The CP & Random Fluctuation Walk

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CP & Critical Phenomena

$$\underbrace{\textit{Collider} \qquad\qquad \textbf{\textit{Fixed target}}}_{\textit{strong interacting matter @ high } T \textit{ \& } \mu_B}$$

In the proximity of *CP*:

- **Matter becomes weakly coupled**
- **Color is no more confined**
- **Chiral symmetry is restored**

➢ **Important:**
Phase transition is associated with breaking of symmetry

Very often: ***CP*** **clarified through** $(\mu_B - T)$ **plane**

scanning of $(\mu_B - T)$ **phase diagram**

CP & Critical Phenomena

A few questions do arise:

- ***CP* meaning?**
- **Basic observables to be measured when *CP* approached?**
- **New knowledge if *CP* approached?**

Answer: in terms of QCD_T @ large distances

N/Perturbative phenomena: χSB & Confinement of color

$\downarrow$ *?relations?* $\downarrow$

Phase transition of χS Restoration Deconfinement

$\downarrow$ **correlations** $\downarrow$

important issue

NO correct solution (massless quarks in the theory)

Effective models, e.g., with topological defects

CP **• Fluctuation measure • Observables**

may be visible $\downarrow$ **through**

Fluctuations of characteristic length ξ of a "critical" mode

Model: **Gauge field is the "critical" mode with mass m**

Excitations above vacuum: narrow flux tubes, $r_s \sim \xi \sim m^{-1}$

(in the center, $r_s \to 0$, scalar condensate vanishes)

Ensemble of a single flux tube system, $N(R)$ configurations of f.t.'s

Partition function $Z_{flux} = \sum_{\beta}\sum_{R} N(R)\exp\left[-\beta\ E(m,R)\right] D\left(|\vec{x}|,\beta;M\right)$

Effective energy: $E(m,R) \sim m^2 R\left[a + b\ln\left(\tilde{\mu}R\right)\right]$ ***GK, 2010***

- *Particles:* Bound states in terms of flux tubes

Two Point Correlation Functions (TPCF)

Observables: At large distances for any correlator

$$\left\langle O(\tau,\vec{x})O(\tau,0)\right\rangle \sim A|\vec{x}|^{c} D\left(|\vec{x}|,\beta;M\right) \text{ as } |\vec{x}|\to\infty$$

$$D\left(|\vec{x}|,\beta;M\right)=\exp\left[-M(\beta)|\vec{x}|\right],\ \ D\left(|\vec{x}|,\beta;M\right)\neq 0 \text{ even at } \beta=\beta_c$$

$M^{-1}(\beta)$ is the measure of screening effect of color electric field

$$\left\langle O(\tau,\vec{x})O(\tau,0)\right\rangle \sim L_W^{-4}-\frac{T}{V}\sigma_0(\beta)\xi^2\left[\frac{1}{\xi}\sqrt{\frac{8\pi}{\sigma_0}}-N\ln\left(\xi\sqrt{2\pi\sigma_0}\right)+\dots\right]!$$ ***GK, 2014***

- ***Small*** $|\vec{x}|$***, - singular behavior.***
- ***Large fluctuations*** ξ***, - TPCF's disappear (CP does approach)***

$SU(3)$ ***gluodynamics*** **vacuum**

$$k_{GL} = \frac{\xi}{l} \sim \frac{m_\phi}{m} \quad \begin{matrix} <1 & (\textit{type I vacuum}, \text{ flux tubes } \textit{attracted}) \\ >1 & (\textit{type II vacuum}, \text{ flux tubes } \textit{repel}) \end{matrix}$$

Scalar fields, dilatons ϕ **(condensate)** remain massive up to the ***CP*** (1^{st} order PT)

- $k_{GL} \to \infty$ **Deconfinement!**
- **If** $k_{GL} = 1$ **parallel strings (carry the same flux) do not interact each other.** $T_c \approx 172\ MeV$, $N_c = 3\ pions$ GK 2014

Singularity of $Z_{flux} \Rightarrow$ $$k_{GL} \geq \frac{3}{4}\frac{\alpha(\beta)}{\xi m_{q\bar{q}}}\left[1 + \frac{4}{3}\underbrace{\frac{\xi^2}{\alpha(\beta)\beta}}_{\text{Type-II vac}} M(\beta)\frac{L_W}{R}\right]$$ ***low bound***

$\downarrow$ **Type-I vac** $\quad$ $\downarrow$ **Type-II vac** $\quad \to \infty \quad as \quad \xi \to \infty$

To experiment: Observation of correlations between two bound states (strings) is rather useful & instructive to check the ***CP*** is approached!

