Precision predictions of WW process

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WG1 & WG2 working meeting

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Outline

- Statistical precision
- Available theoretical calculations and their precision
- Monte Carlo codes from LEP
- Inclusion of Non-Factorisable corrections into YFS scheme
- Summary

W mass from W^+W^- threshold cross section

Statistical precision

Cross section accuracy

$$\Delta\sigma_{WW} = \frac{\sigma_{WW}}{\sqrt{N}}$$

and the W mass accuracy [CERN YR Phys at LEP2 vol. 1 p. 161]

$$\Delta M_W = \Delta\sigma_{WW}\left|\frac{dM_W}{d\sigma_{WW}}\right| = \sqrt{\sigma_{WW}}\left|\frac{dM_W}{d\sigma_{WW}}\right|\frac{\sqrt{\sigma_{WW}}}{\sqrt{N}}$$

At the WW thresh. the sensitivity factor is estimated to be

$$\sqrt{\sigma_{WW}}\left|\frac{dM_W}{d\sigma_{WW}}\right| \sim 0.91\mathrm{GeV}/\sqrt{\mathrm{pb}}$$

The physics goals are set to 10/ab and 3×10^7 events at WW threshold

Experimental precision

With the $\sigma_{WW} = 3\text{pb}$ we obtain

$$\Delta M_W = 0.91 \frac{\text{GeV}}{\sqrt{\text{pb}}} \frac{\sqrt{3\ \text{pb}}}{\sqrt{3 \times 10^7}} = 0.3\ \text{MeV} = 3.6 \times 10^{-6} M_W$$

whereas the accuracy of the total cross section is

$$\frac{\Delta\sigma_{WW}}{\sigma_{WW}} = \frac{1}{\sqrt{3 \times 10^7}} = 0.02\%$$

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Theoretical precision – Status at LEP2

$\mathcal{O}(\alpha)$ EW corrections

- LPA (Leading Pole Appr.) of YFSWW3: only terms with two W-bosons
- DPA (Double Pole Appr.) of RacoonWW: LPA plus full $4f + \gamma$ real corrs.
- IBA (Improved Born Appr.): only universal corrs. to CC03 (Coulomb etc.)
- At the *WW* thresh. the DPA/LPA provide 25% corrs. to the Born
- Complete $\mathcal{O}(\alpha)$ corrs. to $e^+e^- \to 4f$ calculated for selected final states [Denner et al. arXiv:0502063] with the following conclusions:
- At 161 GeV difference DPA/LPA vs. full 4f $\mathcal{O}(\alpha)$ is $\sim 2\%$ of Born.
  Above the threshold this difference drops to below 0.5%.
  The difference DPA/LPA vs. IBA is $\sim 0.2\%$ of Born at 161 GeV.

Missing corrections

Missing corrs. according to [Denner et al. arXiv:0502063]

- Higher order ISR – feasible in YFS exponentiation approx.
- Higher order EW corrs. dominated by $\alpha^2 \log(m_e^2/s)$ estimated at $\leqslant$ **0.1**%
- Higher order Coulomb effect $\sim 0.2\%$

Finally, the QCD effects must be also included: $\mathcal{O}(\alpha_S)$ corrs. including matching with PS, B–E and colour reconnection.

Overall EW precision of $\mathcal{O}(\alpha)$ result

total cross section: **few** $\times$ **0.1%** [Denner et al. arXiv:0502063]

NNLO corrections

- With the method of **unstable particle effective theory** the dominant NNLO corrections to four fermion process ($\mu^- \bar{\nu}_\mu u \bar{d}$) were calculated in [Actis et al. arXiv:0807.0102].
- They are nick-named N$^{3/2}$LO^{EFT} because in EFT a different expansion parameter is used to count the strength of particular corrections.
- The effect of these N$^{3/2}$LO^{EFT} corrections on W mass is 3 MeV and on total cross-section 0.2 % at the threshold.
- The drawback of the EFT method is that it provides **inclusive** results only.

