

The generation of B-mode and circular polarization of cosmic photons due to NonCommutative space-Time background

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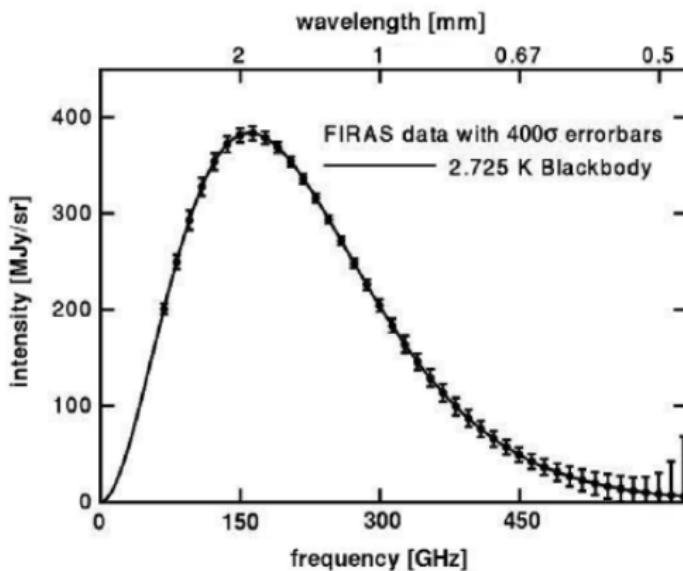
Based on: arXiv:1605.09045

With S.Batebi, M. Haghigat, R.Mohammadi

ICHEP 2018, Seoul
July 05, 2018

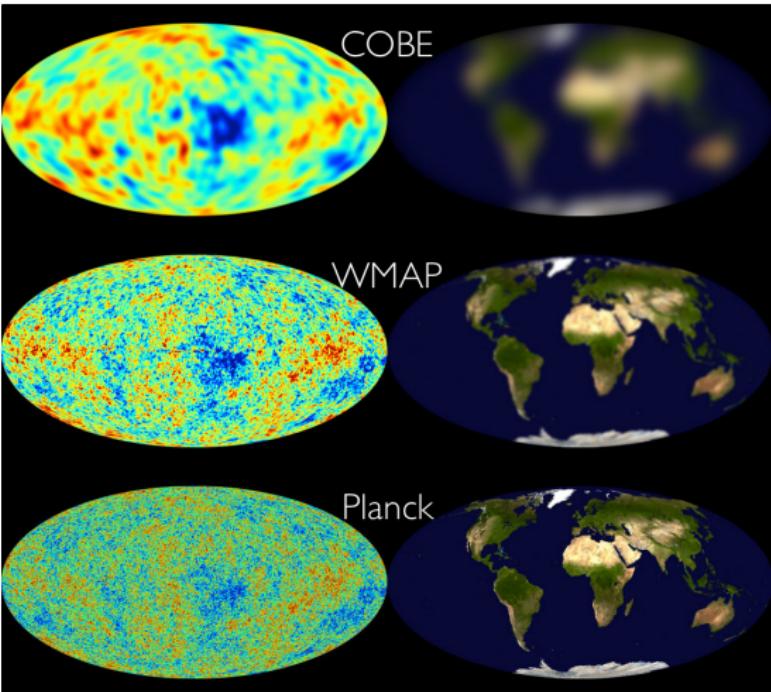
Origin of CMB

- universe initially in a hot dense state. cools as expands.
- Photons and baryons decouple approximately 380,000 years after Big Bang.
- Photon background visible as the Cosmic Microwave Background (CMB)
- Temperature is isotropic to few part.



