Casimir scaling and Yang-Mills glueballs

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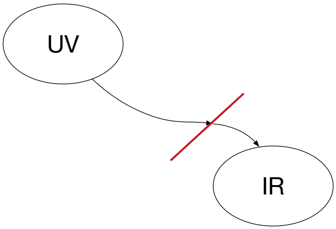
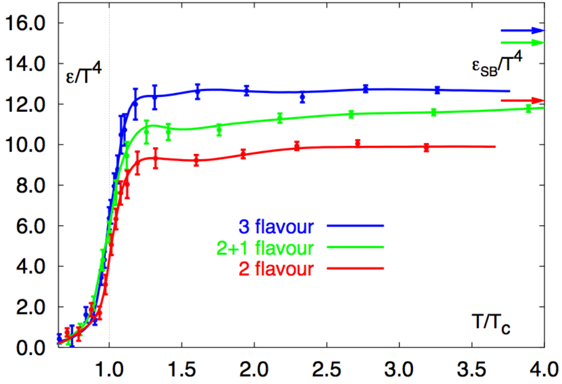
- With J.-W. Lee, B. Lucini, M. Piai, and D. Vadacchino
  -Based on arXiv:1705.00286, Phys.Lett. B775 (2017) 89
- Also with E. Bennet et al. arXiv:1712.04220, JHEP 1803 (2018) 185.

Introduction

- Solving QCD is hard, because quarks and gluons are not right degrees of freedom at low energy, though fundamental.
- Microscopic degrees are often vastly different from the macroscopic degrees of freedom due to phase transition.



- Yang-Mills theories with a few matter fields are believed to exhibit a confining phase.



- The thermal average of the Polyakov loop is an order parameter of the confinement phase transition.

$$L(\tau) = \langle l(\vec{x}) \rangle_\beta \equiv \left\langle \frac{1}{N_c} \mathrm{Tr} P e^{i \int_0^\beta A_0(\tau, \vec{x}) \mathrm{d}\tau} \right\rangle_\beta = e^{-\beta E_q} .$$

- Center symmetry: $g(\beta, \vec{x}) = h\, g(0, \vec{x}),\ h \in Z_{N_c}$.

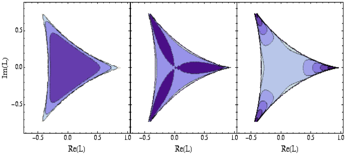


Figure: Asakawa et al PRL '13

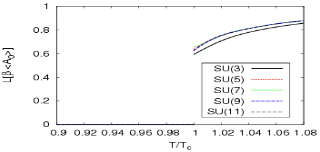
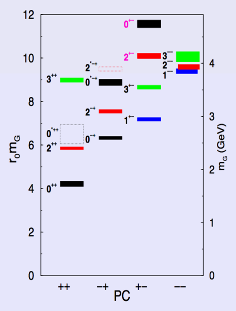
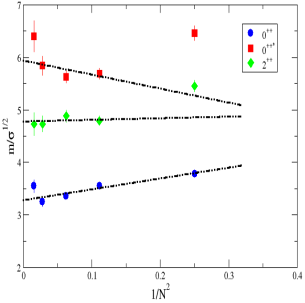


Figure: Braun et al EPJC '10

Glueball mass

- Calculating glueball mass is tantamout to showing the confinement gap:





Glueball mass

- Large N behavior for 4d $\mathrm{SU}(N)$ (Lucini+Teper+Wenger '04):

0^{++}	$\frac{m}{\sqrt{\sigma}} = 3.28(8) + \frac{2.1(1.1)}{N^2}$
0^{++*}	$\frac{m}{\sqrt{\sigma}} = 5.93(17) - \frac{2.7(2.0)}{N^2}$
2^{++}	$\frac{m}{\sqrt{\sigma}} = 4.78(14) + \frac{0.3(1.7)}{N^2}$