$\mathcal{O}(\alpha)$ WW Monte Carlo codes

- RacoonWW [Denner et al. hepph/0104057] Contains $e^+e^- \to 4 fermions$ Born-level process, complete real correction $e^+e^- \to 4 fermions + \gamma$ and $\mathcal{O}(\alpha)$ EW virtual corrections in Double Pole Approx. ISR radiation is based on LL QED structure functions to second order with soft photon exponentiation.
- YFSWW3 v.1.16 [Jadach et al. hepph/0103163] generates signal process $e^+e^- \to W^+W^- \to 4 fermions$ up to $\mathcal{O}(\alpha)$ in production and decay separately. It includes multiphotonic radiation from the production part in the YFS framework. The hard ISR is corrected to third order in the LO approximation. The FSR in W-decay is handled by PHOTOS.
- The difference between the two implementations (DPA vs. LPA) of the virtual plus soft $\mathcal{O}(\alpha)$ corrections to W-pair production is below 0.01% for the total cross section, whereas, the overall agreement of these two codes, including all physical effects is of the order of 0.3% for the total cross section at 200 GeV.

4-fermion Monte Carlo codes

- KoralW 1.42 [Jadach et al. hepph/9906277] contains complete four fermion background to the process $e^+e^- \rightarrow 4\,fermions$ at the Born level. Radiation is limited to multiphoton YFS-type ISR corrected to third order in the LO approximation. FSR is done by PHOTOS.
- KandY = KoralW+YFSWW combined 4-fermion and $\mathcal{O}(\alpha)$ WW
- WPHACT [E. Accomando et al. hep-ph/0204052] all 4fermion final states with imaginary Fermion Loop gauge restoring scheme.
- Other 4fermion codes (not used at LEP)
- No MC code for $\mathcal{O}(\alpha)$ 4fermion process

Single W Monte Carlo code

- WINHAC [Jadach et al. hepph/0302065] generates single W process $q\bar{q} \rightarrow W^{\pm} \rightarrow f_1 f_2$ up to first order. The multiphotonic radiation in YFS scheme is included in the decay of the W-boson. This radiation can be adapted to the YFSWW3 code.

Towards high precision MC for W's

- MC code with $\mathcal{O}(\alpha)$ 4fermion process – feasible, calculations exist
- MC code with $\mathcal{O}(\alpha^2)$ 4f processes – calculation out of reach ??
- Continue LEP2 strategy: MC with $\mathcal{O}(\alpha^2)$ separate corrections in WW production and in W decays, i.e. Double Pole approx. Calculations feasible ??
- How precise it could be? Lets do naive math

$$\mathcal{O}(\alpha)_{SP} \sim \Big(\mathcal{O}(\alpha)_{4f} - \mathcal{O}(\alpha)_{DP}\Big) \sim 2\%\ Born\ \sim 8\%\ \mathcal{O}(\alpha)_{DP}$$

If $\mathcal{O}(\alpha^2) \leqslant 0.2\%$ and $\mathcal{O}(\alpha^2)_{SP} \sim 8\%\ \mathcal{O}(\alpha^2)$

then $\mathcal{O}(\alpha^2)_{SP} \sim 0.016\%$

Towards high precision MC for W's

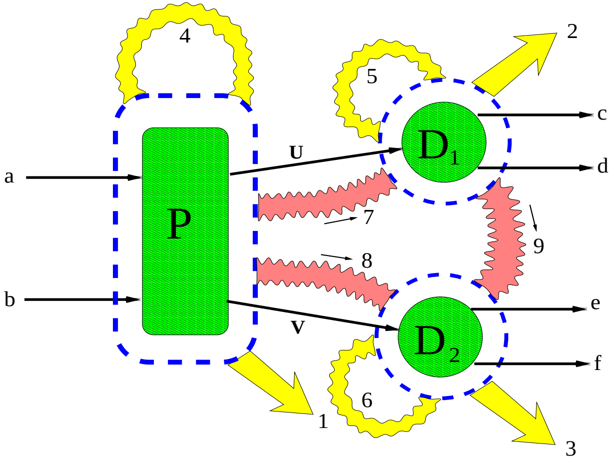
There is one more contribution that we can include

- The missing interferences P-D and D-D to the WW graphs can be included **to all orders in soft approximation** within extended YFS scheme. It was done in KKMC for Z resonance
- How much of the 4f loop corrections is due to interferences Production-Decay and Decay-Decay in the signal graphs ?
- W's can emit photons – complication w.r.t. Z-boson.
- Does it make sense to exponentiate W's as there is no IR singularities for them? Yes! W's are almost stable particles!

Exponentiation for charged resonances

An unfinished LEP2 project ...

