CKM metrology

Sébastien Descotes-Genon

Laboratoire de Physique Théorique
CNRS & Univ. Paris-Sud, Université Paris-Saclay, Orsay, France

Physics of HL-LHC, and perspectives at HE-LHC,
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The CKM matrix

In SM, flavour dynamics related to weak charged transitions which mix quarks of different generations

Encoded in unitary CKM matrix \( V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \)

- 3 generations \( \implies \) 1 phase, only source of \( CP \)-violation in SM
- Wolfenstein parametrisation, defined to hold to all orders in \( \lambda \) and rephasing invariant

\[ \lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2} \]
\[ A^2 \lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2} \]
\[ \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \]

\( \implies \) 4 parameters describing the CKM matrix
Extracting the CKM parameters

- \( CP \)-invariance of QCD to build hadronic-indep. \( CP \)-violating asym. or to determine hadronic inputs from data
- Statistical framework to combine data and assess uncertainties
- Hadronic part treated as nuisance parameters

\[
V = \begin{pmatrix}
  u & d & s & b \\
  n & e^- & \ell^- & \ell^- \\
  D & \ell^- & \ell^- & \ell^- \\
  B^0 & B_s & B & t \\
  B^0 & B_s & B & B^0
\end{pmatrix}
\]

- \( B(b) \to D(c)\ell\nu \)
- \( B(b) \to \pi(u)\ell\nu \)
- \( M \to \ell\nu, M \to N\ell\nu \)

**Exp. uncert.**

**Theoretical uncertainties**

| Tree  | \( B \to DK \) \( \gamma \) | \( B(b) \to D(c)\ell\nu \) | \( |V_{cb}| \) vs form factor \( F^{B\to D} \) (OPE) |
|-------|------------------------------|------------------------------|----------------------------------|
| Loop  | \( B \to J/\psi K_s \) \( \beta \) | \( \epsilon_K \) (\( K \) mix) | \( (\bar{\rho}, \bar{\eta}) \) vs \( B_K \) (bag parameter) |
|       | \( B \to \pi\pi, \rho\rho \) \( \alpha \) | \( \Delta m_d, \Delta m_s \) (\( B_d, B_s \) mix) | \( |V_{tb}V_{tq}| \) vs \( f_B^2B_B \) (bag param) |

S. Descotes-Genon (LPT-Orsay)
The inputs

frequentist ($\sim \chi^2$ minim.) + Rfit scheme for theory uncert.

data = weak $\otimes$ QCD $\rightarrow$ Need for hadronic inputs (mostly lattice)

- $|V_{ud}|$: superallowed $\beta$ decays
- $|V_{us}|$: $K_{\ell 3}$
  - $K \rightarrow \ell \nu, \tau \rightarrow K \nu_{\tau}$
- $|V_{us}/V_{ud}|$: $K \rightarrow \ell \nu/\pi \rightarrow \ell \nu, \tau \rightarrow K \nu_{\tau}/\tau \rightarrow \pi \nu_{\tau}$
- $\epsilon_K$: PDG
- $|V_{cd}|$: $D \rightarrow \mu \nu, D \rightarrow \pi \ell \nu$
- $|V_{cs}|$: $D_s \rightarrow \mu \nu, D_s \rightarrow \tau \nu, D \rightarrow \pi \ell \nu$
- $|V_{ub}|$: inclusive and exclusive $B$ semileptonic
- $|V_{cb}|$: inclusive and exclusive $B$ semileptonic
  - $B \rightarrow \tau \nu$: $(1.24 \pm 0.22) \cdot 10^{-4}$

- $|V_{ub}/V_{cb}|$: $\Lambda_b$ semileptonic decays
- $\Delta m_d$: last WA $B_d - \bar{B}_d$ mixing
- $\Delta m_s$: last WA $B_s - \bar{B}_s$ mixing
- $\beta$: last WA $J/\psi K(\star)$
- $\alpha$: last WA $\pi \pi, \rho \pi, \rho \rho$
- $\gamma$: last WA $B \rightarrow D(\star) K(\star)$

PRC79, 055502 (2009)