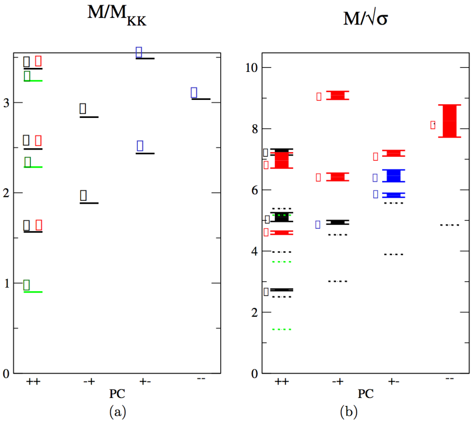
Glueball mass

- Large N limit of lattice data:

$$\left.\frac{M_{0^{++}}}{\sqrt{\sigma}}\right|_{N\to\infty} = \sqrt{2\eta} = 3.28 \quad \to \quad \eta(\mathrm{lattice}) = 5.4$$

Glueball mass

- Holographic QCD Calculations (Brower et al 2000):



Glueball mass

- The string tension in Witten-Sakai-Sugimoto model:

$$\sigma = \frac{1}{2\pi l_s^2}\sqrt{-g_{tt}g_{xx}}\bigg|_{u=u_{\rm KK}} = \frac{\lambda}{27\pi}M_{\rm KK}^2 \,.$$

- Taking $\lambda = 17$ (HRYY '07), we find

$$\eta(\mathrm{hQCD}) = 2.49\left(\frac{M_{0^{++}}}{M_{\rm KK}}\right)^2 = 5.6$$

$$\eta(\mathrm{lattice}) = 5.4$$

- To understand the glueball spectrum is to solve YM theory.
- Finding any regularity especially in the ground state glueball will help us to understand the confinement in YM theory.
- We conjecture a universal ratio in pure YM theories:

$$\eta(0^{++}) = \frac{m_{0^{++}}^2}{\sigma} \cdot \frac{C_2(F)}{C_2(A)}.$$

- It is universal, independent of gauge groups. It depends only on the dimensionality of space-time.

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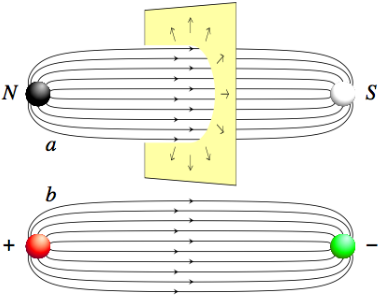
Universal ratio and confinement

- Confinement and monopole condensation - dual Meissner effect (Mandelstam, 't Hooft 74, Seiberg-Witten '94)
- Monopole condensation - 't Hooft operator $\langle\phi\rangle \sim \kappa$, the intrinsic scale, responsible to all dimensional quantities in confining phase.



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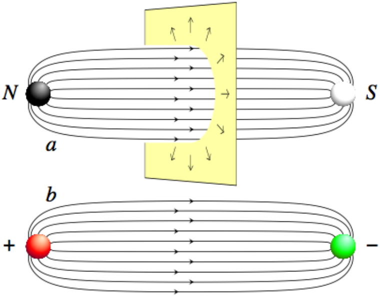

Universal ratio and confinement

- If the monopole condensation is responsible for the confinement, the string tube picture should hold.