Absolute temperature of CMB = 2.7255 K

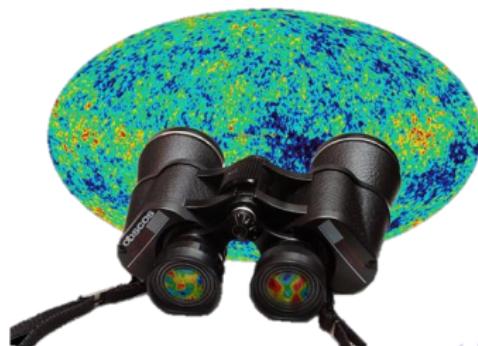
- **COBE**: Cosmic Background Explorer [1989-1993]
Measured absolute temperature
First full sky spectrum of anisotropies.
- **WMAP** Wilkinson Microwave Anisotropy Probe [2001-2010]
Only anisotropies, not temperature.
- **Planck** [2009-2013] only anisotropies, not temperature.



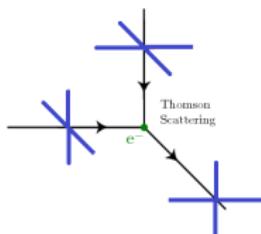
Temperature anisotropy

Quantum fluctuations in field of inflation:

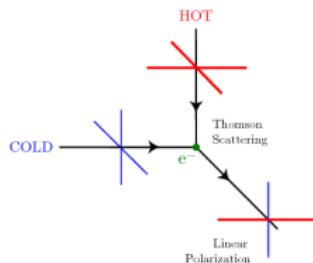
- Scalar perturbation:
spatial variations in the baryon density (dominant effect, seeds of structure we see today)
- Tensor perturbation:
Primordial gravitational waves (produced by an epoch of inflation)



CMB Polarization

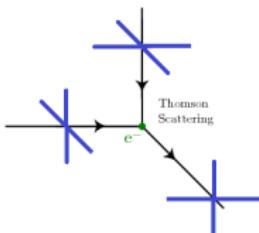


The outgoing light is unpolarized

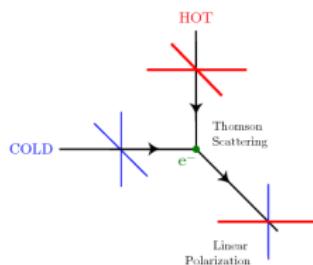


The outgoing light is polarized

CMB Polarization



The outgoing light is unpolarized



The outgoing light is polarized

Polarization can arise from:

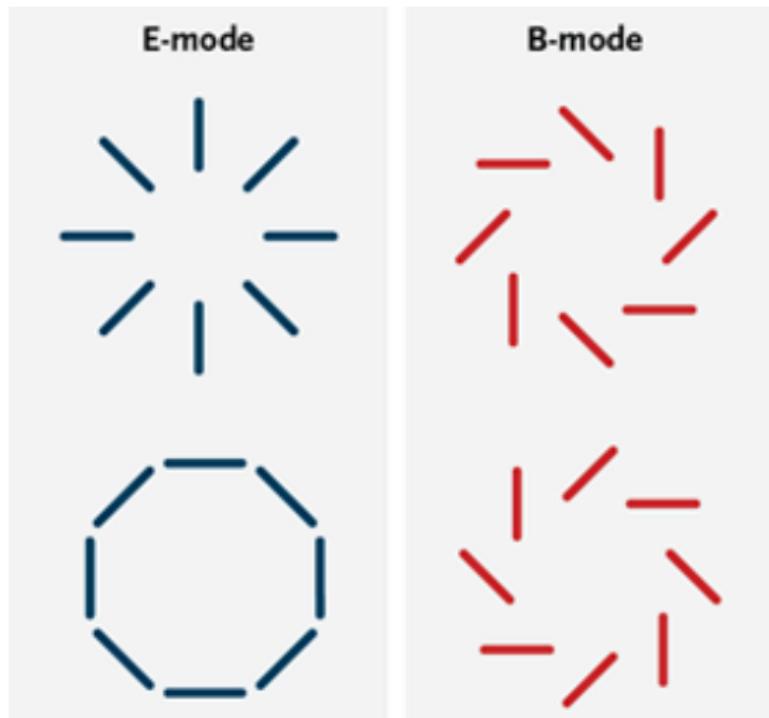
- Gravitational wave
- Density perturbation

How we can specify which perturbation cause polarization?

