Contribution of QCD Condensates to the OPE of Green Functions of Chiral Currents

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Introduction

- What do we do?
    - We study QCD at low energies.
- What do we need?
    - Chiral perturbation theory (χPT) and Resonance chiral theory (RχT).
- What is it?
    - Effective description of low-energy QCD.
    - χPT for $E \leq M_\rho$.
        - Spontaneous breaking of the chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry down to $\mathrm{SU}(3)_V$ in QCD leads to the presence of Goldstone bosons.
        - We identify them with the octet of pseudoscalar mesons (π, K, η) as the lightest hadronic observable states.
    - RχT for $M_\rho \leq E \leq 2\,\mathrm{GeV}$.
        - RχT increases the number of degrees of freedom of χPT by including massive $\mathrm{U}(3)$ multiplets of vector $V(1^{--})$, axial-vector $A(1^{++})$, scalar $S(0^{++})$ and pseudoscalar $P(0^{-+})$ resonances.
- What is it good for?
    - To study important theoretical and phenomenological aspects of QCD.
- What do we use?
    - Green functions of chiral currents.

Green functions of chiral currents

- The amplitudes of physical processes can be computed using LSZ reduction formula from the Green functions, the time ordered products of quantum fields.
- Three-point Green function:

$$\int \mathrm{d}^4x \, \mathrm{d}^4y \, e^{-i(p\cdot x+q\cdot y)} \left\langle 0 \middle| \mathrm{T}\left[\mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(0)\right] \middle| 0 \right\rangle .$$

- $\mathcal{O}_i(x_i)$ stand either for
  - vector and axial-vector currents:

$$V_\mu^a(x) = \overline{q}(x)\gamma_\mu T^a q(x) \, , \qquad A_\mu^a(x) = \overline{q}(x)\gamma_\mu\gamma_5 T^a q(x) \, ,$$

  - or scalar and pseudoscalar densities:

$$S^a(x) = \overline{q}(x) T^a q(x) \, , \qquad P^a(x) = i\overline{q}(x)\gamma_5 T^a q(x) \, .$$

- Nontrivial three-point Green functions in QCD:
  - Set I: $\langle SSS \rangle$, $\langle SPP \rangle$, $\langle VVP \rangle$, $\langle AAP \rangle$, $\langle VAS \rangle$, $\langle VVS \rangle$, $\langle AAS \rangle$, $\langle VAP \rangle$.
  - Set II: $\langle VVA \rangle$, $\langle AAA \rangle$, $\langle VVV \rangle$, $\langle ASP \rangle$, $\langle AAV \rangle$, $\langle VSS \rangle$, $\langle VPP \rangle$.

Green functions of chiral currents: Examples

- $\langle VVP \rangle$ Green function:

$$\left[\Pi_{VVP}(p,q;r)\right]^{abc}_{\mu\nu} = \Pi_{VVP}(p^2,q^2,r^2)d^{abc}\varepsilon_{\mu\nu(p)(q)}\,.$$

  - Direct connection to many phenomenologically important quantities:
    - Transition formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}(p^2,q^2,r^2) = \frac{2r^2}{3B_0F}\Pi^{\mathrm{R}\chi\mathrm{T}}_{VVP}(p^2,q^2,r^2)\,.$
    - Decays $\rho\to\pi\gamma$, $\pi(1300)\to\gamma\gamma$, $\pi(1300)\to\rho\gamma\,.$
    - Significant importance in study of rare π^0 decays: $\pi^0\to e^+e^-\gamma$, $\pi^0\to e^+e^-\,.$
    - Hadronic contribution to light-by-light scattering and $g-2$.

