ICHEP 2018, Seoul

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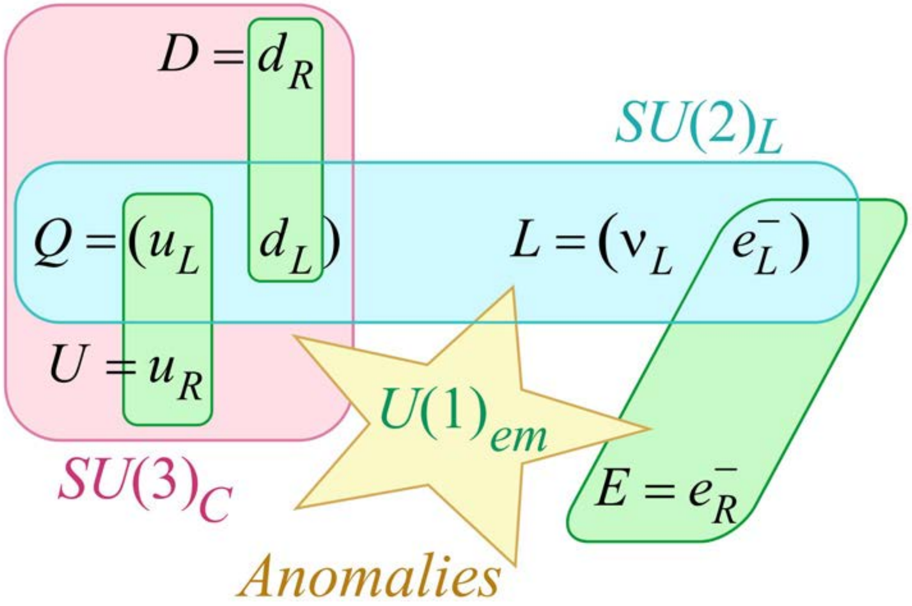
Towards a new paradigm for quark-lepton unification

Christopher Smith

Introduction

A. Flavors and unification

From the gauge point of view, fermions are well unified:



From the flavor point of view, the situation is far less satisfactory:

$$\mathcal{L}_{Yukawa} = \bar{U}\mathbf{Y}_u QH + \bar{D}\mathbf{Y}_d QH^\dagger + \bar{E}\mathbf{Y}_e LH^\dagger$$

Unrelated 3x3 complex matrices

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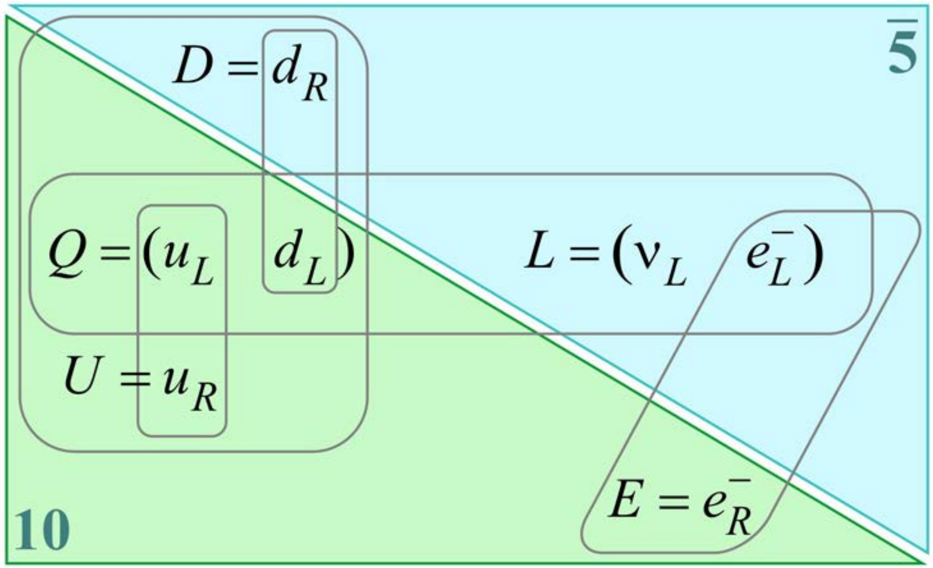

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$$\mathcal{L}_{Yukawa} = -\frac{1}{4}\overline{\chi}_{10}^{C} Y_{10} \chi_{10} h_5 + \sqrt{2}\,\overline{\psi}_{\overline{5}}^{C} Y_5 \chi_{10} h_5^{\dagger}$$

$$Y_{10} = Y_{10}^{T} = Y_u \qquad\qquad Y_5 = Y_e^{T} = Y_d \rightarrow \frac{m_d}{m_s} = \frac{m_e}{m_\mu} \text{ ????}$$

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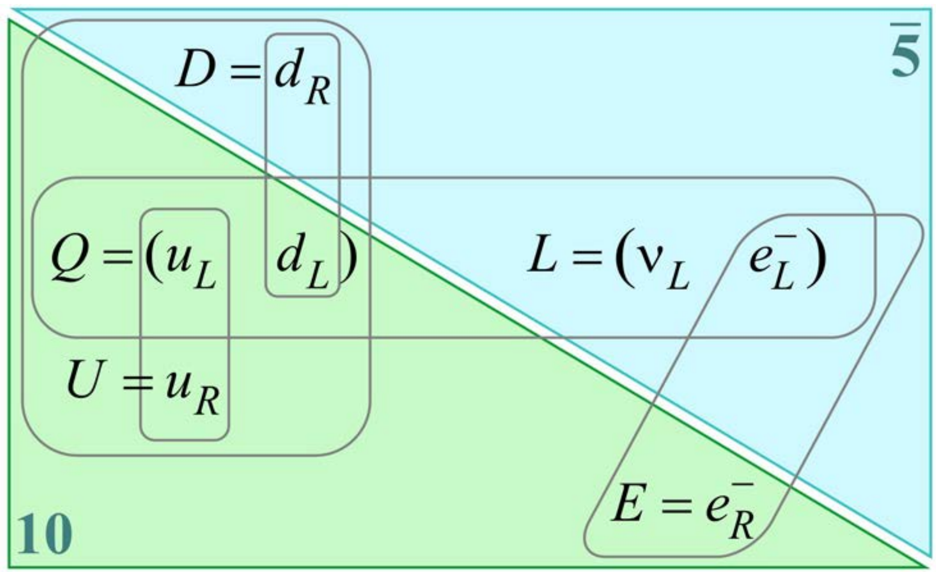

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$$\mathbf{Y}_{10} = \mathbf{Y}_{10}^{T} = \mathbf{Y}_u \qquad \mathbf{Y}_5 - 3\mathbf{Y}_5' = \mathbf{Y}_e^{T}\ , \ \mathbf{Y}_5 + \mathbf{Y}_5' = \mathbf{Y}_d$$

B. Minimal Flavor Violation

D'Ambrosio, et al. '02

$\mathbf{Y}$ = Yukawas SM

NP $\mathbf{A}$ = Anything else

The three families of quarks/leptons have identical gauge interactions

→ flavor symmetry: $G_F = U(3)^5 = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$

This symmetry is explicitly broken by all the flavor couplings, e.g.,

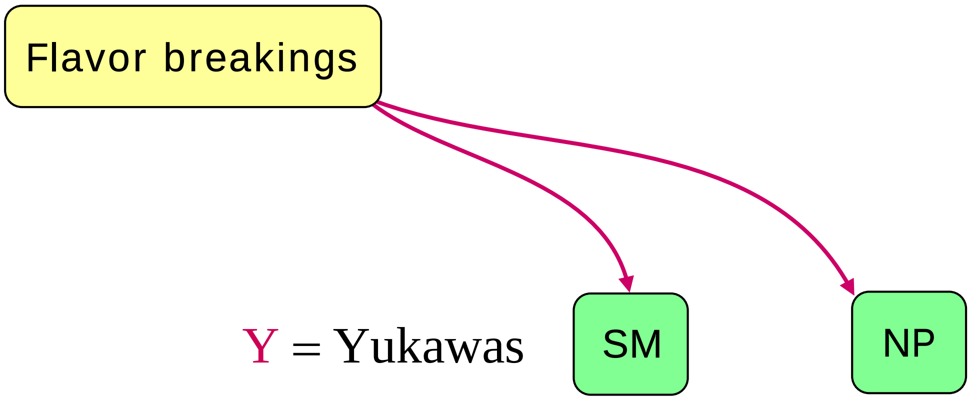
$$\mathcal{L}_{Yukawa} = U\mathbf{Y}_u QH + D\mathbf{Y}_d QH^\dagger + E\mathbf{Y}_e LH^\dagger$$

Chivukula,
Georgi '87

B. Minimal Flavor Violation

D'Ambrosio, et al. '02

Assume some NP mechanism is at the origin of all the flavor structures.



