Electroweak bosonic 2-loop corrections to Z-pole precision observables: present status

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WG1 & WG2 working meeting

CERN, 9 January 2018

Theoretical playground

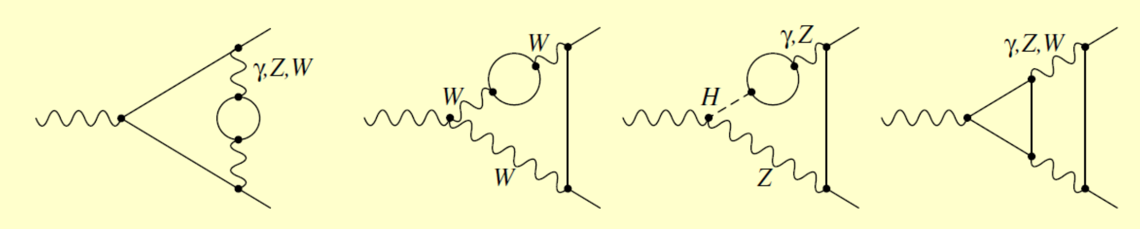
EWPOs (electroweak pseudo-observables):

$$\begin{gathered}
\{\Gamma_{Zff},\, \Gamma_{Zbb}\} \quad \longrightarrow \Gamma_{Z_{\mathrm{tot}}} \\
R_\ell \\
R_{c,b} \\
A^f_{\mathrm{FB}},\, A^f_{\mathrm{LR}} \\
\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}}, \sin^2\theta^{lept}_{\mathrm{eff}}
\end{gathered}$$

In future (Alain's FCC Berlin talk) sensitivity of experiments by a factor 20-100 better compared to LEP physics.

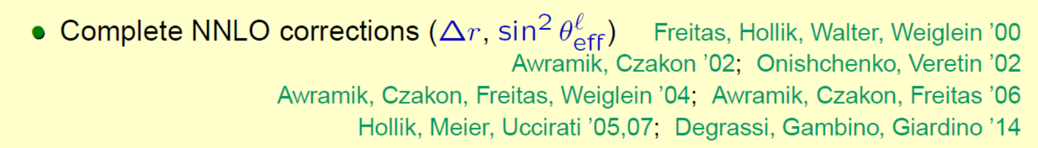
Published results on EWPOs in the SM @NNLO (1)

Known corrections ($\Delta\rho$, $\sin^2\theta^{\rm f}_{\rm eff}, g_V, g_A$):



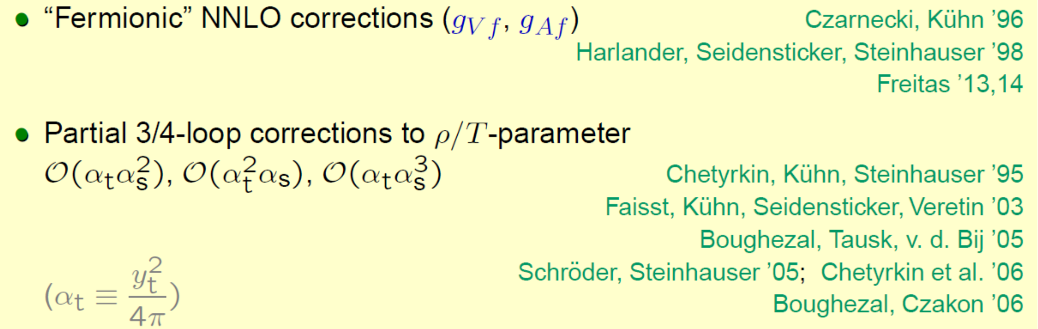
Fermionic (with fermions loops) and bosonic (rest)

Published results on EWPOs in the SM @NNLO (2)



- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\rm eff}$) Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02; Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14

"Bosonic" NNLO corrections for $\sin^2\theta^{\rm b}_{\rm eff}$ - Dubovyk, Freitas, JG, Riemann, Usovitsch '16



