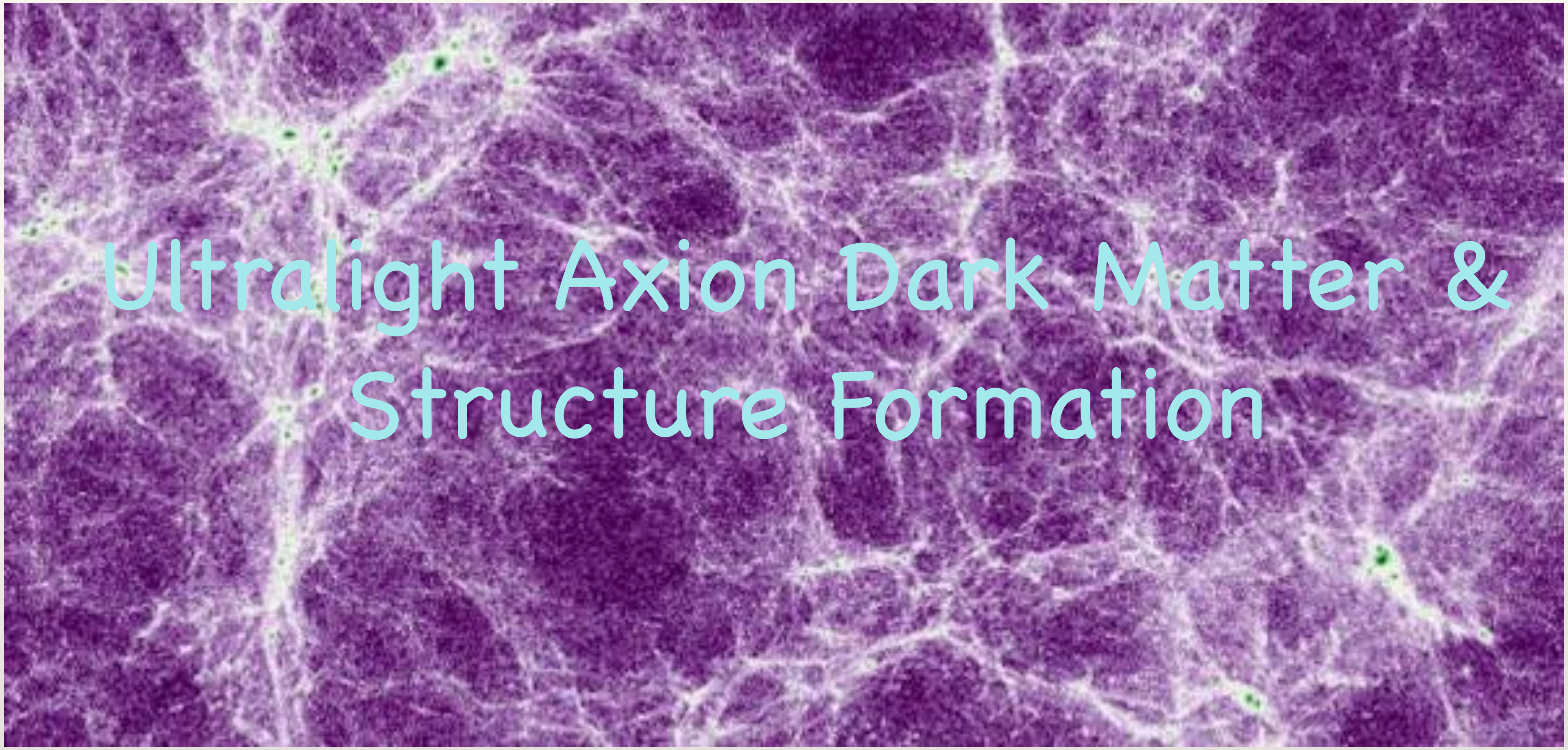


Ultralight Axion Dark Matter & Structure Formation

Kingman Cheung NTHU/NCTS/Konkuk

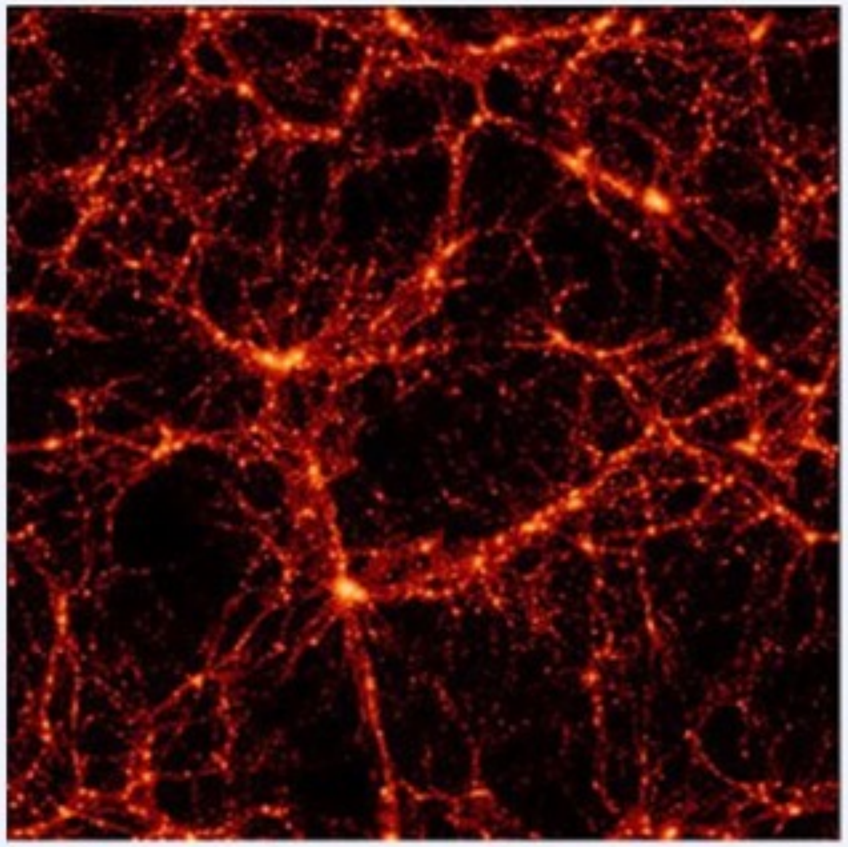
Jiajun Zhang, Yue-Lin Tsai, Jui-Lin Kuo, Ming-Chung Chu, KC — 1611.00892, AstroPhys. J

+ Hantao Liu — 1708.04389, AstroPhys. J

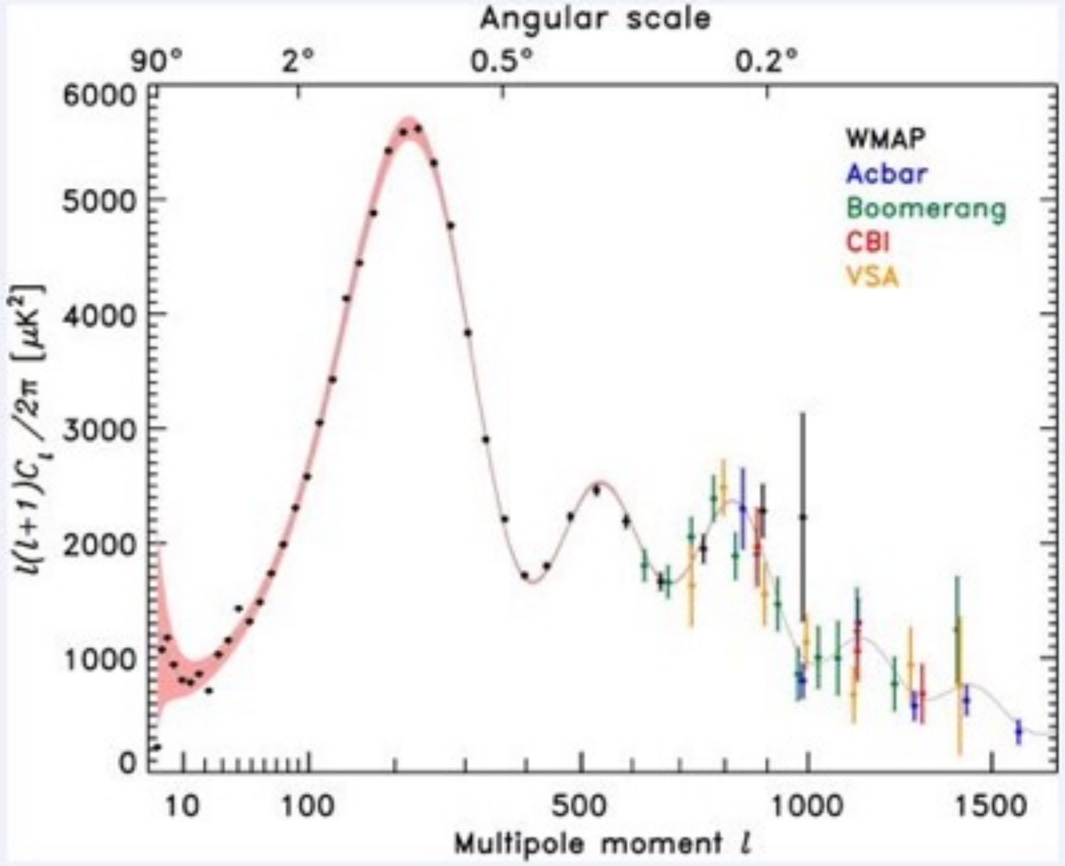
Evidence for Dark Matter

clustering of galaxies (LSS)
sensitive to amount of DM

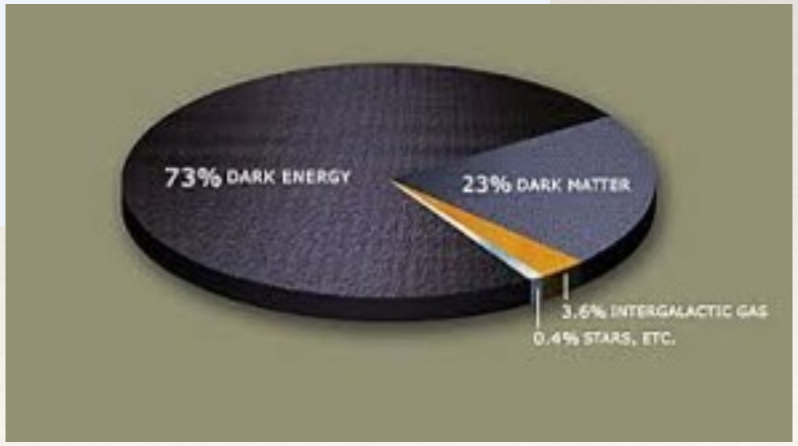
Virgo Consortium



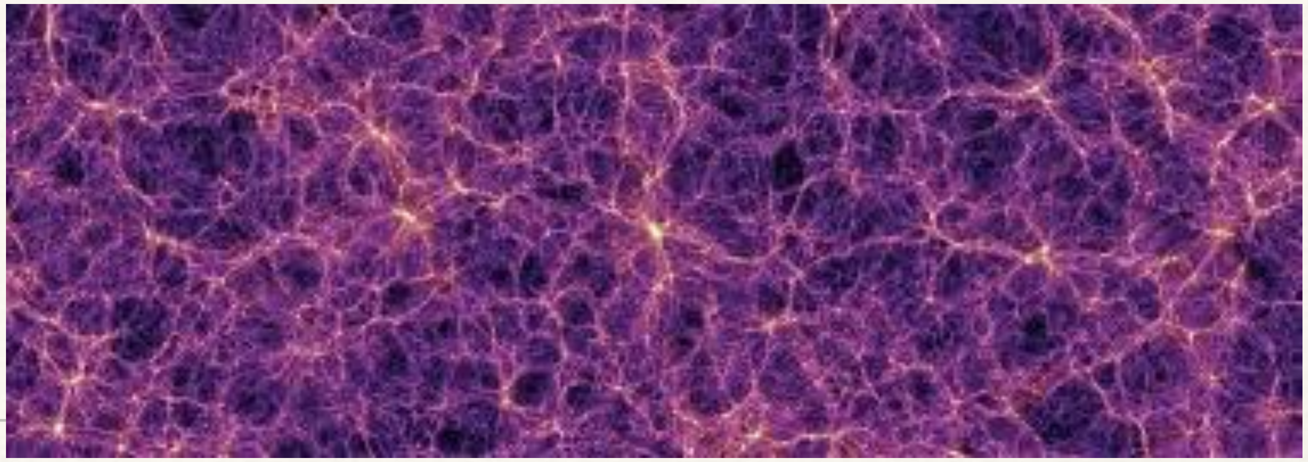
angular power spectrum of CMB
sensitive to baryonic, DM, DE

