Probing Reheating Temperature with primordial GW:

its application to Gauss-Bonnet inflation

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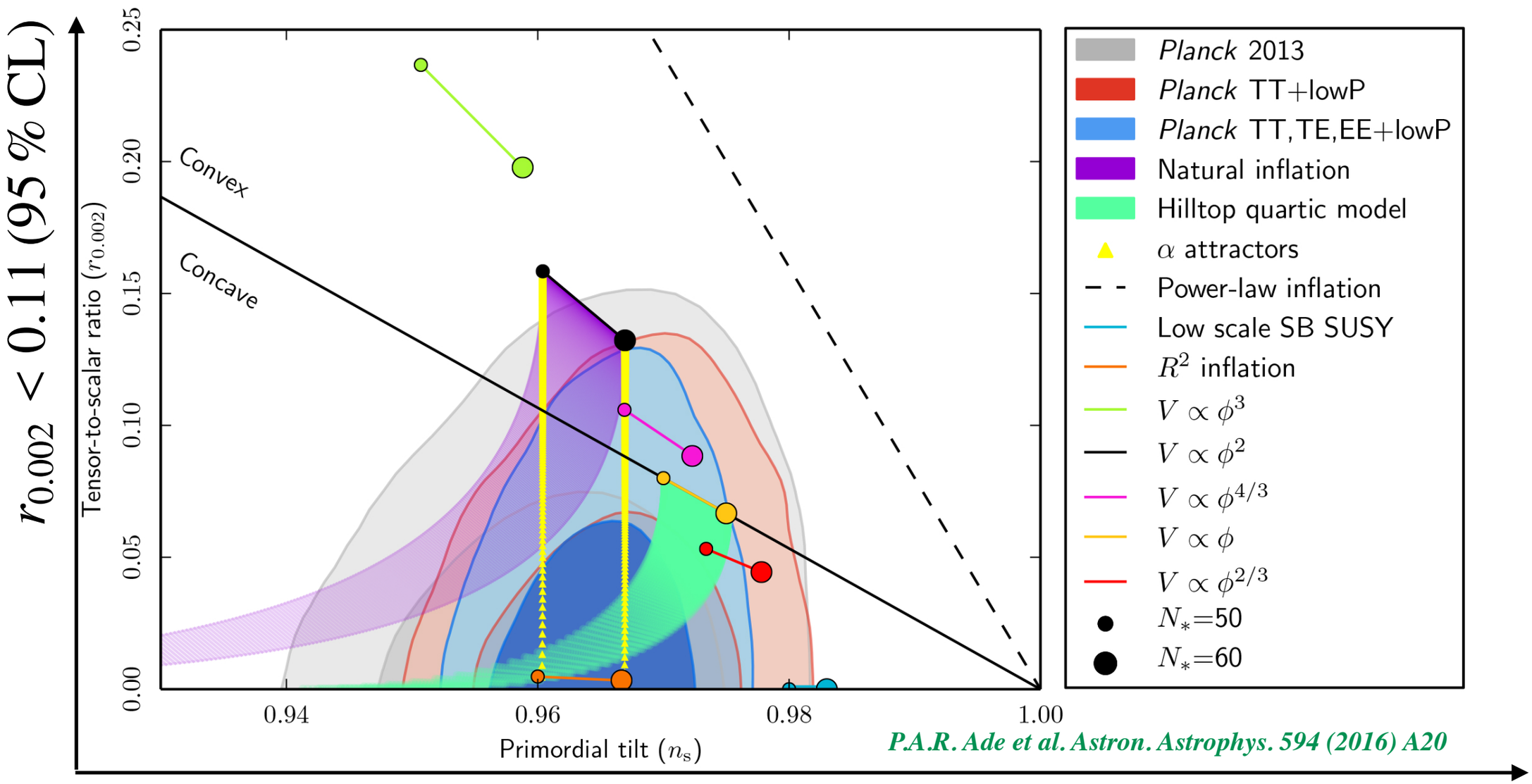
- ***We have hundreds of inflation models that are being consistent with the observational data***



P.A.R. Ade et al. Astron. Astrophys. 594 (2016) A20

$$n_s = 0.9655 \pm 0.0062 \ (68\,\%\ \mathrm{CL},\ \textit{Planck}\ \mathrm{TT{+}lowP})$$

- ***The simplest scenario is based upon a single field, which is minimally coupled to a gravity, with a flat potential;***

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right],$$

- ***In light of both the current and future observations, extended models of inflation seem to be more promising!***

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$$\Downarrow$$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \boxed{- \frac{1}{2}\xi(\phi)R^2_{\rm GB}}\right],$$

$$R^2_{\rm GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

- ***In the early universe, approaching the Planck era, it is quite natural to consider corrections like this.***

Thus, we are interested in understanding the effects of this additional term

- ***during inflation and reheating***
- ***its contribution to the Primordial GW spectra***

- **Here, it is important to have:** $\xi(\phi) \neq const\,.$

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- **because of the background EoM in flat FRW universe:**

$$ds^2 = -dt^2 + a^2\left(dr^2 + r^2 d\Omega^2\right),$$

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot\phi^2 + V + 12\dot\xi H^3\right),$$

$$\dot H = -\frac{\kappa^2}{2}\left[\dot\phi^2 - 4\ddot\xi H^2 - 4\dot\xi H\left(2\dot H - H^2\right)\right],$$

$$\ddot\phi + 3H\dot\phi + V_\phi + 12\xi_\phi H^2\left(\dot H + H^2\right) = 0\,,$$

- **Observable quantities are obtained as,**

$$\mathscr{P}_S(k) = \mathscr{P}_S(k_*)\left(\frac{k}{k_*}\right)^{n_S-1}, \qquad \mathscr{P}_T(k) = \mathscr{P}_T(k_*)\left(\frac{k}{k_*}\right)^{n_T}$$

$$n_S - 1 \simeq -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}, \qquad n_T \simeq -2\epsilon\,, \qquad r \simeq 8(2\epsilon - \delta_1)\,,$$

where $\epsilon \equiv -\frac{\dot H}{H^2}, \quad \eta \equiv \frac{\ddot H}{H\dot H}, \quad \zeta \equiv \frac{\dddot H}{H^2\dot H}, \quad \delta_1 \equiv 4\kappa^2\dot\xi H\,, \quad \delta_2 \equiv \frac{\ddot\xi}{\dot\xi H}, \quad \delta_3 = \frac{\dddot\xi}{\dot\xi H^2}\,.$

S. Koh, B. H. Lee, W. Lee and GT, PRD 90, 063527 (2014)

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$$n_T = -2\epsilon$$

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n_T can be either negative or positive. If it is positive (negative), then the tensor power spectrum is called "blue-tilted" ("red-tilted").