Particle Correlations: *Size of the particle source*

- Possible approach to ***CP*** study through spatial correlations of final state particles */Bose statistics/*
- Size effect of space composed of "hot" particles $\Rightarrow$ derive theoretical formulas for $2-, \ldots, n-$ particle ***distribution-correlation functions*** (stochastic, chaotic behavior)

✓ *Stochastic scale (size)* ***L*** *in C's Bose-Einstein* **GK (2008-2010)**

$$C_2(q,\lambda) \approx \eta(n)\left[1+\lambda(\nu)e^{-q^2 L_{st}^2}\right], \quad \eta(n) = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}$$

event-to-event fluctuations

Particle coherence function:

$$\lambda(\nu) = 1/(1+\nu)^2, \quad 0<\nu<\infty, \quad n \sim V\int d\omega\, m^2 \frac{1}{e^{(\omega-\mu)\beta}-1}$$

OBSERVABLES?

BEC –consequence of hadronization due to phase transition from free **q, g** *to hadrons* (cross-over)

The scaling form C_2 is ***important*** to predict behavior of observables @ **CP**

$L_{st} \to \infty \; as \; T \to T_c, \; \mu \to \mu_c$ indicate the vicinity of **CP**

Observable, e.g., $k_T^2 = \dfrac{1}{\nu(n)\; T^3\; L_{st}^5}, \quad k_T = \left| \vec{p}_{T_1} + \vec{p}_{T_2} \right|$

Exp. confirmed (size decreasing with k): CMS (2010-11), ALICE (2011), ATLAS (2015)

Coherence λ ***measured***. ***Theoretical study is important!***

$$0 \; < \lambda\left[\nu(n)\right] \le \; 1$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

fully coherent phase chaotic *(critical behavior: **BM** to **AM**,* **cross-over walk**)

Random Fluctuation Walk *(cross-over **BM** $\Rightarrow$ **AM**)*

Model 1D x-oriented $(0 < x < \infty)$

$$P\left(x;\bar{\lambda},\mu_c\right) = p\left(\bar{\lambda}\right)\sum_{j=0}^{\infty}\bar{\lambda}^j \frac{1}{2\sqrt{\pi t}}\left[e^{-y_-^{2j}/4t} + e^{-y_+^{2j}/4t}\right]$$

$$y_\pm^j = x\mu_c \pm a^j,\ a = \left(\mu/\mu_c\right) > 1,\ \ t = l\mu_c \text{ (lattice spacing)}$$

$$\underbrace{lim}_{\substack{l\to 0\\ \mu_c\neq 0}} P\left(x;\bar{\lambda},\mu_c\right) = p\left(\bar{\lambda}\right)\sum_{j=0}^{\infty}\bar{\lambda}^j\left[\delta\left(x\mu_c - a^j\right) + \delta\left(x\mu_c + a^j\right)\right]$$

$p\left(\bar{\lambda}\right) = 0.5\left(1-\bar{\lambda}\right),\ \ 0 < \bar{\lambda} \le 1,$ NC: $2\left(p + \bar{\lambda}p + \dots + \bar{\lambda}^j p + \dots\right) = 1$

The limit $\bar{\lambda} \to 1 \Rightarrow$ broad behavior of P: *vicinity of **CP** is approached*

$\bar{\lambda} \to 0,\ P\left(x;\bar{\lambda} \to 0\right) \to 1/2$. Trivial

Random fluctuations & vacuum

Vicinity of **CP**: ***theory conformal***, **scalar dilaton field** ϕ

$$\bar{\lambda} \rightarrow \lambda(\nu) = \left[1 + \nu(n)\right]^{-1}$$ CBE

$$\nu(n) \approx \frac{1}{n\ k_{GL}^2} O\left(\frac{m_\phi^2}{m^2}\right)$$ ***stochastic influence strength***

Dual superconductor QCD vacuum *GK 2014*

$$k_{GL} \sim \frac{m_\phi}{m_B} \begin{cases} <1 \quad vacuum \quad type-I,\ two\ flux\ tubes\ attracted \\ >1 \quad vacuum \quad type-II,\ two\ flux\ tubes\ repel \end{cases}$$

CP: $k_{GL} \rightarrow \infty$ as $\xi \sim m_B^{-1} \rightarrow \infty$ fluctuation length

