Interference effects in very precise measurement of muon charge asymmetry at FCCee

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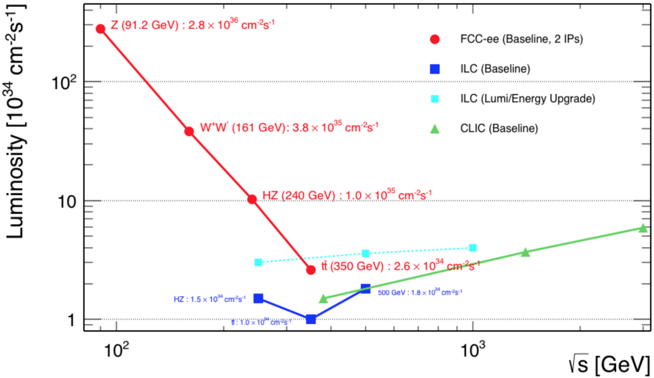
Partly supported by the grants of *Narodowe Centrum Nauki* **2016/23/B/ST2/03927**

Prepared for FCCee meeting, CERN, Jan. 9-th, 2018

FCCee as Z-factory: 1.5×10^{12} Z's/year! $10^5 \times$ LEP

Luminosities and centre-of mass energies



LEP record at the Z
$2.3 \cdot 10^{31}\ cm^{-2}s^{-1}$

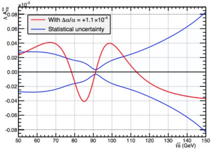
LEP2 record
$\approx 10^{32}\ cm^{-2}s^{-1}$

INTRODUCTION

- $M_Z, G_F, \alpha_{QED}(0)$ outweigh other data in the "testing power" in the SM overall fit to experimental data
- However, $\alpha_{QED}(Q^2 = 0)$ is ported to $\alpha_{QED}(Q^2 = M_Z^2)$ using low energy QCD data -> this limits its usefulness beyond LEP precision.
- Patrick Janot has proposed (arxiv:1512.05544) another observable, $A_{FB}(e^+e^- \to \mu^+\mu^-)$ at $\sqrt{s_\pm} = M_Z \pm 3.5 GeV$, with a similar "testing profile" in the SM overall fit as $\alpha_{QED}(M_Z^2)$, but could be measured at high luminosity FCCee very precisely. (It is advertised as "determining $\alpha_{QED}(M_Z^2)$" from $A_{FB}(\sqrt{s_\pm})$".)
- However, A_{FB} near $\sqrt{s_\pm}$ is varying very strongly, hence is prone to large QED corrections (for instance ISR).
- In particular A_{FB} away from Z peak gets also a direct sizable contributions from **QED initial-final state interference, nickname IFI**.
- It is therefore necessary to re-discuss how efficiently these trivial but large QED effects in A_{FB} can be controlled and/or eliminated.

The aim is to reduce QED uncert. to $\delta A_{FB}(e^+e^- \to \mu^+\mu^-) < 3 \times 10^{-5}$

- Presently $\Delta\alpha_{QED}(M_Z)/\alpha_{QED} \simeq 1.1 \times 10^{-4}$ (using low energy e^+e^- data).
- Recent studies using the same method of dispersion relations are quoting possible improvements down to $\Delta\alpha/\alpha \simeq (0.5 - 0.2) \times 10^{-4}$.
- To be competitive A_{FB} has to provide $\Delta\alpha/\alpha < 10^{-4}$
- Using Fig.4 of arxiv:1512.05544 paper by Patrick Janot



$\Delta\alpha/\alpha < 10^{-4}$ translates into $\Delta A_{FB} < 3 \times 10^{-5}$

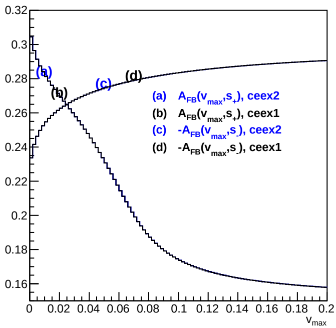
- LEP era estimate of QED uncertainty in A_{FB} outside Z peak was $\sim 2.5 \times 10^{-3}$, see “The LEP-2 MC Workshop 2000”, arxiv:0007180.
- Its improvement by at least factor 200 sounds as a very ambitious goal!
- Encouraging precedent: for QED photonic corrs. to Z-lineshape ($\sim 30\%$), its uncertainty reduced down to $\delta\sigma/\sigma \simeq 3 \times 10^{-4}$, (Jadach,Skrzypek,Martinez, Phys.Lett.B280(1992)129)!