Class-I: $n_T < 0, \longrightarrow \epsilon > 0,$

$$n_T = -2\epsilon$$

Class-II: $n_T > 0, \longrightarrow \epsilon < 0,$

$$\begin{cases} \xi_\phi > -\frac{3}{4\kappa^4}\frac{V_\phi}{V^2}, & \text{for} \quad V_\phi > 0, \\ \xi_\phi < -\frac{3}{4\kappa^4}\frac{V_\phi}{V^2}, & \text{for} \quad V_\phi < 0. \end{cases}$$

$$\epsilon = \frac{1}{2\kappa^2}\frac{V_\phi}{V}\left(\frac{V_\phi}{V} + \frac{4}{3}\kappa^4\xi_\phi V\right).$$

$$\begin{cases} \xi_\phi < -\frac{3}{4\kappa^4}\frac{V_\phi}{V^2}, & \text{for} \quad V_\phi > 0, \\ \xi_\phi > -\frac{3}{4\kappa^4}\frac{V_\phi}{V^2}, & \text{for} \quad V_\phi < 0. \end{cases}$$

Model-I: $$V(\phi) = \frac{V_0}{\kappa^4}(\kappa\phi)^n, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n},$$

Model-II: $$V(\phi) = \frac{1}{\kappa^4}\left[\tanh(\kappa\phi) + \sqrt{\mu}\,\mathrm{sech}(\kappa\phi)\right]^2, \quad \xi(\phi) = \frac{3\left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}}\sinh(\kappa\phi)\right]}{4\left[\sqrt{\mu} + \sinh(\kappa\phi)\right]^2},$$

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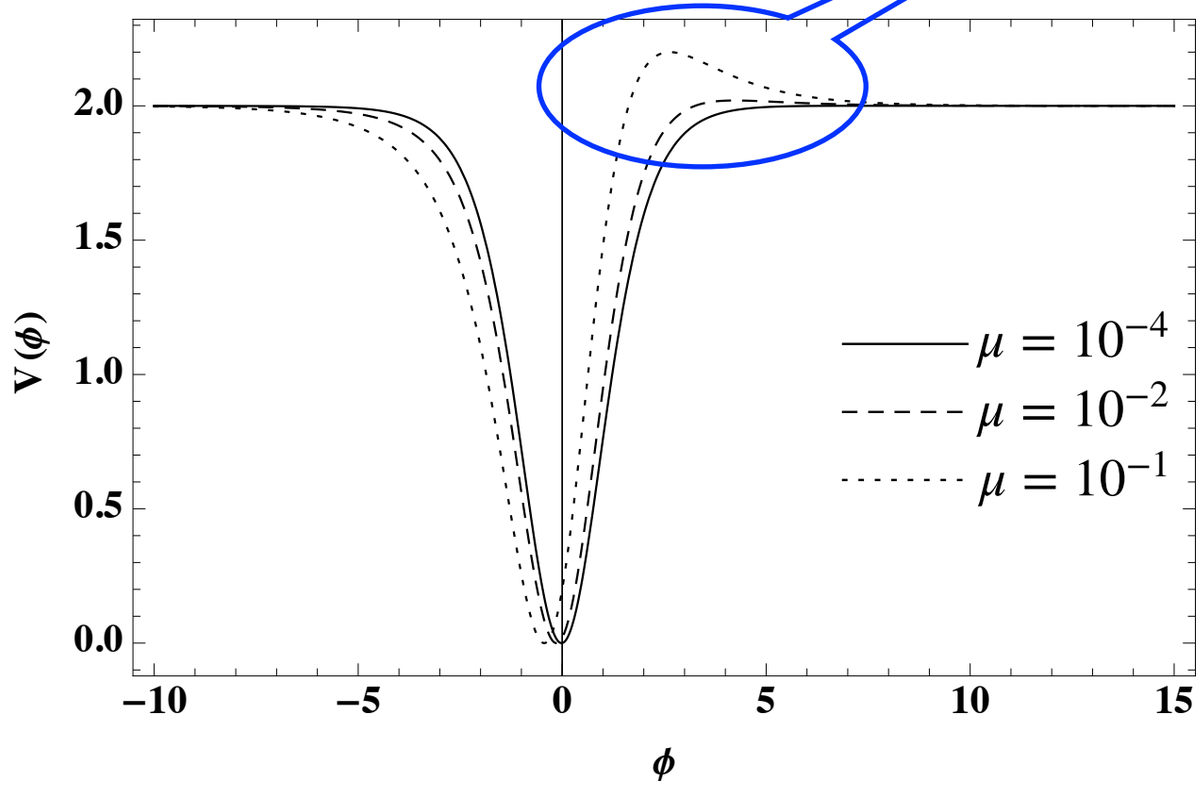

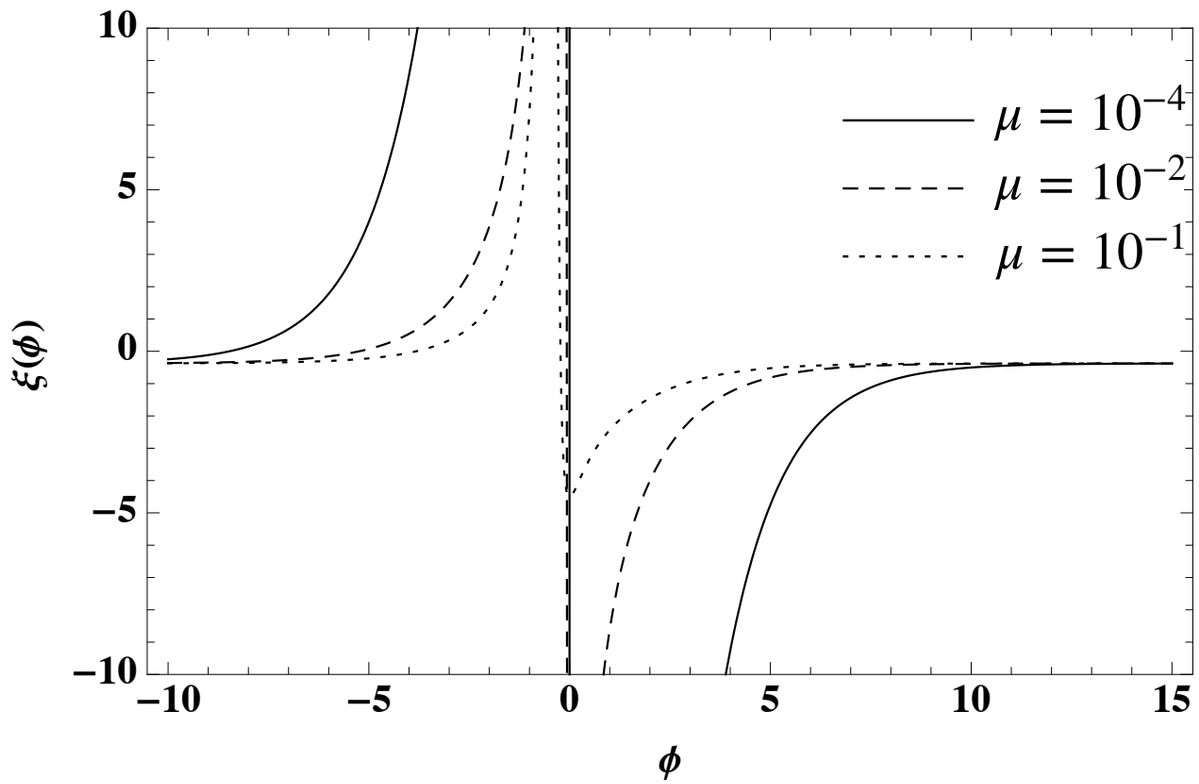

Model-I: $V(\phi) = \frac{V_0}{\kappa^4}(\kappa\phi)^n, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n}, \quad \Longrightarrow \quad \alpha \equiv \frac{4}{3}V_0\xi_0$

