Prediction for the Cosmological Constant and Constraints on SUSY GUTS: Status Report for Resummed Quantum Gravity

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OUTLINE

- Introduction
- Overview of Resummed Quantum Gravity
- Planck Scale Cosmology
- An Estimate of Λ
- An Open Question?
- Einstein-Heisenberg Consistency Condition
- Constraints on SUSY GUTs
- Outlook

Inroduction

- WHAT IS RESUMMATION?
  - FAMILIAR SUMMATION: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
  - RESUMMATION:

$$\sum_{n=0}^{\infty} C_n \alpha_s^n \begin{cases} = F_{\mathrm{RES}}(\alpha_s) \sum_{n=0}^{\infty} B_n \alpha_s^n, & \text{EXACT} \\ \cong G_{\mathrm{RES}}(\alpha_s) \sum_{n=0}^{\infty} B'_n \alpha_s^n, & \text{APROX.} \end{cases}$$

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Inroduction

- A LITTLE HISTORY: 1988 ICHEP-Munich Conference Dinner, ONE YEAR BEFORE LEP DATA TAKING THAT LED, BY PRECISION PHYSICS, TO THE
  ‘t HOOFT-VELTMAN (1999) EW AND GROSS-WILCZEK-POLITZER (2004) QCD NOBEL PRIZES IN PHYSICS:
  F. Berends and I considered, ’How Accurate Can Exponentiation (RESUMMATION) Really Be?’
- Would It Limit or Enhance Precision for a Given Level of Exactness: LO, NLO, NNLO, ... ?

Inroduction

- ’Two’ Realizations in Literature:
  Jackson-Scharre(JS)(APPROX) vs YFS (EXACT)
- JS $\rightarrow$ ’limit to precision’
- YFS $\rightarrow$ ’no limit to precision’
- See 1989 CERN Yellow Book article: Frits was almost convinced, but not completely!
- Today, the analogous discussion continues to a new paradigm: quantum gravity

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Introduction

- IS QUANTUM GRAVITY (Einstein-Hilbert Theory) CALCULABLE IN RELATIVISTIC QFT?
- STRING THEORY: NO. You need superstrings, supersymmetric one-dimensional objects of Planck length size, 1.62×10^{-33} cm.

String Theory

- Advantages:

  UV FINITENESS,

  THEORY OF EVERYTHING: $SU_{2L} \otimes U_1 \otimes QCD \otimes QG$,

  APPLIED PHYSICS IN PURE MATHEMATICS, ...
- Disadvantages:

  So Many Solutions ~ 10^{500} and counting $\Rightarrow$ ANTHROPICITY, ...

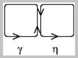
- quark: QFT - • SST -

  0 size $\ell_{Pl} = 1.62 \times 10^{-33}\ cm$

Inroduction

- LOOP QUANTUM GRAVITY: NO. You need Planck length size loops that are the fundamental constructs for quantum gravity.

Loop Quantum Gravity

- Advantages:
  Suspected UV FINITENESS,
  LINEARIZED QG Constraints, ...
- Disadvantages:
  UNKNOWN SEMI-CLASSICAL LIMIT,
  QUESTIONS OF PRINCIPLE, ...
- graviton: QFT - • LQG - 

....

0 size $\quad \ell_{Pl} = 1.62 \times 10^{-33}\ cm$

Inroduction

- HORAVA-LIFSHITZ THEORY: NO. You need anisotropic scaling at Planck length scales:
  - Time and space differ by a factor of z in how they scale at Planck length distances with $z = 3$ in the original proposal– this violates local Lorentz invariance.

