Contribution of QCD Condensates to the OPE of Green Functions of Chiral Currents

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Introduction

- **What do we do?**
  - We study QCD at low energies.

- **What do we need?**
  - Chiral perturbation theory ($\chi$PT) and Resonance chiral theory ($R\chi T$).

- **What is it?**
  - Effective description of low-energy QCD.
  - $\chi$PT for $E \leq M_\rho$.
    - Spontaneous breaking of the chiral $SU(3)_L \times SU(3)_R$ symmetry down to $SU(3)_V$ in QCD leads to the presence of Goldstone bosons.
    - We identify them with the octet of pseudoscalar mesons ($\pi, K, \eta$) as the lightest hadronic observable states.
  - $R\chi T$ for $M_\rho \leq E \leq 2\text{ GeV}$.
    - $R\chi T$ increases the number of degrees of freedom of $\chi$PT by including massive $U(3)$ multiplets of vector $V(1^{--})$, axial-vector $A(1^{++})$, scalar $S(0^{++})$ and pseudoscalar $P(0^{-+})$ resonances.

- **What is it good for?**
  - To study important theoretical and phenomenological aspects of QCD.

- **What do we use?**
  - Green functions of chiral currents.
The amplitudes of physical processes can be computed using LSZ reduction formula from the Green functions, the time ordered products of quantum fields.

Three-point Green function:

$$\int d^4x \, d^4y \, e^{-i(p \cdot x + q \cdot y)} \langle 0 | T[O_1(x)O_2(y)O_3(0)] | 0 \rangle.$$ 

$O_i(x_i)$ stand either for

- vector and axial-vector currents:
  
  $$V_\mu^a(x) = \bar{q}(x)\gamma_\mu T^a q(x), \quad A_\mu^a(x) = \bar{q}(x)\gamma_\mu \gamma_5 T^a q(x),$$

- or scalar and pseudoscalar densities:
  
  $$S^a(x) = \bar{q}(x)T^a q(x), \quad P^a(x) = i\bar{q}(x)\gamma_5 T^a q(x).$$

Nontrivial three-point Green functions in QCD:

- Set I: $\langle SSS \rangle, \langle SPP \rangle, \langle VVP \rangle, \langle AAP \rangle, \langle VAS \rangle, \langle VVS \rangle, \langle AAS \rangle, \langle VAP \rangle$.
- Set II: $\langle VVA \rangle, \langle AAA \rangle, \langle VVV \rangle, \langle ASP \rangle, \langle AAV \rangle, \langle VSS \rangle, \langle VPP \rangle$. 

Green functions of chiral currents: Examples

\( \langle VVP \rangle \) Green function:

\[
\left[ \Pi_{VVP}(p,q;r) \right]^{abc}_{\mu\nu} = \Pi_{VVP}(p^2, q^2, r^2) d^{abc} \varepsilon_{\mu\nu}(p)(q) .
\]

Direct connection to many phenomenologically important quantities:

- Transition formfactor \( \mathcal{F}_{\pi^0\rightarrow\gamma\gamma}^{R\chi T} (p^2, q^2, r^2) = \frac{2r^2}{3B_0 F} \Pi_{VVP}^{R\chi T}(p^2, q^2, r^2) . \)
- Decays \( \rho \rightarrow \pi\gamma, \pi(1300) \rightarrow \gamma\gamma, \pi(1300) \rightarrow \rho\gamma . \)
- Significant importance in study of rare \( \pi^0 \) decays: \( \pi^0 \rightarrow e^+e^-\gamma, \pi^0 \rightarrow e^+e^- . \)
- Hadronic contribution to light-by-light scattering and \( g-2 . \)

\( \langle VVA \rangle \) Green function:

\[
\left[ \Pi_{VVA}(p,q;r) \right]^{abc}_{\mu\nu\rho} = d^{abc} \left[ w_L \varepsilon_{\mu\nu}(p)(q) r_{\rho} + w_T^{(1)} \Pi_{\mu\nu\rho}^{(1)} + w_T^{(2)} \Pi_{\mu\nu\rho}^{(2)} + w_T^{(3)} \Pi_{\mu\nu\rho}^{(3)} \right] .
\]

- The tensor part is nontrivial [M. Knecht et al. '04].
- The structure is thus given by longitudinal formfactor \( w_L \) (fixed by the anomaly) and three formfactors \( w_T^{(1)}, w_T^{(2)}, w_T^{(3)} . \)
- Related to the decay of axial resonance \( f_1(1285) \rightarrow \rho\gamma . \)
Special interest of ours: odd-intrinsic parity sector of QCD.