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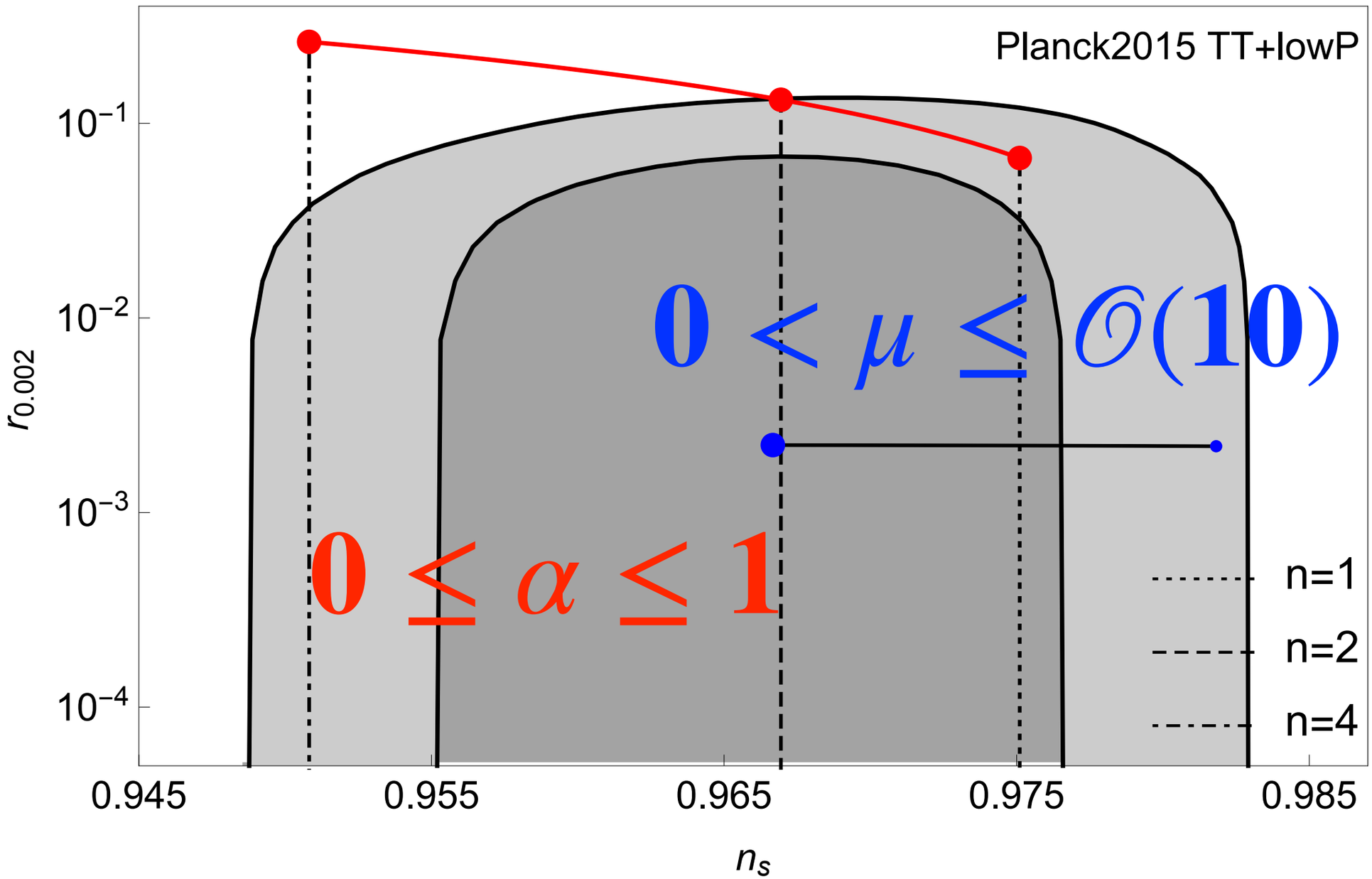

- **In addition,** $n_T < 0,$ **and** $n_T > 0.$

Part-II:
Primordial GW spectrum

- ***The pGWs are described by the tensor part of the pert. metric***

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right],$$

- ***The strength of GW is characterized by their energy spectrum***

$$\boxed{\Omega_{GW}(k) = \frac{1}{\rho_{\rm crit}}\frac{d\rho_{GW}}{d\ln k},}$$

$$\rho_{GW} = \frac{M_p^2}{4}\int d\ln k \left(\frac{k}{a}\right)^2 \frac{k^3}{\pi^2}\sum_\lambda \langle h^\dagger_{\lambda,k} h_{\lambda,k}\rangle.$$

$$\rho_{\rm crit} = 3H_0^2 M_p^2$$

$$\boxed{\Omega_{GW}(k) = \frac{k^2}{12H_0^2}P_T(k),}$$

$$P_T \equiv \frac{k^3}{\pi^2}\sum_\lambda \langle h^\dagger_{\lambda,k} h_{\lambda,k}\rangle = \mathcal{T}^2(k)\mathcal{P}_T(k).$$

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_*)\left(\frac{k}{k_*}\right)^{n_T + \frac{\alpha_T}{2}\ln(k/k_*)},$$

$$h''_{\lambda,k} + 2\frac{z'_T}{z_T}h'_{\lambda,k} + k^2 c_T^2 h_{\lambda,k} = 0,$$

$$\boxed{h_0^2\Omega_{GW} = \frac{3h_0^2}{32\pi^2 H_0^2\tau_0^4 f^2}\Omega_m^2\mathcal{T}_1^2\left(\frac{f}{f_{\rm eq}}\right)\mathcal{T}_2^2\left(\frac{f}{f_{\rm th}}\right) r\,\mathcal{P}_S\left(\frac{f}{f_*}\right)^{n_T + \frac{\alpha_T}{2}\ln(f/f_*)},}$$

- ***Numerical results:***

$$h_0^2\Omega_{GW} = \frac{3h_0^2}{32\pi^2 H_0^2 \tau_0^4 f^2}\Omega_m^2 \mathcal{T}_1^2\left(\frac{f}{f_{\rm eq}}\right)\mathcal{T}_2^2\left(\frac{f}{f_{\rm th}}\right) r\,\mathcal{P}_S\left(\frac{f}{f_*}\right)^{n_T+\frac{\alpha_T}{2}\ln(f/f_*)} ,$$