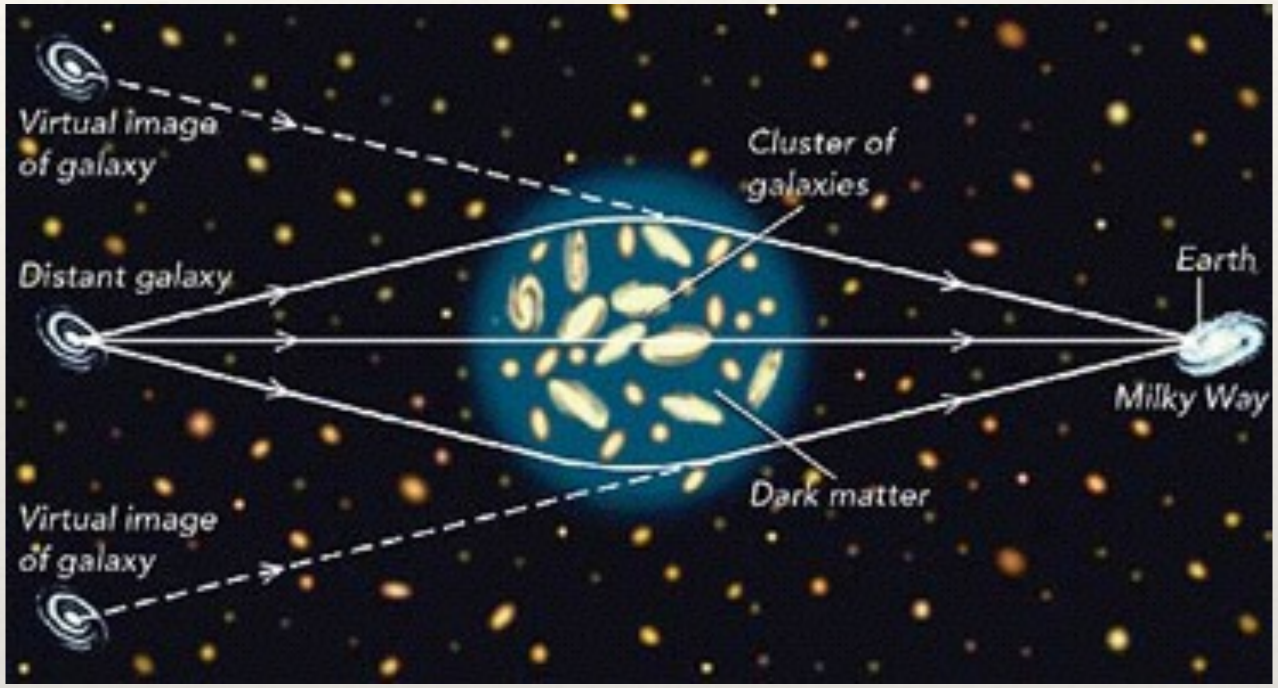


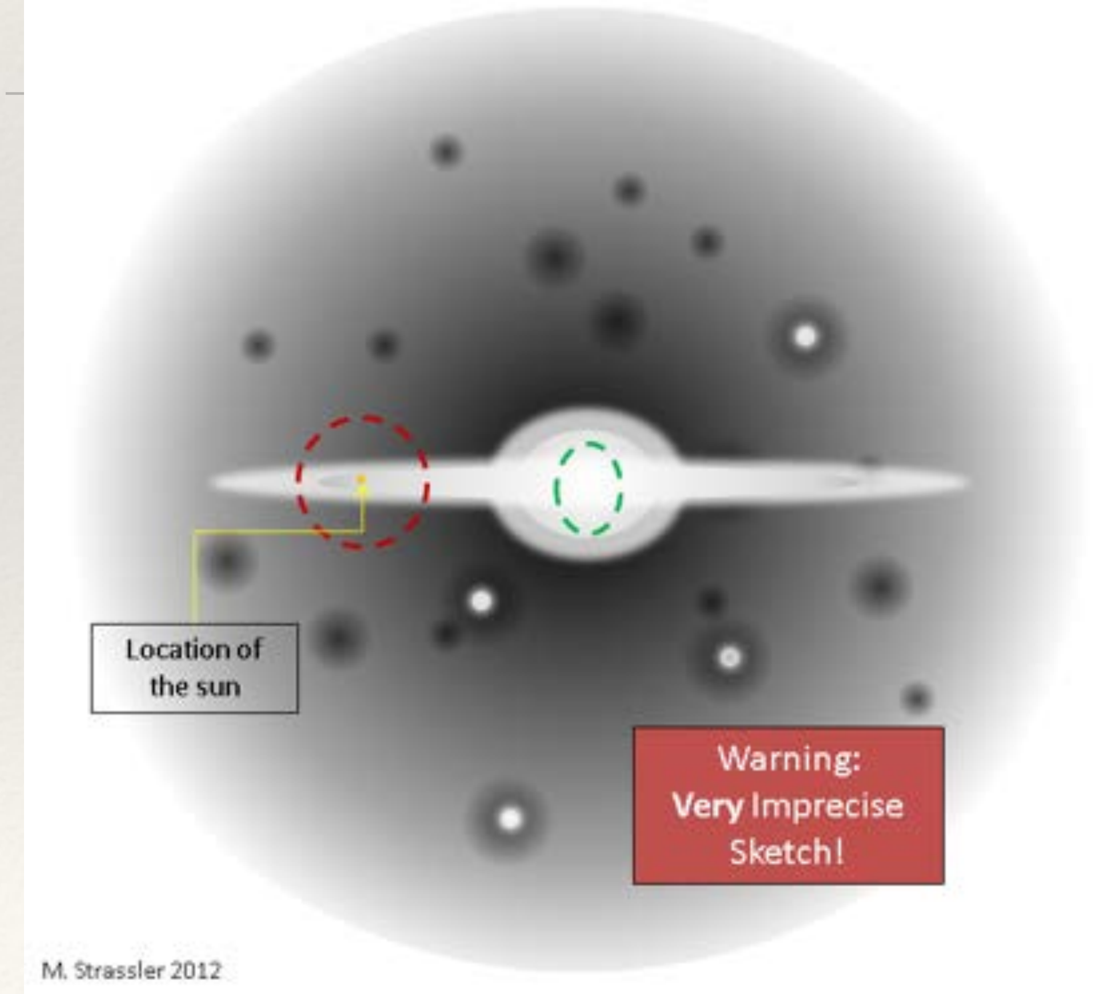
Add breakdown of matter content here



Yet the identity of Dark Matter is unknown







We only know it exists throughout the Universe

Dark Matter Candidates

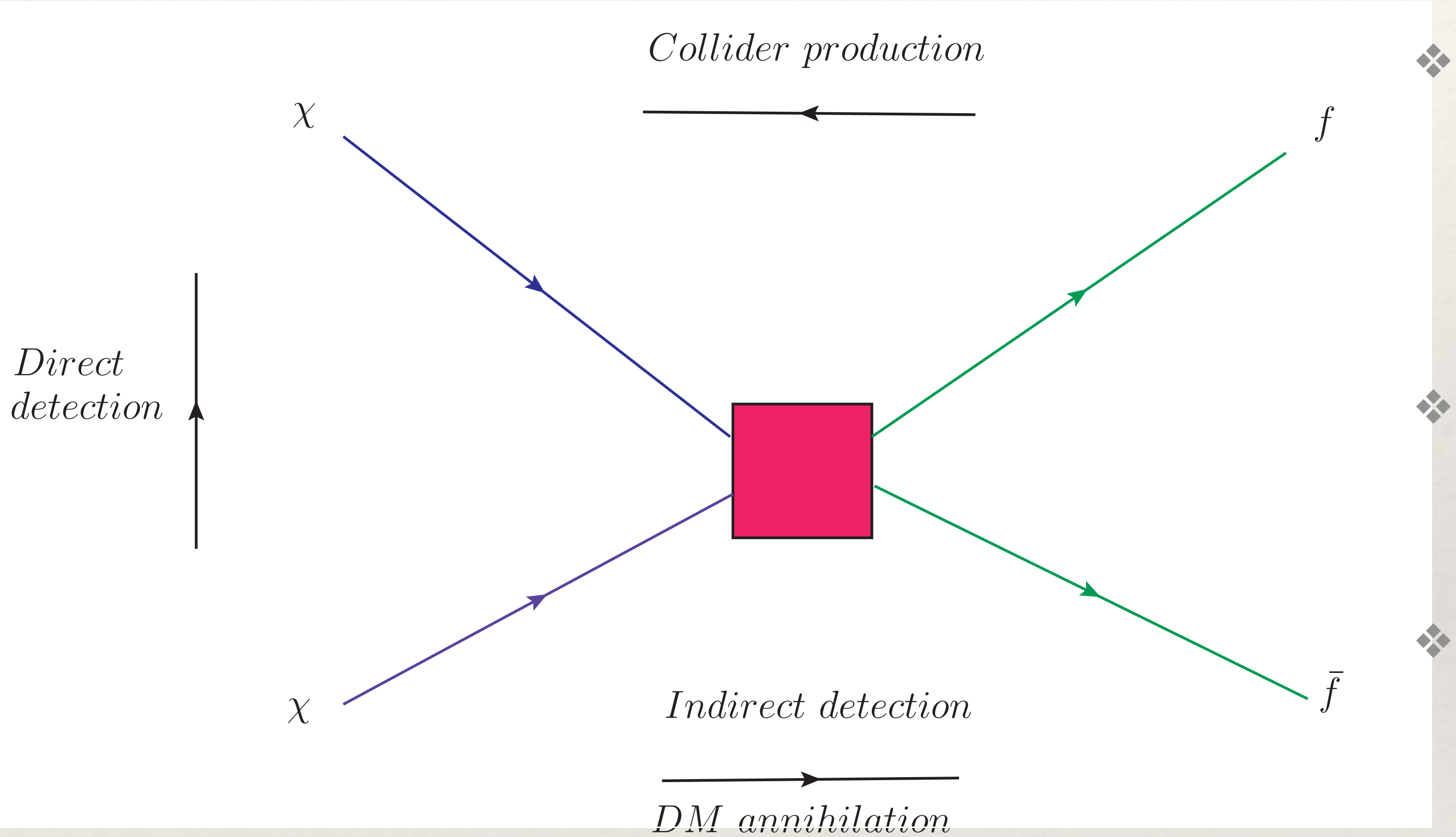
- WIMP — weakly interacting massive particles, e.g., LSP in SUSY, LTP in little Higgs models,LKP in UED model, hidden-sector fermions, …

- Ultra-weak — axion, axino, gravitino, RH neutrinos, …

- Decaying dark matter

A lot of experiments are designed to detect WIMPs, but so far … …

Strategies for hunting Dark Matter



- **Direct Detection: SM + DM -> SM DM ::** measurements of recoil energies of nuclei
- **Collider: SM+SM -> DM DM ::** measurements of missing energies
- **Indirect Detection: DM DM -> SM SM ::** measurements of e+, gamma rays, neutrinos,… from annihilation of DM

Astrophysical Structures: DM DM -> DM DM ::

galaxy formation, rotational velocity, CMB, … …

CDM Has a Missing Satellite Problem

Springel et al. 2001

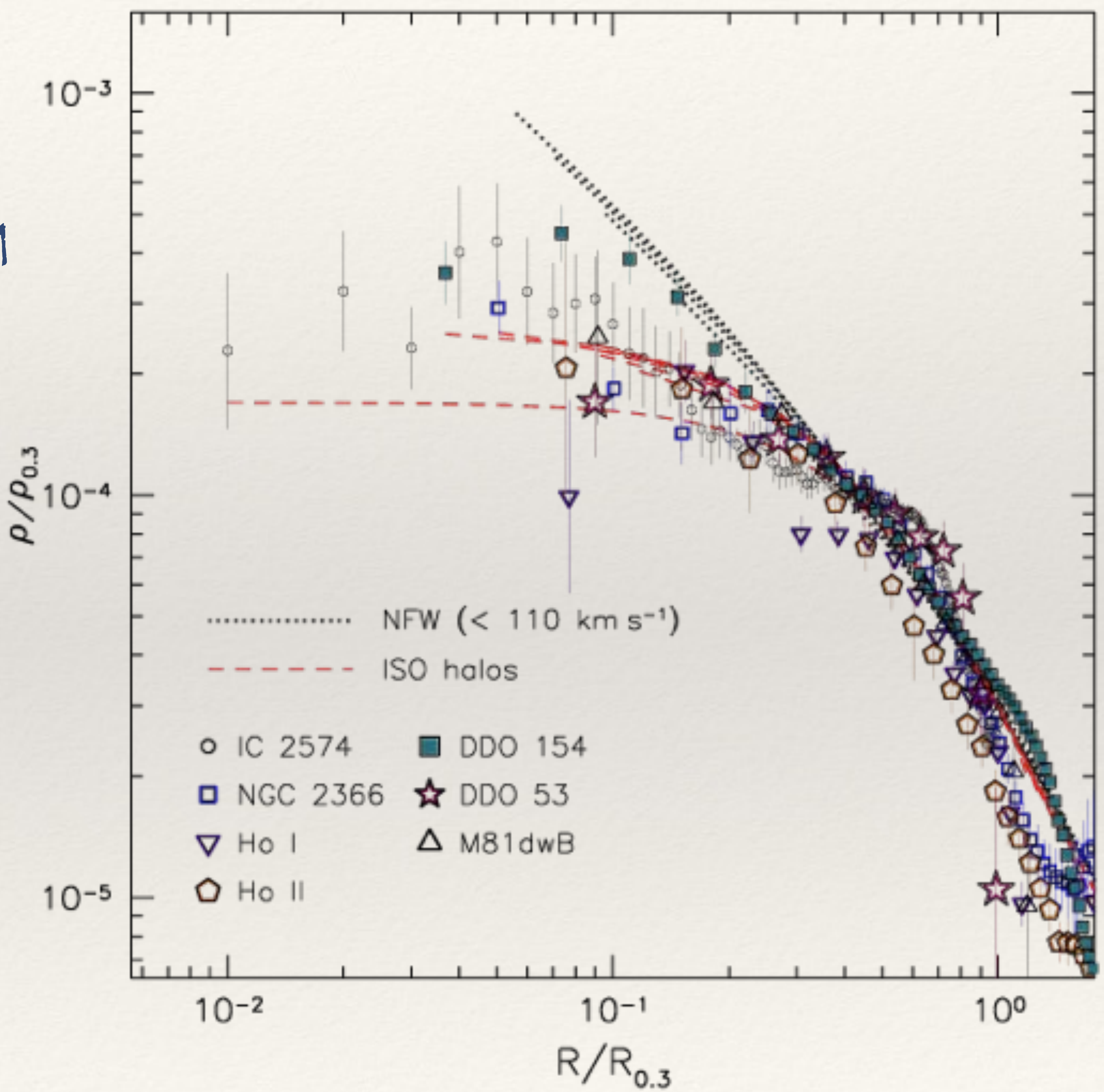
CDM predicts large numbers of subhalos (~100-1000 for a Milky Way-sized galaxy)

Milky Way only has 23 known satellites

What happened to the rest of them?

