SSB in tensor theories and matrices

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Tensor models appear in the context of

- Entanglement.
- Quantum gravity description $d > 2$.
- SYK and holography. Recently SYK has been linked to tensor models [Witten'16]. Holography ($d > 2$?)

Definition of color TM

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Tensors with no additional symmetry assumed

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$$\Phi = \Phi_{i_1 i_2 \ldots i_d} \; e^{i_1} \otimes \cdots \otimes e^{i_d}, \quad e^{i_k} \in \mathbb{C}^{N_k}, \quad i_k = 1, \ldots, N_k.$$

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Under the action of $G_d \equiv U(N_1) \otimes \cdots \otimes U(N_d)$

$$\begin{aligned} \Phi_{j_1 j_2 \dots j_d} &= \sum_{i_1, \dots, i_d} (g_1)^{i_1}_{j_1} \cdots (g_d)^{i_d}_{j_d} \Phi_{i_1 \dots i_d} \\ \overline{\Phi}^{j_1 j_2 \dots j_d} &= \sum_{i_1, \dots, i_d} (\overline{g_1})^{j_1}_{i_1} \cdots (\overline{g_d})^{j_d}_{i_d} \overline{\Phi}^{i_1 \dots i_d}. \end{aligned}$$

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The action of the free theory $\quad S = \Phi_{i_1 i_2 \ldots i_d} \overline{\Phi}^{i_1 i_2 \ldots i_d}.$

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- $n=2,\ d=3$

$$\longrightarrow\{\Phi_{i_1^1 i_2^1 i_3^1}\Phi_{i_1^2 i_2^2 i_3^2}\overline{\Phi}^{i_1^1 i_2^1 i_3^1}\overline{\Phi}^{i_1^2 i_2^2 i_3^2},\Phi_{i_1^1 i_2^1 i_3^1}\Phi_{i_1^2 i_2^2 i_3^2}\overline{\Phi}^{i_1^1 i_2^1 i_3^2}\overline{\Phi}^{i_1^2 i_2^2 i_3^1},$$
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- $n=3 \longrightarrow 11$ invariants.

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- $n = 4 \longrightarrow 43$ invariants.

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By means of representation theory (and some work):

$$\dim\{\mathcal{O}_n^{G_d-\text{Inv}}\} = \sum_{\substack{|\mu_1|,\ldots,|\mu_d|=n \\ l(\mu_k)\leq N_k}} g^2_{\mu_1,\ldots,\mu_d}.$$

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$g_{\mu_1,\ldots,\mu_d}$ are the **Kronecker coefficients**. Branching coefficients in the restriction $U(N_1\cdot\dots\cdot N_d)\to U(N_1)\times\dots\times U(N_d)$.

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For multimatrix models we have

$$\dim\{\mathcal{O}_n^{U(N)-\mathrm{Inv}}\} = \sum_{\substack{\mu\vdash n \\ \nu_1\vdash n_i}} (c^{\mu}_{\nu_1,\ldots,\nu_d})^2, \quad n = n_1+\cdots+n_d$$

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$c^{\mu}_{\nu_1,\ldots,\nu_d}$ are **LR numbers**. Branching coefficients in the restriction

$$S_n \to S_{n_1}\times\cdots\times S_{n_d}$$

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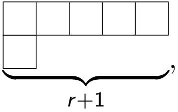

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$$g_{\mu(r)\nu\lambda} = \sum_{\substack{\gamma\vdash r\\ \rho\vdash n-r}} c^{\nu}_{\rho\gamma}\, c^{\lambda}_{\rho\gamma'}, \qquad \mu(r) = \underbrace{\ydiagram{5,1}}_{r+1},$$

At least in the hook sector and for $d=3$ tensor models there is a non-trivial relation between the spectra of matrix and tensor models.

Effective (multi-)matrix theories from tensor theories

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$$\int d\Phi d\overline{\Phi} \exp\left(i \int d^4x \{\mathcal{L}(\Phi(x))\} \right)$$
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- $\Phi^s(x)\overline{\Phi^s}(x) = \frac{1}{d!}\sum_{\sigma\in S_d} \Phi_{i_{\sigma(1)},\dots,i_{\sigma(d)}}(x)\overline{\Phi}^{i_1,\dots,i_d}(x)$

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$$\Phi^s_{j_1 j_2 \dots j_d} = \frac{1}{d!} \sum_{\substack{i_1,\dots,i_d \\ \sigma \in S_n}} g^{i_1}_{j_1} \cdots g^{i_d}_{j_d} \Phi_{i_{\sigma(1)} \dots i_{\sigma(d)}}, \quad g \in U(N)$$

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$$B_a(x) = i\big(\overline{\Phi^s}(x)T_a\Phi^s(x) - \Phi^s(x)T_a^k\overline{\Phi^s}(x)\big), \quad a = 1,\dots,N^2.$$

So they group into $d-1$ multiplets of N^2 elements each.

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So they group into $d-1$ multiplets of N^2 elements each.
- The multiplets organize into matrices transfroming in the adjoint:

$$Z^i_j(x) = \sum_a B_a(x)(T_a)^i_j.$$

Summary of SSB and applications

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$$\Downarrow$$

$$\mathrm{Diag}[U(N)] \longrightarrow \Phi^{s}_{i_1,\ldots,i_d}(x) + \{(Z_1)^i_j(x), \ldots, (Z_{d-1})^i_j(x)\} + \mathrm{Diag}[U(N)] - \text{singlets}$$

(non-solvable, holographic interpretation)

Thanks!