$\downarrow$

- penetration depth of color-electric field/radius of the flux tube

Large distances (from ***BM*** to ***AM***)

To smooth the particularity of $P(x;\lambda,\mu_c)$

$$P(x;\lambda,\mu_c) \to G(k;\lambda,\mu_c) = 2p(\lambda)\sum_{j=0}^{\infty}\lambda^j \cos\left(\frac{k}{\mu_c}a^j\right),\ G(0;\lambda)=1$$

Asymptotic behavior (sharp increasing of L_{st}) / or $k \to 0$ (IR)

Fluctuation length through the even moments of the order $2s$:

$$\xi^2_{(2s)}(\lambda) \sim m_{(2s)}(\lambda) = \frac{\partial^{2s} G(k;\lambda,\mu_c)}{\partial k^{2s}}\Big|_{k=0}$$

QCD vacuum will influence ξ up to cross-over:

unified process of phase transition between *BM* and *AM*

BM & ***AM*** mixed phases

- Infinite # of divergent (singular) terms in $G(k;\lambda,\mu_c)$
- Not suitable to describe an arbitrary phase of excited matter
  Why?

Because wide range of λ, μ; singularity @ $k \ll \mu_c \quad (k \to 0)$

- To find non-analytical part @ $k \approx 0$ *(large distances)*

$G(k;\lambda,\mu_c) = G_{BM}(k;\lambda,\mu_c) + G_{AM}(ak;\lambda,\mu_c)$ linear non-homog. eq.

BM $\quad G_{BM}(k;\lambda,\mu_c) = 2p(\lambda)\boldsymbol{cos}(k/\mu_c)$ regular if $k \approx 0$, for all λ

AM $\quad G_{AM}(k;\lambda,\mu_c) = \lambda G(ak;\lambda,\mu_c)$, for $a^{-2} < \lambda < 1 \qquad a = \frac{\mu}{\mu_c} > 1$

$G_{AM}(k;\lambda,\mu_c) \to 0,\ if\ \lambda \to 0$ The phase with ***BM*** does exist only

Fluctuation length

ξ : analytical and non-analytical behavior at all possible λ & μ

BM: $$G_{BM}\left[k;\lambda(S),\mu_c\right]=1+\sum_{s=1}^{S}(-1)^s\frac{1}{(2s)!}\xi_{(2s)}^2\left[\lambda(S)\right]k^{2s}$$

analytical (regular) @ $0<\lambda<1$ *with finite* S

AM: $$G_{AM}\left[k;\lambda(S),\mu_c\right]=1+\sum_{s=S}^{\infty}(i)^{2s}\frac{1}{(2s)!}\xi_{(2s)}^2\left[\lambda(S)\right]k^{2s}$$

non- analytical (singular) for $\xi_{(2s)}$ *@* $s\geq S$

infinite series $\Downarrow$ replaced by ***k*-power** form

$$G_{AM}\left[k;\lambda(S),\mu_c\right]=C\left[\lambda(S)\right]\left|\frac{k}{\mu_c}\right|^{\alpha[\lambda(S)]}Q(|k|)$$

Solution for AM phase

$$G_{AM}\left(k;\lambda,\mu_c\right)\sim\frac{i^{\alpha-2}}{\Gamma\left(\alpha-1\right)}\xi^2_{(\alpha-2)}\left(\lambda\right)\left|\frac{k}{\mu_c}\right|^{\alpha}\frac{1}{\log a}\sum_{m=0}^{\infty}b_m\cos\left[2\pi m\frac{\log\left(\left|k\right|\right)}{\log a}+\vartheta_m\right]$$

Divergent if a) $\lambda\to 1$ because of $\xi\left(\lambda\right)$

b) $a=\frac{\mu}{\mu_c}\to 1$ from above

The **AM** disappears if $\lambda\to 0$

Conclusions

1. Theoretical search for **CP,** cross-over between **BM** & **AM**
   Random stochastic (chaotic) walk with respect to quantum correlations of identical particles
2. Main points: ***vacuum*** k_{GL}, $\lambda(\nu)$, $m_{(2s)}(\lambda) \sim \xi^2_{(2s)}(\lambda)$, $a = (\mu / \mu_c) > 1$.
3. Solution: ***mixed phase*** **BM** (regular) + **AM** (singular).
4. Smooth $\lambda(\nu) \Rightarrow$ **BM**