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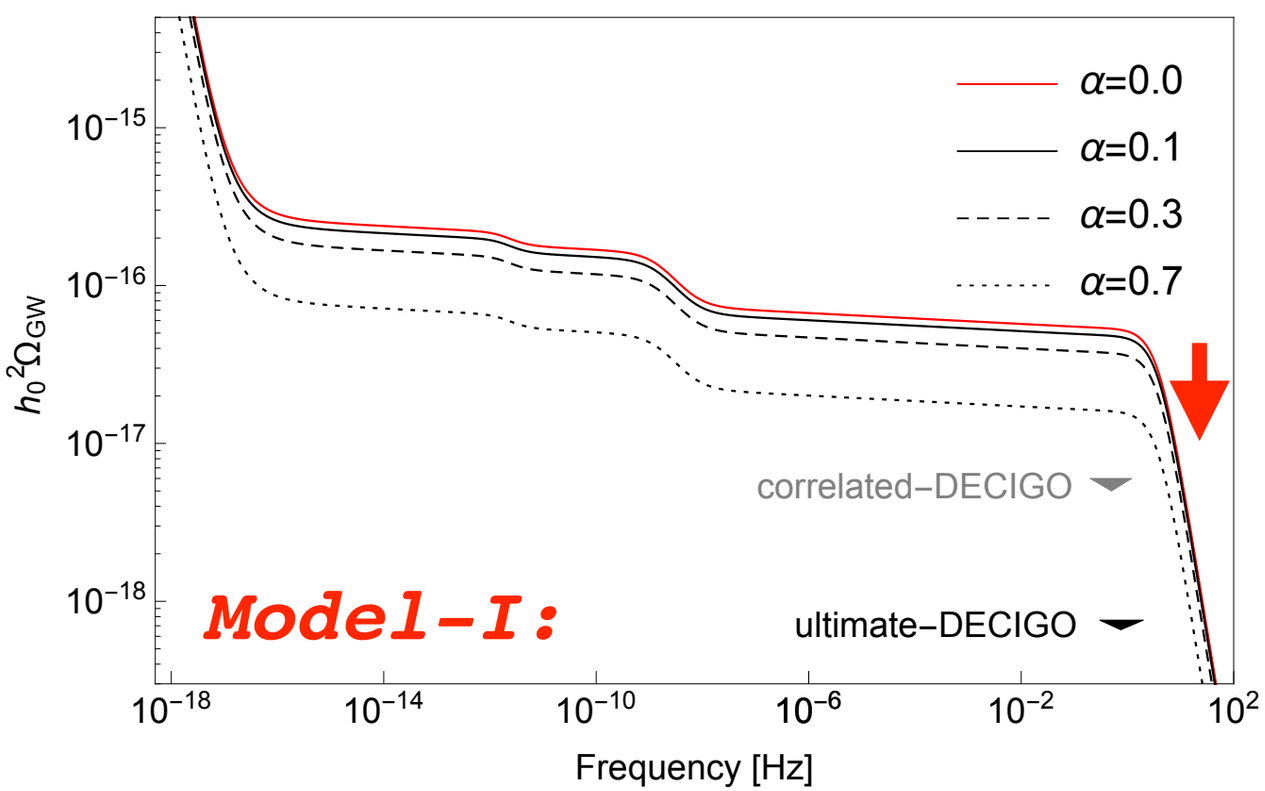
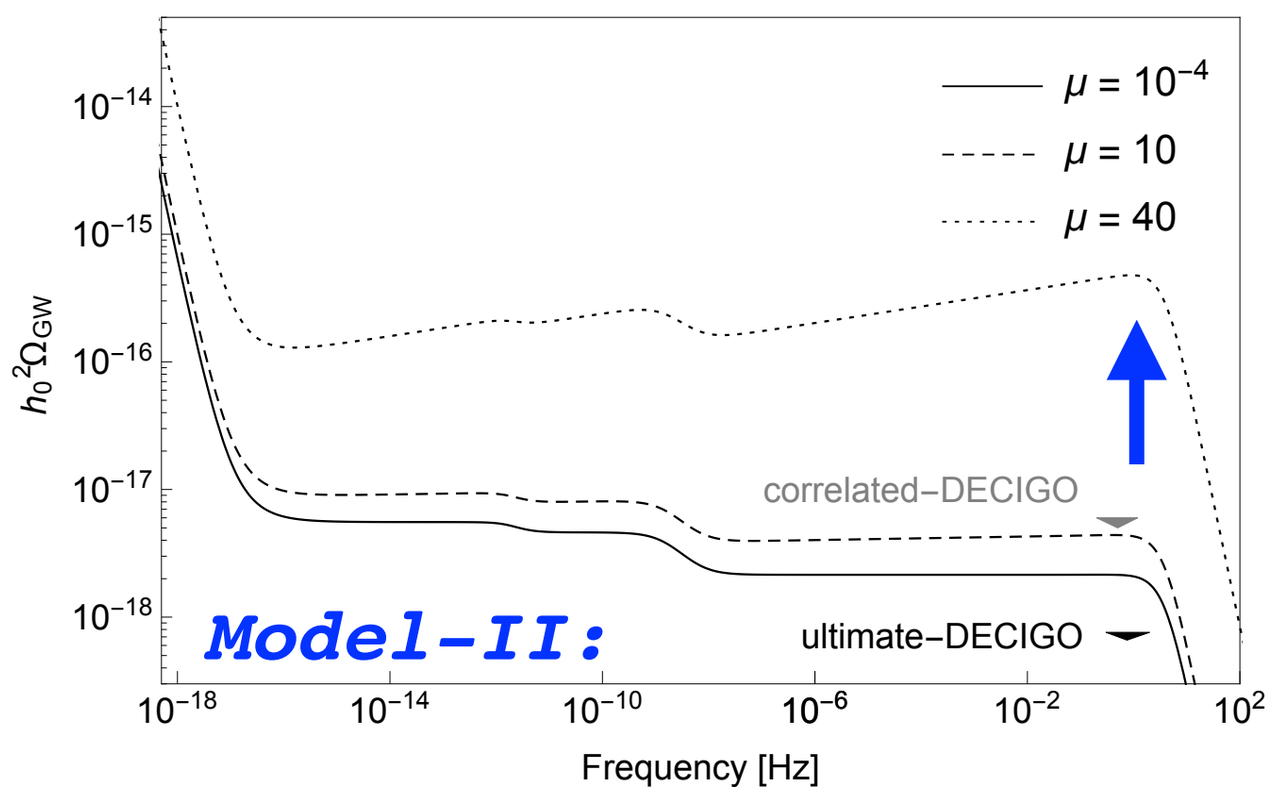
Model-II: $V(\phi) = \frac{1}{\kappa^4}\left[\tanh(\kappa\phi) + \sqrt{\mu}\,\mathrm{sech}(\kappa\phi)\right]^2 , \quad \xi(\phi) = \frac{3\left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}}\sinh(\kappa\phi)\right]}{4\left[\sqrt{\mu} + \sinh(\kappa\phi)\right]^2} ,$





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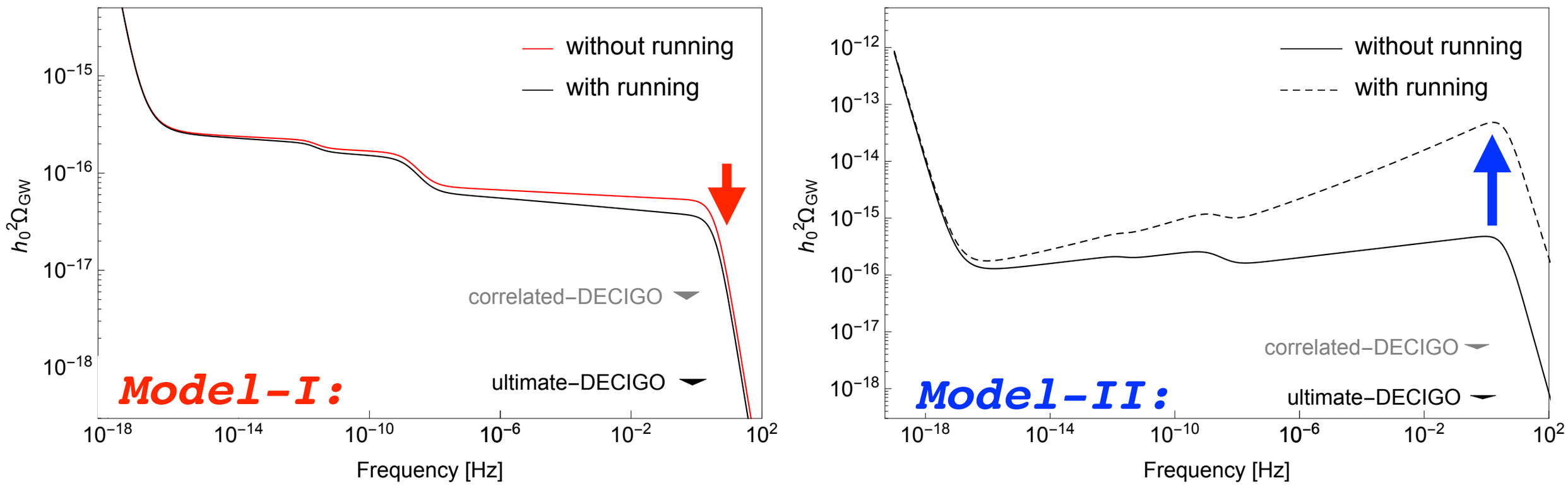