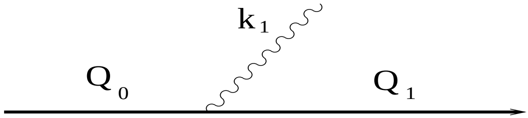
All virtual and real emissions, in soft limit



The above is our aim! How to get there?

Factoring photon emission

Single emission from the internal W line:



Noticing that $Q_0^2 - Q_1^2 = 2k_1 Q_0 - k_1^2 = 2k_1 Q_1 + k_1^2$ we may write:

$$\frac{1}{(Q_0^2 - M^2)(Q_1^2 - M^2)} = \frac{1}{(2k_1 Q_1 + k_1^2)(Q_1^2 - M^2)} + \frac{1}{(-2k_1 Q_0 + k_1^2)(Q_0^2 - M^2)}$$

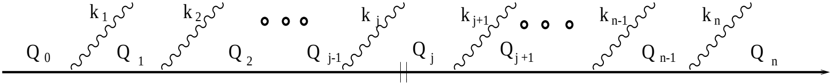
where M is complex mass of W.

It looks like sum of two on-shell emission factors times pole term.

LHS: IR-finite!

RHS: Difference of two IR-divergent terms!

Multiple emission from the internal W line



In the soft photon limit we find general formula

$$\sum_{permut.} \frac{1}{(Q_0^2-M^2)(Q_1^2-M^2)\ldots(Q_n^2-M^2)} =$$

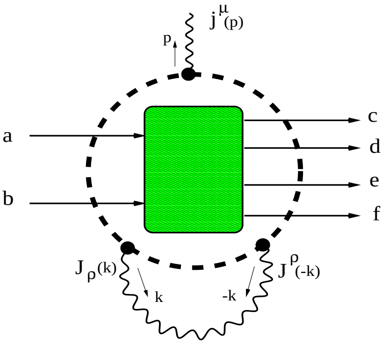
$$= \sum_{\wp=(P,D)^n} \prod_{\wp_i=P} \frac{1}{(Q_\wp+k_i)^2-Q_\wp^2} \times \frac{1}{Q_\wp^2-M^2} \times \prod_{\wp_k=D} \frac{1}{(Q_\wp-k_i)^2-Q_\wp^2},$$

where

$$Q_\wp = Q_0 - \sum_{\wp_i=P} k_i = Q_n + \sum_{\wp_i=D} k_i.$$

It looks like sum of 2^n on-shell emission factors times pole term!

Notation: EM real and virtual current



$$j^\mu(k) = ie \sum_{X=a,b,c,d,e,f} Q_X \theta_X \frac{2p_X^\mu}{2p_X k}$$

$$J^\mu(k) = \sum_{X=a,b,c,d,e,f} \hat{J}_X^\mu(k),$$

$$\hat{J}_X^\mu(k) \equiv Q_X \theta_X \frac{2p_X^\mu + k^\mu \theta_X}{k^2 + 2p_X k \theta_X + i\varepsilon}$$

Virtual lines are pair-contracted giving S-factors:

$$S(k) = J(k) \circ J(k) = \sum_{\substack{X=a,b,c,d,e,f \\ Y=a,b,c,d,e,f}} J_X(k) \circ J_Y(k),$$

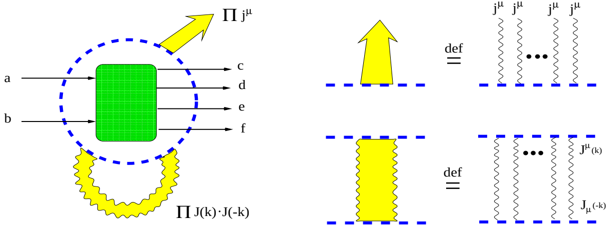
where Q_X is charge, $\theta = +1, -1$ for initial, final state and

$$J_X(k) \circ J_Y(k) \equiv J_X(k) \cdot J_Y(-k), \text{ for } X \neq Y,$$

$$J_X(k) \circ J_X(k) \equiv J_X(k) \cdot J_X(k).$$

(Exactly as in YFS61)