HORAVA-LIFSHITZ Quantum Gravity

- Advantages:
  POWER COUNTING UV RENORMALIZABLE,
  CONNECTIONS TO CONDENSED MATTER, ...
- Disadvantages:
  VIOLATES LOCAL LORENTZ INVARIANCE,
  SCALAR GRAVITON (PATHOLOGY?), ...
- graviton: RQFT - $\underset{0\ \text{size}}{\bullet}$: $\underset{\text{propagator}}{1/(\omega^2 - \vec{k}^2)}$ HLQG - $\underset{0\ \text{size}}{\bullet}$: $\underset{\text{propagator}}{1/(\omega^2 - \vec{k}^2 - G\vec{k}^6)}$,
  $\ell_{Pl} = 1.62 \times 10^{-33}\ cm = \sqrt{G}$

- Our Response: Exact Amplitude-Based Resummation of Feynman's Formulation of Einstein's Theory – Resummed Quantum Gravity (RQG)
- RESULT (1): UV Finiteness!
- RESULT (2): Constraints on SUSY GUT's
- RESULT (3): Prediction for the Cosmological Constant Λ with Relatively Small Theoretical Uncertainty.
- RESULT (4): Consistent with Weinberg's Asymptotic Safety Ansatz, as relaized by Exact Field Space Renormalization Group Program of Reuter *et al.*

- RESULT (5): Consistent with Kreimer's Leg Renormalizability Results ...
- Today we give a report on the status and outlook for this new RQG approach.

Overview of Resummed Quantum Gravity

SM $\Leftrightarrow$ Many Massive Point Particles.
Feynman: spin is an inessential complication – checked. We replace $L^{\mathcal{G}}_{SM}(x)$ with that a free physical Higgs field, $\varphi(x)$, with a rest mass 125 GeV(ATLAS,CMS) $\Rightarrow$ the representative model **{R.P. Feynman, Acta Phys. Pol. 24 (1963) 697;** *Feynman Lectures on Gravitation*, **eds. F.B. Moringo and W.G. Wagner, (Caltech, Pasadena, 1971). }**

$$
\begin{aligned}
\mathcal{L}(x) &= \frac{1}{2\kappa^2} R\sqrt{-g} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\sqrt{-g} \\
&= \quad \frac{1}{2}\left\{h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu_\lambda,\lambda'}\eta^{\sigma\sigma'}\bar{h}_{\mu'\sigma,\sigma'}\right\} \\
&\quad + \frac{1}{2}\left\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right\} - \kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right] \\
&\quad - \kappa^2\left[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}\left(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right) - 2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\right] + \cdots
\end{aligned}
\tag{1}
$$

Overview of Resummed Quantum Gravity

where $\varphi_{,\mu} \equiv \partial_\mu \varphi$ and we have

- $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$,
  $\eta_{\mu\nu} = diag\{1, -1, -1, -1\}$
- $\bar{y}_{\mu\nu} \equiv \frac{1}{2}\left(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho{}^\rho\right)$ for any tensor $y_{\mu\nu}$
- Feynman rules already worked-out by Feynman (*op. cit.*), where we use his gauge, $\partial^\mu \bar{h}_{\nu\mu} = 0$

$\Leftrightarrow$ Quantum Gravity is just another quantum field theory where the metric now has quantum fluctuations as well.

For example, the one-loop corrections to the graviton propagator due to matter loops is just given by the diagrams in the Figure.

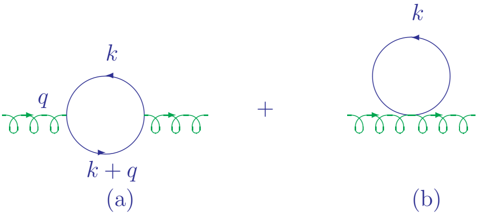


Figure : The scalar one-loop contribution to the graviton propagator.

q is the 4-momentum of the graviton.

QG's BAD UV behavior – unrenormalizable.

Overview of Resummed Quantum Gravity

YFS resum the propagators in the NON-ABELIAN gauge theory of QG:

⇒ from the YFS formula

$$iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_F^{-1}(p) - \Sigma'_F(p)}, \tag{2}$$

we find for Quantum Gravity, proceeding as above, the analogue of

$$\alpha B''_\gamma = \int \frac{d^4\ell}{(2\pi)^4} \frac{-i\eta^{\mu\nu}}{(\ell^2 - \lambda^2 + i\epsilon)} \frac{-ie(2ik_\mu)}{(\ell^2 - 2\ell k + \Delta + i\epsilon)} \frac{-ie(2ik'_\nu)}{(\ell^2 - 2\ell k' + \Delta' + i\epsilon)}\Big|_{k=k'} \tag{3}$$

as $-B''_g(k)$ with

$$B''_g(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \tag{4}$$

for $\Delta = k^2 - m^2 \Rightarrow$ for a scalar field

$$i\Delta'_F(k)|_{YFS-resummed} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)}.$$