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E-mode**B-mode**

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Density perturbation can produce E-mode.

Gravitational wave can produce E-mode and B-mode.

Stokes parameters

Polarization of photon can be described with Stokes parameters:

$$Q, U, V, I$$

Q and U with a rotation's angle ϕ transform to:

$$(Q \pm iU)' = e^{\mp 2i\phi} (Q \pm iU)$$

$p = \sqrt{Q^2 + U^2}$, V, I are frame independent.

A system of photons can be described by the density operator

$$\rho = \frac{1}{2} \begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix} = \frac{1}{2}(I + Q\sigma_3 + U\sigma_1 + V\sigma_2)$$

Compton scattering in standard model

Boltzmann Equation

$$(2\pi)^3 \delta^3(0) (2k^0) \frac{d}{dt} \rho_{ij}(\mathbf{k}) = i \langle [H_{int}(t), D_{ij}(\mathbf{k})] \rangle - \frac{1}{2} \int dt \langle [H_{int}(t), [H_{int}(0), D_{ij}(\mathbf{k})]] \rangle$$

$$i \langle [H_{int}^0(0), D_{ij}^0(\mathbf{k})] \rangle = \int d\mathbf{q} n_e(q) (\delta_{is} \rho_{s'j}(k) - \delta_{js'} \rho_{is}(k)) (\mathcal{M})$$

Kosowsky, Ann.Phys.246 : 49 – 85, (1996)

$$\begin{cases} i \langle [H_{int}^0(0), D_{ij}^0(\mathbf{k})] \rangle = 0 \\ -\frac{1}{2} \int dt \langle [H_{int}(t), [H_{int}(0), D_{ij}(\mathbf{k})]] \rangle \end{cases}$$

Boltzmann equation with scalar perturbation

$$\frac{\partial}{\partial \tau} \Delta_I^{(S)} + \frac{1}{a} i K \mu \Delta_I^{(S)} + 4 \left[\frac{\partial \psi}{\partial \tau} - \frac{1}{a} i K \mu \varphi \right] = \frac{1}{a} \dot{\tau}_{e\gamma} \left[-\Delta_I^{(S)} + \Delta_I^{0(S)} - 4 \mu v_b + \frac{1}{2} P_2(\mu) \Pi \right]$$

$$\frac{\partial}{\partial \tau} \Delta_P^{\pm(S)} + \frac{1}{a} i K \mu \Delta_P^{\pm(S)} = \frac{1}{a} \dot{\tau}_{e\gamma} \left[-\Delta_I^{(S)} + \Delta_I^{0(S)} - 4 \mu v_b + \frac{1}{2} P_2(\mu) \Pi \right]$$

$$\frac{\partial}{\partial \tau} \Delta_V^{(S)} + \frac{1}{a} i K \mu \Delta_V^{(S)} = \frac{1}{a} \dot{\tau}_{e\gamma} \left[-\Delta_V^{(S)} + \frac{3}{2} \mu \Delta_V^{1(S)} \right]$$

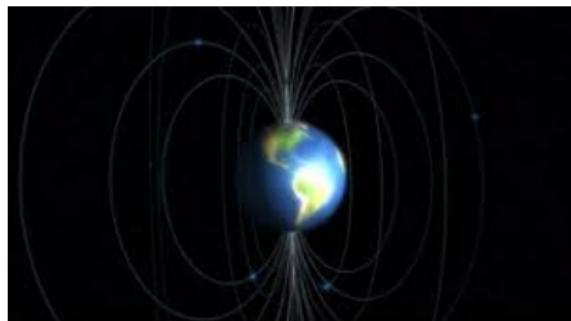
$$\dot{\tau}_{e\gamma} = a n_e \sigma_T, \quad \sigma_T = \frac{8\pi}{3} (\alpha/m)^2$$