- String tension between a static quark and antiquark:

$$\sigma = \kappa^2 C_2(F), \quad \langle \phi \rangle \sim \kappa \,.$$

- The string tension can be calculated from the Wilson loop:

$$\langle W(C) \rangle = \left\langle \frac{1}{N} e^{i \oint_C A} \right\rangle = \exp\left[-\sigma L T + \cdots \right],$$

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Universal ratio and confinement

- Since the gluon is adjoint, we conjecture

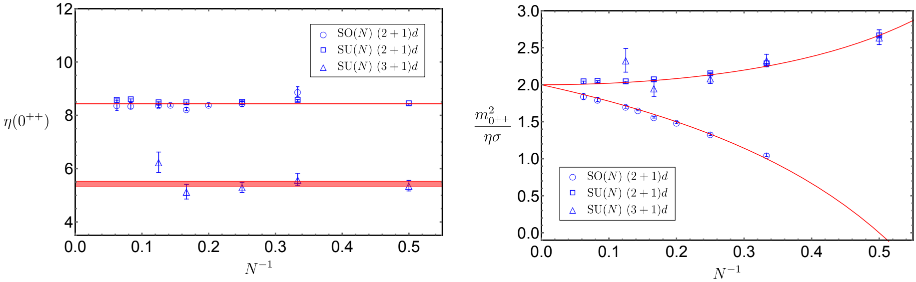
$$m_{0^{++}}^2 = \eta \, \kappa^2 \, C_2(A) \, ,$$

where η is a universal dimensionless number for YM theories.

Universal ratio and confinement

- Numerical evidences from Monte Carlo simulations:

$$\eta(0^{++}) \equiv \frac{m^2_{0^{++}}}{\sigma} \cdot \frac{C_2(F)}{C_2(A)} \approx \begin{cases} 5.41(12)\,, & \text{for } d = 3+1\,, \\ 8.444(15)(90) & \text{for } d = 2+1\,. \end{cases}$$



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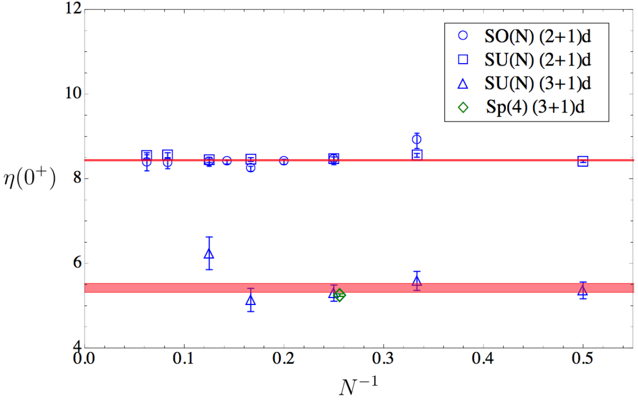
Universal ratio and confinement

- New data by Bennett et al. (arXiv:1712.04220):



Universal ratio and confinement

- Ratio of Casimirs:

$$\frac{C_2(A)}{C_2(F)} = \begin{cases} \frac{2N^2}{N^2-1} \text{ for } \mathrm{SU(N)} \\ \frac{2(N-2)}{N-1} \text{ for } \mathrm{SO(N)} \\ \frac{4(N+1)}{2N+1} \text{ for } \mathrm{Sp(2N)}\,. \end{cases}$$

- In the large-N limit the ratio of Casimirs is universally 2.
- Large-N universality (Lovelace '82; Imoto-Sakai-Sugimoto '10) implies η is at least universal in the large N.

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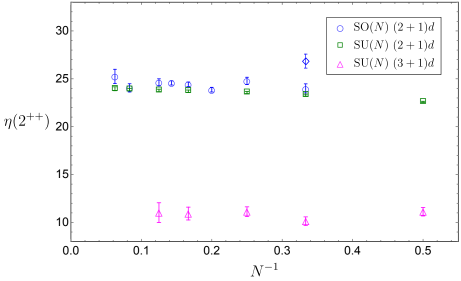
Universal ratio and confinement

- Excited states don't show universal behavior:



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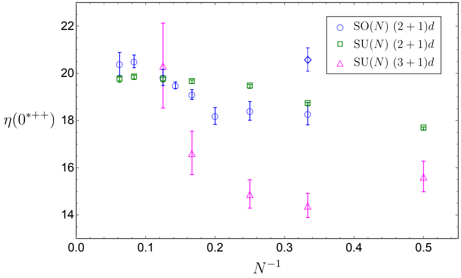
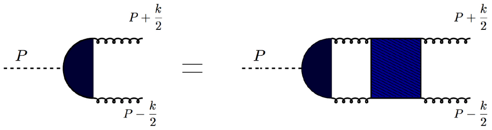