Overview of Resummed Quantum Gravity

$\Rightarrow$

Expand theory with the 'improved Born' propagators

$$iP_{\alpha_1\cdots;\alpha'_1\cdots}\Delta'_F(k)|_{YFS-resummed,\Sigma'_s=0} = \frac{iP_{\alpha_1\cdots;\alpha'_1\cdots}e^{B''_g(k)}}{(k^2-m^2+i\epsilon)} \tag{5}$$

where in the DEEP UV we get

$$B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right), \tag{6}$$

$\Rightarrow$ ALL PROPAGATORS FALL FASTER THAN ANY POWER OF $|k^2|$ $\Rightarrow$ QG IS FINITE (SEE MPL**A17** (2002) 2371;hep-ph/0607198)!

CONTACT WITH ASYMPTOTIC SAFETY APPROACH

- OUR RESULTS IMPLY

$$G(k) = G_N/(1+\frac{k^2}{a^2})$$

$\Rightarrow$ FIXED POINT BEHAVIOR FOR
$k^2 \to \infty$,
IN AGREEMENT WITH THE PHENOMENOLOGICAL ASYMPTOTIC SAFETY APPROACH OF BONANNO & REUTER IN PRD**62**(2000) 043008.

- OUR RESULTS $\Rightarrow$ AN ELEMENTARY PARTICLE HAS NO HORIZON. THIS AGREES WITH BONANNO & REUTER THAT A BLACK HOLE WITH A MASS LESS THAN
$M_{cr} \sim M_{Pl}$
HAS NO HORIZON.
BASIC PHYSICS:
$G(k)$ VANISHES FOR $k^2 \to \infty$.

Planck Scale Cosmology

- Bonanno and Reuter see arXiv.org:0803.2546,and refs. therein – phenomenological approach to Planck scale cosmology: STARTING POINT IS THE EINSTEIN-HILBERT THEORY

$$\mathcal{L}(x) = \frac{1}{2\kappa^2}\sqrt{-g}\,(R - 2\Lambda) \tag{7}$$

PHENOMENOLOGICAL EXACT RENORMALIZATION GROUP FOR THE WILSONIAN COARSE GRAINED EFFECTIVE AVERAGE ACTION IN FIELD SPACE$\Rightarrow$ RUNNING NEWTON CONSTANT $G_N(k)$ AND COSMOLOGICAL CONSTANT $\Lambda(k)$ APPROACH UV FIXED POINTS AS k GOES TO ∞ IN THE DEEP EUCLIDEAN REGIME – $k^2 G_N(k) \rightarrow g_*,\ \Lambda(k) \rightarrow \lambda_* k^2$.

  - Due to the thinining of the degrees of freedom in Wilsonian field space renormalization theory, the arguments of Foot et al.(PLB**664**(2008)199) are obviated.– See also MPLA **25**(2010)607;SHAPIRO&SOLA,PLB682(2009)105

Planck Scale Cosmology

CONTACT WITH COSMOLOGY PROCEEDS AS FOLLOWS: PHENOMENOLOGICAL CONNECTION BETWEEN THE MOMENTUM SCALE k CHARACTERIZING THE COARSENESS OF THE WILSONIAN GRAININESS OF THE AVERAGE EFFECTIVE ACTION AND THE COSMOLOGICAL TIME t, B-R SHOW STANDARD COSMOLOGICAL EQUATIONS ADMIT(see also Bonanno et al.,1006.0192) THE FOLLOWING EXTENSION:

$$
\begin{aligned}
(\frac{\dot{a}}{a})^2 + \frac{K}{a^2} &= \frac{1}{3}\Lambda + \frac{8\pi}{3}G_N\rho \\
\dot{\rho} + 3(1+\omega)\frac{\dot{a}}{a}\rho &= 0 \\
\dot{\Lambda} + 8\pi\rho\dot{G}_N &= 0 \\
G_N(t) &= G_N(k(t)) \\
\Lambda(t) &= \Lambda(k(t))
\end{aligned} \tag{8}
$$

FOR DENSITY ρ AND SCALE FACTOR $a(t)$.