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B. Minimal Flavor Violation

Nikolidakis,CS ‘07
Colangelo,Nikolidakis,CS ‘08
CS ‘11

Imagine there is a single fundamental breaking term X at the high energy scale ← minimality

Flavor breakings

SM

NP

$$\mathrm{Y} = x_0 1 + x_1 \mathrm{X} + x_2 \mathrm{X}^2 \qquad x_i \sim \mathcal{O}(1)$$

$$\mathrm{A} = x_0' 1 + x_1' \mathrm{X} + x_2' \mathrm{X}^2 \qquad x_i' \sim \mathcal{O}(1)$$

← naturality

Remark: finite expansions thanks to Cayley-Hamilton identities:

$$\mathrm{X}^3 - \langle \mathrm{X} \rangle \mathrm{X}^2 + \tfrac{1}{2} \mathrm{X} (\langle \mathrm{X} \rangle^2 - \langle \mathrm{X}^2 \rangle) = \tfrac{1}{3} \langle \mathrm{X}^3 \rangle - \tfrac{1}{2} \langle \mathrm{X} \rangle \langle \mathrm{X}^2 \rangle + \tfrac{1}{6} \langle \mathrm{X} \rangle^3$$

B. Minimal Flavor Violation

Nikolidakis,CS '07
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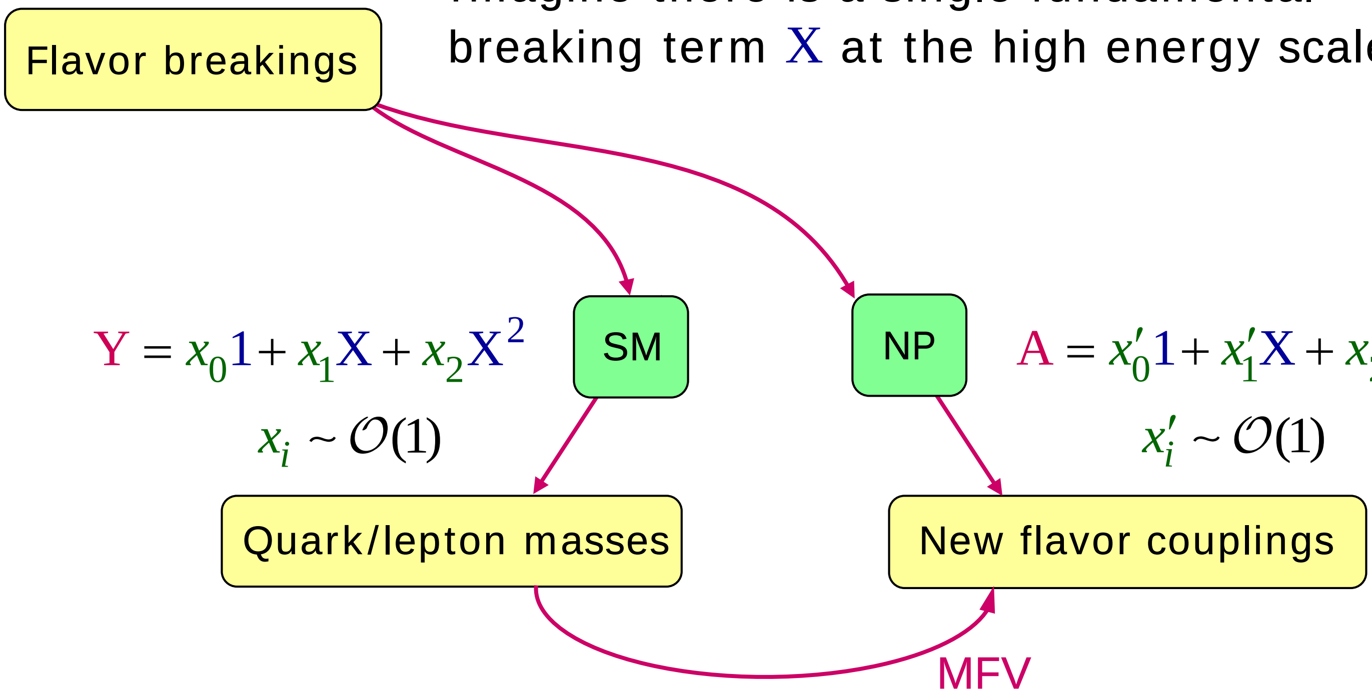
Imagine there is a single fundamental breaking term X at the high energy scale ← minimality



$$Y = x_0 1 + x_1 X + x_2 X^2 \qquad A = x'_0 1 + x'_1 X + x'_2 X^2$$

$$x_i \sim \mathcal{O}(1) \qquad x'_i \sim \mathcal{O}(1)$$

← naturality

Then, low-energy flavor couplings are redundant and obey MFV relations:

$$A = a_0 1 + a_1 Y + a_2 Y^2 \quad \text{or} \quad Y = b_0 1 + b_1 A + b_2 A^2 \quad \text{with} \quad a_i, b_i \sim \mathcal{O}(1)$$

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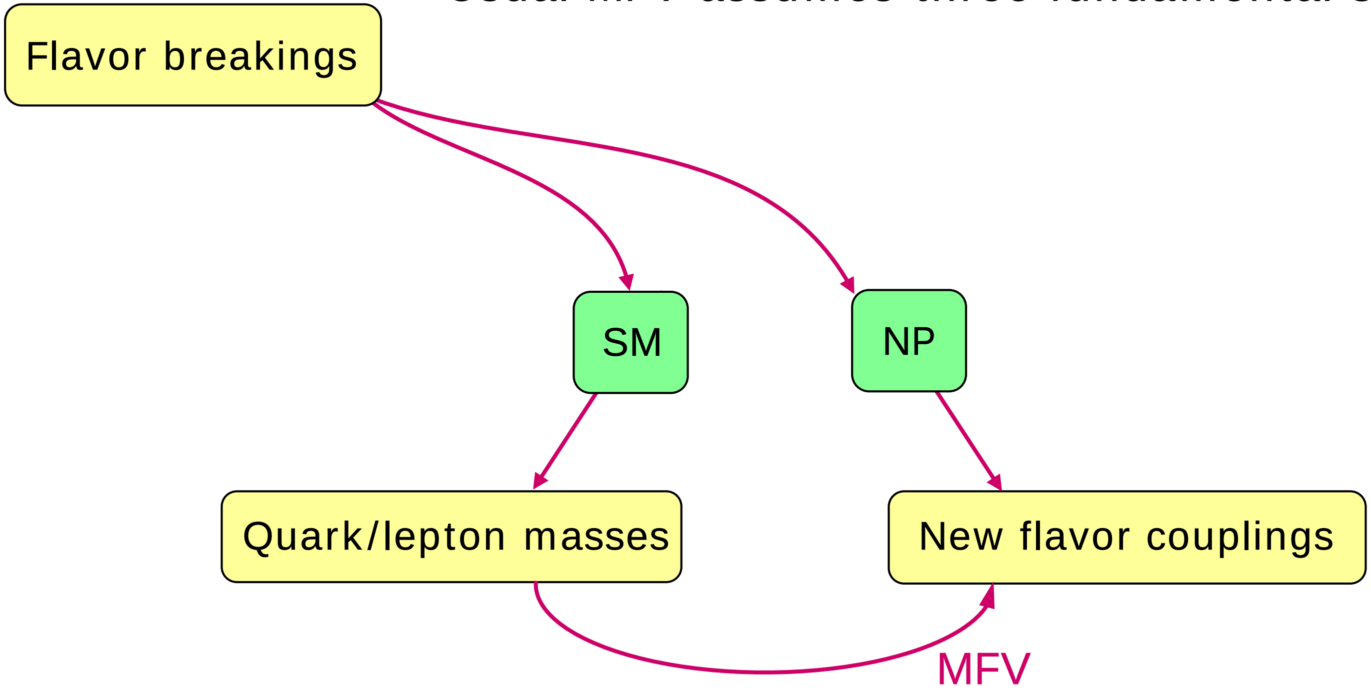
Usual MFV assumes three fundamental spurions X, Y, Z.