$$\Delta_P^\pm(\mathbf{k}) = Q(\mathbf{k}) \pm i U(\mathbf{k})$$

$$\Pi \equiv \Delta_I^{2(S)} + \Delta_P^{2(S)} + \Delta_P^{0(S)}$$

Background fields

- Background fields:
 - Trivial background: Electric and Magnetic field
 - non- trivial background: Non-Commutative space time



Motivations

- Noncommutative relation

$$x^\mu \rightarrow \hat{x}^\mu : [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \neq 0$$

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Extension of Heisenberg algebra with

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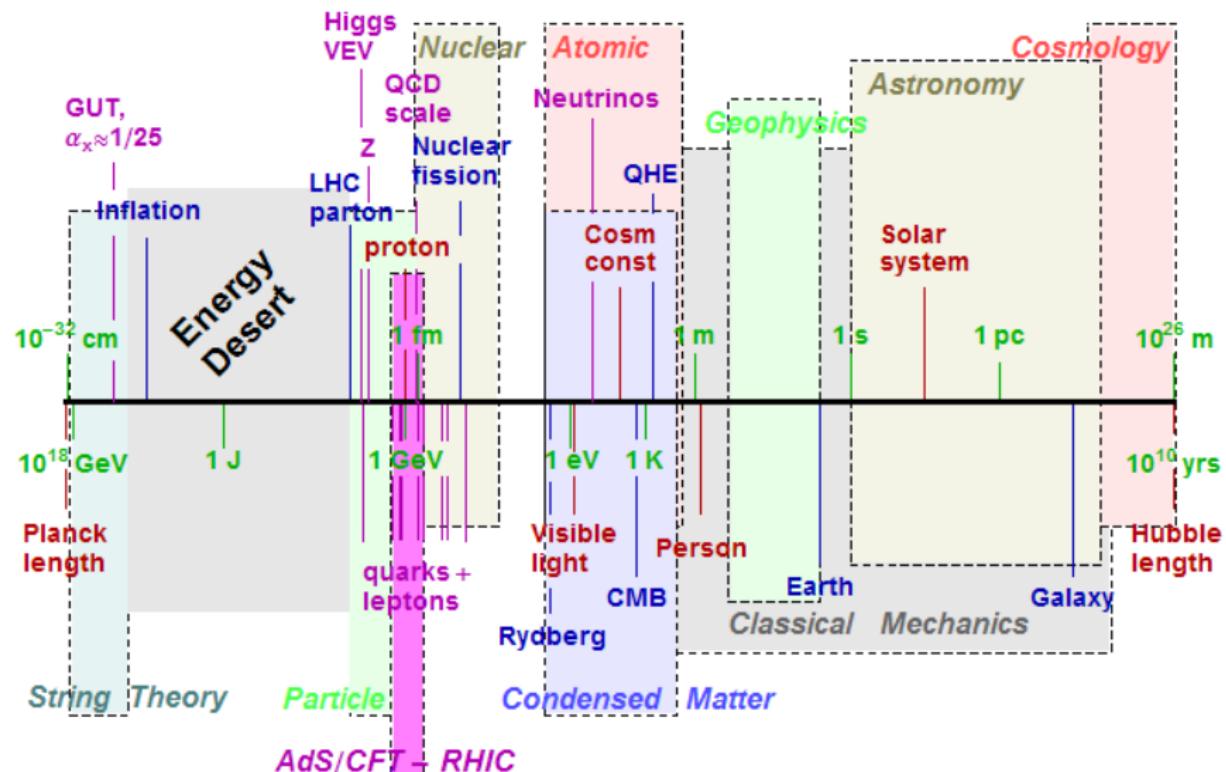
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Implying existence of a minimal length in nature $\sqrt{\theta}$

- To find a natural cut-off for the divergences which plagued quantum electrodynamics (QED).