Standard Yennie-Frautschi-Suura-1961, 6 external legs



$$M^{\mu_1\mu_2\cdots\mu_m}(k_1, k_2, ..., k_m) =$$

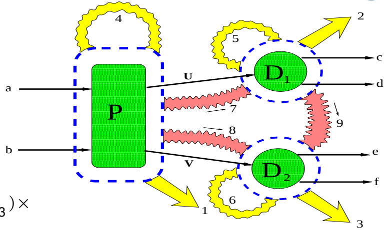
$$= \mathcal{M} \prod_{l=1}^{m} j^{\mu}(k_l) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int \frac{i}{(2\pi)^3} \frac{d^4 k_i}{k_i^2 - \lambda^2 + i\varepsilon} J^{\mu}(k_i) \circ J_{\mu}(k_i)$$

$$= \mathcal{M} \prod_{l=1}^{m} j^{\mu}(k_l)\ e^{\alpha B_6},$$

$$B_6 = \int \frac{i}{(2\pi)^3} \frac{d^4 k}{k^2 - \lambda^2 + i\varepsilon} J(k) \circ J(k).$$

NEW!!! 6 external legs + 2 internal lines (resonaces)

For a given assignment (permutation) of real photons to P, D_1 and D_2 we find for matrix element:



$$
M^{\mu_{1_1}\dots\mu_{3n_3}}_{n_1 n_2 n_3}(\{k\}) = \mathcal{M}_0 \prod_{i_1=1_1}^{n_1} j_P^{\mu_{i_1}}(k_{i_1}) \prod_{i_2=1_2}^{n_2} j_{D_1}^{\mu_{i_2}}(k_{i_2}) \prod_{i_3=1_3}^{n_3} j_{D_2}^{\mu_{i_3}}(k_{i_3}) \times
$$

$$
\sum_{n_4=0}^{\infty}\frac{1}{n_4!}\prod_{i_4=1}^{n}\int\frac{i}{(2\pi)^3}\frac{d^4k_{i_4}}{k_{i_4}^2-m_\gamma^2+i\varepsilon}J_P(k_{i_4})\circ J_P(k_{i_4}) \sum_{n_5=0}^{\infty}\frac{1}{n_5!}\prod_{i_5=1}^{n}\int\frac{i}{(2\pi)^3}\frac{d^4k_{i_5}}{k_{i_5}^2-m_\gamma^2+i\varepsilon}J_{D_1}(k_{i_5})\circ J_{D_1}(k_{i_5})
$$

$$
\sum_{n_6=0}^{\infty}\frac{1}{n_6!}\prod_{i_6=1}^{n}\int\frac{i}{(2\pi)^3}\frac{d^4k_{i_6}}{k_{i_6}^2-m_\gamma^2+i\varepsilon}J_{D_2}(k_{i_6})\circ J_{D_2}(k_{i_6}) \sum_{n_7=0}^{\infty}\frac{1}{n_7!}\prod_{i_7=1}^{n}\int\frac{i}{(2\pi)^3}\frac{d^4k_{i_7}}{k_{i_7}^2-m_\gamma^2+i\varepsilon}J_P(k_{i_7})\circ J_{D_1}(k_{i_7})
$$

$$
\sum_{n_8=0}^{\infty}\frac{1}{n_8!}\prod_{i_8=1}^{n}\int\frac{i}{(2\pi)^3}\frac{d^4k_{i_8}}{k_{i_8}^2-m_\gamma^2+i\varepsilon}J_P(k_{i_8})\circ J_{D_2}(k_{i_8}) \sum_{n_9=0}^{\infty}\frac{1}{n_9!}\prod_{i_9=1}^{n}\int\frac{i}{(2\pi)^3}\frac{d^4k_{i_9}}{k_{i_9}^2-m_\gamma^2+i\varepsilon}J_{D_1}(k_{i_9})\circ J_{D_2}(k_{i_9})
$$

$$
\frac{1}{(p_{cd}+K_2-K_7+K_9)^2-M^2}\;\frac{1}{(p_{ef}+K_3-K_8-K_9)-M^2},
$$

Sum up for P, D_1 and D_1 as in YFS61

$$
= \mathcal{M}_0 \prod_{i_1=1_1}^{n_1} j_P^{\mu_{i_1}}(k_{i_1}) \prod_{i_2=1_2}^{n_2} j_{D_1}^{\mu_{i_2}}(k_{i_2}) \prod_{i_3=1_3}^{n_3} j_{D_2}^{\mu_{i_3}}(k_{i_3})
$$