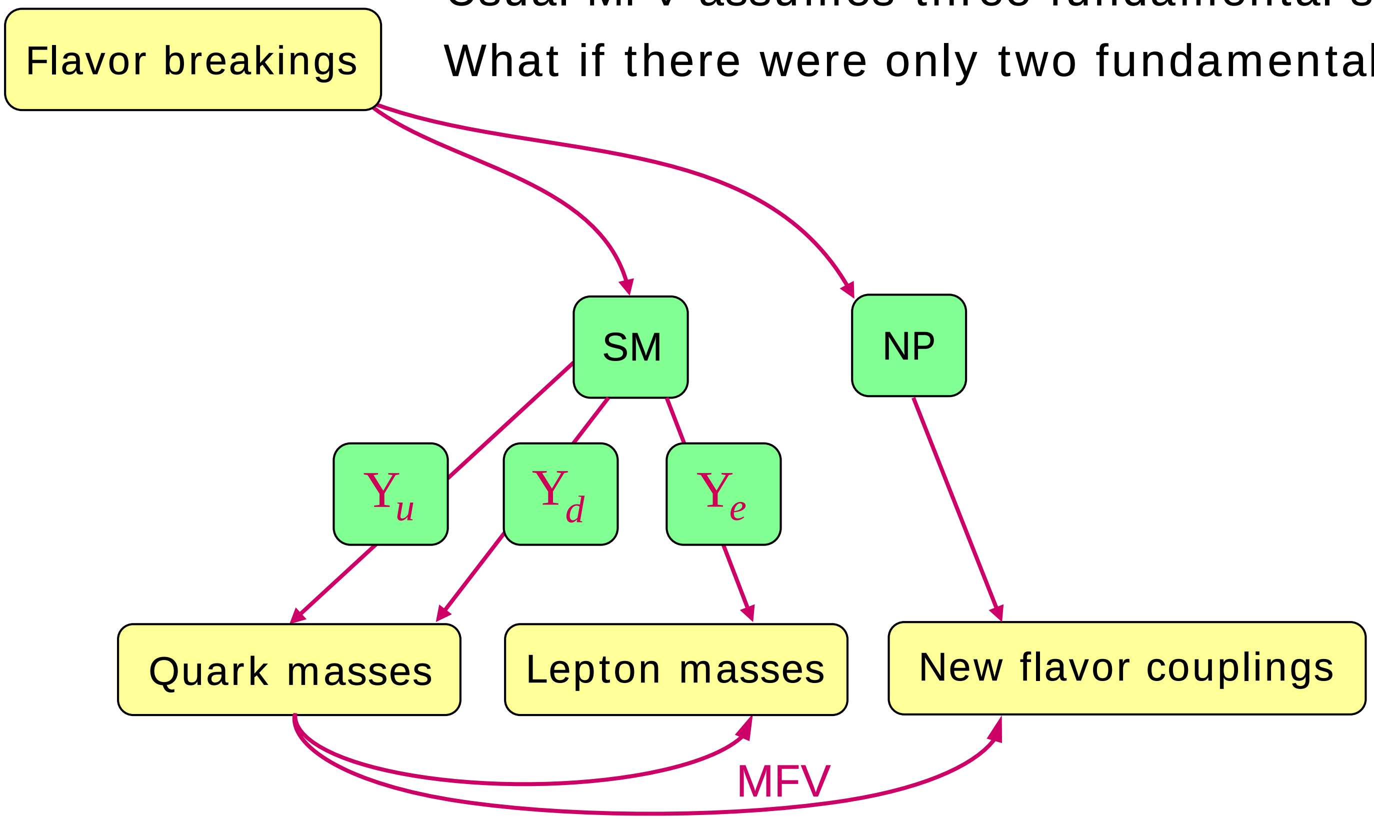


Trade X, Y, Z for Y_u, Y_d, Y_e and express new couplings in terms of them.

C. Going beyond the usual MFV...

CS ‘16

Usual MFV assumes three fundamental spurions $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$.
What if there were only two fundamental spurions $\mathbf{X},\mathbf{Y}$?



At low energy, the SM couplings must satisfy $\mathrm{Y}_e = F(\mathrm{Y}_u, \mathrm{Y}_d)$.

[Neutrinos kept massless here]

- Outline

I. Flavor perspective on Yukawa unification

II. Geometric MFV

III. Application to the MSSM

IV. Application to minimal SU(5)

Based on arXiv:1612.03825 (JHEP)

I. Flavor perspective on Yukawa unification

A. Setting up MFV for Yukawas

To proceed, a choice must be made about the fundamental spurions.

Flavor symmetry: $G_F' = U(3)^3 = U(3)_{Q=L} \times U(3)_U \times U(3)_{D=E}$

Spurions: $\mathrm{Y}_u, \mathrm{Y}_d$ Known in the same gauge basis:

$\xrightarrow{G_F'}$ $v\mathrm{Y}_u = m_u V_{CKM}\,,\; v\mathrm{Y}_d = m_d\,.$

Expansion:

$$\mathrm{Y}_e = x_0 \mathrm{Y}_d \cdot \big(1 + x_1\mathrm{A} + x_2\mathrm{B} + x_3\mathrm{B}^2 + x_4\{\mathrm{A},\mathrm{B}\} + x_5\mathrm{BAB} + x_6 i[\mathrm{A},\mathrm{B}] + x_7 i[\mathrm{A},\mathrm{B}^2] + x_8 i(\mathrm{BAB}^2 - \mathrm{B}^2\mathrm{AB})\big)$$

$$(\mathrm{A} \equiv \mathrm{Y}_d^\dagger \mathrm{Y}_d, \mathrm{B} \equiv \mathrm{Y}_u^\dagger \mathrm{Y}_u)$$

These choices match SU(5), up to an irrelevant transposition.

B. Setting up MFV for Yukawas

The MFV basis is nearly singular → Very large coefficients in general.

Mercoli,CS '09

Assuming alignment of the lepton and down-quark mass basis:

$$Y_e \equiv 0.2 Y_d \cdot \left(1 + 10^8 Y_d^\dagger Y_d - 10^{11} (Y_d^\dagger Y_d)^2\right) \quad \text{for } \tan\beta = 50.$$

[from $m_{e,\mu,\tau} \equiv 0.2 m_{d,s,b} \left(1 + 10^8 m_{d,s,b}^2 - 10^{11} m_{d,s,b}^4\right)$]

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Allowing for subsequent SVD, keeping only three terms ($\tan\beta = 50$):

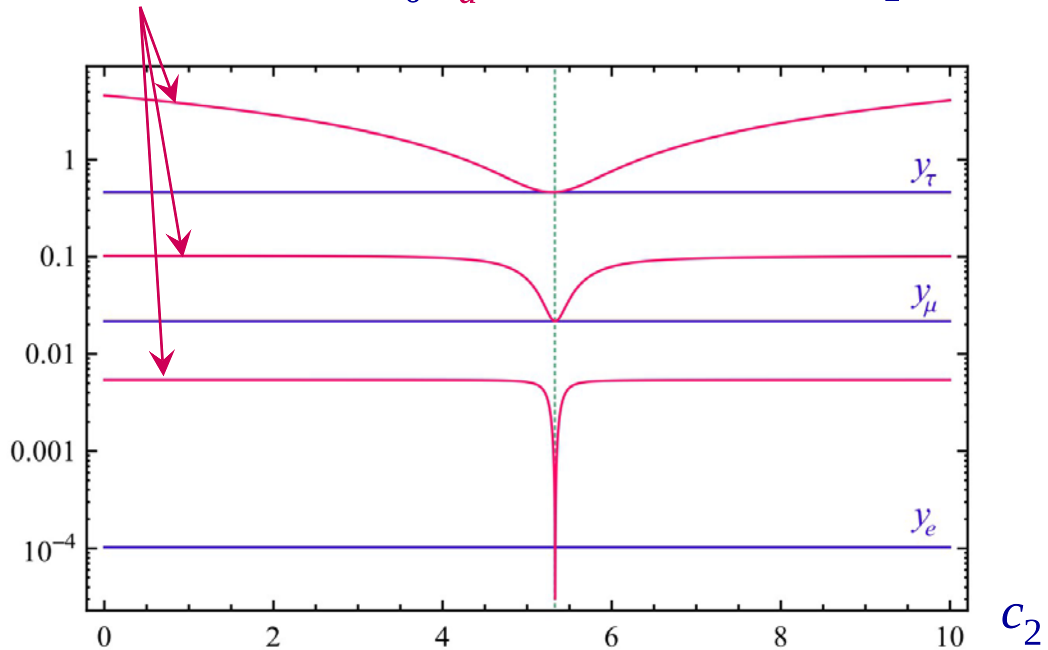
$$\mathrm{Y}_e = c_0 \mathrm{Y}_d \cdot \left(1 + c_1 \mathrm{Y}_u^\dagger \mathrm{Y}_u + c_2 \mathrm{Y}_d^\dagger \mathrm{Y}_d\right)$$

	c_0	c_1	c_2
Masses at M_Z	8.6	-1.8	1.2
SM at M_{GUT}	22	6	-20
MSSM at M_{GUT}	20	-7.9	5.3

Remarkable that reasonable coefficients are possible at all!!!