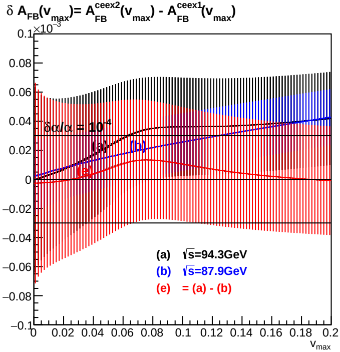
QED (photonic) correction effects in $A_{FB}(e^+e^- \to \mu^+\mu^-)$

General features

- Pure ISR (initial state radiation) indirect influence due to reduction of $\sqrt{s}$. Non-soft h.o. missing corrs. under very good control, see next slide.
- Pure FSR (final state radiation) for sufficiently inclusive event selection (cut-offs) generally small, but cut-off dependence has to be controlled with high quality MC.
- Direct contribution of IFI (initial-final state interference) is suppressed at the peak but sizable off-peak.
- IFI effect comes from non-trivial matrix-element, even in the soft-photon approximation.
- KKMC Monte-Carlo program (J.S., Ward, Wąs, Phys.Rev. D63 (2000)) is the most sophisticated tool to calculate all the above effects.

Estimate QED ISR uncertainty in A_{FB} at $\sqrt{s} \sim M_Z \pm 3$GeV

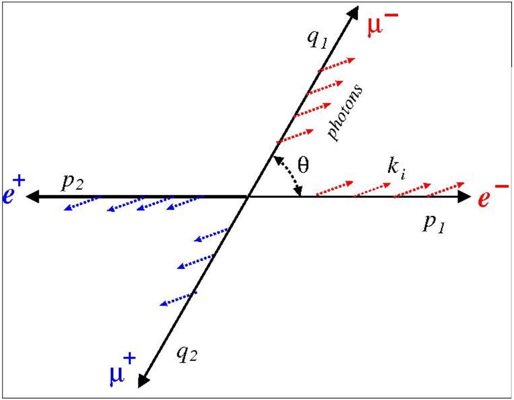


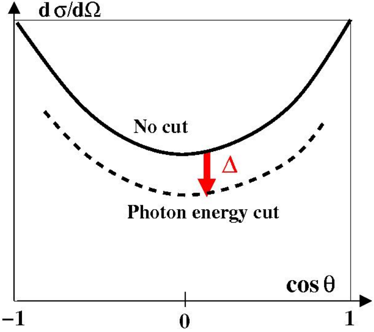


- Cut on energy of all photons $v < v_{\max}$, $v \equiv 1 - \frac{M^2_{\mu\mu}}{s} \simeq \sum_i \frac{2E_i^\gamma}{\sqrt{s}}$
- Examined downgrade non-soft of QED M.E. from CEEX2 to CEEX1
- For photon cut-off below $v_{\max} = 0.06$ we get $\delta A_{FB} < 3 \cdot 10^{-4}$.
- Looks good, but to be x-checked, also using semianalytical *KKsem*.
- Important contribution from e^+e^- soft pairs not included!!!
- Statistical errors overestimated (MC weight differences)

A general understanding of the IFI

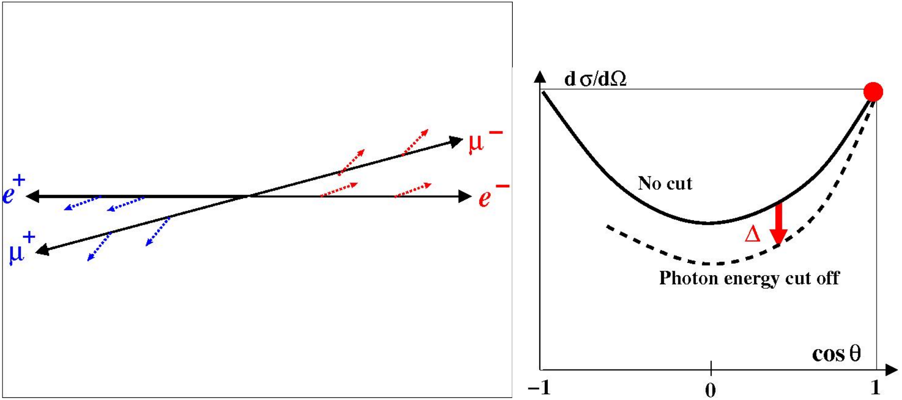
- In $e^- e^+ \rightarrow \mu^- \mu^+$ not only e^- gets annihilated, but also its accompanying elmgt. field of charge -1. New elmg. field of charge -1 is created along μ^-.
- At **wide angles** these two processes are independent sources of real photos. The effect of cut on photon energy is essentially θ-independent.