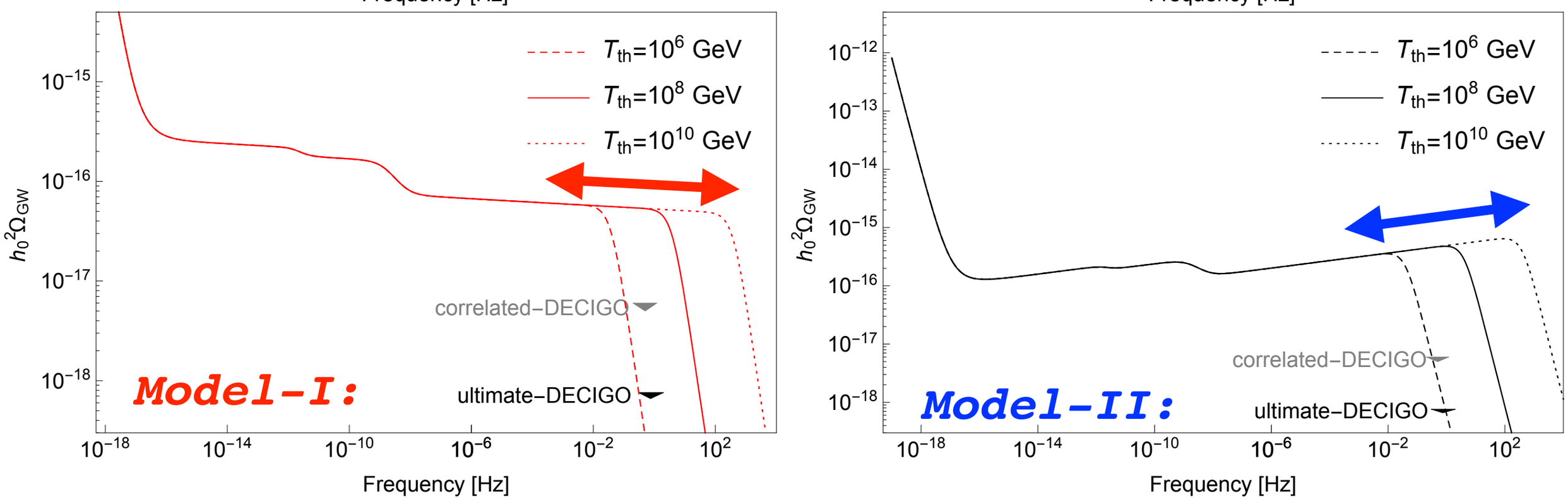

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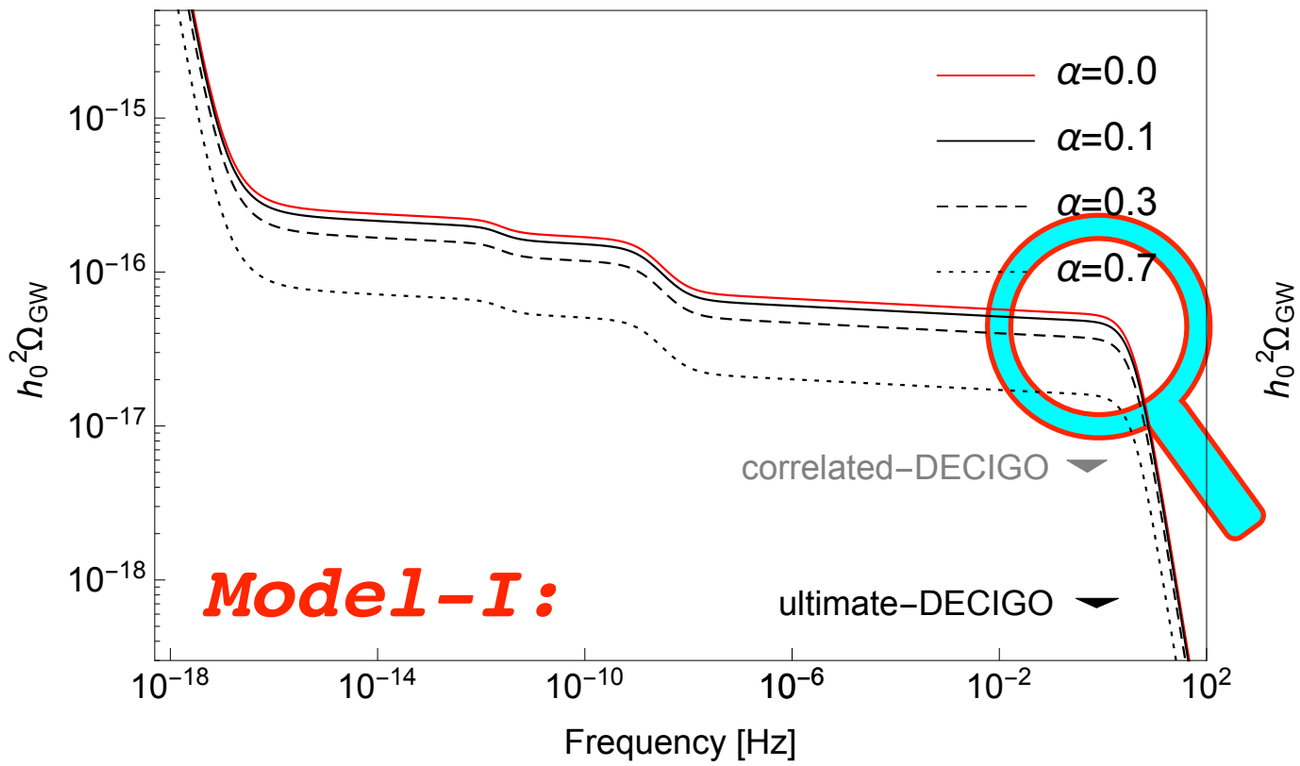