B. On the anatomy of a fine-tuning

If we plot the SVD of $Y_e = c_0 Y_d \cdot X$, with $X = 1 + c_1 Y_u^\dagger Y_u + c_2 Y_d^\dagger Y_d$:



At the physical point: $$|X| \approx \begin{pmatrix} 1 & 0.0005 & 0.01 \\ 0.0005 & 1 & 0.06 \\ 0.01 & 0.06 & 0.004 \end{pmatrix}$$

Very delicate cancellation $1 + c_1 y_t^2 + c_2 y_b^2 \approx 0$!

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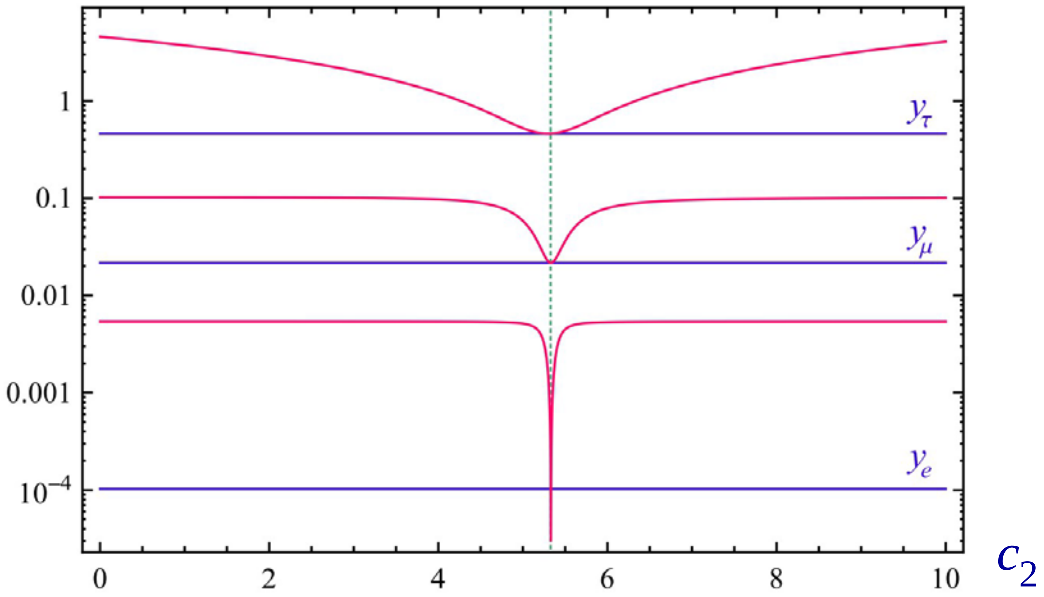


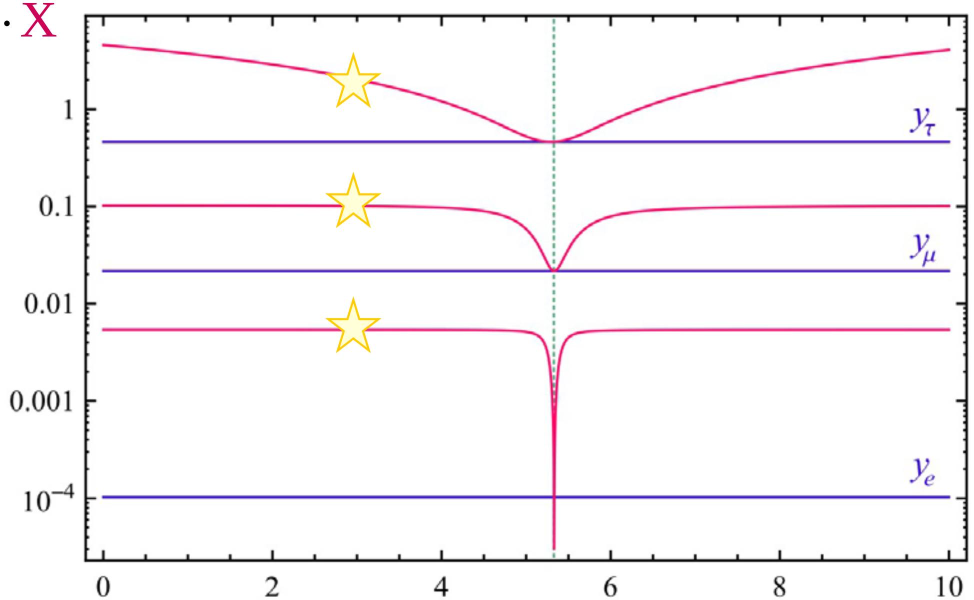
Mathematically, there is a singularity within the natural $c_{1,2}$ ranges:

$$\det Y_e = c_0^3 \times \det Y_d \times \det X \;\Rightarrow\; \det X \approx 1 + c_1 \langle Y_u^\dagger Y_u \rangle + c_2 \langle Y_d^\dagger Y_d \rangle \approx 0$$

No polynomial relation between $Y_{e,u,d}$ will ever be natural!

C. The twisted persona of the leptons

$$Y_e = c_0 Y_d \cdot X$$



$$v g_E Y_e g_L^\dagger = m_e \ , \quad c_2 = 2.5$$: Small CKM-like mixings

$$|g_E| \approx \begin{pmatrix} 1.000 & 0.0002 & 0.00005 \\ 0.0002 & 1.000 & 0.005 \\ 0.00006 & 0.005 & 1.000 \end{pmatrix} \qquad |g_L| \approx \begin{pmatrix} 1.00 & 0.005 & 0.04 \\ 0.002 & 0.98 & 0.18 \\ 0.04 & 0.18 & 0.98 \end{pmatrix}$$

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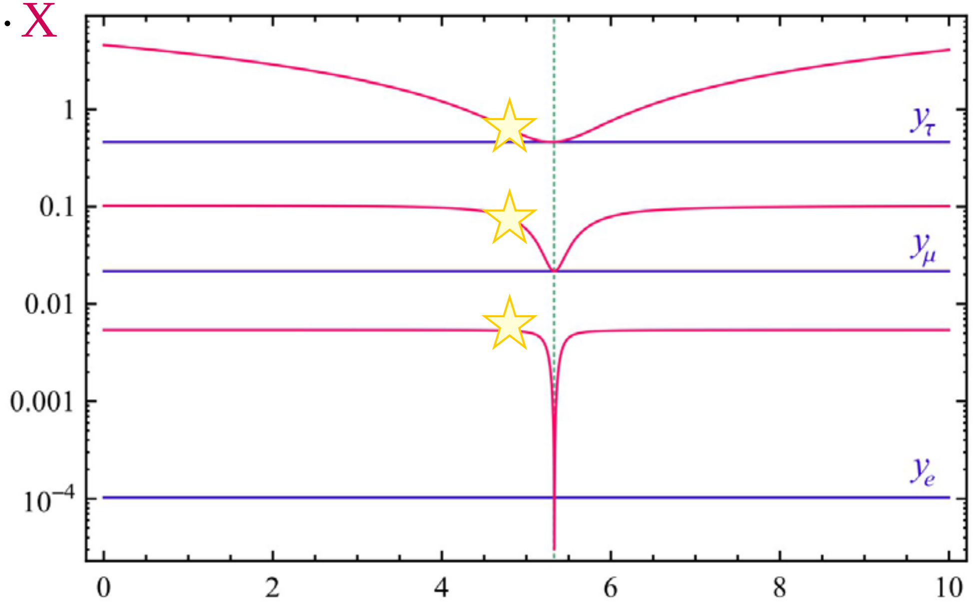
$\mathrm{Y}_e = c_0 \mathrm{Y}_d \cdot \mathrm{X}$



$v g_E \mathrm{Y}_e g_L^\dagger = m_e \;, \quad c_2 = 4.8$: Large mixing close to the SVD reordering

$$|g_E| \approx \begin{pmatrix} 1.000 & 0.008 & 0.0001 \\ 0.008 & 0.99 & 0.11 \\ 0.002 & 0.11 & 0.99 \end{pmatrix} \qquad |g_L| \approx \begin{pmatrix} 0.98 & 0.13 & 0.14 \\ 0.01 & 0.71 & 0.70 \\ 0.20 & 0.69 & 0.70 \end{pmatrix}$$

$$g_E \cdot \mathrm{Y}_d^\dagger \cdot \mathrm{X}^2 \cdot \mathrm{Y}_d \cdot g_E^\dagger \qquad g_L \cdot \mathrm{X} \cdot \mathrm{Y}_d^\dagger \mathrm{Y}_d \cdot \mathrm{X} \cdot g_L^\dagger$$

