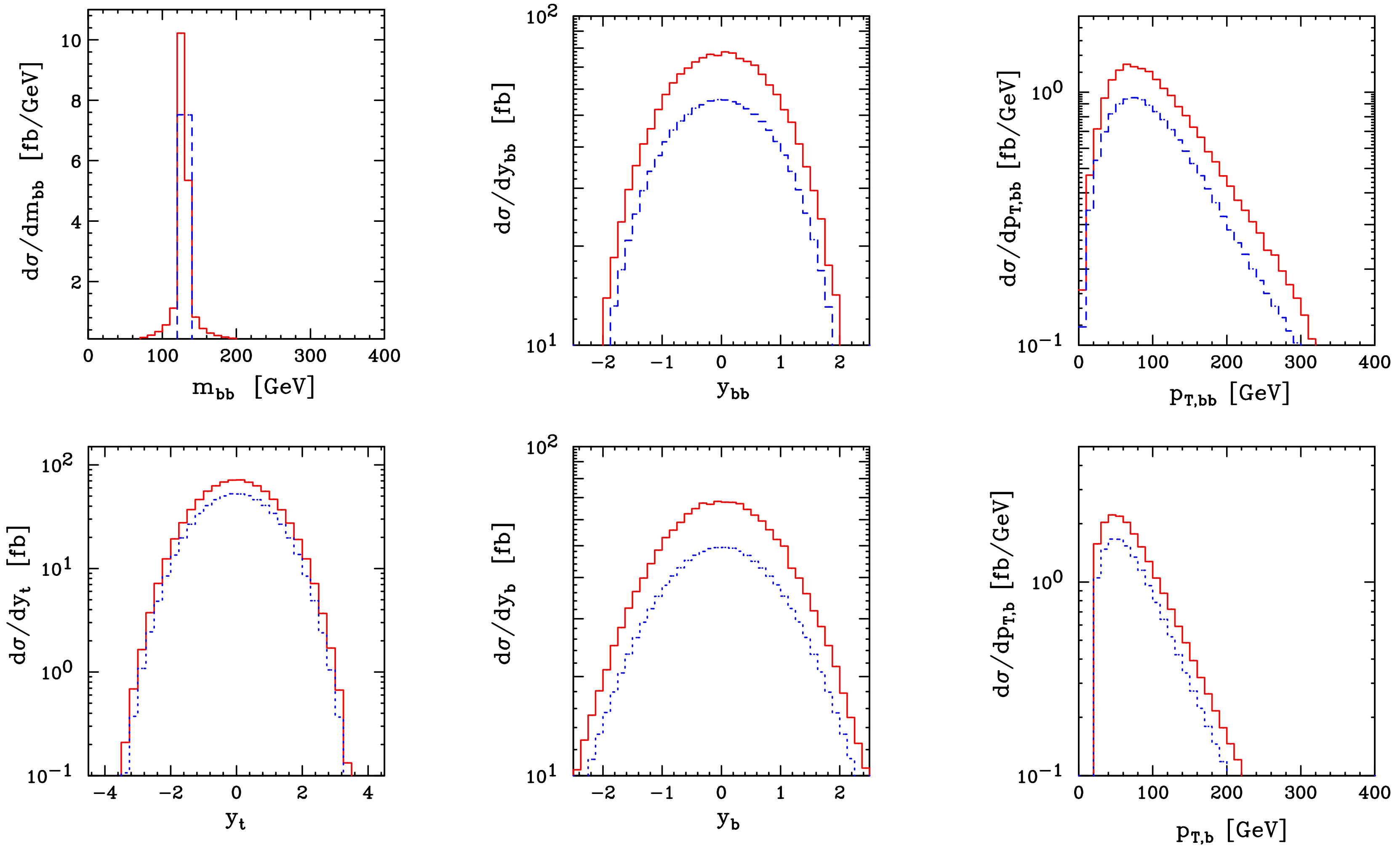
Scale dependence reduced:
**33% @ LO** down to **10% @ NLO**
**28% @ NLO** with **jet veto** of 50 GeV