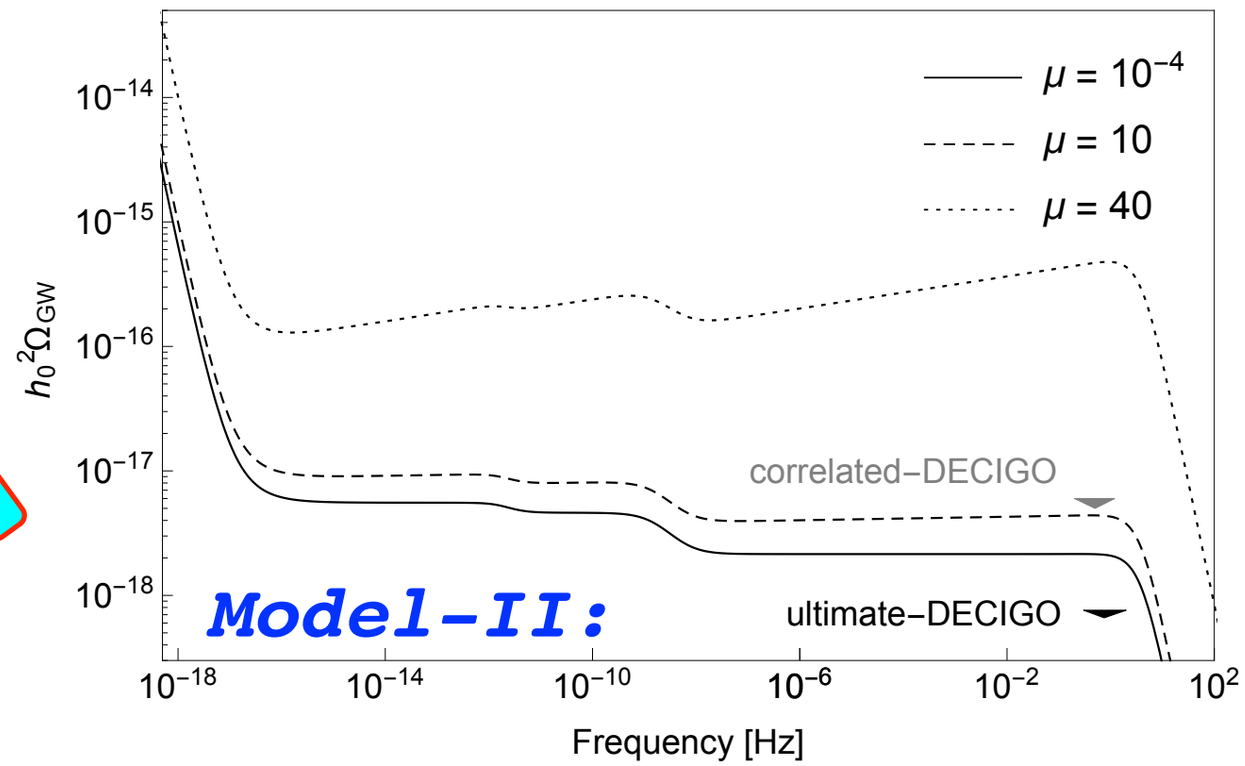

$$\mathcal{T}_1^2\left(\frac{k}{k_{\rm eq}}\right) = 1 + 1.65\left(\frac{k}{k_{\rm eq}}\right) + 1.92\left(\frac{k}{k_{\rm eq}}\right)^2, \qquad \mathcal{T}_2^2\left(\frac{k}{k_{\rm th}}\right) = \left[1+\gamma\left(\frac{k}{k_{\rm th}}\right)^{\frac{3}{2}} + \sigma\left(\frac{k}{k_{\rm th}}\right)^2\right]^{-1},$$

- ***Once the pGWs from inflationary origin are detected:***

$$k_{\rm th} = 1.7\times 10^{13}\,\textbf{Mpc}^{-1}\left(\frac{g_{*s}(T_{\rm th})}{106.75}\right)^{\frac{1}{6}}\left(\frac{T_{\rm th}}{10^6\,\textbf{GeV}}\right)$$

Part-III:
Constraints on Reheating

- ***By assuming,***
  - ***constant equation-of-state during reheating,*** $\omega_{\rm th} = const,$
  - ***no entropy production after the end of reheating***

we calculate the duration of reheating and the thermalization temperature at the end of reheating,

$$N_{\rm th} = \frac{4}{3\omega_{\rm th}-1}\left[\ln\left(\frac{k}{a_0 T_0}\right) + \frac{1}{3}\ln\left(\frac{11 g_{*s}}{43}\right) + \frac{1}{4}\ln\left(\frac{30\lambda_{\rm end}}{\pi^2 g_*}\right) + \frac{1}{4}\ln\left(\frac{V_{\rm end}}{H_*^4}\right) + N_*\right].$$

$$T_{\rm th} = \left(\frac{30\lambda_{\rm end} V_{\rm end}}{\pi^2 g_*}\right)^{\frac{1}{4}} e^{-\frac{3}{4}(1+\omega_{\rm th})N_{\rm th}}. \quad \textbf{where} \quad \lambda_{\rm end} = \left.\frac{6}{6-2\epsilon-\delta_1(5-2\epsilon+\delta_2)}\right|_{\phi=\phi_{\rm end}}$$

- ***In our numerical study, we consider following models:***

Model-I: $$V(\phi) = \frac{V_0}{\kappa^4}(\kappa\phi)^n, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n}, \quad \Longrightarrow \quad \alpha \equiv \frac{4}{3}V_0\xi_0$$

Model-II: $$V(\phi) = \frac{1}{\kappa^4}\left[\tanh(\kappa\phi) + \sqrt{\mu}\,\mathrm{sech}(\kappa\phi)\right]^2, \quad \xi(\phi) = \frac{3\left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}}\sinh(\kappa\phi)\right]}{4\left[\sqrt{\mu}+\sinh(\kappa\phi)\right]^2},$$

- ***Numerical results:***

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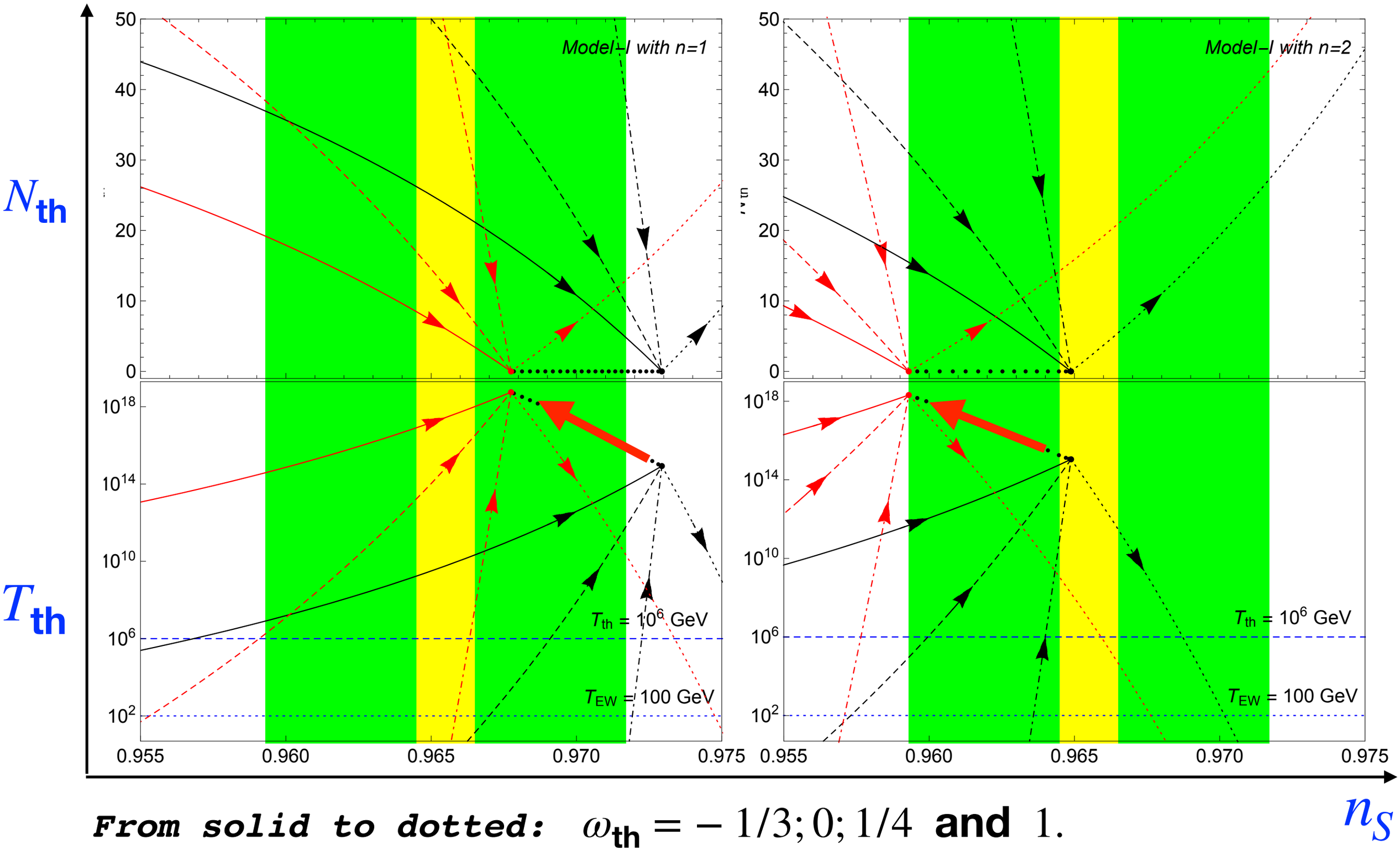