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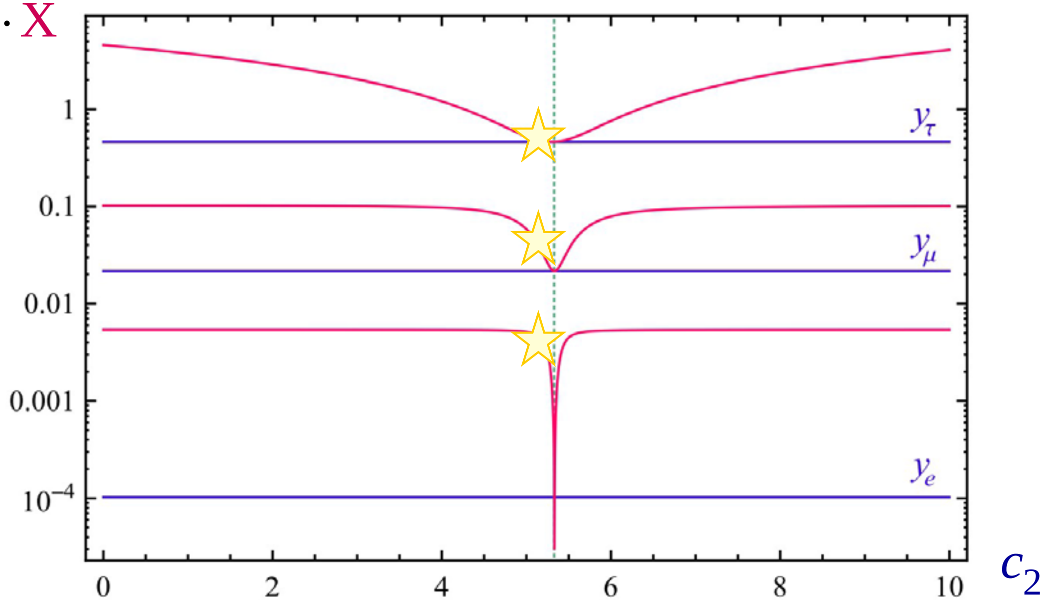
$$Y_e = c_0 Y_d \cdot X$$

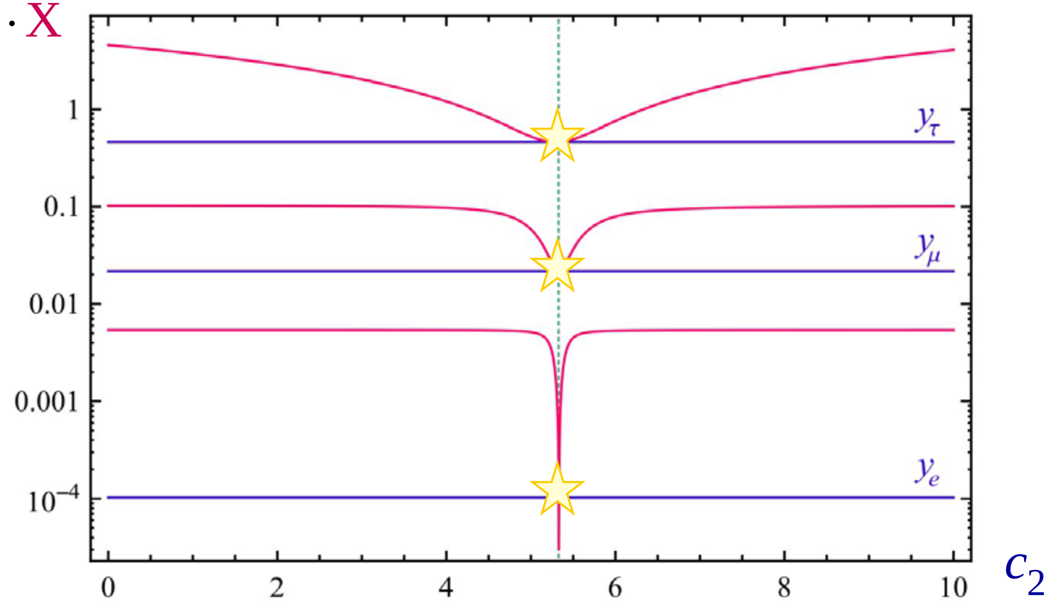


$v g_E Y_e g_L^\dagger = m_e$, $c_2 = 5.2$: Towards a second SVD reordering

$$|g_E| \approx \begin{pmatrix} 0.97 & 0.09 & 0.002 \\ 0.08 & 0.98 & 0.20 \\ 0.02 & 0.20 & 0.98 \end{pmatrix} \qquad |g_L| \approx \begin{pmatrix} 0.80 & 0.56 & 0.20 \\ 0.03 & 0.28 & 0.96 \\ 0.59 & 0.77 & 0.21 \end{pmatrix}$$

C. The twisted persona of the leptons

$$Y_e = c_0 Y_d \cdot X$$

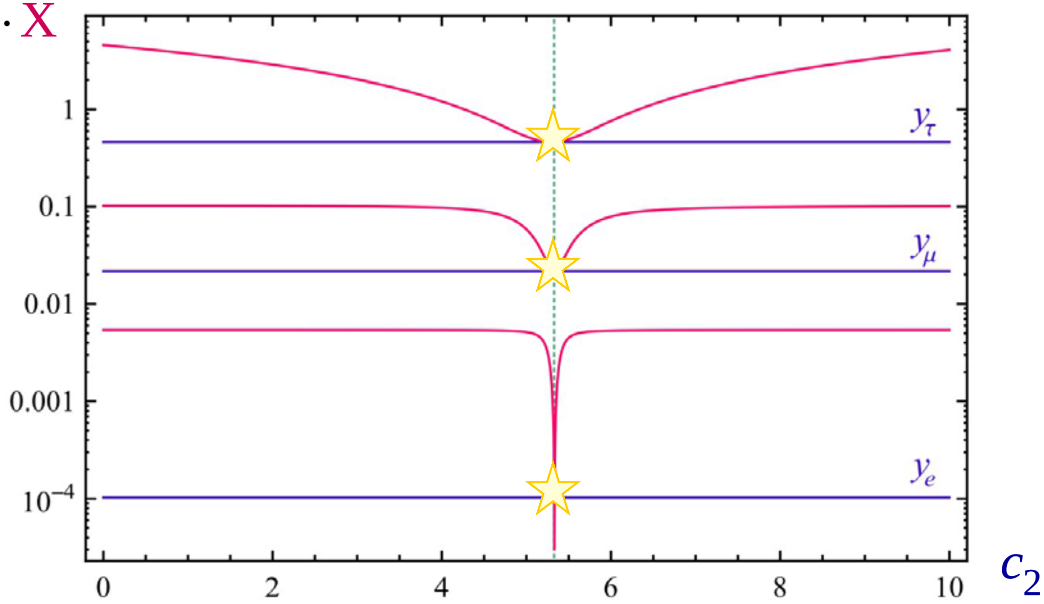


$v g_E Y_e g_L^\dagger = m_e$, $c_2 = 5.3$: At the physical point, CKM-like mixings

$$|g_E| \approx \begin{pmatrix} 0.97 & 0.24 & 0.002 \\ 0.24 & 0.95 & 0.22 \\ 0.06 & 0.21 & 0.98 \end{pmatrix} \qquad |g_L| \approx \begin{pmatrix} 0.03 & 0.98 & 0.20 \\ 0.06 & 0.20 & 0.98 \\ 1.00 & 0.02 & 0.07 \end{pmatrix}$$

C. The twisted persona of the leptons

$$\mathrm{Y}_e = c_0 \mathrm{Y}_d \cdot \mathrm{X}$$



$v g_E \mathrm{Y}_e g_L^\dagger = m_e$, $c_2 = 5.3$: At the physical point, twisted leptons!

$$|\mathrm{X}| \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}^{gauge} \approx \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}^{phys}$$

Only singularity at natural $c_{1,2}$ values

Top partner is the lightest

II. Geometric MFV

A. Pseudo fine tuning thanks to the geometric expansion

Polynomial expansions are fine-tuned: What about infinite series?

$$\mathrm{X} = 1 + \eta \mathrm{Y}_u^\dagger \mathrm{Y}_u + \eta^2 (\mathrm{Y}_u^\dagger \mathrm{Y}_u)^2 + \eta^3 (\mathrm{Y}_u^\dagger \mathrm{Y}_u)^3 + \ldots = \frac{1}{1 - \eta \mathrm{Y}_u^\dagger \mathrm{Y}_u}$$

If η is large enough: $\mathrm{X} = \begin{pmatrix} \frac{1}{1-\eta y_u^2} \approx 1 & & \\ & \frac{1}{1-\eta y_c^2} \approx 1 & \\ & & \frac{1}{1-\eta y_t^2} \approx 0 \end{pmatrix}$.