Snyder, *Phys Rev*, 71(1) : 38(1947); Heisenberg(1954)



<https://www.Princeton.edu/physics/research/high-energy-theory/gubser-group/outreach/energy-scales-in-physics/>

Non-commutative Standard model

Seiberg- Witten approach

- Gauge group

$$SU(3) \star SU(2) \star U(1)$$

- The number of Non-Commutative Fields are the same as commutative fields.
- Fields are expanded in order of commutative fields.

X.Calmet, et al., Eur.Phys.J.C23 : 363 – 376, (2002)

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \equiv i\frac{1}{\Lambda_{\text{NC}}^2}$$

$$(f \star g)(x) = f(x) e^{\frac{i}{2} \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu} g(x) = f(x)g(x) + \frac{i}{2} \theta^{\mu\nu} \frac{\partial f(x)}{\partial x^\mu} \frac{\partial g(x)}{\partial x^\nu} + \mathcal{O}(\theta^2)$$

Seiberg-Witten map

$$\begin{aligned}
 \widehat{\psi} &= \psi - \frac{1}{2} \theta^{\alpha\beta} V_\alpha \partial_\beta \psi + \frac{i}{8} \theta^{\alpha\beta} [V_\alpha, V_\beta] \psi + \mathcal{O}(\theta^2), \\
 \widehat{V}_\mu &= V_\mu + \frac{1}{4} \theta^{\alpha\beta} \{ \partial_\alpha V_\mu + F_{\alpha\mu}, V_\beta \} + \mathcal{O}(\theta^2), \\
 \widehat{\Phi} &= \Phi + \frac{1}{2} \theta^{\alpha\beta} V_\beta \left(\partial_\alpha \Phi - \frac{i}{2} (V_\alpha \Phi - \Phi V'_\alpha) \right) \\
 &\quad + \frac{1}{2} \theta^{\alpha\beta} \left(\partial_\alpha \Phi - \frac{i}{2} (V_\alpha \Phi - \Phi V'_\alpha) \right) V'_\beta + \mathcal{O}(\theta^2),
 \end{aligned}$$

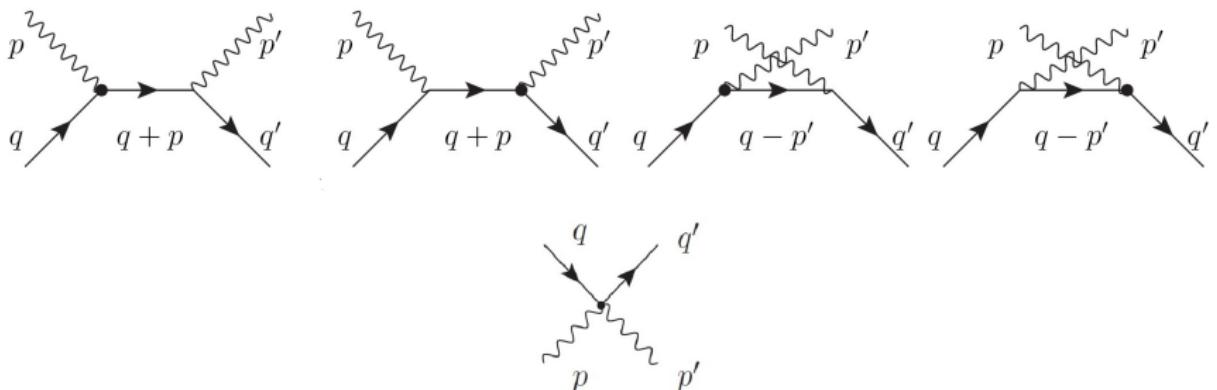
$$S_{\text{NCSM}} = S_{\text{fermions}} + S_{\text{gauge}} + S_{\text{Higgs}} + S_{\text{Yukawa}}$$