- "Fermionic" NNLO corrections ($g_{V\,f}$, $g_{A\,f}$) Czarnecki, Kühn '96
  Harlander, Seidensticker, Steinhauser '98
  Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2)$, $\mathcal{O}(\alpha_{\rm t}^2\alpha_{\rm s})$, $\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^3)$ Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05; Chetyrkin et al. '06
  Boughezal, Czakon '06

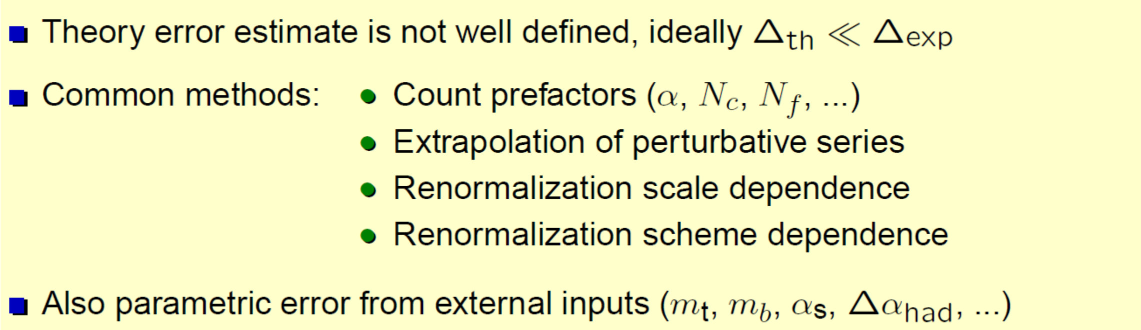
  $(\alpha_{\rm t} \equiv \frac{y_{\rm t}^2}{4\pi})$

Current uncertainities, Ayres: 1604.00406

	Experiment	Theory error	Main source
M_{W}	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_{\mathrm{s}}$
Γ_{Z}	2495.2 ± 2.3 MeV	0.5 MeV	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$, $\alpha\alpha^2_{\mathrm{s}}$
σ^0_{had}	41540 ± 37 pb	6 pb	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$R_b \equiv \Gamma^b_{\mathrm{Z}}/\Gamma^{\mathrm{had}}_{\mathrm{Z}}$	0.21629 ± 0.00066	0.00015	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$\sin^2\theta^\ell_{\mathrm{eff}}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_{\mathrm{s}}$

Work in progress

Error estimations, Ayres: 1604.00406



- Theory error estimate is not well defined, ideally $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs (m_{t}, m_b, α_{s}, $\Delta\alpha_{\text{had}}$, ...)

Error estimation for Γ_Z, Ayres: 1604.00406

1. Geometric series

$$\begin{aligned}
\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) &\sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha^2) \sim 0.26\ MeV \\
\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) &\sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.3\ MeV \\
\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) &\sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.23\ MeV \\
\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) &\sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s^2) \sim 0.035\ MeV \\
\mathcal{O}(\alpha_{bos}^2) &\sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1\ MeV}
\end{aligned}$$

2. Parametric prefactors

$$\begin{aligned}
\mathcal{O}(\alpha_{bos}^2) &\sim \Gamma_Z\mathcal{O}(\alpha^2) \sim 0.1\ MeV \\
\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) &\sim \Gamma_Z\frac{\alpha n_q}{\pi}\alpha_s^2 \sim 0.29\ MeV
\end{aligned}$$

Total: $\delta\Gamma_Z \sim \mathbf{0.5}$ **MeV**

Future projections, Ayres: 1604.00406

	Measurement error			Intrinsic theory	
	ILC	CEPC	FCC-ee	Current	Future[†]
M_W [MeV]	3–4	3	1	4	1
Γ_Z [MeV]	0.8	0.5	**0.1**	0.5	**0.2**
R_b $[10^{-5}]$	14	17	6	15	7
$\sin^2\theta^\ell_\mathrm{eff}$	1	2.3	**0.6**	4.5	**1.5**

Table: Projected experimental and theoretical uncertainties for some electroweak precision pseudo-observables.