Actually, the large top mass ensures: $(\mathrm{Y}_u^\dagger \mathrm{Y}_u)^n \approx \langle \mathrm{Y}_u^\dagger \mathrm{Y}_u \rangle^{n-1} \mathrm{Y}_u^\dagger \mathrm{Y}_u$

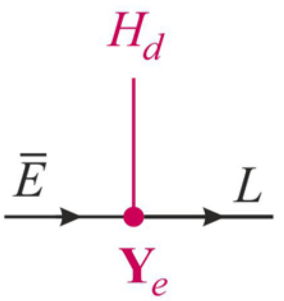
$$\mathrm{X} = \sum_{n=0}^{\infty} \eta^n (\mathrm{Y}_u^\dagger \mathrm{Y}_u)^n = 1 + \frac{\eta}{1 - \eta \langle \mathrm{Y}_u^\dagger \mathrm{Y}_u \rangle} \mathrm{Y}_u^\dagger \mathrm{Y}_u \xrightarrow{\eta \to \infty} 1 - \frac{1}{\langle \mathrm{Y}_u^\dagger \mathrm{Y}_u \rangle} \mathrm{Y}_u^\dagger \mathrm{Y}_u$$

Precisely the fictitious fine-tuning we need!

B. Toy model with vector-like lepton doublets

Toy model to sum the MFV series outside its radius of convergence.

1- Start with $\mathbf{Y}_e = \gamma \mathbf{Y}_d$ in the Lagrangian, for some coefficient γ.



B. Toy model with vector-like lepton doublets

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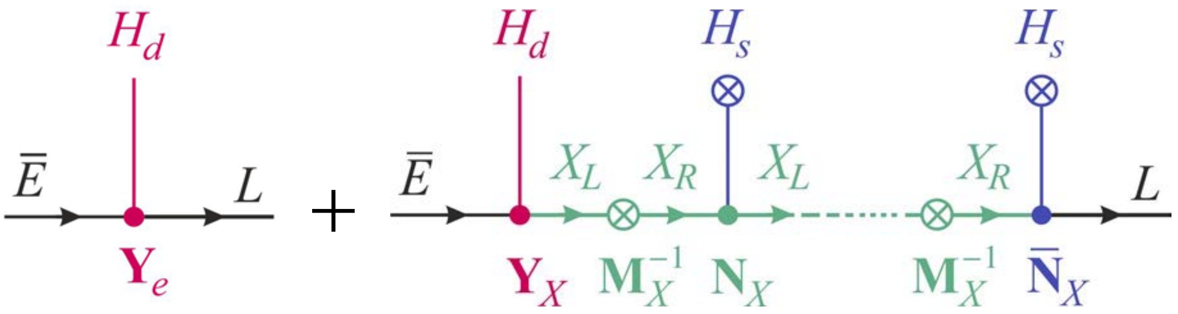
2- Engineer an infinite tower of seesaw-like contributions:



– Weak doublet of vector-like leptons $X_{L,R}$.

– Scalar singlet H_s with vev v_s.

$$Y_e^{eff} = \gamma Y_d - Y_X \frac{1}{M_X + N_X H_s} \bar{N}_X H_s$$

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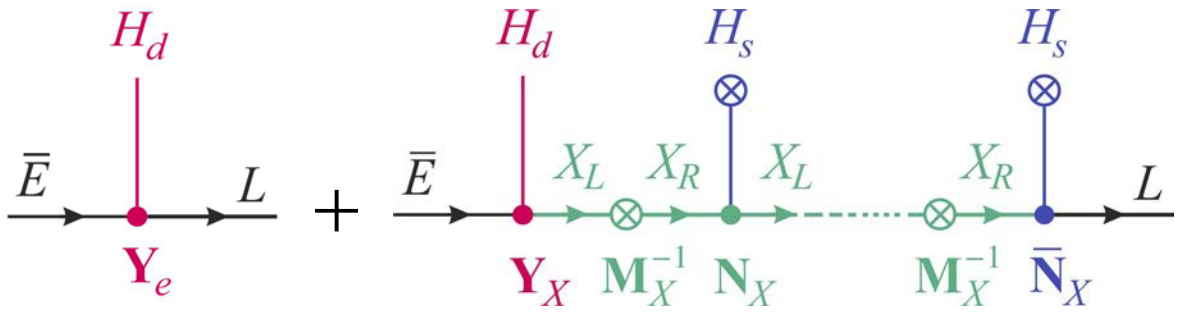


3- Impose MFV on the heavy vector lepton couplings:

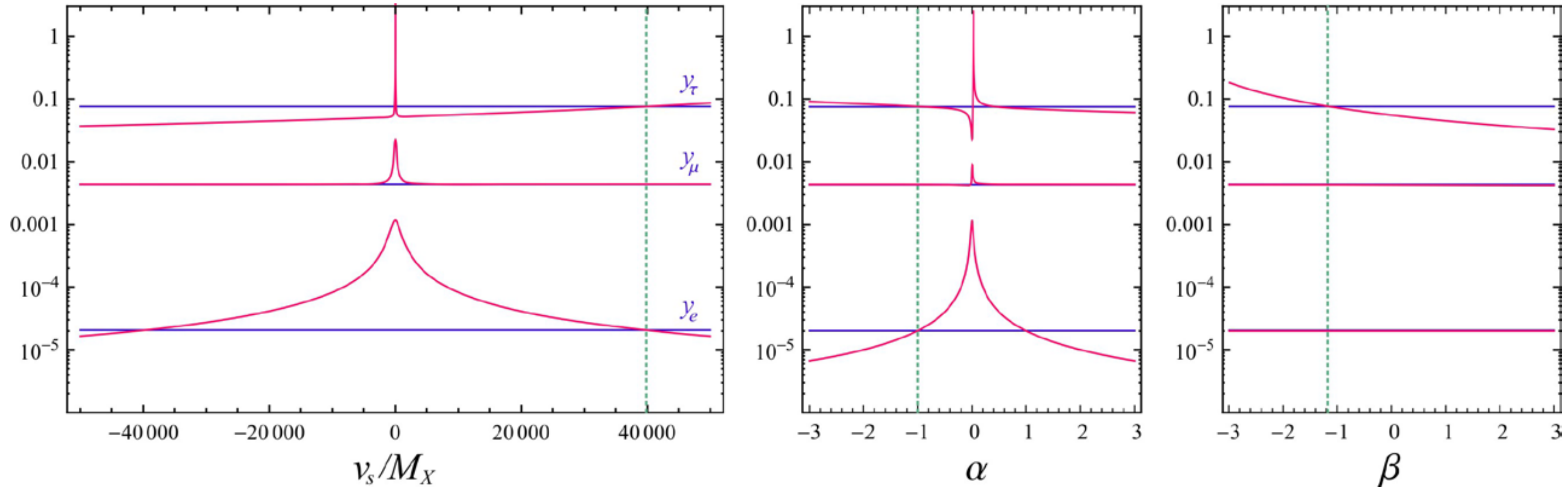
$$G_F = U(3)_{Q=L=X_{L,R}} \times U(3)_U \times U(3)_{D=E} \quad \begin{cases} \mathrm{M}_X = M_X \mathbf{1} \\ \mathrm{Y}_e = \mathrm{Y}_X = \gamma \mathrm{Y}_d \\ \mathrm{N}_X = \bar{\mathrm{N}}_X = \alpha \mathrm{Y}_u^\dagger \mathrm{Y}_u + \beta \mathrm{Y}_d^\dagger \mathrm{Y}_d \end{cases}$$

B. Toy model with vector-like lepton doublets

Dressed lepton Yukawa coupling:

$$\mathrm{Y}_e^{eff} = \gamma \mathrm{Y}_d \frac{1}{1+(v_s / M_X)(\alpha \mathrm{Y}_u^\dagger \mathrm{Y}_u + \beta \mathrm{Y}_d^\dagger \mathrm{Y}_d)}$$

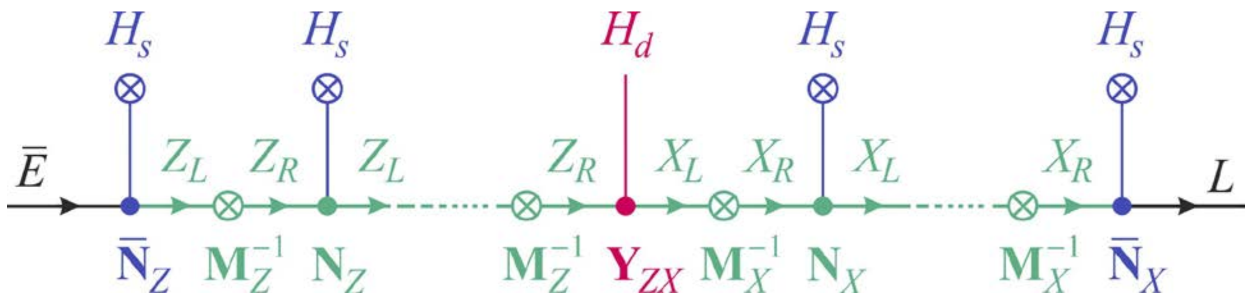
No more fine-tuning, all parameters natural, and free dynamical factor:



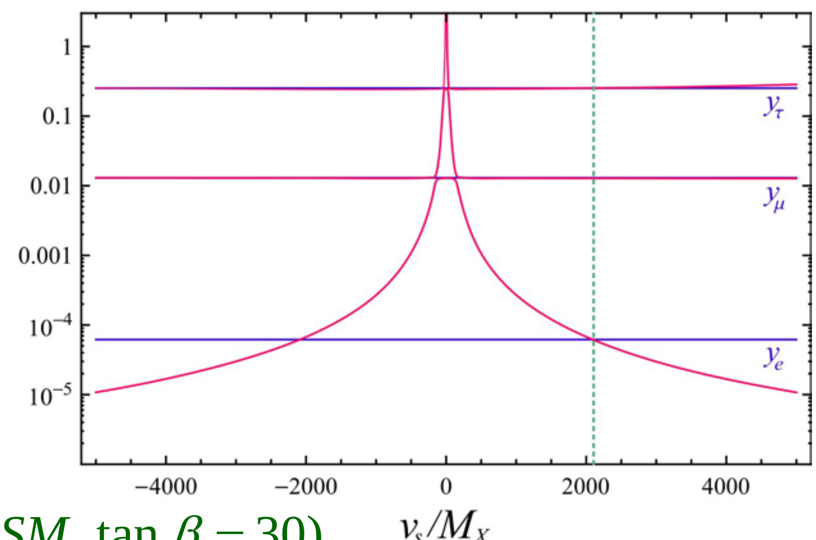
$(MSSM, \tan\beta = 10)$

C. Adding vector-like lepton singlets

Further adding weak singlet vector-like leptons $Z_{L,R}$:



$$\mathrm{Y}_e^{eff} = \frac{1}{1+(v_s/M_Z)(\varepsilon \mathrm{Y}_d \mathrm{Y}_d^\dagger)} \gamma \mathrm{Y}_d \frac{1}{1+(v_s/M_X)(\alpha \mathrm{Y}_u^\dagger \mathrm{Y}_u + \beta \mathrm{Y}_d^\dagger \mathrm{Y}_d)}$$



$(MSSM, \tan\beta = 30)$

Full twisting of the leptons:

$$|g_{E,L}| \approx \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} + \mathcal{O}(0.01)$$

III. Application to the MSSM

A. Geometric MFV for sfermion soft breaking terms

Consider the squark mass term: $\mathcal{L}_{squark} = \begin{pmatrix}\tilde{u}_L^\dagger & \tilde{c}_L^\dagger & \tilde{t}_L^\dagger\end{pmatrix} \cdot \mathrm{m}_Q^2 \cdot \begin{pmatrix}\tilde{u}_L \\ \tilde{c}_L \\ \tilde{t}_L\end{pmatrix}$

Usual MFV: $\mathrm{m}_Q^2 = m_0^2 (1 + \alpha_1 \mathrm{Y}_u^\dagger \mathrm{Y}_u + \alpha_2 \mathrm{Y}_d^\dagger \mathrm{Y}_d + \ldots)$

Typical spectrum: 1^{st}, 2^{nd} generations degenerate,

3^{rd} generation slightly split.

Geometric MFV: $\mathrm{m}_Q^2 = m_0^2 \dfrac{1}{1 - \eta(\alpha_1 \mathrm{Y}_u^\dagger \mathrm{Y}_u + \alpha_2 \mathrm{Y}_d^\dagger \mathrm{Y}_d + \ldots)}$

When η is large, 3^{rd} generation squark(s) are much lighter:

$$\mathrm{m}_Q^2 \to m_0^2 (1 - \langle \mathrm{Y}_u^\dagger \mathrm{Y}_u \rangle^{-1} \mathrm{Y}_u \mathrm{Y}_u^\dagger) \approx \mathrm{diag}(m_0^2,\, m_0^2,\, 0)$$

A. Geometric MFV for sfermion soft breaking terms

In the squark sector

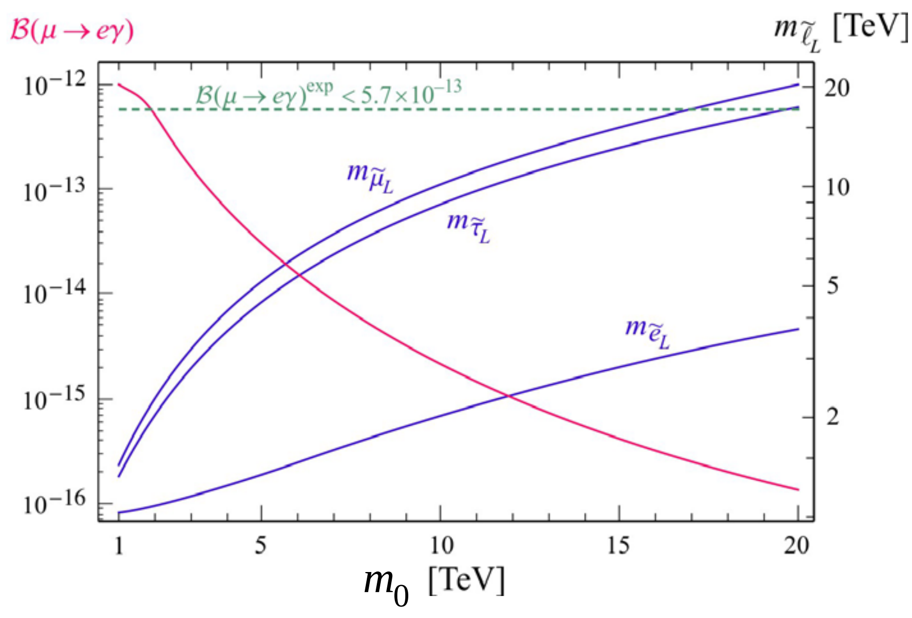
After RGE down, NSUSY-like squark spectrum:

Brummer,Kraml, Kulkarni,CS, '14

- $A_{u,d}$ geometric structure washed out,
- stop quark(s) and possibly left sbottom remain much lighter,
- MFV remains at all scales → No problem with FCNC!

In the slepton sector:

- Lightest sleptons $\tilde{e}^{phys}_{R,L} = \tilde{\tau}^{gauge}_{R,L}$
- Lepton-slepton misaligned.
  (but mixing tuned by CKM)



Caution: PMNS is still absent since $m_\nu = 0$!

B. What about R-parity violation?

Nikolidakis, CS '07

The flavor symmetry $U(3)_{Q=L} \times U(3)_U \times U(3)_{D=E}$ with only $\mathbf{Y}_{u,d}$ as spurions:

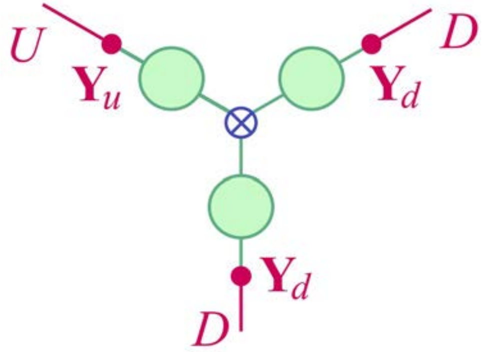
Forbids $\mathcal{L}$ violation but allows for $\mathcal{B}$ violation: $\mathcal{W}_{RPV} \supset \lambda''^{IJK} U^I D^J D^K$

Holomorphy ? → $\lambda''^{IJK} = \lambda \varepsilon^{LMN} \mathbf{Y}_u^{IL} \mathbf{Y}_d^{JM} \mathbf{Y}_d^{KN}$

Csaki, Grossman, Heidenreich '11

With a geometric behavior, holomorphy in $\mathbf{Y}_{u,d}$ is lost, but:

$$\lambda''^{IJK} = \lambda \varepsilon^{LMN} (X_U \cdot \gamma \mathbf{Y}_u \cdot X_Q)^{IL} (X_D \cdot \gamma \mathbf{Y}_d \cdot X_Q)^{JM} (X_D \cdot \gamma \mathbf{Y}_d \cdot X_Q)^{KN}$$



Numerically: holomorphy comes back after the RG evolution starting from this MFV input.

Holomorphy is a very strong attractor!