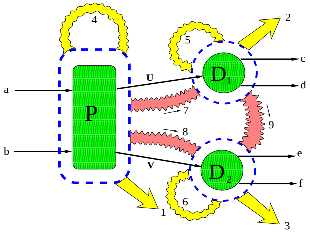
$$
e^{\alpha B_P}\; e^{\alpha B_{D_1}}\; e^{\alpha B_{D_2}}
$$

$$
\sum_{n_7=0}^{\infty} \frac{1}{n_7!} \prod_{i_7=1}^{n} \int \frac{i}{(2\pi)^3} \frac{d^4 k_{i_7}}{k_{i_7}^2 - m_\gamma^2} J_P(k_{i_7}) \circ J_{D_1}(k_{i_7})
$$

$$
\sum_{n_8=0}^{\infty} \frac{1}{n_8!} \prod_{i_8=1}^{n} \int \frac{i}{(2\pi)^3} \frac{d^4 k_{i_8}}{k_{i_8}^2 - m_\gamma^2} J_P(k_{i_8}) \circ J_{D_2}(k_{i_8})
$$

$$
\sum_{n_9=0}^{\infty} \frac{1}{n_9!} \prod_{i_9=1}^{n} \int \frac{i}{(2\pi)^3} \frac{d^4 k_{i_9}}{k_{i_9}^2 - m_\gamma^2} J_{D_1}(k_{i_9}) \circ J_{D_2}(k_{i_9})
$$

$$
\frac{1}{(U_2 - K_7 + K_9)^2 - M^2} \; \frac{1}{(V_3 - K_8 - K_9) - M^2},
$$



where $\alpha B_X = \int \frac{i}{(2\pi)^3} \frac{d^4 k}{k^2 - m_\gamma^2 + i\varepsilon} J_X(k) \circ J_X(k)$, $X = P, D_1, D_2$.

Now tricky point:

In the soft photon limit:

$$\frac{1}{(U_2 - K_7 + K_9)^2 - M^2} \simeq \frac{1}{U_2^2 - M^2 - 2U_2K_7 + 2U_2K_9}$$

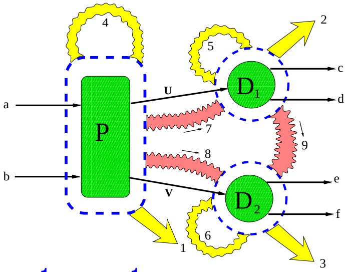
$$= \frac{1}{U_2^2 - M^2} \frac{1}{1 - \sum_{i_7} \frac{2U_2k_{i_7}}{U_2^2 - M^2} + \sum_{i_9} \frac{2U_2k_{i_9}}{U_2^2 - M^2}}$$

$$= \frac{1}{U_2^2 - M^2} \prod_{i_7} \frac{1}{1 - \frac{2U_2k_{i_7}}{U_2^2 - M^2}} \prod_{i_9} \frac{1}{1 + \frac{2U_2k_{i_9}}{U_2^2 - M^2}}$$

$$\simeq \frac{1}{U_2^2 - M^2} \prod_{i_7} \frac{U_2^2 - M^2}{(U_2 - k_{i_7})^2 - M^2} \prod_{i_9} \frac{U_2^2 - M^2}{(U_2 + k_{i_9})^2 - M^2}$$

leading to final result CEEX result, see next slide...

CEEX for narrow resonances



$$M^{\mu_1 \dots \mu_n}(k_1, k_2, \dots, k_n) =$$

$$= \sum_{\wp \in (P, D_1, D_2)^n} \mathfrak{M}_0 \prod_{i=1}^{n} j_{\wp_i}^{\mu_i}(k_i)\; e^{B_{10}^{\rm CEEX}(U_\wp, V_\wp)} \frac{1}{U_\wp^2 - M^2} \frac{1}{V_\wp^2 - M^2},$$

where

$$U_\wp = p_{cd} + \sum_{\wp_i = D1} k_i, \quad V_\wp = p_{ef} + \sum_{\wp_i = D2} k_i,$$