- $\langle VVA \rangle$ Green function:

$$\left[\Pi_{VVA}(p,q;r)\right]^{abc}_{\mu\nu\rho} = d^{abc}\left[w_L\varepsilon_{\mu\nu(p)(q)}r_\rho + w_T^{(1)}\Pi^{(1)}_{\mu\nu\rho} + w_T^{(2)}\Pi^{(2)}_{\mu\nu\rho} + w_T^{(3)}\Pi^{(3)}_{\mu\nu\rho}\right].$$

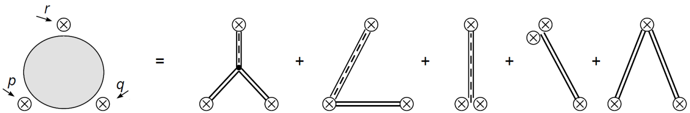
  - The tensor part is nontrivial [M. Knecht *et al.* '04].
  - The structure is thus given by longitudinal formfactor w_L (fixed by the anomaly) and three formfactors $w_T^{(1)}$, $w_T^{(2)}$, $w_T^{(3)}\,.$
  - Related to the decay of axial resonance $f_1(1285)\to\rho\gamma\,.$

RχT at NLO in the odd-intrinsic parity sector of QCD

- Special interest of ours: odd-intrinsic parity sector of QCD.
  - $\langle VVP\rangle$, $\langle AAP\rangle$, $\langle VAS\rangle$, $\langle VVA\rangle$, $\langle AAA\rangle$.
- We assume the saturation of dynamics with the lightest resonances.
- At the NLO, relevant Lagrangian in the odd-intrinsic parity sector was formulated for the first time in [K. Kampf and J. Novotný '11]:

$$\mathcal{L}^{(6)}_{\mathrm{R}\chi\mathrm{T}} = \sum_X \sum_i \kappa_i^X \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta}\varepsilon^{\mu\nu\alpha\beta}\,.$$

  - X stands for the resonance fields V, A, S, P, double-resonance fields VV, AA, SA, SV, VA, PA, PV and triple-resonance fields VVP, VAS, AAP.
  - $\mathcal{L}^{(6)}_{\mathrm{R}\chi\mathrm{T}}$: 67 operators and 67 corresponding unknown couplings κ_i^X in total.
- Topology of the Feynman diagrams (the crossing is implicitly assumed):



- Goal: express κ_i^X in terms of known parameters (F, F_V, F_A, M_V, M_A etc.).
- Our approach: high-energy behaviour of Green functions within OPE.

Operator Product Expansion

Operator Product Expansion

- Framework to study behaviour at high energies.
- For $x, y \to 0$ (i.e. for large external momenta) Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$\begin{aligned}\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(0)\rangle &= C_{\mathbb{1}}\mathbb{1} + C_{\langle\overline{q}q\rangle}\langle\overline{q}q\rangle + C_{\langle G^2\rangle}\langle G_{\mu\nu}G^{\mu\nu}\rangle \\ &+ C_{\langle\overline{q}Gq\rangle}\langle\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle + C_{\langle 4q\rangle}\langle\overline{q}\,\Gamma_1\,q\overline{q}\,\Gamma_2\,q\rangle \\ &+ C_{\langle G^3\rangle}\langle G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu}\rangle f^{abc} + \ldots\end{aligned}$$

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- QCD condensates (with dimension $D \leq 6$):
  - Quark condensate.

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  - Quark condensate.
  - Gluon condensate.
  - Quark-gluon condensate.
  - Four-quark condensate.
  - Three-gluon condensate (vanishes in the chiral limit).