K factor of **K = 1.38** (**K = 0.76**)
NLO QCD Corrections **38%** (**24%**)

*Bevilacqua, Czakon, Garzelli, Hameren, Papadopoulos, Pittau, Worek '10 (Les Houches 2009)*

# pp -> ttH -> ttbb @ LHC

- Differential cross section, bb pair, single bottom & top kinematics, LO & NLO



# pp -> ttbb @ LHC

- Integrated cross sections and scale dependence, *Permille level agreement !*

| Process | $\sigma^{\rm LO}_{[23,\ 24]}$ [fb] | $\sigma^{\rm LO}$ [fb] | $\sigma^{\rm NLO}_{[23,\ 24]}$ [fb] | $\sigma^{\rm NLO}_{\alpha_{\rm max}=1}$ [fb] | $\sigma^{\rm NLO}_{\alpha_{\rm max}=0.01}$ [fb] |
|---|---|---|---|---|---|
| $q\bar{q} \to t\bar{t}b\bar{b}$ | 85.522(26) | 85.489(46) | 87.698(56) | 87.545(91) | 87.581(134) |
| $pp \to t\bar{t}b\bar{b}$ | 1488.8(1.2) | 1489.2(0.9) | 2638(6) | 2642(3) | 2636(3) |

| $\xi \cdot m_t$ | $1/8 \cdot m_t$ | $1/2 \cdot m_t$ | $1 \cdot m_t$ | $2 \cdot m_t$ | $8 \cdot m_t$ |
|---|---|---|---|---|---|
| $\sigma^{\rm LO}$ [fb] | 8885(36) | 2526(10) | 1489.2(0.9) | 923.4(3.8) | 388.8(1.4) |
| $\sigma^{\rm NLO}$ [fb] | 4213(65) | 3498(11) | 2636(3) | 1933.0(3.8) | 1044.7(1.7) |

$$\sigma_{\rm LO} = (\mathbf{1489.2 \pm 0.9})\ \mathbf{fb}$$

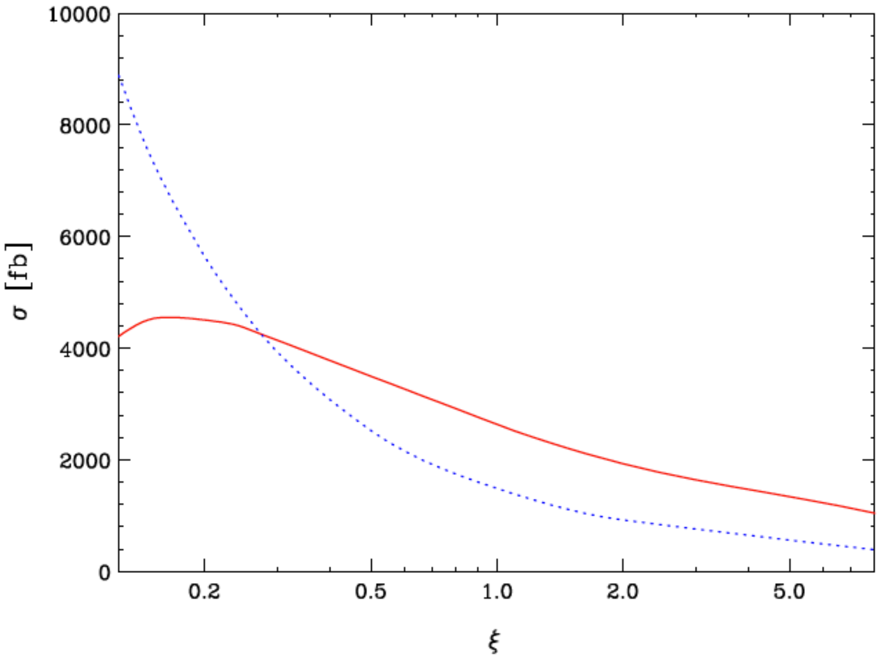
$$\sigma_{\rm NLO} = (\mathbf{2636 \pm 3})\ \mathbf{fb}$$

*Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09*
*Bredenstein, Denner, Dittmaier, Pozzorini '08, '09*

Scale dependence reduced:
**70% @ LO** down to **33% @ NLO**
K factor of **K = 1.77**
for quarks initial states only **K = 1.03**
With jet veto of 50 GeV **K = 1.20**

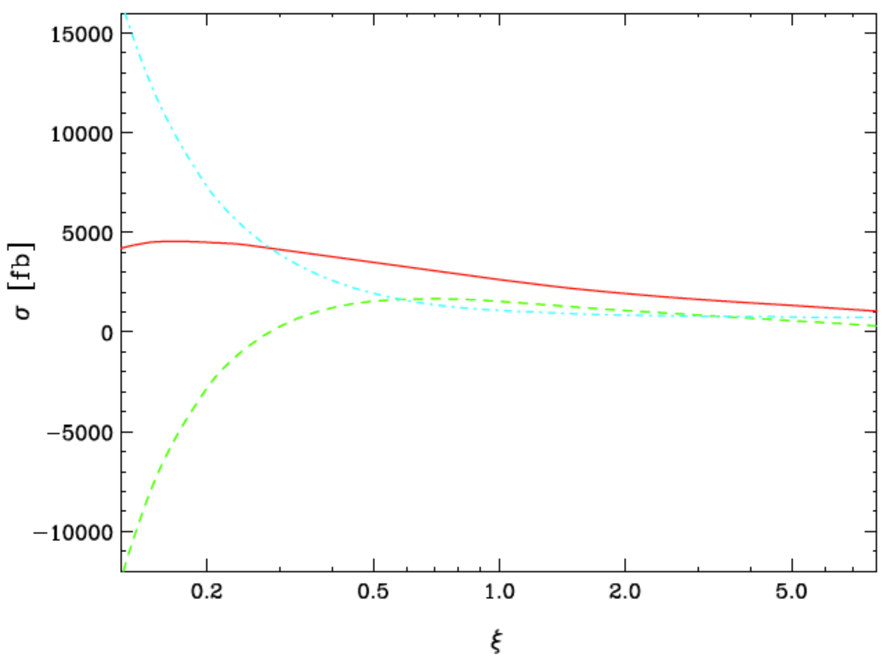
# *pp -> ttbb @ LHC*

- Scale dependence graphically




Varying scale up or down by a factor two changes cross section by **70% @ LO** and by **33% @ NLO**

Scale dependence at NLO decomposed into contribution of *Virtual Corrections* & *Real Radiation*




*Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09*

# *pp -> ttbb @ LHC*

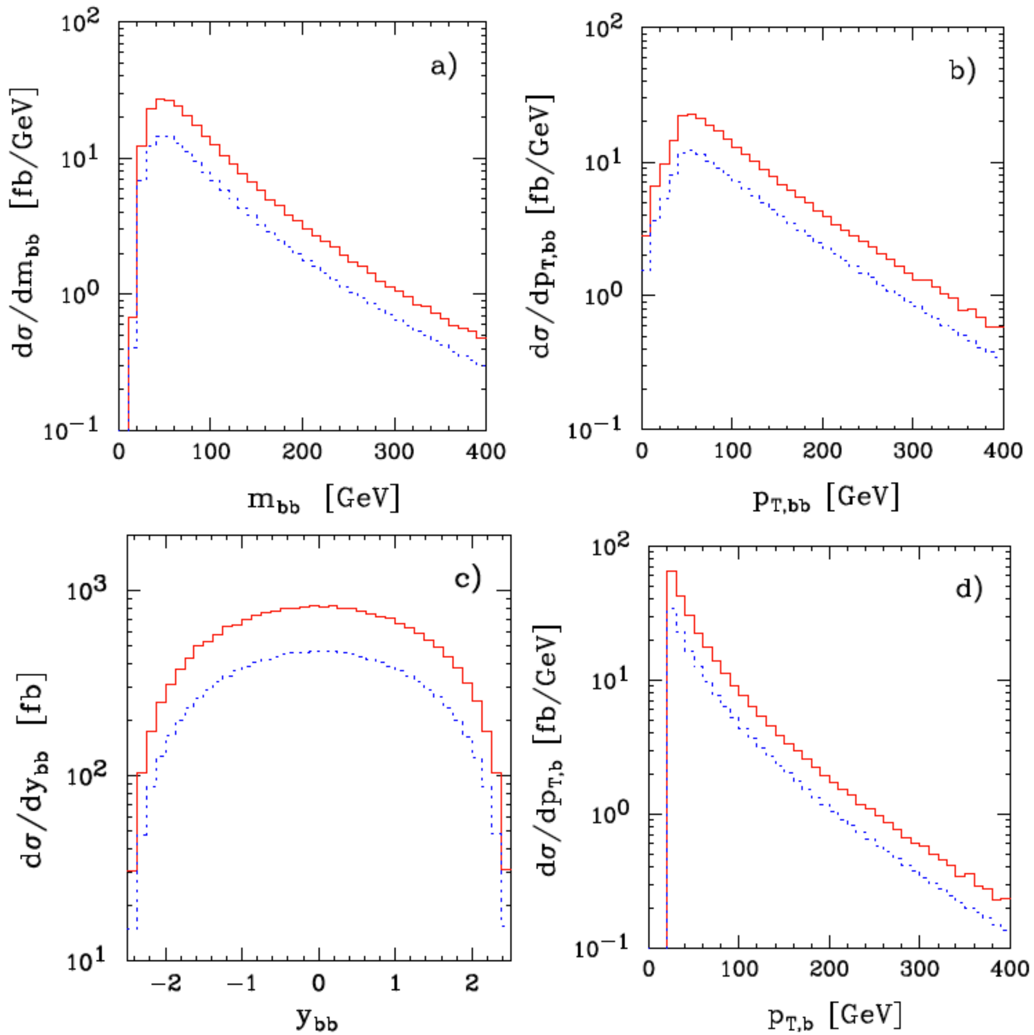
- Differential cross sections

- b-jet pair kinematics
  - Invariant mass
  - Transverse momentum
  - Rapidity distribution