- $f_+(0) = 0.9645 \pm 0.0015 \pm 0.0045$
- $f_K = 155.2 \pm 0.2 \pm 0.6$ MeV
- $f_K/f_\pi = 1.1952 \pm 0.0007 \pm 0.0029$
- $\hat{B}_K = 0.7615 \pm 0.0027 \pm 0.0137$
- $f_{D_s}/f_D = 1.175 \pm 0.001 \pm 0.004, f_{D^+ \rightarrow \pi}(0)$
- $f_{D_s} = 248.2 \pm 0.3 \pm 1.9$ MeV, $f_{D^+ \rightarrow K}(0)$
- $|V_{ub}| \cdot 10^3 = 4.01 \pm 0.08 \pm 0.22$
- $|V_{cb}| \cdot 10^3 = 41.00 \pm 0.33 \pm 0.74$
- $f_{B_s}/f_{B_d} = 1.205 \pm 0.003 \pm 0.006$
- $f_{B_s} = 224.0 \pm 1.0 \pm 2.0$ MeV

integrals of $\Lambda_b$ form factors

- $B_{B_s}/B_{B_d} = 1.023 \pm 0.013 \pm 0.014$
- $B_{B_s} = 1.320 \pm 0.016 \pm 0.030$

no penguin pollution

isospin

GLW/ADS/GGSZ

as well as inputs on $m_t, m_c, \alpha_s(M_Z)$
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CKM metrology

ICHEP 16

Current status

$$A = 0.825^{+0.007}_{-0.011}$$

$$\lambda = 0.2251^{+0.0003}_{-0.0003}$$

$$\bar{\rho} = 0.160^{+0.008}_{-0.007}$$

$$\bar{\eta} = 0.350^{+0.006}_{-0.006}$$

(68% CL)
Consistency of the KM mechanism

Validity of Kobayashi-Maskawa picture of $CP$ violation

$CP$-allowed only

$CP$-violating only

Tree only

Loop only

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Pulls for various observables (included in the fit or not)

If Gaussian errors, uncorrelated, random vars of mean 0 and variance 1

Here correlations, and some pulls = 0 due to the Rfit model for syst

No indication of significant deviations from CKM picture [and no sign here of deviations in relation with B-anomalies]
A basis for prospective


- Central values chosen to be all consistent within SM
- Gaussian uncertainties based on 2013 extrapolations
  - *Lattice QCD at the Intensity Frontier*, T. Blum et al.

Stage I of this paper: $\sim$ 2020, LHCb 7 fb$^{-1}$, Belle II 5 ab$^{-1}$
  
  *not completely off the target for theory and LHCb inputs*

Stage II of this paper: $\sim$ 2025, LHCb 50 fb$^{-1}$, Belle II 50 ab$^{-1}$

Stage II good starting point to include HL-LHC projections
Updating to HL LHC projections

Two HL LHC projections

- Phase 1: \( \sim 2025 \), LHCb 27 fb\(^{-1}\), CMS/ATLAS 300 fb\(^{-1}\)
- Phase 2: \( \sim \) Upgrade 2, LHCb 300 fb\(^{-1}\), CMS/ATLAS 3000 fb\(^{-1}\)

Start from ”old” Stage II exploiting the new HL-LHC projections

- HL-LHC: uncertainties from Phase 1 and Phase 2
- Theory inputs (mainly lattice): uncertainties from Stage II
- Other exp inputs (mainly Belle II): uncertainties from Stage II

Work in progress with certainly wrong numbers!
I am using inputs from other WG and guesstimates…
They will certainly change but are useful to organise the discussions!
### HL-LHC inputs

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Current</th>
<th>Unc. Phase 1</th>
<th>Unc. Phase 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin 2\beta$</td>
<td>0.017</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.3°</td>
<td>1.5°</td>
<td>0.35°</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}/V_{cb}</td>
<td>$</td>
<td>7%</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>31 mrad</td>
<td>14 mrad</td>
<td>4 mrad</td>
</tr>
<tr>
<td>$m_t$</td>
<td>0.4%</td>
<td>0.4%</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

- $m_t$: Uncertainty related to the definition (MC mass versus $\overline{\text{MS}}$ mass)
- $|V_{ub}/V_{cb}|$: Assumptions on lattice determination of $\Lambda_b$ form factors
- $\beta$ and $\phi_s$: What to do with penguin pollution?
  - $SU(3)$ flavour: $|\Delta \beta| \leq 0.5^\circ$ [R. Fleischer et al., M. Ciuchini et al.]
  - OPE + factorisation: $|\Delta \beta| \leq 0.3^\circ$, $|\Delta \phi_s| \leq 1^\circ$ [U. Nierste et al.]
  - other approaches? forget about the problem?
Other experimental inputs (1)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Current</th>
<th>Unc. (Phases 1 and 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>2.3°</td>
<td>1°</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
<td>$</td>
</tr>
<tr>
<td>$\Delta m_d$</td>
<td>0.4 %</td>
<td>0.4 %</td>
</tr>
<tr>
<td>$\Delta m_s$</td>
<td>1.2 %</td>
<td>1.2 %</td>
</tr>
<tr>
<td>$</td>
<td>\epsilon_K</td>
<td>$</td>
</tr>
<tr>
<td>$B \to \tau \nu$</td>
<td>24 %</td>
<td>12 %</td>
</tr>
<tr>
<td>$B \to \mu \nu$</td>
<td>–</td>
<td>24 %</td>
</tr>
</tbody>
</table>