Matching RχT to OPE with two large momenta

Example: $\langle VVA \rangle$ Green function

- Calculation in RχT at NLO leads to the formfactors [TK, K. Kampf and J. Novotný '18]:

$$
\begin{aligned}
w_L(p^2,q^2,r^2) &= \frac{N_c}{8\pi^2 r^2}\,,\\
w_T^{(1)}(p^2,q^2,r^2) &= -\frac{2\sqrt{2}F_V\left[\kappa_{17}^V(p^2+q^2-2M_V^2)-\sqrt{2}F_V\kappa_3^{VV}\right]}{(p^2-M_V^2)(q^2-M_V^2)}\,,\\
w_T^{(2)}(p^2,q^2,r^2) &= -\frac{2\sqrt{2}F_V(p^2-q^2)(2\kappa_{12}^V+\kappa_{16}^V-\kappa_{17}^V)}{(p^2-M_V^2)(q^2-M_V^2)}\,,\\
w_T^{(3)}(p^2,q^2,r^2) &= \frac{2\sqrt{2}F_V(p^2-q^2)}{(p^2-M_V^2)(q^2-M_V^2)}\left(2\kappa_{11}^V+2\kappa_{12}^V-\kappa_{17}^V-\frac{\sqrt{2}F_A\kappa_5^{VA}}{r^2-M_A^2}\right).
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- Phenomenologically important formfactor $w_T(Q^2)$:

$$
w_T(Q^2) = -16\pi^2\left[w_T^{(1)}(-Q^2,0,-Q^2)+w_T^{(3)}(-Q^2,0,-Q^2)\right].
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- Idea: expand $w_T(Q^2)$ in terms of Q^2 up to $\mathcal{O}\left(\frac{1}{Q^8}\right)$.

$\langle VVA \rangle$ Green function: $\langle V^*VA \rangle$

- Why? We know OPE for $\langle V^*VA \rangle$ [P. Colangelo *et al.* '12]:

$$w_T(Q^2) = \frac{N_c}{Q^2} + \frac{128\pi^3\alpha_s\chi\langle\overline{q}q\rangle^2}{9Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right).$$

  - One momentum soft, two momenta large.
- Comparison leads to a system of equations:

$$\frac{N_c}{64\pi^2 F_V} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = 0\,,$$

$$\frac{F_V\kappa_3^{VV} - F_A\kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = -\frac{N_c}{64\pi^2 F_V}\,,$$

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$\langle VVA \rangle$ Green function: Coupling constants constraints

- It is possible to extract the following coupling constants constraints:

$$\kappa_{11}^{V} + \kappa_{12}^{V} = -\frac{N_c}{64\sqrt{2}\pi^2 F_V}\,, \quad \kappa_3^{VV} = -\frac{N_c M_V^4}{64\pi^2 M_A^2 F_V^2}\,, \quad \kappa_5^{VA} = \kappa_3^{VV}\frac{F_V}{F_A}\,.$$

  - Since it is not possible to solve the system of equations completely, the relevance of the constraints should be taken with caution!
- For example, determination of κ_5^{VA}:
  - Numerically: $\kappa_5^{VA} = -0.086$.
  - From the decay $f_1(1285) \to \rho\gamma$: $\kappa_5^{VA} = -0.062 \pm 0.030$.
- Using the constraints for VVP we can also determine:

$$\kappa_2^{VV} = \frac{1}{64 F_V^2}\left(F^2 - \frac{N_c M_V^4}{8\pi^2 M_A^2}\right), \quad \kappa_3^{PV} = -\frac{F^2}{32\sqrt{2} d_m F_V}\left[1 + \frac{N_c M_V^2}{8\pi^2 F^2}\left(\frac{M_V^2}{M_A^2} - 1\right)\right].$$

  - However: BABAR dictates $\kappa_3^{PV} = -\frac{F^2}{32\sqrt{2} d_m F_V}(1 + \delta_{\rm BL})$ with the value $\delta_{\rm BL} = -0,055 \pm 0.025$ from VVP.
- However, our prediction from VVA gives $\delta_{\rm BL} = -1.342\,$.