\langle VVP \rangle, \langle AAP \rangle, \langle VAS \rangle, \langle VVA \rangle, \langle AAA \rangle.

We assume the saturation of dynamics with the lightest resonances.

At the NLO, relevant Lagrangian in the odd-intrinsic parity sector was formulated for the first time in [K. Kampf and J. Novotný '11]:

\[ \mathcal{L}_{R\chi T}^{(6)} = \sum_X \sum_i \kappa_i^X \hat{\mathcal{O}}_{i \mu \nu \alpha \beta} \varepsilon^{\mu \nu \alpha \beta}. \]

\(X\) stands for the resonance fields \(V, A, S, P\), double-resonance fields \(VV, AA, SA, SV, VA, PA, PV\) and triple-resonance fields \(VVP, VAS, AAP\).

\(\mathcal{L}_{R\chi T}^{(6)}\): 67 operators and 67 corresponding unknown couplings \(\kappa_i^X\) in total.

Goal: express \(\kappa_i^X\) in terms of known parameters \((F, F_V, F_A, M_V, M_A\) etc.).

Our approach: high-energy behaviour of Green functions within OPE.
Operator Product Expansion
Framework to study behaviour at high energies.

For $x, y \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(0) \rangle = C_1 1 + C_{\langle \bar{q}q \rangle} \langle \bar{q}q \rangle + C_{\langle G^2 \rangle} \langle G_{\mu\nu}G^{\mu\nu} \rangle + C_{\langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \rangle} \langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \rangle + C_{\langle 4q \rangle} \langle \bar{q}\Gamma_1 q\bar{q}\Gamma_2 q \rangle + C_{\langle G^3 \rangle} \langle G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu} \rangle f^{abc} + \ldots$$
Framework to study behaviour at high energies.

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$$\langle O_1(x)O_2(y)O_3(0) \rangle = C_1 1 + C_{\langle \bar{q}q \rangle} \langle \bar{q}q \rangle + C_{\langle G^2 \rangle} \langle G_{\mu\nu} G^{\mu\nu} \rangle$$

$$+ C_{\langle \bar{q}Gq \rangle} \langle \bar{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle + C_{\langle 4q \rangle} \langle \bar{q} \Gamma_1 q \bar{q} \Gamma_2 q \rangle$$

$$+ C_{\langle G^3 \rangle} \langle G_{\mu\nu}^a G_{\nu\sigma}^b G_{\sigma\mu}^c \rangle f^{a b c} + \ldots$$

Perturbative contribution.
Operator Product Expansion

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- Perturbative contribution.
- QCD condensates (with dimension $D \leq 6$):
  - Quark condensate.
Framework to study behaviour at high energies.

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\]

Perturbative contribution.

QCD condensates (with dimension \( D \leq 6 \)):
- Quark condensate.
- Gluon condensate.
Framework to study behaviour at high energies.

For $x, y \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$
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$$

Perturbative contribution.

QCD condensates (with dimension $D \leq 6$):

- Quark condensate.
- Gluon condensate.
- Quark-gluon condensate.
Operator Product Expansion

- Framework to study behaviour at high energies.
- For $x, y \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with $c$-number coefficients:

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(0) \rangle = C_1 1 + C_{\langle \overline{q} q \rangle} \langle \overline{q} q \rangle + C_{\langle G^2 \rangle} \langle G_{\mu\nu} G^{\mu\nu} \rangle + C_{\langle \overline{q} G q \rangle} \langle \overline{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle + C_{\langle 4q \rangle} \langle \overline{q} \Gamma_1 q \overline{q} \Gamma_2 q \rangle + C_{\langle G^3 \rangle} \langle G^{a\mu}_{\mu} G^{b\nu}_{\nu} G^{c\sigma}_{\sigma} \rangle f^{abc} + \ldots$$

- Perturbative contribution.
- QCD condensates (with dimension $D \leq 6$):
  - Quark condensate.
  - Gluon condensate.
  - Quark-gluon condensate.
  - Four-quark condensate.
Framework to study behaviour at high energies.