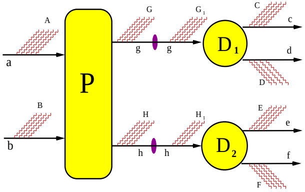
$$\alpha B_{10}^{\rm CEEX}(U,V) = \alpha B_P + \alpha B_{D_1} + \alpha B_{D_2} + \alpha B_{P \otimes D_1}(U) + \alpha B_{P \otimes D_1}(U) + \alpha B_{D_1 \otimes D_2}(U,V),$$

$$\alpha B_{P \otimes D_1}(U) = \int \frac{i}{(2\pi)^3} \frac{d^4 k}{k^2 - m_\gamma^2 + i\varepsilon} J_P(k) \circ J_{D_1}(k) \frac{U^2 - M^2}{(U-k)^2 - M^2},$$

$$\alpha B_{P \otimes D_1}(V) = \int \frac{i}{(2\pi)^3} \frac{d^4 k}{k^2 - m_\gamma^2 + i\varepsilon} J_P(k) \circ J_{D_2}(k) \frac{V^2 - M^2}{(V-k)^2 - M^2},$$

$$\alpha B_{D_1 \otimes D_2}(U,V) = \int \frac{i}{(2\pi)^3} \frac{d^4 k}{k^2 - m_\gamma^2 + i\varepsilon} J_{D_1}(k) \circ J_{D_2}(k) \frac{U^2 - M^2}{(U+k)^2 - M^2} \frac{V^2 - M^2}{(V-k)^2 - M^2}.$$

Closer look at 1-real-photon case



$$\mathcal{M}_1^{(0)\,\mu_1}(k) \simeq$$

$$\frac{1}{p_{cd}^2-M^2}\;\frac{1}{p_{ef}^2-M^2}\left\{ Q_a\frac{2p_a^\mu}{2p_a k} + Q_b\frac{2p_b^\mu}{2p_b k} - Q_g\frac{2p_g^\mu}{2p_g k} - Q_h\frac{2p_h^\mu}{2p_h k}\right\}$$

$$+\frac{1}{(p_{cd}+k)^2-M^2}\;\frac{1}{p_{ef}^2-M^2}\left\{ Q_g\frac{2p_g^\mu}{2p_g k} - Q_c\frac{2p_c^\mu}{2p_c k} - Q_d\frac{2p_d^\mu}{2p_d k}\right\}$$

$$+\frac{1}{p_{cd}^2-M^2}\;\frac{1}{(p_{ef}+k)^2-M^2}\left\{ Q_h\frac{2p_h^\mu}{2p_h k} - Q_e\frac{2p_e^\mu}{2p_e k} - Q_f\frac{2p_d^\mu}{2p_f k}\right\}$$

$$=\frac{1}{p_{cd}^2-M^2}\;\frac{1}{p_{ef}^2-M^2}\left\{ j_P^\mu + \frac{p_{cd}^2-M^2}{(p_{cd}+k)^2-M^2}\, j_{D_1}^\mu + \frac{p_{ef}^2-M^2}{(p_{ef}+k)^2-M^2}\, j_{D_2}^\mu\right\}$$

For $k^0 < \Gamma$: Normal YFS small limit, Emission from *W*'s cancels out!

For $k^0 > \Gamma$: Each Intermediade *W* is present twice (4+3+3=10 sources),
Energy shift in W propagator proprly coherently accounted for,
Three gauge-invariant currects for production and 2 decays.

Summary of new exponentiation scheme

Interferences between production and two decays of W's can be implemented to infinite order in diffrential distributions

- It is extension of YFS61 scheme at the amplitude level (CEEX)
- It is exact in the soft photon limit
- Recoil in W propagators is properly described
- $\mathcal{O}(\alpha)$ corrs. can be added to the β functions
- EEX-type scheme (ready for YFSWW3) without P-D, D-D interferences can be derived from general formula by dropping non-diagonal products of currents
- Works for single-W as well
- The CEEX case of Z-boson is already implemented in KKMC (as reweighted EEX)

Conclusions and outlook

- Monte Carlo scheme for W-pair production is proposed based on extension of YFS61 method.
  It includes NF interferences P-D and D-D.
- Important question: how much of the complete corrs. is reproduced by these NF exponentiated interferences?
- $\mathcal{O}(\alpha^2)$ calculations for the Double-Pole production and decay parts are needed ! Feasible ??
- $\mathcal{O}(\alpha^1)$ for $e^- e^+ \to 4f$ must be implemented in MC with explicit split into DP+SP.
- Exponentiated ISR with $\mathcal{O}(\alpha^3)$ LL mandatory

The overall precision tag $\sim 10^{-4}$!?

