$\theta = \pi$ in $\mathrm{SU}(N)/\mathbb{Z}_N$ Theory

Takao Suyama (KEK)

Based on collaboration with

R.Kitano and N.Yamada (KEK).

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An application:

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We give an alternative argument for SSB of CP based on finite size corrections.

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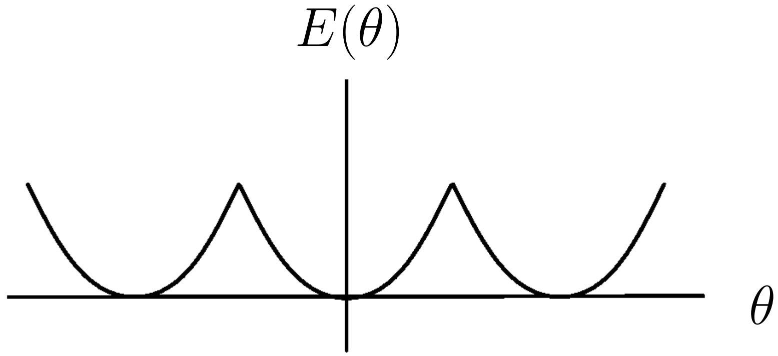

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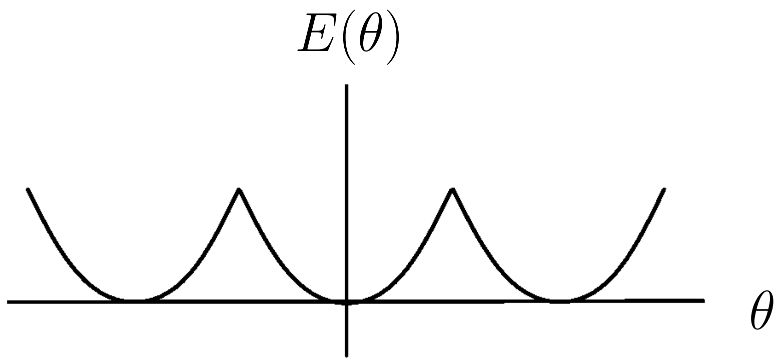

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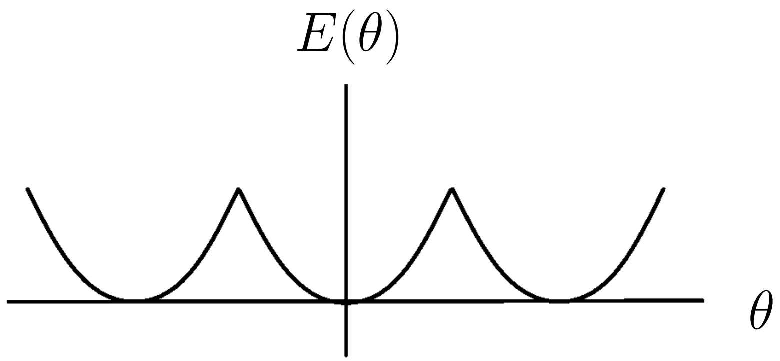

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Also for finite N?

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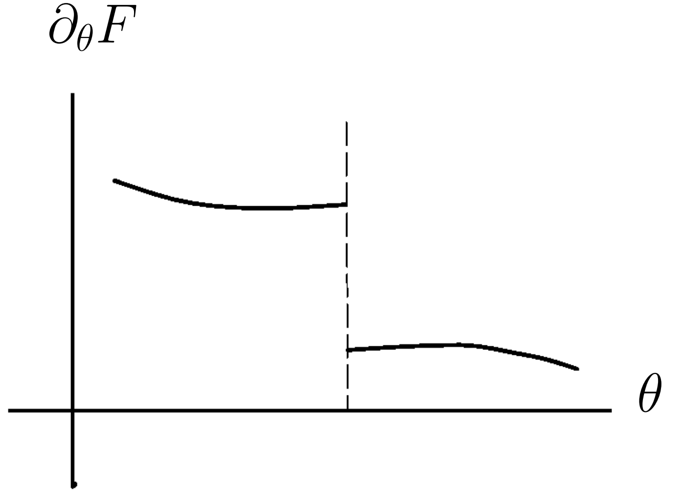
Instead, we look for a large finite volume corrections to $\partial_\theta F$.



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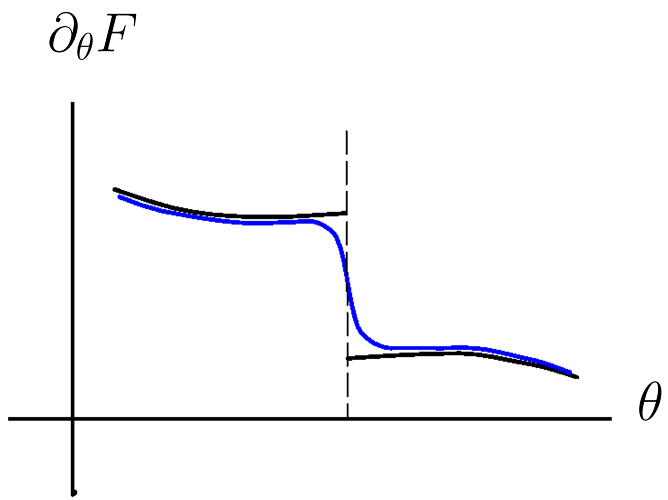

Physical quantities are analytic for a finite volume.

$\Rightarrow$ Finite V correction becomes large near transition points.

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$$\partial_\theta F(\theta, V) - \partial_\theta F(\theta, \infty) \ \sim \ e^{-\Delta V^{1/4}},$$

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In either case, it is surprising!

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We estimate $g(2\pi, V) - g(0, V)$ for $\mathrm{SU}(N)/\mathbb{Z}_N$ theory.

Let $F(e, m, \theta, V)$ be the free energy in the presence of e electric fluxes and m magnetic fluxes. **['t Hooft 79]**

The partition function is

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What we want to estimate is

$$g(2\pi, V) - g(0, V) \;=\; -\frac{1}{V}\log Z(2\pi, V) + \frac{1}{V}\log Z(0, V).$$

Assume the confinement at $\theta = 0$. This implies **['t Hooft 79]**

$$F(0, m, 0, V) \;\to\; 0, \qquad F(m, m, 0, V) \;\to\; \infty. \qquad (V \to \infty)$$

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$\Rightarrow$ **Spontaneous CP violation!**

(Or $\Delta = 0$.)

Assume further that there is only one transition in $[0, 2\pi]$ in both $\mathrm{SU}(N)$ theory and $\mathrm{SU}(N)/\mathbb{Z}_N$ theory.

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$\mathrm{SU}(N)$ **theory has 2π periodicity, and is CP invariant at $\theta = 0$.**

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Equally exciting if there are multiple transitions in $[0, 2\pi]$.

Summary

- **CP at $\theta = \pi$ is spontaneously broken in bosonic YM.**
- **It is related to a 1st order phase transition.**
- **Finite size correction implies spontaneous CP violation or vanishing mass gap.**

Open issues

- **Numerical simulation of $\mathrm{SU}(N)/\mathbb{Z}_N$ theory.**
- **Detailed investigation of CP^N model.**
- **Adding matter, phase diagram.**
- **etc.**