- $\alpha$: measurements from Belle II et LHC, but main improvements for $\alpha$ expected from Belle II (main limitation on $\alpha$ from neutral modes) and isospin-breaking systematics
  
  [O. Deschamps et al, arXiv:1705.02981]

- $|V_{ub}|$ and $|V_{cb}|$: assumptions about both lattice improvements and convergence of inclusive and exclusive decays

- What about $K \to \pi \nu \nu$ (constraint on $\bar{\eta}$) ? and more generally...
\[ |V_{ud}|, |V_{us}|: \text{ no expected change from leptonic and semileptonic decays (apart from lattice)} \]

\[ |V_{cd}|, |V_{cs}|: \text{ any exp improvement from the charm sector (leptonic and semileptonic } D \text{ and } D_s \text{ decays)}? \]
### Lattice inputs

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Current</th>
<th>Unc. (Phases 1 and 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_K )</td>
<td>0.4 %</td>
<td>0.4 %</td>
</tr>
<tr>
<td>( f_{K \rightarrow \pi}(0) )</td>
<td>0.3 %</td>
<td>0.2 %</td>
</tr>
<tr>
<td>( B_K )</td>
<td>1.6 %</td>
<td>0.5 %</td>
</tr>
<tr>
<td>( f_{B_s} )</td>
<td>1 %</td>
<td>0.4 %</td>
</tr>
<tr>
<td>( B_{B_s} )</td>
<td>3 %</td>
<td>0.8 %</td>
</tr>
<tr>
<td>( f_{B_s}/f_{B_d} )</td>
<td>0.6 %</td>
<td>0.4 %</td>
</tr>
<tr>
<td>( B_{B_s}/B_{B_d} )</td>
<td>1.9 %</td>
<td>0.5 %</td>
</tr>
<tr>
<td>( m_c )</td>
<td>1 %</td>
<td>1 %</td>
</tr>
</tbody>
</table>

- Are these guesstimates realistic for lattice extrapolations?
  - *hep-ph/1309.2293 Stage I guesstimates ≃ now, as expected*
- If leptonic and semileptonic charm decays included, extrapolations for decay constants and form factors needed
- What about isospin and QED corrections for leptonic and semileptonic decays?
  - *known and around 1% for K, but for D and B?*
Phase 2 (Preliminary !)

\[
\begin{align*}
\gamma & \quad \Delta m_d & \Delta m_s & \varepsilon_K \\
\alpha & \quad \rho & \beta & |V_{ub}|
\end{align*}
\]

- excluded area has CL > 0.95
- sol. w/ cos 2\(\beta\) < 0
- (excl. at CL > 0.95)
### CKM metrology (Preliminary !)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Current (No discr)</th>
<th>Unc. Phase 1</th>
<th>Unc. Phase 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>1.1% (1.4%)</td>
<td>0.7%</td>
<td>0.7%</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.2% (2.6%)</td>
<td>2.4%</td>
<td>2.3%</td>
</tr>
<tr>
<td>$\bar{\rho}$</td>
<td>4.6% (5.9%)</td>
<td>2.6%</td>
<td>1.4%</td>
</tr>
<tr>
<td>$\bar{\eta}$</td>
<td>1.7% (2.6%)</td>
<td>1.1%</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

- In brackets, same uncertainties as now, but central values adjusted to have perfect agreement of the constraints
- $A$ and $\lambda$ driven by $|V_{ud}|$, $|V_{us}|$, $|V_{cb}|$ inputs
- Possibility to play the game with restricted subsets (tree, loop only)
Playing with subsets: Current

- CP-allowed only
- CP-violating only
- Tree only
- Loop only
Playing with subsets : Phase 1 (Preliminary !)

\[ \text{CP-allowed only} \]

\[ \text{CP-violating only} \]

\[ \text{Tree only} \]

\[ \text{Loop only} \]
Playing with subsets : Phase 2 (Preliminary !)

\begin{equation*}
\Delta m_u & \Delta m_d \\
\Delta m_d & \Delta m_s
\end{equation*}

\begin{itemize}
\item \textit{CP-allowed only}
\item \textit{CP-violating only}
\item \textit{Tree only}
\item \textit{Loop only}
\end{itemize}

\begin{itemize}
\item $\sin 2\beta < 0$
\item $\sin 2\beta > 0$
\end{itemize}

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Uncertainty on a few quantities (Preliminary !)