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$$\frac{-e^2 Q_f^2}{2} \theta_{\mu\nu\rho} (k_1^\rho - k_2^\rho),$$

$$i e Q_f \gamma_\mu + \frac{1}{2} e Q_f [(\not{p}_{\text{out}} \theta \not{p}_{\text{in}}) \gamma_\mu - (\not{p}_{\text{out}} \theta)_\mu (\not{p}_{\text{in}} - m_f) - (\not{p}_{\text{out}} - m_f)(\theta \not{p}_{\text{in}})_\mu]$$

" B.Melic, et al., Eur.Phys.J.C42 : 483 – 497, (2005)



$$i\mathcal{M}^\theta = i\mathcal{M}_1^\theta + i\mathcal{M}_2^\theta + i\mathcal{M}_3^\theta + i\mathcal{M}_4^\theta + i\mathcal{M}_5^\theta$$

$$\begin{aligned}
 &= \frac{e^2 Q_f^2}{4p \cdot q} \bar{u}_f(q') \left[\left(\epsilon_s(\hat{p}) \dot{q} \cdot \theta \cdot (q + p) - \dot{q} \cdot \theta \cdot \epsilon_s(\hat{p}) \dot{p} \right) (\not{q} + \not{p} + m_f) \epsilon_s(p) \right. \\
 &\quad \left. + \epsilon_s(\hat{p}) (\not{q} + \not{p} + m_f) \left(\epsilon_s(p) (q + p) \cdot \theta \cdot q - q \cdot \theta \cdot \epsilon_s(p) \not{p} \right) \right] u_r(q) \\
 &- \frac{e^2 Q_f^2}{4\dot{p} \cdot q} \bar{u}_f(\dot{q}) \left[\left(\epsilon_s(p) \dot{q} \cdot \theta \cdot (q - \hat{p}) + \dot{q} \cdot \theta \cdot \epsilon_s(p) \not{p} \right) (\not{q} - \not{p} + m) \epsilon_s(\hat{p}) \right. \\
 &\quad \left. + \epsilon_s(p) (\not{q} - \not{p} + m_f) \left(\epsilon_s(\hat{p}) (q - \hat{p}) \cdot \theta \cdot q + q \cdot \theta \cdot \epsilon_s(\hat{p}) \not{p} \right) \right] u_r(q) \\
 &- \frac{e^2 Q_f^2}{2} \bar{u}_f(q') \epsilon_s^\mu(p') (p + p')^\rho \theta_{\mu\nu\rho} \epsilon_s^\nu(p) u_r(q)
 \end{aligned}$$

Boltzmann equation in NC space-time

$$\frac{d}{d\tau} \Delta_I^{(S)} + iK\mu \Delta_I^{(S)} + 4[\dot{\psi} - iK\mu\varphi] = C_{e\gamma}^I$$

$$\frac{d}{d\tau} \Delta_P^{\pm(S)} + iK\mu \Delta_P^{\pm(S)} = C_{e\gamma}^{\pm} - iv_b \kappa_{NC}^{\pm} \Delta_V^{(S)}$$

$$\frac{d}{d\tau} \Delta_V^{(S)} + iK\mu \Delta_V^{(S)} = C_{e\gamma}^V + i \frac{v_b}{2} \left[\kappa_{NC}^{-} \Delta_P^{+(S)} + \kappa_{NC}^{+} \Delta_P^{-(S)} \right]$$

$$\kappa_{NC}^{\pm} = a(\tau) \frac{3}{4} \frac{\sigma^T}{\alpha} \frac{m_e^2}{\Lambda_{NC}^2} \bar{n}_e \sum_{f=e,p} \frac{m_f}{k^0} (C \pm iD)$$