YFSWW3+KoralW look like a good starting point

Backup

$\mathcal{O}(\alpha^1)_{\rm CEEX}$ with **incomplete** $\mathcal{O}(\alpha\Gamma_W/M_W)$

$$\mathfrak{M}_n^{(1)\,\mu_1,\mu_2,\ldots,\mu_n}(k_1,k_2,\ldots,k_n) = \sum_{\wp\in\{P,D_1,D_2\}^n} e^{\alpha B_{10}^{\rm CEEX}(U_\wp,V_\wp)}$$

$$\times\left\{\hat\beta_0^{(1)}(U_\wp,V_\wp)\prod_{i=1}^n j_{\{\wp_i\}}^{\mu_i}(k_i) + \sum_{j=1}^n \hat\beta_{1\{\wp_j\}}^{(1)\mu_j}(U_\wp,V_\wp,k_j)\prod_{i\neq j} j_{\{\wp_i\}}^{\mu_i}(k_i)\right\}$$

IR-finite $\hat\beta$'s from $\mathcal{O}(\alpha^1)$ Feynman diagram calculations (without NF):

$$\hat\beta_0^{(1)}(U,V) = \left[e^{-\alpha B_{10}^{Fact}(U,V)} M_0^{(1)}(U,V)\right]_{\mathcal{O}(\alpha^1)} = M_0^{(1)}(U,V) - B_{10}^{Fact}(U,V),$$

where $B_{10}^{Fact}(s_1,s_2) = \alpha B_P + \alpha B_{D_1} + \alpha B_{D_2}$,

$$\hat\beta_{1\{\pi\}}^{(1)\mu}(U_\pi,V_\pi,k) = M_{1\{\pi\}}^{(1)\mu}(U_\pi,V_\pi,k) - j_\pi^\mu(k) M_0^{(0)}(U_\pi,V_\pi),\quad \pi = P, D_1, D_2.$$

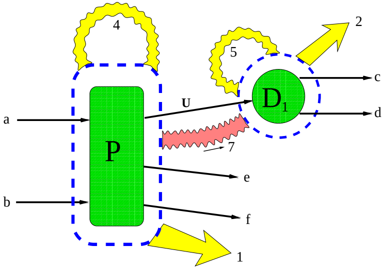
IR cancellation occur automaticaly after phase space integration:

$$\sigma = \sum_{n=0}^{\infty}\frac{1}{n!}\int_{m_\gamma} d\Phi_{4+n}(k_1\ldots k_n)\sum_{spin}|e^{\alpha B_{10}(m_\gamma)}\mathfrak{M}_n(k_1\ldots k_n)|^2.$$

Non-Factorisable (NF) included in the soft-photon approximation (as in all existing calculations in the literature); good enough for differential distributions (disregarding normalization).

This is Double-Pole (DP) part, Single-Pole (SP) $\mathcal{O}(\Gamma_W/M_W)$ next slide

$\mathcal{O}(\Gamma_W/M_W)$ Single-Pole $\mathcal{O}(\alpha^0)_{\rm CEEX}$



$$\mathcal{M}_n^{(0)\,\mu_1,\mu_2,\dots,\mu_n}(k_1,k_2,\dots,k_n) =$$

$$= \sum_{\wp\in\{P,D_1\}^n} e^{\alpha B_8^{\rm CEEX}(U_\wp)}\; \hat{\beta}_0^{(0)}\left(U_\wp\right) \prod_{i=1}^{n} j^{\mu_i}_{\{\wp_i\}}(k_i)$$

Here $\hat{\beta}_0^{(0)}(U)$ is Single-Pole component of Born.
DP at $\mathcal{O}(\alpha^1)_{\rm CEEX}$ and SP at $\mathcal{O}(\alpha^0)_{\rm CEEX}$ to be added coherently!