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+ C_{\langle G^{3} \rangle} \langle G_{\mu\nu}^{a}G_{\nu\sigma}^{b}G_{\sigma\mu}^{c} \rangle f^{abc} + \ldots
\]

Perturbative contribution.

QCD condensates (with dimension \( D \leq 6 \)):

- Quark condensate.
- Gluon condensate.
- Quark-gluon condensate.
- Four-quark condensate.
- Three-gluon condensate (vanishes in the chiral limit).
Matching $R \chi T$ to OPE with two large momenta
Calculation in RχT at NLO leads to the formfactors [TK, K. Kampf and J. Novotný ’18]:

\[ w_L(p^2, q^2, r^2) = \frac{N_c}{8\pi^2 r^2}, \]

\[ w_T^{(1)}(p^2, q^2, r^2) = - \frac{2\sqrt{2}F_V [\kappa_{17}^V(p^2 + q^2 - 2M_V^2) - \sqrt{2}F_V \kappa_{3}^V]}{(p^2 - M_V^2)(q^2 - M_V^2)}, \]

\[ w_T^{(2)}(p^2, q^2, r^2) = - \frac{2\sqrt{2}F_V (p^2 - q^2)(2\kappa_{12}^V + \kappa_{16}^V - \kappa_{17}^V)}{(p^2 - M_V^2)(q^2 - M_V^2)}, \]

\[ w_T^{(3)}(p^2, q^2, r^2) = \frac{2\sqrt{2}F_V (p^2 - q^2)}{(p^2 - M_V^2)(q^2 - M_V^2)} \left(2\kappa_{11}^V + 2\kappa_{12}^V - \kappa_{17}^V - \frac{\sqrt{2}F_A \kappa_{5}^V}{r^2 - M_A^2}\right). \]
Calculation in $R\chi T$ at NLO leads to the formfactors \cite{TK, K. Kampf and J. Novotný '18}:

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\begin{align*}
    w_L(p^2, q^2, r^2) &= \frac{N_c}{8\pi^2 r^2}, \\
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    w_T^{(2)}(p^2, q^2, r^2) &= -\frac{2\sqrt{2}F_V (p^2 - q^2)(2\kappa_{12}^V + \kappa_{16}^V - \kappa_{17}^V)}{(p^2 - M_V^2)(q^2 - M_V^2)}, \\
    w_T^{(3)}(p^2, q^2, r^2) &= \frac{2\sqrt{2}F_V (p^2 - q^2)}{(p^2 - M_V^2)(q^2 - M_V^2)} \left( 2\kappa_{11}^V + 2\kappa_{12}^V - \kappa_{17}^V - \frac{\sqrt{2}F_A \kappa_5^V}{r^2 - M_A^2} \right).
\end{align*}
\]

Phenomenologically important formfactor $w_T(Q^2)$:

\[
w_T(Q^2) = -16\pi^2 \left[ w_T^{(1)}(-Q^2, 0, -Q^2) + w_T^{(3)}(-Q^2, 0, -Q^2) \right].
\]
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\begin{align*}
  w_L(p^2, q^2, r^2) &= \frac{N_c}{8\pi^2 r^2}, \\
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  w_T^{(2)}(p^2, q^2, r^2) &= -\frac{2\sqrt{2} F_V (p^2 - q^2)(2\kappa_{12}^V + \kappa_{16}^V - \kappa_{17}^V)}{(p^2 - M_V^2)(q^2 - M_V^2)} , \\
  w_T^{(3)}(p^2, q^2, r^2) &= \frac{2\sqrt{2} F_V (p^2 - q^2)}{(p^2 - M_V^2)(q^2 - M_V^2)} \left( 2\kappa_{11}^V + 2\kappa_{12}^V - \kappa_{17}^V - \frac{\sqrt{2} F_A \kappa_{5}^{V A}}{r^2 - M_A^2} \right).
\end{align*}
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Phenomenologically important formfactor $w_T(Q^2)$:

\[
w_T(Q^2) = -16\pi^2 \left[ w_T^{(1)}(-Q^2, 0, -Q^2) + w_T^{(3)}(-Q^2, 0, -Q^2) \right].
\]

Idea: expand $w_T(Q^2)$ in terms of $Q^2$ up to $O\left(\frac{1}{Q^8}\right)$. 