Bernon, CS '14

IV. Application to minimal SU(5)

A. Flavor troubles in minimal SU(5)

The minimal flavor content is not compatible with observed masses:

$$\mathcal{L}_{Yukawa} = -\frac{1}{4}\overline{\chi}_{10}^{C}\mathbf{Y}_{10}\chi_{10}h_5 + \sqrt{2}\overline{\psi}_{\bar{5}}^{C}\mathbf{Y}_5\chi_{10}h_5^{\dagger} \longrightarrow \mathbf{Y}_e^T = \mathbf{Y}_d \Leftrightarrow \frac{m_d}{m_s} = \frac{m_e}{m_\mu}$$

This can be cured by adding a third Yukawa coupling:

$$\mathcal{L}_{Yukawa} = -\frac{1}{4}\overline{\chi}_{10}^{C}\mathbf{Y}_{10}\chi_{10}h_5 + \underbrace{\sqrt{2}\overline{\psi}_{\bar{5}}^{C}\mathbf{Y}_5\chi_{10}h_5^{\dagger} + \frac{\sqrt{2}}{\Lambda}\overline{\psi}_{\bar{5}}^{C}\mathbf{Y}_5'\chi_{10}H_{24}h_5^{\dagger}} + \ldots$$

$$\mathbf{Y}_5 - \frac{3v_{24}}{2\Lambda}\mathbf{Y}_5' = \mathbf{Y}_e^T \ , \quad \mathbf{Y}_5 + \frac{v_{24}}{\Lambda}\mathbf{Y}_5' = \mathbf{Y}_d$$

But then, MFV fails in SU(5) because $U(3)_{Q=U=E} \times U(3)_{D=L}$ is too small:

$$\mathrm{m}_{10}^2 = m_0^2(c_0 1 + c_1\mathbf{Y}_{10}^{\dagger}\mathbf{Y}_{10} + \underbrace{c_2\mathbf{Y}_5^{\dagger}\mathbf{Y}_5 + c_3\mathbf{Y}_5^{\dagger}\mathbf{Y}_5'}_{\text{Not fixed in terms of fermion masses \& CKM}} + \ldots)$$

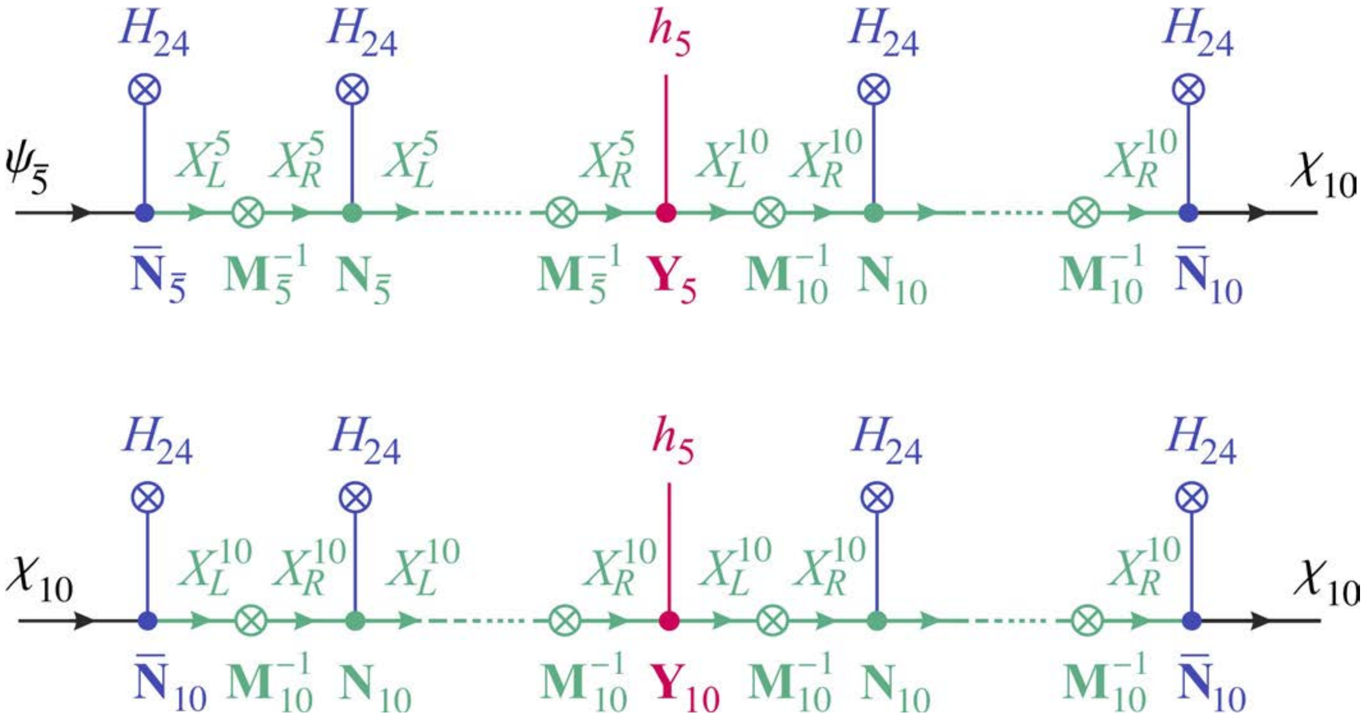
Not fixed in terms of fermion masses & CKM

Unknown and a priori generic mixing matrices threaten FCNC.

B. Towards dynamical flavor unification

We know: $Y_5' = F(Y_{10}, Y_5)$ is possible if F is not a finite polynomial.

Let us try a vector-like model, with new $X^5_{L,R}, X^{10}_{L,R}$ fermions:



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$$\begin{cases} \mathrm{Y}_u = F^1_{10} \cdot \mathrm{Y}_{10} \cdot F^{-1/4,T}_{10} \\ \mathrm{Y}_d = F^1_5 \cdot \mathrm{Y}_5 \cdot F^{-1/4,T}_{10} \\ \mathrm{Y}^T_e = F^{-3/2}_5 \cdot \mathrm{Y}_5 \cdot F^{-3/2,T}_{10} \end{cases} \qquad F^\alpha_R = \frac{1}{1 + \alpha \dfrac{v_{24}}{M_R} \mathrm{N}_R}$$

Simple system, but incredibly difficult to solve:

Unknowns: $\mathrm{Y}_{5,10}$, $M_{5,10}$, and the MFV parameters in $\mathrm{N}_{5,10}$.

Constraints: SVD of $\mathrm{Y}_{u,d,e}$, and CKM mismatch between $\mathrm{Y}_{u,d}$.

Requirement: Natural solution + absence of fine-tuning.

Solutions found only in the no-mixing limit (not very illuminating).

Conclusion

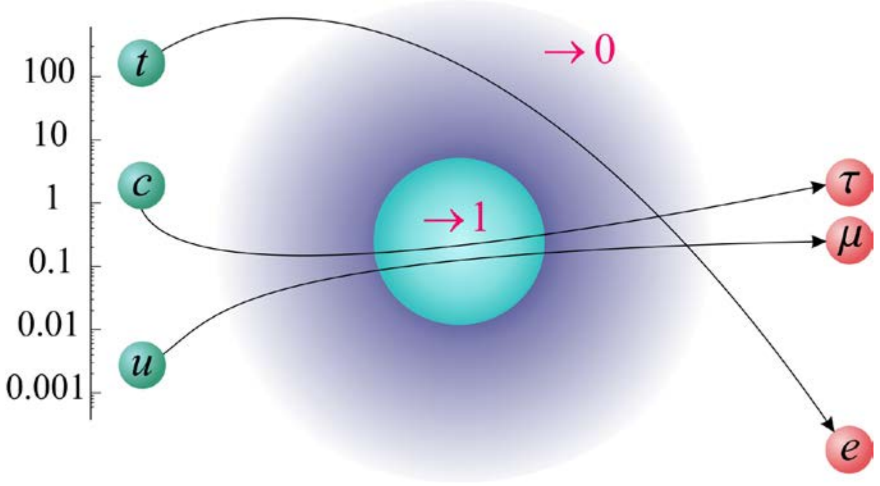
1. There could be only two fundamental flavor structures (+neutrinos)

   The three $\mathrm{Y}_{e,u,d}$ are redundant: they can be related!

   Finite polynomial relationship necessarily fine-tuned.

   Geometric MFV to achieve this naturally.

2. Third-generation partners of the top are the lightest



$$\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}^{phys} \approx \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}^{gauge}$$

Light stops: $M_{\tilde{Q}}^2 \approx m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

3. Perspectives: Real and complete dynamical implementation(s).

   Consequences for models of neutrino masses.

   Numerical solutions yet to be found in minimal SU(5).