Derivation of EEX ready for YFSWW3 from new CEEX

Essential step: Neglect interference between production and decays in

$$\sigma^{\rm CEEX} = \frac{1}{flux} \sum_{n=0}^{\infty} \frac{1}{n!} \int dLips_{4+n}(p_a + p_b; p_c, p_d, p_e, p_f, k_1 \ldots k_n)$$

$$\times \sum_{\wp \in \{P, D_1, D_2\}^n} e^{\alpha B_{10}^{\rm CEEX}(U_\wp, V_\wp)} \times \sum_{\wp' \in \{P, D_1, D_2\}^n} e^{\alpha B_{10}^{\rm CEEX\,*}(U_{\wp'}, V_{\wp'})}$$

$$\times \left\{ \hat\beta_0^{(1)} (U_\wp, V_\wp) \prod_{i=1}^{n} j^{\mu_i}_{\{\wp_i\}}(k_i) + \sum_{j=1}^{n} \hat\beta^{(1)\mu_j}_{1\{\wp_j\}} (U_\wp, V_\wp, k_j) \prod_{i \neq j} j^{\mu_i}_{\{\wp_i\}}(k_i) \right\}$$

$$\times \left\{ \hat\beta_0^{(1)} (U_{\wp'}, V_{\wp'}) \prod_{i=1}^{n} j^{\mu_i}_{\{\wp'_i\}}(k_i) + \sum_{j=1}^{n} \hat\beta^{(1)\mu_j}_{1\{\wp'_j\}} (U_{\wp'}, V_{\wp'}, k_j) \prod_{i \neq j} j^{\mu_i}_{\{\wp'_i\}}(k_i) \right\}^\star$$

Neglect iterference terms $\wp \neq \wp'$ and in $\alpha B_{10}^{\rm CEEX} \to \alpha B_{10}^{\rm EEX}$

Derivation of EEX ready for YFSWW3 from new CEEX

$$\sigma^{\rm CEEX} \to \sigma^{\rm EEX} = \frac{1}{flux} \sum_{n=0}^{\infty} \frac{1}{n!} \int dLips_{4+n}(p_a+p_b; p_c, p_d, p_e, p_f, k_1 \dots k_n)$$

$$\times \sum_{\wp \in \{P,D_1,D_2\}^n} e^{2\alpha\Re B_P + 2\alpha\Re B_{D_1} + 2\alpha\Re B_{D_2}} \prod_{i=1}^{n} |j^{\mu_i}_{\{\wp_i\}}(k_i)|^2$$

$$\times \left\{ |\hat\beta^{(1)}_0(U_\wp, V_\wp)|^2 + \sum_{j=1}^{n} \frac{2\Re\left(\hat\beta^{(1)}_{1\{\wp_j\}}(U_\wp, V_\wp, k_j)\cdot j_{\{\wp_j\}}(k_j)^*\right) + |\hat\beta^{(1)}_{1\{\wp_j\}}(U_\wp, V_\wp, k_j)|^2}{|j_{\{\wp_j\}}(k_j)|^2} \right\}$$

Using Bose symmetry it can be brought to usual EEX (YFS1961) form:

$$\sigma^{\rm EEX} = \sum_{n_0=0}^{\infty} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int dLips_{4+n_0+n_1+n_2}(p_a+p_b; p_c, p_d, p_e, p_f, k_1 \dots k_{n_2})$$

$$\times \frac{1}{n_0!} \prod_{i_1=0}^{n_0} \tilde S_P(k_{i_0}) \; \frac{1}{n_1!} \prod_{i_1=1}^{n_1} \tilde S_{D_1}(k_{i_1}) \; \frac{1}{n_2!} \prod_{i_2=1}^{n_2} \tilde S_{D_2}(k_{i_2})$$

$$\times e^{2\alpha\Re B_P + 2\alpha\Re B_{D_1} + 2\alpha\Re B_{D_2}} \Bigg\{ \bar\beta^{(1)}_0(U, V)$$

$$+ \sum_{j=1}^{n_0} \frac{\bar\beta^{(1)}_{1\{P\}}(U,V,k_j)}{\tilde S_P(k_j)} + \sum_{j=1}^{n_1} \frac{\bar\beta^{(1)}_{1\{D_1\}}(U,V,k_j)}{\tilde S_{D_1}(k_j)} + \sum_{j=1}^{n_2} \frac{\bar\beta^{(1)}_{1\{D_2\}}(U,V,k_j)}{\tilde S_{D_2}(k_j)} \Bigg\},$$

where $U = p_{cd} + \sum_{i_1=0}^{n_1} k_{i_1}$ and $V = p_{ef} + \sum_{i_2=0}^{n_2} k_{i_2}$.

Three independend EEX, one for production and two decays.