WITH ROBERTSON-WALKER METRIC REPRESENTATION

$$ds^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{9}$$

$K = 0, 1, -1 \Leftrightarrow$ RESPECTIVELY FLAT, SPHERICAL AND PSEUDO-SPHERICAL 3-SPACES FOR CONSTANT TIME t FOR A LINEAR RELATION BETWEEN THE PRESSURE p and ρ (EQN. OF STATE)

$$p(t) = \omega\rho(t). \tag{10}$$

Planck Scale Cosmology

FUNCTIONAL RELATIONSHIP BETWEEN MOMENTUM SCALE k AND COSMOLOGICAL TIME t DETERMINED PHENOMENOLOGICALLY VIA

$$k(t) = \frac{\xi}{t} \tag{11}$$

WITH POSITIVE CONSTANT ξ.
Using the UV fixed points for $k^2 G_N(k) = g_*$ and $\Lambda(k)/k^2 = \lambda_*$ B-R SHOW THAT (8) ADMITS, FOR $K = 0$, A SOLUTION IN THE PLANCK REGIME ($0 \leq t \leq t_{\rm class}$, with $t_{\rm class}$ a few times the Planck time t_{Pl}), WHICH JOINS SMOOTHLY ONTO A SOLUTION IN THE CLASSICAL REGIME ($t > t_{\rm class}$) which agrees with standard Friedmann-Robertson-Walker phenomenology but with the horizon, flatness, scale free Harrison-Zeldovich spectrum, and entropy problems solved by Planck scale quantum physics.

PHENOMENOLOGICAL NATURE OF THE ANALYSIS: THE fixed-point results g_*, λ_* depend on the cut-offs used in the Wilsonian coarse-graining procedure.
KEY PROPERTIES OF g_*, λ_* USED FOR THE B-R ANALYSES: they are both positive and the product $g_*\lambda_*$ is cut-off/threshold function independent.

An Estimate of Λ

- In Phys. Dark Univ. **2** (2013) 97, using (5) and (5) we get rigorous cut-off independent values for the fixed points $g_*,\ \lambda_*$ and the following estimate of Λ:

$$\rho_\Lambda(t_0) \cong \frac{-M_{Pl}^4(1+c_{2,eff}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64}\sum_j \frac{(-1)^{F_j}n_j}{\rho_j^2}$$
$$\times \frac{t_{tr}^2}{t_{eq}^2} \times (\frac{t_{eq}^{2/3}}{t_0^{2/3}})^3$$
$$\cong \frac{-M_{Pl}^2(1.0362)^2(-9.194\times 10^{-3})}{64}\frac{(25)^2}{t_0^2}$$
$$\cong (2.4\times 10^{-3}eV)^4, \tag{12}$$

where the age of the universe is $t_0 \cong 13.7\times 10^9$ yrs.

- Compare: $\rho_\Lambda(t_0)|_{\rm expt} \cong ((2.37\pm 0.05)\times 10^{-3}eV)^4$.

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An Open Question

- A MAIN UNCERTAINTY: $t_{\rm tr}$
- B-R: NUMERICAL STUDIES $\Rightarrow t_{\rm tr} \cong 25/M_{Pl}$
- IN GENERAL, A FACTOR of $\mathcal{O}(100)$ IS ALLOWED
- CAN WE DO BETTER?

Einstein-Heisenberg Consistency Condition

- Recently, arXiv:1507.00661, we use the de Sitter space solutions of Duerr et al. to get the Einstein-Heisenberg consistency condition

$$k \geq \frac{\sqrt{5}}{2w_0} = \frac{\sqrt{5}}{2}\frac{1}{\sqrt{3/\Lambda(k)}} \tag{13}$$

from the Heisenberg uncertainty relation $\Delta p \Delta q \geq \frac{1}{2}$, with $\Delta p = k$ and ($w_0 = \sqrt{3/\Lambda}$)

$$(\Delta q)^2 \cong \frac{\int_0^{w_0} dww^2 w^2 < \cos^2\theta >}{\int_0^{w_0} dww^2} = \frac{1}{5}w_0^2. \tag{14}$$

- Violation of (13) ends Planck scale inflation: solving for k_{tr} $\Rightarrow k_{tr} \cong M_{Pl}/25.3$, in agreement with what Bonnano and Reuter suggested from numerical studies.
- $\Rightarrow$ uncertainty on our estimate of ρ_Λ is $\mathcal{O}(10)$.