- single b-jet kinematics
  - Transverse momentum

*LO* & *NLO*

- Relatively small variation compared to the size but shape change important



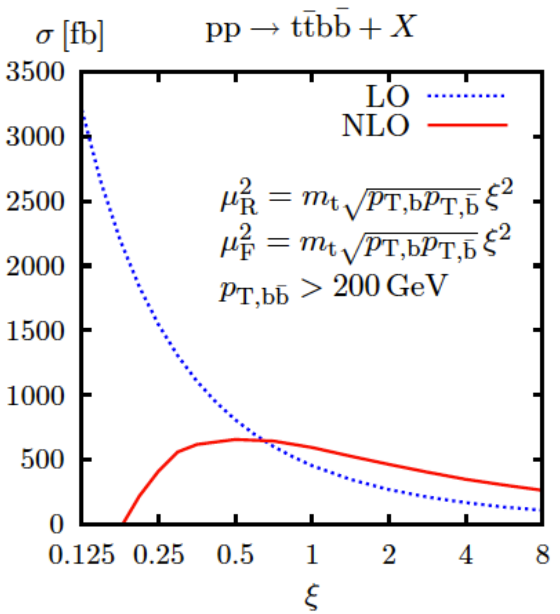

*Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09*

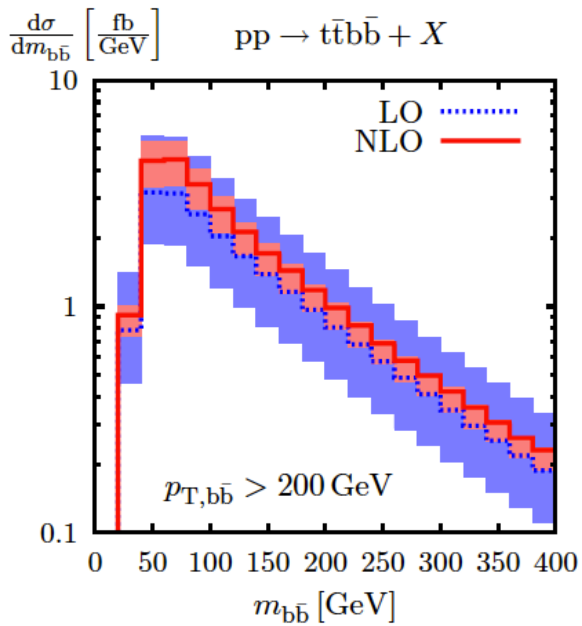
# *pp -> ttbb @ LHC*

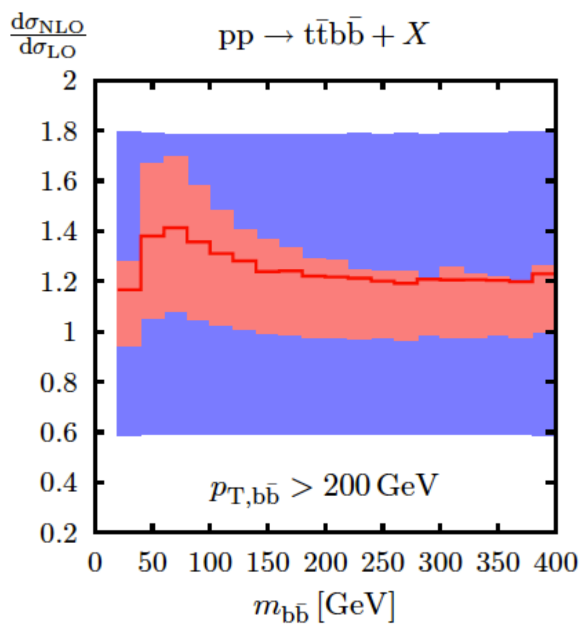
- Broad study:
  - *Cross section in fb*
  - *Dynamic scale*
  - *$m_{bb}$ distribution*
  - *K-factor*

*Bredenstein, Denner, Dittmaier, Pozzorini '10*

| Setup | $m_{b\bar{b},cut}$ | $p_{T,b\bar{b},cut}$ | $p_{jet,veto}$ | $p_{T,b,cut}$ | $y_{b,cut}$ | $\sigma_{LO}$ | $\sigma_{NLO}$ | $K$ |
|---|---|---|---|---|---|---|---|---|
| I | 100 | - | - | 20 | 2.5 | $786.3(2)^{+78\%}_{-41\%}$ | $978(3)^{+13\%}_{-21\%}$ | 1.24 |
| II | - | 200 | - | 20 | 2.5 | $451.8(2)^{+79\%}_{-41\%}$ | $592(4)^{+13\%}_{-22\%}$ | 1.31 |
| III | 100 | - | 100 | 20 | 2.5 | $786.1(6)^{+78\%}_{-41\%}$ | $700(3)^{+0.4\%}_{-19\%}$ | 0.89 |
| IV | 100 | - | - | 50 | 2.5 | $419.4(1)^{+77\%}_{-40\%}$ | $526(2)^{+13\%}_{-21\%}$ | 1.25 |