Cusp-Core Problem

- LCDM simulations predict DM density cusp in center of galaxies, but inconsistent with observations.
- Especially low-mass galaxies.



A Few Possible Solutions

- Baryon physics: efficiency of transforming baryons into stars to be lower in lower-mass systems.
- Some warm DM: its thermal velocity dispersion provides free streaming that suppresses low-mass halos or sub-halos, and also reduce the density cusp at the center.
- DM has self-interactions, reducing the density cusp, form less sub-halos.
- Fuzzy dark matter: large de Broglie wavelength suppresses small-scale structures (Hu et al.).

Ultralight Axion Dark Matter

- Very light m ~ 10^{-22} eV, with de Broglie wavelength ~ h/mv ~ O(kpc). Also called Fuzzy Dark Matter (FDM).
- At large scale ~ O(50 kpc) it behaves like CDM and succeeds in explaining the large scale structures.
- The difference is at relatively small scales ~ O(10 kpc). It "smooths" out density cusp distributions, due to quantum nature of the FDM.
- The quantum pressure of FDM induces a solitonic core of size O(kpc). It explains the "small scale crisis".

Light fields of spin zero

- Consider the action $S=\frac{1}{2}\int d^4x\sqrt{g}g^{\mu\nu}\partial_\mu\phi\partial_\upsilon\phi$, When the mass and self-coupling of a spinless field ϕ are precisely zero, there is an extra symmetry $\phi\to\phi+C$.

- But the candidate of FDM is very nearly massless boson, so it has an **approximate shift symmetry, not an exact one**.

- A axion-like candidate for FDM can be described by:

$$S=\frac{1}{2}\int d^4x\sqrt{g}[\frac{1}{2}F^2g^{\mu\nu}\partial_\mu a\partial_\upsilon a-\mu^4(1-\cos a)]$$

- The mass of a is $m=\frac{\mu^2}{F}$.

For FDM we want $m\sim10^{-22}-10^{-21}eV$.

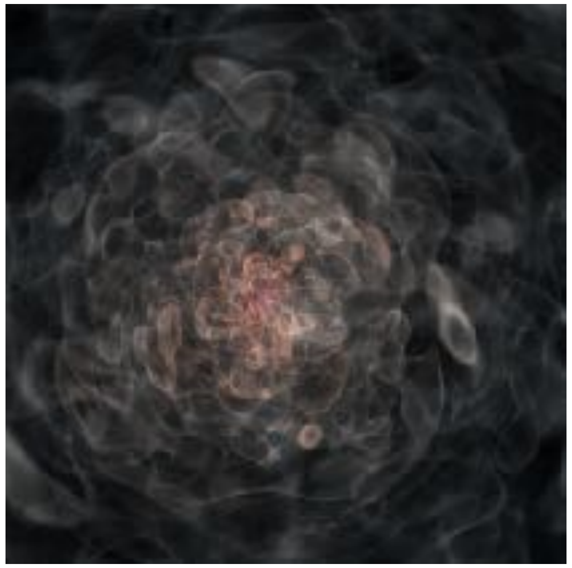
FDM as a superfluid

- It is useful to think the dark matter as a superfluid.

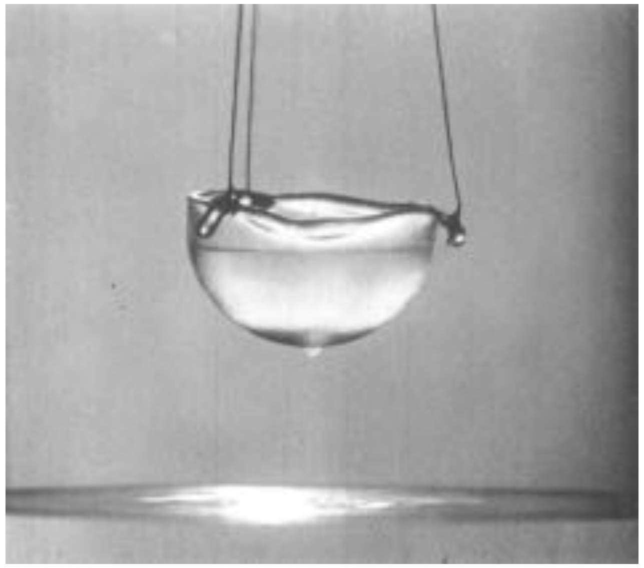
no self interaction (→ dark matter)   no viscosity (→ superfluid)

- Define the fluid density and velocity by:

$$\psi = \sqrt{\frac{\rho}{m}} e^{i\theta} \quad , \quad \vec{v} = \frac{\hbar}{Rm} \nabla \theta = \frac{\hbar}{2miR} \left(\frac{1}{\psi} \nabla \psi - \frac{1}{\psi^*} \nabla \psi^* \right).$$



from arXiv:1606.05151



from Wikipedia

Madelung equation

- By plugging the density and the velocity into the equation of motion in comoving coordinates, we could obtain the Madelung equation.

$$
\begin{cases}
\dot{\rho} + 3H\rho + \dfrac{1}{R}\nabla\cdot(\rho\vec{v}) = 0 \\
\dot{\vec{v}} + H\vec{v} + \dfrac{1}{R}(\vec{v}\cdot\nabla)\vec{v} = -\dfrac{1}{R}\nabla\Phi + \dfrac{\hbar^2}{2R^3m^2}\nabla\cdot\left(\dfrac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)
\end{cases}
$$

The Madelung equations are well-suited to numerical simulation!

Quantum pressure

- Second equation of Madelung equations:

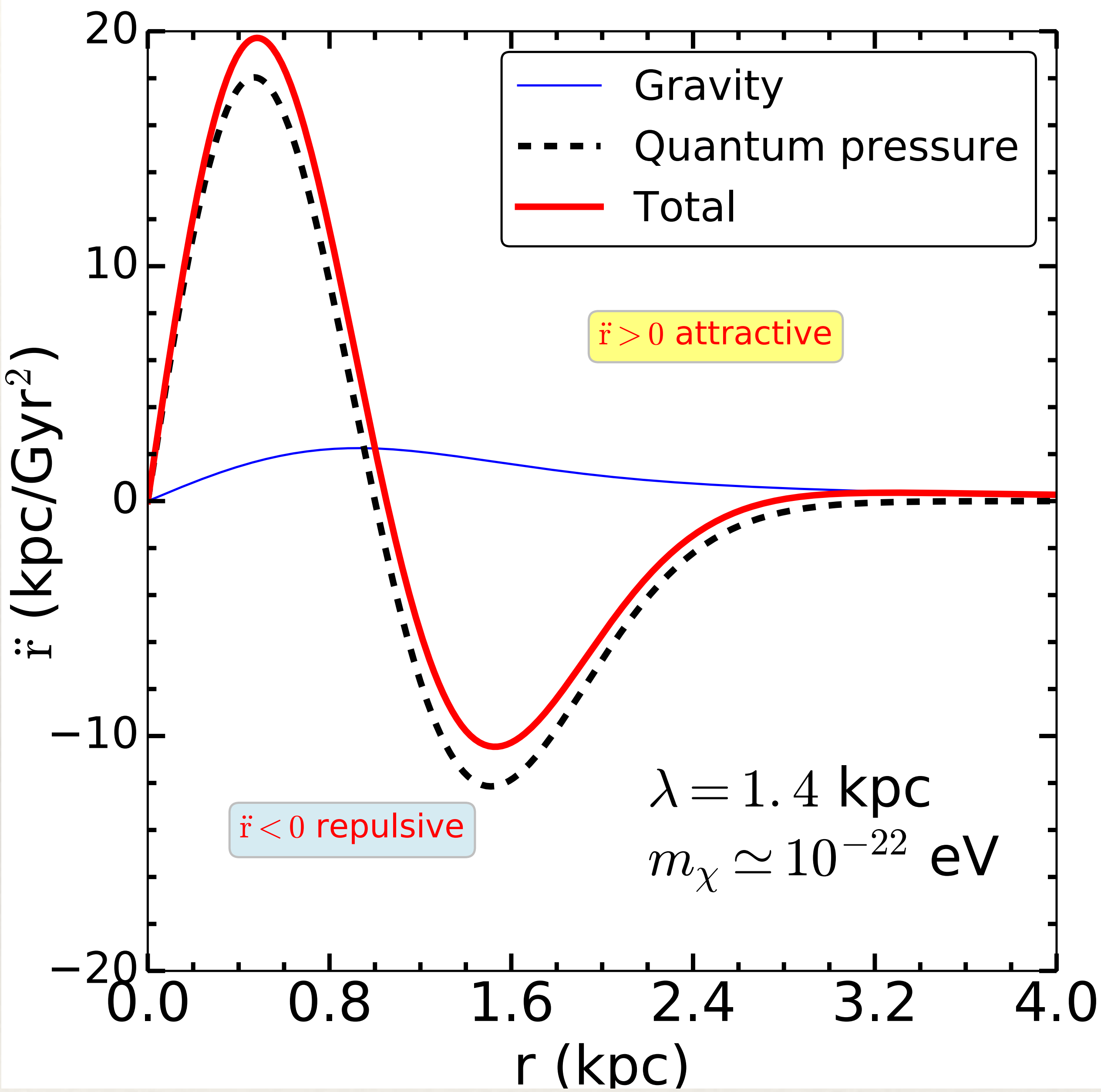
$$\dot{\vec{v}} + H\vec{v} + \frac{1}{R}(\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{R}\nabla\Phi + \frac{\hbar^2}{2R^3m^2}\nabla\cdot\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

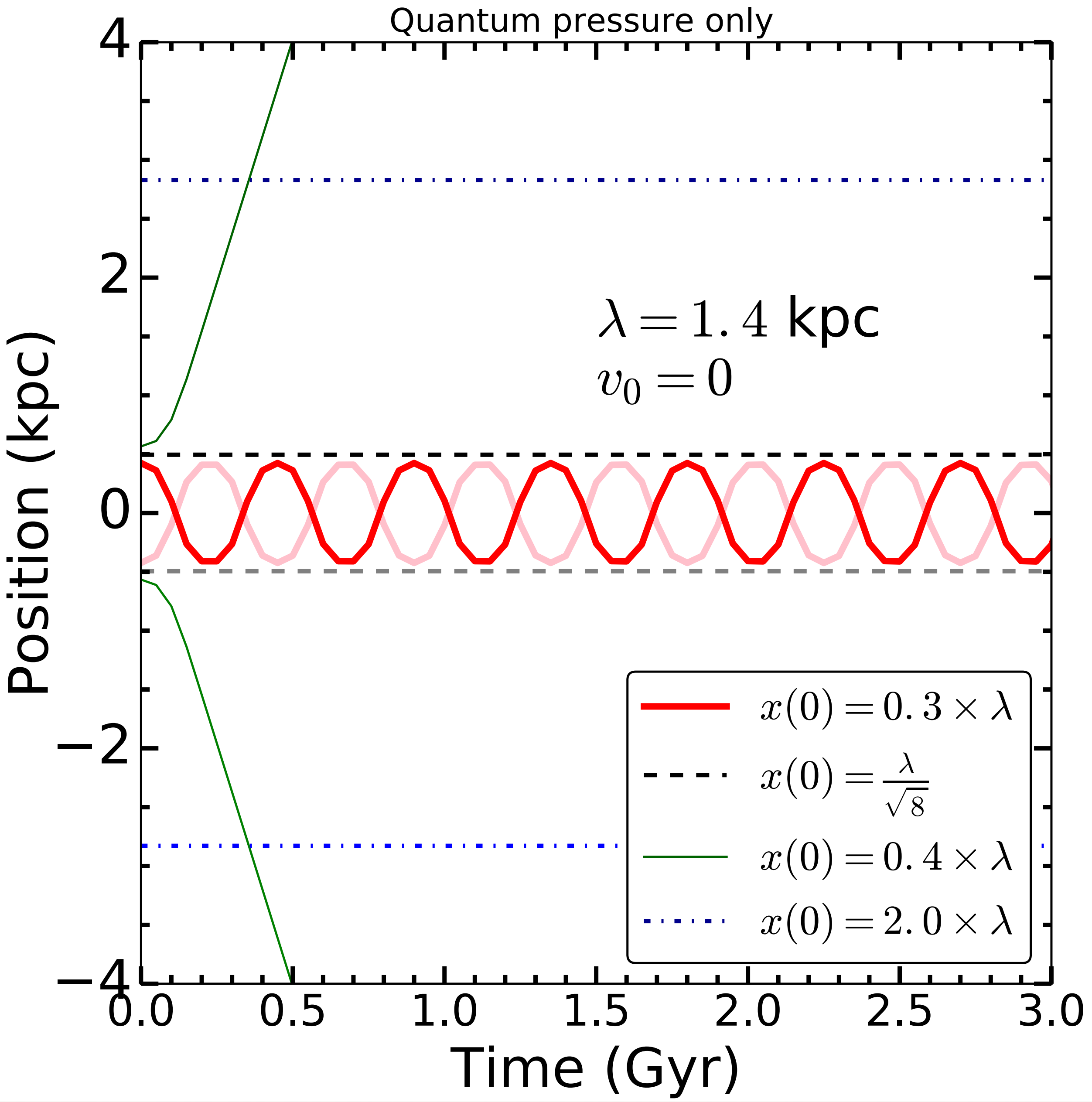
- The quantum pressure arises from a stress tensor :

$$\dot{\vec{v}} + H\vec{v} + \frac{1}{R}(\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{R}\nabla\Phi + \frac{1}{R}\nabla\cdot\vec{\sigma}.$$

By comparing this two equations, we could interpret quantum pressure as $\vec{\sigma} = \frac{\hbar^2}{2R^2m^2}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right).$