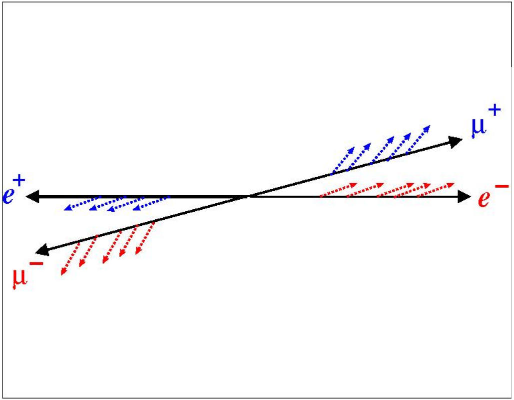
A general understanding of the IFI

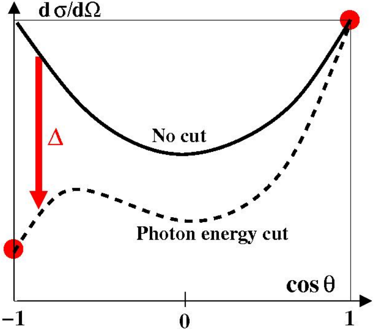
- In $e^- e^+ \to \mu^- \mu^+$ not only e^- gets annihilated, but also its accompanying elmgt. field of charge -1. New elmg. field of charge -1 is created along μ^-.
- μ^- close to initial e^- inherits part of e^- elmg. field $\to$ bremsstrahlung is weaker. Hence for $\theta \to 0$ zero effect due to cut on real photons!



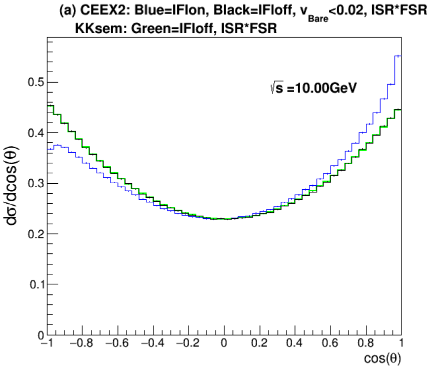
A general understanding of the IFI

- In $e^- e^+ \to \mu^- \mu^+$ not only e^- gets annihilated, but also its accompanying elmgt. field of charge -1. New elmg. field of charge -1 is created along μ^-.
- In the **backward** direction, replacing field of charge -1 with that of $+1$ is "more violent", more real photons $\to$ stronger effect of the cut, dip in $d\sigma/d\Omega$.





IFI effect in the muon angular distri. at $\sqrt{s} = 10GeV,\ M_Z \pm 3.5GeV$ for total photon energy cut $v = 1 - M_{\mu\mu}^2/s < v_{\max} = 0.02$ (KKMC)



- **A few percent effect** seen in the angular distribution.
- Good agreement of KKMC and semianalytical KKsem when IFI is off.
- (Inclusion of IFI in semianalytical KKsem is quite urgent!)

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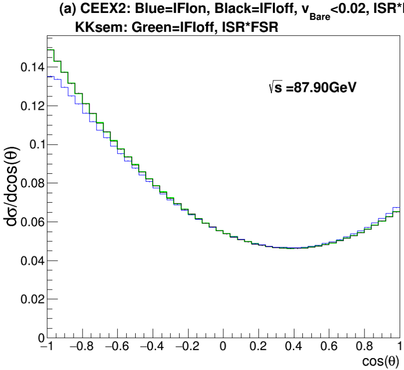