   $\lambda(\nu) \to 1$ (strong particle chaoticity) $\Rightarrow$ **AM**

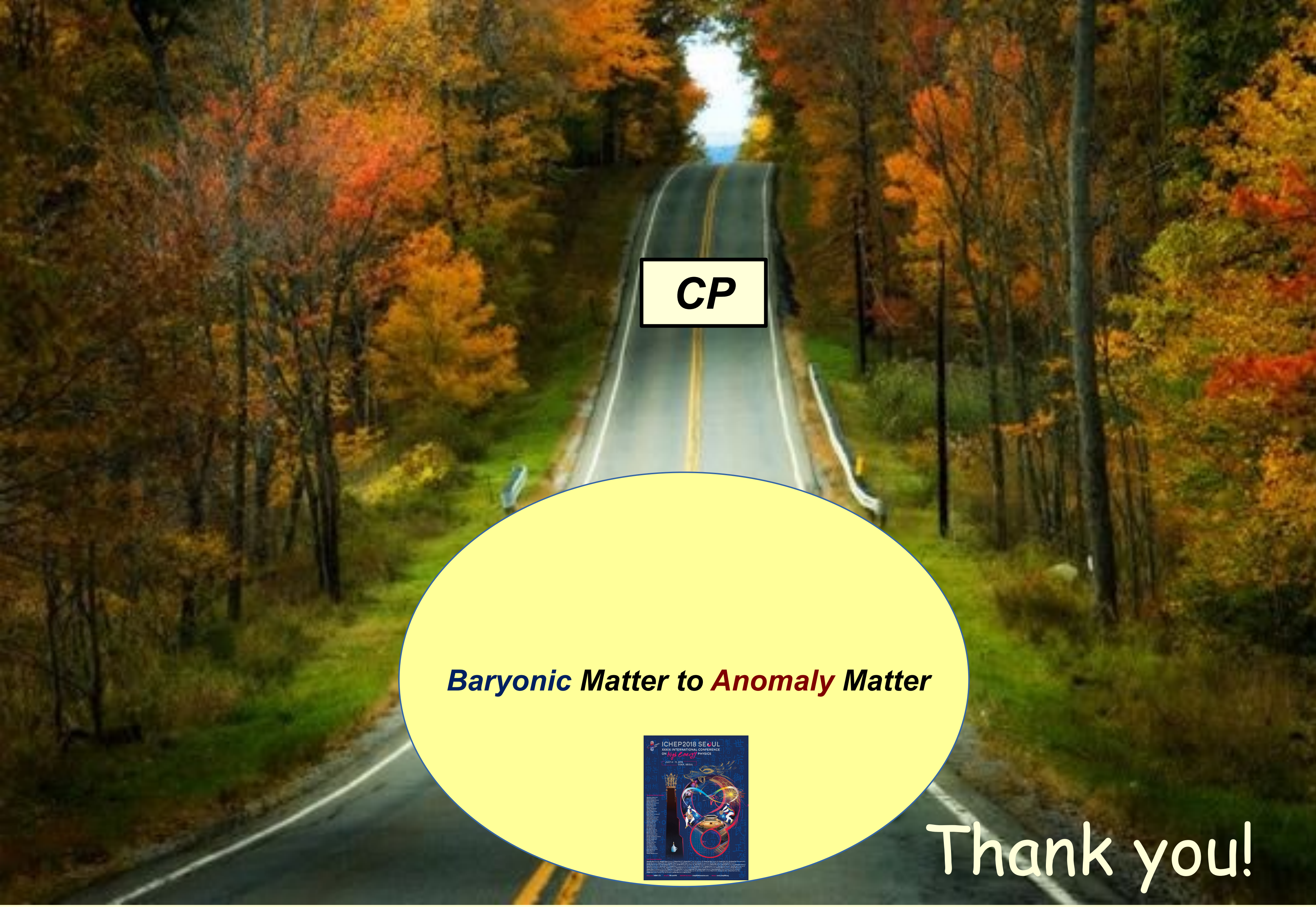
   asymp. behavior at $k \to 0$ (**IR**), large x is approached

   $\Downarrow$

   infinite size of particle source.

5. Close to ***CP*** (i.e. **AM):** $\mu_c < \mu / \left[1 + \frac{1}{n k_{GL}^2} O\left(\frac{m_\varphi^2}{m^2}\right) \right]$

CP (cross-over) $k_{GL} \to \infty$ at $\mu = \mu_c$. No dependence on particle n.



CP

***Baryonic* Matter to *Anomaly* Matter**

Thank you!