Assuming the quantities are SM-like (not necessarily the case...)

<table>
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<th>Unc. Phase 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_s$</td>
<td>32 mrad (46 mrad)</td>
<td>20 mrad</td>
<td>14 mrad</td>
</tr>
<tr>
<td>$B(B_s \to \mu\mu)$</td>
<td>3.5% (4.3%)</td>
<td>2.8%</td>
<td>2.5%</td>
</tr>
<tr>
<td>$B(B_d \to \mu\mu)$</td>
<td>3.6% (4.6%)</td>
<td>2.9%</td>
<td>2.5%</td>
</tr>
<tr>
<td>$B(B_d \to \mu\mu)$</td>
<td>1.1% (1.7%)</td>
<td>0.6%</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

- $\phi_s$ meas. at 14 mrad (Phase 1), 4 mrad (Phase 2)
- $B(B_s \to \mu\mu)$ meas. at 10%-12% (Phase 1), 5%-10% (Phase 2)
- $B(B_d \to \mu\mu)$ meas. at 33%-40% (Phase 1), 10%-20% (Phase 2)
- ratio of $B_{rs}$ meas. at 34%-50% (Phase 1), 10%-20% (Phase 2)
- obs. related to mixing (e.g., $a^s_{SL}$) accessible, but require further extrapolations (scalar bag parameters)
- obs. related to anomalies either dominated by hadronic unc. (BRs) or no CKM contribution (optimised $P_i$, LFU ratios $R_X$)

[J. Charles et al., hep-ph/1309.2293]
Discussion

Inputs for discussion

- Lattice extrapolations
- QED corrections for semileptonic and leptonic decays?
- Penguin pollution for $\beta$ and $\phi_s$?
- Any charm inputs? Other inputs to be added?

CKM metrology

- Improvement in CKM parameters
- Corresponding plots in $\bar{\rho}, \bar{\eta}$ plane
- Other plots of interest (coordinates? set of constraints?)
- Other predictions of interest?
Back-up
Determinations of $|V_{cb}|$ and $|V_{ub}|$

- $|V_{ub}|$ and $|V_{cb}|$ inclusive and exclusive semileptonic
- Global fit of CKM favours excl. $|V_{ub}|_{SL}$ but incl. $|V_{cb}|_{SL}$
- $|V_{ub}|$ from $Br(B \rightarrow \tau \nu)$
- $|V_{ub}/V_{cb}|$ from $\Gamma(\Lambda_b \rightarrow p\mu\nu)/\Gamma(\Lambda_b \rightarrow \Lambda_c\mu\nu)$ (high $q^2$)
- Averaging procedure between incl and excl
Statistical framework

$q = (A, \lambda, \bar{\rho}, \bar{\eta}, \ldots)$ to be determined

- $\mathcal{O}_{\text{meas}} \pm \sigma_{\mathcal{O}}$ experimental values of observables
- $\mathcal{O}_{\text{th}}(q)$ theoretical description in a given model

In case of statistical uncertainties $\sigma_{\mathcal{O}}$, likelihoods and $\chi^2$

\[
\mathcal{L}(q) = \prod_{\mathcal{O}} \mathcal{L}_{\mathcal{O}}(q) \quad \chi^2(q) = -2 \ln \mathcal{L}(q) = \sum_{\mathcal{O}} \left( \frac{\mathcal{O}_{\text{th}}(q) - \mathcal{O}_{\text{meas}}}{\sigma_{\mathcal{O}}} \right)^2
\]

- Central value: estimator $\hat{q}$ max likelihood: $\chi^2(\hat{q}) = \min_q \chi^2(q)$
- Range: confidence level for each $q_0$ ($p$-value for $q = q_0$) by:

\[
\Delta \chi^2(q_0) = \chi^2(q_0) - \min_q \chi^2(q)
\]

assumed to obey $\chi^2$ law with $N = \text{dim}(q)$ to yield CIs

- Pull: comparison of $\chi^2_{\text{min}}$ with and without one measurement

\[
p_{\mathcal{O}} = \sqrt{\min_q \chi^2_{\text{with meas}}(q) - \min_q \chi^2_{\text{without meas}}(q)}
\]

$\implies$ Specific scheme to treat theoretical uncertain (currently Rfit)