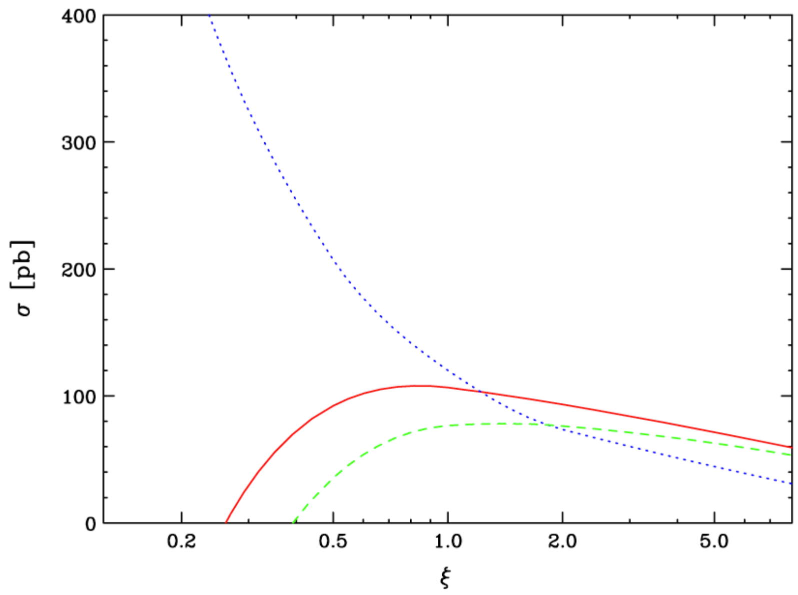



# pp -> ttjj @ LHC

- Scale dependence & integrated cross sections

*Bevilacqua, Czakon, Papadopoulos, Worek '10*

| Process | $\sigma^{\rm LO}$ [pb] | Contribution |
|---|---|---|
| $pp \to t\bar{t}jj$ | 120.17(8) | 100% |
| $qg \to t\bar{t}qg$ | 56.59(5) | 47.1% |
| $gg \to t\bar{t}gg$ | 52.70(6) | 43.8% |
| $qq' \to t\bar{t}qq'$, $q\bar{q} \to t\bar{t}q'\bar{q}'$ | 7.475(8) | 6.2% |
| $gg \to t\bar{t}q\bar{q}$ | 1.981(3) | 1.6% |
| $q\bar{q} \to t\bar{t}gg$ | 1.429(1) | 1.2% |




Scale dependence reduced:
72% @ LO down to 13% @ NLO
54% @ NLO with jet veto of 50 GeV

$$\sigma_{\mathbf{LO}} = (\mathbf{120.17} \pm \mathbf{0.08})\ \mathbf{pb}$$

$$\sigma_{\mathbf{NLO}} = (\mathbf{106.94} \pm \mathbf{0.17})\ \mathbf{pb}$$

$$\sigma^{\mathbf{veto}}_{\mathbf{NLO}} = (\mathbf{76.58} \pm \mathbf{0.17})\ \mathbf{pb}$$

K factor of **K = 0.89** (**K = 0.64**)
Negative shift of **11%** (**36%**)