Example: $\langle VVA \rangle$ Green function
Green function: $\langle V^* V A \rangle$

- Why? We know OPE for $\langle V^* V A \rangle$ [P. Colangelo et al. '12]:

$$w_T(Q^2) = \frac{N_c}{Q^2} + \frac{128\pi^3 \alpha_s \chi \langle \bar{q}q \rangle^2}{9Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right).$$

- One momentum soft, two momenta large.
- Comparison leads to a system of equations:

$$\frac{N_c}{64\pi^2 F_V} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = 0,$$

$$\frac{F_V \kappa_3^{VV} - F_A \kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = -\frac{N_c}{64\pi^2 F_V},$$

$$\frac{F_V \kappa_3^{VV} - F_A \kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) - F_A \kappa_5^{VA} \frac{M_A^2}{M_V^4} = 0,$$

$$\frac{F_V \kappa_3^{VV} - F_A \kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) - F_A \kappa_5^{VA} \frac{M_A^2}{M_V^4} \left(1 + \frac{M_A^2}{M_V^2}\right) = -\frac{2\pi \alpha_s \chi \langle \bar{q}q \rangle^2}{9F_V M_V^4}. $$
\[ \langle VVA \rangle \] Green function: \( \langle V^*VA \rangle \)

- Why? We know OPE for \( \langle V^*VA \rangle \) [P. Colangelo et al. '12]:

\[
\begin{align*}
w_T(Q^2) &= \frac{N_c}{Q^2} + \frac{128\pi^3\alpha_s\chi\langle \bar{q}q \rangle^2}{9Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right).
\end{align*}
\]

- One momentum soft, two momenta large.
- Comparison leads to a system of equations:

\[
\begin{align*}
\frac{N_c}{64\pi^2 F_V} + \sqrt{2}(\kappa^V_{11} + \kappa^V_{12}) &= 0, \\
\frac{F_V \kappa^V_3 - F_A \kappa^V_5}{M^2_V} + \sqrt{2}(\kappa^V_{11} + \kappa^V_{12}) - \frac{F_A \kappa^V_5}{M^4_A} M^2_A &= 0, \\
\frac{F_V \kappa^V_3 - F_A \kappa^V_5}{M^2_V} + \sqrt{2}(\kappa^V_{11} + \kappa^V_{12}) - F_A \kappa^V_5 \left(1 + \frac{M^2_A}{M^2_V}\right) &= -\frac{2\pi\alpha_s\chi\langle \bar{q}q \rangle^2}{9F_V M^4_V}.
\end{align*}
\]
Green function: Coupling constants constraints

- It is possible to extract the following coupling constants constraints:

\[
\kappa_{11}^{V} + \kappa_{12}^{V} = -\frac{N_c}{64\sqrt{2}\pi^2 F_V}, \quad \kappa_{3}^{VV} = -\frac{N_c M_V^4}{64\pi^2 M_A^2 F_V^2}, \quad \kappa_{5}^{VA} = \kappa_{3}^{VV} \frac{F_V}{F_A}.
\]

- Since it is not possible to solve the system of equations completely, the relevance of the constraints should be taken with caution!

- For example, determination of \( \kappa_{5}^{VA} \):
  - Numerically: \( \kappa_{5}^{VA} = -0.086 \).
  - From the decay \( f_1(1285) \rightarrow \rho\gamma \): \( \kappa_{5}^{VA} = -0.062 \pm 0.030 \).

- Using the constraints for \( VVP \) we can also determine:

\[
\kappa_{2}^{VV} = \frac{1}{64 F_V^2} \left( F^2 - \frac{N_c M_V^4}{8\pi^2 M_A^2} \right), \quad \kappa_{3}^{PV} = -\frac{F^2}{32\sqrt{2}d_m F_V} \left[ 1 + \frac{N_c M_V^2}{8\pi^2 F^2} \left( \frac{M_V^2}{M_A^2} - 1 \right) \right].
\]

- However: BABAR dictates \( \kappa_{3}^{PV} = -\frac{F^2}{32\sqrt{2}d_m F_V} (1 + \delta_{BL}) \) with the value \( \delta_{BL} = -0.055 \pm 0.025 \) from \( VVP \).