$\langle VVA \rangle$ Green function: $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}$ formfactor revisited

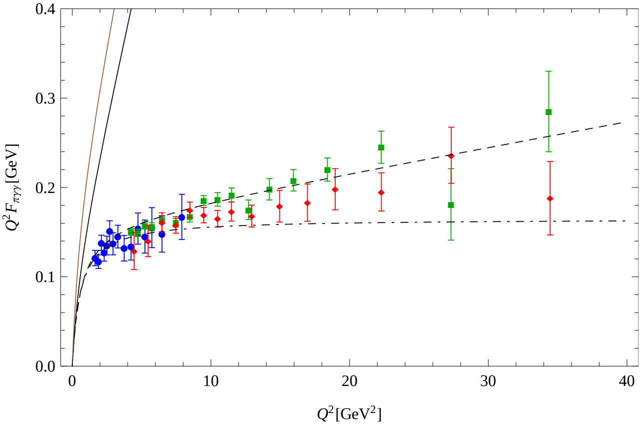
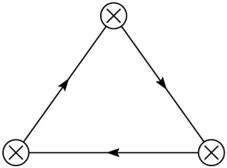


Figure: A plot of BABAR (green), BELLE (red) and CLEO (blue) data fitted with the formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\gamma\gamma}(0,-Q^2;0)$ using the modified Brodsky-Lepage condition. The full black line represents our fit with $\delta_{\mathrm{BL}} = -1.342$, and the full brown line is a fit using the LMD formfactor. The dashed line stands for $\delta_{\mathrm{BL}} = -0.055$ and the dot-dashed line for $\delta_{\mathrm{BL}} = 0$.

Matching RχT to OPE with three large momenta

OPE: Perturbative contribution

- Leading order; for example $\langle VVA \rangle$:



$$= i\,\mathrm{Tr}[T^a T^b T^c] \int \frac{\mathrm{d}^4\ell}{(2\pi)^4} \frac{\mathrm{Tr}[\gamma_\mu \not{\ell} \gamma_\nu (\not{\ell} - \not{q}) \gamma_\rho \gamma_5 (\not{\ell} + \not{p})]}{\ell^2 (\ell - q)^2 (\ell + p)^2} \,.$$

- Anomaly:
  - If vector Ward identities are imposed, the axial Ward identity picks up an extra term.
  - EW for mass fermions: $r^\rho T^{\text{ren.}}_{\mu\nu\rho}(p,q,r;m) = 2m T_{\mu\nu}(p,q,r;m) + \frac{1}{2\pi^2} \varepsilon_{\mu\nu(p)(q)}$.
- Non-renormalization of the $\langle VVA \rangle$ correlator:
  - $\langle VVA \rangle$ is not modified by QCD radiative corrections at two loops.
  - Does it persist at higher orders?
  - Three-loop corrections do not vanish and they are proportional to the QCD β-function [J. Mondejar and K. Melnikov '12].

OPE: Propagation of QCD condensates

- The Fock-Schwinger gauge, $(x-x_0)^\mu \mathcal{A}^a_\mu(x)=0$, allows us to obtain expansion of the nonlocal QCD condensates in terms of local ones:

$$\langle \overline{q}^A_{i,\alpha}(x) q^B_{k,\beta}(y)\rangle \sim \left[\langle\overline{q}q\rangle\delta_{ik} + \frac{g_s}{2^4}\langle\overline{q}Gq\rangle(x-y)^2\delta_{ik} + \langle\overline{q}q\rangle^2\left[F^{\langle\overline{q}q\rangle}(x,y)\right]_{ki}\right]\delta_{\alpha\beta}\delta^{AB}\,,$$

$$g_s\langle \overline{q}^A_{i,\alpha}(x)\mathcal{A}^a_\mu(y) q^B_{k,\beta}(z)\rangle \sim \left[\frac{g_s}{2^4}\langle\overline{q}Gq\rangle\, y^\nu(\sigma_{\nu\mu})_{ki} + \frac{\pi\alpha_s}{3^2}\langle\overline{q}q\rangle^2\left[F_\mu^{\langle\overline{q}\mathcal{A}q\rangle}(x,y,z)\right]_{ki}\right](T^a)_{\beta\alpha}\delta^{AB}\,,$$

with:

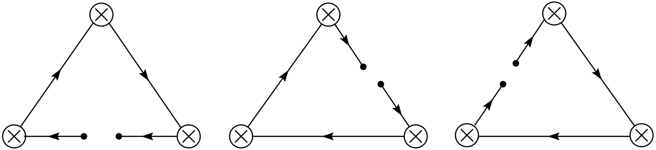
$$\left[F^{\langle\overline{q}q\rangle}(x,y)\right] = \frac{i\pi\alpha_s}{2\cdot 3^4}\left[4(x\cdot y)(\not{x}-\not{y}) - (x^2-y^2)(\not{x}+\not{y})\right],$$

$$\left[F_\mu^{\langle\overline{q}\mathcal{A}q\rangle}(x,y,z)\right] = i(g_{\kappa\lambda}\gamma_\mu - g_{\kappa\mu}\gamma_\lambda)\left(\frac{1}{2}(x+z)^\kappa y^\lambda - \frac{2}{3}y^\kappa y^\lambda\right) - \frac{1}{2}\varepsilon_{(x-z)(y)\mu\nu}\gamma^\nu\gamma_5\,.$$

- Therefore, nonlocal quark and quark-gluon condensates propagate as local quark, quark-gluon and four-quark condensates.
  - "*This effect has been one of the main source of errors in the existing QSSR (QCD spectral sum rules) literature*." [S. Narison '07]
- Our results are in the most general form.
  - So far in the literature, one usually takes one of the coordinates as zero, so the formulas were not applicable for three-point Green functions.

OPE: Quark condensate $\langle \overline{q} q \rangle$

- LO: two contractions between three currents/densities:

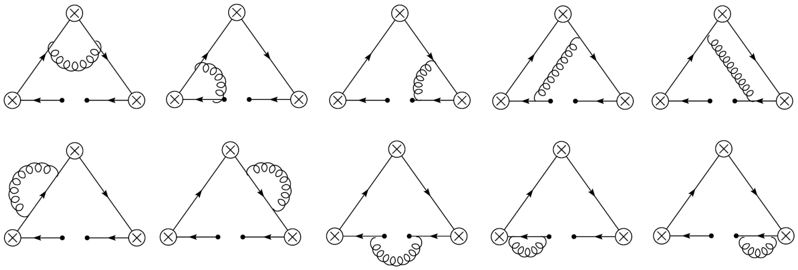


- First nontrivial QCD condensate:
  - Implicates the chiral symmetry breaking.
  - Contributes to the order parameters of the χSB.
- The LO was studied a long time ago.
  - For example: $\langle VVP \rangle$ [B. Moussallam '94]

$$\left[\Pi^{\langle \overline{q} q \rangle}_{VVP}(p,q;r)\right]^{abc}_{\mu\nu} = \frac{\langle \overline{q} q \rangle}{6p^2 q^2 r^2}(p^2+q^2+r^2) d^{abc} \varepsilon_{\mu\nu(p)(q)} \,.$$

OPE: Quark condensate $\langle \bar{q} q \rangle$

- NLO: gluonic corrections at $\mathcal{O}(\alpha_s)$ [M. Jamin and V. Mateu '08].
  - An opportunity to explore the renormalisation dependence of such condensate in full QCD.



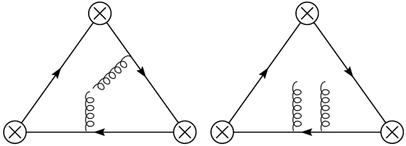
- General structure of the contributions:

$$L_p \log\left(-\frac{p^2}{\mu^2}\right) + L_q \log\left(-\frac{q^2}{\mu^2}\right) + L_r \log\left(-\frac{r^2}{\mu^2}\right) + L_d C_0 + L_c \,.$$