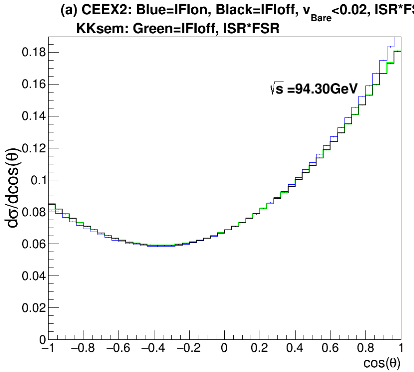

- **A few percent effect** seen in the angular distribution.
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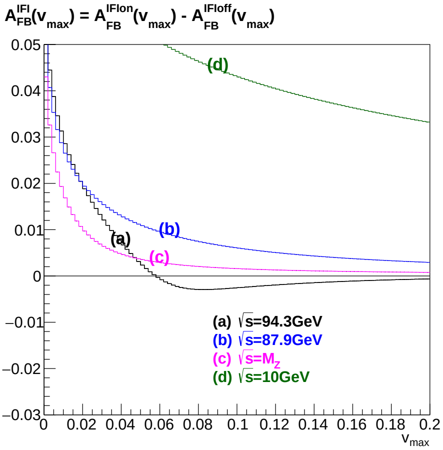
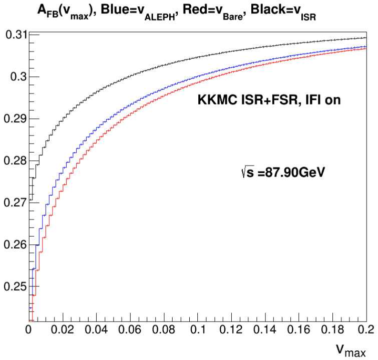
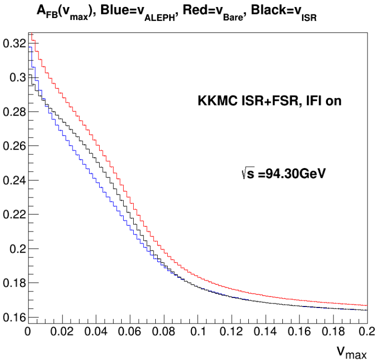
Direct influence of IFI in $A_{FB}(e^+e^- \to \mu^+\mu^-)$ at $\sqrt{s} \sim M_Z \pm 3$GeV



- IFI suppression by $\sim \Gamma/M$ seen comparing $\sqrt{s} = 10$GeV and 91GeV results.
- IFI effect is $\sim 3\%$ at $s_\pm$ ($\sim 1\%$ when combined).
- IFI is huge, compared to the aimed precision $\delta A_{FB} \sim 10^{-5}$
- $\sim \Gamma/M$ suppression dies out for $v_{max} < 0.04$.

How important is the type of kinematic cuts in A_{FB} ?





- v_{ALEPH} is FSR-inclusive, $v_{bare} = 1 - M^2_{\mu\mu}/s$ is FSR-sensitive and v_{ISR} from $M^2_{\mu\mu}$ after ISR before FSR (from MC).
- It matters a lot, $> 1\%$, especially above Z!
- It does not seem to cancel between s_+ and s_-.
- MC like KKMC is mandatory to control/eliminate this effect.
- N.B. Effect of changing definition of muon $\cos\theta$ is completely negligible!

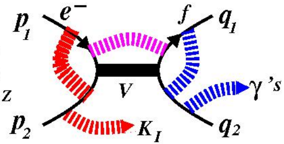
Theoretical uncertainty of soft-resummed IFI contribution to resonant matrix element implemented in KKMC

- Basicaly, soft-resumed M.E. in KKMC looks perfect, but all resummed calculation are to some extent non-unique.
- Pioneering works in the soft-photon resummation for resonant $e + e^-$ annihilation including IFI were done by Frascati group, (Greco et.at. Phys. Lett. B101 (1975) 234, Phys. Lett. B171 (1980) 118.)
- KKMC implements and extends this technique, see ref. [JWW-2001], Jadach,Ward,Wąs, Phys.Rev. D63(2001)113009
- What is badly needed is another calculation of comparable quality in order to test predictions of KKMC.

Multiphoton matrix element in KKMC

Neglecting for clarity non-soft parts it reads (see [JWW-2001]):

$$\sigma(s) = \frac{1}{flux(s)} \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_{n+2} \prod_{i=1}^{n} \int \frac{d^3 k_i}{2k_i^0}\; \mathfrak{M}^{\mu_1,\mu_2,\dots,\mu_n}(k_1,\dots,k_n)\left[\mathfrak{M}_{\mu_1,\mu_2,\dots,\mu_n}(k_1,\dots,k_n)\right]^*$$