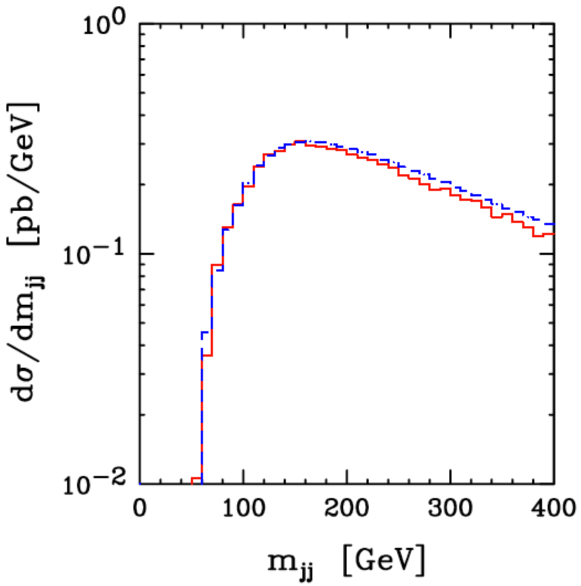
# *pp -> ttjj @ LHC*

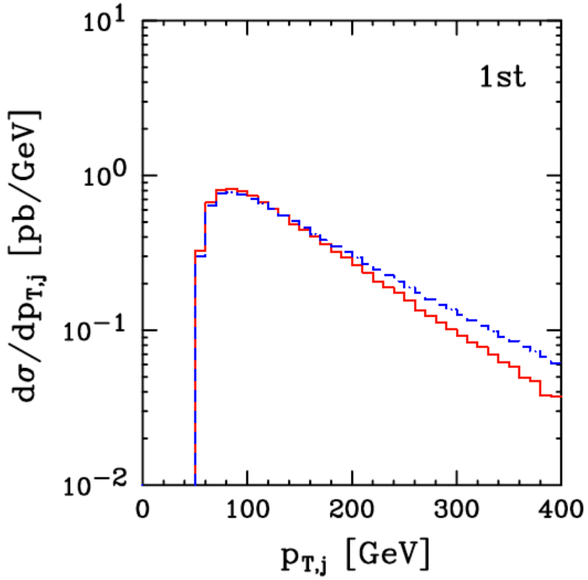
- Differential cross section

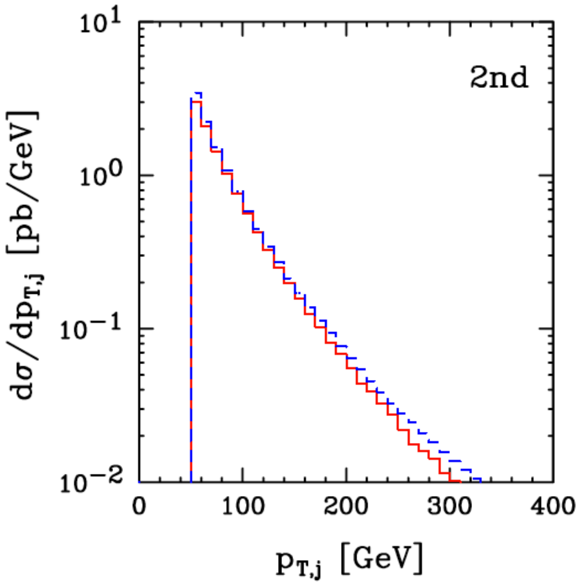
*LO* & *NLO*

- $m_{jj}$ size of the corrections transmitted to distributions for low $p_T$, shapes change for hight $p_T$

- *$p_T$ of 1st hardest & 2nd hardest jet* (ordered in $p_T$) altered shapes up to **39% & 28%** in tails




*Bevilacqua, Czakon, Papadopoulos, Worek '10*

# Summary & Outlook

- Automated approaches:
  - **HELAC-NLO, BLACKHAT/SHERPA, ROCKET/MCFM, GOLEM, ...**
- First results have already been presented:

$$\mathbf{pp \to t\bar{t}b\bar{b},\ pp \to t\bar{t}jj,\ pp(q\bar{q}) \to b\bar{b}b\bar{b}\ pp \to Vjjj,\ pp \to W^+W^+jj,\ pp \to VVjj}$$

- **HELAC-NLO**
  - Complete tool at NLO built around **HELAC-PHEGAS:** **HELAC-1LOOP, CUTTOOLS, ONELOOP & HELAC-DIPOLES**
  - Much wider study for $\mathbf{pp \to t\bar{t}jj}$: variation of the center of mass energy, cone size in jet algorithm, transverse momentum cuts, jet vetoes, ...
  - Other processes from NLO Wishlist under attack
  - Constant improvements in speed and functionality
  - Big step: Matching to parton-shower