[†] Based on estimations for: $\cancel{\mathcal{O}(\alpha^2_{bos})}, \mathcal{O}(\alpha\alpha_s^2), \mathcal{O}(N_f\alpha^2\alpha_s), \mathcal{O}(N_f^2\alpha^3)$

To substantiate my claim: Effective weak mixing angle $\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}}$

- The standard model prediction for the effective weak mixing angle can be written as

$$\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_{\mathrm{b}})$$

- The bosonic electroweak two-loop corrections amount to

$$\boxed{\Delta\kappa_{\mathrm{b}}^{(\alpha^2,bos)} = -0.9855 \times 10^{-4}}$$

DFGRU, Phys.Lett. B762 (2016) 184

Collection of radiative corrections: full stabilization at 10^{-4}!

Order	Value $[10^{-4}]$
α	468.945
$\alpha\alpha_s$	-42.655
$\alpha^2_{\rm ferm}$	3.866
$\alpha^2_{\rm bos}$	**-0.986**

Order	Value $[10^{-4}]$
$\alpha_t^2\alpha_s$	1.362
α_t^3	0.123
$\alpha_t\alpha_s^2$	-7.074
$\alpha_t\alpha_s^3$	-1.196

± 0.001 $\longrightarrow$

Table: *Comparison of different orders of radiative corrections to $\Delta\kappa_{\rm b}$.*

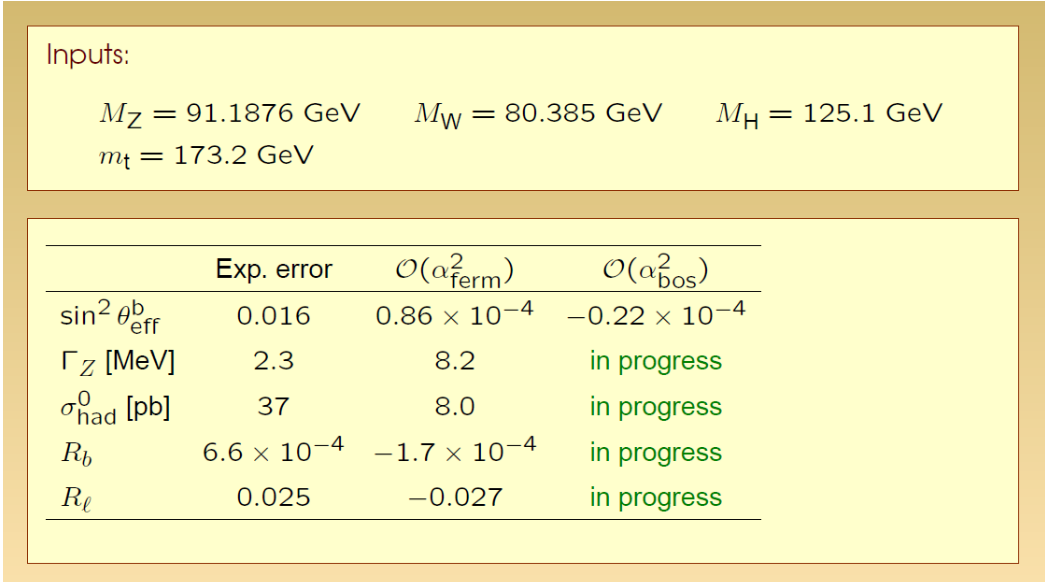
Input Parameters: M_Z, Γ_Z, M_W, Γ_W, M_H, m_t, α_s *and* $\Delta\alpha$

- one-loop contributions [Akhundov, Bardin, Riemann, 1986] [Beenakker, Hollik, 1988]
- two-loop fermionic contributions [Awramik, Czakon, Freitas, Kniehl, 2009]
- two-loop bosonic contributions [Dubovyk, Freitas, JG, Riemann, Usovitsch, 2016]