$$\mathcal{M} = \sum_{V=\gamma,Z}$$


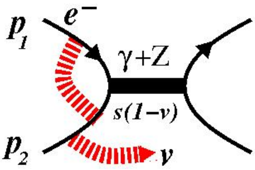
$$\mathfrak{M}^{\mu_1,\dots,\mu_n}(k_1,\dots,k_n) = \sum_{V=\gamma,Z} e^{\alpha B_4(p_i,q_i)+\alpha\Delta B_4^V(P-K_I)} \sum_{\{I,F\}} \prod_{i\in I} j_I^{\mu_i}(k_i) \prod_{r\in F} j_F^{\mu_r}(k_r)\; \mathcal{M}_V^{(0)}(P-K_I)$$

$$j_I^{\mu}(k) = \frac{e}{4\pi^{3/2}}\left(\frac{p_1^\mu}{p_1\cdot k} - \frac{p_2^\mu}{p_2\cdot k}\right),\quad j_F^{\mu}(k) = \frac{e}{4\pi^{3/2}}\left(\frac{q_1^\mu}{q_1\cdot k} - \frac{q_2^\mu}{q_2\cdot k}\right),\quad P = p_1+p_2,\quad K_I = \sum_{i\in I} k_j.$$

- $B_4(p_i,q_i)$ is YFS virtual formfactor. The additional $\alpha\Delta B_4^Z(P) = -2\frac{\alpha}{\pi}\ln\frac{-t}{s}\ln\frac{M_Z^2 - iM\Gamma_Z - (P-K_I)^2}{M_Z^2 - iM_Z\Gamma_Z}$, $\Delta B_4^\gamma = 0$, (Greco et.al. 1974) is mandatory for real/virtual cancellations of $\sim \frac{\alpha}{\pi}\ln\frac{\Gamma_Z}{M_Z}$. (To be improve further?).
- Almost complete $\mathcal{O}(\alpha^2)$ (except penta-boxes) QED virtual and real corrs. and EW $\mathcal{O}(\alpha^1)$ (DIZET) are also included in KKMC.

High precision Z-lineshape QED ISR formula used at LEP

decades of work by: Yennie, Frautschi, Suura, Gribov Lipatov, Kuraev, Fadin, Greco, Pancherini, Srivastava, Jackson, Martin, Berends, Burgers, Jadach, Skrzypek, Ward,...



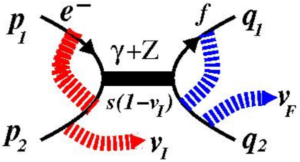
$$\sigma(s, v_{\max}) = \int_0^{v_{\max}} dv\ F(\gamma_I)\gamma_I v^{\gamma_I - 1}\ \sigma_B\big(s(1-v)\big)\ \big[1 + \mathrm{NIR(v)}\big],$$

$$F(\gamma) \equiv \frac{e^{-C_E\gamma}}{\Gamma(1+\gamma)}, \quad \gamma_I = 2\frac{\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$$

- Non-infrared perturbative function NIR(v), for $\delta\sigma/\sigma \simeq 2 \times 10^{-4}$ precision, to be found in J.S.+Skrzypek+Pietrzyk Phys.Lett.B280(1992)129.
- One can add Electroweak corrections to σ_B, 1st order FSR, generalize to $d\sigma/d\Omega$ etc. as it was done in ZFITTER.

KKMC extensively tested with ISR+FSR (IFI off) formula

implemented in semianalytical program KKsem, part of KKMC distribution



$$\frac{d\sigma}{d\Omega}(s,\theta,v_{\max}) = \int dv_I\, dv_F\; \delta(v - v_I - v_F)\theta(v < v_{\max})$$

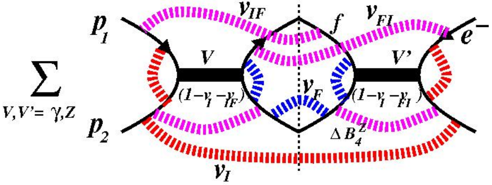
$$\times\, F(\gamma_I)\gamma_I v_I^{\gamma_I - 1}\; F(\gamma_F)\gamma_I v_F^{\gamma_F - 1}\; \frac{d\sigma_0}{d\Omega}(s(1-v_I),\theta)\; \left[1 + \mathrm{NIR}(\mathrm{v_I},\mathrm{v_F})\right],$$

$$v = 1 - (q_1+q_2)^2/s, \quad \gamma_F = 2\frac{\alpha}{\pi}\left(\ln\frac{s}{m_f^2} - 1\right)$$

- In KKsem $d\sigma_0/d\Omega$ is decorated with EW corrections
- For $v_{\max} < 0.2$ definition of θ is not essential.
- Non-IR function $\mathrm{NIR}(v_I, v_F)$ from analytical integration of the MC distributions.
- $\delta(v - v_I - v_F) \to \delta(1 - v - (1-v_I)(1-v_F))$ more realistic for hard emissions.