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Constraints on SUSY GUTS

Note

$$<0|\mathcal{H}|0>\sim\int^{M_{Pl}}\frac{d^3k}{(2\pi)^3}\frac{1}{2}\omega(k)=\int^{M_{Pl}}\frac{d^3k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m_i^2}$$

Raises the question of GUTS: Use SO(10) SUSY GUT Approach of Dev & Mohapatra (PRD82(2010)035014):

Intermediate Stage:

$SU_{2L}xSU_{2R}xU_1XSU(3)^c$

SM Stage at ~2TeV = M_R:

$SU_{2L}XU_1XSU(3)^c$

SUSY Breaking at EW scale M_S:

$U_1XSU(3)^c$

- Possible spectrum(?)

$$
\begin{aligned}
m_{\tilde{g}} &\cong 1.5(10)\mathrm{TeV}\\
m_{\tilde{G}} &\cong 1.5\mathrm{TeV}\\
m_{\tilde{q}} &\cong 1.0\mathrm{TeV}\\
m_{\tilde{\ell}} &\cong 0.5\mathrm{TeV}\\
m_{\tilde{\chi}^0_i} &\cong \begin{cases} 0.4\mathrm{TeV}, & i=1\\ 0.5\mathrm{TeV}, & i=2,3,4 \end{cases}\\
m_{\tilde{\chi}^\pm_i} &\cong 0.5\mathrm{TeV},\ i=1,2\\
m_S &= .5\mathrm{TeV},\ S=A^0,\ H^\pm,\ H_2,
\end{aligned}
$$

$\Longrightarrow$

$$
\begin{aligned}
\Delta_{\mathrm{GUT}} &= \sum_{j\in\{\text{MSSM low energy susy partners}\}} \frac{(-1)^F n_j}{\rho_j^2}\\
&\cong 1.13(1.12)\times 10^{-2}
\end{aligned}
$$

Constraints on SUSY GUTS

$\Longrightarrow$

- Compensate by either (A) adding new susy families with scalars lighter than fermions or (B) allowing the gravitino mass to go to $\sim .05\, M_{GUT} \sim 2\text{x}\, 10^{15}$ GeV.
- For approach (A),

  new quarks and leptons at

  $M_{High} \sim 3.4(3.3)\ \text{x}\ 10^{3}$ TeV,

  scalar partners at $\sim .5$TeV $= M_{Low}$

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What About EW, QCD, GUT Symmetry Breaking Scales?

Consider GUT symmetry breaking:
It gives a $M_{GUT}{}^4/(.01\ M_{Pl}{}^4/64) < 10^{-6}$ correction, which we drop here.
The other breaking scales are even smaller and hence their corrections are even less significant in our result for ρ_Λ.

Covariance Issues for $\dot{\Lambda}$, $\dot{G}_N \neq 0$:

Bianchi's Identity,

$D^{\nu}(\Lambda g_{\nu\mu}+8\pi G_N T_{\nu\mu})=0,$

allows

$$\dot{\rho}+3\frac{\dot{a}}{a}(1+\omega)\rho = -\frac{\dot{\Lambda}+8\pi\rho\dot{G}_N}{8\pi G_N},$$

a more general form of the new Friedmann eqns: qualitatively the same but details differ -- see arXiv:0907.4555,1103.4632,1202.5097....

Our estimate uses the more general form.

OUTLOOK

- Precision Quantum Field Theory: EW, QCD, QG $\equiv$ Control all limits:

  IR ($z \to 1$)

  and

  Collinear ($p_T \to 0$)

  UV limit

- We now have control over all aspects of the QG corrections.
- Toward quantitative understanding of ρ_Λ.