Partial higher-order corrections

$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2)$	Avdeev: 1994,Chetyrkin: 1995
$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^3)$	Schroder: 2005,Chetyrkin: 2006,Boughezal: 2006
$\mathcal{O}(\alpha^2\alpha_{\rm t})$ and $\mathcal{O}(\alpha_{\rm t}^3)$	vanderBij: 2000,Faisst: 2003

Final accords @NNLO



Inputs:

$M_Z = 91.1876$ GeV $\quad M_W = 80.385$ GeV $\quad M_H = 125.1$ GeV

$m_t = 173.2$ GeV

	Exp. error	$\mathcal{O}(\alpha^2_{\text{ferm}})$	$\mathcal{O}(\alpha^2_{\text{bos}})$
$\sin^2\theta^{b}_{\text{eff}}$	0.016	0.86×10^{-4}	-0.22×10^{-4}
Γ_Z [MeV]	2.3	8.2	in progress
σ^0_{had} [pb]	37	8.0	in progress
R_b	6.6×10^{-4}	-1.7×10^{-4}	in progress
R_ℓ	0.025	-0.027	in progress

"Midnight" fresh tables

Input parameters:

Parameter	Value	Parameter	Value
M_Z	91.1876 GeV	$m_\mathrm{b}^{\overline{\mathrm{MS}}}$	4.20 GeV
Γ_Z	2.4952 GeV	$m_\mathrm{c}^{\overline{\mathrm{MS}}}$	1.275 GeV
M_W	80.385 GeV	m_τ	1.777 GeV
Γ_W	2.085 GeV	$\Delta\alpha$	0.05900
M_H	125.1 GeV	$\alpha_\mathrm{s}(M_\mathrm{Z})$	0.1184
m_t	173.2 GeV	G_μ	1.16638×10^{-5} GeV^{-2}

The cherry on the 2-loops EWPOs cake: results (preliminary)

	Γ_Z [MeV]	σ^0_{had} [pb]
$\mathcal{O}(\alpha)$	60.22	-48.86
$\mathcal{O}(\alpha\alpha_s)$	9.11	3.14
$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	1.20	0.48
$\mathcal{O}(N_f^2\alpha^2)$	5.13	-1.03
$\mathcal{O}(N_f\alpha^2)$	3.04	9.09
$\mathcal{O}(\alpha^2_{\mathrm{bos}})$	**0.51**	**1.27**

Compare it with slide 7:

Error estimation for $\mathcal{O}(\alpha^2_{bos}) \sim 0.1$ MeV

Error estimation for $\Gamma_Z \sim 0.5$ MeV

Compare it with slide 8:

FCC-ee$^{exper.\ error}(\Gamma_Z) \sim 0.1$ MeV

FCC-ee$^{theor.\ error}(\Gamma_Z) \sim 0.2$ MeV

Γ_i [MeV]	Γ_e	Γ_ν	Γ_d	Γ_u	Γ_b	Γ_Z
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	0.038	0.059	0.191	0.170	0.190	1.20
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha^2_{\mathrm{bos}})$	**0.017**	**0.019**	**0.058**	**0.057**	**0.167**	**0.51**

How to think about errors estimations

For discussion:

"To carry out these calculations, qualitatively new developments of existing loop integration techniques will be required, but no conceptual paradigm shift."

From MTTD,17 conference proceedings:

"It is important to emphasize that the theory errors are estimates based on experience and several well-motivated but somewhat ambigious principles, so that the true magnitude of the missing higher orders could well be larger [5]. Thus it is generally desirable to have a situation where the theory errors are subdominant compared to the experimental errors."

[5] A. Freitas, Prog. Part. Nucl. Phys. 90 (2016) 201 [arXiv:1604.00406 [hep-ph]].