NEW formula for precision calibration of ISR+FSR+IFI

Eq.(90) in [JWW2001] and in older Frascati works, implemented recently in KKsem



$$\frac{d\sigma}{d\Omega}(s,\theta,v_{\max}) = \sum_{V,V'=\gamma,Z} \int dv\; dv_I\; dv_F\; dv_{IF}\; dv_{FI}\; \delta(v - v_I - v_F - v_{IF} - v_{FI})\theta(v < v_{\max})$$

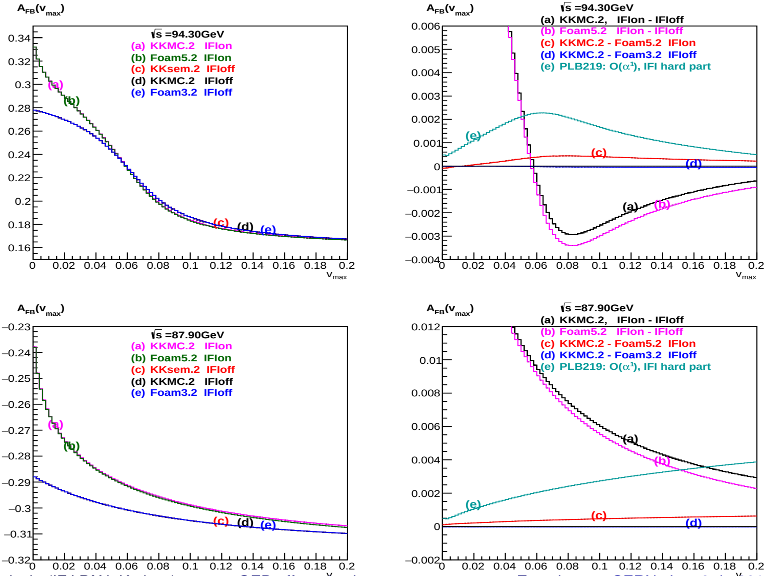
$$\times\; F(\gamma_I)\gamma_I v_I^{\gamma_I - 1}\; F(\gamma_F)\gamma_I v_F^{\gamma_F - 1}\; F(\gamma_{IF})\gamma_{IF} v_{IF}^{\gamma_{IF}-1}\; F(\gamma_{FI})\gamma_{FI} v_{IF}^{\gamma_{FI}-1}$$

$$\times\; e^{2\alpha\Delta B_4^V}\mathcal{M}_V^{(0)}(s(1-v_I-v_{IF}),\theta)\; [e^{2\alpha\Delta B_4^{V'}}\mathcal{M}_{V'}^{(0)}(s(1-v_I-v_{FI}),\theta)]^*\; [1+\mathrm{NIR}(\mathrm{v_I},\mathrm{v_F})],$$

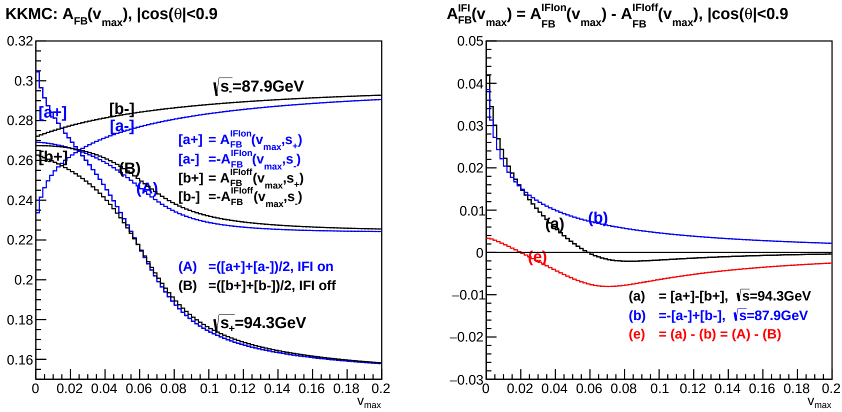
- Convolution of **four** radiator functions (instead of two)!
- Extra virtual formfactor ΔB_4^Z due to IFI for resonant contrib.
- $\gamma_I = Q_e^2 \frac{\alpha}{\pi}[\frac{s}{m_e^2} - 1], \quad \gamma_{IF} = \gamma_{FI} = Q_e Q_f \frac{\alpha}{\pi} \ln\frac{1-\cos\theta}{1+\cos\theta}, \quad F(\gamma) = \frac{e^{-C_E\gamma}}{\Gamma(1+\gamma)}$