$$C = -\hat{w}^i \left(\epsilon_{1i} \cdot v_f \cdot \epsilon_2 + \epsilon_{2i} \cdot v_f \cdot \epsilon_1 \right) \quad D = \hat{w}^i \left(\epsilon_{1i} \cdot v_f \cdot \epsilon_1 - \epsilon_{2i} \cdot v_f \cdot \epsilon_2 \right)$$

$$\kappa_{NC}^{\pm} / (\dot{\tau}_{e\gamma}) \propto \frac{3}{4} \frac{m_f}{\alpha} \left(\frac{10^{-4} eV}{k^0} \right) \left(\frac{m_e}{\Lambda_{NC}} \right)^2 \sim 0.1 \left(\frac{TeV}{\Lambda_{NC}} \right)^2$$

General solution

$$\Delta I(\hat{\mathbf{n}}) = \sum_{lm} a_{I,lm} Y_{lm}(\hat{\mathbf{n}}) \quad \Delta V(\hat{\mathbf{n}}) = \sum_{lm} a_{V,lm} Y_{lm}(\hat{\mathbf{n}})$$

$$\Delta_P^{\pm'(S)}(\hat{\mathbf{n}}) = e^{\mp 2i\phi} \Delta_P^{\pm(S)}(\hat{\mathbf{n}})$$

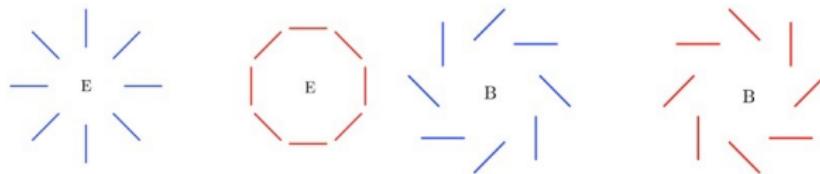
$$\Delta_P^{\pm(S)}(\hat{\mathbf{n}}) = \sum_{lm} a_{\pm 2,lm} {}_{\pm 2} Y_{lm}(\hat{\mathbf{n}})$$

E-mode and B-mode

$$\bar{\partial}^2 \Delta_P^{+(S)}(\hat{n}) = \sum_{lm} \left[\frac{(l+2)!}{(l-2)!} \right]^{1/2} a_{2,lm} Y_{lm}(\hat{n}), \quad \bar{\partial}^2 \Delta_P^{-(S)}(\hat{n}) = \sum_{lm} \left[\frac{(l+2)!}{(l-2)!} \right]^{1/2} a_{-2,lm} Y_{lm}(\hat{n})$$

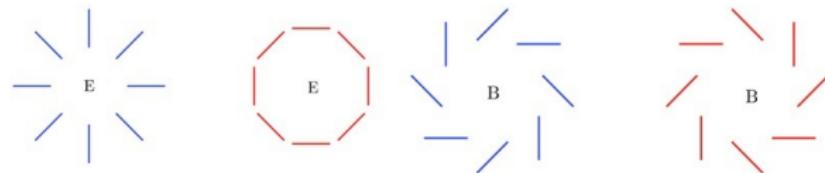
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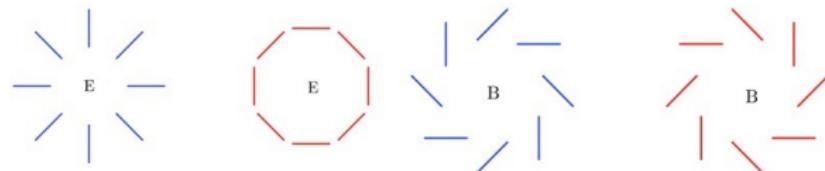


$$\Delta_E^{(S)}(\hat{n}) \equiv -\frac{1}{2} [\bar{\partial}^2 \Delta_P^{+(S)}(\hat{n}) + \partial^2 \Delta_P^{-(S)}(\hat{n})] = \sum_{lm} \left[\frac{(l+2)!}{(l-2)!} \right]^{1/2} a_{E,lm} Y_{lm}(\hat{n})$$

$$\Delta_B^{(S)}(\hat{n}) \equiv \frac{i}{2} [\bar{\partial}^2 \Delta_P^{+(S)}(\hat{n}) - \partial^2 \Delta_P^{-(S)}(\hat{n})] = \sum_{lm} \left[\frac{(l+2)!}{(l-2)!} \right]^{1/2} a_{B,lm} Y_{lm}(\hat{n})$$