From solid to dotted: $\omega_{\rm th} = -1/3; 0; 1/4$ **and** $1.$

- ***Numerical results:***

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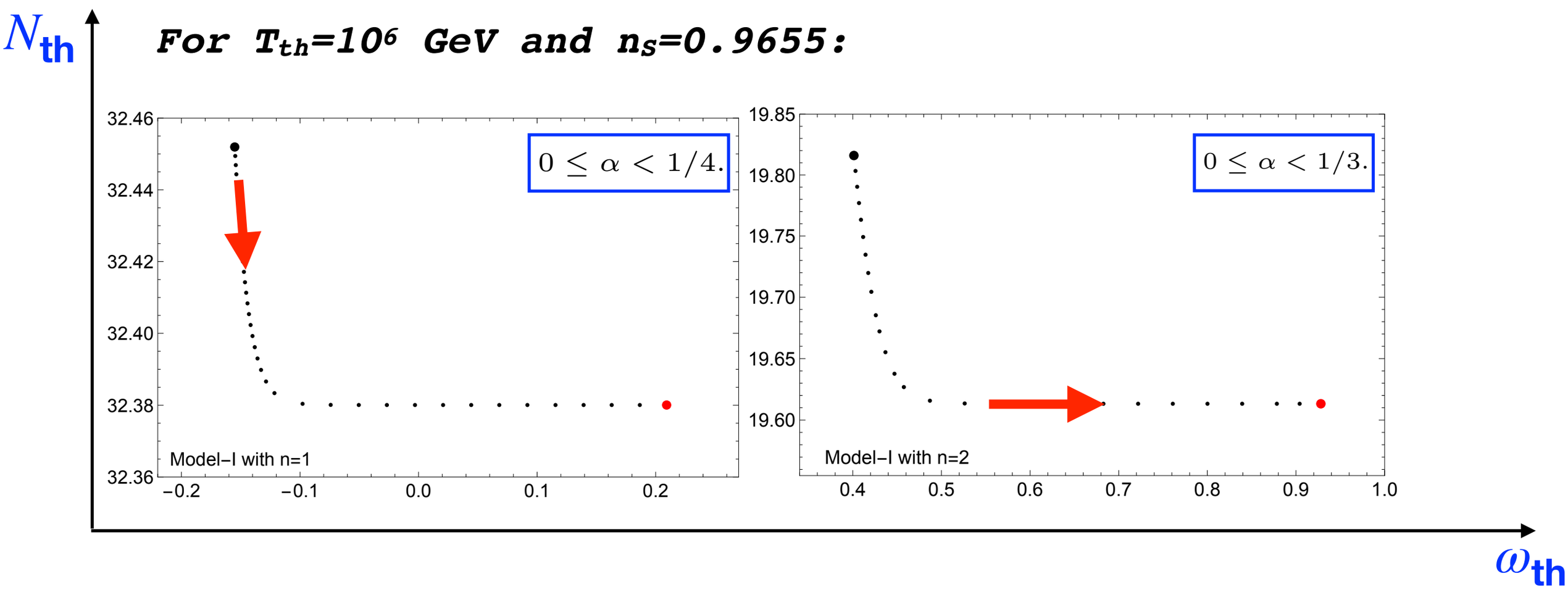
$$N_{\rm th} = \frac{4}{3\omega_{\rm th}-1}\left[-60.0085 + \frac{1}{4}\ln\left(\frac{3\lambda_{\rm end}}{100\pi^2}\right) + \frac{1}{4}\ln\left(\frac{V_{\rm end}}{H_*^4}\right) + N_*\right]$$

$$\lambda_{\rm end} = \left.\frac{6}{6-2\epsilon-\delta_1(5-2\epsilon+\delta_2)}\right|_{\phi=\phi_{\rm end}} > 0; \quad 0 \le \alpha \le 1 \quad \Rightarrow \quad 0 \le \alpha < \frac{n}{2n+2}\,.$$

For $T_{th}=10^6$ GeV and $n_s=0.9655$:



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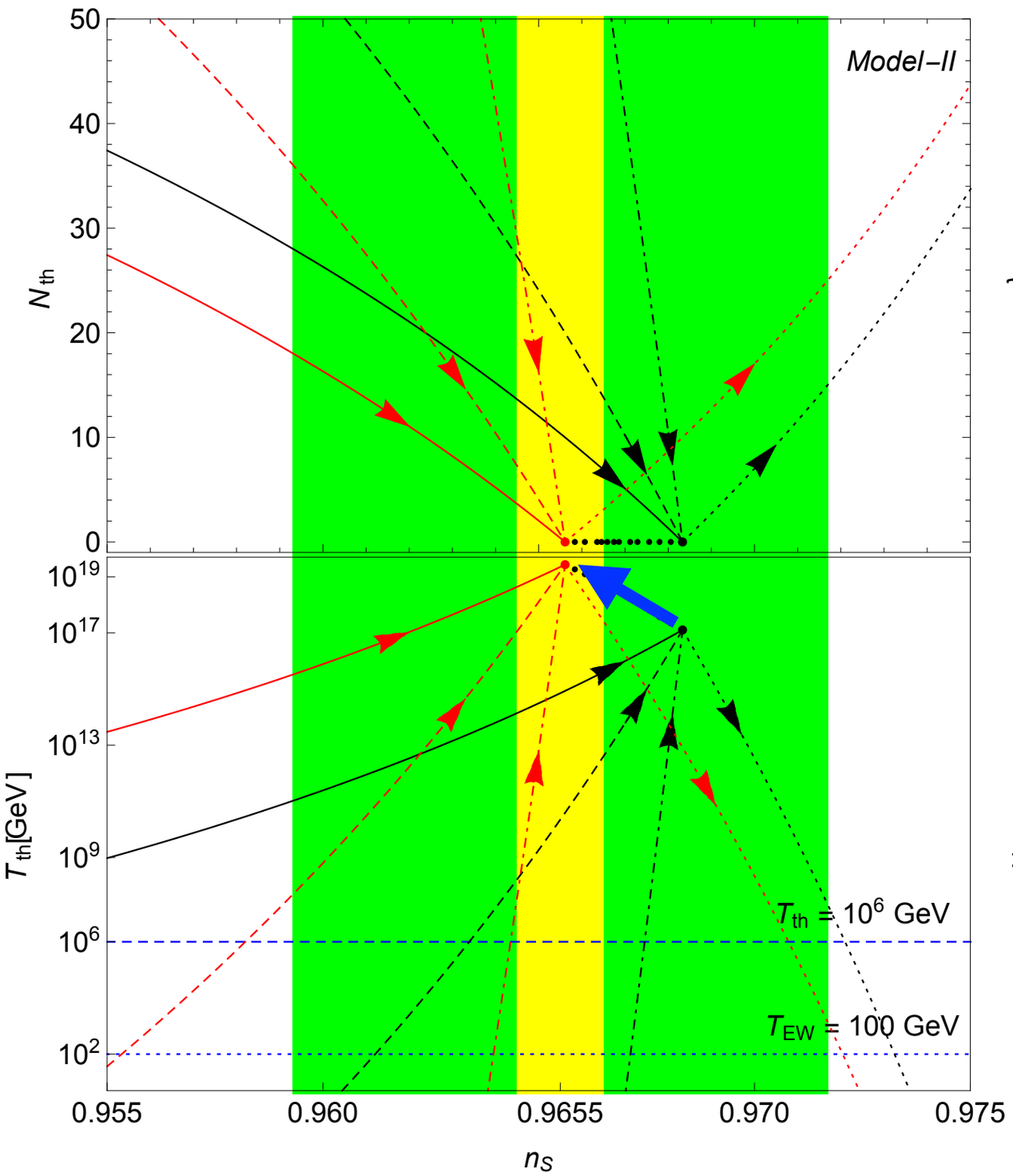