The acceleration due to quantum pressure, gravity between TWO particles



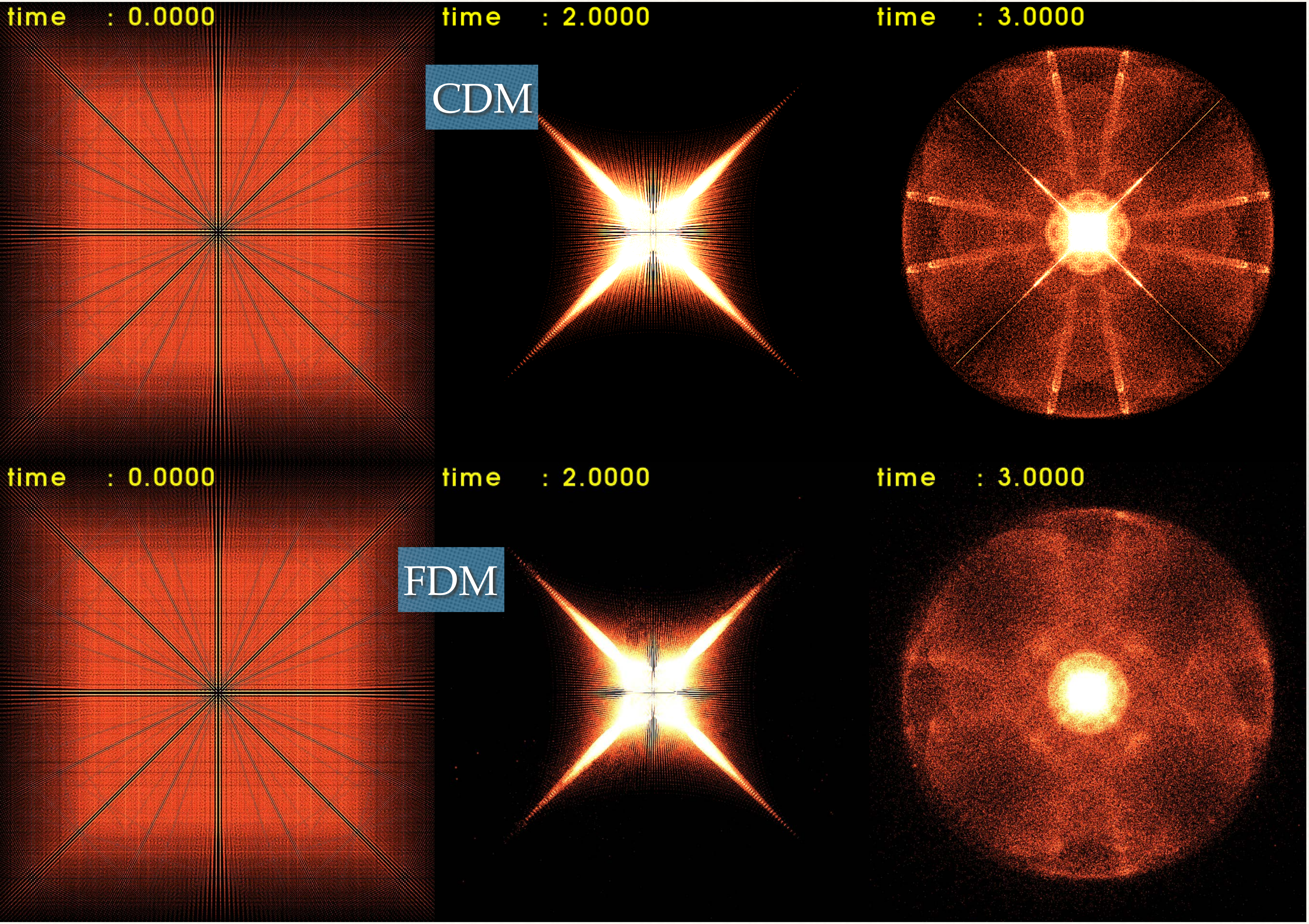


Attractive for r ≤ $\lambda/\sqrt{2}$

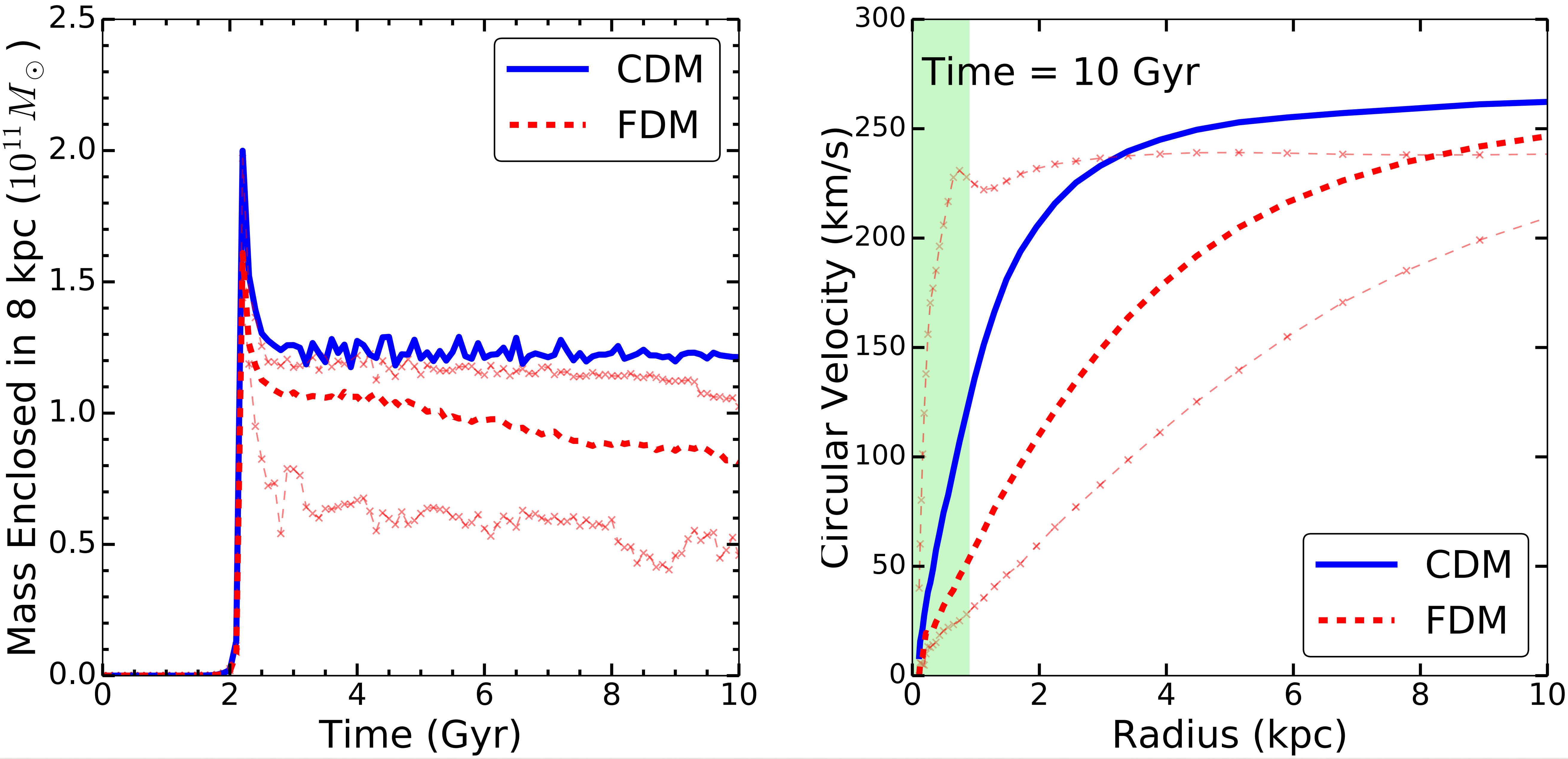
Repulsive for r ≥ $\lambda/\sqrt{2}$

N-body Simulations

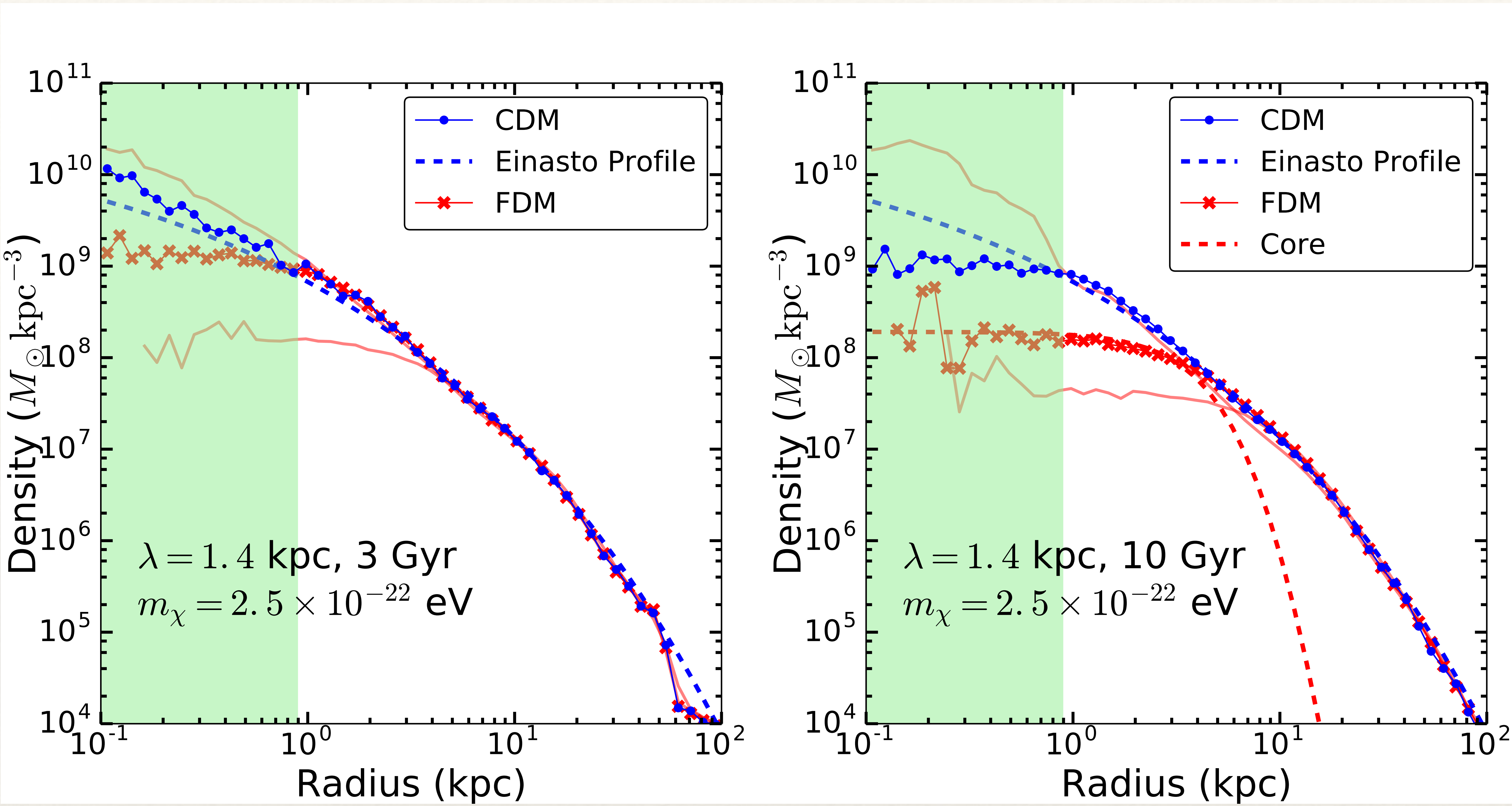
- Modify **Gadget2** to include effects of quantum pressure
- Set up a self-collapsing system — a cubic box of side 400 kpc, 10^6 simulation particles, each has a mass of 10^6 solar mass.
- All particles start from rest and the system collapses due to self-gravity to form a stable self-gravitationally-bound virialized halo at the center.
- The final virialized halo for FDM depends sensitively on the initial slight "push" given to the system.
- We compare FDM and CDM halos.
- m ~ 2.45 . 10^{-22} eV, wavelength ~ 1.4 kpc



time : 0.0000
time : 2.0000
time : 3.0000
CDM
time : 0.0000
time : 2.0000
time : 3.0000
FDM



FDM1 halo has a higher density in inner core than CDM
FDM2 has a lower density



FDMs develop a solitonic core. Beyond the core the same as Einasto.