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$$a_{E,lm} = -(a_{2,lm} + a_{-2,lm})/2 \quad a_{B,lm} = i(a_{2,lm} - a_{-2,lm})/2$$

Power Spectrum

$$C_{EI} = \frac{1}{2l+1} \sum_m \langle a_{E,l'm'}^* a_{E,lm} \rangle , \quad C_{BI} = \frac{1}{2l+1} \sum_m \langle a_{B,l'm'}^* a_{B,lm} \rangle$$
$$C_{VI} = \frac{1}{2l+1} \sum_m \langle a_{V,l'm'}^* a_{V,lm} \rangle$$

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$$C_{VI} \approx \frac{1}{2l+1} \int d^3\mathbf{K} P_V(\mathbf{K}) \sum_m \left| \int d\Omega Y_{lm}^* \int_0^{\tau_0} d\tau \dot{\tau}_{e\gamma} e^{ix\mu - \tau_{e\gamma}} C \tilde{\kappa} \Delta_P^{(S)} \right|^2$$

$$\approx \tilde{\kappa}^2 C_{PI}$$

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experimental value: $C_{PI} \equiv 0.1 \mu K^2$

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$$\tilde{\kappa}_{min} < \tilde{\kappa} < \tilde{\kappa}_{max}$$

$$\tilde{\kappa}_{max} = \frac{3}{4} \frac{m_e + m_p}{T^0} \frac{1}{\alpha} \frac{m_e^2}{\Lambda^2} \simeq 3 \times 10^{-1} (20 \text{ TeV}/\Lambda)^2$$

$$\tilde{\kappa}_{min} = \frac{3}{4} \frac{m_e + m_p}{T^{iss}} \frac{1}{\alpha} \frac{m_e^2}{\Lambda^2} \simeq 3 \times 10^{-4} (20 \text{ TeV}/\Lambda)^2$$

Constrain on power spectrum

C_{VI} power spectrum

$$\tilde{\kappa}_{min}^2 C_{PI} \leq C_{VI} \leq \tilde{\kappa}_{max}^2 C_{PI}$$

$$\left\{ \begin{array}{l} 10^{-3} \mu K^2 \leq C_{VI} \leq 10^3 \mu K^2 \\ 0.01 n K^2 \leq C_{VI} \leq 0.01 \mu K^2 \end{array} \right. \quad \left. \begin{array}{l} \Lambda_{NC} \sim 1 \text{ TeV} \\ \Lambda_{NC} \sim 20 \text{ TeV} \end{array} \right\}$$

C_{BI} power spectrum

$$C_{BI}^S \propto \tilde{\kappa}^2 C_{VI}$$

$$\tilde{\kappa}_{min}^4 C_{PI} \leq C_{BI}^S \leq \tilde{\kappa}_{max}^4 C_{PI}$$

$$\left\{ \begin{array}{l} 10 n K^2 \leq C_{BI}^S \leq 10 m K^2 \\ 10^{-4} p K^2 \leq C_{BI}^S \leq 10^{-4} \mu K^2 \end{array} \right. \quad \left. \begin{array}{l} \Lambda_{NC} \sim 1 \text{ TeV} \\ \Lambda_{NC} \sim 20 \text{ TeV} \end{array} \right\}$$

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Observed B-mode

$$C_{BI}^{ob} \sim 0.01 \mu K^2$$

Thank you for your attention