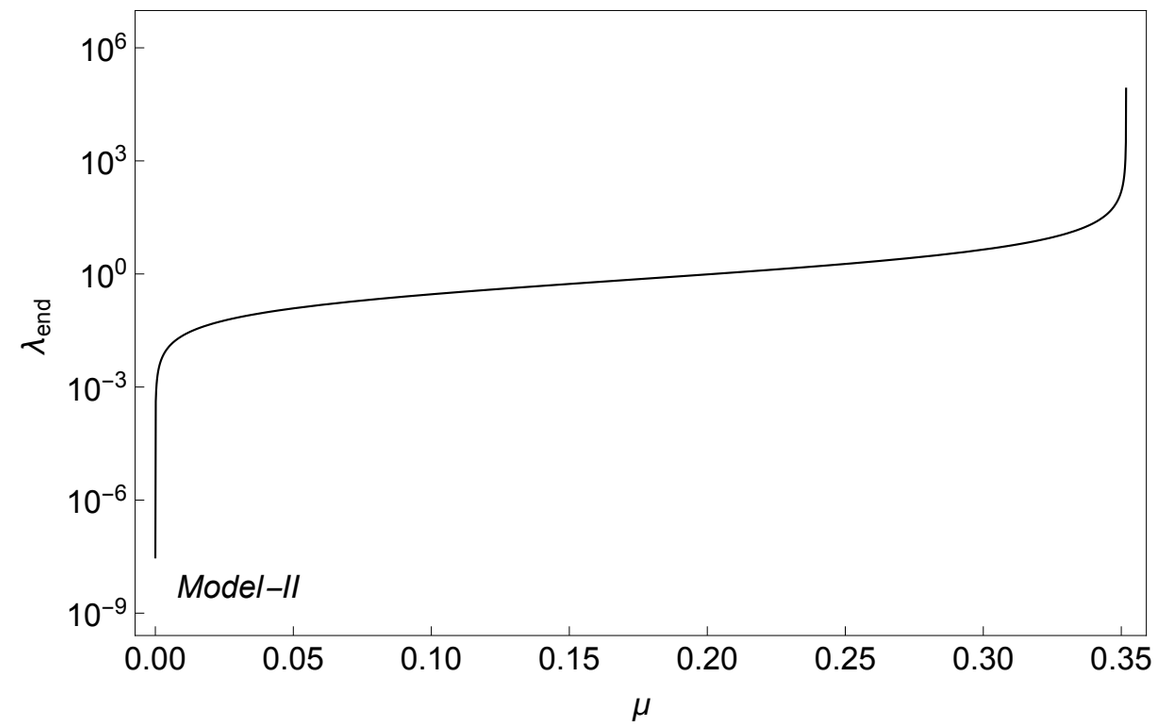

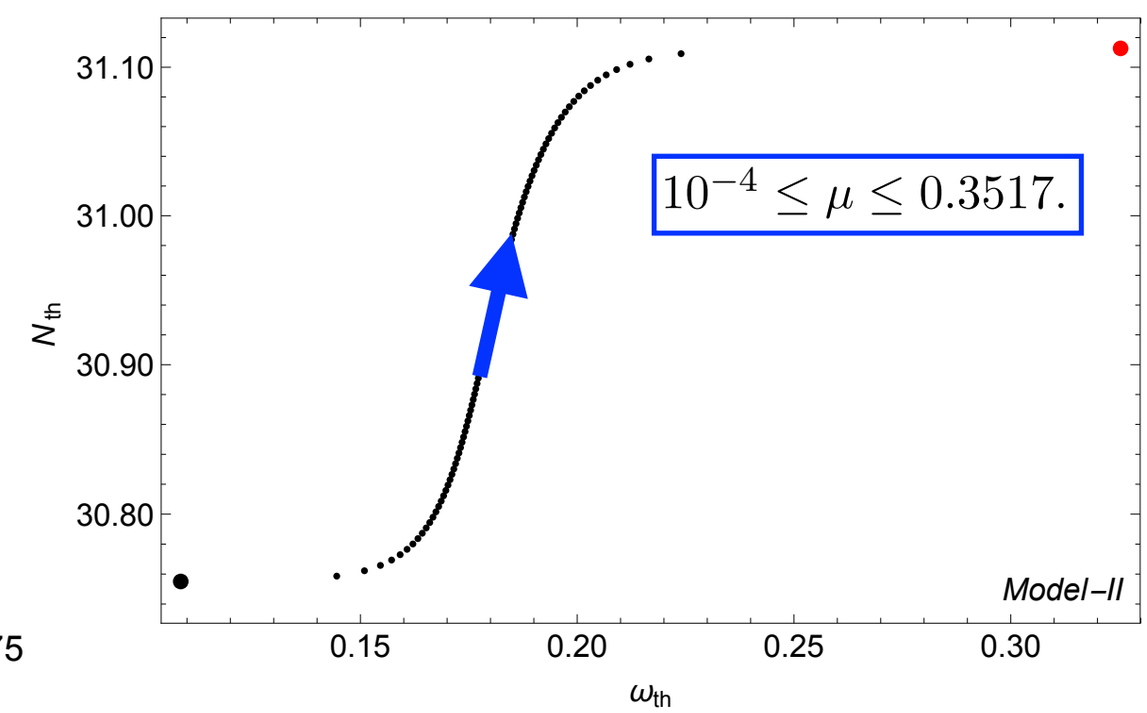

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CONCLUSSION:

- ***Inflationary models with a Gauss-Bonnet term are consistent with observational data,***
- ***These models predict both red- and blue-tilted inflationary tensor power spectrum,***
- ***Primordial GW spectrum suppresses (enhances) for $n_T<0$ ($n_T>0$),***
- ***Once pGWs are detected (DECIGO), T_{th} can be determined hence the other parameter of reheating including N_{th} and ω_{th} can also be determined.***
- ***T_{th} significantly increases in the presence of the GB term***
- ***Moreover, reheating can be used as an additional constraint to inflationary models!***

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THANK YOU FOR YOUR KIND ATTENTION!