Solitonic Core of FDM

The FDM develops a solitonic core after 10 Gyr. The core can be fitted by

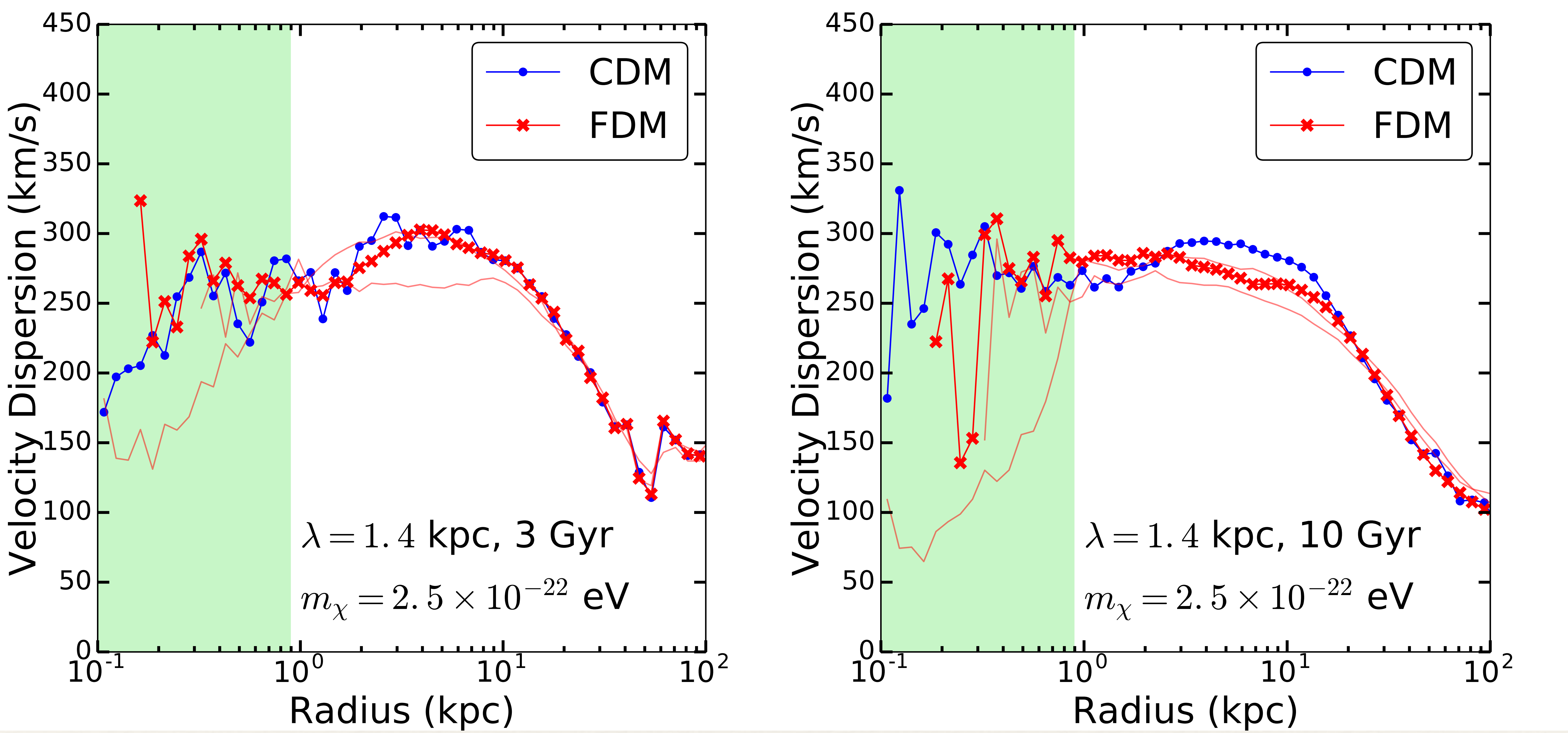
$$\rho_c(r) \simeq \rho_b \rho_0 [1 + 0.091(\frac{r}{r_c})^2]^{-8},$$

$$\rho_0 \simeq 3.1 \times 10^6 (\frac{2.5 \times 10^{-22}\,\mathrm{eV}}{m_\chi})^2 (\frac{\mathrm{kpc}}{r_c})^4 \frac{M_\odot}{\mathrm{kpc}^3}.$$

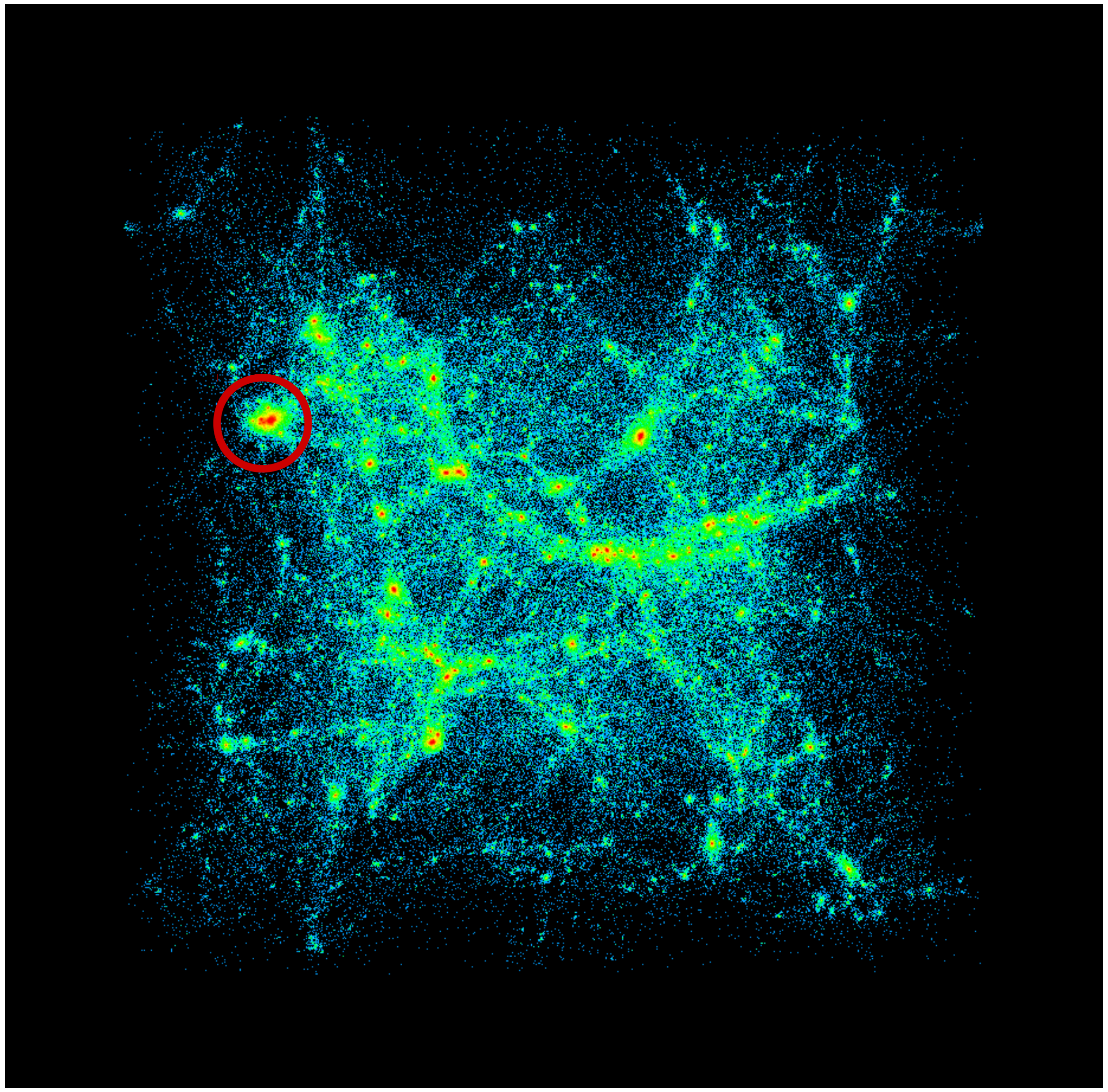
FDM: $\rho_b = 5000$ and $r_c = 3\,\mathrm{kpc}$

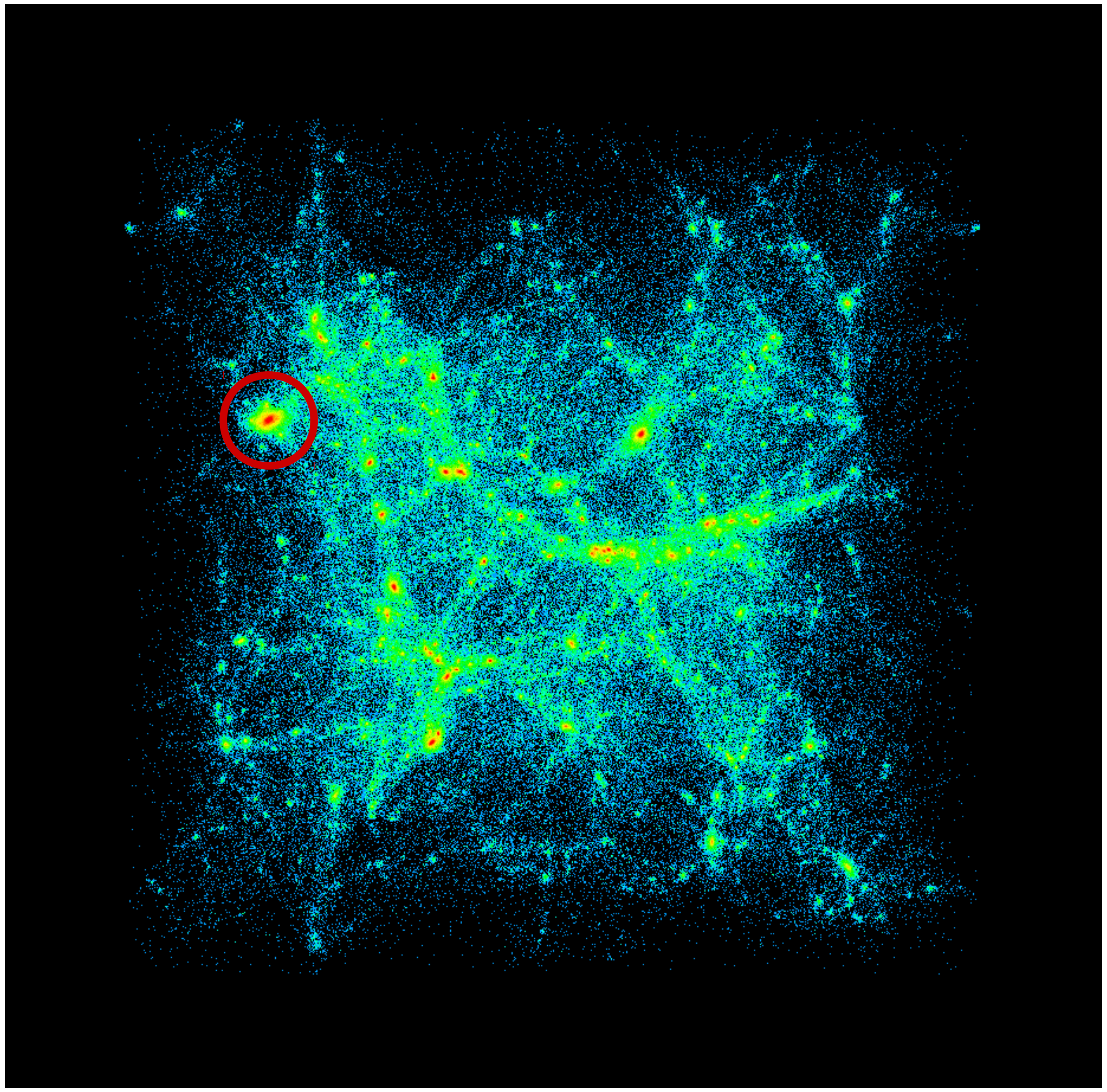
FDM: solitonic core within 3 kpc with lower density than CDM due to repulsive quantum pressure

Velocity Dispersion Profile



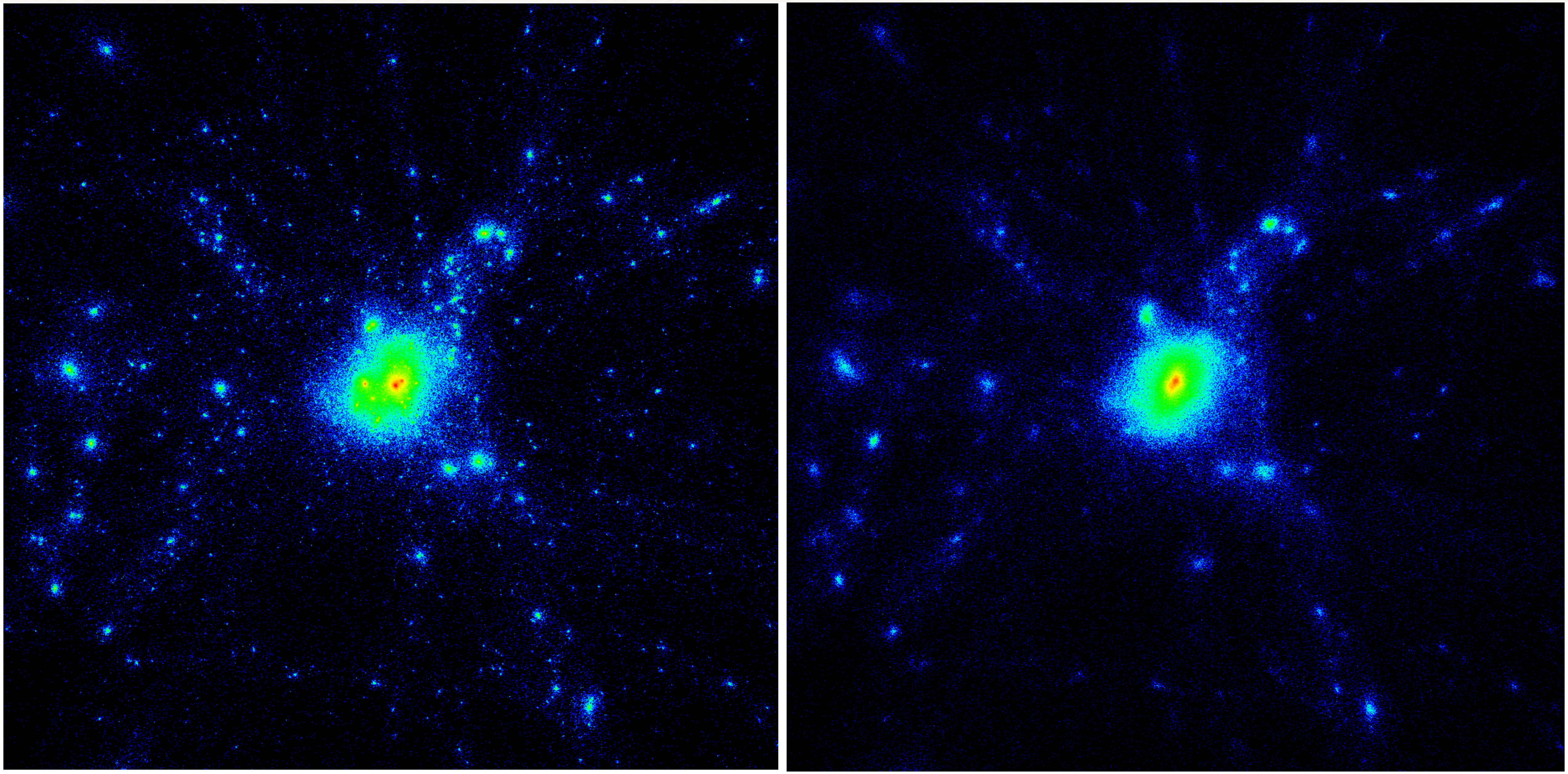
The FDM has less mass within 3 kpc, so have smaller velocity dispersion than CDM.