- However, our prediction from \( VVA \) gives \( \delta_{BL} = -1.342 \).
Green function: $\mathcal{F}_{\pi^0 \rightarrow \gamma \gamma}^R \chi_T$ formfactor revisited

Figure: A plot of BABAR (green), BELLE (red) and CLEO (blue) data fitted with the formfactor $\mathcal{F}_{\pi^0 \rightarrow \gamma \gamma}^R \chi_T (0, -Q^2; 0)$ using the modified Brodsky-Lepage condition. The full black line represents our fit with $\delta_{BL} = -1.342$, and the full brown line is a fit using the LMD formfactor. The dashed line stands for $\delta_{BL} = -0.055$ and the dot-dashed line for $\delta_{BL} = 0$. 
Matching $R\chi T$ to OPE with three large momenta
Leading order; for example $\langle VVA \rangle$:

\[ = i \operatorname{Tr}[T^a T^b T^c] \int \frac{d^4 \ell}{(2\pi)^4} \frac{\operatorname{Tr}[\gamma_\mu \ell \gamma_\nu (\ell - q) \gamma_\rho \gamma_5 (\ell + p)]}{\ell^2 (\ell - q)^2 (\ell + p)^2}. \]

Anomaly:

- If vector Ward identities are imposed, the axial Ward identity picks up an extra term.
- EW for mass fermions: $r^\rho T^\mu \nu\rho^\rho (p, q, r; m) = 2mT^\mu \nu (p, q, r; m) + \frac{1}{2\pi^2} \varepsilon^\mu \nu (p)(q)$.

Non-renormalization of the $\langle VVA \rangle$ correlator:

- $\langle VVA \rangle$ is not modified by QCD radiative corrections at two loops.
- Does it persist at higher orders?
- Three-loop corrections do not vanish and they are proportional to the QCD $\beta$-function [J. Mondejar and K. Melnikov '12].
The Fock-Schwinger gauge, \((x - x_0)^\mu A_\mu^a(x) = 0\), allows us to obtain expansion of the nonlocal QCD condensates in terms of local ones:

\[
\langle \bar{q}^A_i, \alpha(x) q^B_k, \beta(y) \rangle \sim \left[ \langle \bar{q}q \rangle \delta_{ik} + \frac{g_s}{24} \langle \bar{q}Gq \rangle (x - y)^2 \delta_{ik} + \langle \bar{q}q \rangle^2 \left[ F^{\langle \bar{q}q \rangle}(x, y) \right]_{ki} \right] \delta_{\alpha\beta} \delta^{AB},
\]

\[
g_s \langle \bar{q}^A_i, \alpha(x) A_\mu^a(y) q^B_k, \beta(z) \rangle \sim \left[ \frac{g_s}{24} \langle \bar{q}Gq \rangle y^\nu (\sigma_{\nu\mu})_{ki} + \frac{\pi \alpha_s}{32} \langle \bar{q}q \rangle^2 \left[ F^{\langle \bar{q}Aq \rangle}_\mu(x, y, z) \right]_{ki} \right] (T^a)_{\beta\alpha} \delta^{AB},
\]

with:

\[
[F^{\langle \bar{q}q \rangle}(x, y)] = \frac{i \pi \alpha_s}{2 \cdot 3^4} \left[ 4(x \cdot y)(\not{x} - \not{y}) - (x^2 - y^2)(\not{x} + \not{y}) \right],
\]

\[
[F^{\langle \bar{q}Aq \rangle}_\mu(x, y, z)] = i(g_{\kappa\lambda} \gamma_\mu - g_{\kappa\mu} \gamma_\lambda) \left( \frac{1}{2} (x + z)^\kappa y^\lambda - \frac{2}{3} y^\kappa y^\lambda \right) - \frac{1}{2} \varepsilon(x-z)(y)_{\mu\nu} \gamma^\nu \gamma_5.
\]

Therefore, nonlocal quark and quark-gluon condensates propagate as local quark, quark-gluon and four-quark condensates.

"This effect has been one of the main source of errors in the existing QSSR (QCD spectral sum rules) literature." [S. Narison '07]

Our results are in the most general form.