Bethe-Salpeter equations

- The Bethe-Salpeter amplitude for a color-singlet bound state:

$$\Gamma_R^{\mu\nu}(x_1, x_2; P) = \langle 0 | \, TA^{\mu\, a}(x_1) A^{\nu\, a}(x_2) \, | R(P, \lambda) \rangle \ ,$$

$$\left[\partial^2 - P^2\right] \chi_P(x) = \int \mathrm{d}^4 y \, V(x-y) \chi_P(y) \, ,$$



Bethe-Salpeter equations

- To get the Regge behavior for the radial excitations, we take

$$V(x-y) = \frac{1}{2} k\, x^2\, \delta^4(x-y)\,.$$

- If the string tube picture holds for the glueball state,

$$\sqrt{k} \sim \sigma(A) = C_2(A)\kappa^2\,.$$

- The glueball mass becomes $(n = 1, 2, \cdots)$

$$M_{0^{++}}^2 \sim\ C_2(A)\kappa^2\,(n+1)\,.$$

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Bethe-Salpeter equations

- Corrections to the area law: for $|x| \gg \kappa^{-1}$

$$\frac{1}{2}\left(C_2(A)\kappa^2 x - \frac{\alpha}{x}\right)^2 \approx \frac{1}{2}C_2(A)^2\kappa^4 x^2 - \alpha\, C_2(A)\kappa^2\,,$$

where $\alpha = \pi/12$ is the universal coefficient of Lüscher term.

- The ground state glueball mass then gets a correction as

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Scale anomaly

- If the scale anomaly is parametrically small, compared to the condensate of the order parameter, then there should be a Nambu-Goldstone boson associated with scale symmetry:
- By Goldstone theorem, the dilatation current should create a state, dilaton:

$$\langle 0 | \, D^{\mu} \, | D(p) \rangle = -f_D p^{\mu} e^{-ip\cdot x} \, ;$$

- The scale anomaly in pure YM theory is given as

$$\partial^{\mu} D_{\mu} = -\frac{\beta(g)}{2g} F^{a}_{\mu\nu} F^{a\,\mu\nu} \, .$$

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- The anomalous WT identity relates:

$$\int_x \langle 0|\, \mathrm{T}\, \theta^\mu_\mu(x)\theta^\nu_\nu(y)\, |0\rangle = -4 \left\langle \theta^\mu_\mu(y) \right\rangle = -16\mathcal{E}_{\rm vac}\,.$$

- Since all the gluons contribute to the vacuum energy equally and additively,

$$4\mathcal{E}_{\rm vac} = \left\langle \theta^\mu_\mu \right\rangle = -\beta\, C_2(A)\kappa^4\,.$$

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$$\int_x \langle 0| \mathrm{T}\, \theta^\mu_\mu(x)\theta^\nu_\nu(y) |0\rangle \approx f_D^2 m_D^2\,.$$

- We have PCDC relation:

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$$m^2_{0^{++}}(= m^2_D) \propto \beta\, C_2(A)\kappa^2\,.$$

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QCD sum rules

- In lattice the glueball mass is extracted from the correlators of $\mathcal{O}_S(x) = \alpha_s F^a_{\mu\nu} F^{a\,\mu\nu}$:

$$\Pi_S(x) = \langle 0 | \, \mathrm{T}\, \mathcal{O}_S(x) \mathcal{O}_S(0) \, | 0 \rangle = \sum_n c_n \, e^{-m_n |x|} \, ,$$

- QCD sum rule for the zero moment (Novikov1980) becomes

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Conclusion and Outlook

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