CDM FDM

No difference on large scale! (Expected!)



CDM FDM

Much less small scale structure for FDM!

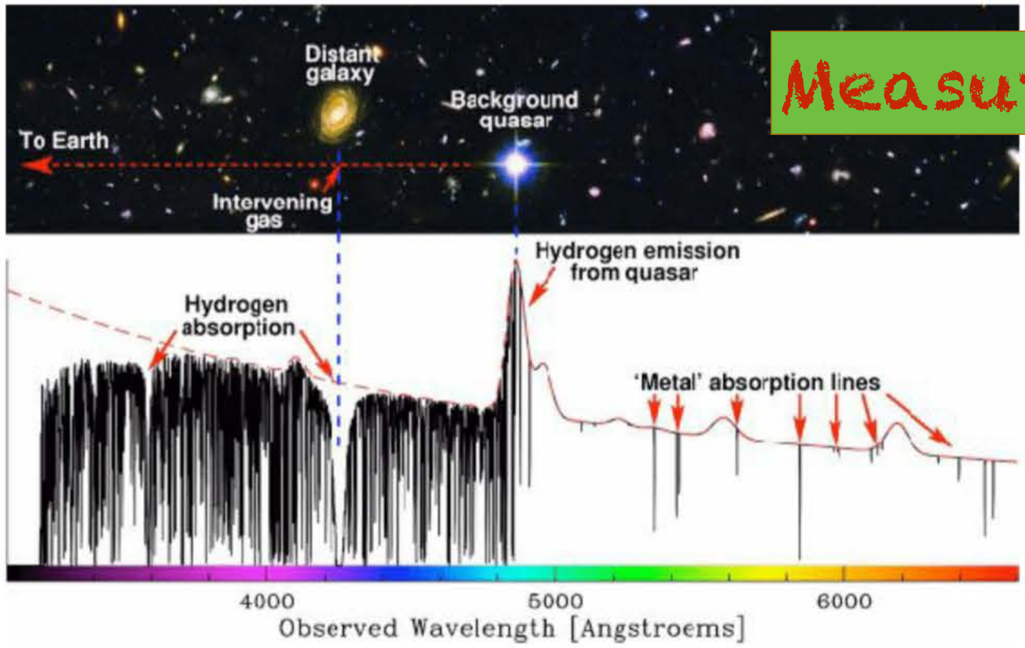
Cosmological Simulations

- Run large scale cosmological simulations of size $(50\ h^{-1}\ \mathrm{Mpc})^3$, number of simulations particles 512^3.
- Start at z=99 with a linear FDM power spectrum to the current z=0.
- Project the 3D power spectrum on 1D flux spectrum, and compare with the Lyman-Alpha Forest data.

What is Lyman-alpha forest?

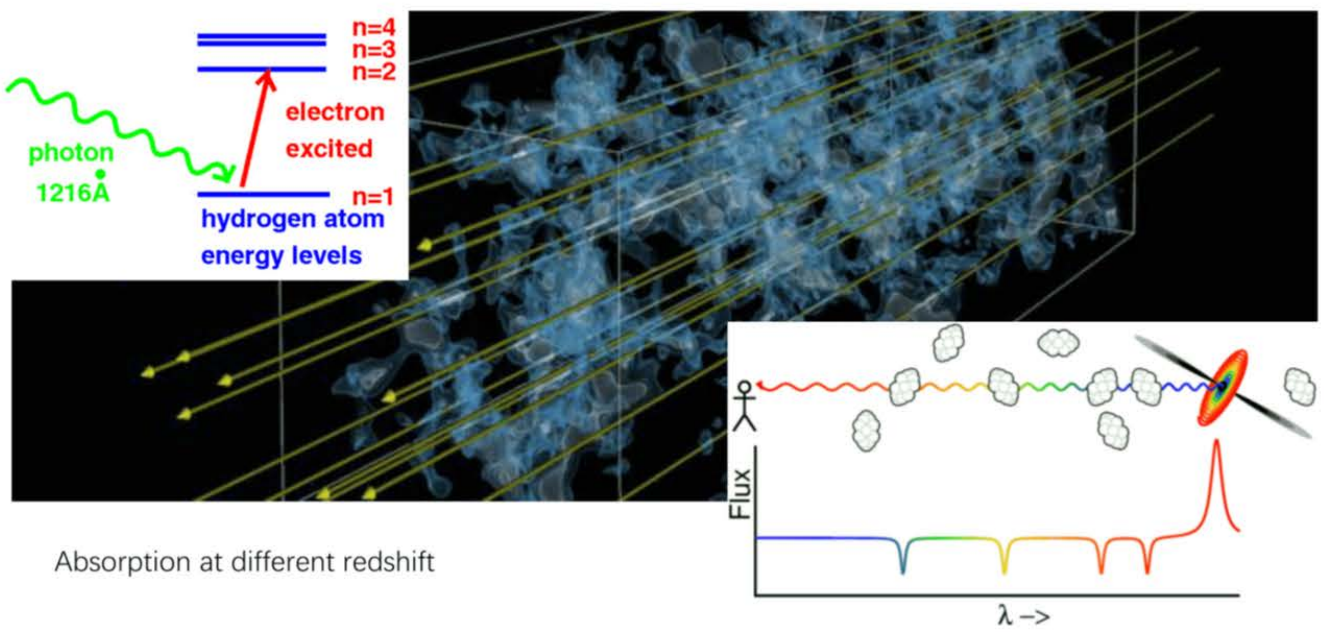
- The Lyman-alpha forest is **a series of absorption lines in the spectra of distant galaxies and quasars** from the Lyman-alpha electron transition of the neutral hydrogen atom.

Measurements



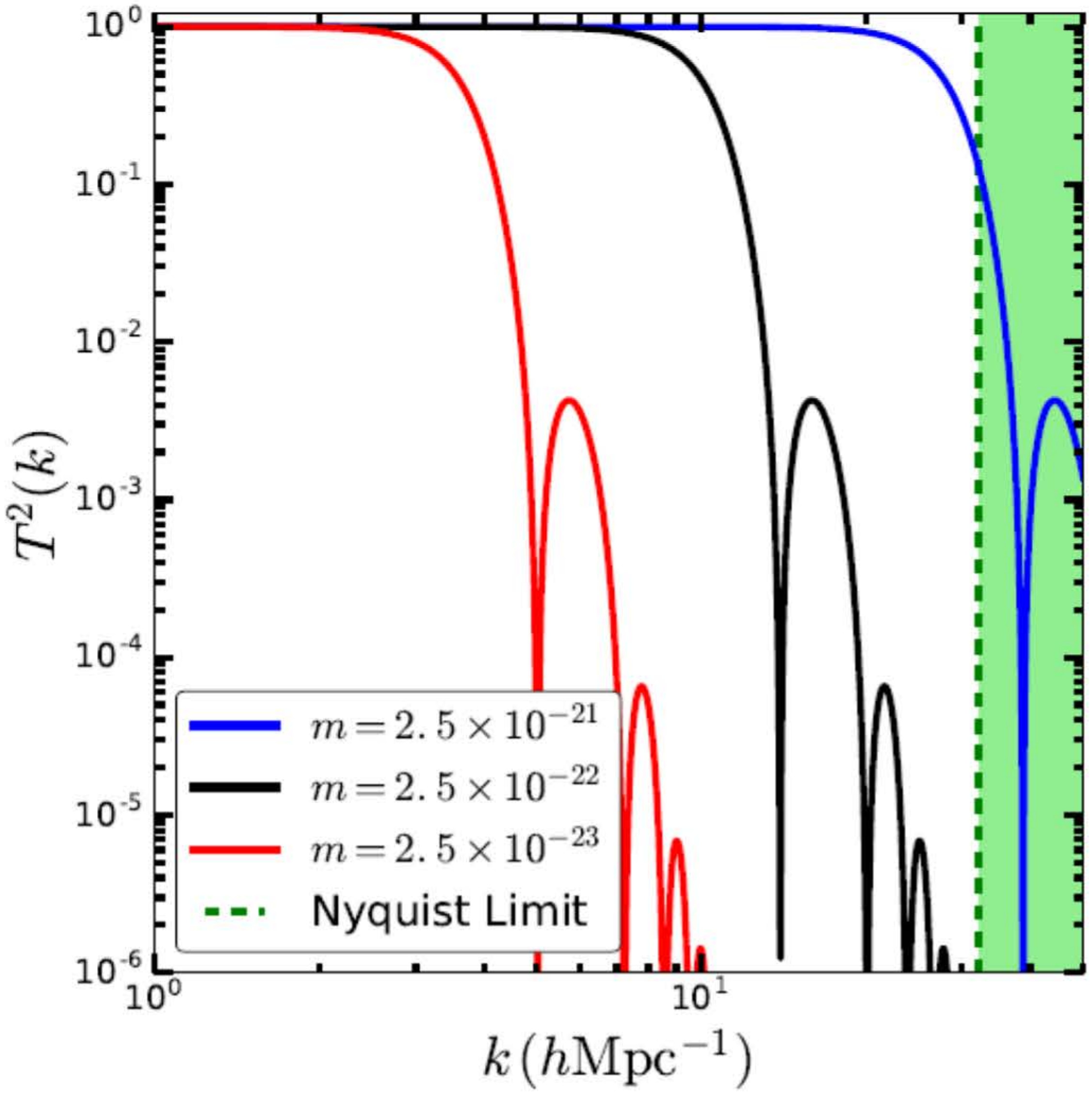
by Joe Like

Idea



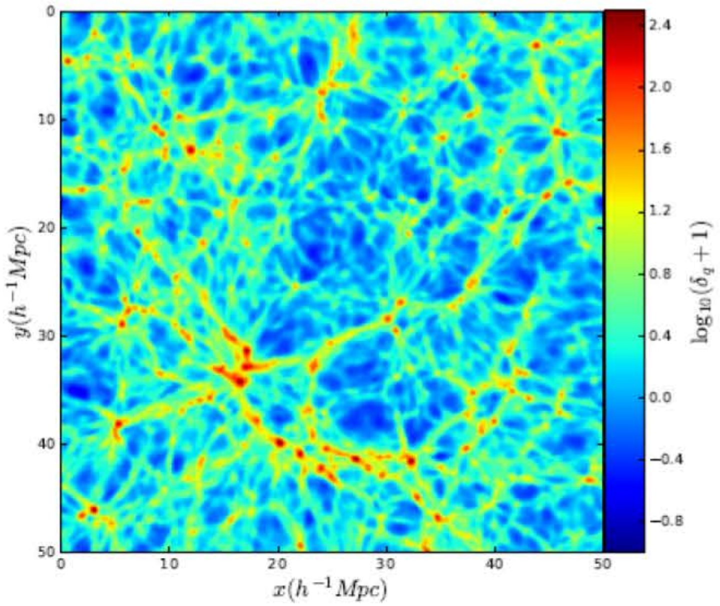
Absorption at different redshift

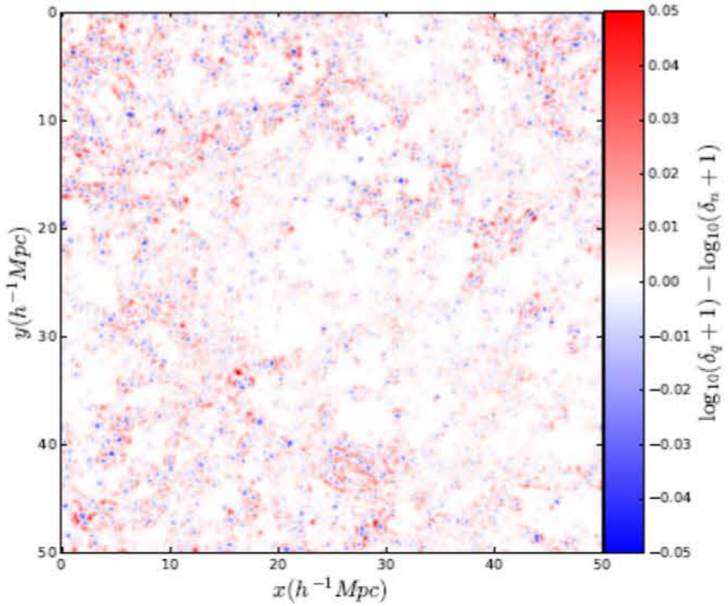
FDM cosmological simulation



$$P_F(k) = \left[\frac{T_{\mathtt{FDM}}(k)}{T_{\mathtt{CDM}}(k)}\right]^2 P_C(k) = T^2(k)P_C(k)$$

$$\ddot{\boldsymbol{r}} = \frac{4M\hbar^2}{M_0 m_\chi^2 \lambda^4 a^2}\sum_j \mathcal{B}_j \exp\left[-\frac{2|\boldsymbol{r}-\boldsymbol{r_j}|^2}{\lambda^2}\right]\left(1-\frac{2|\boldsymbol{r}-\boldsymbol{r_j}|^2}{\lambda^2}\right)(\boldsymbol{r_j}-\boldsymbol{r}).$$