IFI from KKMC tested using new KKfoam at the $\delta A_{FB} \sim 10^{-4}$ level

v_{max} = cutoff on total photon energy in units of the beam energy



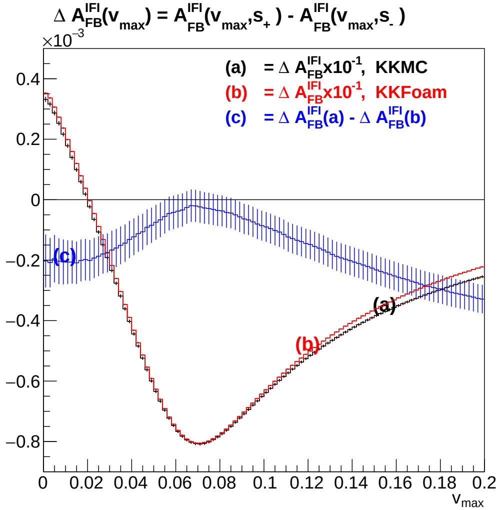
$A_{FB}(v_{\max}, s_{\pm})$ from KKMC with $\mathcal{O}_{exp.}(\alpha^2)$ ISR+FSR and $\mathcal{O}_{exp.}(\alpha^1)$ IFI.



Results from KKfoam look the same.

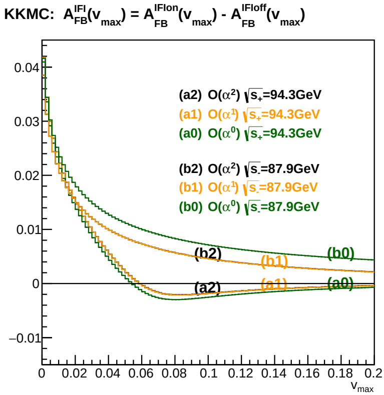
Let us chack the differences KKMC-KKfoam. See next slide.

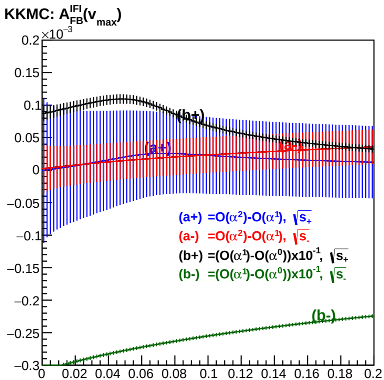
Differences between $\Delta A_{FB}^{IFI}(v_{max})$ from KKMC and KKfoam $\sim 2 \cdot 10^{-4}$.



More work needed on the improvement of KKfoam:
inclusion of complete $\mathcal{O}(\alpha^1)$ hard non-soft IFI component.

IFI component in $A_{FB}(s_{\pm})$ from KKMC





IFI component in $A_{FB}(s_{\pm})$ obtained using KKMC program with three types of the increasingly sophisticated QED matrix element, $\mathcal{O}_{exp.}(\alpha^{i})$, $i = 0, 1, 2$.

Precision $\delta A_{FB}^{IFI} \sim 3 \cdot 10^{-3}$ seems to be within reach...

Summary

- The influence of IFI on A_{FB} is huge, as compared to precision scale aimed at FCCee.
- Strong $\sqrt{s}$ dependence of A_{FB} near $M_Z \pm 3.5 GeV$ matters (ISR).
- However, IFI could be calculated in perturbative QED very precisely, thanks to power of the semi-soft photon resummation, similarly as huge QED correction to Z lineshape.
- IFI effect is strongly dependent on the type and strength of kinematic cuts – good quality MC implementation is mandatory.
- KKMC simulates soft (hard) real photons including IFI in an almost perfect way.
- New encouraging results from KKfoam/KKMC comparisons.
- More work needed to cross-check KKMC and get more/better quantitative results down to $\delta A_{FB} \sim 10^{-5}$ needed for FCCee.