So far in the literature, one usually takes one of the coordinates as zero, so the formulas were not applicable for three-point Green functions.
OPE: Quark condensate $\langle \bar{q} q \rangle$

- **LO:** two contractions between three currents/densities:

  ![LO diagrams]

  *First nontrivial QCD condensate:*
  - Implicates the chiral symmetry breaking.
  - Contributes to the order parameters of the $\chi_{SB}$.

  *The LO was studied a long time ago.*
  - For example: $\langle VVP \rangle$ [B. Moussallam '94]

\[
[\Pi_{VVVP}^{\langle \bar{q} q \rangle} (p, q; r)]^{abc}_{\mu\nu} = \frac{\langle \bar{q} q \rangle}{6p^2q^2r^2 (p^2 + q^2 + r^2)} d^{abc} \varepsilon_{\mu\nu}(p)(q).
\]
OPE: Quark condensate $\langle \bar{q}q \rangle$

- NLO: gluonic corrections at $\mathcal{O}(\alpha_s)$ [M. Jamin and V. Mateu '08].
  - An opportunity to explore the renormalisation dependence of such condensate in full QCD.

General structure of the contributions:

$$L_p \log \left( -\frac{p^2}{\mu^2} \right) + L_q \log \left( -\frac{q^2}{\mu^2} \right) + L_r \log \left( -\frac{r^2}{\mu^2} \right) + L_d C_0 + L_c.$$
OPE: Gluon condensate $\langle G_{\mu\nu}G^{\mu\nu} \rangle$

- Massless quark propagator in external gluon field (Fock-Schwinger gauge):
  
  $$S(x, y) = S_0(x, y) + S_1^{\alpha\beta}(x, y)G_{\alpha\beta}(0) + S_2(x, y)G_{\alpha\beta}(0)G^{\alpha\beta}(0).$$

- Fourier transform needed to convert the result into $p$-representation.
- Results, for example for $\langle VVA \rangle$ (and $\langle AAA \rangle$), are surprisingly simple [TK, K. Kampf and J. Novotný '18]:

  $$w_T^{(1)} = -\frac{\alpha_s \langle G^2 \rangle}{96\pi} \frac{p^2(r^2 - 4q^2) + p^4 + q^2(q^2 + r^2)}{p^4q^4r^2},$$

  $$w_T^{(2)} = -\frac{\alpha_s \langle G^2 \rangle}{96\pi} \frac{(p^2 - q^2)(p^2 + q^2 + r^2)}{p^4q^4r^2} = -w_T^{(3)}.$$

- Cancelation of logarithmic terms!
OPE: Quark-gluon and four-quark condensates

- **Quark-gluon condensate:** \( \langle \bar{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle \).
  - Given by a propagation of nonlocal quark-gluon (left) and quark (right) condensates.

- **Four-quark condensate:** \( \langle 0 | \bar{q} \Gamma_1 q \bar{q} \Gamma_2 q | 0 \rangle \sim \langle \bar{q}q \rangle^2 \).
  - \( \Gamma_1, \Gamma_2 \): combination of \( \{ 1, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu} \} \) with \( \{ 1, T^a \} \), that preserves the Lorentz invariance.
  - Given by a perturbative contribution (left) and propagation of nonlocal quark (middle) and quark-gluon (right) condensates.
Conclusion

- We calculated OPE of all 3-point Green functions of chiral currents for all momenta large.
- At high energies, the Green functions are given in terms of QCD condensates \((D \leq 6)\):
  - \(\langle \bar{q}q \rangle\),
  - \(\langle G_{\mu\nu}G^{\mu\nu} \rangle\),
  - \(\langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \rangle\),
  - \(\langle \bar{q}\Gamma_1 q\bar{q}\Gamma_2 q \rangle\).
- Our interest: odd sector of QCD

\[
\mathcal{L}^{(6)}_{R\chi T} = \sum_X \sum_i \kappa_i^X \hat{O}_i \chi X \alpha_\beta \varepsilon^{\mu\nu\alpha\beta}.
\]
- Our goal: match OPE with \(R\chi T\).
  - Unclear how to deal with logarithmic terms!
  - Infinite tower of resonances!
Thank you for your attention!