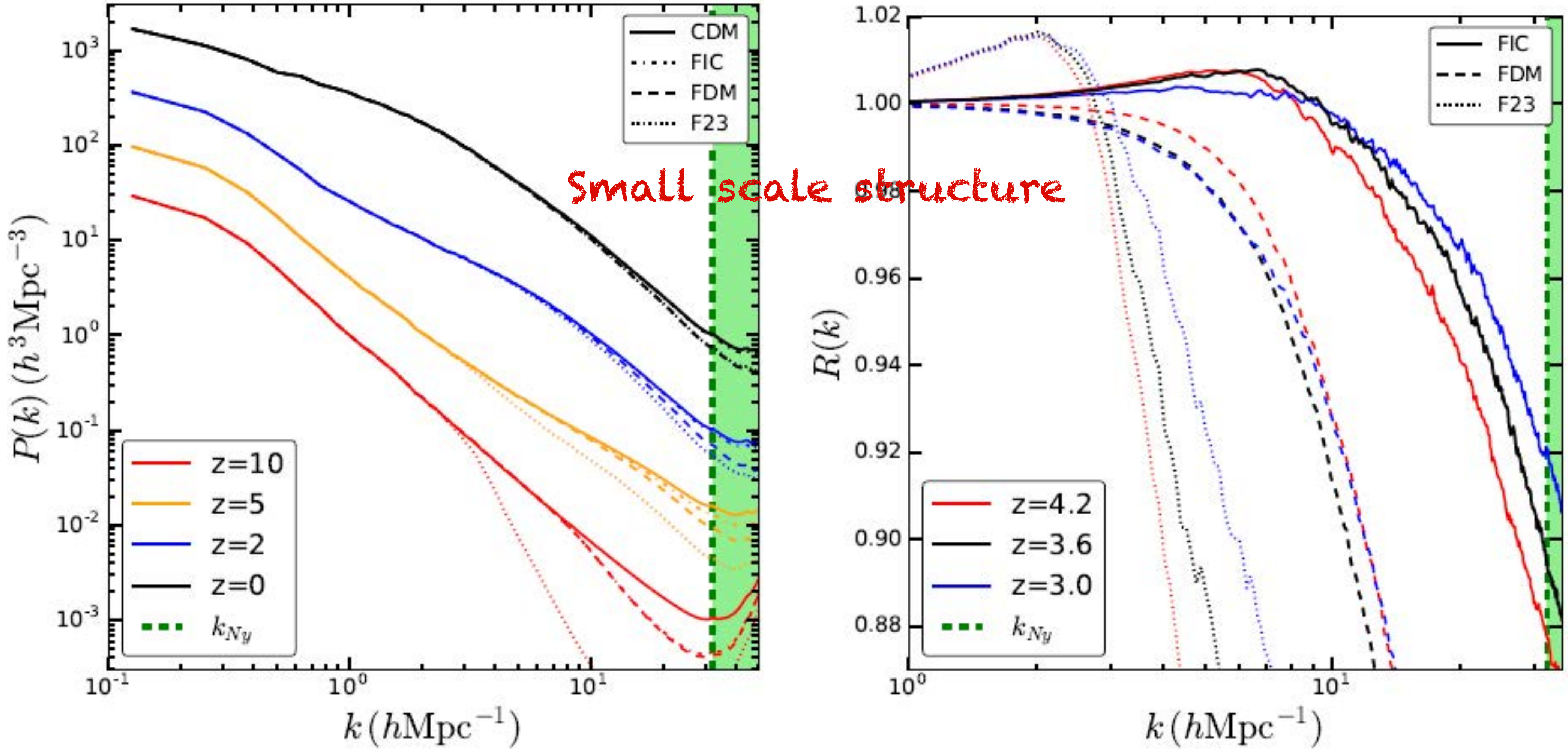




Time for N-body simulation Movie...

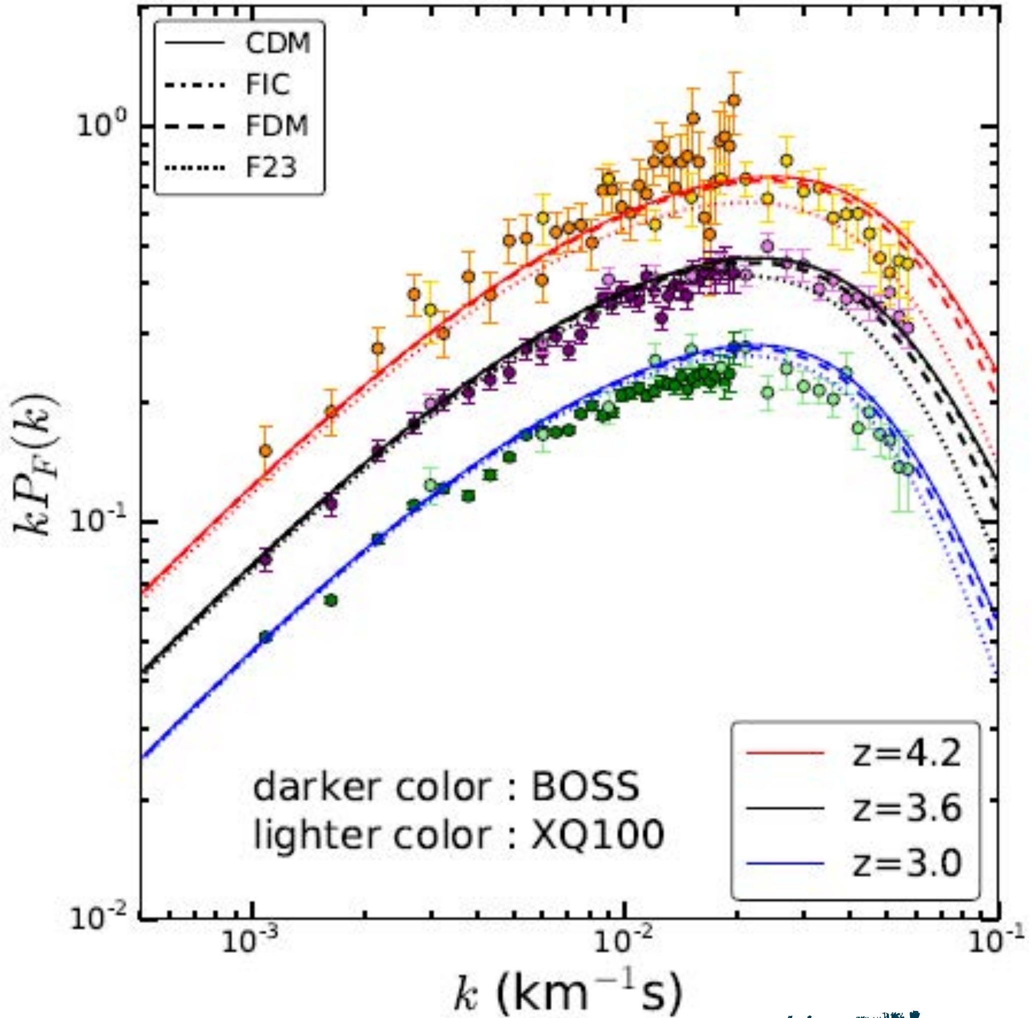
Abbreviation	Description
CDM	The standard ΛCDM N-body simulation.
FIC	The simulation similar to **CDM** but using the FDM initial condition.
FDM	The ΛFDM simulation with $m_\chi = 2.5 \times 10^{-22}\,\mathrm{eV}$, the FDM initial condition and the additional acceleration from quantum pressure.
F23	The simulation similar to **FDM** but with $m_\chi = 2.5 \times 10^{-23}\,\mathrm{eV}$.

Comparision: Matter Power Spectrum



The lower mass the larger discrepancy between Lambda CDM and FDM.

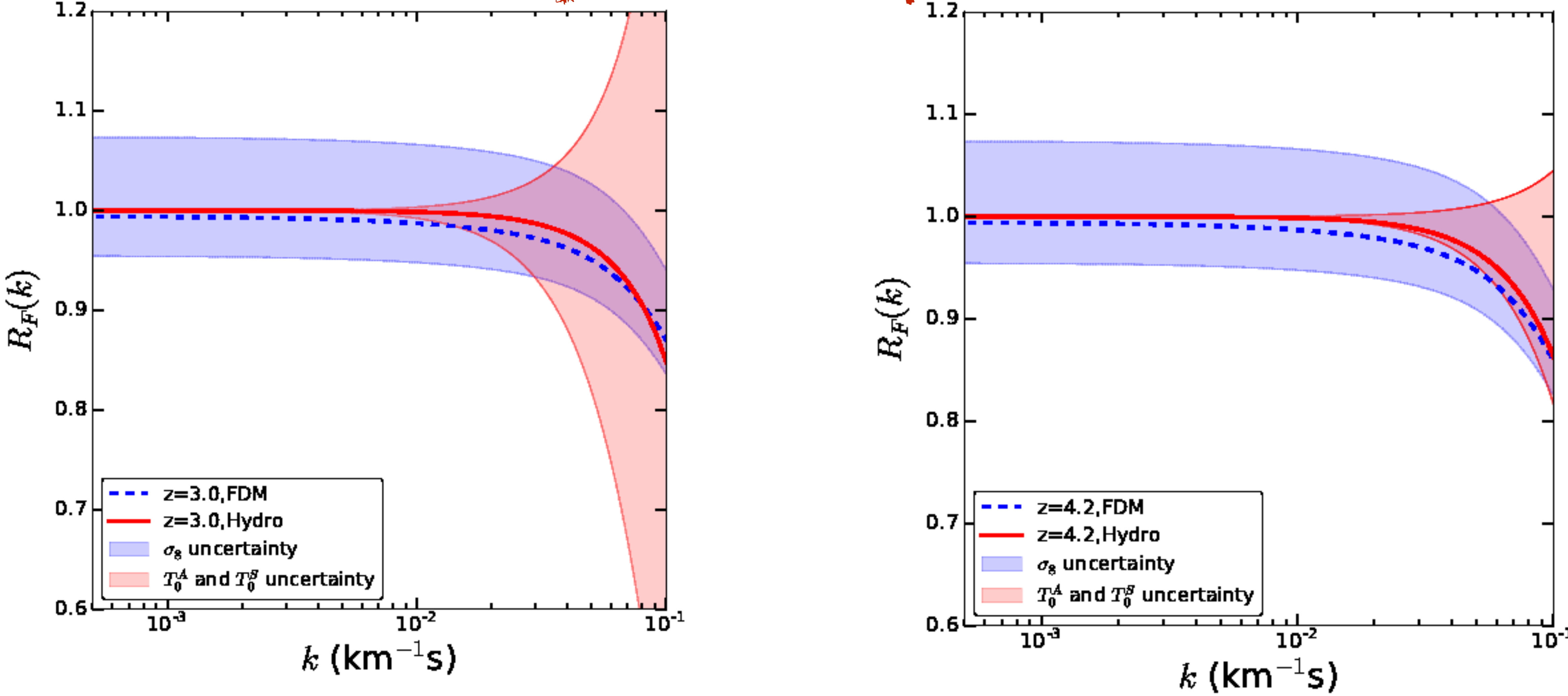
FDM fits Lyman-alpha forest?



Simulation	$\mathcal{F}$ at $z = 3.0$	$\mathcal{F}$ at $z = 3.6$	$\mathcal{F}$ at $z = 4.2$	Total χ^2	$\delta\chi^2$
BOSS					
CDM	0.910	0.932	1.092	193.69	0.00
FIC	0.911	0.933	1.093	193.34	-0.35
FDM	0.920	0.946	1.183	186.35	-7.34
F23	0.939	0.989	1.183	177.08	-16.61
XQ-100					
CDM	0.884	0.979	0.946	51.54	0.00
FIC	0.885	0.982	0.950	51.10	-0.44
FDM	0.910	1.013	0.975	49.33	-2.21
F23	0.959	1.103	1.119	53.78	2.24
$k > 0.02\,\mathrm{km}^{-1}s$					
CDM	0.853	0.970	0.899	23.68	0.00
FIC	0.854	0.973	0.904	23.73	0.05
FDM	0.883	1.013	0.934	25.56	1.88
F23	0.942	1.133	1.108	32.22	8.54

The 1e-23 eV FDM is strongly disfavored.
The 1e-22 eV FDM is hard to say being excluded at this level (no hydro-simulation).

FDM fits Lyman-alpha forest?