OPE: Gluon condensate $\langle G_{\mu\nu}G^{\mu\nu}\rangle$

- Massless quark propagator in external gluon field (Fock-Schwinger gauge):

$$S(x,y) = S_0(x,y) + S_1^{\alpha\beta}(x,y)G_{\alpha\beta}(0) + S_2(x,y)G_{\alpha\beta}(0)G^{\alpha\beta}(0)\,.$$



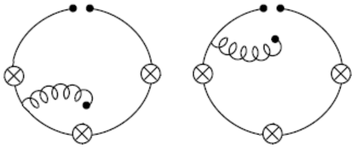
- Fourier transform needed to convert the result into p-representation.
- Results, for example for $\langle VVA\rangle$ (and $\langle AAA\rangle$), are suprisingly simple [TK, K. Kampf and J. Novotný '18]:

$$w_T^{(1)} = -\frac{\alpha_s\langle G^2\rangle}{96\pi}\frac{p^2(r^2-4q^2)+p^4+q^2(q^2+r^2)}{p^4q^4r^2}\,,$$
$$w_T^{(2)} = -\frac{\alpha_s\langle G^2\rangle}{96\pi}\frac{(p^2-q^2)(p^2+q^2+r^2)}{p^4q^4r^2} = -w_T^{(3)}\,.$$

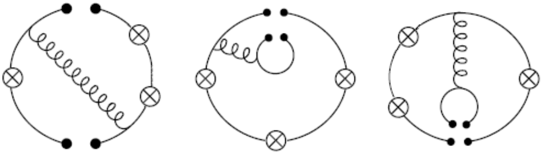
- Cancelation of logarithmic terms!

OPE: Quark-gluon and four-quark condensates

- Quark-gluon condensate: $\langle \overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle$.
  - Given by a propagation of nonlocal quark-gluon (left) and quark (right) condensates.



- Four-quark condensate: $\langle 0|\overline{q}\,\Gamma_1\,q\overline{q}\,\Gamma_2\,q|0\rangle \sim \langle\overline{q}q\rangle^2$.
  - Γ_1, Γ_2: combination of $\{\mathbb{1},\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5,\sigma_{\mu\nu}\}$ with $\{\mathbb{1},T^a\}$, that preserves the Lorentz invariance.
  - Given by a perturbative contribution (left) and propagation of nonlocal quark (middle) and quark-gluon (right) condensates.

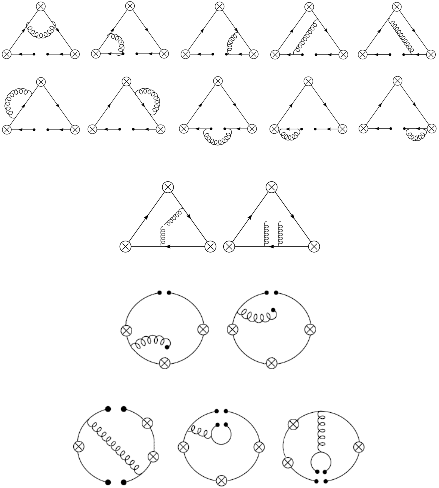


Conclusion

- We calculated OPE of all 3-point Green functions of chiral currents for all momenta large.
- At high energies, the Green functions are given in terms of QCD condensates ($D \leq 6$):
  - $\langle \overline{q} q \rangle$,
  - $\langle G_{\mu\nu} G^{\mu\nu} \rangle$,
  - $\langle \overline{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle$,
  - $\langle \overline{q}\, \Gamma_1\, q \overline{q}\, \Gamma_2\, q \rangle$.
- Our interest: odd sector of QCD

$$\mathcal{L}^{(6)}_{\mathrm{R}\chi\mathrm{T}} = \sum_X \sum_i \kappa_i^X \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta} \varepsilon^{\mu\nu\alpha\beta} \,.$$

- Our goal: match OPE with RχT.
  - Unclear how to deal with logarithmic terms!
  - Infinite tower of resonances!



Thank you for your attention!