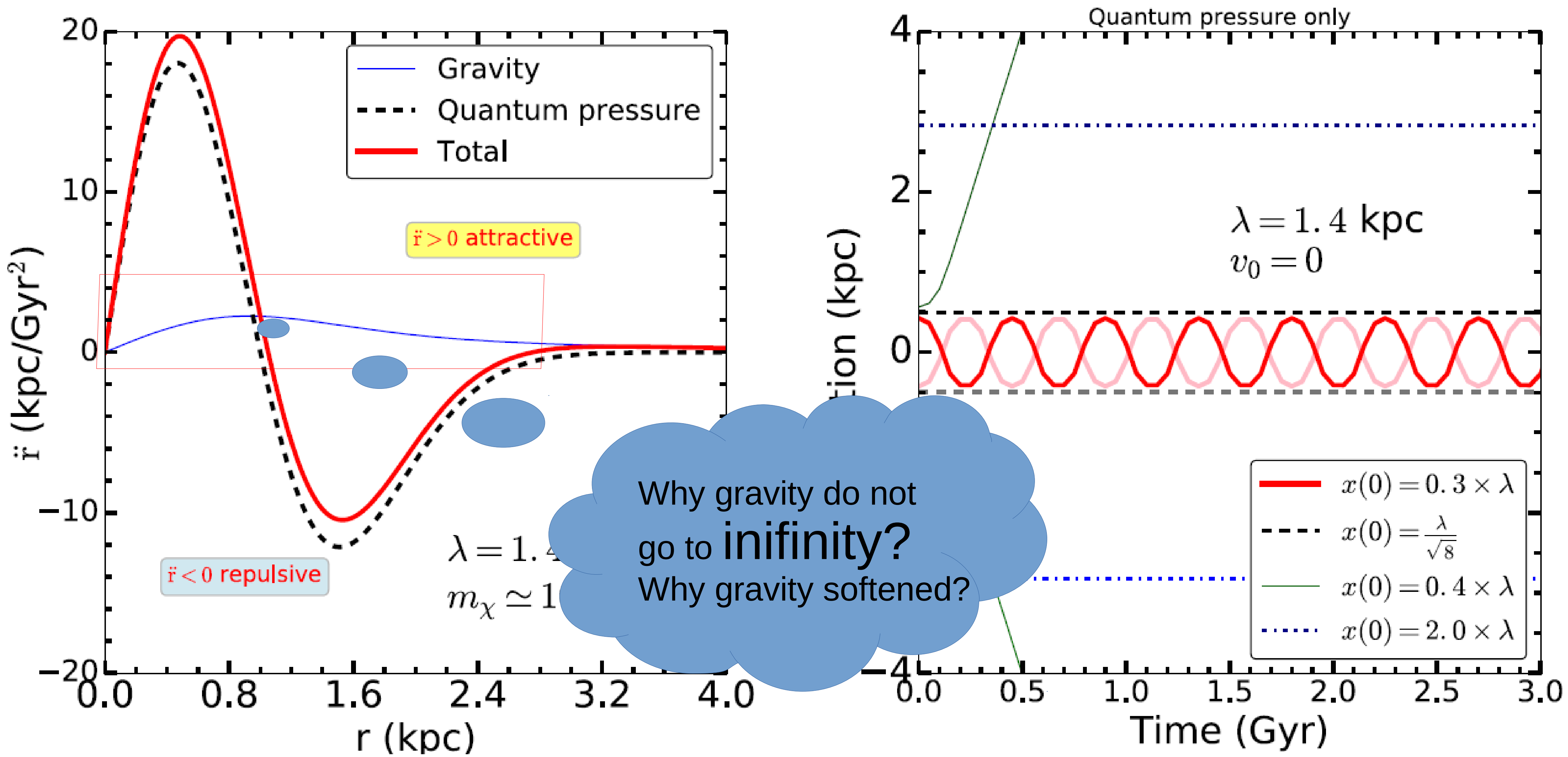


Hydro simulation uncertainty is huge and the effects from QP can be hidden.

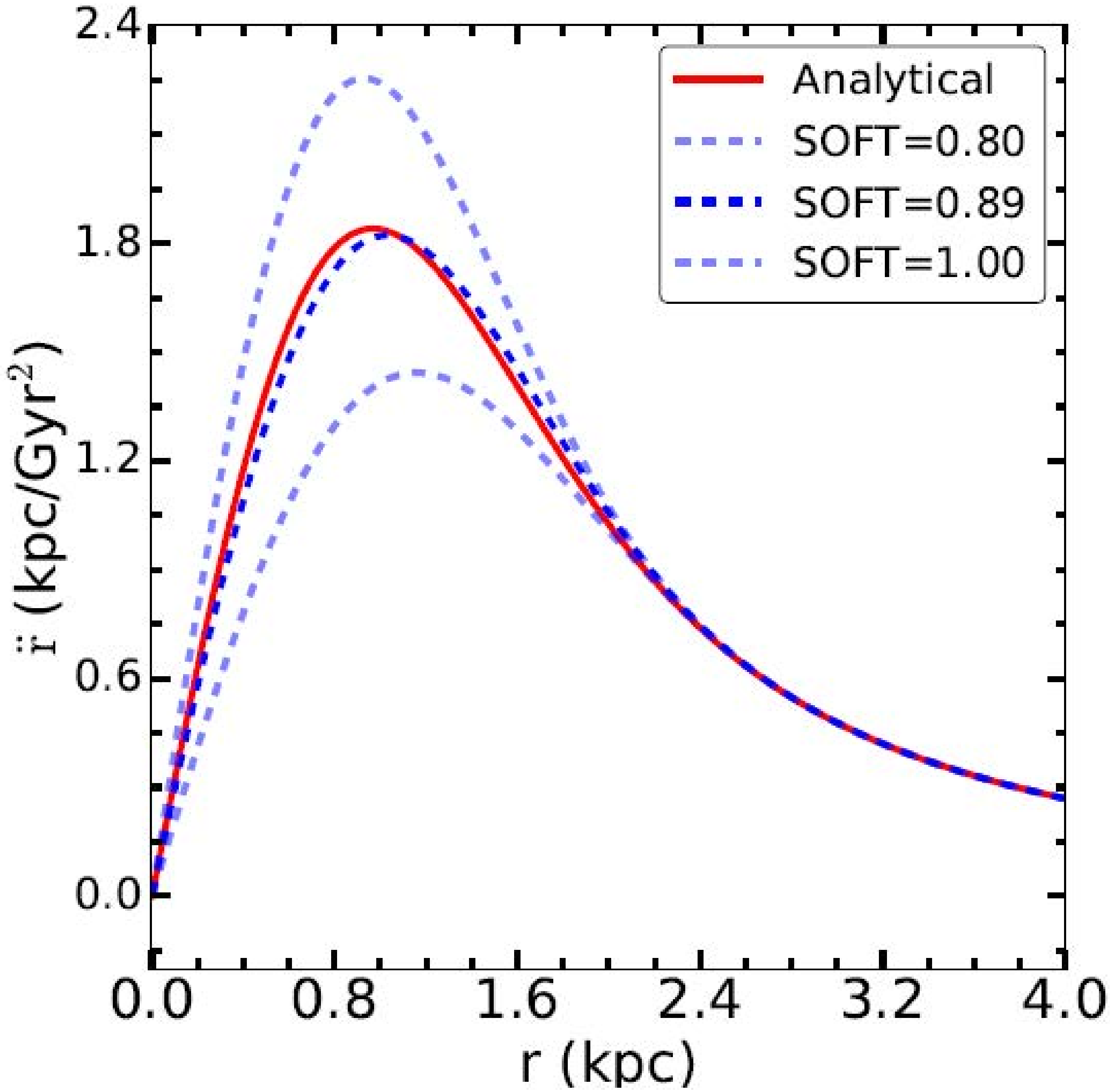
Conclusions

- FDM can solve the cusp-core problem with solitonic core.
- Missing satellite problem may be alleviated because low-mass sub halos are much smaller in FDM than CDM. FDM sub halos are always disrupted by tidal field. The power spectrum of FDM density perturbation is suppressed at small mass scale.
- We need cosmological simulations to show that FDM can solve the missing satellite and too-big-to-fail problems.
- The lower mass bound on FDM is about a few times 10^{-22} eV, due to an observation on the reionization history of Universe.

Backup Slides



SOFTening length: a numerical paramter for CDM, a physical paramter for FDM!



$$\ddot{r} = \frac{GM(<r)}{r^3} r,$$

$$\frac{M(<r)}{M(r=\infty)} = \frac{\int_0^r \exp(\frac{-2r^2}{\lambda^2}) 4\pi r^2 dr}{\int_0^\infty \exp(\frac{-2r^2}{\lambda^2}) 4\pi r^2 dr}$$

$$= \mathrm{erf}(\frac{\sqrt{2}r}{\lambda}) - 1.13 \exp(\frac{-2r^2}{\lambda^2}) \frac{\sqrt{2}r}{\lambda}.$$

Plug it in Gadget2!

Schrodinger-Possion Equations

The nature of FDM can be well described by the Schrödinger-Poisson equations,

$$i\hbar\frac{d\Psi}{dt} = -\frac{\hbar^2}{2m_\chi}\boldsymbol{\nabla}^2\Psi + m_\chi V\Psi,$$

$$\boldsymbol{\nabla}^2 V = 4\pi G m_\chi |\Psi|^2.$$

The wave function Ψ can be written as

$$\Psi = \sqrt{\frac{\rho}{m_\chi}}\exp(\frac{iS}{\hbar})$$

in terms of the number density $\frac{\rho}{m_\chi}$, while we can define the gradient of S to be DM momentum,

$$\boldsymbol{\nabla} S = m_\chi \boldsymbol{v}.$$ (

Particle-particle Implementation of Quantum Pressure

the mass density ρ can be discretized as

$$\rho(\boldsymbol{r}) = \sum_i m_i \delta(\boldsymbol{r} - \boldsymbol{r_i}), \quad \delta(\boldsymbol{r} - \boldsymbol{r_i}) = \frac{1}{2\sqrt{2}\lambda^3\pi^{3/2}} \exp(-\frac{2|\boldsymbol{r} - \boldsymbol{r_i}|^2}{\lambda^2}),$$

$$K_\rho = \int \frac{\hbar^2}{2m_\chi^2} (\boldsymbol{\nabla}\sqrt{\rho})^2 d^3x. \qquad m_j \ddot{q}_j = -\frac{\partial K_\rho}{\partial q_j}$$

Additional acceleration due to quantum pressure:

$$\ddot{\boldsymbol{r}} = \frac{4M\hbar^2}{M_0 m_\chi^2 \lambda^4} \sum_j \exp\left[-\frac{2|\boldsymbol{r} - \boldsymbol{r_j}|^2}{\lambda^2}\right] (1 - \frac{2|\boldsymbol{r} - \boldsymbol{r_j}|^2}{\lambda^2})(\boldsymbol{r_j} - \boldsymbol{r}).$$

After solving the Schrödinger-Poisson equations, from the real and imaginary parts of the soluti
one can obtain the continuity equation,

$$\frac{d\rho}{dt} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0,$$

and the momentum-conservation equation,

$$\frac{d\boldsymbol{v}}{dt} + (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v} = -\boldsymbol{\nabla}(Q + V),$$

where we have defined the quantum pressure as

$$Q = -\frac{\hbar^2}{2m_\chi^2} \frac{\boldsymbol{\nabla}^2 \sqrt{\rho}}{\sqrt{\rho}}.$$

To discuss the effects of quantum pressure, the Hamiltonian w/o gravity:

$$H = \int \frac{\hbar^2}{2m_\chi} |\boldsymbol{\nabla}\Psi|^2 d^3x = \int \frac{\rho}{2} |\boldsymbol{v}|^2 d^3x + \int \frac{\hbar^2}{2m_\chi^2} (\boldsymbol{\nabla}\sqrt{\rho})^2 d^3x.$$

$$T = \int \frac{\rho}{2} |\boldsymbol{v}|^2 d^3x = \sum_j \frac{1}{2} m_j (\frac{dq_j}{dt})^2, \qquad K_\rho = \int \frac{\hbar^2}{2m_\chi^2} (\boldsymbol{\nabla}\sqrt{\rho})^2 d^3x.$$

K.E. P.E. term from quantum pressure

But it was only in the dilute limit. We need a correction factor.

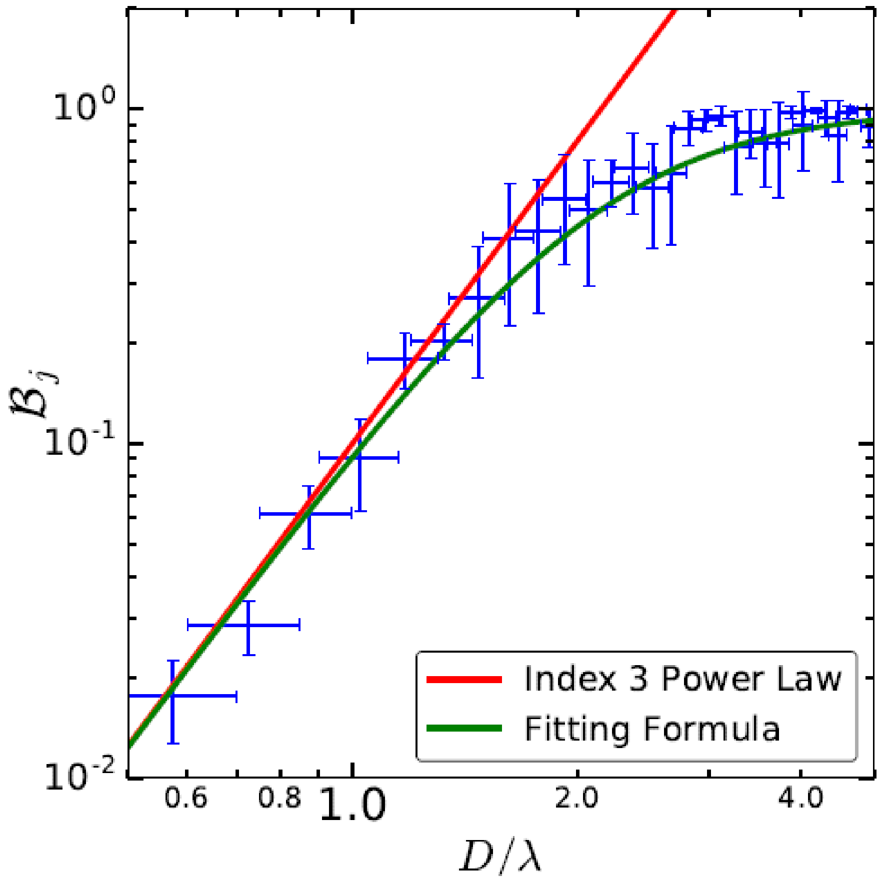
$$\ddot{\boldsymbol{r}} = \frac{4M\hbar^2}{M_0 m_\chi^2 \lambda^4} \sum_j \mathcal{B}_j \exp\left[-\frac{2|\boldsymbol{r}-\boldsymbol{r_j}|^2}{\lambda^2}\right](1-\frac{2|\boldsymbol{r}-\boldsymbol{r_j}|^2}{\lambda^2})(\boldsymbol{r_j}-\boldsymbol{r}).$$

$$\mathcal{B}_j = (D/\lambda)^3/(10+(D/\lambda)^3).$$

Correction in the high density environment!

D^3 is the mean volumn of particles.

When D is small means high number density.



Simulating by putting a large number of particles in a cubic box, and calculate the ratio. Bj -> 1 for D very large.